THE IMPACT OF TECHNOLOGICAL CHANGE

ON THE DISTRIBUTION OF LABOR INCOME

by

Dagobert L. Brito and Michael D. Intriligator

THE IMPACT OF TECHNOLOGICAL CHANGE

ON THE DISTRIBUTION OF LABOR INCOME

Dagobert L. Brito*
Rice University

and

Michael D. Intriligator
University of California, Los Angeles

Abstract

To address the question of the impact of technological change on the distribution of labor income, we develop a model of an economy in which two types of workers, "skilled" workers and "unskilled" workers furnish different types of labor. The social welfare function, to be maximized by government redistribution, is given as the sum of the utilities of the two types of workers, where the government cannot distinguish between workers of different types. The initial technology for this economy is given by an aggregate production function where the two types of workers enter in the production process in a symmetric way. Assuming perfect competition, all individuals will consume identical bundles, yielding a first-best solution with no need for redistribution, all levels of consumption and leisure being equal. We then introduce into this "Garden of Eden" a "serpent" in the form of a new technology for which skilled workers can produce the good using this technology without using any of the unskilled workers. In this new economy the old technology uses both types of workers, while the new technology uses only skilled workers. Using this framework we prove that an increase in the marginal product of labor of the skilled workers in the new technology will change the market equilibrium in such a way that all skilled workers will benefit at the expense of the unskilled workers, social welfare may be reduced, and the level of output at the competitive equilibrium will increase as a result of the introduction of the new technology. If government redistribution is introduced then the government can achieve a first-best allocation. The optimal tax is a lump-sum tax on the skilled workers and a subsidy for the unskilled workers with a zero marginal tax on wages of both types of workers. Furthermore, there exists a level of productivity of the skilled workers in the new technology such that, for a welfare function defined on consumption and leisure of both types of workers it is optimal to eliminate the industry employing the unskilled workers, who will not be employed.

1. Introduction

This paper represents an attempt to address the question of the impact of technological change on the distribution of labor income and employment. It appears to be true, at least to the point at which it can be considered a stylized fact, that the distribution of income in certain advanced industrialized economies, such as those of the United States and the United Kingdom, is becoming less equal. Technology is changing in a fashion that will lead to a distribution of income that may be politically unstable. It appears to be true that the distribution of income in the United States is becoming less equal. Some have argued that part of this increase in inequality has been due to changes in the structure of taxes, while others have pointed to differences in educational opportunities available to various groups in society. No doubt these factors explain part of the observed change. However, there are some other changes not related to those factors: manufacturing jobs are being eliminated in favor of service sector jobs, and there appears to be movement of people in business and in engineering schools away from production and manufacturing to finance and to more exotic types of engineering. There is the growing gap in the United States between the earnings of college graduates and high school graduates. An increasing number of jobs once filled by high school graduates, for example, now require some college training or even a college degree.[1]

The educated portion of the labor force is becoming a far more important component of the economy. In a recent study of the American and Swedish economies, Eliasson and his collaborators the Industrial Institute of Economic and Social Research have concluded that *information* is becoming the dominant component in the advanced economies.

> There is a goods producing engine somewhere underneath this deep structure. But it is no longer only "manufacturing" that moves and supports the rest of the of the economy. A large chunk of typical information activities has to be added . . . Together information design activities accounted for at least 8 percent of total employment in an advanced industrialized economy and contributed about as much to GNP in 1985, as we measure it . . . . With the exception of parts of

manufacturing, these activities employ the most well educated members of the labor force.[2]

They go on to argue that it is not an "unreasonable proposition that . . . [in the U.S.] more than half of the economic value, perhaps even 75 percent of production value [is] being generated by one quarter of the people in the labor market."[3] This is not a statement about unequal distribution of wealth or income in the U.S. but rather a statement about the economic value of what is actually produced and by whom; it is a proposition that is neutral with respect to the appropriate distribution of income or wealth.

Similar observations about the fundamental change in the nature of work can be found in the 1991 papal encyclical *Centesimus Annus*. Pope John Paul II notes that "[w]hereas at one time the decisive factor of production was the land and later capital - understood as a total complex of the instrument of production - today the decisive factor is increasingly man himself, that is, his knowledge, especially his scientific knowledge, his capacity of interrelated and compact organization as well as his ability to perceive the needs of others and to satisfy them."[4] He goes on to note that in many third world countries a majority do not have the capability to acquire the necessary human capital and are marginalized.[5] This paper does not directly address the issue of development in the third world. However, there are two observations which can be made. First, inasmuch as the sectors in the developed world that employ unskilled labor are declining in importance, the ability of third world countries to compete in the world economy is reduced. Second, human capital is very mobile. Thus, it is not surprising to observe the migration of third world individuals with high human capital to markets where their skills are rewarded, the "brain drain."[6]

To analyze these stylized facts and to study the impact of technological change on the distribution of labor income and employment, we develop a very simple model of an economy with two types of workers. The two types of workers can be interpreted as "skilled" workers and "unskilled" workers. With the initial technology the workers enter in the production process in a

symmetric way, so, assuming perfect competition, all individuals will consume identical bundles, yielding a first-best solution with no need for redistribution. We then introduce a new technology, such as robots, for which the skilled workers can produce the good using this new technology without using any of the unskilled workers. Unskilled workers cannot be employed in this new technology, substitution of the two types of workers in the new technology is not possible, and unskilled workers cannot be trained to become skilled workers. These assumption differs from the assumptions typically made in the optimal tax literature that follows the Mirrlees tradition.[7] In that literature, the assumption is usually made that all types of labor are perfect substitutes if denominated in effectiveness units. Individuals differ in the amount of effective labor that they can supply per hour worked. Thus, even though individuals have identical preference on goods and leisure, these preferences differ when defined over goods and effective labor The assumption that unskilled workers cannot be trained to become skilled workers is clearly controversial. We will get into the debate of heredity versus the environment in the development of human capital. If the problems pointed out by this paper can, in fact, be addressed by investment in human capital, so much the better.

After the introduction of the new technology, there are two sectors using different technologies to produce the same good. The old technology uses both types of workers, while the new technology uses only skilled workers. Using this framework we analyze the impacts of the introduction of the new technology on the distribution of income and the role of government in redistributing income. Among our results, we show that if the wage of skilled labor in the new technology sector is sufficiently large, it is socially optimal to eliminate the old technology sector altogether, leading to the unemployment of all unskilled workers. A historical example of such a change is agriculture. In 1920, 27 percent of the United States labor force was employed directly in agriculture. The mechanization of agriculture led to the elimination of most unskilled workers from farming, and now only 2 percent of the labor force is employed in agriculture. There was a migration of former farmer workers to the cities. In many cases this has resulted in high societal cost in terms of unemployment and instability of families. An even earlier example is Rome in the

second century B. C. Free men were not able to complete with slave labor and moved to Rome, creating a class, "the *plebs urbana* [which] was allowed to continue its aimless existence as the pampered drone of the Rome world."[8]

There are four questions we want to address: First, what is the impact of the new technology on the distribution of income if there is no government intervention? Second, what is the impact of the new technology on output and aggregate welfare if there is no government intervention? Third, what is the impact of government intervention if government is restricted to an optimal income tax? Fourth, is it possible that the unskilled workers might become economically unemployable?

2. The Model

To address the impact of technological change on the distribution of labor income we will use a very simple model that captures the stylized facts that we want to explore. We will assume that there are two types of workers, "skilled" workers and "unskilled" workers, furnishing different types of labor and deriving utility from consumption and leisure. Skilled workers can furnish skilled or unskilled labor, but unskilled workers can furnish only unskilled labor. There are two sectors that produce a single good, and one sector is specialized in that it only employs skilled workers. Thus skilled workers are free to move between the two sectors while unskilled workers are not mobile.

The two types of workers, α and β, are indexed by i, for which i = α refers to skilled workers while i = β refers to unskilled workers.[9] The two types of workers have identical utility functions defined on a single good and leisure, however, they furnish different types of labor.[10] Letting C_i be the consumption and L_i be the labor supplied by a type i worker, the utility function of α-worker of type i is given by a concave function $u(C_i, L_i)$, is the amount of leisure available to a type i worker, i = α, β, and where leisure is treated as a normal good.

It will be assumed that there are N workers of each type and, without loss of generality, that $N = 1$. The social welfare function, to be maximized by government redistribution, is given as the total utility obtained by the two types of workers

(1) $$W = u(C_\alpha, L_\alpha) + u(C_\beta, L_\beta)$$

where the government cannot distinguish between skilled workers and unskilled workers. Thus there is nothing to prevent a skilled worker from pretending to be an unskilled worker for the purpose of income redistribution.

The initial technology for this economy is given by the symmetric aggregate linearly homogeneous production function

(2) $$Y = F(L_\alpha, L_\beta)$$

where L_i is the total labor supply of type i workers and total output is consumed by the two types of workers. Without loss of generality we will assume that if $L_\alpha = L_\beta$, then the marginal products must be equal in equilibrium with the output divided between the two types of workers.

(3) $$F_1(L_\alpha, L_\beta) = F_2(L_\alpha, L_\beta) = 1$$

(4) $$Y = C_\alpha + C_\beta,$$

where $F_1(L_\alpha, L_\beta)$ and $F_2(L_\alpha, L_\beta)$ are the marginal products of skilled and unskilled labor respectively. Assuming perfect competition, then, via symmetry, the solution is trivial. This is a first-best solution with no need for redistribution, since all levels of consumption and leisure are equal.

Now consider the introduction into this "Garden of Eden" of a "serpent" in the form of a new technology. The new technology allows the skilled workers to produce the good without using any of the unskilled workers. The new technology is linear. The new technology can be

considered as the introduction of a new process, such as robots, that replaces unskilled workers in the production process. This new economy is described by the following system of equations:

$$(5) \qquad Y_1 = F(L_{\alpha 1}, L_\beta)$$

$$(6) \qquad Y_2 = M\,L_{\alpha 2}$$

$$(7) \qquad Y_1 + Y_2 = C_\alpha + C_\beta$$

$$(8) \qquad L_\alpha = L_{\alpha 1} + L_{\alpha 2}\,.$$

Here the two sectors are using different technologies to produce the same good: Y_j is the output of sector j for j = 1,2; $L_{\alpha 1}$ in (5) is the total labor supply of skilled workers in sector 1, which uses the same joint linearly homogeneous technology as before; L_β in (5) is the total labor supply of unskilled workers, which are employed only in sector 1; and $L_{\alpha 2}$ in (6) is the total labor supply of skilled workers in sector 2, which uses a linear technology, which does not use the unskilled labor, and which has the constant marginal product of labor (of skilled labor) M. In (7) the total product is allocated as consumption between the two types of labor, while in (8) the total supply of labor of skilled workers, L_α, is the sum of the supply of skilled workers allocated to the two sectors.

If M, the marginal product of labor of skilled workers in sector 2 is large enough then some fraction of the skilled workers will shift to the new technology. Since skilled workers and unskilled workers are complements, this shift will *lower* the marginal productivity of unskilled workers.

3. Market Allocation

We first address the free market case and prove three propositions based on the model of two types of workers in an initial technology which, following the technological change, includes both the initial technology that uses both types of labor and a new linear technology that uses only skilled workers:

Proposition 1: An increase in the marginal product of labor of the skilled workers in the new technology will change the market equilibrium in such a way that all skilled workers will benefit at the expense of the unskilled workers .

Proof:

Let w_α be the wage of the skilled workers and w_β be the wage of the unskilled workers. The marginal product of the skilled workers must equal the wage in both sectors, so from the assumption that the production function is linearly homogeneous

$$(9) \qquad F_1(\frac{L_{\alpha1}}{L_\beta},1) = w_\alpha = M$$

and since the marginal product of the unskilled workers must equal their wage

$$(10) \qquad F_2(1,\frac{L_\beta}{L_{\alpha1}}) = w_\beta.$$

From (9), $\frac{\partial w_\alpha}{\partial M} = 1 > 0$. Since from diminishing returns $F_{11}(\frac{L_{\alpha1}}{L_\beta},1) < 0$ and $F_{22}(1,\frac{L_\beta}{L_{\alpha1}}) < 0$, and it follows that $\frac{\partial w_\beta}{\partial M} < 0.$ •

Proposition 2: If the supply of both types of labor is fixed, the level of output at the competitive equilibrium will increase as a result of the introduction of the new technology.

Proposition 3: In the competitive equilibrium, the utility of the skilled workers will increase and the utility of the unskilled workers will decrease as a result of the introduction of the new technology.

The second proposition is trivial since one factor is more productive and there is no change in the level of employment of the factors. The third proposition is almost as simple, the budget set of the skilled workers after an increase in their productivity expands (contains their previous budget set), while the budget set of the unskilled workers after an increase in the productivity of the skilled workers shrink (is contained in their previous budget set).

If, in Proposition 2, we drop the assumption that the supply of labor is fixed, then the change in output depends on the changes in the supply of labor. It is then possible that the change in the demand for leisure as a result of the wage changes can lead to *lower* total output Y.

4. Non-Market Allocation and Optimal Taxation

An alternative way of addressing the problem of technological change is to consider the result of government intervention in the allocation of income so as to maximize social welfare. One possible case is that in which the government can intervene in such a way so that it can directly allocate all factors, as in Sadka (1976b). This case is unrealistic, however, in that it requires that the government have complete information and control. Nevertheless, it provides a comparison to the allocation by optimal taxation, in which case the government is bound by the individual's optimizing behavior. It can be shown that under reasonable assumptions the solution results in skilled workers working more than unskilled workers. If consumption and leisure are complements, the solution will also result in unskilled workers having higher levels of consumption. Thus in the full information case a skilled worker always has less utility than an unskilled worker. This is similar to the result obtained by Sadka.

If we now assume that the government cannot or will not distinguish between α-workers and β-workers then, since an α-worker can always pretend to be a β-worker, the government faces the constraint that the utility of an α-worker must be at least as great as he/she could achieve consuming a β-worker's bundle. This constraint implies that $u(C_\alpha, L_\alpha) - u(C_\beta, L_\beta) \geq 0$, the *self-selection constraint*.[11] There is only one such constraint because of the assumed asymmetry between the α-workers and the β-workers.

The Lagrangian for the maximization of welfare in (1) subject to (5),(6), (7), (8) and the self-selection constraint can be written

$$(11) \qquad L = u(C_\alpha, L_\alpha) + u(C_\beta, L_\beta) + p[F(L_{\alpha 1}, L_\beta) + ML_{\alpha 2} - C_\alpha - C_\beta]$$
$$+ q[L_\alpha - L_{\alpha 1} - L_{\alpha 2}] + \lambda[u(C_\alpha, L_\alpha) - u(C_\beta, L_\beta)]$$

where p and q are the multipliers associated with the goods and α-labor constraints and λ is the multiplier associated with the the self-selection constraint. The Kuhn-Tucker conditions, obtained by differentiating the Lagrangian with respect to C_α, C_β, L_α, L_β, $L_{\alpha1}$, $L_{\alpha2}$, p, q and λ respectively, are

(12a) $\quad (1 + \lambda)u_1(C_\alpha, L_\alpha) - p \leq 0; \qquad\qquad C_\alpha[(1 + \lambda)u_1(C_\alpha, L_\alpha) - p] = 0$

(12b) $\quad (1 - \lambda)u_1(C_\beta, L_\beta) - p \leq 0; \qquad\qquad C_\beta[(1 - \lambda)u_1(C_\beta, L_\beta) - p] = 0$

(12c) $\quad (1 + \lambda)u_2(C_\alpha, L_\alpha) + q \leq 0; \qquad\qquad L_\alpha[(1 + \lambda)u_2(C_\alpha, L_\alpha) + q] = 0$

(12d) $\quad (1 - \lambda)u_2(C_\beta, L_\beta) + pF_2(L_{\alpha1},L_\beta) \leq 0; \qquad L_\beta[(1 - \lambda)u_2(C_\beta, L_\beta) + pF_2(L_{\alpha1},L_\beta)] = 0$

(12e) $\quad pF_1(L_{\alpha1},L_\beta) - q \leq 0; \qquad\qquad L_{\alpha1}[pF_1(L_{\alpha1},L_\beta) - q] = 0$

(12f) $\quad pM - q \leq 0; \qquad\qquad L_{\alpha2}[pM - q] = 0$

(12g) $\quad F(L_{\alpha1},L_\beta) + ML_{\alpha2} - C_\alpha - C_\beta \geq 0; \qquad p[F(L_{\alpha1},L_\beta) + ML_{\alpha2} - C_\alpha - C_\beta] = 0$

(12h) $\quad L_\alpha - L_{\alpha1} - L_{\alpha2} \geq 0; \qquad\qquad q[L_\alpha - L_{\alpha1} - L_{\alpha2}] = 0$

(12i) $\quad u(C_\alpha, L_\alpha) - u(C_\beta, L_\beta) \geq 0; \qquad\qquad \lambda[u(C_\alpha, L_\alpha) - u(C_\beta, L_\beta)] = 0.$

Proposition 4: If both types of worker are employed the optimal marginal tax on both types of workers is zero. If the marginal product,M, of the α-workers is highter than the marginal product of the β-workers , the optimal tax consists of a lump sum transfer from the α-workers to the β-workers.

Proof:

The solution requires that C_α, C_β and $L_\alpha > 0$, so (12a), (12c) and (12f) imply that

(13) $$-\frac{u_2(C_\alpha, L_\alpha)}{u_1(C_\alpha, L_\alpha)} = \frac{q}{p} \geq M$$

Further, if $L_\beta > 0$, equations (12b), (12d) and (12e) imply

(14) $$-\frac{u_2(C_\beta, L_\beta)}{u_1(C_\beta, L_\beta)} = F_2(L_{\alpha1},L_\beta).$$

If $\lambda > 0$, it follows from (12i) that $u(C_\alpha, L_\alpha) - u(C_\beta, L_\beta) = 0$. This together with (13) and (14) implies that if $M > F_2(L_{\alpha 1}, L_\beta)$, then $C_\alpha > C_\beta$ and $L_\alpha > L_\beta$. •

This result is general and does not depend on any particular form of the utility function. However, it <u>does</u> depend on the assumptions that an α-worker is no more productive in β tasks than an β-worker and that both types of workers have identical preferences. Suppose that β-workers have a utility function of the form $v(C,L)$ and the marginal rate substitution associated with $v(C,L)$ is different than the marginal rate of substitution associated with $u(C,L)$ for all C and L. Equation (14) then becomes,

$$(15) \qquad -\frac{v_2(C_\beta,L_\beta) - \lambda u_2(C_\beta,L_\beta)}{v_1(C_\beta,L_\beta) - \lambda u_1(C_\beta,L_\beta)} = F_2(L_{\alpha 1},L_\beta)$$

The marginal rate of substitution for the β-workers is not equal to the marginal rate of transformation and they are paying a positive marginal tax.

Proposition 4: If government redistribution using an optimal income tax is introduced then employment of the unskilled workers is a strictly declining function of the marginal product of the skilled workers. If the marginal product of the skilled workers has an upper bound there exists a level of the marginal product of the skilled workers in the new technology such that it is socially optimal to eliminate the industry employing the unskilled workers, who will not be employed.

Proof:

Letting T be the amount of the transfer, the tax and subsidy schedules can be written

$$(16) \qquad C_\alpha = ML_\alpha - T$$

$$(17) \qquad C_\beta = F_2(L_{\alpha 1},L_\beta)L_\beta + T = w_\beta(M)L_\beta + T.$$

The form of the tax schedule implies that a β-worker is solving a concave problem so quasi-concavity of preferences and the Maximum Theorem imply that C_β is a continuous function of M^{12}. linear homogeneity of $F(L_{\alpha 1},L_\beta)$ implies that $\frac{dw_\beta(M)}{dM} < 0$.

It follows from the envelope theorem that

(18)
$$\frac{dW}{dM} = L_{\alpha 2} > 0$$

where W, welfare, is the maximand in (11). But $W = u(C_\alpha, L_\alpha) + u(C_\beta, L_\beta)$ and the self-selection constraint implies that $u(C_\alpha, L_\alpha) = u(C_\beta, L_\beta)$, so

(19)
$$\frac{du(C_\beta, L_\beta)}{dM} = \frac{L_{\alpha 2}}{2}.$$

Since the β-individual is on a higher indifference curve, the income effect (given the strict normality of leisure) would increase the unskilled workers' demand for leisure. Similarly, $\frac{\partial w_\beta(M)}{\partial M} < 0$ implies that since leisure is now less expensive, the substitution effect would also reduce L_β.[13] Now suppose that the marginal product of the skilled labor, M^*, is such that $F_1(L_{\alpha 1}, L_\beta) = F_1(\frac{L_{\alpha 1}}{L_\beta}, 1) \leq M^*$ for all values of $\frac{L_{\alpha 1}}{L_\beta}$. Thus, $M > M^*$ would imply that $L_\beta = 0$. •

5. Conclusions

We have used a very simple model to study an economy characterized by the following stylized facts: first, labor is heterogeneous and there are two types labor, skilled and unskilled; second, there is asymmetry in substitution of the two types labor in the production process; and third, the demand for skilled labor is growing as a result of technical change. We have addressed four issues. First, what is the impact of the new technology on the distribution of income if there is no government intervention? Second, what is the impact of the new technology on output and aggregate welfare if there is no government intervention? Third, what is the impact of government intervention if government is restricted to an optimal income tax? Fourth, is it possible that the unskilled workers might become economically unemployable?

We have found that under competitive market conditions an increase in the marginal product of labor of the skilled workers in the new technology will change the market equilibrium in such a way that all skilled workers benefit at the expense of the unskilled workers. Social welfare may decline even though output increases. These effects of the introduction of new technology

could be mitigated if the government introduces an optimal income tax. If government redistribution is introduced, then, under the special assumption of identical preferences, the government can achieve a first-best allocation. The optimal tax is a lump sum tax on skilled workers and subsidy for unskilled workers with a zero marginal tax on wages for both types of workers. The utility of both groups is equal; however, there is a critical threshold level of productivity of the skilled workers in the new technology beyond which unskilled workers became redundant as the sector that uses them is eliminated in favor of the sector using skilled workers, and it is socially optimal to eliminate the industry employing the unskilled workers, who will not be employed.

Perfect competition and optimal taxation are perhaps the two extremes in the possible allocative processes. Yet they both result in a decline in the employment of unskilled workers as the marginal product of the skilled workers increases. Perfect competition without government redistribution of income results in the unskilled workers becoming increasingly marginalized. Optimal taxation results in a more optimistic outcome. In that case, the feasible transfers are sufficient to maintain the utility of the unskilled workers at a level comparable to the skilled workers. This outcome should be qualified by the observation that there are some social, political and physiological benefits to employment that are not captured in an economic model that treats leisure as a normal good. An economy in which a large fraction of the population is economically redundant - no matter how prosperous - is unfortunately reminiscent of Rome's "bread and circuses" and, at the same time of Huxley's *Brave New World.*

Our model is very simple, and all the results would be qualified in more complicated models. The results of our model simply highlight the implication of some stylized facts. Inasmuch as these stylized facts reflect reality, the implications should be explored using more complex models which will probably require numerical solutions.

References

Bhagwati J. and M Partington, Eds.(1976), *Taxing the Brain Drain*, Amsterdam, North Holland.

Brito, D. L., J. Hamilton, S. Slutsky, and J.E. Stiglitz (1990), "Pareto Efficient Tax Structures," *Oxford Economic Papers*, 42:61-77.

Brown, C., M. Reich, and D. Stern (1990), "Skills and Security in Evolving Employment Systems: Observations from Case Studies," University of California, Berkeley, paper prepared for presentation at the conference "Changing Occupational Skill Requirements: Gathering and Assessing the Evidence," Brown University, June 5-6, 1990.

Bureau of Labor Statistics (1990), *Outlook 2000*, Bulletin 2352, Bureau of Labor Statistics, U.S. Department of Labor, Washington, D.C.

Committee on Science, Engineering, and Public Policy (1987), *Technology and Employment*, National Academy of Sciences, National Academy of Engineering, and Institute of Medicine Panel on Technology and Employment, National Academy Press, Washington, D.C.

Eliasson, G., S. Foster, T. Lindberg, T. Pousettes & E. Taymaz (1990), *The Knowledge Based Information Economy*, Almqvist and Wiksell, Stockholm.

John Paul II (1991), encyclical *Centesimus Annus*, Rome.

Kutscher, R. E. and R.W. Riche (1990), "Impact of Technology on Employment and the Workforce -- The U.S. Experience," Bureau of Labor Statistics, U.S. Department of Labor, Washington, D.C., paper prepared for presentation at the World Bank Seminar on Employment and Social Dimension of Economic Adjustment, Washington, D.C., February 27-28, 1990.

Mirrlees, J. A. (1971), "An Exploration in the Theory of Optimal Taxation," *Review of Economic Studies*, 38:175-208.

Sadka, E. (1976a), "On Income Redistribution: Incentive Effects and the Optimal Income Taxation" *Review of Economic Studies*, 43:261-268.

_____ (1976b), "Social Welfare and Income Distribution" *Econometrica*, 44: 1239-1252.

Smith, R. E. (1955), *The Failure of the Roman Republic*, Cambridge, Cambridge University Press.

Stiglitz, J.E. (1982), "Self Selection and Pareto Efficient Taxation," *Journal of Public Economics*, 17:213-240.

Uchitelle, L. (1990), "Surplus of College Graduates Dims Job Outlook for Others," *The New York Times*, June 18, 1990.

[1] See Committee on Science, Engineering, and Public Policy Technology and Employment, (1987), Brown, Reich, and Stern (1990), Bureau of Labor Statistics (1990), Kutscher and Riche (1990) and Uchitelle (1990)

[2] G. Eliasson, S. Foster, T. Lindberg, T. Pousettes and E. Taymaz (1990).

[3] *Id* at 43.

[4] John Paul II (1991) §32.

[5] *Id* § 33.

[6] See Bhagwati and Partington, Eds, (1976).

[7] See Mirrlees (1971).

[8] See Smith (1955), pp. 61-63.

[9] For related models, involving skilled (α) and unskilled (β) workers in the context of Pareto efficient taxation see Sadka (1976a, 1976b); Stiglitz (1982); and Brito, Hamilton, Slutsky, and Stiglitz (1990).

[10] Any results where this assumption is important will be noted.

[11] See Stiglitz (1982).

[12] The Maximum Theorem implies that C_β is upper hemi-continuous and the quasi-concavity of preferences implies that C_β is single-valued.

[13] This result can also be obtained by the more direct (but tedious) fashion of reducing equations (12-a) - (12-i) to

$$\frac{\partial C_\alpha}{\partial L_\alpha} = \frac{\partial C_\alpha}{\partial (L_{\alpha 1} + L_{\alpha 2})} = M$$

$$\frac{\partial C_\beta}{\partial L_\beta} = F_2(L_{\alpha 1}, L_\beta)$$

$$F(L_{\alpha 1}, L_\beta) + ML_{\alpha 2} - C_\alpha - C_\beta = 0$$

$$F_1(L_{\alpha 1}, L_\beta) = M$$

$$u(C_\alpha, L_\alpha) - u(C_\beta, L_\beta) = 0.$$

This reduced system can be differentiated with respect to M and solved to yield

$$\frac{\partial L_\beta}{\partial M} = \frac{\{F_{21}\frac{\partial^2 C_\alpha}{\partial C_\alpha \partial L_\alpha} u_1(C_\beta, L_\beta) + u_1(C_\alpha, L_\alpha)\frac{\partial^2 C_\beta}{\partial L_\beta^2}[\frac{\partial^2 C_\beta}{\partial C_\beta \partial L_\beta}(F_1 - L_{\alpha 2}F_{11}) + M(F_{11} + \frac{\partial^2 C_\alpha}{\partial L_\alpha^2})]\}}{\{\frac{\partial^2 C_\alpha}{\partial C_\alpha \partial L_\alpha} u_1(C_\beta, L_\beta)M[\frac{\partial^2 C_\beta}{\partial L_\beta^2}F_{11} + F_{12}F_{21}] + u_1(C_\alpha, L_\alpha)\frac{\partial^2 C_\beta}{\partial C_\beta \partial L_\beta}F_2 F_{11}\frac{\partial^2 C_\alpha}{\partial L_\alpha^2}\}} < 0.$$