GENERAL EQUILIBRIUM ANALYSIS OF REGIONAL FISCAL INCIDENCE

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February 1982

Forthcoming in H.E. Scarf and J.B.Shoven (eds.)
Applied General Equilibrium Analysis
(University of Chicago Press, Chicago, 1982)

We are grateful to Glenn Graves, Charles McLure,Jr., Herbert Scarf, and participants of the NBER Conference on Applied General Equilibrium Analysis (San Diego, California 1981) for Comments. The National Science Foundation (Grant DAR-78-26309) has provided generous research support.

Working Paper #236
University of California,
Los Angeles

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Section 1

INTRODUCTION

Two important and influential themes in applied general equilibrium analysis were initiated by Harberger [1962] and Scarf [1967] [1973], respectively: the use of analytic approximations to problems of differential tax incidence, and the use of numerical simulation techniques to study arbitrary general equilibrium models. Our paper discusses the application of general equilibrium numerical simulation techniques to the analysis of the regional incidence of fiscal policy.

Our general objective is to present a framework of analysis that incorporates the major elements of: (i) the analytical approaches to tax incidence effects developed by Harberger [1962], Mieszkowski [1966] [1969] [1972], and McLure [1969] [1970] [1971]; (ii) the implementation of constructive numerical approaches to general equilibrium comparative statics by Scarf [1967] [1973] and Shoven and Whalley [1972]; and (iii) the use of inter-industry transactions data in Input-Output models by Leontief [1966] and many others. Our specific objective is to provide an operational counterpart to the contributions of Mieszkowski and McLure, in the sense that Shoven and Whalley [1972] and Shoven [1976] have provided a counterpart to Harberger's work.

The structure of our paper is as follows. In Section 2 we examine the basic general equilibrium framework underlying the recent literature, noting the more important and sensitive theoretical issues raised by that framework. Section 3 presents our proposed General Equilibrium Numerical Input-Output

(GENIO) framework for the analysis of these and other issues. A number of extensions to the earlier numerical simulation literature are noted. Finally in Section 4 we present several illustrative applications of our framework. The specific context for the work, and the regional partition used in the empirical application, concerns California and the Rest of the United States.

Section 2

THEORETICAL ISSUES

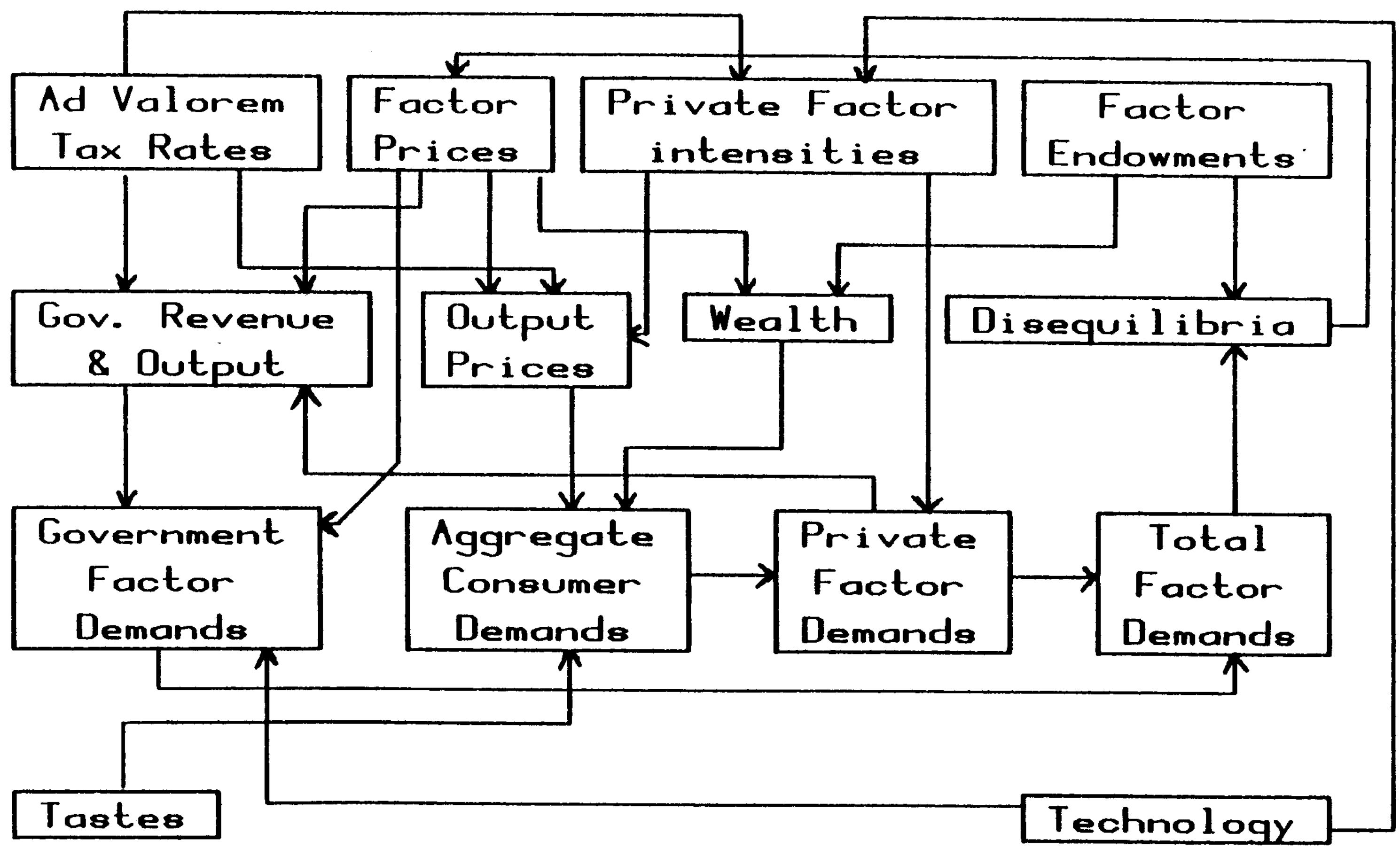
2.1 BASIC GENERAL EQUILIBRIUM INTERACTIONS

In this section we introduce the overall equilibrium structure of our approach to the analysis of fiscal incidence. It is pedagogically useful to proceed in the rough algorithmic sequence adopted in searching for a new general equilibrium. Figure 1 illustrates this sequence in a simple flow chart. We see in Section 3 that this particular "recursive" sequence of calculations has a number of computational advantages.

Assume that some imaginary Auctioneer or Middleman proposes a vector of factor prices, expressed in terms of labor units (labor is the "numeraire"). Assuming constant returns to scale production functions[1], cost-minimizing producers will choose to allocate factors in the same ratio for all levels of output: there is an efficient "recipe" for producing a loaf of bread, and the input proportions of that recipe do not vary with the number of loaves produced. The assumption of profit maximization leads to the equality of (long-run) marginal cost with marginal revenue; the assumption of "small" price-taking economic agents implies that marginal revenue is not a function of the output-level decision of any single entrepreneur. The further assumption of

[1] We presume that this production function is Leontief with respect to all inputs (intermediate and final), but that the primary factor input is a composite input that is produced with a Cobb-Douglas or C.E.S. specification. The efficient choice of primary inputs is therefore separable from the use of intermediate inputs. When we refer to Cobb-Douglas or C.E.S. production functions below it is assumed that we are referring just to the production of the composite primary input.

Basic General Equilibrium Interactions



free entry into any industry leads to the key "zero-profit condition" allowing us to translate the factor-prices and factor allocations prevailing into commodity prices. If we weight the amount of factor "i" used to produce a unit of output "j" (denoted F(i,j)) by the numeraire price of each factor "i", and sum over all factors "i", we have the marginal and average[2] cost of producing commodity "j".

It follows from our earlier presumption of profit-maximizing producers, who are free to commit or remove resources from any given industry, that this average cost of producing the good is also equal to the price that it must be offered at. Producers offering the good at a higher price would earn "pure profits", attracting more producers into the industry; their added production, in aggregate, causes an excess supply at the old output price leading to a reduction in output price in order to clear the output market. Well-informed buyers will not purchase from the buyer offering the higher price, forcing producers offering the higher prices to lower their prices. All of the above is "action" that is presumed to occur behind the scenes of our model -- we merely invoke the final equilibrium condition of zero-profits. More succinctly, we have the condition that

$$p(j) = F(1,j)*p(1) + F(2,j)*p(2) + \ldots\ldots + F(n,j)*p(n),$$

where we assume "n" factors in the economy. This condition holds for each output good "j".

Note again the sequence of the algorithm: given the n-dimensional vector of factor prices we derive the cost-minimizing allocation of factors in the production of each and every output good. These F(i,j) terms are typically the

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[2] The marginal is equal to the average cost because of the constant returns assumption. Note also that we ignore the use of intermediate inputs in this introductory discussion (with no essential loss in generality).

result of lengthy calculations, depending on the structure and form of the production function for good "j" that is assumed. What is most important is that they are not related to the amount of "j" being produced; we may therefore proceed in one "closed" algorithmic step from the prevailing factor prices to the implied relative commodity prices.

Households are assumed to be endowed with certain physical quantities of the various factors. We assume that these factors are inelastically supplied. Each household is assumed to determine their "spending-power" in terms of the numeraire by valuing these factors at prevailing factor prices. Households are assumed to allocate their spending-power so determined and decide which commodities they wish to purchase. For this "calculation", they need three pieces of information: their preferences (embodied in the specification of their utility functions), the numeraire value of their factor endowment (given from their physical endowments and the prevailing factor prices), and finally relative commodity prices (which are derived in the way discussed earlier). By aggregating the planned purchases of each household for each commodity, we arrive at a vector of final demands[3].

Market equilibrium in the commodity markets is attained in an exceedingly simple way -- producers' just produce the quantity demanded in aggregate by households. This assumption determines the amount of output of each good "j" that is to be produced at the given factor prices, and therefore the derived demand for factors as inputs in that production.

Ignoring the influence of government activities for the time being, we proceed to consider the equilibrium of factor markets. This is where the lack of foresight of the Auctioneer reveals itself, and the "search" for a general

[3] Note that household demands are not constrained to have unitary income-elasticities.

equilibrium effectively begins. There was nothing in our preceding discussion to guarantee that the derived demand for factors as the result of the production of commodities would equal the existing endowment of factors. The initial set of factor prices was indeed arbitrary[4]. An obvious loop now occurs in the algorithm: if the derived demand for factors is not equal (within some close tolerance range) to the current period endowment of the factor in each and every factor market, we must try another set of factor prices and try to get closer to an equilibrium. Computationally the search for a general equilibrium solution can be seen as a problem in minimizing the sum of the absolute values of the factor market disequilibria as a (highly non-linear) function of the prevailing factor prices. It is possible to use standard algorithmic techniques to solve this problem (e.g., Newton-Raphson methods). Irrespective of how the solution is found, the important point at this stage is that it is identified by examining factor-market disequilibria only[5].

There is much more to our framework than the simple interactions mentioned above, and these are introduced in due course below. One central feature of the models, however, should be noted -- the role allowed Government. In terms of the hypothetical sequence of decisions and events depicted above, Government is assumed to enter at virtually all stages. The initial factor prices are "loaded" by some ad valorem tàx on factors, affecting both the efficient

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[4] Note that the starting values for the comparative static simulations of such models are not arbitrary in our framework. The "previous" solution or "benchmark equilibrium" provides a natural place to begin the search for a solution based on some <u>perturbation</u> of the conditions defining that initial equilibrium. Moreover, we introduce several results in Kimbell and Harrison [1982] and Section 3.3 below that allow an <u>analytic</u> solution of a wide class of fully-dimensioned general equilibrium models. These solutions serve as excellent starting-values for more general models, as we explain and demonstrate in due course below.

[5] This point is now widely recognized -- see Fullerton, Shoven and Whalley [1978; p. 45] and the earlier references cited there.

factor allocation decisions of producers and the spending-power of given physical factor endowments of households (thereby affecting the distribution of spending-power directly). Similar taxes may be exacted on commodities, producing a direct "loading" on the prevailing relative commodity prices. The direct impact of this is on the allocation of spending-power by households amongst the various commodities. This of course causes secondary influences in the factor markets, due to the derived demand for factors. The influence of the Government, however, is felt in one further powerful manner -- the way in which the revenue from this taxation[6] is spent. We assume that the Government produces certain public goods. The essential distinction between private and public goods, for our purposes, is the nature of their consumption -- private goods are rivalrously consumed[7], public goods are not[8]. Naturally the production of public goods, requiring primary factor inputs, influences factor markets. The point to be made by the qualitative distinction between private and public goods is that the existing stock of the latter is a predetermined argument of the utility functions of private sector agents. Government provision of Education is a prime, and properly controversial, example of some substitutability existing between a private good (Private Education) and a public good (Government Education). Our framework can deal explicitly with all of these influences of Government.

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[6] Not to mention the value of the physical endowment of factors that the Fiscal Household may own.

[7] That is, "if you want to eat that banana you buy it!".

[8] That is, one can contrast the views of different household types about the existing provision of particular public goods. HAWK: "I think we have too little Defence expenditure!"; DOVE: "I think we have too much Defence expenditure ..."; QUAKER: "Shouldn't have any!". The point is that there is simply no way that these household types may contemporaneously choose to "enjoy" more or less of the public good than exists.

2.2 REGIONAL INCIDENCE ANALYSIS

The numerous analyses of tax incidence built on the original Harberger [1962] model have been surveyed by Mieszkowski [1969], Break [1974], and McLure [1975]. This theoretical tradition has firmly established the advantages of general equilibrium analysis for a large number of policy issues. In this section we review several developments in terms of this tradition, identifying areas in which numerical simulation techniques may usefully contribute. We are particularly concerned with issues germane to regional incidence analysis.

2.2.1 Factor Specificity and Mobility

Within the Harberger tradition, McLure [1969] [1970] [1971] and Mieszkowski [1966] [1972] have examined the effects of assuming region-specific factors on the standard tax incidence results[9]. The notion of factor mobility in our framework can be usefully viewed as a particular configuration of sector-specific factoral usage. This is indeed how most of the literature has modelled factor mobility (see Section 3.2 below for further discussion).

The key result here is that regional taxes levied on regionally mobile factors (working capital, for example) will tend to fall on those factors that are specific to the region (land, for example). Clearly one would also expect the final incidence of regional taxes levied on region-specific factors to also fall on those factors. To the extent that the commodity prices of locally produced goods are impacted, there are said to be local "excise tax effects" on the industry arising from the factor tax changes[10]. If, additional-

[9] We shall often refer to a Harberger-Mieszkowski-McLure, or HMM, tradition in regional fiscal incidence analysis. We specifically refer here to the extensions of the Harberger [1962] model by Mieszkowski [1967] [1969] [1972] and McLure [1969] [1970] [1971]. The choice of these three authors as representative of a long theoretical tradition is due to Break [1974; p.131].

ly, other regions import goods from the taxing region then there is "tax exporting" in the sense that consumers in other regions bear some of the burden of the regional tax. If local product prices change but few goods are exported then there are excise tax effects with insignificant tax exporting.

These are seminal ideas[11], and are central to any incidence analysis of regional taxes. We illustrate their importance in two applied simulations in Section 4 of the effects of regional property tax changes and energy tax changes. It should be noted that McLure [1975; p. 145] acknowledges that the original analyses producing these results

> suffered from the necessity to lump the entire output of each region together into one aggregate commodity and the concomitant inability to consider partial taxes on only some products or some uses of a given factor in the region.

We see in our simulations that numerical techniques are not similarly constrained.

Virtually identical ideas have appeared recently in an emerging theory of tariff incidence in international trade theory. Notable recent contributions include Dornbusch [1974], Clements [1979], Sjaasted [1980] and Sjaasted and Clements [1982]. The key result of this literature is that the burden of a tariff on importables may fall mainly on exportables, rather than local non-traded goods[12]. This incidence depends on the import tariff elasticity of the

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[10] McLure [1977; p.74] notes an important clarification of the popular expression "excise tax effects": "Whereas true excises are imposed upon <u>products</u> and distort only consumption decisions, differentials in property taxes (between regions or types of products) distort both production and consumption decisions. [...] Differential property taxes are identical to excise taxes only if factor proportions are fixed." We shall see in Section 4 the importance of the notion of excise tax effects in the interpretation of our simulations.

[11] See Mayer [1974], Mussa [1974] and Jones [1975] for a discussion of related ideas in the international trade literature.

[12] It is interesting to contrast McLure's [1975; p. 146] assessment of Miesz-

price of local non-traded goods. If this elasticity is close to unity[13] then the incidence of the tariff falls largely on exportables. If this elasticity is somewhat less than unity the incidence is shared by exportables and non-traded goods[14].

A number of alternative assumptions about the regional specificity of factors of production are widely used in incidence analysis. Heckscher-Ohlin models of international trade assume that factors may move freely between sectors in any region, but may not move between regions. A popular variant on these assumptions in the international trade literature is the Ricardo-Viner class of models that assume that one factor is able to move freely between sectors and another factor (or set of factors) is specific to each domestic sector. Again, no inter-regional factor mobility is assumed. A less popular variant on the Heckscher-Ohlin tradition, identified by Caves [1971], allows factors that are specific to certain sectors to move freely between regions

kowski's [1972] extension of the Harberger model: "... seen as a national tax levied at some uniform average rate, the local property tax reduces the net return to capital by roughly the amount of the tax, rather than being 'shifted' to consumers or reducing the rents paid on land. On the other hand, due to the mobility of capital, local differentials in property tax rates not matched by service differentials are likely to have what Mieszkowski calls 'excise tax effects' by raising (or lowering) the cost of capital to the local area imposing differentially higher (lower) taxes on property. Such excise effects are likely to be particularly important for so-called 'home' products, as opposed to 'export' goods." The empirical presumption obtained from the international trade application has been rather different (a heavier burden on exportables industries), but the analytical thrust has been similar.

[13] Which would be the case if importables and non-tradeables are close substitutes in local production and consumption.

[14] Sjaasted and Clements [1982] present some interesting evidence on the sensitivity of estimates of this elasticity to alternative parameter choices in an explicit general equilibrium model. This elasticity is a composite of four elasticities -- those of the demand and supply of non-traded goods with respect to the prices of importables and non-traded goods. It is therefore sensitive to the determinants of these elasticities. The simulations of the ORANI model reported in Dixon et. al. [1979] imply that some

providing they do not move between "different" sectors. Multi-national mining companies, for example, tend to have an expertise in one sector and are highly mobile geographically.

Traditional HMM incidence analysis assumes one factor that is perfectly mobile between regions and one factor that is perfectly immobile. As noted earlier, however, the extent of inter-sectoral mobility was not really an issue here -- the output of each region was aggregated into one output with no explicit discussion of its composition. We return to this issue below.

Three important modelling implications of our discussion may now be drawn. The first is that it is desirable to be able to contrast the results of incidence analyses of a particular policy that assume differing configurations of regional factor specificity. Apart from providing some common ground between international trade and HMM theoretical approaches, such contrasts often have considerable interpretive value with respect to model results. Factor specificity and immobility is widely regarded as a reasonable characterization of the "short-run", with inter-sectoral and inter-regional mobility a reasonable characterization of the "long-run". We need not dwell on the limitations of such characterizations; the important point is that they do have substantial "story-telling" value in a general equilibrium analysis.

A second point, related to the first, is that we would like to be able to <u>readily</u> contrast alternative configurations on the spectrum of factor specificity and mobility. We shall see in Section 3 that numerical techniques are not limited to those configurations popularly adopted for analytical convenience.

83% of an equiproportional tariff change on 112 sectors ends up as an excise tax on exporting industries. These simulations, however, were intended to be illustrative of the policy potential of ORANI, and did not incorporate any sensitivity analysis with respect to underlying parameters.

The third general implication of dealing with factor specificity is the issue of computational feasibility. A general equilibrium model of a single region assuming a mere ten sectors will have two factors if capital and labor are both sectorally mobile, and eleven factors if labor is sector-specific and capital is mobile. This modest enough extension raises the dimension of the search for an equilibrium from one relative factor price to ten relative factor prices. A model of two regions assuming ten sectors in each region will have two factors if capital and labor are regionally and sectorally mobile, three factors if labor is regionally immobile but still sectorally mobile within each region, and twenty-one factors if labor is regionally mobile and sector specific. The reader can easily extend this counting exercise to the case of three or more original factors (e.g., energy, water), and three or more regions of interest. With current computer technology and standard solution algorithms virtually any of these modestly realistic extensions would involve "considerable" computational expense. We propose a solution algorithm in Kimbell and Harrison [1982] and Section 3.3 below that allows rapid solutions to numerical models of the dimensions noted above.

2.2.2 Regional Differentiation

The importance of intermediate inputs in international trade theory has grown in recent years; see Chang and Mayer [1973], Jones [1975], and Burgess [1980], inter alia. The theory of effective protection has a number of general equilibrium features[15] and is widely applied for policy purposes. It embodies the simple and important message that tariffs act as a subsidy to importables pro-

[15] Along with a number of weaknesses from a general equilibrium perspective -- see Cordon [1971; Ch.6], Kreinin, Ramsey and Kmenta [1971], Evans [1971], and Johnson [1971; Part IV].

duction and a tax on exportables production, since imported goods will typically be used directly or indirectly as intermediate inputs to exportables production. The relevance of this point for the extent of excise tax effects and tax exporting in regional incidence analysis is clear. Melvin [1979] indeed employs the effective protection methodology to estimate the incidence of the U.S. and Canadian corporate income tax structure on sectoral commodity prices. Interindustry trade flows have been allowed for in a number of recent general equilibrium models -- see Dresch, Lin, and Stout [1977], Treddenick and Boadway [1977], Fullerton, Shoven and Whalley [1978], DeFina [1980], and Fullerton, King, Shoven and Whalley [1981].

Following Kuenne [1963; p.398] we can divide spatial models into two basic types:

> those which take the spatial structure of the economy as given and which seek to determine the equilibrium patterns of prices and flows of goods over space, and those which add to these latter unknowns the spatial structure itself.

As we shall see below in Section 3.2, the use of an inter-regional input-output (IO) accounting framework to calibrate our numerical model allows us to deal operationally with a regionally differentiated economy -- the first type of spatial model referred to by Kuenne. The data on regional demand and value-added allows tastes and production functions to vary from region to region. Similarly, the interindustry trade data allows the "same" sector in different regions to have different (direct and indirect) own-region requirements of intermediate inputs.

One important idea in the analysis of regional fiscal incidence, provided by Tiebout [1956], focusses squarely on the possible endogeneity of the spatial fiscal structure. In Tiebout's analysis households "vote with their feet" by choosing a region in which to reside. Their choice of location depends on the public service mix[16] that a given region provides and the taxes it levies. In other words, households seek an optimal mix of regional taxes and local public goods. If there are a large number of small regions, each offering its own public service bundle and tax rate, the equilibrium location of residents may provide a solution that resembles the choices households would have made if they had fully revealed their preferences. Kimbell, Shih and Shulman [1979] discuss the relevance of Tiebout mobility for the general equilibrium analysis of the Proposition 13 fiscal reform in California. It is possible to study such residential locational decisions in our framework if they are assumed to be made independently of the computation of a general equilibrium in our regionally differentiated model. In other words, we can solve for a given spatial configuration of households, separately "ask" households[17] whether they would like to move to another region[18], move households to their preferred region[19], and then re-solve the model given this conditionally preferred spatial distribution of households. After a sequence of such migrations, one would

[16] Edel and Sclar [1974] found that although the Tiebout hypothesis appeared to be valid for school spending, no locational decisions appeared to have been made with respect to road maintenance expenditures for several communities in the Boston area. Thus, the composition of public services may be as important as the general level of public services in determining locational decisions.

[17] Which may be distinguished on the basis of their tastes, their endowments, and their current location.

[18] One basis for this decision could be the "true" cost of living for any particular household in any particular region. This evaluation could be made given the general equilibrium solution previously obtained. One may, therefore, observe households moving to regions with lower wages (for example) if they place a heavy enough weight (in their explicit utility function) on that region's local public good.

[19] Some pragmatic distributed-lag mechanism could be used to avoid complete abandonment of any one region in a single period. One would tend to find economic forces slowing such heavy migration patterns down over several periods (e.g., with region-specific labor, outward migration would, ceteris paribus, tend to drive up local wages).

expect to obtain a stationary pattern of relative prices and no migratory inclinations. Modelling Tiebout mobility in terms of sequential comparative static general equilibrium solutions permits some explicit analysis of locational issues in terms of our general equilibrium framework[20].

A number of recent studies have extended general equilibrium models in the Scarf tradition to deal with contemporaneous locational decisions. Important contributions in this area include MacKinnon [1974], King [1977], Arnott and MacKinnon [1977(a)] [1977(b)] [1978], and Richter [1978(b)] [1980].

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[20] Richter [1978(a)] provides a direct extension of the Scarf framework to deal with regional locational decisions in the Tiebout sense.

Section 3

A GENERAL EQUILIBRIUM NUMERICAL INPUT-OUTPUT FRAMEWORK

3.1 OVERVIEW

In this section we introduce the numerical framework that we have developed to deal with several of the issues identified in Section 2. Although our approach is quite traditional, there are a number of novelties that we present in some detail.

The first major difference in our approach concerns the use of the general equilibrium structure described in Section 2.1 and illustrated in Figure 1. The advantage of this structure, in relation to the computational structure originally proposed by Scarf [1967], is that certain computational redundancies are removed for the class of models we happen to be interested in[21]. Specifically, the critical dimension of our search for a solution is the number of independent factor prices, which is just the number of primary factors in the model less one. The number of produced commodities has no fundamental influence on the search dimension, allowing us to build and solve models that utilize the detailed data available from Input-Output models on sectoral transactions. It should be noted that the structure of Scarf's algorithm can easi-

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[21] Scarf's structure is more general than ours, but we have no foreseeable need to use that full generality for the problems that concern us. Certain aspects of that generality (such as the ability to handle joint production) raises problems in many applications. This does not justify ignoring such phenomena in general, but it does rationalize our desire to avoid them in certain applications.

ly be modified to remove this computational redundancy[22].

The second major difference concerns the algorithm used to solve for the general equilibrium. We have developed an exceedingly simple procedure, along the homely lines of the Walrasian Auctioneer tatonnement process, that displays several dramatic properties. The first such property is that, for a specified class of general equilibrium economies, the solution is a closed-form <u>analytic</u> solution. This means that one can simply compute the exact solution algebraically without <u>any</u> iterations. This result is independent of the number of primary factors and produced commodities. Since it is an analytic solution there is, of course, no question of finding "good" starting-values for factor prices. The dependence of this result on particular features of the model is noted below, and a proof that it is indeed the analytic solution offered in Kimbell and Harrison [1982]. The second result is even more remarkable -- this algorithm appears to be an excellent approximation to some undefined analytic solution for more general cases. In short, it is extremely fast for very general cases. What this second result means is that we may routinely solve for quite large problems. The implication here is that we are no longer limited by technique and/or computational resources to assume only two or three primary factors. This allows us to assume very detailed patterns of factor specificity and mobility in particular applications, as required by the analysis of regional fiscal incidence.

The third feature of our framework concerns the use of existing data from an Input-Output accounting framework to calibrate the model. Arguably, one of the most important developments in this field since the original contribution by Scarf [1967] has been the use of observed data to provide an empirical

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[22] By re-defining the relevant unit simplex over factor-price and excess factor-demand space.

characterization of some kind in general equilibrium models. Input-Output transactions tables are now routinely generated[23] for many countries and regions. Such tables provide the basic data necessary to minimally calibrate general equilibrium models -- the value added rows provide data on payments to primary factors and government (indirect taxes) by sector, the final demand columns provide expenditure data by domestic households and foreigners by sector, and the inter-industry transactions provide data on intermediate imput requirements by sector. With extraneous estimates of certain key elasticities it is possible to identify all the necessary parameters of a simple general equilibrium model. For certain exercises this data are all that is needed; for other problems it is necessary to have recourse to essentially hypothetical parameter values. The important feature of our model is that it is flexible enough to incorporate an arbitrary Input-Output transactions table, providing it satisfies simple "adding-up" requirements.

In Section 3.2 we discuss the use of a general Input-Output accounting system in obtaining the "basic" data for our model; in Section 3.3 the crucial algorithmic features of our framework; and in Section 3.4 a number of immediate extensions to our framework.

[23] Albeit with long delays. Several official agencies are now exploring non-survey methods for updating these tables more frequently.

3.2 THE INPUT-OUTPUT ACCOUNTING FRAMEWORK

3.2.1 Basic Data

The typical Input-Output (IO) table provides all of the data necessary to calibrate a general equilibrium model allowing substitution in production and consumption. If extraneous estimates of certain elasticities of substitution can be obtained, from thin-air or a literature survey, it is possible to calibrate more general (nested) functional forms with widely available IO data. In this secton we discuss the use of such data, as a prelude in Section 3.4 to a discussion of the concept and computation of a benchmark equilibrium.

Figure 2 illustrates the basic IO accounting framework[24]. We shall be concerned with the use of three blocks of data from this table: the value-added rows, the final demand columns, and the matrix of inter-industry transactions.

Each value added row can be interpreted as the (value) payments to a particular factor -- it is common to aggregate the rows for Profit Income, Interest Income, and Depreciation to obtain the payments to the single factor Working Capital. The Employee Compensation row provides data on payments to the factor Labor. The row that is typically titled Indirect Business Taxes can be viewed for the moment as a general tax on Value Added as a whole; we discuss its use in model-equivalent "ad valorem" form below. It is important to note that these rows provide sectoral data on factor payments, invariably for an exhaustive series of sectors. Imports often appear as a row in Input-Output tables, typically with some distinction being drawn between Competitive Imports[25] and

[24] We deal throughout with Industry-by-Industry tables that are presumed to be expressed in producer's prices. The GEMTAP model of Fullerton, Shoven and Whalley [1978] illustrates the use of Industry-by-Commodity absorption matrices, and the ORANI model of Dixon et. al. [1981] the possible use of data on various pricing "margins".

[25] Usually defined as imports in sectors that also show some non-trivial domestic production level.

	Intermediate Purchases					Final Demand				E = Gross Output
	1	. . .	j	. . .	n	C	I	G	E	
1	X_{11}	. .	X_{1j}	. .	X_{1n}	C_1	I_1	G_1	E_1	X_1
.	.		.		.	.	.	.	.	.
Producing .	.		.		.	.	.	.	.	.
Sector i	X_{i1}		X_{ij}		X_{in}	C_i	I_i	G_i	E_i	X_i
.	.		.		.	.	.	.	.	.
.	.		.		.	.	.	.	.	.
n	X_{n1}	. .	X_{nj}	. .	X_{nn}	C_n	I_n	G_n	E_n	X_n
Employee Compensation	EC_1	. .	EC_j	. .	EC_n					
Profits	PR_1	. .	PR_j	. .	PR_n					
Interest	IN_1	. .	IN_j	. .	IN_n					
Depreciation	DE_1	. .	DE_j	. .	DE_n					
Indirect Bus. Taxes	BT_1	. .	BT_j	. .	BT_n					
Imports	IM_1	. .	IM_j	. .	IM_n					
E = GROSS OUTLAYS	X_1	. .	X_j	. .	X_n					

Legend for Final Demand Components: C = Consumption; I = Investment; G = Government Expenditure; and E = Gross Exports.

Figure 2: BASIC INTERINDUSTRY ACCOUNTING SYSTEM

Non-Competitive Imports[26]. The treatment of Imports happens to be quite important for certain general equilibrium problems, particularly those involving open economies and trading regions. The ORANI model of Dixon et. al. [1981] is particularly sensitive to the assumed treatment of imports[27]. We return to the treatment of Imports in the context of an Inter-Regional IO table below.

The final demand columns of the IO table provide data on expenditure in value units. The "household" types typically distinguished include Consumers (Consumption), Investors (Gross Private Capital Formation), Federal Government (Expenditure on Current and Capital account), State and Local Government, and Foreigners (Gross Exports). Some tables provide further disaggregation of government expenditure, but this is not common. In any case, these categories at least provide the basis for more detailed attempts to disaggregate final expenditure on the basis of data from Household Expenditure Surveys and/or Taxation data. This block of data, as provided by the typical IO table, is probably the one that is least satisfactory for general equilibrium purposes[28].

The final block of data provided by the IO table is of course the industry-by-industry matrix of intermediate input requirements. Each column of this matrix, divided element-wise by the value of that sector's total input usage, shows the direct per unit requirements of every other sector's output. It is common to assume that the intermediate input technology is of the standard fixed-coefficient Leontief type, and we follow convention here.

[26] Usually defined as imports in sectors that show no domestic production. Note that these definitions do not often relate to the pragmatic methods used to distinguish the two types of Imports. In some cases tables based on alternative methods of allocation (Direct and Indirect) are provided.

[27] This is entirely appropriate, given the open nature of the Australian economy.

[28] One possible alternative is to substitute the results of systems-wide estimates of particular demand systems for the available IO Consumption data.

Data on individual household endowments is not obtainable from typical IO tables, and popular calibration procedures recognize this by adjusting the distribution of endowments across household types until the necessary budget constraints are satisfied.

3.2.2 Inter-Regional Data

It is possible to adapt essentially the same IO accounting framework to develop Inter-Regional transactions data. The typical procedure is to generate, by one method or another, matrices of direct own-region requirements and direct other-region requirements for each region considered. If we consider the simple case of just two regions, the former matrix represents the per unit requirements of each sector that are assumed to be met from own-region gross production; the latter matrix shows the per unit requirements of each sector that are assumed to be met by imports from the other region[29]. We shall not concern ourselves here with the methods used to construct such Inter-Regional tables[30], but rather with the way in which it allows us to distinguish regions in terms of factor usage, expenditure patterns, and intermediate input technology.

Figure 3 illustrates the general Inter-Regional IO accounting framework used in this study for studying the regional incidence of government tax and expenditure policies. Regions are distinguished from sectors by the use of superscripts rather than subscripts. The R matrices represent own-region inter-sectoral transactions, and the T matrices represent inter-regional intersectoral transactions. Similarly, the FD matrices represent final demand for

[29] Whether or not they were produced in the other region.

[30] General references on the empirical construction of inter-regional IO tables include Richardson [1972], Riefler [1973], and Round [1978].

		Intermediate Purchases Calif.	Intermediate Purchases ROUS	Final Demand	E = GROSS OUTPUT
Producing Region	California	R^{11}	T^{12}	FD^{1}	X^{1}
	ROUS	T^{21}	R^{22}	FD^{2}	X^{2}
Value Added		VA^{1}	VA^{2}		
Imports		IM^{1}	IM^{2}		
E = GROSS OUTLAYS		X^{1}	X^{2}		

Notes: Superscripts refer to regions (California = 1, ROUS = 2). See text for further explanation.

Figure 3: INTER-REGIONAL INPUT-OUTPUT ACCOUNTING SYSTEM

the net output of each sector in each region, and the VA matrices represent value-added generated by each sector in each region. Figure 2 provides the accounting detail that Figure 3 glosses. Thus each regional FD matrix is n-by-4 (since we earlier distinguished four components of sectoral final demand), and each regional VA matrix is 5-by-n (since we distinguish five types of sectoral value-added). Each R and T matrix is n-by-n, since we distinguish n sectors in each region.

Note that this inter-regional IO accounting framework is <u>qualitatively</u> identical to the accounting framework presented earlier (contrast Figure 2). The first block of sectors refer to one region (California, here) and the second block of sectors to the other region (Rest Of U.S., or ROUS for short, here). Thus we distinguish the production of each good by the region in which that production is assumed to occur. This is the so-called "Armington" assumption, named after Armington [1969]. It has been employed recently by Whalley [1980] and Brown and Whalley [1980] in an international context. One important area for future research is to allow goods that are distinguished solely by region to have a differential (and invariably higher) elasticity of substitution in consumption than goods that are produced in any single region. Our present models assume no such "nesting" of utility functions. One attractive feature of such nesting is the ability to replicate popular international trade assumptions about "small" economies (regions) effectively facing given relative commodity prices in some sectors.

As Isard [1951] emphasized many years ago, the use of such models of the spatial economy allows one to deal with a rich assortment of regional differentiation in tastes (each block of sectors has a particular expenditure pattern), factor usage (each block of sectors has distinct value added shares), own-region intermediate production processes (each diagonal block of the inter-industry matrix is distinct), and inter-regional trade flows (each off-diagonal block of the inter-industry matrix is sector-specific). However, it is crucial that the limitations of this approach for the study of spatial issues be identified. Isard [1951; p. 328] notes that:

> Constant production coefficients, interregionally viewed, mean unchanging supply channels. The model thus forces the assumption that the spatial extent of each line of production in each region is fixed with respect to its inputs. In this respect, no spatial lengthening or shortening of any line of production can be tolerated. Viewed otherwise, transport outlays or, in physical terms, distance inputs per unit of output of any line in any region are not permitted to change. Also, as a corollary, roughly fixed price ratios are assumed, so that substitution of distance inputs for other inputs, of transport outlays for other outlays, is precluded. In these respects, the underlying structure of the space-economy is held rigid.

As noted earlier in Section 2, we prefer to view our approach as dealing explicitly with many issues of regional differentiation, and effectively ignoring the issues raised by allowing contemporaneous regional location decisions.

3.2.3 Factor Specificity and Mobility

A simple extension of the basic IO accounting framework, coupled with the powerful algorithm introduced in Kimbell and Harrison [1982] and discussed below, allow us to deal with relatively large problems and somewhat more "realistic" configurations of primary inputs. This extension is intended to allow us to deal with factor specificity in a general and intuitive manner. Suppose that we wished to distinguish the Labor employed in California from the Labor employed in the ROUS, to view Labor as a region-specific or immobile factor. The procedure adopted is to view the payments by ROUS sectors to Californian Labor as a null row vector, and the payments by California sectors to ROUS Labor as a null row vector. Viewing California Labor and ROUS Labor as two distinct factors, each would be entitled to a separate row in the value added block in the Inter-Regional IO accounting framework. Figure 4 illustrates this matrix configuration. Note that no adding-up constraints have been violated, by construction, and yet we have defined Labor as being region-specific.

This simple procedure obviously generalizes to allow any configuration of sector-specific factor usage[31]. Note also that many theorists have interpret-

[31] For certain computational reasons (avoidance of "divide-by-zero" error messages) it is sometimes necessary to place numerical equivalents of an infinitisemal "epsilon" in the place of zero. This constraint refers only to the first factor, which plays an important algorithmic role in parameterizing production functions (see below); in general, it makes no noticeable difference to the numerical solution whether or not these zeroes are indeed just epsilons. Less substantively, some economists like to think of every input to a C.E.S. production function as being economically essential, and may be offended by the use of zeroes. To repeat: it makes no computational difference.

		Intermediate Purchases Calif.	Intermediate Purchases ROUS	Final Demand	E = GROSS OUTPUT
Producing Region	California	R^{11}	T^{12}	FD^{1}	X^{1}
	ROUS	T^{21}	R^{22}	FD^{2}	X^{2}
California Labor		EC^{1}	$\bar{0}$		
ROUS Labor		$\bar{0}$	EC^{2}		
Other Value Added		OVA^{1}	OVA^{2}		
Imports		IM^{1}	IM^{2}		
E = GROSS OUTLAYS		X^{1}	X^{2}		

Notes: "California Labor" refers to Employee Compensation paid to labor directly working for California sectors.

Figure 4: SPECIFIC FACTORS IN THE ACCOUNTING SYSTEM

ed the two extreme cases of specific factors and sectorally mobile factors in terms of "short-run" and "long-run" resource allocation outcomes[32]. Our procedure for dealing with specific factors allows a simple contrast of these two types of outcomes, simply by altering the way the data are constructed. Moreover, it is an equally straightforward matter to deal with intermediate, "medium-run", configurations as desired. This opens the way to dealing with problems of great complexity and, depending on one's attitude to such modell-

[32] Amano [1977] correctly points out that "... the emphasis on specific factors may lead to a danger of circular reasoning unless one can successfully explain the mechanism that determines the relative supply of those factors." (p.131).

ing exercises, realism.

The computational effect of this important extension is to raise the dimensionality of the search for an equilibrium. As noted earlier, we believe that the algorithm discussed below allows us to proceed to use these extensions without undue concern over computational expense.

3.2.4 Parameter Identification

There has been an increasing trend in the numerical simulation literature towards providing empirical versions of the economy under study. For many policy purposes this is clearly important, if for no other reason than their presumed relevance in the eyes of those who will act upon them. On a slightly more subtle level, however, budget constraints must "add-up" for one to even define a general equilibrium solution. This is a transparent problem if one is willing to use purely hypothetical numbers, since the solution algorithm we propose in Figure 1 automatically ensures that budget constraints are met.

The truly important step in the generation of an empirical characterization occurs when the model is able to replicate the essential outcomes of the data it is fed. Specifically, we have in mind as "essential" the chosen factor intensities for each sector, the allocation of each household's spending-power across commodities, and the aggregate allocation of final demand across commodities. The first item ensures that we do end up producing Secondary Energy with non-trivial amounts of Primary Energy. The second item is important for any study that is concerned with the distributional implications of any comparative static exercises with a general equilibrium model[33]. The last of these

[33] This is because the expenditure pattern of particular households is a critical input into any determination of the implied effect on their ("true", or functional) cost-of-living. It is common in standard computations of the incidence of particular taxes to distinguish their incidence by the level

items ensures that we do not end up consuming a great deal of New York wine relative to California wine[34].

Seminal contributions in the area of "general equilibrium accounting" for the United States, the United Kingdom, and Canada are provided by Fullerton, Shoven and Whalley [1978], Piggott and Whalley [1983], and St-Hillaire and Whalley [1980]. They have each successfully integrated wide varieties of national data to generate benchmark equilibria against which counter-factual simulations may be compared[35]. The problem of choosing parameter values for the underlying production and utility functions in a general equilibrium model may be usefully viewed from the perspective of the econometric identification of a system of structural equations. We use the available data on factor shares (by factor and sector) and expenditure shares (by sector and household type) along with the "identifying restrictions" of choice theory in order to work "backwards" to the parameters of the production and utility function that would generate those shares. Piggott and Whalley [1983; Ch.4] provide an excellent exposition of these standard calibration procedures.

The generation of an empirical general equilibrium model requires considerable attention to accounting detail and niceties, and a liberal measure of common-sense in deciding what is "essential" to the proposed application. Our

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of household income (viz., how regressive or progressive is this tax?), and one can address these issues by distinguishing expenditure patterns by level of household income. Similarly, development theorists are increasingly concerned with the effects of alternative development strategies on income distribution within the nation. This concern, powerfully expressed in Chenery et. al. [1974], is reinforced by the patterns of substitutability (particularly with respect to food prices) in household demand identified in Lluch, Powell and Williams [1977] for less-developed economies.

[34] Which is an offensive proposition, aesthetically and literally.

[35] Fullerton, Shoven and Whalley [1978] also provide a facility to incorporate more recent data than their benchmark year. Whalley [1980] uses similar methods in an international context.

efforts have been directed towards making the generation of such empirical characterizations revolve around the generation of reliable inter-regional input-output transactions data, on the simple rationale that one rarely has much else to work with.

3.3 ALGORITHMIC FEATURES

3.3.1 The Analytic Factor Price Solution

The central thrust of our efforts to develop a faster and more accurate solution algorithm for general equilibrium models has been to sacrifice generality for closed form analytic solutions to the various sub-problems implied by a general equilibrium model. For the class of GE model considered here, the only iterative process involves the determination of equilibrium factor prices. Use of neoclassical production functions, as extended by fixed-coefficient intermediate input requirements, means that output prices are analytically determined by cost-minimization and the zero-profit assumption[36].

For a class of general equilibrium models, to be specified below, the closed-form solution is given by the following Analytic Factor Price Solution:

$$P(f) = \left(K(f)/K(1) \right)^{1/s} \left(X(f)/X(1) \right)^{-1/s}$$

for f = 2 to NFAC (the number of factors in the model), with P(1) = 1.0 and

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[36] More general linear activity specifications, such as used by Scarf [1967] [1973], permit multiple outputs but also include output prices in the price vector space as additional dimensions. The linear activity approach does not have to define some goods as factors on an *a priori* basis, since a given good is a "factor" if it has a negative coefficient in any linear activity. We accept this restriction that factors be stipulated *a priori* as worth the gain in reduced dimensionality. Similarly, our way of specifying the role of government implies that the government budget constraint is imposed analytically and (unlike the approach of Fullerton, Shoven and Whalley [1978]) does not add another dimension to the search.

$$K(f) = \sum_{g=1}^{NCOM} a(g)^{s-1} B(g)^{s} d(g,f)^{s} T(g,f)^{-s},$$

and where P(f) is the price of factor f, D(f) is the demand for factor f, X(f) is the (perfectly inelastic) aggregate supply of factor f, s is the elasticity of substitution in production (uniform across all industries) and consumption, a(g) is the efficiency parameter for producing good g, NCOM is the number of produced goods (or commodities), B(g) is the distribution parameter for good g in the CES utility function of the (single) consumer, d(g,f) is the distribution parameter of factor f in the CES production function for good g, and T(g,f) is unity plus the fractional rate of taxation on the use of factor f in industry g. The price of the first factor is taken without loss of generality to be the numeraire.

The Analytic Factor Price Solution is known for models that are general with respect to: (i) any number of factors and goods; (ii) any pattern of distribution parameters in the single-level CES production functions or (single) utility function; (iii) any pattern of efficiency parameters in the production function; and (iv) any arbitrary pattern of factor taxes across factors and producing sectors. The exact solution does not apply to models with: (i) more than one private household; (ii) any interindustry (input-output) flows; (iii) elasticities of substitution in production that vary from sector to sector; (iv) an elasticity of substitution in consumption different from the (uniform) elasticity of substitution in production, and (v) government factor demands that are not proportional to aggregate private industry factor demands.

The Analytic Factor Price Solution exhibits a property that we will call System-Wide Separability, since it implies that any changes in the tax rates on (non-numeraire) factor f alter <u>only</u> the price of factor f. Equivalently, a change in the tax rates on the numeraire factor will change all other factor prices by exactly the same proportion. More formally[37],

$$\frac{d\,[\,P(f)/P(k)\,]}{d\,[\,T(g,j)\,]} = 0,$$

for f, k, and j not equal, and for all such triples of f, k, and j. This property does not imply that the price of factor f adjusts enough to capitalize fully the tax changes -- there can still be excise tax effects via changes in goods prices -- but the burden of increased taxes on one factor produce utterly no <u>relative</u> factor price changes among the other factors.

The Analytic Factor Price Solution leads to equations for factor intensities and output as follows:

$$F(g,f) = [\,d(g,f)^{s}\ X(f)\,] \,/\, [\,K(f) * Z\,]$$

$$Q(g) = b(g)^{s} * Z$$

$$\text{where}\quad Z = [\,\sum_{f=1}^{NFAC} d(g,f)^{s}\ K(f)^{-r}\ X(f)^{r}\,]^{1/r}.$$

The allocation of factor f to the production of good g, A(g,f), is simply the product of the relevant factor intensity and output:

[37] <u>Proof</u>: Notice that T(g,j) is not an argument of the formula for P(f)/P(k).

$$A(g,f) = F(g,f) * Q(g)$$

$$= [\ d(g,f)^{s}\ b(g)^{s}\ X(f)\ ]\ /\ K(f)$$

Suppose the endowment of factor i increases, holding constant all other endowments and parameters. The ratio of the new factor allocation to the old factor allocation, for any arbitrary good g, becomes:

$$A(i,g) = X(i)^{*}\ /\ X(i)$$

$$A(g,f) = 1.00\ ,$$

for all f not equal to i, and where

$$X(i)^{*} = \text{new value of } X(i).$$

For all factors other than i the full allocation across all industries is completely invariant to changes in the endowment of factor i. Substituting for A(g,f) yields the <u>Factor Use Decomposition Formula</u>:

$$(1) \qquad \frac{F^{*}(g,f)}{F(g,f)} * \frac{Q^{*}(g)}{Q(g)} = 1.00 \quad .$$

The ratio of the output of good g <u>after</u> the expansion of the endowment of factor i to the output of good g <u>before</u> the endowment change,

$$Q^{*}(g)\ /\ Q(g),$$

is called the "expansion effect". It shows the proportion by which the use of factor f in producing good g would increase if output changed but factor prices were constant[38]. The ratio of the factor intensities,

$$F^*(g,f) \,/\, F(g,f),$$

is called the "substitution effect". It shows how the demand for factor f in producing good g would have changed, given the factor price changes, holding output constant. The Factor Use Decomposition Formula (expression (1)) shows that the substitution effect is _identically equal to the reciprocal of the expansion effect_ for all factors other than the one which changed, and for each and every industry. Essentially, the special restrictions involved in our Analytic Factor Price Solution are sufficient to make the expansion effect _identically offset_ the substitution effect, leaving allocations other than X(i) unchanged[39].

This result also helps to explain a distinctive feature of the Analytic Factor Price Solution -- the System-Wide Separability property. The CES family of production functions is strongly separable; i.e., the marginal rate of substitution between factors f and j is invariant to changes in any other factor i, holding the inputs of f and j constant. An increase in the endowment of factor i, since it does not alter the amounts of f and j used in any production function (in a general equilibrium), leaves unchanged these marginal rates of substitution. The ratio of factor prices f and j is therefore invari-

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[38] Recall that the class of production functions currently being considered are homothetic and have constant returns to scale.

[39] Note that this result is analogous to the familiar result that in the case of Cobb-Douglas production functions cross-price elasticities of factor demand are identically zero, even though they are not zero for the more general CES case.

ant to changes in the endowment of factor i. System-Wide Separability is therefore simply the result of strong separability of CES production functions, combined with the special result that allocations of factors other than i are invariant to changes in the endowment of factor i.

For any general equilibrium model the sum of factor demands across all industries equals the (inelastic) aggregate factor supply, before and after a change in endowments. In other words:

$$\left[\sum_{g=1}^{NCOM} A^{*}(g,f) \Big/ \sum_{g=1}^{NCOM} A(g,f) \right] = X^{*}(f) / X(f) ,$$

for all f. In our special case:

$$A^{*}(g,f) / A(g,f) = X^{*}(f) / X(f) ,$$

for all f, g pairs. Thus, what is special about our Analytic Factor Price Solution is that the expansion effect is exactly offset by the substitution effect <u>in each and every industry</u>.

Even though the applications presented in Section 4 do not conform strictly to the conditions for a known analytic solution, System-Wide Separability tends to manifest itself repeatedly. Specifically, except for the effects of high tax rates on government revenues, and therefore government factor demands, many other factor prices change by exactly the same proportion relative to the numeraire. That is, a whole block of factor prices show the same percentage impacts.

One feature of our Analytic Factor Price Solution should be noted as particularly relevant for the study of regional fiscal incidence: the arbitrary pattern of distribution parameters in the CES production functions for each sector that is allowed. It is the adoption of alternative configurations of these parameters that allows us to examine alternative assumptions about factor specificity and mobility, as described earlier. Note that although our earlier discussion dealt with value share data and not distribution parameters, the latter may be obtained from the former for <u>given</u> values of the elasticity of substitution for each sector.

3.3.2 Some Familiar Analytic Results

It is possible to analytically derive several standard HMM results with our Analytic Factor Price Solution, accepting the assumptions noted above for the existence of the closed-form solution. To simplify notation let

$$h(g,f) = a(g)^{s-1} \; B(g)^{s} \; d(g,f)^{s} ,$$

and

$$M = K(1)^{-1/s} \; (\, X(f)/X(1) \,)^{-1/s} .$$

Factor prices then become:

$$P(f) = \Big(\sum_{g=1}^{NCOM} h(g,f) \; T(g,f)^{-s} \Big)^{1/s} * M \qquad (2)$$

for f = 2 to NFAC and with P(1) = 1. A standard HMM result emerges from expression (2): if the tax expressions T(g,f) are increased in proportion V for all uses of factor f (e.g., the property tax is increased in all regions) then factor price f changes by exactly the reciprocal of V. This result obtains by substituting V*T(g,f) into expression (2) and factoring out 1/V. Capitalists bear fully a <u>uniform</u> <u>nation-wide</u> increase in property taxes.

There are no excise tax effects in this case. The expression for product prices (obtained from the zero-profit equilibrium condition) becomes with taxes:

$$(3) \qquad P(g) = (1/a(g)) \left(\sum_{f=1}^{NFAC} d(g,f)^{s} \, (T(g,f)*P(f))^{1-s} \right)^{1/(1-s)}$$

for g = 1 to NCOM. Replace P(f) in the expression for product prices by T(g,f)*P(f) to obtain equation (3). But increasing T(g,f) by V, for all f, leads to P(f)/V, so the products (T(g,f)*V)(P(f)/V) = T(g,f)*P(f), and all product prices are invariant to the tax increase that is uniform across all factor uses.

Increasing the tax on <u>one use</u> of factor f is not the same, of course, as increasing taxes on all uses. In the original Harberger [1962] context the corporate profits tax was treated as levied on the use of capital in one sector, and in the Mieszkowski-McLure work on regional tax incidence an increase in property tax on one region is recognized as fundamentally different from a uniform nation-wide tax change.

For expository convenience let h(g,f) = 1.00 in expression (2), for all pairs g and f. In the base case let T(g,f) = 1.00, again for all g and f[40]. As the alternative, let the tax on the use of factor f in industry (or region) g increase by V(g,f). The ratio of factor price f after the tax increase, P(f)', to the price before the tax change, P(f), becomes:

$$\frac{P(f)'}{P(f)} = \left[\frac{G-1}{G} + \frac{1}{G*V(g,f)^{s}} \right]^{1/s}$$

[40] This means that no taxes are levied in the base case.

where G is the number of equally sized industries (or regions). It is obvious that as G grows larger the price of factor f is less and less affected (although it is always lower after the tax increase). The returns to a mobile factor are not significantly affected by a change in taxes on its usage in one small industry (or region).

These familiar results illustrate the potential analytical uses of the closed form solution without any necessary numerical calculations.

3.3.3 The Factor Price Revision Rule

Useful as the Analytic Factor Price Solution may be, it does not appear to generalize to include certain important features of applied general equilibrium models (as noted earlier). It does, however, motivate a simple and powerful iterative algorithm for use in more general problems. Kimbell and Harrison [1982] examine the above propositions in some detail.

The critical step in revising factor prices over iterations involves the following Factor Price Revision Rule:

$$\text{Part 1.} \qquad P(f,i+1) = P(f,i) * \left(D(f)/X(f) \right)^{1/s}$$

for f = 1 to NFAC, and where P(f,i) is now the price of factor f at iteration i; and

$$\text{Part 2. (Renormalize)} \quad P(f,i+1) \Leftarrow P(f,i+1)/P(1,i+1)$$

for f = 2 to NFAC. The price of the first factor is taken without loss of generality to be the numeraire, and is simply re-set to unity.

Notice that the Factor Price Revision Rule is a simple Walrasian rule that raises the price of a factor in excess demand, lowers the price of a factor in excess supply, and leaves unchanged the price of a factor with own-demand equal to own-supply.

The magnitude of the price revision, for a given ratio of demand to supply, is determined by the reciprocal of the elasticity of substitution. This is a <u>crucial discovery that critically influences the speed of convergence</u>. If the elasticity of substitution is very low, say 0.10, then the ratio of demand to supply in market f is raised to the tenth power in determining the price revision factor. If the elasticity of substitution is very high, say 10, then the tenth root of the ratio of demand to supply is the price revision factor. The low elasticity revision rule causes far more "energetic" changes in factor prices than the high elasticity case (for given values of the ratio of demand to supply).

We provide some computational experience with our Factor Price Revision Rule in the next Section.

Section 4

APPLICATIONS

4.1 A TAXONOMIC ILLUSTRATION

4.1.1 Structure of the Economy

In this section we present several simulations of a hypothetical economy consisting of two regions each producing two private goods[41]. Using the "Armington Assumption" noted in Section 3 we therefore distinguish four private commodities. Our hypothetical economy also includes two public goods, as described earlier in Section 2.1. One of these goods is produced with revenues generated by (factor) taxes that are specific to the good and region in which the factor is being used -- this good may be regarded as a local public good. The other public good is produced with revenues generated by (factor) taxes that may vary with thé good and factor being taxed, but are identical across regions -- this good may be regarded as a national public good.

Each of the six commodities in our economy uses four types of primary factor. These are:

i) a factor that is mobile between all regions and sectors;

ii) a factor that is mobile between sectors of the region in which the good is produced, but is not able to move to any other region;

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[41] To avoid possible semantic snarls, we shall use the term "good" to refer to a production activity that may occur in either region (e.g., Agriculture) and the term "commodity" to refer to a production activity in a specific region (e.g., Californian Agriculture).

iii) a factor that is specific to the production of a particular good and is able to move between regions;

iv) a factor that is both specific to a particular good and a particular region.

Using our procedure for defining specific factors, the reader can confirm that our hypothetical economy has one factor of the first type, two factors of the second type (one for each region), three factors of the third type (one for each private good produced and one for the public good), and seven factors of the fourth type (one for each private and public good in each region, and one for the national public good). Our search for a general equilibrium solution therefore involves twelve relative factor prices (we define the completely mobile factor to be the numeraire). This configuration of factor specificity and immobility is designed to provide a complete taxonomy of such cases, extending earlier analyses in the HMM tradition. In addition, of course, we are able to study excise tax effects that may occur within regions and/or between regions.

For ease of interpretation of the simulation results, we have grossly simplified the production structure of this hypothetical economy: no intermediate trade between sectors or regions is assumed, the CES production functions for each good are calibrated with distribution parameters that take the value of "1" or "0" as the nature of the factor indicates, government taxes are assumed to be levied on each factor in each sector in each region (initially) at an "ad valorem" rate of 5 percent, a uniform elasticity of substitution (in production and consumption) of 2.0 is assumed, and each region is assumed to have only one household[42].

[42] Each household is endowed with: two units of the fully mobile factor; one unit of each of the regionally mobile, good specific, factors; two units of

Although conceptually simple and empirically spartan, this model is: (i) large enough to illustrate the algorithmic power of our Factor Price Revision Rule; (ii) taxonomically complete enough to provide several extensions to the HMM literature; and (iii) small enough that we may present a policy simulation in some detail.

4.1.2 Some Computational Experience

Table 1 shows the speed of convergence to the general equilibrium set of (twelve) relative prices in terms of the absolute value of the maximum factor market disequilibria expressed as a percentage of the aggregate endowment of that factor[43]. The first run shown corresponds to the use of the unit vector as an initial guess at the equilibrium factor prices; in this case the Factor Price Revision Rule was able to solve the model described above in <u>nine</u> iterations.

The second run adopts starting values for factor prices that are deliberately disturbed "significantly" away from the (known) equilibrium solution. The sizeable (maximum percent) disequilibrium of 1423.977 (see iteration 1 for Run 2 in Table 1) indicates what our algorithm had to overcome (relative to Run 1). A new solution was found in <u>nine</u> iterations. Indeed, after the shock of the first iteration the Rule tended to follow the convergence path of Run 1.

The third run adopts the unit vector of factor prices as a starting point, but unlike Run 1 assumes that intermediate trade is allowed between commodities. The Leontief Inverse used for this exercise was pseudo-randomly generat-

the own-region-specific factor that is mobile between goods; and one unit of each of the own-region-specific, good specific, factors.

[43] Convergence was assumed when this criterion was less than 0.100.

TABLE 1

Computational Experience

Convergence measured by absolute value of the maximum factor market disequilibrium expressed as a percent of the aggregate endowment.

Iteration	RUN 1	RUN 2	RUN 3	RUN 4
	PERCENT DISEQUILIBRIUM			
1	90.000	1423.977	90.000	63.762
2	43.396	48.348	43.454	17.319
3	19.352	20.534	19.709	4.705
4	7.998	8.112	8.225	1.430
5	3.205	3.153	3.310	0.468
6	1.269	1.225	1.312	0.161
7	0.500	0.477	0.517	0.059
8	0.196	0.186	0.203	
9	0.077	0.073	0.080	

ed[44]. The convergence path again parallels that observed in Run 1, with a solution (quite different from the solution found in Run 1 or Run 2) rapidly found.

The fourth run shown in Table 1 illustrates a common situation -- given the general equilibrium solution found in Run 1 (or Run 2), we search for a new solution after some policy perturbation. In this case we increase the local government tax on the fully mobile factor's use in good 1 in region 1 from 5% to 100%. The Factor Price Revision Rule is able to return us to a general equilibrium within a mere <u>seven</u> iterations.

[44] With the restriction that off-diagonal elements take values between 0.3 and 0.5, and diagonal elements take values between one and 1.500. Run 1 essentially assumes that this is an identity matrix.

4.1.3 Tax Policy Simulations

The simulations discussed below assume that the tax on the fully mobile factor is raised from an ad valorem rate of 5% to 100% on its use in the productions of good 1 in region 1.

It is common in the analytical literature to distinguish tax and expenditure incidence by making some "equal yield" assumptions about one side of the public accounts while altering the other side (e.g., assuming that an industry and/or factor specific tax cut is offset by a general "across-the-board" value-added tax increase of appropriate size). Shoven and Whalley [1977] provide an explicit computational approach to this issue, and Fullerton, Henderson and Shoven [1983] a critique of several numerical applications of the notion of "differential tax incidence". There are two objections to the use of such notions: (i) general equilibrium analysis is designed to deal with interdependencies of revenue sources and expenditures for all agents, and to single out the activities of one agent (Government) for artificial treatment is illogical; and (ii) in a multi-government economy (such as occurs invariably in a regional context) it is not clear what one should assume for the non-tax-change fiscal authority[45].

The correct response to both of these objections is that the notion of "differential tax incidence" is essentially one motivated by pedagogic and expositional concerns[46] and that those concerns should indicate whether or not a

[45] That is, changes in State and Local Government taxes will in general affect the tax revenues of the Federal Government (and vice versa). Ignoring the further complication of fiscal revenue equalization procedures, <u>should</u> one neutralize the impact on the Federal Government, and <u>how</u> should one do so (viz., by an "offsetting" change in State and Local taxes, or an "offsetting" change in Federal taxes, or some combination of both?). The importance of such fiscal interdependence in an applied context is discussed by Kimbell, Shih and Shulman [1979].

[46] This is not strictly true -- some analytical approaches are algebraically

particular procedure is useful. In the present context we shall examine the incidence of a particular tax increase by the region 1 government under two assumptions:

a) that the tax revenue is spent on the production of the local public good in region 1; and

b) that a uniform "across-the-board" value-added change in region 1 government taxes occurs such that its revenue does not change (in numeraire terms) from the base case.

In the first simulation our results represent a combination of tax <u>and</u> expenditure incidence. In the second simulation we make an equal yield assumption with respect to the original taxing authority only (viz., the region 1 government); note that there will be some change in the expenditures of the region 2 government and the national government.

Our hypothetical economy is sufficiently similar to the model for which the Analytic Factor Price Solution is known to exist[47] that we may use that Solution to motivate the nature of some of our simulation results. The nature of factor specificity in our model is that the distribution parameter d(g,f) for factor f in the production function of good g is zero if the factor is unable to be used in that sector, and non-zero otherwise[48]. Recall from equation (2) earlier that if d(g,f)=0 for a particular factor and given g and f associated with a change in taxes T(g,f), then there is <u>no change</u> in the relative price

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intractable without some such assumption (e.g., our own Analytic Factor Price Solution concept, discussed earlier). Our present remarks refer more narrowly to numerical approaches.

[47] In our hypothetical economy government factor demands are <u>not</u> proportional to aggregate private industry factor demands.

[48] In our present model this parameter simply takes the value of zero or one.

of the non-used non-taxed factor[49]. This result appears in one form or another in the simulations reported below. Specifically, there is typically a block of factors whose relative factor prices do not vary greatly from the numeraire or at all from each other.

Tables 2 and 3 present the incidence of an increase, from 5% to 100%, in the region 1 government tax on the use of the fully mobile factor in the production of good 1 in region 1. As noted earlier, we distinguish (a) overall fiscal (tax and expenditure) incidence results, and (b) "differential" equal yield tax incidence results.

The tax increase leads to a shift in private good production away from good 1 in region 1 towards other commodities. In the "equal yield" simulation this shift significantly favours good 2 in region 1 more than the goods in the non-taxed region. These relative shifts are reflected also in the excise tax effects on goods prices. Note the absence of any excise tax effects within the non-taxed region.

The changes in factor prices are shown relative to the taxed factor -- there is a general increase in the relative price of all non-taxed factors. The System-Wide Separability of factor price effects on the taxed and non-taxed factors, discussed earlier in connection with our Analytic Factor Price Solution, is evident in the complete fiscal incidence results. Note the absence of any particular factor price capitalization effects on the factor specific to the tax base (good 1 in region 1).

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[49] If d(g,f)=0 for a given g and f then h(g,f)=0, as defined earlier. The expression M is given parametrically for a change in the taxes of any non-numeraire factor. Finally note that the only element of the summation in equation (2) that changes refers to the particular taxed good g, and that this element will be zero if the corresponding h(g,f) term is zero.

TABLE 2

Incidence of Tax on Fully Mobile Factor (I)

1. Percent change in demand for private goods.

	(a) Region 1	Region 2	(b) Region 1	Region 2
Good 1	-19.9	4.6	-9.0	1.6
Good 2	4.2	4.6	4.7	1.6

2. Percent change in goods prices.

	(a) Region 1	Region 2	(b) Region 1	Region 2
Good 1	22.1	6.8	9.8	3.9
Good 2	7.0	6.8	2.4	3.9

3. Percent changes in factor prices.

	(a) Regionally Mobile	Specific to R1	Specific to R2	(b) Regionally Mobile	Specific to R1	Specific to R2
Goods Mobile	0.0	10.2	9.4	0.0	7.5	4.7
Good 1 Specific	9.3	9.3	9.3	6.2	7.7	4.7
Good 2 Specific	9.3	9.3	9.3	6.2	7.7	4.7
Govt. Specific	24.2	55.3	13.4	3.6	1.2	4.7

When expenditure incidence is included there are sizeable increases in the prices of factors that are specific to government and regionally mobile. The factors that are specific to some region but are able to move into the production of public goods also show a small additional factor price increase. Moreover, this additional increase is higher for the region 1 goods-mobile factor, since it is able to be employed by the taxing authority.

Table 3 presents an account of the detailed movements of the various factors in response to the tax policy. The sectoral movement of each factor as a percentage of the aggregate endowment[50] reveals the extent of the general

[50] Which is constant over the various simulations.

equilibrium interactions present even in this simple hypothetical economy[51]. The mobile factor escapes the taxed sector, finding re-employment largely in the other private sectors. In the complete fiscal incidence simulation the region 1 government draws some of the government-specific regionally mobile factor away from the region 2 government and the national government.

Table 3 also presents an analysis of changes in factor use from the perspective of each private sector. The percent change in the use of each factor type is shown, as well as a decomposition of that change into substitution and expansion effects using the Factor Use Decomposition Formula.

In the complete fiscal incidence simulation the taxed sector uses 65.6% less of the mobile factor after the tax change than it did before, and each of the other private sectors use 19.4% more. In the equal yield simulation good 2 in region 1 uses 15.9% more of the mobile factor, whereas the region 2 sectors only use 9.7% more. Note in the latter simulation that the region 1 sectors use more of their sector-specific regionally mobile factor.

The interpretation of these percent changes is straightforward with the Factor Use Decomposition Formula, introduced earlier as expression (1) and shown in Table 3 in log-ratio form. We are able to distinguish substitution and expansion effects on factor use. Consider the complete fiscal incidence simulation. The expansion effect shows a substantial decline in the production of good 1 in region 1[52] and a small increase in the production of all other private sectors. In the case of the fully specific factors we see that their

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[51] Note that the entire matrix of factor movements for the fully mobile factor sums to zero. Each of the <u>columns</u> of the region-specific sectorally mobile factor movements matrix sum to zero, and each of the <u>rows</u> of the sector-specific regionally mobile factor movements matrix sum to zero.

[52] This is revealed in the 19.9% decline in the demand for this good, noted in Table 2.

TABLE 3

Incidence of Tax on Fully Mobile Factor (II)

4. Factor Movements (as percent of aggregate endowment)

Mobile Factor

	(a) Region 1	Region 2	(b) Region 1	Region 2
Good 1	-15.9	4.7	-8.8	2.3
Good 2	4.7	4.7	3.9	2.3
S&L Govt.	1.2	0.2	0.02	0.08
Fed. Govt.	0.3		0.1	

Region-Specific Sectorally Mobile Factor

	(a) Region 1	Region 2	(b) Region 1	Region 2
Good 1	-0.8	-0.1	0.2	0.0
Good 2	-0.8	-0.1	0.2	0.0
S&L Govt.	1.7	0.1	-0.2	0.0
Fed. Govt.	-0.04	0.01	-0.2	0.0

Sector-Specific Regionally Mobile Factor

	(a) Region 1	Region 2	Fed.Govt.	(b) Region 1	Region 2	Fed.Govt.
Good 1	0.0	0.0	--	1.4	-1.4	--
Good 2	0.0	0.0	--	1.4	-1.4	--
Government	28.3	-8.3	-19.8	-2.3	1.1	1.3

5. Factor Changes for Each Private Producing Sector

(a) Sector	Factor Type 1	2	3	4	(b) Factor Type 1	2	3	4
Region 1								
- Good 1	-65.6	-0.01	-1.7	-0.01	-36.3	2.8	0.4	0.0
- Good 2	19.4	-0.01	-1.7	-0.01	15.9	2.8	0.4	0.0
Region 2								
- Good 1	19.4	-0.01	-0.1	-0.01	9.6	-2.8	0.02	0.0
- Good 2	19.4	-0.01	-0.1	-0.01	9.6	-2.8	0.02	0.0

6. Factor Use Decomposition

(a) Sector	Substitution Effects 1	2	3	4	Expansion Effects	(b) Substitution Effects 1	2	3	4	Expansion Effects
Region 1										
- Good 1	-0.85	0.22	0.20	0.22	-0.22	-0.36	0.12	0.10	0.09	-0.09
- Good 2	0.14	-0.05	-0.06	-0.04	0.04	0.10	-0.02	-0.04	-0.05	0.05
Region 2										
- Good 1	0.13	-0.05	-0.05	-0.05	0.05	0.08	-0.05	-0.02	-0.02	0.02}
- Good 2	0.13	-0.05	-0.05	-0.05	0.05	0.08	-0.05	-0.02	-0.02	0.02

LEGEND:

Factor Type	Description
1	Fully Mobile (between regions and sectors)
2	Sector-Specific Regionally Mobile
3	Sector-Mobile Regionally Immobile
4	Fully Specific (to a region and a sector)

substitution effects, by definition, offset the corresponding expansion effects exactly. Note also the extent to which the substitution effects for the two partially mobile factors roughly offset the expansion effect for their sector[53]. The (negative) substitution effect for the fully mobile factor in the taxed sector reinforces the (negative) expansion effect. In the non-taxed private sectors the same reinforcement occurs, albeit qualitatively different. The overall sign-pattern of the substitution effects is intuitive -- the taxed sector uses more of the non-taxed factor and less of the taxed factor, and vice versa for the non-taxed sectors.

Now consider the decomposition of factor use changes in the equal yield simulation. The sign-patterns of the expansion and substitution effects are qualitatively the same as in the complete fiscal incidence simulation. However, we see that the substitution effects more than outweigh the expansion effects in several cases (most notably for the sector-specific regionally mobile factor).

4.2 A SIMPLE EMPIRICAL MODEL

An empirical model of the California and U.S. economies is under development by the UCLA Business Forecasting Project. This model has been brought to a prototype form which includes a 10-sector intermediate good matrix for California and the Rest of the U.S., with explicit trade between the two regions. It represents consumer demand with only one household type per region. Since it lacks any distribution of income, it cannot have any meaningful progressive income tax system. Eventually, the model will be used to evaluate the impacts

[53] This approximate offset lies at the heart of the System-Wide Separability Property, and illustrates why our Factor Price Revision Rule incorporates the "first-order" properties that are exact in the Analytic Factor Price Solution.

of a major tax cut in property taxes in California, known as Proposition 13. At present we limit our focus to a reduction in factor taxes on capital.

The empirical representation of California and the Rest of the U.S. (ROUS) uses the 10 sectors shown in Table 4. Each industry typically buys from each other industry in its own region and from industries in the other region. There are five sources of final demands by industry: consumers in each region, state and local governments in each region, and the federal government[54]. For convenience and lack of data, consumers and state and local governments are assumed to buy goods directly only from their own region. Since these industries use intermediate goods from the other region there is, of course, induced demand for the output of industries in the other region as well. The federal government demands goods from both regions.

Each industry uses two primary substitutable factors: capital, which is mobile across regions, and own-region labor. There are three factors: capital, California labor, and ROUS labor.

The case we examine is one in which the ad valorem factor tax rates levied by state and local governments in California on capital used by industries producing goods in California are reduced by 50 percent. The 50 percent reduction in the ad valorem tax rate on capital used in industries in California, made by state and local governments in California, leads to an increase in the price of the mobile capital factor relative to wages in California. Alternatively, since the price of capital is the numeraire, California wages fall 4.3 percent with the tax rate reduction.

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[54] Exports and imports from foreign countries are not explicitly modelled, but are included in consumption.

TABLE 4

Impacts of Capital Tax Reduction on Relative Prices

Percent change relative to numeraire.

Factor/ Sector	California	Rest of U.S.
	Factor Prices	
Capital	0.0	
Labor	-4.3	.005
	Output Prices	
Agriculture	-2.7	-.054
Mining	-3.5	-.018
Construction	-3.4	-.001
Food Processing	-4.6	-.003
Non-Durables	-4.5	-.050
Durables	-3.4	-.030
Trans. & Util.	-4.1	-.041
Real Estate	-6.7	-.015
Services	-4.3	-.024
Public Enter.	-3.8	-.006

Note: Capital is the numeriare, hence its price does not change. Moreover, it is not region-specific.

Induced wage effects in the Rest of the U.S. are negligible -- wages in the ROUS rise 0.005 percent relative to the price of capital. The capitalization effects are therefore strongly concentrated in the mobile factor and the California specific factor.

If the tax reduction induced full capitalization in the price of capital then there would be no excise tax effects. However, the mobility of capital leads to excise tax effects, especially for goods produced in California. The prices of goods produced in California fall (relative to the price of capital) for two reasons. First, the reduction in factor taxes lowers their costs of production even without changes in factor prices. Second, wage rates drop. Industries that are relatively capital intensive (and which have relatively high capital factor taxes) receive proportionately higher reductions in capital costs, but benefit less from the wage reduction. Industries that are relatively labor intensive also receive benefits in reduced wage costs. The largest price reduction in California industries is for Real Estate which is highly capital intensive.

Output prices in the ROUS drop slightly even though there are no directly relevant tax changes, no change in the price of capital, and a very slight increase in the price of labor in the ROUS. The reason is that industries in the ROUS use intermediate goods imported from California, and the prices of these goods drop (as discussed above).

Section 5

CONCLUSION

Two major analytic traditions, those of Harberger-Mieszkowski-McLure and Scarf-Shoven-Whalley, form the basis of our efforts to develop an empirical model of the incidence of fiscal policy on the California economy. Regional tax incidence theory leads naturally to the consideration of factors specific to regions and/or industries. This increase in numbers of factors, however, has previously been severely constrained by computational costs that increase rapidly with more dimensions. Our pursuit of techniques for dealing with large dimensionality led to the discovery of an admittedly special case that requires no computational search, since the exact algebraic solution is available. Explorations of a number of regional tax incidence issues can be pursued without the use of computer simulations if these restrictions are appropriate. Furthermore, in more general cases the iterative algorithm suggested by the closed-form solution provides a rapid and cost-effective solution technique.

The analytic formulae, and the applications presented, show a strong tendency for the first order effects of factor tax changes to concentrate in blocks of variables, with variables outside of the directly affected sectors changing relative to the directly affected variables but not relative to one another. A large tax cut in one given region, such as the property tax reductions involved in Proposition 13, does not appear to propagate to other regions with significant force in the model specified. Other issues of regional fiscal incidence, specifically those that affect trade between regions or nations, need not necessarily show such strongly concentrated impacts.

Analysts of regional taxation and expenditure issues have been constrained by a lack of empirical models that stem from the Harberger-Mieszkowski-McLure tradition. Available models, however well they may serve specific policy purposes such as short-run revenue forecasting, generally lack solid analytic foundations and strong equilibrating market-like mechanisms. Our purpose is to enlarge the techniques regional fiscal analysts can use to examine the important longer-run issues that general equilibrium models seems well suited for. With the aid of such techniques, a great deal of applied work remains to be done within existing theoretical and empirical general equilibrium traditions.

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