A TIME SERIES MODEL OF HOUSING INVESTMENT IN THE U.S.

by

ROBERT TOPEL

UNIVERSITY OF CALIFORNIA, LOS ANGELES

and

SHERWIN ROSEN

UNIVERSITY OF CHICAGO

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Robert Topel and Sherwin Rosen*

I. INTRODUCTION

The housing market is especially attractive for studying investment behavior. Housing investment is one of the most volatile components of national product, and understanding its sources of variability may illuminate the sources of volatility in other forms of investment in the economy. In addition, housing investment and allied data are among the most complete and conceptually clean sources for studying investment. This is perhaps the only investment goods market where data on the valuation of an additional unit of capital are directly available, and are adjusted for quality change. In neoclassical investment theory, the value or price of an additional unit of capital serves as a sufficient statistic embodying all current and expected future information on the sources of value of capital services. The model studied here is constructed to exploit and test this fundamental hypothesis.

The basic conception of the problem is an ancient one, and bears a close relationship to more modern adjustment cost and q-theory models. Since investment is a trivial fraction of existing housing stock, the instantaneous supply of housing services is essentially inelastic. Maintain the hypothesis that the market for housing services is instantaneously cleared. Furthermore, assume that the market for housing stock also clears. Then the price of a standard house must be determined so that agents are content to hold the existing stock. The rate of additions to the stock at that time are determined by supply decisions of a construction sector. A supplier of new homes can sell as many as desired at the prevailing market price in a competitive market. If the price of a house is high relative to production cost then more houses will be built. The housing investment decision is completely decentralized through decisions of the construction sector in this market, and issues of demand for investment (as opposed to the demand for capital services) never arise. This is a supply theory of investment. But if investment is supply determined, there must be rising supply price for aggregate investment to be determinate.

Preliminary evidence of rising supply price in the housing investment sector is presented in the next section. The main content of this evidence is that factor prices and construction costs are positively correlated with the level of construction activity, which in turn is positively correlated with the price of houses. This is crudely indicative of an essentially demand driven industry in which factors of production are, at least to a degree, inelastically supplied to the industry as a whole. This fact and other elements of diminishing returns imply rising supply price of new investment, which is investigated in detail in subsequent analysis.

Sections III and IV lay out the conceptual model based on this preliminary evidence. The rudiments of this model are structured as above: there is a demand function for housing services and a supply function for new investment. A structural relationship linking flow or rental prices of housing services to house (stock) prices is necessary to close the model. In this respect we maintain the rational expectations hypothesis that the observed stock price is the expected discounted present value of current and future rental prices. It is in this way that current price serves as a "sufficient statistic" embodying all available information. We show that this model is in all respects equivalent to an adjustment cost theory of investment. However, there is the substantial difference that it is completely decentralized through a market mechanism, with the observed price of housing serving as the relevant valuation for investment decisions. Section IV presents an important extension of this type of model, to approximate a situation in which there is a difference between the short and long-run supply elasticities of new houses. The model is changed in interesting and nontrivial ways. Sufficiency of the current stock price for investment is lost, but a richer set of possibilities, including endogenous generation of cyclical response patterns to exogeneous disturbances, are added. This case is favored by the empirical estimates presented in section V.

Our main estimates relate to the supply price elasticity of new housing. The long run supply elasticity is approximately 3.0. The short run, quarter-to-quarter elasticity of supply is also found to be quite substantial, in the neighborhood of 1.0. These large supply elasticities are one of the reasons why investment activity in housing is so volatile and responds so quickly to new information. Attempts to estimate the demand function and the intertemporal arbitrage condition are far less successful. This is partly due to data limitations (the available housing stock series is inadequate), and also due to unexplainable movements in the price series itself. The enormous and sustained increase in the relative price of housing during the 1975-80 period is not readily explained by the exogenous variables available to us or by the assumption of stable expectations about capital gains. Some, but not all of this increase can be explained by portfolio considerations in the presence of income tax advantages to holding housing assets in an inflationary environment. There appear to be important additional causes of this relative shift in demand which are not yet understood.

II. CONSTRUCTION ACTIVITY AND CONSTRUCTION COSTS

Some of the time-series data used in the subsequent analysis are shown in the accompanying charts. Our primary focus in this work is on starts (which are very highly correlated with measures of housing investment, but due to measurement and data consistency problems, we, like most others in this field, prefer to use starts as our index of investment). There are roughly three "episodes" in this series. The series takes a gentle downward course in the 1963-70 period, with a few wiggles associated with the 1967 and 1970 recessions. The next episode shows a big spurt in activity in the 1970-73 period followed by a decline of about equal magnitude during 1973-75. Another boom and bust is exhibited in the 1975-82 period, with a peak sometime in 1978. To get a feeling for the volatility of real activity in this industry note that the (quarterly) trough-to-peak levels in the second and third episodes differ by a factor of 2.0. During a building boom construction activity might double and during a bust it might fall in half. The turning points in this series are similar to those in the general business cycle. This correlation is made fairly clear in matching the starts series with the first difference in real consumption in the economy (a rough index of changes in real wealth) over this period.[1]

The relative prices of new houses are shown on the next chart. The nominal price index for new homes is a hedonically adjusted "house of 1977 characteristics" prepared by BEA. The mean sales price was $19,000 in 1963 and $85,000 in 1982, close to an 8 percent nominal rate of growth. The chart shows real prices, using the CPI (less the shelter component to avoid the measurement problems in that component during the inflationary period in the 1970s). The overall trend in real price is closer to one percent over the period as a whole, but is punctuated by a remarkable 25 percent increase during 1975-79. In fact the land price index implicit in this series increased by approximately 50 percent during those four years, similar to the increases observed in valuations of agricultural land.

There is an obvious positive correlation between movements in starts and house prices. This is more pronounced in the 1970-82 period than before, but there is also more action in both series in that later subperiod. Construction activity tends to increase when prices rise and it tends to decrease when prices decline. However, the timing is somewhat different: construction activity appears to turn down prior to downturns in prices.

Additional informal evidence on rising supply price is presented in the charts, which show the behavior of various dimensions of building costs. A major materials component of new house construction is lumber. Total lumber consumption is highly correlated with starts, and the real price of lumber closely tracks lumber consumption. The simplest and best interpretation of these facts is that there is rising supply price of lumber to the residential construction industry.

Labor costs represent a major share of total construction costs. The construction sector is well known to exhibit the most volatile employment behavior in virtually any industry (Topel and Ward [1985]). Furthermore, this has been true well before the modern era; for example, Smith remarked upon it in The Wealth of Nations. The correlation between starts and total hours of construction labor (not shown) is quite large (however, starts leads employment by a few quarters). Further, the bulk of total hours variability is taken up by variations in employment: hours per employed person shows remarkably little variation. This is consistent with a fairly fluid market in which people move in and out of the industry in response to changes in demand for labor. Competing uses for residential construction labor include nonresidential construction, the timing of which is a year or so out of phase with residential construction activity; and a significant maintenance and allied use of these skills in the industrial and service sector. There are frequent transitions of workers among these sectors. Rising supply price of construction labor is to be expected under these circumstances, with relative wage changes among sectors providing the economic signals for workers to move from one sector to another. The real wage series shown in the charts at least are consistent with this interpretation. Construction wage movements are strongly related to manufacturing wage changes, reflecting the opportunity costs of construction labor supply. There is also a noticeable positive comovement between construction wage rates and construction activity.

The charts show the time-series for the real Boeckh index of residential construction cost, which combines wage and materials costs in one summary measure. The pattern of the cost index closely follows the pattern of construction wage rates and real activity. That both prices and costs are positively correlated with output is consistent with a rising supply curve mapped out by shifts in demand; that is, with rising supply price of investment. Notice also that construction costs increase relative to housing prices in the first (1963-69) episode, perhaps reflecting a shift in supply in that episode.

III. A MODEL OF HOUSING INVESTMENT

The model has its origins in the work of Walras, and much later by Friedman [1963] and Tobin [1969] -- see also Witte [1963], Foley and Sidrausky [1971], Engle and Foley [1975], Mussa [1977], Sheffrin [1983], and Poterba [1984] among many others. We deal with a linear structure for analytical tractability and present a deterministic (perfect foresight) formulation to illustrate the key ideas. To avoid expository distractions, which are well treated in the literature, we also ignore the special and peculiar income tax provisions of home ownership, though these factors are included in the empirical work where necessary. The basic model consists of three equations and some boundary conditions.

First is the demand function for the services of existing stock. Let $K(t)$ denote the stock of existing capital (homes) at time t, and let $R(t)$ be the implicit rental value of a unit of stock. Then the demand function for home services, assuming service flows proportional to stock, is

$$R = \alpha K + x \tag{1}$$

where α is the inverse of the slope of the demand curve, with $\alpha < 0$. $x(t)$ is a vector of exogenous variables (and coefficients) affecting the demand for housing services, including per capita income, the demographic composition of the population, operating and maintenance costs, and the like.

The second equation is the supply of new stock. Let $P(t)$ be the competitive equilibrium price of a unit of housing stock. Let $I(t)$ be the gross flow of new construction. If the construction sector is competitively organized and if the representative firm has a neoclassical cost function of output I of the form $C(I, y)$ where y is a vector of cost shifters and C_1 and C_{11} are positive, then output decisions are determined by the familiar price equals marginal cost rule: $P(t) = C_1(I, y)$. The (short) delay in the construction time of new homes is ignored. Taking a linear approximation to the marginal condition and assuming exponential decay $\dot{K} + \delta K = I$ where $\dot{K} = dK/dt$ and δ is the depreciation rate, we have the investment equation:

$$(2) \qquad I = \dot{K} + \delta K = \beta P + y$$

where $\beta > 0$ is the inverse of the marginal cost function, the slope of the supply curve, and $y(t)$ is a vector of variables (and coefficients) affecting the supply conditions of factors of production that are specialized to the industry and the prices of factors which are unspecialized. These would include such things as the opportunity cost of construction labor (e.g., the manufacturing wage), and the short-term interest rate reflecting the cost of capital to construction firms.

The model is closed by connecting stock and flow prices. Several possibilities might be entertained, but we concentrate on the only internally consistent alternative -- rational expectations (equivalent to perfect foresight in deterministic models). Thus

$$(3) \qquad R = (r + \delta)P - \dot{P}$$

where $\dot{P} = dP(t)/dt$ and r is the rate of interest. This is the familiar condition that the imputed flow price of a unit of capital is the amortized stock price, including allowances for interest, depreciation and capital gains. We also require a boundary condition that the future price of capital is bounded:

$$\lim_{t\to\infty} P(t)e^{-(r+\delta)t} = 0 \quad . \tag{4}$$

The boundary condition in (4) requires that the sequences x(t) and y(t) are themselves bounded in their rates of growth by exponential functions of order r: the forcing variables in x and y cannot forever grow (or decline) at rates exceeding the interest rate, though of course they can do so for finite periods of time. Integrating (3) and using the boundary condition (4) yields the familiar asset pricing equation

$$P(t) = \int_t^\infty R(s)e^{-(r+\delta)(s-t)}ds \quad . \tag{5}$$

The price of a unit of stock at time t is the discounted present value of its anticipated stream of future market equilibrium rentals. Finally, the model is closed by specifying the initial stock $K(0) = K_0$.

Equations (1) - (4) provide a complete description of competitive market equilibrium, conditional on the course of the forcing variables x(t) and y(t) and the initial condition. Appendix A shows that these conditions duplicate a surplus maximizing social planning problem in the presence of adjustment costs: the decentralized competitive market solution is pareto efficient and price signals provide the correct information for private decision makers so long as rational expectations hold true. The reason for this is the sufficient statistic interpretation of market asset prices. The market price conveys the social value of an increment of investment -- the discounted present value of the marginal utility of an additional unit of housing services, because marginal utilities are reflected in competitive rental prices. A competitive housing sector which builds new houses up to the point where price equals marginal cost therefore duplicates a necessary condition for social efficiency, that marginal social cost equals marginal social benefit.

Empirical investment models based on internal adjustment costs have had difficulties in accomodating price and value data. The rising supply price model is, as the appendix shows, equivalent to an external adjustment cost formulation in which market price/value data plays its traditional role. We believe that the revival of interest in q-type models lies, at least in part, in bringing the traditional price signals back into central focus (Summers [1981]). The difference between adjustment cost and q-theory models lies in a difference between marginal and average (see Hayashi [1982], Able [1980]). The price-equals-marginal-cost characterization of supply in our model implies that "marginal q" is identically unity (in a competitive market), whereas q-theory proper has been less precise on supply conditions. However, the spirit of the two approaches has marked similarities in spite of these differences in detail. The virtue of studying the housing market in this connection is precisely the direct availablilty of price data, which has already been used to excellent advantage by Poterba [1984] to study the influence of certain tax treatments of housing using a similar analytical framework as this one. And of course the role of price in housing investment has been for years a central focus of empirical investigations of the housing sector though without the dynamic optimization paraphenalia (see the fine survey by Weicher [19]).

We are now prepared to analyze the dynamical properties of this market. Substituting (3) into (1) yields a first order system in P(t) and K(t)

$$
(6)\qquad
\begin{aligned}
(r + \delta)P(t) - \dot{P}(t) &= \alpha K(t) + x(t) \\
\dot{K}(t) + \delta K(t) &= \beta P(t) + y(t)
\end{aligned}
$$

along with the boundary condition (4) and the initial condition. The phase plane is useful for illustrating some general properties of this system. First set $x(t) = x$ and $y(t) = y$, some constant values. The arrows in the figure show all possible trajectories of the general solution that satisfy both equations in (6). However, the two heavy arrows are the only ones that also satisfy the terminal condition (we assume for simplicity that costs of demolition and accumulation of capital are symmetric). Thus for constant values of x and y, the market ultimately achieves a steady state. This experiment is related to the general solution to system (6).

More interesting are the particular solutions. The phase plan can be used to give a rough and ready analysis of the complete response characteristics of the system, using a "gluing and pasting" algorithm of the trajectories in figure 1 (Able [1982], Judd [1985]). Consider an experiment in which there is an anticipated pulse in demand at time T in the future persisting for an interval Δt, which returns to its initial level thereafter. y(t) remains at its initial constant level for all t. This is similar to a distributed lag response experiment. However, in this problem there will also be a distributed <u>lead</u> because rational agents act in anticipation of future demand changes when there is rising supply price of investment. The reason is that it is too expensive to wait for the demand change to occur prior to acting on that information. With rising supply price it pays to build ahead of anticipated demand since rapid investment is penalized by greater cost.

The key to analyzing this experiment in the phase plane is to recognize that the solution must be consistent from the backward point of view. System (6) will apply with $x(t) = x$ and $y(t) = y$ for all $t > T + \Delta t$. Therefore we must be on a stable trajectory of figure 1 in that final phase of the problem. However, for the interval $T < t < T + \Delta t$ the system is described by (6) with $x(t)$ at a larger value. For this interval we must find the unique unstable trajectory of the altered system which arrives at the known stable trajectory of the original system at exactly time $t = T + \Delta t$, and glue the two together. Finally, we know that we must be back in the original system (with $x(t)$ at its original value) for $t < T$. Therefore we must find another unstable trajectory of the original system which arrives at the now known second phase trajectory of the perturbed system at exactly time $t = T$. The solution is pasted together in figure 2, assuming an initial capital stock at $\overline{K}$. The pulse in demands sets things off on the trajectory marked A, which corresponds to an unstable path of the original system. At $t = T$, this path joins arrow B and that trajectory takes over for t in the interval $(T, T + \Delta t)$. Arrow B is an unstable trajectory of the system where the $\dot{P} = 0$ line has been shifted up to reflect a larger value of x in the second phase. Finally, at $t = T + \Delta t$, the stable path of the original system is joined and capital converges to its initial value.

The whole solution is described by the circular motion in the figure. This translates in the (P, t) plane to a price "bubble" in figure

3. The price of housing immediately begins to rise in anticipation of the increase in demand, and keeps rising up to $t = T$. This signals construction firms to increase the stock of housing before the demand shock is realized. New construction reaches its maximal rate at time T if Δt is relatively small. At that point demand jumps up. However, agents perceive the transitory nature of the shock, and price begins falling at that point (for small Δt), slowing down the rate of increase of new construction, but not sufficiently to keep the stock from growing. However, at some time during the $T + \Delta t$ interval, price falls enough to reduce investment to a level that makes the capital stock reach a peak and then start to fall. In fact $P(t)$ falls below its initial level in this phase, which signals that the stock must decline. At $t = T + \Delta t$, the final phase is begun: the price gradually works its way back up to its initial level and the capital stock asymptotically falls to its initial level.[2] The investment path $I(t)$ follows the same qualitative course as that of $P(t)$. It is also clear what must happen to the rental price $R(t)$ in this experiment. Since demand for housing services is unchanging in the period prior to T, the increase in the stock makes $R(t)$ actually fall in this phase. In the second phase R jumps up and reaches its peak at a level higher than its initial value. The capital stock has been "overbuilt" in this phase, so R must drop below its initial level in the third phase and then gradually work its way back up to its initial level as $K(t)$ falls back to $\overline{K}$. Hence the intertemporal pattern of $R(t)$ would appear as the silhouette of a "sombrero".

Analyzing the trajectories in this way allows qualitative analysis of all kinds of experiments, e.g., permanent anticipated changes in r, x and y, transitory changes, unanticipated changes,and so forth. Responses to more complicated changes could be found by approximating them by step functions and linking trajectories in the pasting manner depicted above. However, since we have assumed a linear structure, analytical solutions are easily calculated. We turn to this task next.[3]

Define the differential operator D as $Db(t) = db(t)/dt$, $D^2b(t) = d^2b(t)/dt^2$, etc. In this notation, (6) can be written as a matrix differential equation

$$(7) \quad \begin{bmatrix} D - (r + \delta) & \alpha \\ -\beta & (D + \delta) \end{bmatrix} \begin{bmatrix} P \\ K \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

The determinant of the matrix is $D^2 - rD + [(\alpha\beta - \delta(r + \delta)]$. Solving (7) yields two independent second order differential equations, one in P and the other in K:

$$(8) \quad \begin{aligned} &[D^2 - rD + (\alpha\beta - \delta(r + \delta))]P(t) = g(t) \\ &[D^2 - rD + (\alpha\beta - \delta(r + \delta))]K(t) = h(t) \end{aligned}$$

where

$$g(t) = -dx(t)/dt - \delta x(t) - \alpha y(t)$$

$$h(t) = dy(t)/dt - (r + \delta)y(t) - \beta x(t)$$

The forcing functions in (8) contain elements of both demand shifters and supply shifters. Notice also that the homogeneous parts of (8) are identical. This is one aspect of cross-equation-restrictions inherent in problems of this type.

Use the quadratic formula to factor the left hand sides of (8). The characteristic equation is

$$\lambda^2 - r\lambda + (\alpha\beta - \delta(r + \delta)) = 0$$

so

(9) $$\lambda = [r \pm (r^2 - 4(\alpha\beta - \delta(r + \delta)))^{1/2}]/2 \quad .$$

Since $\alpha < 0$ and $\beta > 0$, both roots are real. Furthermore, one is positive and one is negative. Take λ_1 to be the negative (stable) root and take λ_2 to be the positive (unstable) root. Then (8) can be written as

(10) $$(D - \lambda_1)(D - \lambda_2)P(t) = g(t)$$

(11) $$(D - \lambda_1)(D - \lambda_2)K(t) = h(t) \quad .$$

Solving these equations requires inversion of the operator $(D - \lambda)$.

The inverse $(D - \lambda)^{-1}$ operating on some function $b(t)$ has both a backward and forward representation: The backward form is

(12) $$(D - \lambda)^{-1}b(t) = c_1e^{\lambda t} + \int_0^t b(\tau)e^{-\lambda(\tau-t)}d\tau$$

where c_1 is some arbitrary constant; and the forward form is

$$(D - \lambda)^{-1}b(t) = c_2e^{\lambda t} - \int_t^{\infty} b(\tau)e^{-\lambda(\tau-t)}d\tau \quad . \tag{13}$$

where c_2 is another constant. These formulas are verified by applying $(D - \lambda)$ to both sides to obtain $b(t)$ itself. To solve (10) and (11) and produce a stable system it is natural to take the positive root forward and the negative root backward (Sargent [1979]). To illustrate, let us solve (11).

$$(D - \lambda_1)K(t) = (D - \lambda_2)^{-1}h(t)$$

$$= c_2e^{\lambda_2 t} - \int_t^{\infty} h(\tau)e^{\lambda_2(t-\tau)}d\tau \quad ,$$

using (13). However, c_2 must be zero from the transversality condition and figure 1. Therefore,

$$(D - \lambda_1)K(t) = - \int_t^{\infty} h(\tau)e^{\lambda_2(t-\tau)}d\tau$$

and

$$K(t) = -(D - \lambda_2)^{-1} \int_t^{\infty} h(\tau)e^{\lambda_2(t-\tau)}d\tau$$

$$= c_1e^{\lambda_1 t} - \int_0^t \int_s^{\infty} h(\tau)e^{\lambda_2(s-\tau)} e^{\lambda_1(t-s)}dsd\tau$$

from (12). Reversing the order of integration and using the initial condition $K(0) = K_0$ gives the complete solution

$$(14) \qquad K(t) = e^{\lambda_1 t}\left[K_0 + \frac{1}{\lambda_2-\lambda_1}\int_0^\infty h(\tau)e^{-\lambda_2 t}\,d\tau\right]$$

$$- \frac{1}{\lambda_2-\lambda_1}\int_0^t h(\tau)e^{\lambda_1(t-\tau)}\,d\tau - \frac{1}{\lambda_2-\lambda_1}\int_t^\infty h(\tau)e^{\lambda_2(t-\tau)}\,d\tau.$$

Equation (14) is useful for studying the equilibrium response pattern for any intertemporal pattern of exogenous supply and demand shocks. It also shows why the unstable trajectory comes into the geometry of figure 2.

Notice from the second and third terms in (14) that the equilibrium position of K(t) is a weighted sum of past and future information. The weights decline exponentially in both directions, so current information in and around the neighborhood of t itself gets the most weight. The "speed" of response depends on these weighting functions. If λ_2 and λ_1 are very large in absolute value then current data h(t) gets most of the weight and the adjustment speed is very rapid. If they are small then the response is distributed over a longer interval.

From the definition (9) we see that λ_2 and $-\lambda_1$ are increasing in the depreciation rate δ. Response speed is increasing in δ because there are less overhanging effects of existing stocks. Indeed as δ get increasingly large the model gets very close to a conventional flow market in which long- and short-run elasticities of supply (and demand) are identical. λ_2 and $-\lambda_1$ are also increasing in $\alpha\beta$. β indexes the responsiveness of investment to housing prices. The housing investment supply function is more elastic the larger is β so the sensitivity and speed of response to current data is correspondingly larger. However, adjustments are also more rapid the larger is $-\alpha$, which is the inverse slope of flow demand. Adjustments are more responsive the more inelastic is the demand for housing services because inelastic flow demand provokes larger asset price responses to changes in stocks, and it is these price signals that guide supply decisions.

The effects of the interest rate are somewhat more complex. The positive root λ_2 is increasing in r but $|\lambda_1|$ is decreasing in r. Equation (14) implies that future events are given less weight because λ_2 is larger. However, the legacy of the past carries greater weight because the exponential term in the backward integral does not die out as rapidly. All in all, the system is less responsive to exogenous data as r increases. That the future is weighted less heavily means that anticipated future events have smaller influence on current asset price signals for investment decisons. Furthermore it can be shown (see below) that for permanent (step function) impulses of the forcing variables, the rate of investment is proportional to the differences between current and ultimate stock, with constant of proportionality $-\lambda_1$, which is decreasing in r.

An expression similar to (14) applies for the complete solution to P(t) and is omitted. Three methods are available for estimation. One by Hansen and Sargent [1980] estimates the complete solutions such as (14). This has the virtue of maintaining the transversality condition, but with the cost of additional assumptions about the nature of the forcing process. It also does not exploit market price and value data in determining investment, though these connections are implicit in the derivations of the complete solutions. Instead, we estimate the structural form (6). This has the virtue of being less sensitive to stochastic and specification assumptions about x(t) and y(t) and allows investigation of a direct and familiar behavioral relationship: the elasticity of supply is a key parameter in the investment process. The cost of this is that the transversality condition is not imposed. Something of a halfway house would be provided by estimating the independent forms in (8) (Zellner and Palm [1974]) though again the transversality condition is not imposed, and we do not attempt to do so here.

IV. RISING SUPPLY PRICE AND SHORT-RUN ADJUSTMENT COSTS

A disadvantage of a cost structure based on rising supply price alone is that it does not make the Marshallian distinction between short-run and long-run supply responses: the industry supply curve is fixed, and has no time-dimension. This assumption gives an industry version of the adjustment cost theory of investment, but is unlikely to be valid in the present problem (and perhaps in others as well), because supply is likely to be more inelastic in the short-run.

One way of handling these issues formally would be to specify with care and precision the nature of the short- and long-run supply conditions of factors of production to the industry. Thus, for example, labor does not move costlessly in and out of the industry. Neither does capital. Short-run factor supplies are less elastic than long-run supplies. To go in this direction, however, requires introducing additional state variables into the analysis, which increases the complexity of the model, especially for empirical work. Instead we adopt a more tractable alternative where supply conditions of factors are approximately incorporated into an expanded cost function which includes the rate of change of industry output. Short-run output supply inelasticity is implied by cost penalties to rapid changes in the level of construction activity. Specify costs as

(15) $$C(I, \dot{I}, y)$$

where $C_1 > 0$, $C_{11} > 0$ as before, and $C_2 \geq 0$ and $C_{22} \geq 0$. Strict inequalities for the last two derivatives capture the effect under discussion.

The representative firm cannot maximize instantaneous profits in choosing its level of construction activity because of the presence of the term $\dot{I}$ in (15). Instead it must maximize its expected present value:

$$\int_0^{\infty} [P(t)I(t) - C(I(t), \dot{I}(t), y(t))]e^{-rt}dt.$$

The Euler condition for this problem is

(16) $$P(t) - C_1(\cdot) = rC_2(\cdot) - dC_2(\cdot)/dt.$$

If C_2 is identically zero, then the right hand side of (16) vanishes and section III applies. Otherwise, the presence of increasing marginal adjustment costs for the firm creates a wedge between price and marginal cost. The main implication of "internal" adjustment costs is that the responsiveness of investment to changes in asset values is dampened and smoothed relative to the simpler model. In addition, the current price of capital no longer serves as the sufficient statistic: expectations of the future course of prices as well as the current price affect current investment decisions.

To illustrate these points, linearize the marginal costs in (16):

$$(17) \qquad C_1(I, \dot{I}, y) = C_1 + C_{11}I + C_{12}\dot{I} + C_{13}y$$

$$C_2(I, \dot{I}, y) = C_2 + C_{12}I + C_{22}\dot{I} + C_{23}y$$

where the C_i and C_{ij} are positive constants. Carrying out the differentiation in (16) and substituting from (17) yields the linearized Euler equation (assuming $C_{23} = 0$ for convenience)

$$(18) \qquad C_1 + rC_2 + (C_{11} + rC_{12})I(t) + rC_{22}\dot{I}(t) - C_{22}\ddot{I}(t) + C_{13}y(t) = P(t).$$

If $C_{22} = 0$, (18) is equivalent to (2) with $\beta = (C_{11} + rC_{12})^{-1}$. If $C_{22} > 0$, (18) embodies a much richer set of possibilities than (2). Write (18) as:

$$(19) \qquad [1 + r\beta D - \beta D^2]I(t) = (\beta/C_{22})P(t) - (\beta/C_{22})[C_1 + rC_2 + C_{13}y] = \theta(t)$$

where $\beta = C_{22}/(C_{11} + rC_{21})$.

Equation (19) defines investment supply. It embodies the standard distinction between short and long run supply responses and also the modern distinction of differential response to permanent and transitory shocks. To see this, consider a <u>conceptual</u> (partial equilibrium) experiment in which the right hand side of (19) -- written as $\theta(t)$ -- is, for the moment, treated as exogenous. Dividing (19) by β and rearranging

$$(20) \qquad (D^2 - rD - 1/\beta)I(t) = -\theta(t)/\beta$$

or

$$I(t) = (D - \lambda_3)^{-1}(D - \lambda_4)^{-1}(-\theta(t)/\beta) \tag{21}$$

where $\lambda_3 < 0$ and $\lambda_4 > 0$ are real numbers that solve $\lambda^2 - r\theta - 1/\beta = 0$:

$$\lambda = (r \pm \sqrt{r^2 + 1/\beta})/2 . \tag{22}$$

Following exactly the same steps as those leading to (14) above yields

$$I(t) = e^{\lambda_3 t}(c_0 - \frac{1}{\lambda_4 - \lambda_3}\int_0^{\infty} (\theta(\tau)/\beta)e^{-\lambda_4 \tau} d\tau) + \frac{1}{\lambda_4 - \lambda_3}[\int_0^t (\theta(\tau)/\beta)e^{\lambda_3(t-\tau)} d\tau + \int_t^{\infty} (\theta(\tau)/\beta)e^{\lambda_4(t-\tau)} d\tau] \tag{23}$$

so the forcing data affects current investment through a backward and forward exponential "window." The weight function on $\theta(t)$ is increasingly concentrated on current data as β approaches zero because $-\lambda_3$ and λ_4 increase without bound, from (22). This happens when C_{22} approaches zero, by the definition of β.

$\theta(t)$ is linear in $P(t)$ from (19). Suppose $y(t)$ is constant and $P(t) = P_1$, a constant. Then $\theta(t) = \theta_1$ is also constant, and $I(t)$ eventually settles down to its long-run level $I(t) = \theta_1 = I_1$, which is a point on the long-run supply curve of investment. Take this point as an initial condition (so $c_0 = \theta_1$, from (23)) and suppose new information comes in that the price of housing will <u>permanently</u> increase from P_1 to P_2. Then $\theta(t)$ jumps from θ_1 to, say, θ_2, and $I(t)$ asymptotes to $\theta_2 = I_2$, another point on the long-run supply curve. Substituting $\theta(t) = \theta_2$ in the intergrals in (23) and integrating yields the path by which $I(t)$ travels from θ_1 to θ_2:

$$I_t = \theta_2 - (\theta_2 - \theta_1)e^{\lambda_3 t}.$$

The response is exponential, with the largest response at the beginning and the smallest at the end. Familiar manipulations lead to $\dot{I}(t) = -\lambda_3(I_2 - I(t))$, where I_2 is the target to which $I(t)$ converges in the long-run when $P(t) = P_2$. This flexible accelerator form is common from early discussions of adjustment cost models (Eisner and Strotz [1963], Lucas [1967], Gould [1968], Treadway [1969]). The adjustment speed is governed by $-\lambda_3$, which is decreasing in r and increasing in β, from (22). Adjustments are speedier the smaller is C_{22}: there is no distinction between short- and long-run supply when C_{22} is zero.

This experiment can be depicted in the familiar Marshallian diagram if various short-runs are identified with specific intervals of time and the long-run with an arbitrarily long interval. Long-run supply connects the points (I_1,P_1) and (I_2,P_2) in the investment-price plane. Short-run supply curves are spun out of the point (I_1,P_1) and are less elastic than long-run supply, with the elasticity increasing as time goes by.

Now consider another conceptual experiment in which the price rises from P_1, its initial value, to P_2 for a finite interval of time T, after which it returns to its initial value. We shall say that the price disturbance is more permanent the larger is T and is more transitory the smaller is T. Equation (23) implies that the initial response is smaller the more transitory the price impulse.

Differentiating (23) with respect to t and evaluating the derivative at T = 0 yields an expression for the initial response:

$$\dot{I}(0) = \lambda_3\theta_1 + \int_0^{\infty} (\theta(\tau)/\beta)e^{-\lambda_4\tau}d\tau \tag{24}$$

which, for the postulated square wave pulse in P(t) becomes

$$\dot{I}(0) = -\lambda_3\ (\theta_2 - \theta_1)(1 - e^{-\lambda_4 T}). \tag{25}$$

The impact response $\dot{I}(0)$ is increasing in T, that is, the more permanent the pulse. (25) shows that the difference in impact responses to permanent and transitory shocks is decreasing in λ_4. Thus as C_{22} approaches zero the term in T in (25) vanishes -- λ_4 grows very large -- and there is no difference in initial response to permanent and transitory changes in price. How long must the pulse in P(t) last for the impact response to be m% of the impact response to a permanent change in price? Equation (25) provides a ready answer. The pulse must have length $T^* = -\ln(1 - m)/\lambda_4$. T* is decreasing in λ_4, or in C_{22}, which is another way of saying that differences between short and long-run responses to price changes vanish as internal adjustment costs get small. For example, if m = .95, then $T^* = 3/\lambda_4$, so if $\lambda_4 = 3$ an anticipated price pulse of one year's duration essentially simulates a permanent impact response.

The specification in (19) is of more than academic interest. We were led to it not only because the simpler model does not correspond to short-run/long-run differences in supply that we know are present in most other goods in the economy, but also because of certain anomolies in the fit of the simpler theory to the data. The cost of this generality, as is clear from (23), is that current P(t) is no longer sufficient for the supply decision: the path of P(t) as well as its current value determines supply decisions when there are internal adjustment costs for firms on top of rising supply price. This presents additional difficulties of estimation, but our estimates definitely favor this case.

The experiments in (20)-(25) are conceptual because P(t) is endogenous in the full market equilibrium: current investment affects future rentals, which in turn influences P(t). If there are neither adjustment nor transactions costs on the demand side of the market, then asset pricing theory requires that equation (9) remains valid for this model as well as for the earlier one. The price of a house is its discounted anticipated future rental, independent of the technology of supply. Differentiating (9) with respect to t and substituting for $\dot{K}$ yields a second order equation for P(t) which conforms to (19). The differential equations that fully characterize competitive equilibrium are

$$\text{(a)} \quad [1 + r\beta D - \beta D^2]I(t) - \frac{\beta}{C_{22}} P(t) = -(\beta/C_{22})(C_1 + rC_2 + C_{13}y)$$

(26)

$$\text{(b)} \quad [1 + rBD - BD^2]P(t) - \alpha BI(t) = B(D + \delta)x$$

where $B = [\delta(\delta + r)]^{-1}$.

System (26) may be transformed to standard first-order matrix form by defining two new variables for $\dot{I}$ and $\ddot{I}$. The transformed system is 4x4 so the solution must extract four roots rather than two. Two of these roots are explosive and two are stable. Some of the formalities are sketched in Appendix B and are summarized here:

(i) The complete solution has both forward- and backward-looking parts on the forcing data (x, $\dot{x}$, y, $\dot{y}$, etc.), analogous to (14). Now each part has two exponential weighting functions rather than one, but the two explosive roots look forward and the stable roots look backward. Hence this model preserves the building-ahead-of-anticipated-demand feature of the simpler model of section III. It is clear from the basic economics that incentives for building ahead of demand are greater when there are internal adjustment cost ($C_{22} > 0$) than when these cost are not present.

(ii) In distinction to the simpler model of section III, where the roots are necessarily real, it is possible for the roots of (26) to have imaginary parts. In this case both the forward and backward weight functions in the complete solution exhibit damped sinusoidal patterns, and the model generates <u>endogenous</u> damped cyclicality to noncyclical impulses. This contrasts with the simpler rising supply price model in section III, where cyclicality occurs only if the forcing variables are themselves cyclical.[4] A necessary condition for endogenous cyclicality is that the demand for housing services be sufficiently price inelastic. Then small changes in housing stock provoke large changes in rentals and in housing prices, and may lead to some high frequency overshooting of responses to nonsinusoidal supply and demand shocks.

(iii) If the roots of (26) are real, internal adjustment costs added to rising supply price considerations tend to reduce the responsiveness of investment to supply and demand shocks, and to smooth and spread out the responses over time. If demand is so inelastic that the roots are complex, the smoothed and spread-out responses at lower frequencies remain, but additional endogeous variability of response is added at higher frequencies.

V. ESTIMATION: SUPPLY

The model is estimated with quarterly time-series data on investment in new single family homes in the United States (see Appendix C for data sources). We focus here on the structure of supply decisions. The data cover the twenty-one year period 1963I-1983IV. The empirical form of the supply side of the model that makes no distinction between short and long run supply responses, is

$$I_t = \alpha + \beta P_t + \gamma y_t + v_t \,, \tag{27}$$

where I_t denotes new single family housing units started during quarter t, P_t is the (real) hedonic price index for 1977 quality homes, and y_t is a vector of observable variables that shift marginal cost. Unobserved cost shifters account for v_t, and these are assumed to be orthogonal to observable supply and the demand shifters, y_t and x_t, unless otherwise specified. Summary statistics for variables entering (27) are reported in the last row in table 1.

Alternative formations of (27) are shown in table 1. With some differences in detail and specification the estimates in table 1 are similar to those reported by Poterba [1984] and serve to corroborate his results.

We report two sets of specifications. The first ignores any autoregressive structure in the residuals, while the second allows the residuals to follow an AR(2) process. The method of estimation is instrumental variables using current and lagged exogenous variables as instruments due to the endogeneity of P_t. Thus the estimators are consistent under alternative serial correlation structures to those reported here.[5] The data are not seasonally adjusted; instead we have included seasonal dummies in the regression (not shown), plus another shifter for the severe winter of 1979, to control for the impact of weather in the new housing production function. The real price variable in table 2 includes the real value of the site as well as the structure, though similar results are obtained when only the structure price is used. It should also be noted that the first-stage instrumenting equation for P_t displays an R^2 of about .95 in all cases. This "overfitting" of prices, common in time series, means that the point estimates in the first half of table 2 differ little from least squares.

The impact of house prices on new construction is of primary interest. Ignoring adjustment costs other than rising supply price, we find strong supply respones to a change in the price of new homes. Evaluating the point estimates at sample means, the implied supply elasticity ranges between 1.4 and 2.2. Furthermore, the estimated price effects are not sensitive to the specification of the error process. These relatively large magnitudes appear to be consistent with the volatile nature of investment in this sector as compared to the much smoother price series (illustrated in section II).

The estimated response of investment to changes in interest rates is surprising in terms of the framework of the model.[6] In all models that we have estimated, we find a strong response of housing starts to changes in both the real rate of interest and expected inflation, and the hypothesis that the nominal rate of interest affects supply decisions cannot be rejected. The reported specifications include both the ex ante real rate of interest and the implied expected three month rate of price inflation. In these and in subsequent models, the two components of the nominal rate have very similar negative effects on supply decisions. To fix ideas on magnitudes, the estimates in row 5 of the table imply that a one-point increase in the annual real rate of interest reduces new construction by about 8.0 percent, with a similar effect for a point increase in the rate of inflation. When the model is extended to include both current and lagged effects of these variables, both have statistically significant (and nearly identical) negative effects on current supply.

The effects of interest rates on housing construction has often been discussed in the literature (e.g., Muth [1960]), but we remain surprised by this finding because most of these effects should be embodied in house prices in an ideal housing market. There are coherent reasons why changes in the nominal rate of interest may shift the demand for housing; for example the structure of fixed rate mortgages imlies that higher nominal interest rates increase current real interest payments on mortgage loans (Kearl [1979]). These demand-side effects should reduce investment by causing prices to fall. They should have no direct effects in and of themselves, yet our result is that supply is shifted by the nominal rate, price held constant. One interpretation of this is that there are measurement problems in the price series.[7] Another is that the lag structure of the model (27) is too simple. A third is that the observed price series is not market clearing: fluctuations in the nominal rate signal changes in the ability to sell new homes at the current price, and hence they are added dimensions of the real price facing builders.

This last interpretation is supported by the finding that time-to-sale affects construction activity. The variable denoted "Months" in table 1 is the median time on the market for new single family houses that are for sale in month t. It is included to control for the amount of time that builders hold completed new housing units before they are sold. It therefore reflects a real cost (or a reduction in price). Again, our finding is that price alone is not sufficient for the supply decision: new investment decreases when builders have to hold the existing stock longer.[8] Though our estimates of supply price elasticities are not very sensitive to the inclusion of this variable, these results suggest that the pure auction model of trade in homogeneous units of capital is not completely accurate. Since incentives to inventory units of housing must be minor at best, the data suggest that builders cannot liquidate units instantaneously at the observed market price. Search by buyers among heterogeneous units may play a role in explaining this finding, and remains to be investigated.

We experimented with other cost shifters in the supply function, including the Boeckh index of construction input costs, the manufacturing wage, and the average wage of construction workers. None had important effects. To illustrate, row (4) of the table reports estimates that control for the hourly wage of construction workers. After instrumenting to account for rising supply price of labor to the industry, we find no significant evidence that fluctuations in wages were important exogenous cost shifters during the period of our data. Rather, wage fluctuations are endogenously determined from shifts in the derived demand for labor.

Rows (5) and (6) of table 1 report variants of the rising supply price model when the errors follow an AR(2) process. The main results are not much different than in the unrestricted case, though substantial serial correlation in the residuals is found. This may be a characteristic of unobserved cost shifters, or it may indicate that the model is misspecified by the omission of internal adjustment costs.

The discrete-time analogue of (19) is

$$I_t = \beta_0 + \beta_1 I_{t-1} + a\beta_1 E_t I_{t+1} + \beta_2 P_t + \beta_3 y_t + v_t \tag{28}$$

where E_t is the expectation operator given current (period t) information, <u>a</u> is a discount factor, and the coefficient β_1 is non-negative and increasing in the degree of interval adjustment costs (C_{22} in the notation of section IV). Of course $\beta_2 > 0$ and $\beta_3 < 0$. The appearance of I_{t-1} and EI_{t+1} in (28) adds some econometric complications. We continue to assume that some elements of the vector of cost shifters, y, are unobserved, which accounts for the error term in (28). Note that the expectation $E_t I_{t+1}$ is unobserved and, as before, P_t is endogenous. To write (28) in an estimatable form, exploit the hypothesis of rational expectations and replace the unobserved expectation with its realization, I_{t+1}:

$$I_t = \beta_0 + \beta_1 I_{t-1} + a\beta_1 I_{t+1} + \beta_2 P_t + \beta_3 y_t + v_t - a\beta_1 \varepsilon_{t+1} \tag{29}$$

where $\varepsilon_{t+1} = I_{t+1} - E_t I_{t+1}$ is orthogonal to the information at t under rational expectations. By construction I_{t+1} and the composite error term are correlated, and I_{t+1} is not exogenous. To proceed, we assume that $E(x_{t-j} v_t) = E(y_{t-j} v_t) = 0$ at all lags j, so that lagged supply and demand shifters are valid instruments for I_{t+1} and P_t. Note that if v_t follows some arbitrary time series process then I_{t-1}, I_{t+1} and P_{t-1} are also correlated with the error, so that in general these supply and demand shifters are the only valid instruments. Thus under our assumptions, instrumental variables applied to (29) produces consistent estimates of the parameters of interest.

Denoting the composite errors in (29) as $\eta_t = v_t - a\beta_1\varepsilon_{t+1}$, the error covariance at lag one is

$$E(\eta_t\eta_{t-1}) = E(v_tv_{t-1}) - a\beta_1E(v_t\varepsilon_t) \quad . \tag{30}$$

Innovations to marginal cost shifters in v_t are components of the forecast error ε_t so that the expectation (30) is generally non-zero. This is related to a point made by Hansen [1982]. Since $E(v_t\varepsilon_t)$ is positive, the errors in equation (29) will be negatively correlated at lag one even if v_t is white noise. If, in addition, the errors v_t are correlated, this negative correlation in η_t may persist at higher lags. For example, if v_t is AR(1) with parameter μ, then

$$E(\eta_t\eta_{t-j}) = \mu^j\sigma_{vv} - \mu^{j-1}\sigma_{v\varepsilon} = \mu^{j-1}E(\eta_t\eta_{t-1}) \ . \tag{31}$$

We allow the errors in (29) to follow the process characterized by (31).[9] Denote the associated error covariance matrix by Ω. Denote the matrix of instruments by Z and the right hand variables in (29) by Y. Then the asymptotic covariance matrix of the instrumental variables estimator applied to (29) is

$$V(\hat{\beta}) = [Y'Z(Z'Z)^{-1}Z'Y]^{-1}Y'Z(Z'Z)^{-1}Z'\Omega Z(Z'Z)^{-1}Z'Y[Y'Z(Z'Z)^{-1}Z'Y]^{-1} \tag{32}$$

Applying (31) in the case of first-order correlation in v_t, a consistent estimate of μ is obtained from

$$\hat{\mu} = \frac{\Sigma\hat{\eta}_t\hat{\eta}_{t-2}}{\Sigma\hat{\eta}_t\hat{\eta}_{t-1}} \tag{33}$$

which allows construction of a consistent estimate of the covariance matrix Ω and hence construction of a consistent estimate of $V(\hat{\beta})$.

If the errors v_t are truly AR(1), then it might seem appropriate to quasi-difference the data using the autoregressive parameter μ. In this case the model becomes

$$I_t = (1 + \mu a\beta_1)^{-1}[\beta_0(1 - \mu) + (\mu + \beta_1)I_{t-1} - \mu\beta_1 I_{t-2} + a\beta_1 I_{t+1} + \beta_2(P_t - \mu P_{t-1}) + \beta_3(y_t - \mu y_{t-1}) + \eta_t - \mu\eta_{t-1}], \tag{34}$$

$$\eta_t - \mu\eta_{t-1} = u_t - a\beta_1\varepsilon_{t+1} + \mu a\beta_1\varepsilon_t \tag{35}$$

where u_t is a white noise process. Equation (34) can be estimated by instrumenting the relevant variables and imposing the nonlinear restrictions that are implied across the parameters. In this case, μ is estimated directly with the other parameters of the model. However, the appearance of the forecast error ε_t in (35) means that current demand and supply shifters, y_t and x_t, can no longer be treated as exogenous: these variables are in the information set at t, and innovations to these series are components of

ε_t. Thus y_t in (34) must also be instrumented and current demand shifters must be dropped from the instrument list. On the other hand, lags of investment and price are valid instruments under this assumption, so some tradeoff in efficiency is involved. For this reason, we report estimates of the parameters of (29) in both differenced (equation (34)) and nondifferenced form.[10]

Table 2 reports the estimates for the adjustment cost model. Rows (1) – (4) are based on equation (29), using only lagged supply and demand shifters as instruments for investment and price. No specific time series process for v_t is assumed for these point estimates, though in calculating the asymptotic standard errors from (32) the parameter μ from (33) is used. In all specifications of the model, the error covariance at lag one is negative, as is clearly possible from (30) or (31) if the covariance between v and ε is sufficiently large. This does not imply that the "true" errors, v_t, are negatively serially correlated: the estimated autoregressive parameter for v_t, based on (33), is always positive. Positive autocorrelation is plausible if v_t represents unobserved cost shifters, as assumed. When the quasi-differenced form of the model is estimated (rows (5) and (6)) and μ is estimated directly, the autoregressive parameter is slightly smaller, though still positive.

The main result in table 2 is that the simple time-invariant rising supply price model of table 1 is rejected: the estimated internal adjustment cost parameter is numerically large and always more than triple its estimated standard error.[11] We estimate slightly smaller adjustment costs in the quasi-differenced form of the model, but the fundamental finding of the importance of these costs is not affected. The estimated effect of the current asset price on current investment is correspondingly smaller, indicating differences in the response of current investment to permanent and transitory changes in price, as well as differences between short and long run production adjustments. The estimated tradeoff between adjustment costs and the response of investment to a change in the asset price is somewhat sensitive to the model's specification: the model in row (1) of the table shows large adjustment costs and only a small current investment response to price changes, but when lagged interest rates and median time on the market are included in the model the immediate response of investment increases substantially. Excluding row (1), the long run effects of price on investment are not much different among the models in table 2.

To illustrate these effects along the lines of section IV, consider the one-sided solution to the stochastic difference equation (29):

$$(36) \qquad I_t = \frac{\beta_0\kappa}{a\beta_1} + \frac{\kappa}{a} I_{t-1} + \frac{\beta_2\kappa}{a\beta_1} \underset{t}{E} \sum_{i=0}^{\infty} \kappa^i P_{t+i} + \frac{\beta_3\kappa}{a\beta_1} \underset{t}{E} \sum_{i=0}^{\infty} \kappa^i y_{t+i}$$

where κ is the forward stable root of the characteristic polynomial of (29), given by

$$\kappa = \frac{1}{2\beta_1} \{1 - \sqrt{1 - 4a\beta_1^{\,2}}\} \quad .$$

Note that as β_1 rises to 1/2, κ approaches unity, so the effect of lagged investment is increasing in β_1. Applying (36), the <u>current</u> impact of an unanticipated unit pulse in price that is thereafter expected to last exactly T periods is

$$\left.\frac{dI_t}{dP_t}\right|_T = \frac{\beta_2\kappa}{a\beta_1} \sum_{i=0}^{T} \kappa^i = \frac{\beta_2}{a\beta_1}\frac{\kappa}{1-\kappa}(1-\kappa^T) , \tag{37}$$

which is increasing in T. Note that even for T = 1, the estimated response exceeds the estimated coefficient on price reported in the table because the experiment in (36) allows <u>future</u> levels of planned investment to adjust optimally.

As in section IV, the time path of investment in response to a permanent change in P allows comparison between short and long run supply response. Straightforward calculations show this time path to be

$$\frac{dI_{t+j}}{dP} = \frac{\beta_2}{a\beta_1}\frac{\kappa}{1-\kappa}\frac{1}{1-\kappa/a}(1-\frac{\kappa}{a})^{j+1} \tag{38}$$

which is increasing in j. The change in long-run equilibrium levels of investment is obtained by letting $j \rightarrow \infty$

$$\frac{dI^*}{dP} = \frac{\beta_2}{a\beta_1}\frac{\kappa}{1-\kappa}\frac{1}{1-\kappa/a}.$$

Table 3 reports supply responses for models (1), (2), and (6) in table 2. For ease of interpretation, the effects are expressed as elasticities evaluated at sample means. For example, the first entry of 0.72 corresponds to equation (37) with $T = 1$, $\beta_1 = .496$, and $\beta_2 = 805.76$. For these parameters the impact of adjustment costs on supply decisons is large ($\kappa = .82$), so adjustments are spread over a long period of time (see panel B). The current response to a permanent price increase has an elasticity of 4.0, and the long run supply elasticity is nearly 24.0. These estimates are very large because no allowance is made for time-to-sale effects and β_1 is apparently overestimated in that specification. The other two models produce smaller though still important effects of adjustment costs. In these models, a permanent price increase has about a 50 percent more impact on current investment than a one-period price shock. However, almost all of this difference is accounted for by a relatively short disturbance lasting one year. In our judgement the best estimate of the long run elasticity of supply is for model (2), which produces an elasticity of 2.76. For comparison, the rising supply price model in table 1 yielded an elasticity of 2.08 for both long and short run price changes.

VI. DEMAND

A large portion of our time and effort on this project has been devoted to the demand side of the market, represented by the first equation in (6), after making adjustments for income tax features of home ownership in the rent-stock price equation (3) -- see appendix C.[12] Per capita income, unemployment rates, the rate of family formation, and fuel prices (a proxy for operating costs) serve as observable demand shifters (x(t)). We do not subscribe to a labor theory of value, so this section will be brief. All attempts to estimate this equation have been unsuccessful. In retrospect this failure is perhaps not surprising because of two serious data problems.

Estimating the demand equation in (6) requires constructing a time-series for the stock K_t using the perpetual inventory method. Available benchmarks are subject to substantial error. Furthermore, data on renovation and maintenance components of investment of existing homes are not of the comparable quality as the new investment series. Measured investment represents such a small fraction of existing stock that the imputed stock series is practically indistinguishable from a pure trend over the sample period. There is simply not enough intertemporal variation in the constructed stock series to provide usable information for estimating the slope of a demand curve.

This difficulty is conceptually circumvented by taking quasi-differences and estimating the discrete analog of (26b). Then only directly measured I_t, not K_t, appears in the equation. But this solution introduces another problem that is equally serious. Prices appear in second difference form in (26b), and differencing compounds measurement errors and data timing problems (see footnote 7) in the price series. It reduces the signal-to-noise ratio to remarkably small proportions, especially given the extensive statistical manipulations required to deal with simultaneity and serial correlation induced from differencing. The resulting estimates of (26b) are wildly unstable and unacceptably sensitive to minor changes in specification and data period. They are worthless. Major research effort must be made to generate more meaningful data before this can be taken as serious evidence against the basic framework of the model.

Figure 4 shows a constructed time series for the ex post implicit rental price R_t imputed from the tax adjusted equivalent of (3) using the new house price series in section V. The scaling of R_t is in index form because P_t is a price index. The noise in this series is to be expected from the importance of capital gains in the formula, which varies from quarter to quarter. A more disturbing aspect of figure 4 from the point of view of theory is the low average level of implicit rent during the 1974-79 period, when the relative house prices increased dramatically. Feldstein [1982] has plausibly argued that inflation increases the subsidy to home ownership due to the personal income tax. This makes housing a more attractive investment and causes housing prices to rise. Figure 4 shows that the argument needs more detail. If the demand for housing services remained unchanged, the first order effect of the greater subsidy is capitalized in house prices. Rents would remain unchanged at given stocks. There is a second order effect on rents because the increase in prices signals more construction: rents fall as the stock rises. However, this decline should be small because new construction adds only small increments to the stock and the price elasticity for housing services is not inelastic (it is thought to be around unity). Figure 4 shows that rents fell substantially during 1974-79 and were actually zero or negative in many quarters.

The series shown is ex-post, not ex-ante anticipated real rent. Therefore it is possible that the ex ante series would show a much shallower decline, if the market on average underpredicted capital gains over this period (due to underprediction of general price inflation; recall that ex post real interest rates were negative in this period). Our attempts to estimate ex ante rent using instrumental variables methods produced identical results as in figure 4, due to the overfitting-of-prices mentioned above. We simply have no way of testing whether expected gains calculations approximate "true" expectation over this volatile period.

There are other possibilities. First, the asset pricing equation is built on the premise of certainty equivalence. Rosen, Rosen and Holz-Eakin (1984) have argued that the increase in relative house prices was accompanied by increased uncertainty. This is no doubt true, but it would call for an increased risk premium in the mortgage interest rate.[13] There is no persuasive evidence that real mortgage rates rose over the period or that mortgage credit was seriously rationed. Since much of this risk would be borne by creditors, it remains to be explained why banks and mortgage institutions would be willing to eat so much of it without commensurate compensation. Second, the decline in R_t may reflect excessive speculation based on differential inflationary expectations. House financing charges looked very cheap (even negative) to those persons anticipating very high rates of inflation. Gains from trade would exist if others had the opposite expectations. There is evidence that transactions and roll over of the stock of existing homes increased during this period. Finally, the big drop in R_t could reflect a decline in demand for services of single family homes. The productivity slowdown, increased fuel prices, declining rates of family formation, increasing marital instability and declining fertility rates all work in the direction of reduced demand. But these changes could not produce negative rentals and were not exclusively confined to 1974-79. They cannot fully account for the pattern in figure 4.

VII. SUMMARY AND CONCLUSIONS

This work has explored the connection between prices and investment in new single family dwelling units in the U.S. during the past 20 years. Informal evidence that investment is positively correlated with factor prices and construction costs and also with housing prices supports a view of investment based on rising supply price in a competitive construction sector. A formal model of this idea was spelled out and shown to be related to adjustment cost and q-theory. Its main predictions are consistent with what is generally known about this market, that investment in new housing increases when the demand for new housing services increases or when the marginal cost of construction declines. Housing construction falls off when interest rates rise. There are more subtle implications about timing and current response to anticipated future chanegs in demand and supply as well. The most important practical implication of this simple model is that all these events are filtered through a decentralized market mechanism in which asset prices play their familiar resource allocating role. This mechanism is socially efficient if expectations are rational because asset prices serve as sufficient statistics embodying all available information. This model implies a direct and important behavioral supply relationship linking current asset prices to current investment that is easy to estimate.

This simplicity, however, comes at a considerable cost because it rests on the key assumption that there are no differences between short- and long-run elasticities of supply. Short run immobility of factors tends to provoke such differences. These are approximated by a model in which internal adjustment costs are superimposed on increasing marginal cost. This model simulates one in which long-run housing supply is more elastic than short-run supply. Furthermore, the response to transitory changes in demand and supply shifters and interest rates is smaller than the response to permanent changes. The sufficiency of current price data for supply decisions is lost in this enriched model. Builders must anticipate future prices in making current construction decisions when supply is not time-invariant. Nevertheless, a conceptually straightforward, though computationally complex procedure for estimating the supply function is available in this case.

The empirical findings strongly support the view that housing prices influence the level of construction activity. The estimates definitely favor a model in which short-run supply is less elastic than long-run supply, but they also show that the differences between short- and long-run supply converge rapidly, within the time frame of a year. There are good economic reasons for this. Labor is not highly specific to the house construction industry and huge seasonal and cyclical fluctuations in construction activity promote a type of adaptability and built-in flexibility in industrial organization that enables resources to flow in and out rapidly in quick response to changing conditions. We expect less adaptability for other investment goods industries in the economy and therefore greater differences between short- and long-run supply than are found in the home builiding industry. Since q-theory is essentially a version of the model in which supply is time invariant (so only "current q" matters), we expect an improvement in the fit of these models from the considerations raised in this work.

Inadequacies of data have not allowed meaningful estimates of the demand side of this market. However, our empirical investigation has uncovered some apparent anomolies with asset pricing theory that will have to be incorporated into a more complete model. First, nominal interest rates affect supply decisions. Many investigators have attributed this to credit market imperfections, and it is certainly true that the interest rate measures used here do not include elements of availability and rationing of credit. Even so, most of these factors should be reflected in house prices and work themselves out in that indirect way rather than directly. A related finding is that the time required to sell a finished house exerts an important direct effect on housing investment. Incomplete data we have uncovered on total house sales suggests that the level of construction activity is highly correlated with transactions volume and overall sale activity among existing homes. Asset pricing theory of homogeneous goods makes no reference whatsoever to the volume of transactions. Whatever the technology of supply all available current information should be embodied in current house prices: the volume of overall transactions activity or time-to-sale should have no independent effects. We conjecture that analysis of heterogeneity of dwelling units and search and matching considerations between buyers and sellers will be necessary to fully understand these findings. However that works out, convincing evidence has been presented that the supply of new housing has substantial price elasticity. Surely elastic supply price is an important consideration for understanding the great variability in housing construction activity.

REFERENCES

Abel, Andrew B., "Empirical Investment Equations: An Integrative Framework," in K. Brunner and Alan Meltzer (eds.), Carnegie-Rochester Conference Series on Public Policy XII (1980), pp. 39-91

________, "Dynamic Effects of Permanent and Temporary Tax Policies in a q Model of Investment," Journal of Monetary Economics 9 (1982): 353-73.

Barro, Robert J. and Chaipat Sahasakul, "Measuring the Average Marginal Tax Rate from the Individual Income Tax," University of Rochester, 1983.

Benhabib, Jess and K. Nishimura, "On the Uniqueness of Steady States in an Economy with Heterogeneous Capital Goods," International Economic Review 20 (1979), 59-82.

Benveniste, L.M. and J.A. Scheinkman, "On the Differentiability of the Value Function in Dynamic Models of Economics," Econometrica 47 (1979), 727-32.

Eisner, Robert and Robert H. Strotz, "Determinants of Business Investment," in Commission on Money and Credit, Impacts of Monetary Policy, Englewood Cliffs, N.J.: Prentice-Hall, 1963.

Engle, Robert F. and Duncan K. Foley, "On Asset Price Model of Aggregate Investment," International Economic Review 16 (1975), 625-47.

Fama, Eugene F. and Michael R. Gibbons, "Inflation, Real Returns and Capital Investment," Journal of Monetary Economics 9 (1982), 297-323.

Feldstein, Martin, "Inflation, Tax Rules and the Accumulation of Residential and Nonresidential Capital," Scandanavian Journal of Economics 84 (1982), 293-311.

Foley, Duncan K. and Miguil Sidrauski, Monetary and Fiscal Policy in a Growing Economy, New York: Macmillan, 1971.

Friedman, Milton, Price Theory: A Provisional Text. Chicago: Aldine Publishing Company, 1963.

Gould, John P., "Adjustment Costs in the Theory of Investment of the Firm," Review of Economic Studies 35 (1968), 1029-54.

Hansen, Lars Peter, "Large Sample Properties of Generalized Method of Moment Estimators," Econometrica 50 (1982), 1029-54.

Hansen, Lars Peter and T.J. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations," Journal of Economic Dynamics and Control 2 (1980), 7-46.

Hayashi, Fumio, "Tobin's Marginal and Average q: A Neoclassical Interpretation," Econometrica 50 (1982), 213-24.

Hendershott, Patric H. and Sheng Cheng Hu, "The Allocation of Capital Between Residential and Nonresidential Uses," National Bureau of Economic Research, 1981.

Judd, Kenneth L., "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model," Journal of Political Economy 93 (1985), 298-319.

Kearl, James, "Inflation, Mortgages and Housing," Journal of Political Economy 87 (1979), 1115-38.

Lucas, Robert E., Jr., "Optimal Investment Policy and the Flexible Accelerator," International Economic Review 8 (1967), 78-85.

________, "Optimal Investment with Rational Expectations," in R.E. Lucas, Jr. and T.J. Sargent (eds.), Rational Expectations and Econometric Practice, Vol. I, Minneapolis: University of Minnesota Press, 1981.

Mussa, Michael, "External and Internal Adjustment Costs and the Theory of Aggregate and Firm Investment," Economica 44 (1977), 163-78.

Muth, Richard, "The Demand for Nonfarm Housing," in A.C. Harberger (ed.), The Demand for Durable Goods, Chicago: University of Chicago Press, 1960.

Poterba, James M. "Tax Subsidies to Owner-Occupied Housing: An Asset Market Approach," Quarterly Journal of Economics 99 (1984), 729-52.

Rosen, Harvey S., Kenneth T. Rosen and Douglas Holtz-Eakin, "Housing Tenure, Uncertainty and Taxation," Review of Economics and Statistics 66 (1984), 405-16.

Sargent, Thomas J., Macroeconomic Theory, New York: Academic Press, 1979.

Sheffrin, Steven M. Rational Expectations, Cambridge: Cambridge University Press, 1983.

Summers, Lawrence H., "Taxation and Corporate Investment: A q-Theory Approach," Brookings Papers on Economic Activity, 1981, 67-127.

Tobin, James, "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit and Banking 1 (1969), 15-29.

Topel, Robert and Michael Ward, "Job Mobility and the Careers of Young Men," Graduate School of Business, University of Chicago, 1985.

Topel, Robert and Kevin M. Murphy, "Estimation and Inference in Two-Step Econometric Models," Journal of Business and Economic Statistics (1985).

Treadway, Arthur B., "On Rational Entrepreneurial Behavior and the Demand for Investment," Review of Economic Studies 36 (1969), 227-39.

Walras, Leon, Elements of Economics

Witte, James G., "Micro-foundations of the Social Investment Function," Journal of Political Economy 71 (1963), 441-56.

Zellner, Arnold and Franz Palm, "Time Series Analysis and Simultaneous Equation Econometric Models," Journal of Econometrics 2 (1974), 17-54.

FOOTNOTES

*UCLA and University of Chicago, respectively. We are indebted to Lars Hansen, Kevin M. Murphy and Michael Mussa for helpful discussions and to the National Science Foundation for research support. Andy Sparks and David Ross provided excellent research assistance.

[1]Large seasonal variations in construction activity mask longer term fluctuations. Annual data are shown to highlight the major cyclical movements, though quarterly data are used in the empirical work. Construction activity in the summer is twice as large as in the winter, so seasonals rival cyclical components in amplitude. Since this industry is geared to moving substantial resources in and out from quarter-to-quarter, we expect this built-in flexibility to carry over to longer term fluctuations and sustain a relatively large elasticity of supply. This intuition is confirmed in the estimates.

[2]These statements require symmetry in costs between construction and demolition. Analysis is more complicated if investments are irreversible or if stock reductions involve a different cost structure than stock additions. These refinements are ignored because they are well known and because negative gross investment is not observed in the aggregate data.

[3]The method described does not cover the case of a variable interest rate because then one of the coefficients in (6) is time-varying. However, the pasting algorithm covers this case so long as the time path of $r(t)$ can be approximated by step functions. For example, the reader is invited to verify that the qualitative solution for an anticipated positive pulse in $r(t)$ is very similar to the experiment described in figure 3, but with opposite signs. If the pulse is not anticipated (a "surprise"), the first unstable arrow does not apply -- the system jumps immediately to the second unstable arrow. In either case, changes in interest rates have the usual predictable consequences for construction activity and asset prices in this model: increasing interest rates reduce investment and decrease prices.

[4]It is well known that complex roots and endogenous cyclicality do not arise in most economic problems where there is a single state variable. The model of section III is a primary example. Higher order dynamics must be built into the system or there must be several state variables to produce complex roots. The current problem is an example of the former, since costs may be thought of as a function of $\dot{K}$ and $\ddot{K}$. Benhabib and Nishimura [1979] present an interesting recent example of the latter.

[5]The instruments are listed at the bottom of the table. We chose not to include lagged endogenous variables in the instrument list due to the possibility of more complicated error structures than the one we estimate.

[6]The real rate used is the one-step-ahead forecast from an estimated AR(2) regression in the first differences of the real rate, following a method of Fama and Gibbons [1982]. Since r_t is estimated, standard errors must be corrected. The methods derived by Topel and Murphy [1985] are used for this.

[7]There may be selection problems in these data because they are constructed from actual transactions. For example, if there are no sales in a particular location in some quarter, that location gets no weight in the price index. We do not view this as a serious problem for ascretaining the overall connection between construction and price movements, because it approximates the appropriate "marginal" concept. However, it may interfere with making more detailed inferences concerning timing and lags. Also, approximately 25 percent of units are built on contract and the rest for the market at large. This introduces some noise, for our purposes, in linking starts with prices on a quarter-to-quarter basis. Experiments with one-quarter leads and lags of prices and with two-quarter price averages revealed that the estimates of supply parameters are insensitive to these refinements.

[8]We have considered the case where Months is endogenous. The results when this variable is instrumented differ trivally from those reported here.

[9]This assumption affects only the calculations of standard errors in equation (32) with our estimation procedure. Point estimates are independent of assumptions about the error process for v_t.

[10]Higher order AR process for v_t were also estimated, but the results were not materially different from those reported in table 2.

[11]In estimating β_1 the coefficients on I_{t-1} and $E_t I_{t+1}$ are restricted to differ only by the discount factor $\underline{a}$, which was set at .98. This restriction was not rejected in any form of the model at any conventional significance level. The estimates are insensitive to the choice of $\underline{a}$ in this neighborhood, that is, $\underline{a}$ is not precisely estimated.

[12]The stochastic version of (3) is in the form of the efficient-market-hypothesis in finance. R_t plays the same role as the dividend yield on a stock. Asset prices do not necessarily follow a martingale (random walk) unless the dividend return is properly accounted. Single family housing is largely owner-occupied so R_t is not observed but is a regression function of K_t and x_t, by (1). Thus housing prices will not follow a random walk unless K_t and x_t do (which is implausible in this model). In fact the price series for new houses is well described by an AR(2) process over the entire sample period.

[13]For dramatic illustration, suppose agents expect the market to crash at some random time in the future, and that if it does crash, housing will lose all value thereafter. $f(t)$ is the probability that the market will crash at t and $1 - F(t)$ is the probability it will survive longer than t, with $f(t) = F'(t)$. Then the probability of death at t given survival up to t is $f(t)/(1-F(t)) = h(t)$. It follows that $1 - F(t) = \exp -\int_0^t h(\tau)d\tau$. If a house survives exactly t periods an owner will receive value $\int_0^t R(s)e^{-rs}ds$. This occurs with probability $f(t)$, so the expected value of a house under these circumstances is

$$\int_0^\infty f(t)\int_0^t R(s)e^{-rs}dsdt = \int_0^\infty (1 - F(t))R(t)e^{-rt}dt = \int_0^\infty R(t)\exp(-rt - \int_0^t h(\tau)d\tau)dt.$$

The possibilities of a crash increase the effect rate of discount and require an extra risk premium. If $h(\tau) = h$ for all t the connection between stock and flow prices becomes $R = P(r + \delta + h - \dot{P}/P)$.

APPENDIX A

Compare the text model (1) - (5) with the conditions necessary to maximize social surplus in the presence of adjustment costs. Consider a central planner choosing to maximize the social surplus of housing services over an infinite horizon and facing a social adjustment cost technology $C(I)$, with positive and increasing marginal cost: $C', C'' > 0$. Let $F(K)$ be the inverse demand function for housing services at any time. Then the total flow valuation of housing stock level K is the area under the demand curve or $\int_0^{K(t)} F(z)dz = V(K(t))$.

Following Lucas [1981] an altruistic social planner chooses a net investment sequence $\dot{K}(t)$ to maximize the present value of net social surplus,

$$\int_0^{\infty} [\int_0^{K(t)} F(z)dz - C(\dot{K}(t) + \delta K(t))]e^{-rt}dt \tag{A.1}$$

subject to the boundary condition $K(0) = K_0$. One necessary condition for maximization of (A.1) is the Euler condition (recalling $I = \dot{K} + \delta K$)

$$F(K(t)) - (\delta + r)C'(I(t)) = \frac{d}{dt} C'(I_t) \quad . \tag{A.2}$$

The other is a transversality condition: $\lim_{t\to\infty} C'(I)e^{-rt} = 0$, which provides a boundary condition for solving (A.2). The solution is

$$C'(I(t)) = \int_t^{\infty} F(K(s))e^{-(r+\delta)(s-t)}ds \tag{A.3}$$

Now $F(K(t))$ is nothing more than the demand price for the services of the stock at t, or $R(t)$, so the right hand side of (A.3) is the marginal social value of a unit of stock. The left hand side is its marginal social cost. Thus (A.3) implies that if intertemporal prices are consistent in the sense of condition (3), then competitive market production decisions that equate marginal construction costs to the price of a house are surplus maximizing. In a competitive market, (A.2) is equivalent to (3), and (A.3) is equivalent to (5). Equation (2) -- the market supply function -- reflects the price equals marginal cost conditions since the integral in (A.3) is $P(t)$. Finally equation (1) is just the definition of $F(K)$. The decentralized market solution duplicates the surplus maximization problem, so long as the market can enforce boundary condition (4). If we assume a set of rational speculators, then the transversality condition must be satisfied to keep their wealth bounded, though we have little to add to the extensive literature on this particular point (e.g., see Scheinkman).

Early applications of adjustment cost theory proceeded by approximating the Euler equation and deriving flexible accelerator formulations of investment, though with inadequate treatment of expectations. Another approach estimates the complete solution to (A.2) based on quadratic approximations to $F(K)$ and $C(I)$. Neither method exploits information on the market value of a unit of capital. A third method, related to q-theory, is based on an envelope theorem type result. Write the maximized value of the expression in (A.1) as $V(K_0)$. Then $V(K_0)$ is the market value of a "firm" with capital stock K_0 and the gradient of this function gives the shadow price of an additional unit of capital (Mussa [1977]; Ben Veniste and Scheinkman [1979]). This provides a basis for estimating the "demand price" for a unit of capital from stock and bond market data relating to "average-q" (e.g., Summers [1981]). The virtue of the housing market data at our disposal is that it gives a direct reading of $V'(K_0)$, which is simply the price of a house and which is directly observed in the data. No fancy manipulations are needed to estimate the structural equation (2).

APPENDIX B

We sketch the nature of the solution to (26). Define the operator $\Delta = rD - D^2$. Writing (26) in matrix form and inverting shows that the pair of second order equations can be written as two fourth order equations, one for I(t) and the other for P(t). The one for I(t) is

$$\text{(B.1)} \qquad [(1 + B\Delta)(1 + \beta\Delta) - \frac{\alpha B\beta}{C_{22}}]I(t) = f(t).$$

where

$$\text{(B.2)} \qquad f(t) = \frac{\beta}{C_{22}}[B(\dot{x} + \delta x) - (1 + B\Delta)(C_1 + rC_2 + C_{13}y)].$$

Factoring the polynomial in Δ in (B.1) as $(\Delta - \rho_1)(\Delta - \rho_2)$, the solution for investment is of the form

$$\text{(B.3)} \qquad I(t) = [(\Delta - \rho_1)(\Delta - \rho_2)]^{-1}f(t),$$

with characteristic roots

$$\text{(B.4)} \qquad \rho = \frac{-(B+\beta) \pm \sqrt{[(B+\beta)^2 - 4B\beta(1 - \frac{\alpha B\beta}{C_{22}})]}}{2B\beta}$$

Unlike section III, these roots may be either real or complex. In either case, corresponding to each root ρ_i there are roots $(\lambda_{i1}, \lambda_{i2})$ that solve the polynomial $D^2 - rD + \rho_i = 0$, and which therefore satisfy $\lambda_{i2} = r - \lambda_{i1}$. Thus, of the four roots two, say λ_{11} and λ_{21}, are positive and larger than r (or have positive real parts), and two are negative, say λ_{12} and λ_{22} (or have negative real parts). If the ρ-roots are complex, then the λ-roots must be also.

Using the identity $\Delta = rD - D^2$, some algebra establishes the operator in (B.3) is as

$$[(\Delta - \rho_1)(\Delta - \rho_2)]^{-1} = \frac{1}{\rho_1 - \rho_2}\Big\{\frac{1}{\lambda_{11} - \lambda_{12}}[(D - \lambda_{11})^{-1} - (D - \lambda_{12})^{-1}] - \frac{1}{\lambda_{21} - \lambda_{22}}[(D - \lambda_{21})^{-1} - (D - \lambda_{22})^{-1}]\Big\}$$

where the operators $(D - \lambda)^{-1}$ were defined in section III. Applying these formulas and the transversality condition that investments have finite present value, the complete solution for current investment is

$$\text{(B.5)}\quad I(t) = \frac{-1}{(\rho_1 - \rho_2)(\lambda_{11} - \lambda_{12})}\int_t^\infty f(s)e^{-\lambda_{11}(s-t)}ds + \frac{1}{(\rho_1 - \rho_2)(\lambda_{21} - \lambda_{22})}\int_t^\infty f(s)e^{-\lambda_{21}(s-t)}ds$$

$$+ A_1 e^{\lambda_{12}t} - \frac{1}{(\rho_1 - \rho_2)(\lambda_{11} - \lambda_{12})}\int_0^t f(s)e^{-\lambda_{12}(s-t)}ds$$

$$+ A_2 e^{\lambda_{22}t} + \frac{1}{(\rho_1 - \rho_2)(\lambda_{21} - \lambda_{22})}\int_0^t f(s)e^{-\lambda_{22}(s-t)}ds$$

where A_1 and A_2 are constants that can be determined by specifying the initial conditions $I(0)$ and $\dot{I}(0)$. Since λ_{12} and λ_{22} have negative real parts, these terms vanish for large t.

The solution (B.5) is of interest for studying investment dynamics because it is not possible to use phase-plane methods. The definition of f(t) in (B.2) implies that increases in the demand shifters increase the forcing function while increasing cost shifters reduce it. Examining (B.5), the weight on f(s) for arbitrary s is, when the roots are real,

$$\text{(B.6)} \qquad \frac{1}{\rho_1-\rho_2}\left[\frac{e^{-\lambda_{21}(s-t)}}{2\lambda_{21}-r} - \frac{e^{-\lambda_{11}(s-t)}}{2\lambda_{11}-r}\right] \quad \text{for} \quad s > t$$

$$\frac{1}{\rho_1-\rho_2}\left[\frac{e^{-\lambda_{22}(s-t)}}{2\lambda_{21}-r} - \frac{e^{-\lambda_{12}(s-t)}}{2\lambda_{11}-r}\right] \quad \text{for} \quad s < t$$

making use of the fact that $\lambda_{12} = r - \lambda_{11}$. To evaluate these effects, let ρ_1 be the large root. Then $\rho_i = \lambda_{i1}(r - \lambda_{i1})$ implies, for real roots, that $r < \lambda_{21} < \lambda_{11}$, so the weight on the forward looking part in the first expression in (B.6) is positive. The building-ahead-of-anticipated-future-demand feature of the simpler model is preserved here. A similar argument for the negative roots establishes that λ_{12} is smaller than λ_{22}. Since the (s-t) term is negative in the backward looking second expression in (B.6), the overall expression is negative. The second term in that expression is larger than the first. For example, events that increased demand in the past tend to make investment negative at the present time, reflecting the natural response to the previous building that occurred to meet those prior increases in demand. All-in-all the distributed lag and lead responses to anticipated demand and supply shifters are qualitatively similar to those of section III. However, the responses to a similar type of demand shift are not as large due to the added adjustment cost. Internal adjustment costs on top of rising supply price dampen the output responses to shifts in demand.

These effects are less obvious when the roots are complex, say $\rho_1 = a + bi$ and $\rho_2 = a - bi$. In this case the four λ-roots of the characteristic polynomial in (B.1) occur in complex conjugate pairs, so that $\lambda_{11} = \lambda_0 + \gamma i$, $\lambda_{21} = \lambda_0 - \gamma i$ where $\gamma = b(r - 2\lambda_0)^{-1}$ and $\gamma < 0$. Using the identity $e^{ix} = \cos x + i\sin x$, the lead and lag effects in (B.5) are

$$\text{(B.7)}\quad \begin{aligned} &\frac{e^{-\lambda_0(s-t)}}{\gamma(2\lambda_0-r)[(2\lambda_0-r)^2+4\gamma^2]}\{2\gamma\cos\gamma(s-t) + (2\lambda_0 - r)\sin\gamma(s-t)\}, \quad s \geq t \\ &\frac{e^{-(r+\lambda_0)(s-t)}}{\gamma(2\lambda_0-r)[(2\lambda_0-r)^2+4\gamma^2]}\{2\gamma\cos\gamma(t-s) + (2\lambda_0 - r)\sin\gamma(t-s)\}, \quad s \leq t \end{aligned}$$

$\gamma < 0$ implies that these effects on period t investment are always positive when s is sufficiently close to t, and their magnitude is declining in s - t, as above. Curiously however, these effects are <u>negative</u> for some values of s. The weight functions have wiggles which drop off in a damped sinusoidal manner. At certain leads and lags increases in the flow demand for capital may <u>reduce</u> current investment. The response to a predicted or past positive demand disturbance can produce damped oscillations in investment in which production actually falls below its long-run equilibrium level in some periods. Inspection of the definitions of the roots shows that oscillations in investment can occur only when demand is sufficiently inelastic.

APPENDIX C

Time series data used in the empirical work were obtained from the following sources:

1. New Single Family Housing Prices: The price data are obtained from a survey conducted by the Bureau of the Census since 1963. The data are for new single family homes actually sold during the reference period. Series are available that both include and exclude the estimated site value. The index itself is obtained from a hedonic regression of actual price data on a vector of house characteristics in each year, and the index used refers to predictions from this procedure using characteristics of a standard 1977-quality house. Source: U.S. Bureau of the Census, New One-Family Houses Sold and for Sale, Construction Reports c25.

2. Investment: Housing starts are new one-unit structures on which construction was started during the reference period. Source: U.S. Bureau of the Census, Construction Reports, series c20.

3. Interest Rates: The nominal rate of interest at time t for the supply function is the three month treasury bill rate quoted on the last day of period t-1. These are taken from the quote sheets of Salomon Brothers, and are available in machine readable form from the Center for Research in Securities Prices at the University of Chicago. The decomposition of the nominal rate into a real and expected inflation components is described in footnote 6. Mortgage interest rates for first mortgage loans on single family homes are published by the Federal Home Loan Bank Board. The series used refers to the effective interest rate on twenty-five-year maturity loans with a loan to price ratio of 25 percent.

4. Months: Median time (in months) on the market for new single family housing units that are sold during the reference period. Source: unpublished data obtained from the Bureau of the Census.

5. Boeckh Cost Index for small residential structures: A weighted average of construction input prices, with weights depending on survey responses. Source: U.S. Department of Commerce, Bureau of Industrial Economics, Construction Review.

6. Lumber Price Index: Source: U.S. Bureau of Labor Statistics, Producer Prices and Price Indexes.

7. Personal Consumption Expenditures. Source: U.S. Bureau of Economic Analysis, The National Income and Product Accounts of the U.S.

8. Hourly wage rates for construction and manufacturing: U.S. Department of Labor, Bureau of Labor Statistics, Employment and Earnings, various issues.

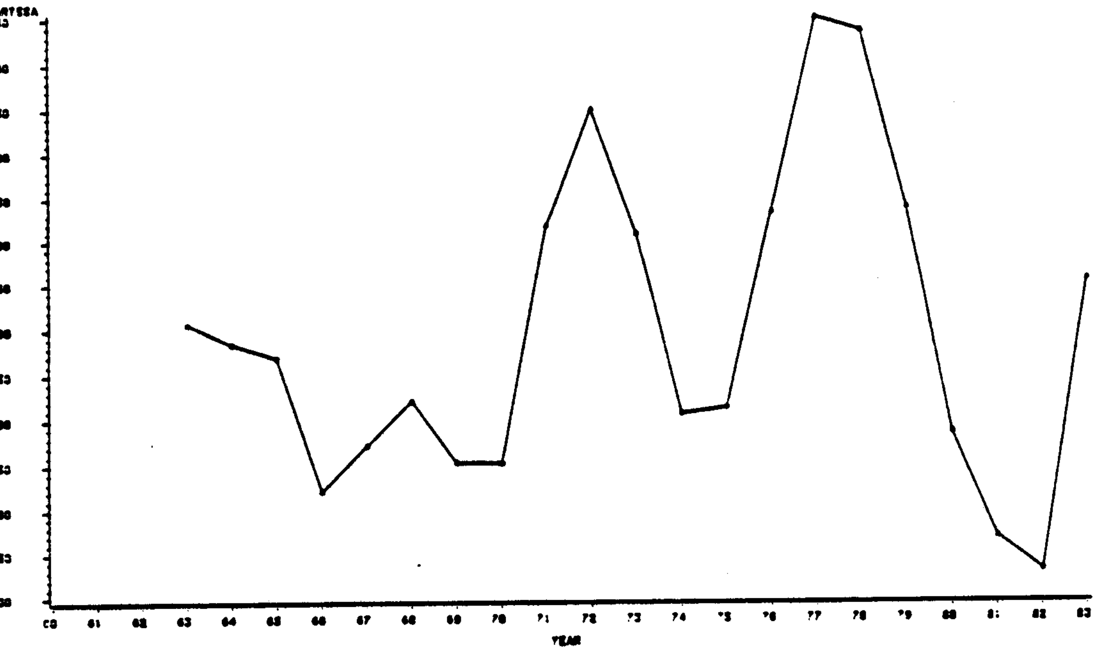
9. Families: The number of married couple family households. Source: U.S. Bureau of the Census, Current Population Reports, series p-20.

10. Fuel Price Index: Source: U.S. Department of Labor, Bureau of Labor Statistics, Monthly Labor Review.

11. Real Implicit Rental Price: Define the income-tax-adjusted real interest rate as $r_t = (1 - \tau_t)i_t - \pi_t$, where i_t is the nominal mortgage interest rate, τ_t is the marginal income tax rate, and π_t is the rate of inflation. Familiar manipulations of the present value formula yield the anticipated real rent as the expected present value of a round-trip buy and sell transaction over one quarter (ignoring transactions costs): $R_t = P_t - E_t P_{t+1}(1-\delta)/(1+r_t)$, where P_t is the real asset price described above and δ is the quarterly depreciation rate, calculated at .0035 per quarter from a perpetual inventory method. This expression ignores

maintenance expenditures and property taxes, and assumes full mortgage finance and no taxation of capital gains (see Hendershott and Hu [1981] for a discussion of those refinements). The ex post numbers shown in figure 4 replace the expectation with realized values, and assume that capital gains are taxed at rate τ_t. Assuming no taxation of capital gains yields a series with the same general appearance but with more pronounced fluctuations and a much larger drop in rent during 1974-79. Several alternative estimates of τ_t were tried. One is Barro and Sahasakul's [1983] estimates of the average marginal tax rate; the other is the estimated tax bracket that makes tax-free municipal bonds a marginally profitable investment. τ_t is set at 0.3 in figure 4. The time-series character of the R_t series is insensitive to these differences in taxes.

HOUSING STARTS



DEMAND VARIABLES

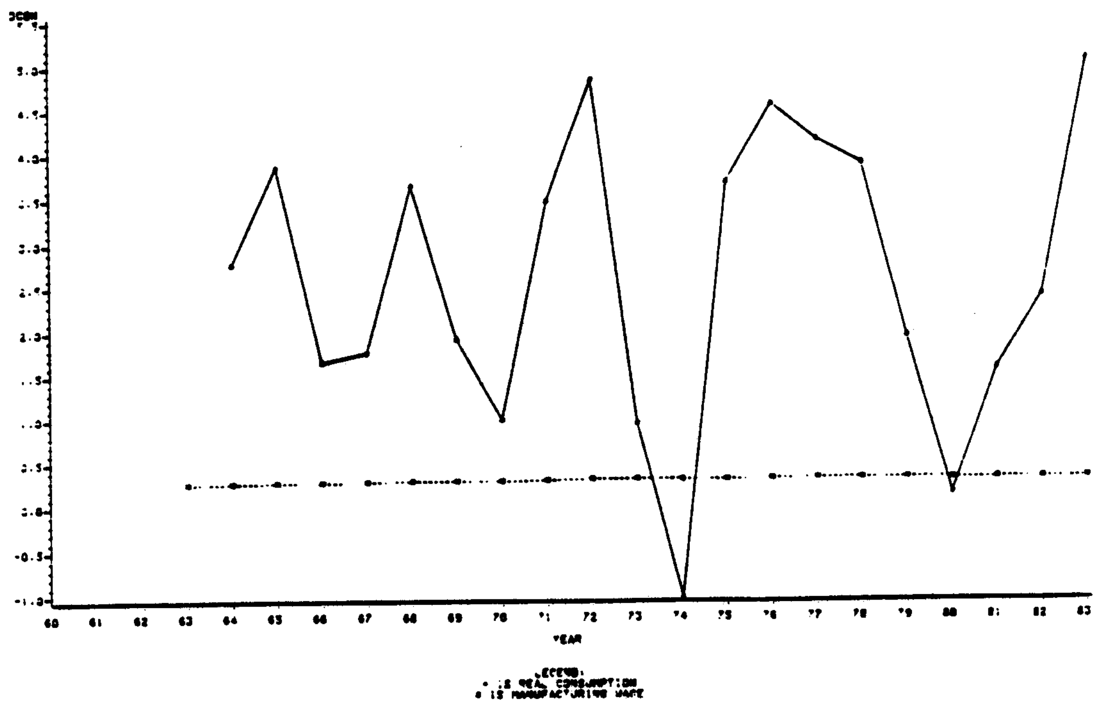
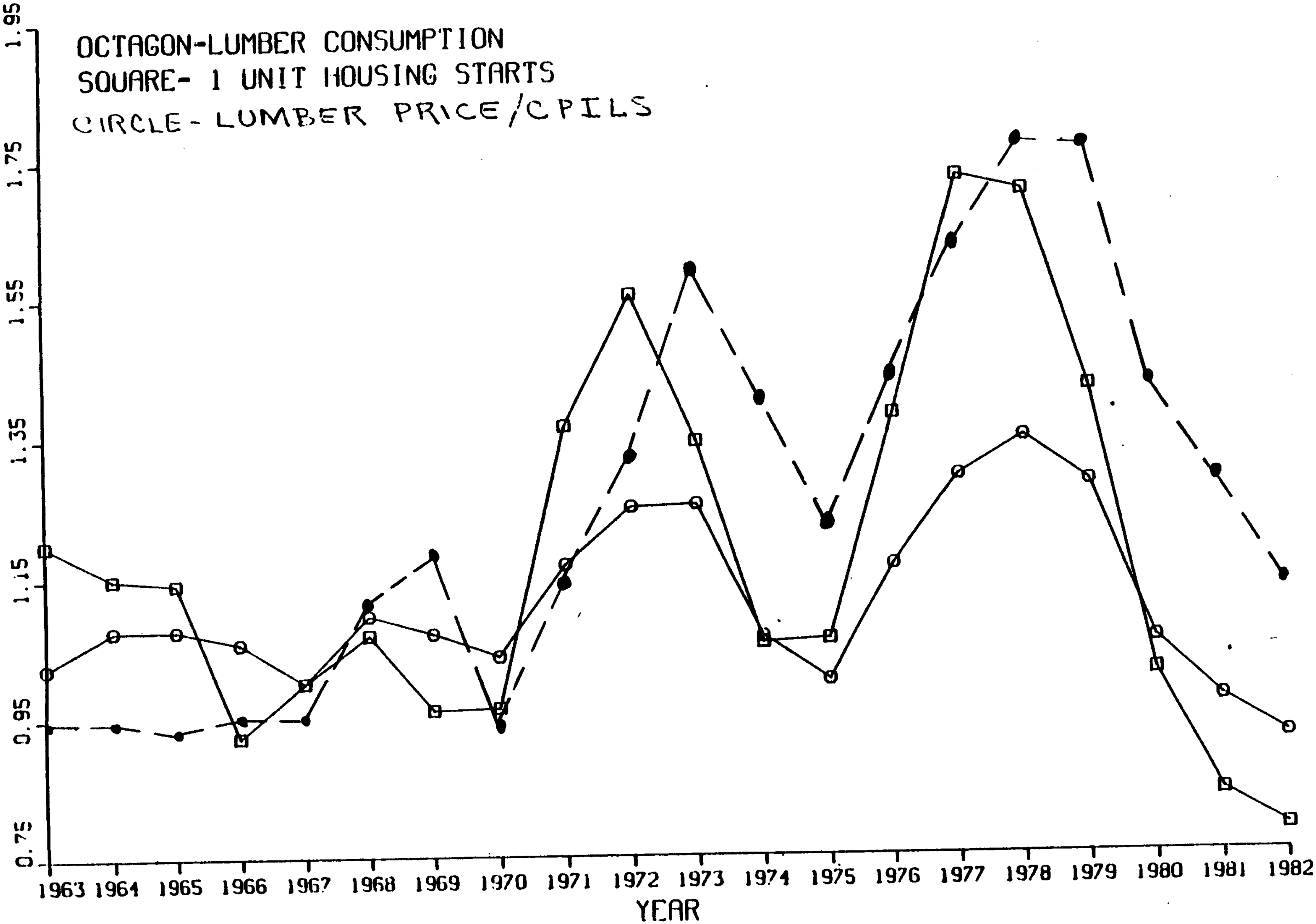


Chart 1

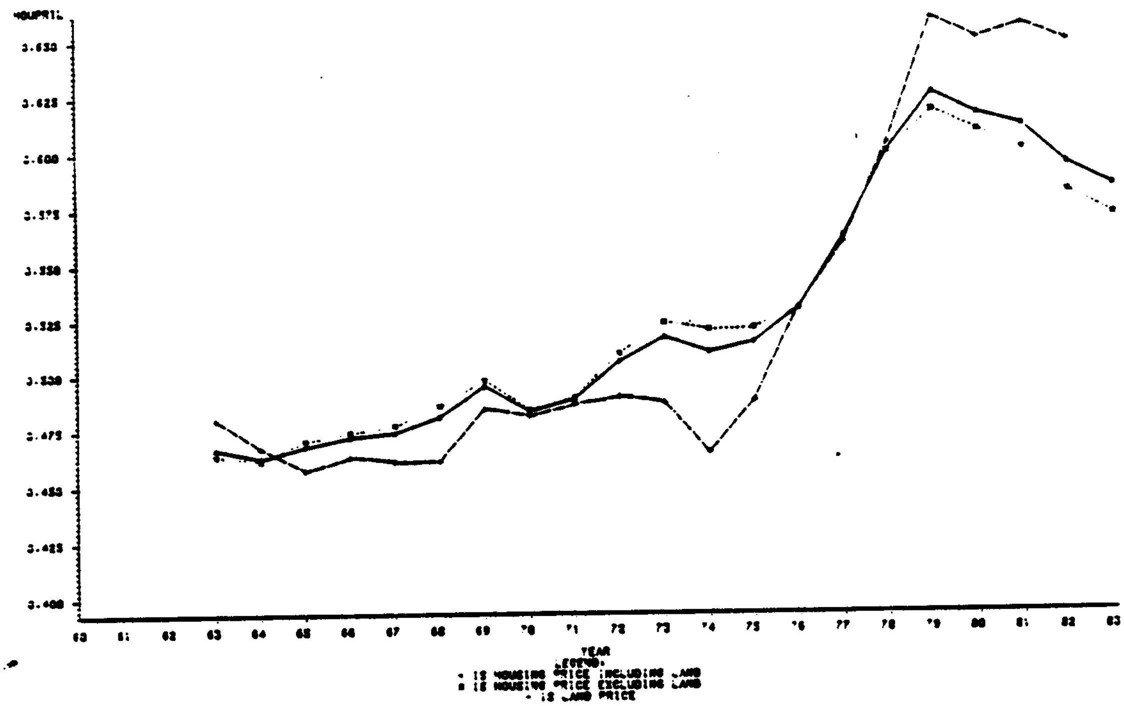
ANNUAL STARTS AND LUMBER CONSUMP., 1967=100



SOURCE: BLS

Chart 2

HOUSING PRICE INDICES



COST SERIES

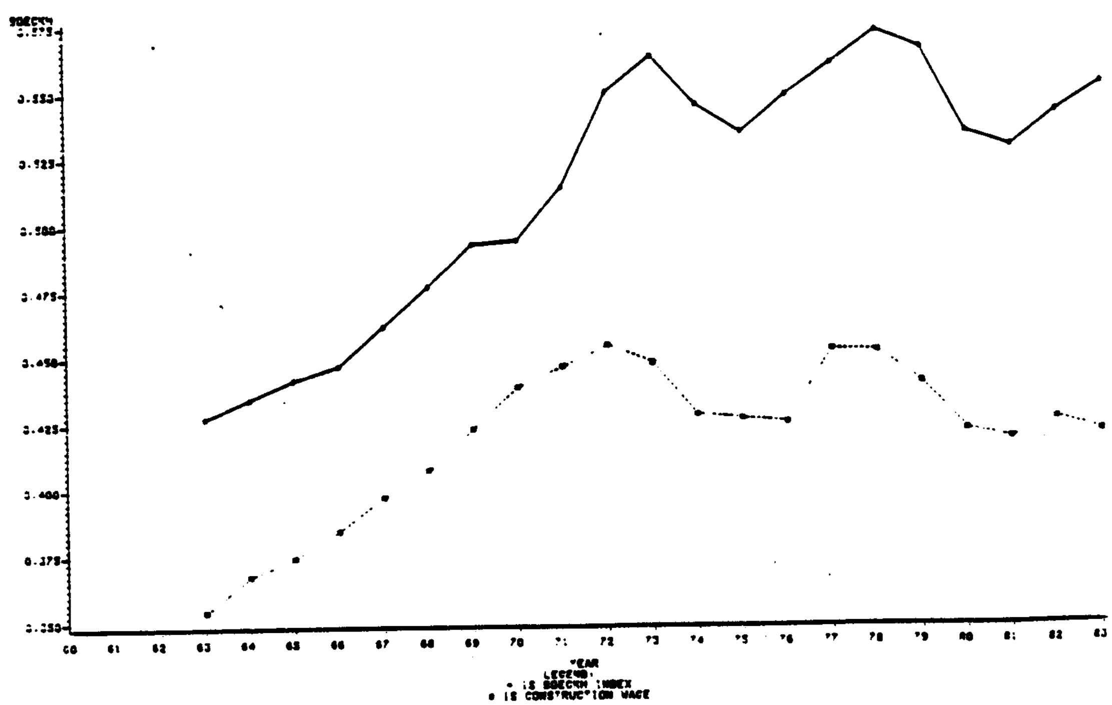


Chart 3

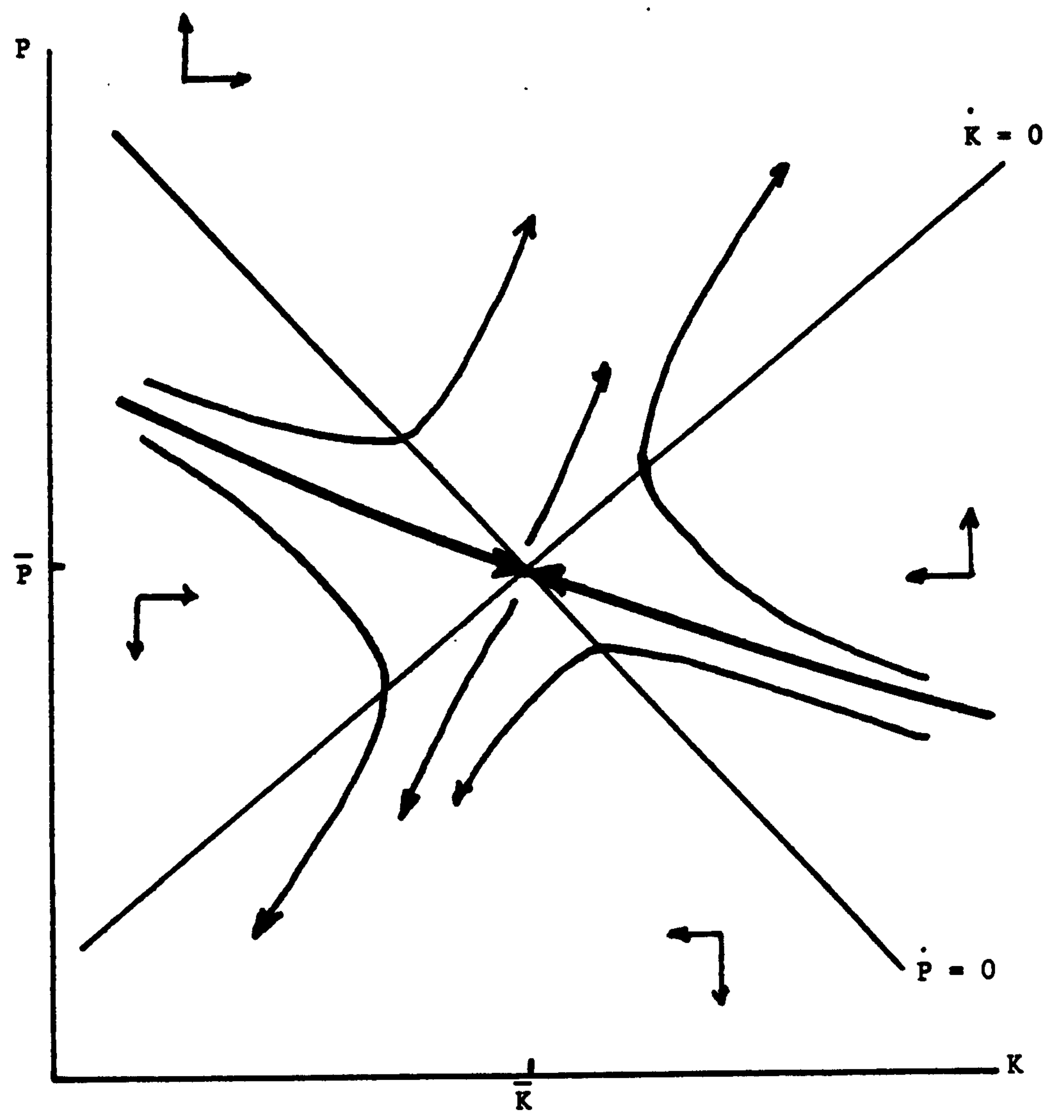


FIGURE 1

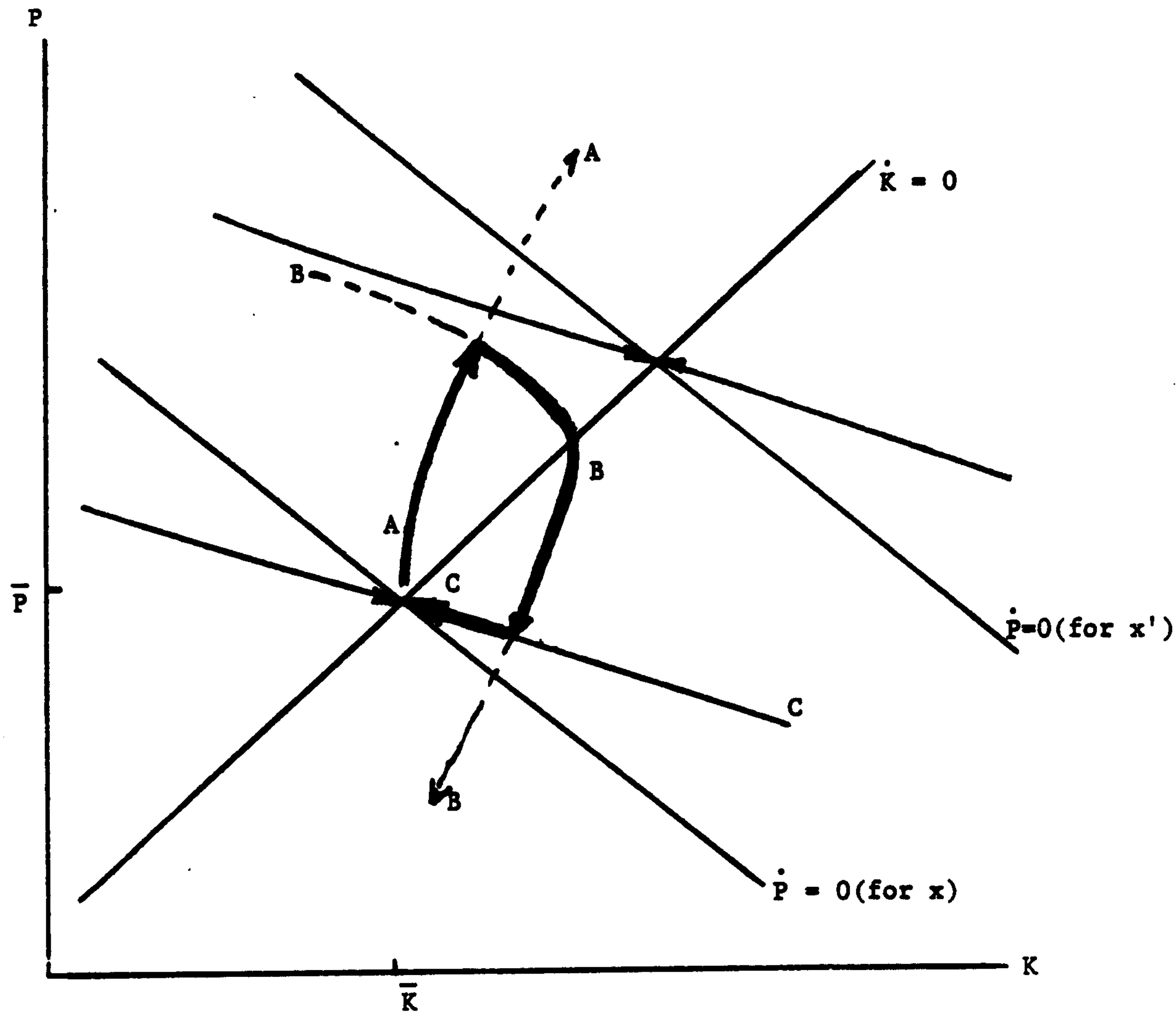


FIGURE 2

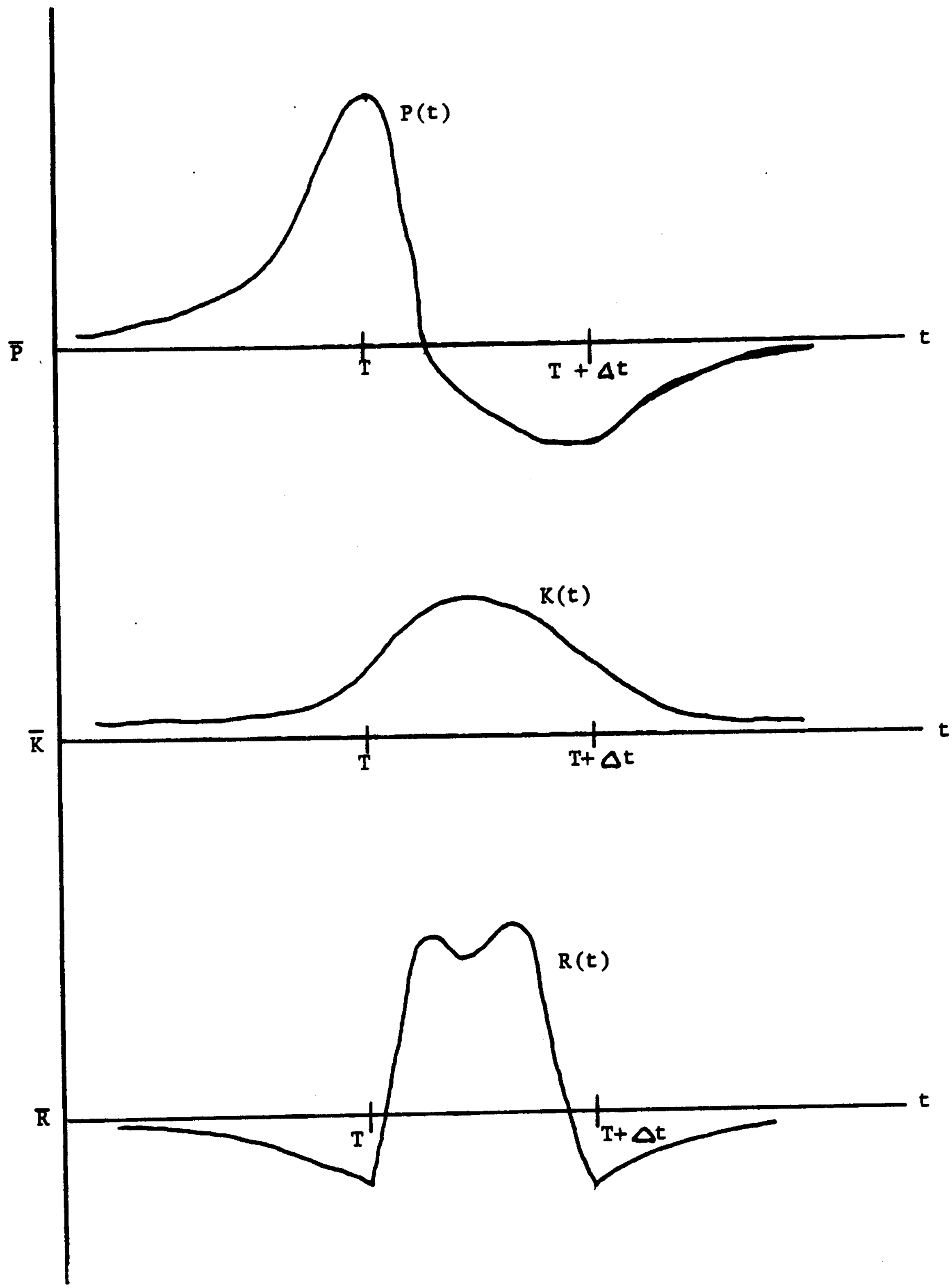
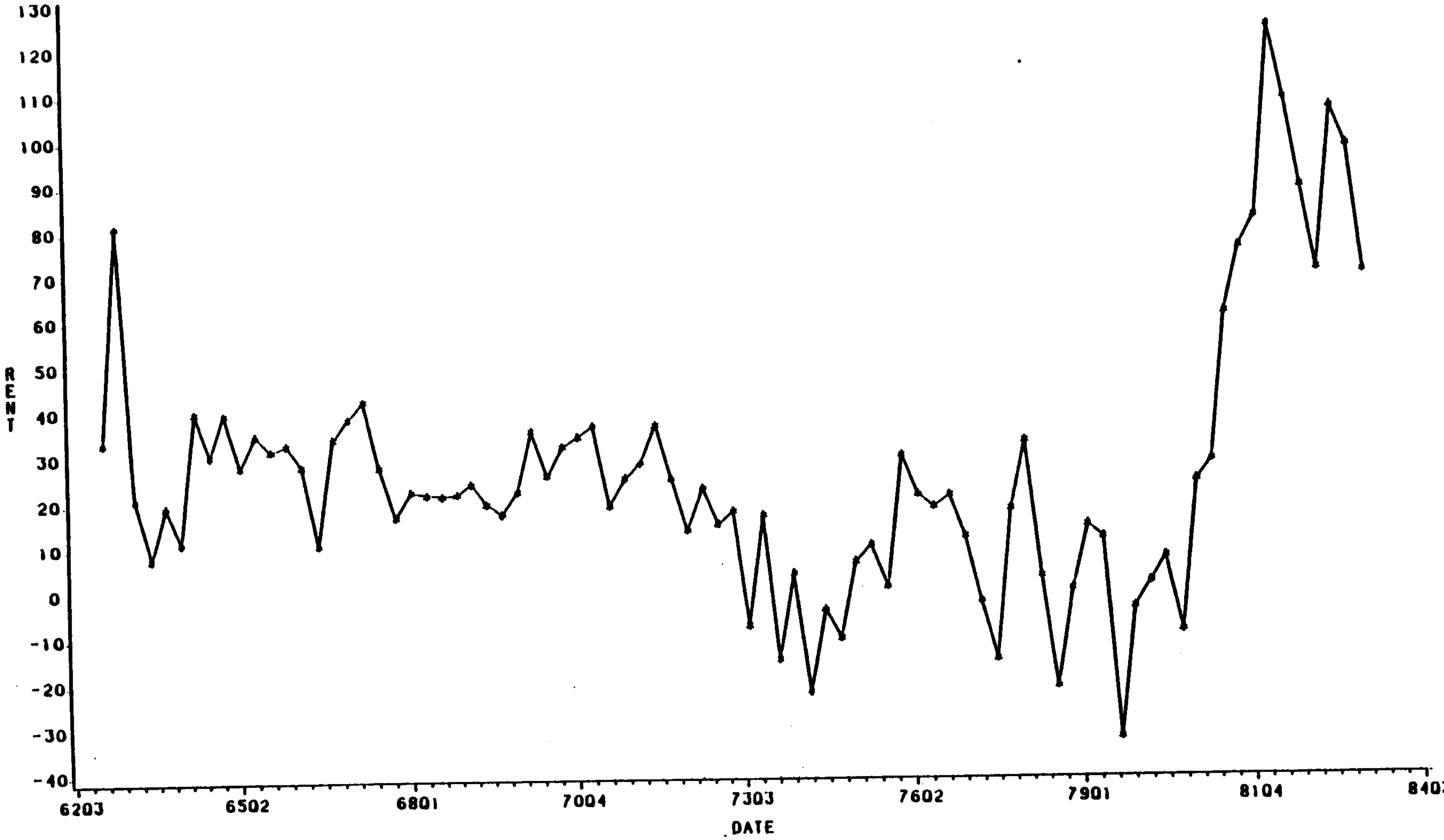


FIGURE 3



Rent includes capital gain component

FIGURE 4

Table 1

Two Stage Least Squares Estimates of Housing Investment:
Rising Supply Price Model, 1963I-1984IV
(dependent variable is quarterly single family housing starts)

	P_t	$E_{t-1}r_t$	$E_{t-1}\pi_t$	$E_{t-2}r_{t-1}$	$E_{t-2}\pi_{t-1}$	$Months_t$	$Wage_t$	Trend	1979.I	Intercept	AR(1)	AR(2)	R^2	SE
1.	4110.64 (984.83)	-452.86 (46.36)	-370.93 (42.26)	-	-	-	-	1.54 (1.97)	-73.87 (168.29)	-809.31 (414.82)	-	-	.76	154.5
2.	3925.87 (648.95)	-273.20 (35.65)	-240.52 (30.84)	-	-	-304.21 (31.07)	-	1.20 (1.31)	-135.88 (112.12)	-365.17 (277.47)	-	-	.89	102.0
3.	3709.95 (615.83)	-142.24 (62.49)	-115.03 (47.56)	-177.28 (68.32)	-177.34 (53.97)	-236.73 (35.73)	-	2.41 (1.30)	-130.50 (109.12)	-340.74 (262.79)	-	-	.91	96.3
4.	4049.33 (660.02)	-266.72 (36.35)	-242.83 (30.82)	-	-	-291.15 (34.10)	59.74 (66.20)	0.66 (1.43)	-142.28 (108.16)	-667.17 (430.04)	-	-	.90	101.5
5.	2589.62 (1177.54)	-313.62 (53.12)	-244.21 (44.48)	-	-	-	-	1.67 (2.60)	-215.98 (92.14)	-222.77 (506.30)	.69 (0.11)	-.16 (0.11)	.91	109.1
6.	3693.73 (920.85)	-131.06 (44.04)	-120.38 (32.72)	-133.05 (46.87)	-136.82 (37.52)	-261.59 (35.70)	-	1.31 (2.13)	-207.37 (62.89)	-302.21 (399.36)	.58 (0.11)	.13 (0.11)	.93	69.87
Mean (s.d.)	.527 (.055)	0.20 (0.66)	1.50 (0.90)	0.20 (0.70)	1.50 (1.00)	1.67 (0.49)	2.83 (0.13)	43.5 (23.90)	.012 (.110)	-	-	-	-	-

Note: Asymptotic standard errors in parentheses.

Variable definitions: P_t is the hedonic price index for new, 1977 quality single family homes sold in quarter t; $E_{t-1}r_t$ is the real quarterly rate of interest; $E_{t-1}\pi_t$ is the expected rate of inflation (see footnote 6); Months is the median time on the market since the beginning of construction for houses that are for sale in quarter t; Wage is the average hourly wage of construction workers; 1979-I is a dummy variable for the winter of 1979. All data are seasonally unadjusted.

Instruments: Variables used as instruments are current and lagged values of interest rates on 25-year term mortgages, aggregate real consumption expenditure (as a proxy for permanent income), an index of family formation, and an energy price index.

Table 2

Instrumental Variables Estimates of Housing Investment:
Adjustment Cost Model, 1963I-1983IV
(dependent variable is quarterly single family housing starts)

	P_t	I_{t-1}, $\rho E_{t-1}, I_{t+1}$	$E_{t-1}r_t$	$E_{t-1}\pi_t$	$E_{t-2}r_{t-1}$	$E_{t-2}\pi_{t-1}$	$Months_t$	$Wage_t$	Trend	1979.I	Inter-cept	Corr $(\eta_{t_1}\eta_{t-1})$	μ	S.E.
1.	805.76 (348.79)	.496 (0.04)	-35.78 (32.44)	-35.12 (27.87)	-	-	-	-	-0.69 (0.41)	-274.82 (83.02)	-580.71 (100.51)	-.29	.41	74.19
2.	1699.21 (510.25)	.339 (0.05)	-89.52 (35.75)	-85.85 (30.82)	-	-	-144.31 (32.68)	-	0.22 (0.79)	-230.94 (75.21)	-392.27 (163.98)	-.01	.45	68.31
3.	1945.41 (555.76)	.274 (0.06)	-50.16 (25.75)	-37.42 (36.75)	-95.05 (25.31)	-103.67 (39.93)	-131.03 (34.35)	-	1.07 (0.96)	-210.55 (74.00)	-367.07 (182.73)	-.36	.44	66.98
4.	1496.96 (491.28)	.356 (0.05)	-75.14 (32.20)	-72.37 (28.98)	-	-	-131.99 (28.86)	-5.1 (24.94)	0.20 (0.76)	-237.74 (74.41)	-364.72 (244.05)	-.15	.43	67.85
5.	1676.37 (606.65)	.300 (0.04)	-154.86 (43.85)	-143.05 (37.83)	-	-	-	-	0.25 (1.00)	-274.93 (84.56)	-657.14 (216.59)	-.20	.29	78.28
6.	2130.20 (589.47)	.209 (0.05)	-109.62 (66.05)	-100.25 (55.14)	-80.07 (67.36)	-79.66 (57.29)	-52.73 (37.50)	-	0.78 (0.97)	-232.41 (75.55)	-579.02 (194.16)	0.22	.34	69.45

Note: Asymptotic standard errors in parentheses.

Variable definitions: I_{t-1} and I_{t+1} denote starts from quarters t-1 and t+1 respectively. For definitions of other variables see notes to table 1.

Table 3

Estimated Supply Elasticities for
Permanent and Transitory Price Changes
(evaluated at sample means)

	A: Current Response to a Price Shock Lasting T Quarters				B: Response by Quarter T to a Permanent Price Increase			
	T=1	T=4	T=8	T=∞	T=1	T=4	T=8	T=∞
Model 1: λ=.82	0.72	2.18	3.15	3.94	3.94	12.27	18.22	23.83
Model 2: λ=.34	1.04	1.64	1.68	1.68	1.68	2.69	2.76	2.76
Model 3: λ=.22	1.18	1.51	1.51	1.51	1.52	1.93	1.93	1.93

Note: λ is the forward-stable root of the characteristic polynominal of Euler equation (5.4).