

Straight Time and Overtime in Equilibrium*

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1. INTRODUCTION

This paper describes a dynamic equilibrium model of straight-time and over-time shift work, one that is intended to be consistent with the models of shift work used earlier by Lucas [1970], Sargent and Wallace [1974], and Sargent [1978]. Those earlier papers did not describe an equilibrium, but instead were devoted to analyzing one side of a market in which a firm confronts a given real wage process and an exogenously given wage premium schedule for shift work. One purpose of the present paper is to describe an equilibrium setting in which the premium schedule emerges from the interaction of technology and preferences. In addition to endogenizing the premium schedule, our model building efforts aim to explain several features of the second moments of U.S. postwar time series of aggregates, as summarized by a vector autoregression. We are interested in interpreting the observed patterns of response to innovations in measures of straight time employment, overtime employment, capital, and consumption. In particular, we want to interpret how overtime employment appears to adjust more rapidly than straight time employment. Simultaneously, we also seek to interpret the observations on real wage movements, labor productivity, and the capital labor ratio stressed by Lucas [1970].^{1,2} We also use our model to interpret some of Trejo's [1986] recent empirical findings on the irrelevance of the overtime provisions of the Fair Labor Standards Act.

Our model is a linear-quadratic stochastic optimal growth model, a descendent of earlier models of Brock and Mirman [1972], Kydland and Prescott [1982] and Hansen [1985]. The model is in the tradition of "real business cycle models" since the time series generated by this model display fluctuations in response to an exogenous technology shock. One key substantive modification relative to those earlier models is that our model incorporates a version of Lucas's work-shift structure. Another modification is that we use a version of the multivariate signal extraction problem to model formally the vector autoregression of

the data available to econometricians, which we assume to be corrupted by measurement errors. Measurement errors and productivity shocks interact in an intricate way to generate both the innovations and the predictable part of the vector autoregression computed by the econometrician.

Our linear-quadratic optimal stochastic growth model is itself parameterized profusely. However, we use a device of Kydland and Prescott [1982] to induce a mapping which makes the many parameters of the linear-quadratic model a function of a much smaller number of interpretable parameters. This smaller set of parameters describes the preferences and technology of a parsimoniously parametrized nonlinear stochastic optimal growth model. We view the underlying nonlinear model as an instrument for deriving the associated linear-quadratic model, the latter model being the one whose statistical properties we shall describe precisely. If the parameters of the underlying nonlinear model are to be interpretable, the linear-quadratic model has to be a good approximation to it, in a sense that we shall make precise. We propose and implement (in the appendix) some measures of the quality of this approximation.

2. THE MODELING STRATEGY

We have created a model with a small number of economically interpretable parameters describing preferences, technology and measurement error processes. Although the number of interpretable parameters is small, the model restricts a very large number of second moments of the observed processes. The model is created in six steps. First, a nonlinear, recursive social planning problem is described, the solution of which is the competitive equilibrium for a stochastic growth model. Second, a quadratic approximation around the stationary state of an associated nonstochastic model is taken, and used to construct a linear-quadratic social planning problem. The objective function of the quadratic problem

has many parameters, but they are each functions of a much smaller number of parameters in the original problem. Third, the linear-quadratic social planning problem is solved for an optimal linear feedback law. Fourth, linear approximations are formed expressing each observable (output, consumption, wage rates, etc.,) of interest as a linear function of the state variables. The second, third and fourth steps combined give a “linear systems theory” representation for the observables. Fifth, this linear systems model is augmented by a linear model for measurement errors in the observables. Sixth, a Wold-moving average representation is obtained for the error-ridden observables of the model. This Wold-moving average representation determines the population vector autoregression for the observables.

These steps are motivated by our desire to build a model with a small number of interpretable parameters that restrict a large number of second moments of observables. Our focusing on second moments, and our wish for tractability explain our contentment with a linear model. Our desire for a model with a small number of interpretable parameters explains our choosing to begin with something other than a linear-quadratic model. There are approximations at two steps of the construction described above, at the second and fourth steps. In the appendix, we describe some ways of measuring the errors induced by these approximations. It is necessary that each of these approximation errors be small if interpretations of our results in terms of the parameters of the original nonlinear-quadratic model are to be valid.³

This modeling strategy interprets the population moments of the observables in terms of two sorts of parameters: (a) parameters describing the technology and preferences that determine the behavior of the *economic agents* in the model, and (b) parameters describing the second moments of errors in the *measurements* available to the econometrician.

3. AN EQUILIBRIUM MODEL OF SHIFT PREMIA

The artificial economy is populated by a continuum of identical households on the unit interval. Each household has preferences given by the utility function

$$U(c_t, l_t) = \log c_t + A \log l_t, \quad (1)$$

where $A > 0$, c_t is consumption at date t , and l_t is leisure at date t . The endowment of leisure is unity in each period, which constrains the choice of l_t to satisfy $0 \leq l_t \leq 1$. The choice of l_t is further restricted by the following specification of work-shifts. Let $1 > h_1 + h_2 > h_1 > 0$. Here h_1 is the length of a "straight-time shift," while h_2 is the length of an "overtime shift." The shift lengths h_1 and h_2 are taken as given.⁴ At each date t , the household's consumption set is restricted along the leisure axis so that only three selections of leisure are feasible: $l_t = 1$, residing in a state of unemployment; $l_t = 1 - h_1$, working straight time; and $l_t = 1 - h_1 - h_2$, working straight time plus the overtime shift.⁵

Following Richard Rogerson [1984] and Gary Hansen [1985], we convexify the consumption set by adding employment lotteries to the commodity space. In particular, we suppose that during period t the representative household chooses a probability π_{1t} of working h_1 hours, a probability π_{2t} of working $h_1 + h_2$ hours, and a probability $1 - \pi_{1t} - \pi_{2t}$ of working zero hours. The household's expected utility is then⁶

$$\begin{aligned} & \pi_{1t} [\log c_t + A \log(1 - h_1)] \\ & + \pi_{2t} [\log c_t + A \log(1 - h_1 - h_2)] \\ & + (1 - \pi_{1t} - \pi_{2t}) [\log c_t + A \log 1]. \end{aligned}$$

Rearranging the above expression shows that expected utility is

$$\log c_t + \pi_{1t} A \log(1 - h_1) + \pi_{2t} A \log(1 - h_1 - h_2). \quad (2)$$

Ex post π_{2t} is the fraction of people working $(h_1 + h_2)$ hours, which we denote n_{2t} . This is the fraction of individuals who work overtime. Similarly, $\pi_{1t} + \pi_{2t}$ is the fraction

of people who work during the first h_1 hours in period t . We denote this fraction as n_{1t} .⁷ With this notation, the preference function (2) of the representative agent can be written

$$\log c_t - a_1(n_{1t} - n_{2t}) - a_2 n_{2t} \quad (3)$$

where

$$a_1 = -A \log(1 - h_1) \quad \text{and} \quad a_2 = -A \log(1 - h_1 - h_2). \quad (4)$$

In addition to the continuum of households, there is a single firm with access to a technology described by

$$c_t + x_t \leq y_t \quad (5)$$

$$y_t = z_t k_t^\theta \left\{ h_1 \left[n_{1t} - \frac{d}{2} (n_{1t} - n_{1t-1})^2 \right]^{1-\theta} + h_2 n_{2t}^{1-\theta} \right\}, \quad d > 0, \quad 0 < \theta < 1 \quad (6)$$

$$k_{t+1} = (1 - \delta)k_t + x_t \quad 1 > \delta > 0 \quad (7)$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1} \quad 1 > \rho > 0 \quad (8)$$

In (6), z_t is a technology shock governed by a first order autoregression (8), where ϵ_t is distributed identically and independently across time according to a log-normal distribution with mean $1 - \rho$ and variance σ_ϵ^2 . This implies that the unconditional mean of z is one. In (7), x_t is a gross investment rate, and k_t is the capital stock. The right side of (6) is a Cobb-Douglas production function, where effective units of straight time labor are given by $n_{1t} - \left(\frac{d}{2}\right) (n_{1t} - n_{1t-1})^2$. The term $\frac{d}{2}(n_{1t} - n_{1t-1})^2$ is subtracted from n_{1t} in order to model costs of rapidly adjusting straight time employment. We have included this term with $d > 0$ since otherwise (if $d = 0$) the model has the property that n_{2t} is a constant multiple of n_{1t} . This implies that the ratio of n_{1t} to n_{2t} does not vary over the cycle. This, however, is not consistent with findings from U.S. time series data (see section 4). The right side of (6) expresses output at t as the sum of output obtained from the straight time shift, $(h_1 z_t k_t^\theta \left[n_{1t} - \frac{d}{2} (n_{1t} - n_{1t-1})^2 \right]^{1-\theta})$, and output obtained from the overtime shift,

$(h_2 z_t k_t^\theta n_{2t}^{1-\theta})$. We have assumed that it is costly to adjust straight time employment, but not to adjust overtime employment. This assumption is reasonable given that overtime workers are selected from among those working straight time, so that the adjustment costs related to their employment have already been borne. The right side of equation (6) is a version of the production function used by Lucas [1970], Sargent and Wallace [1974] and Sargent [1978], which is obtained by integrating an instantaneous production function over the "day." [Note the way the shift lengths h_1 and h_2 appear in (6)].

The social planning problem is to choose a contingency plan for $(c_t, x_t, n_{1t}, n_{2t})$ to maximize⁸

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log c_t - a_1(n_{1t} - n_{2t}) - a_2 n_{2t}] \quad (9)$$

subject to (4), (5), (6) (7) and (8). This is a recursive problem that can in principle be solved using discounted dynamic programming. The *state* for this problem is the vector $(1, z_t, k_t, n_{1t-1}) = X_t$. The solution of the problem is a time-invariant decision rule expressing $(c_t, x_t, n_{1t}, n_{2t})$ as a function of the state $(1, z_t, k_t, n_{1t-1})$.

We use the preceding social planning problem as an instrument for deriving a linear-quadratic model. We use the method of Kydland and Prescott [1982] to derive the linear-quadratic model by proceeding as follows. First, form a nonstochastic version of the above problem, by replacing z_t in (6) for all t with its unconditional expectation Ez . Obtain the first order conditions for this problem and solve for the stationary point. Second, substitute the constraints (5) and (6) into the one-period return function (3), and take the first two terms of a Taylor series expansion about the stationary point as an approximation to the return function. Let the *controls* be denoted $u_t = (n_{1t}, n_{2t}, x_t)$. Then the second order Taylor series approximation about the stationary state yields the approximation

$$\begin{aligned} & \log c_t - a_1(n_{1t} - n_{2t}) - a_2 n_{2t} \\ & \approx X_t' q X_t + u_t' r u_t + 2u_t' w X_t \end{aligned} \quad (10)$$

where q , r , and w are matrices determined by the Taylor series. Note that this procedure determines the matrices q , r , and w as functions of the parameters (A, h_1, h_2, θ, d) . The transition law of the state is linear and is given by

$$\begin{bmatrix} 1 \\ z_{t+1} \\ k_{t+1} \\ n_{1t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & (1-\delta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ z_t \\ k_t \\ n_{1t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} n_{1t} \\ n_{2t} \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \epsilon_t \quad \text{or}$$

$$X_{t+1} = aX_t + b u_t + g\epsilon_t \quad (11)$$

Given the preceding definitions of (q, r, w, a, b, g) , we study the social planning problem, to maximize

$$E \sum_{t=0}^{\infty} \beta^t \{ X_t' q X_t + u_t' r u_t + 2u_t' w X_t \} \quad (12)$$

subject to (11). This is a discounted optimal regulator problem whose solution is the time-invariant decision rule

$$u_t = -f X_t. \quad (13)$$

where $f = (b' P b + r)^{-1} (a' P b + w)$ and P is the unique limit of iterations on the matrix Ricatti equation

$$P_{t+1} = q + a' P_t a - (a' P_t b + w') (r + b' P_t b)^{-1} (b' P_t a + w),$$

as $t \rightarrow \infty$, starting from $P_0 = [0]$. Substituting (13) into (11) gives the "closed loop system"

$$X_{t+1} = (a - b f) X_t + g \epsilon_{t+1}. \quad (14)$$

Given X_t and $u_t = -f X_t$, the values of a variety of other variables can be determined. Equations (6) and (5) can be used to solve for y_t and c_t as functions of the elements of

(X_t, u_t) . The straight time wage rate, w_{1t} , is obtained as h_1^{-1} times the derivative with respect to n_{1t} of the right side of (6). The overtime wage rate w_{2t} , is obtained as h_2^{-1} times the derivative with respect to n_{2t} of the right side of (6).⁹ The variable manhours is then obtained via

$$h_t = h_1 n_{1t} + h_2 n_{2t} \quad (15a)$$

while average hourly earnings is obtained by

$$\bar{w}_t = (h_1 w_{1t} n_{1t} + h_2 w_{2t} n_{2t}) / h_t. \quad (15b)$$

Average productivity is given by

$$pr_t = y_t / h_t. \quad (16)$$

The preceding paragraph describes how to obtain the variables $(y_t, c_t, w_{1t}, w_{2t}, \bar{w}_t, h_t, pr_t)$ as *nonlinear* functions of the state vector X_t . In order to reap the benefit of linear systems theory, it is convenient to replace these nonlinear functions with linear ones. We can accomplish this by taking linear approximations about the steady state of each of the functions described in the preceding paragraph. Doing this gives the approximation¹⁰

$$\begin{bmatrix} y_t \\ c_t \\ w_{1t} \\ w_{2t} \\ h_t \\ \bar{w}_t \\ pr_t \end{bmatrix} = H X_t \quad (17)$$

Our model for the observable time series is formed by combining (13), (14) and (17).

That is, our model is

$$X_{t+1} = (a - bf)X_t + g \epsilon_t \quad (14)$$

$$\begin{bmatrix} y_t \\ c_t \\ w_{1t} \\ w_{2t} \\ h_t \\ \bar{w}_t \\ pr_t \\ n_{1t} \\ n_{2t} \\ x_t \end{bmatrix} = C X_t \quad (18)$$

where $C = \begin{bmatrix} H \\ -f \end{bmatrix}$. Let Y_t' denote the vector $(y_t, c_t, w_{1t}, w_{2t}, h_t, \bar{w}_t, pr_t, n_{1t}, n_{2t}, x_t)$. Then the model can be represented as

$$X_{t+1} = (a - bf)X_t + g\epsilon_{t+1} \quad (14)$$

$$Y_t = C X_t \quad (18)$$

Letting L be the lag operator, (18) and (14) imply the following model for the observables

$$Y_t = C[I - (a - bf)L]^{-1}g \epsilon_t \quad (19)$$

or

$$Y_t = w(L)\epsilon_t \quad (20)$$

where $w(L) = C[I - (a - bf)L]^{-1}g$. Since ϵ_t is a *scalar* white noise, (20) expresses the (10×1) vector stochastic process Y_t as a (10×1) vector ($w(L)$ is 10×1) of distributed lags of the single white noise ϵ_t . Representation (19) or (20) implies that the stochastic process for Y_t is singular — the spectral density of $\{Y_t\}$ is of rank one at all frequencies. This implies that each variable within Y_t can be expressed as an *exact* ($\bar{R}^2 = 1$) linear function of past, present and future values of any other variable within Y_t . This strong implication is untenable.

The implication that Y_t is a process whose spectral density is of rank one can be avoided if we assume that all or most components of Y_t are measured with error. Specifically, we posit that the measured data Y_t^m are linked to the true variables Y_t by the model

$$Y_t^m = Y_t + v_t \quad (21)$$

where

$$v_t = Dv_{t-1} + \eta_t \quad (22)$$

Here D is a matrix of parameters governing the serial correlation of the measurement errors v_t , and η_t is a vector white noise satisfying

$$E\eta_t\eta'_{t-s} = \begin{cases} O & s \neq 0 \\ R & s = 0 \end{cases}$$

where R is a positive definite matrix. We also assume that η_t is orthogonal to ϵ_s for all t and s .

Combining (19), (21), and (22) yields the following model for the observables

$$Y_t^m = C[I - (a - bf)L]^{-1}g \epsilon_t + (I - DL)^{-1}\eta_t, \quad (23)$$

which is in the form of a one-unobservable index model.¹¹ From (23) one can obtain the Wold representation for Y_t^m , which is of the form

$$Y_t^m = \gamma(L)a_t \quad (24)$$

where a_t is a (10×1) vector white noise with

$$a_t = Y_t^m - \hat{E} \left[Y_t^m | Y_{t-1}^m, Y_{t-2}^m, \dots \right],$$

$$\gamma(L) = \sum_{j=0}^{\infty} \gamma_j L^j, \quad \gamma_0 = I, \quad \sum_{j=0}^{\infty} \text{tr} \gamma_j \gamma_j' < +\infty.$$

Operating on both sides of (24) with the inverse of $\gamma(L)$ gives the vector autoregression for Y_t^m

$$\gamma(L)^{-1}Y_t^m = a_t, \quad (24')$$

where a_t is the innovation in the vector Y_t^m . For the economy under study, empirical estimates of vector autoregressions and "innovation accountings" (see Sims [1980]) converge to the corresponding objects in representation (24) or (24'), as the sample size grows. The second moments of the Y_t^m process are characterized by representation (24) in a form convenient for linear prediction.

Sargent [1986] describes recursive calculations for obtaining the parameters of $Ea_t a_t' = S$ and $\gamma(L)$ as a function of the parameters $\{\sigma_\epsilon^2, (a - bf), R, D\}$. These calculations define

a mapping from the economically interpretable parameters underlying $\{\sigma_\epsilon^2, (a - bf), R, D\}$ to the parameters of $(\gamma(L), S)$ that describe the second moments of the observable data.

The next section of this paper describes some of the second moments of series for n_{1t} and n_{2t} synthesized from U.S. time series, as described by a vector autoregression. The succeeding section describes an illustrative equilibrium for our model.

4. SOME EVIDENCE

In this section we describe the evolution of some empirical counterparts to the variables n_{1t} and n_{2t} as implied by a vector autoregression. In the next section, we will actually compute an example equilibrium for our artificial economy which delivers time series for n_{1t} and n_{2t} whose stochastic behavior can be compared to those summarized by the empirical vector autoregression.

We synthesized time series for n_{1t} and n_{2t} by using data from the Department of Commerce's *Current Population Survey* on total hours at work for all industries (THRS_t) and average weekly hours for all industries (AVHRS_t).¹² We interpret THRS_t as measuring $n_{1t}h_1 + n_{2t}h_2$, and AVHRS_t as measuring $(n_{1t}h_1 + n_{2t}h_2)/n_{1t}$. Consequently, we estimate n_{1t} as

$$n_{1t} = \frac{\text{THRS}_t}{\text{AVHRS}_t}, \quad t = 47, 1; \dots; 85, 2$$

Then for given estimates for h_1 and h_2 , we can obtain n_{2t} from

$$n_{2t} = \frac{n_{1t}(\text{AVRS}_t - \hat{h}_1)}{\hat{h}_2}, \quad t = 47, 1; \dots; 85, 2.$$

We estimated \hat{h}_1 and made up a value for \hat{h}_2 . To obtain an estimate of h_1 , we noted that average weekly hours worked by production workers averaged 40 hours over the period 56,1 to 85,3, while average weekly over-time hours of production averaged 3 hours. In other

words, $\hat{h}_1 + \frac{n_{2t}}{n_{1t}} \hat{h}_2$ averaged to 40 while $\frac{n_{2t} \hat{h}_2}{n_{1t}}$ averaged to 3, leading us to set \hat{h}_1 , at 37. The data we are using are silent about \hat{h}_2 ; we arbitrarily set \hat{h}_2 at 10.

Figure 1 plots the time series for n_{1t} and n_{2t} . The figure reveals that n_{2t} fluctuates more than n_{1t} . This greater smoothness in n_{1t} than n_{2t} lead Sargent [1978] to the interpretation that adjustment costs for straight-time employment exceed those for over-time employment. We can summarize this differential smoothness in a different way by inspecting representations associated with a vector autoregression. Figures 2 and 3 show the response of n_{1t} , n_{2t} and k_t to an innovation in y_t . To produce these plots, a vector autoregression was estimated for a vector Y_t , consisting of real GNP, n_{1t} , n_{2t} and k_t .¹³ Next, a Wold moving average representation (MAR), which can be viewed as estimating a version of eq. (24) for this system, was formed by inverting the estimated autoregressive representation. The components of the innovations in the Wold MAR are contemporaneously correlated. We use the method suggested by Sims (1980) to transform the Wold MAR to a MAR which is in terms of innovations that have orthogonal components. This method is sensitive to the ordering of the four variables in the vector Y_t . We placed y_t first in the ordering. We then calculated the response of the system to an innovation in y_t .

Figures 2 and 3 show that the response of capital is most drawn out and the response of overtime is the least. In addition, the response of overtime employment is more immediate than the response of straight time employment. Our model interprets this last pattern by setting $d > 0$.

5. AN EXAMPLE OF AN EQUILIBRIUM

This section describes some sample calculations designed to illustrate our model, and some of its potential eventually to be a vehicle for interpreting data. We proceeded by

making informed “guesses” at the values of the underlying parameters of the nonlinear growth model, and the parameters of the measurement error processes. So far, we have not adjusted these parameters in an effort to get a better match to the data. Later, we plan to pursue such adjustments ruthlessly, by setting down a metric (*e.g.*, one induced by a likelihood function) and estimating parameters by finding values that make the metric attain a minimum. The calculations described here are intended as a prolegomenon to pursuing estimation, and as a way of indicating whether the model might match the data.

We set the parameters of the nonlinear growth model at the following values:¹⁴

$$h_1 = 0.46 \quad h_2 = 0.13$$

$$\theta = 0.36 \quad d = 15$$

$$A = 2 \quad \rho = 0.95$$

$$\beta = 0.99 \quad \delta = 0.025$$

$$\sigma_\epsilon = 0.0075$$

The associated linear-quadratic social planning problem has the following parameters:

$$r = \begin{bmatrix} -10.3595 & -0.3394 & 1.0087 \\ -0.3394 & -0.7224 & 0.4508 \\ 1.0087 & 0.4508 & -1.3398 \end{bmatrix}$$

$$q = \begin{bmatrix} 0 & 1.5767 & 0.0674 & 0 \\ 1.5767 & -0.9043 & -0.0099 & 0 \\ 0.0674 & -0.0099 & -0.0038 & 0 \\ 0 & 0 & 0 & -9.2428 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.8287 & 0.3704 & -2.7376 \\ -0.2125 & -0.0950 & 1.1007 \\ -0.0091 & -0.0041 & 0.0470 \\ 9.2428 & 0 & 0 \end{bmatrix}$$

Recall that the state is the vector $X_t = [1, z_t, k_t, n_{1t-1}]$ while the controls are $u_t = [n_{1t}, n_{2t}, x_t]$. The optimal linear decision rule is given by the parameters of

$$f = \begin{bmatrix} -0.0452 & -0.1229 & 0.0055 & -0.8044 \\ -0.0043 & -0.2902 & 0.0112 & 0.0420 \\ 0.7807 & -0.7681 & 0.0132 & -0.5383 \end{bmatrix}$$

The optimal closed loop system is given by

$$(a - bf) = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0.0500 & 0.9500 & 0 & 0 \\ -0.7807 & 0.7681 & 0.9618 & 0.5383 \\ 0.0452 & 0.1229 & -0.0055 & 0.8044 \end{bmatrix}$$

The eigenvalues of $(a - bf)$ are equal to $(1, 0.95, 0.9398, 0.8264)$. The stationary values of the state variables described by the nonstochastic linear closed loop system $X_{t+1} = (a - bf)X_t$ can be determined to be $[1, 1, 8.4257, 0.6207]$. These are the same as the stationary values of the state variables of the original nonlinear nonstochastic growth model.

These stationary values of the nonstochastic linear closed loop system determine the unconditional means of the variables generated by the stochastic closed loop linear system $X_t = (a - bf)X_{t-1} + g\epsilon_t$. For some of the key variables of our system, these means are as follows:

| | |
|-------|--------|
| k | 8.4257 |
| n_1 | 0.6207 |
| n_2 | 0.1737 |
| w_1 | 1.6366 |
| w_2 | 2.5884 |

Note that the ratio of the mean of w_2 to the mean of w_1 is 1.58, which is to be compared to the “time and a half” for overtime that we seek to explain. To us, it is encouraging that we get close to “time and a half” with little coaxing.

Figure 4 displays the response functions of k_t and z_t to an innovation ϵ_t . That is, Figure 4 displays the second and third rows of the operator $[I - (a - bf)L]^{-1}g$, tracing out the response of the state to an innovation ϵ_t in the technology stock. Figure 5 gives the corresponding responses for z_t again, and also n_{1t} and n_{2t} . Notice that the response of k_t to an innovation ϵ_t is the most drawn out, followed by n_{1t} , while the response of n_{2t} is the shortest lived. Compare this to responses in Figures 2 and 3. The relatively quicker

response of n_{2t} than n_{1t} is a consequence of the adjustment cost specification, and the relatively large value for d of 15. Compare the first and second rows of f , which indicate that n_{1t} is much more highly dependent on lagged n_{1t} than is n_{2t} (a coefficient of -0.8044 compared with $.0420$). This quicker response of n_{2t} to an innovation ϵ_t makes n_2 less highly positively serially correlated than n_{1t} .

Figures 6, 7 and 8 display some of the results of simulating the system(19) above, namely

$$Y_t = C[I - (a - bf)L]^{-1}g \epsilon_t, \quad (19)$$

where ϵ_t is a log-normally distributed white noise with mean zero and variance $(0.0075)^2$. Figure 6 plots realizations of y_t, c_t and x_t . Figure 7 plots realizations of straight time and overtime employment n_{1t} and n_{2t} . Figure 7 can be compared to Figure 1 which is constructed using actual data. Figure 8 plots realization of w_{1t} and w_{2t} . Note how c_t appears to be a less volatile series than x_t . Reflecting this, the coefficient of variation of x_t is 0.0632, while that of c_t is 0.0134.¹⁵ The coefficient of variation of n_{1t} is 0.0101 while that of n_{2t} is 0.0202, a reflection of the impulse response functions reported in Figure 5. (At this point, the volatility of n_2 given the chosen parameter values is too small to be consistent with the data described in section 4. Also, at this point, the *amplitude* of the response of n_1 in Figure 5 is too large relative to the amplitude of the response of n_2 to be consistent with the data.)

We now introduce measurement errors into our example. Let the vector of measured series be

$$Y_t^m = \begin{bmatrix} k_t^m \\ n_{1t}^m \\ n_{2t}^m \\ y_t^m \\ c_t^m \\ x_t^m \end{bmatrix},$$

and assume the model (21)–(22). We set the matrix D describing the autoregression of measurement errors v_t equal to a matrix of zeros, except for the (1,1) element which we

set equal to 0.95. Thus, the measurement error in each variable was assumed serially independent, except for the error in measuring capital, which followed the process

$$v_t^k = 0.95v_{t-1}^k + \eta_t^k$$

We set the measurement error variances to be the following fractions of the variances of the *true* variables:

Parameters of R matrix

| variable | measurement error variance as a fraction of variance in true variables |
|----------|---|
| k | 0.15 |
| n_1 | 0.05 |
| n_2 | 0.05 |
| y | 0.01 |
| c | 0.07 |
| x | 0.07 |

Thus, we assumed that output is best measured, while c , x , and k are relatively poorly measured. We assumed that the off-diagonal elements of the matrix R are all zero, so that the measurement errors are orthogonal.¹⁶

Having set the parameters of the multivariate signal extraction model described above, we performed the calculations described in Sargent [1986] to compute the Wold representation (24), namely

$$Y_t^m = \gamma(L)a_t, \tag{24}$$

where a_t is an innovation process for Y_t^m .

Figure 9 depicts a simulation of the error ridden series y_t^m , c_t^m and x_t^m . This is a simulation of system (24), where the shocks a_t are drawn from a multivariate normal distribution with covariance matrix $Ea_t a_t'$. (The covariance matrix $Ea_t a_t'$ is determined

during the process of computing the Wold representation (24).) The standard deviations of the innovations in Y_t^m (the square roots of the diagonal elements of $Ea_t a_t'$) are given by [0.0173, 0.0015, 0.0016, 0.0051, 0.0024, 0.0050]. Figure 10, 11 and 12 depict parts of $\gamma(L)$ associated with responses to one-standard deviation innovations in measured output, consumption, and investment, respectively.

Figures 10, 11 and 12 indicate that the best measured series — output — displays the strongest feedback to other series. This stronger feedback is a consequence of the assumed values of the measurement error (see Sargent [1986]). These figures indicate that this model is one for which the data on Y_t^m would spuriously appear to conform to an investment accelerator, for reasons described by Sargent [1986].

6. WAGES AS VALUES OF ALTERNATIVE BUNDLES OF STATE CONTINGENT COMMODITIES AND THE IRRELEVANCE OF THE FAIR LABOR STANDARDS ACT.

In the version of the model studied in section 5, time and a half for overtime is close to being an equilibrium outcome. In point of fact, in the U.S., time and a half for overtime is imposed on specified industries by the Fair Labor Standards Act (FLSA). Our interpretation of time and a half for overtime as a competitive equilibrium outcome is interesting only if it can be argued that the FLSA is redundant, in the sense that it imposes no restrictions on equilibrium allocations or shadow prices. Stephen Trejo [1986] has studied the hypothesis that the FLSA is redundant in just this sense, and has implemented empirical tests of the hypothesis which he interprets as containing some evidence against the hypothesis.

In this section, we reinterpret some of Trejo's empirical findings in light of our model and argue that more must be known about the particular state contingent bundle that

the wage data are regarded as pricing before Trejo's finding can be regarded as evidence against the redundancy of the FLSA. This exercise highlights some hazards that occur more generally in interpreting wage data, due to the alternative possible "bundles" of state contingent payments which a given wage series can be interpreted as representing.

In our model, one decentralization and pricing scheme that can support the social planning solution occurs when workers are paid a straight time real wage of w_{1t} for the first h_1 hours of work, and an overtime wage of w_{2t} when they work an additional h_2 hours. A worker who actually works overtime, thereby working $h_1 + h_2$ hours, then receives $w_{1t}h_1 + w_{2t}h_2$. Trejo notes that such a worker would be indifferent between this two-part compensation scheme, and simply receiving a single daily wage rate of $w_t^N \equiv (w_{1t}h_1 + w_{2t}h_2)/(h_1 + h_2)$ per hour worked. Notice that:

$$w_t^N = \frac{w_{1t} \left(h_1 + \frac{w_{2t}}{w_{1t}} h_2 \right)}{h_1 + h_2}. \quad (25)$$

Trejo uses the definition of w_t^N in (25) to motivate a test of the hypothesis that the legal imposition of $w_{2t}/w_{1t} = 1.5$ by the Fair Labor Standards Act is irrelevant. Trejo defines a variable

$$V = \frac{w_t^N}{w_{1t}} = \frac{h_1 + \frac{w_2}{w_1} h_2}{h_1 + h_2} \quad (26)$$

where for Trejo $w_2/w_1 = 1.5$. Notice that V can be computed solely from information about the shift lengths h_1 and h_2 . He then represents (25) as

$$\log w_1 = \log w^N - \log V. \quad (27)$$

On the hypothesis that the legal restriction is irrelevant, (27) represents the relation between the straight time wage w_1 in an industry covered by the FLSA, and the single daily wage w_t^N that could be paid by an uncovered industry.

Trejo proceeded to construct an empirical test as follows. For each worker i in a collection of N workers indexed by i , by using (27) and information on the shift lengths of

h_1 and h_2 worked by worker i he constructed a measure of V , call it V_i . He also had data on the straight time wage rate received by worker i , w^i ; a vector of worker characteristics X_i ; and a dummy variable d_i which took on the value 1 only if a worker was covered by FLSA and worked overtime, and zero otherwise. Trejo then computed the regression

$$\log w^i = X_i\beta + \alpha[\log V_i]d_i + \epsilon_i \quad (28)$$

where ϵ_i is a least squares residual. Equation (28) is intended to nest an econometric version of (27) as a special case. Here $X_i\beta$ is intended to capture wage differences across individuals due to factors other than whether or not the industry is covered under the FLSA. On the hypothesis that the restriction is irrelevant, Trejo expected α to be equal to -1 . In fact, he estimated α to be in the range of $-.57$ to $-.32$, depending on the year to which the data corresponded.

The wage w_t^N defined in (25) is to be interpreted as the wage that would be paid only to workers who actually work overtime. However, we can imagine another price system which supports our equilibrium allocation and which leads to another concept of average daily wage. In particular, suppose that all workers participate in the lottery and receive the single wage

$$w_t^h = \frac{w_{1t}h_1(\pi_{1t} + \pi_{2t}) + w_{2t}h_2\pi_{2t}}{h_1(\pi_{1t} + \pi_{2t}) + h_2\pi_{2t}}$$

or

$$w_t^h = \frac{w_{1t}h_1n_{1t} + w_{2t}h_2n_{2t}}{h_1n_{1t} + h_2n_{2t}}.$$

The numerator of w_t^h is the expected daily wage of workers who participate in the lottery, while the denominator is expected daily hours. All workers, not just those who actually work overtime, can be regarded as receiving the daily wage rate w_t^h .

In Trejo's regressions, what difference would it make in interpretation if the observed wage data in industries where time and a half is not paid to workers who work overtime corresponded to a measure of concept w^h rather than concept w^N ? We use our model at

the steady state values associated with the parameters described in section 5 to compute how w^h would be related to V (as defined by Trejo) at that steady state. The steady state values of the various variables are

$$\begin{aligned} h_1 &= .46, h_2 = .13, n_1 = .6219, n_2 = .1733, \\ w_1 &= 1.6361, w_2 = 2.5920, w^h = 1.7059, \\ w^N/w_1 &= V = 1.13. \end{aligned}$$

From these values we compute α from the equation

$$\log w_1 = \alpha \log V + \log w^h. \quad (29)$$

With the above values, we obtain $\alpha = -.35$. Had we revised our definition of V to be $\frac{w^h}{w_1}$ in (29), we would have obtained the value $\alpha = -1$ expected by Trejo. But, using Trejo's measure of V , we obtain a value of α in the upper end of the range of values actually obtained by Trejo.¹⁷

Thus, on the view that the recorded wage data are w_t^h , being payments for accepting exposure to the lottery of engaging in overtime work, Trejo's regressions contain little evidence against the hypothesis that the FLSA restrictions are redundant when interpreted in light of our model.

7. SUMMARY

This paper uses an idea of Rogerson [1984], as previously applied by Hansen [1985], in order to interpret observations on employment and hours. We restrict the feasible consumption set in a way designed to generate an equilibrium setting consistent with the specialization of Lucas' [1970] model used by Sargent and Wallace [1974] and Sargent [1978]. We impose up front, as an assumption, constant and exogeneous shift lengths h_1 and h_2 , and interpret this imposition in terms of a restriction on the feasible consumption

set. Thus, although our model is one of general equilibrium, it is hardly general as a specification.¹⁸ The economy moves along as though workers face lotteries across states of unemployment, straight-time employment, and straight-time plus overtime employment. For parameter values that were "made up," simulations of the model generate shadow prices for straight-time and overtime employment that exhibit a premium factor of about 1.58 for overtime.

APPENDIX

Approximation Quality

Let $V_q(X) = X'PX$ be the optimal value function for our linear-quadratic problem of maximizing (12) subject to (11). Now consider

$$V_1(X_t) = \max \{ \log c_t - a_1(n_{1t} - n_{2t}) - a_2n_{2t} + \beta E V_q(X_{t+1}) \} \quad (A1)$$

where the maximization is over $u_t = (n_{1t}, n_{2t}, x_t)$ and is subject to (5), (6), (7), and (8). Let the right side of (A1) be attained by the decision rule $u_t = \phi(X_t)$. The optimal linear rule given by the solution, (13), of the approximating problem is $u_t = -f X_t$. For various values of X_t , we form both $\phi(X_t)$ and $-f X_t$, and then compute a measure of the distance between the two decision rules. If the distance is large, we will regard the approximation to be a bad one. If the distance is small, we regard the experiment as not putting the approximation under suspicion.

To implement this procedure, we must choose a set of points, X_t , in the state space at which to compute $-f X_t$ and $\phi(X_t)$. To obtain a grid of points, we used a simulation of 500 points from the optimal closed loop system derived in section 5.¹⁹ This procedure has the advantage of assigning weight to regions of the state space according to the probability of visiting them.

To measure the distance between corresponding components of $-f X_t$ and $\phi(X_t)$ we used four measures of distance. Letting d be the decision for a given decision variable from rule $-f X_t$ and d' be the decision from $\phi(X_t)$, we define for each decision variable (n_1, n_2, x) the following functions:

$$\rho_1(d, d') = \max_X |d(X) - d'(X)| \quad (A2)$$

$$\rho_2(d, d') = \text{mean}_X |d(X) - d'(X)| \quad (A3)$$

$$\rho_3(d, d') = \max_X \{100 \times |d(X) - d'(X)|/d(X)\} \quad (A4)$$

$$\rho_4(d, d') = \text{mean}_X \{100 \times |d(X) - d'(X)|/d(X)\}. \quad (A5)$$

The results from carrying out this exercise are summarized in the following table:

| | Decision Variable | | |
|-----------------|-----------------------|-----------------------|-----------------------|
| | n_1 | n_2 | x |
| $\rho_1(d, d')$ | 1.80×10^{-4} | 1.28×10^{-3} | 1.56×10^{-3} |
| $\rho_2(d, d')$ | 3.89×10^{-5} | 2.74×10^{-4} | 3.74×10^{-4} |
| $\rho_3(d, d')$ | 2.85×10^{-2} | .696 | .720 |
| $\rho_4(d, d')$ | 6.22×10^{-3} | .155 | .172 |

The quality of the linear approximation in the fourth step of our model building process can also be evaluated numerically. For artificial realizations of the random process ϵ_t , the system $X_{t+1} = (a - bf)X_t + g\epsilon_{t+1}$ can be computed. Then each of w_{1t} , w_{2t} , \bar{w}_{2t} , h_t , pr_t can be computed as nonlinear functions of X_t , as can the linear approximations to them. The \bar{R}^2 between the realizations of these variables based on the nonlinear functions and the realizations based on the linear approximations can be computed. With the variance of ϵ_t specified in the text, this check gave an \bar{R}^2 extremely close to unity for our model. Indeed, it was via these regressions that we estimated the parameter H in eq. (17).

FOOTNOTES

1. For annual U.S. data, logarithmic least squares regressions of aggregate output on manhours and capital yield constant or slightly increasing returns to manhours, and slightly negative returns to capital. See Lucas [1970]. Robert E. Hall has compiled similar findings for data disaggregated by industry (see Hall [1986]).
2. An alternative effort to ours is Kydland and Prescott's [1986].
3. Christiano [1986] has studied how well the solution to a linear-quadratic problem approximates the solution to a class of problems with logarithmic objective functions and Cobb-Douglas constraints (problems of the class studied by Brock and Mirman [1972] and Long and Plosser [1983]). This setting has the advantage that Christiano can solve analytically both the linear quadratic problem and the nonlinear problem. Exploiting this advantage, Christiano is able to characterize precisely some properties of the estimates of the parameters of the nonlinear quadratic model based on the intermedating linear quadratic model.
4. Sargent and Wallace [1974] and Sargent [1978] not only take h_1 and h_2 as given, but also take the ratio of the overtime wage to the straight-time wage as given from outside the analysis. Taking h_1, h_2 and the wage ratios as given amounts to adopting a specialization of the wage premium schedule which Lucas [1970] takes as exogenous to the firm. In terms of Lucas' specification, the present paper endogenizes some but not all aspects of the premium schedule. We continue to impose h_1 and h_2 exogenously, reinterpreting their imposition in terms of a restriction on agents' consumption set. Given the shift lengths, the model determines equilibrium wage rates across shifts.
5. Compare the specification of the household's consumption set with the more restric-

tive one in Hansen [1985].

6. This uses the fact that, since preferences are separable in consumption and leisure, the consumption level chosen in equilibrium is independent of whether the individual works zero, h_1 , or $h_1 + h_2$ hours.
7. Note that π_{1t} is the probability of working h_1 hours, and *only* h_1 hours, while π_{2t} is the probability of working precisely $h_1 + h_2$ hours. We assume that all of those who work overtime at t also work the straight time shift. Thus, $\pi_{1t} + \pi_{2t}$ is the fraction of people working the first shift.
8. We have also worked with a modified model generated by altering (9) to be

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t - a_1(n_{1t} - n_{2t}) - a_2 n_{2t} - \frac{d_2}{2} (n_{2t} + \alpha n_{2t-1})^2 \right] \quad (9')$$

where $0 < \alpha < 1$ and $d_2 > 0$. The extra term in (9') represents a preference for not being exposed in two successive periods to a high probability of working overtime. Setting α close to unity and making d_2 large will tend to reduce the serial correlation of the equilibrium n_{2t} process. Kydland and Prescott [1982, 1986] have a term similar to this in their social planner's preference function. We do not, however, report results based on this model in this paper.

9. The straight time wage rate w_{1t} is given by h_1^{-1} times $\partial y_t / \partial n_{1t}$, where

$$\frac{\partial y}{\partial n_{1t}} = z_t k_t^\theta \left\{ h_1 (1 - \theta) \left[n_{1t} - \frac{d}{2} (n_{1t} - n_{1t-1})^2 \right]^{-\theta} \cdot [1 - d(n_{1t} - n_{1t-1})] \right\}.$$

The overtime rate w_{2t} is given by h_2^{-1} times $\partial y_t / \partial n_{2t}$, where

$$\frac{\partial y}{\partial n_{2t}} = z_t k_t^\theta h_2 (1 - \theta) n_{2t}^{-\theta}.$$

Note that since agents are identical and markets complete, all households are paid the same amount independent of whether they work straight time, overtime, or not at all. However, we are free to consider a decentralization of this economy where households get paid for the work they do according to the above wage structure and, in addition, have access to an insurance market. This is similar to the decentralization discussed in the appendix to Hansen [1985].

10. For the calculations reported in section 5 of this paper, we used the following method for obtaining this linear approximation. Using a pseudo-random number generator to produce a realization of 200 observations for ϵ_t , we computed a 200 observation realization of X_t using (14). Then we solved for realizations $(y_t, c_t, w_{1t}, w_{2t}, \bar{w}_t, h_t, pr_t)$ as nonlinear functions of X_t . We used this 200 observation sample to regress $(y_t, c_t, w_{1t}, w_{2t}, h_t, \bar{w}_t, pr_t)$ linearly against X_t to estimate the matrix H in (17). For each of the seven regressions, \bar{R}^2 exceeded 0.999, which made us comfortable with the quality of these linear approximations.
11. See Sargent and Sims [1977]. Altug [1985] has noted that Kydland and Prescott's [1982] model augmented with measurement error and some orthogonality assumptions delivers an unobservable index model.
12. THRS is the series LHOURS from the Citibase data tape. Similarly AVHRS is the series LHCH.
13. The capital stock series is nonresidential equipment and structures. This series, as well as real GNP, for the period 47,1 to 85,2 was taken from the Citibase data tape. We estimated a vector autoregression with eight lags, a constant term, and a trend.
14. The values chosen for the parameters $(\theta, \beta, \rho, \delta, A)$ are the same as the values of the same set of parameters used by Kydland and Prescott [1982,1986] and Hansen [1985]. These values were selected by Kydland and Prescott based on evidence from growth

observations and results of studies using microdata (see Prescott [1986]). The value chosen for σ_ϵ^2 is consistent with measurements in Prescott [1986]. The adjustment cost parameter d is a free parameter which was set equal to 15 so that the model mimics certain features of the impulse response functions described in the previous section.

The parameters h_1 and h_2 are calculated from the estimates \hat{h}_1 and \hat{h}_2 of the previous section. We assumed that households have 80 hours per week of discretionary time which can be allocated between market and non-market activities. From this we set $h_1 = (\hat{h}_1/80) = 0.46$ and $h_2 = (\hat{h}_2/80) = 0.13$.

15. The “coefficient of variation” is the standard deviation of a variable divided by its mean.
16. If data on y_t^m are constructed as the *sum* of x_t^m and c_t^m , then the measurement errors for $\{x_t^m, c_t^m, y_t^m\}$ cannot all be uncorrelated. We have explored alternative assumptions about the correlation structure of these measurement errors, but do not report on these in this paper.
17. Note that an additional implication of our regarding measured wages in Trejo’s work as corresponding to w^h is that the value of α should not differ if the dummy variable d_i is altered to be 1 for *all* in FLSA covered workers, whether or not they work overtime.
18. It hardly bears mentioning that “general equilibrium” need not entail generality, and that building general equilibrium models does not permit dispensing with “ad hoc” (“for this purpose”) assumptions. The assumptions made in this paper are indisputably *ad hoc*, and are frankly tailored for the purposes announced in the introduction to this paper. Our assumptions’ status in terms of general equilibrium theory rests on their being interpretable in terms of restrictions on preferences, endowments,

technology, and equilibrium concept (or mechanism).

19. We used as initial conditions the stationary values of the state variables described by the nonstochastic version of the closed loop system.

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N1 AND N2 FROM U.S TIME SERIES

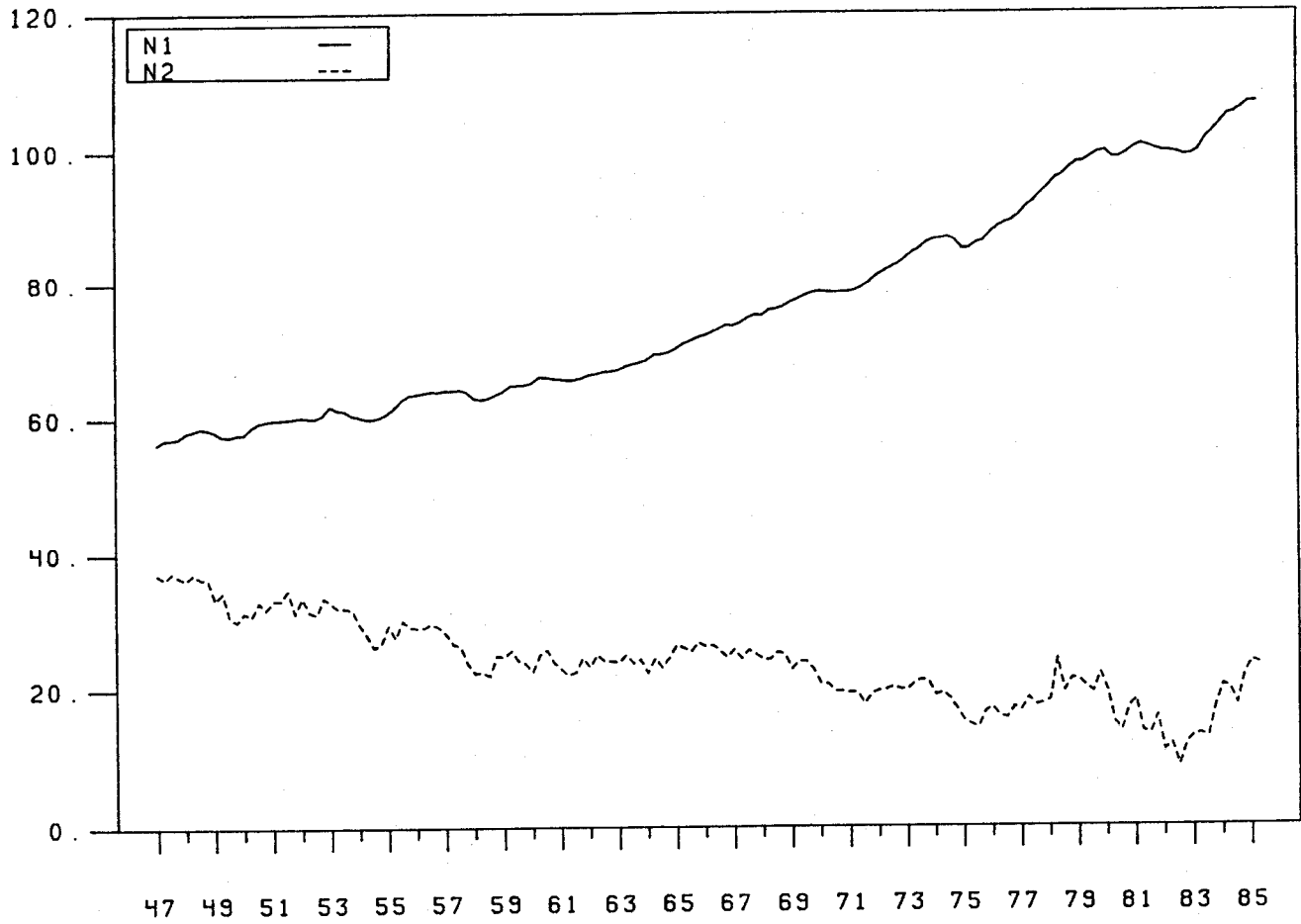


FIGURE 1

INNOVATION IN OUTPUT

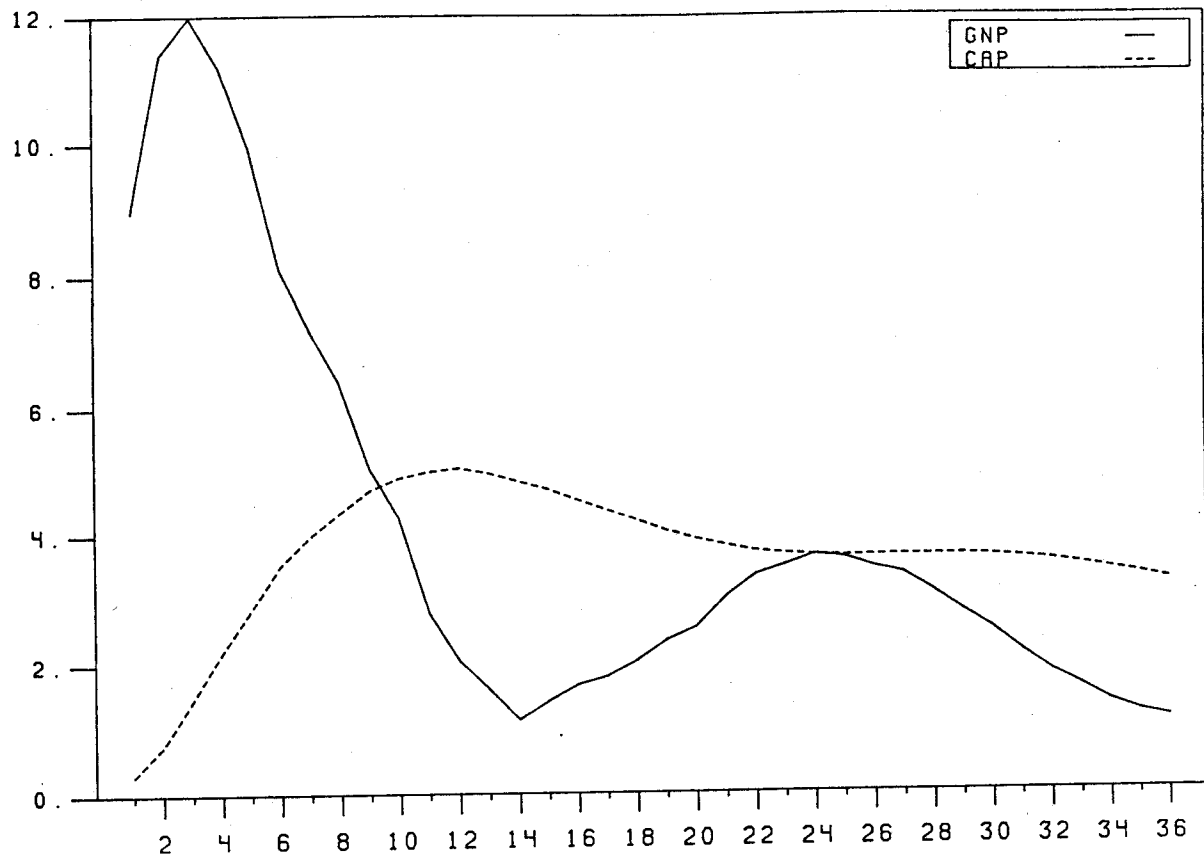


FIGURE 2

RESPONSE TO INNOVATION IN OUTPUT

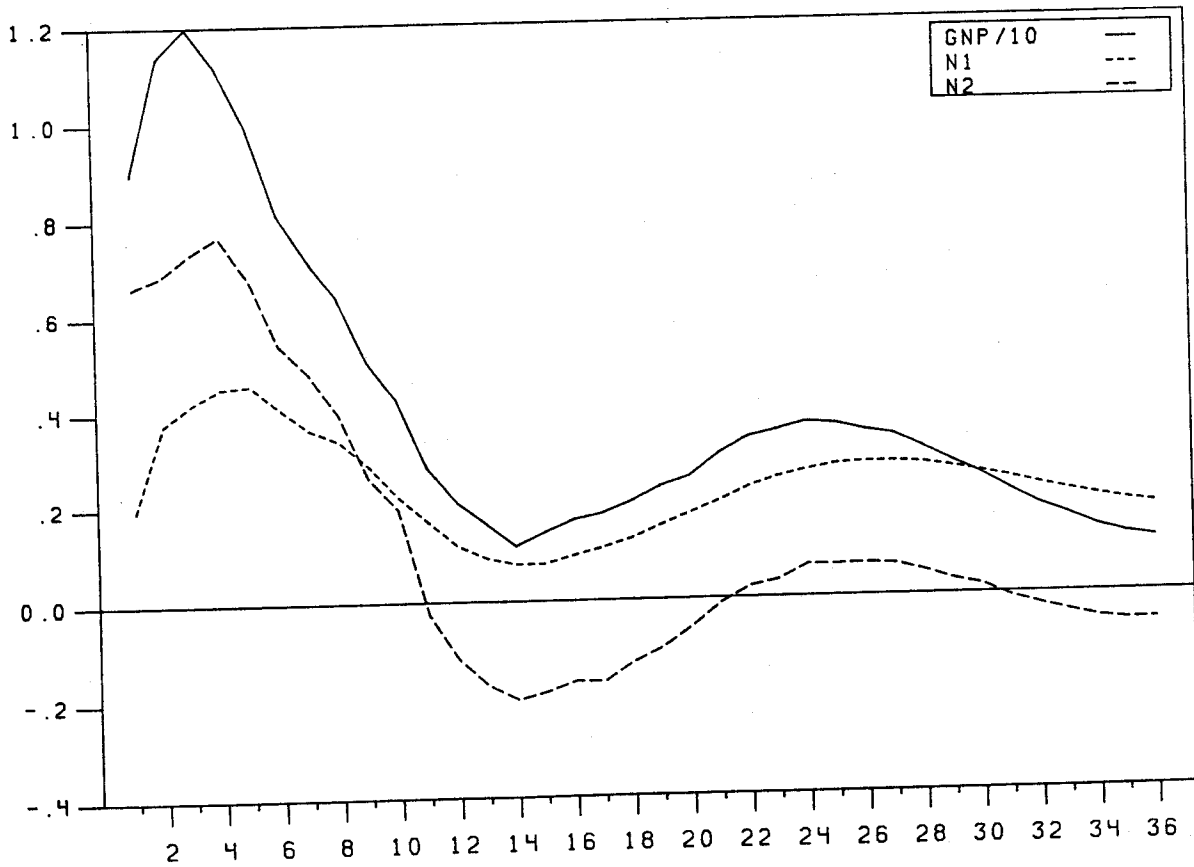


FIGURE 3

RESPONSE TO AN INNOVATION IN Z

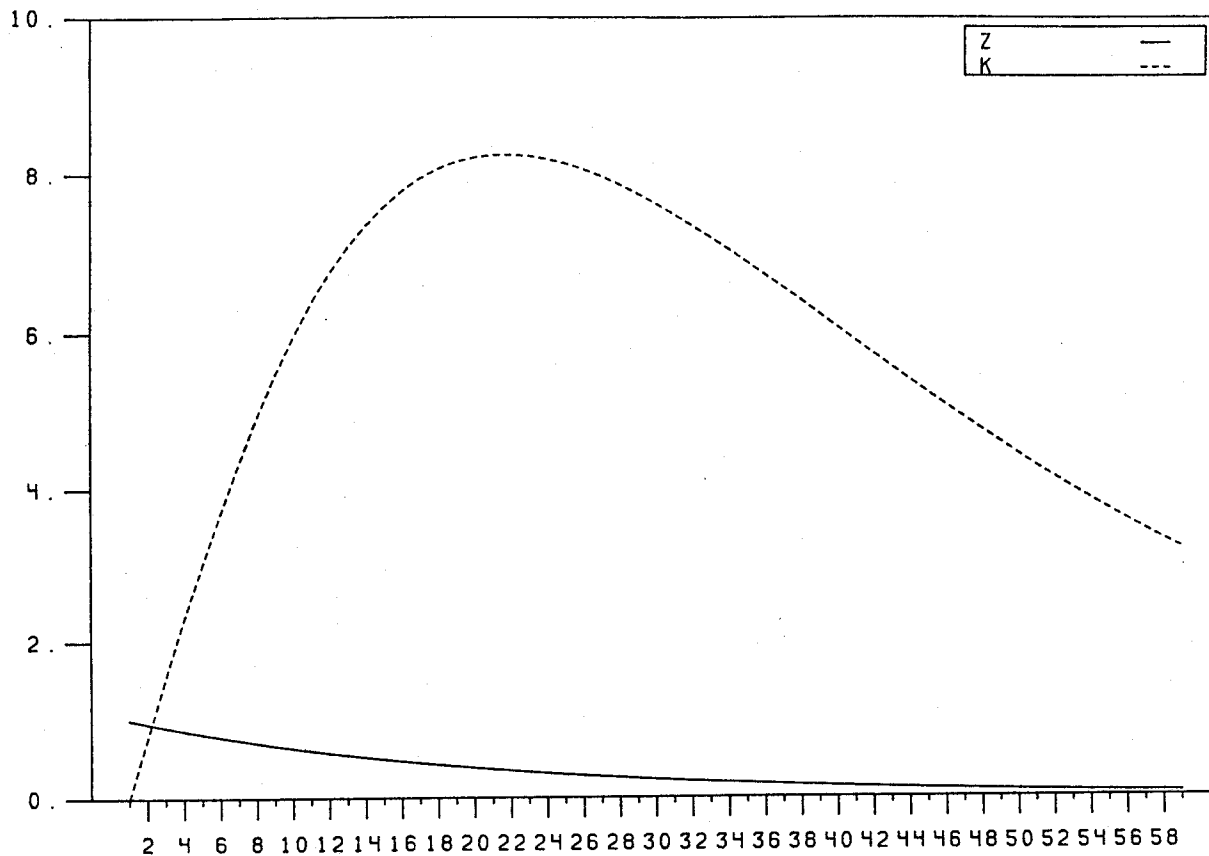


FIGURE 4

RESPONSE TO AN INNOVATION IN Z

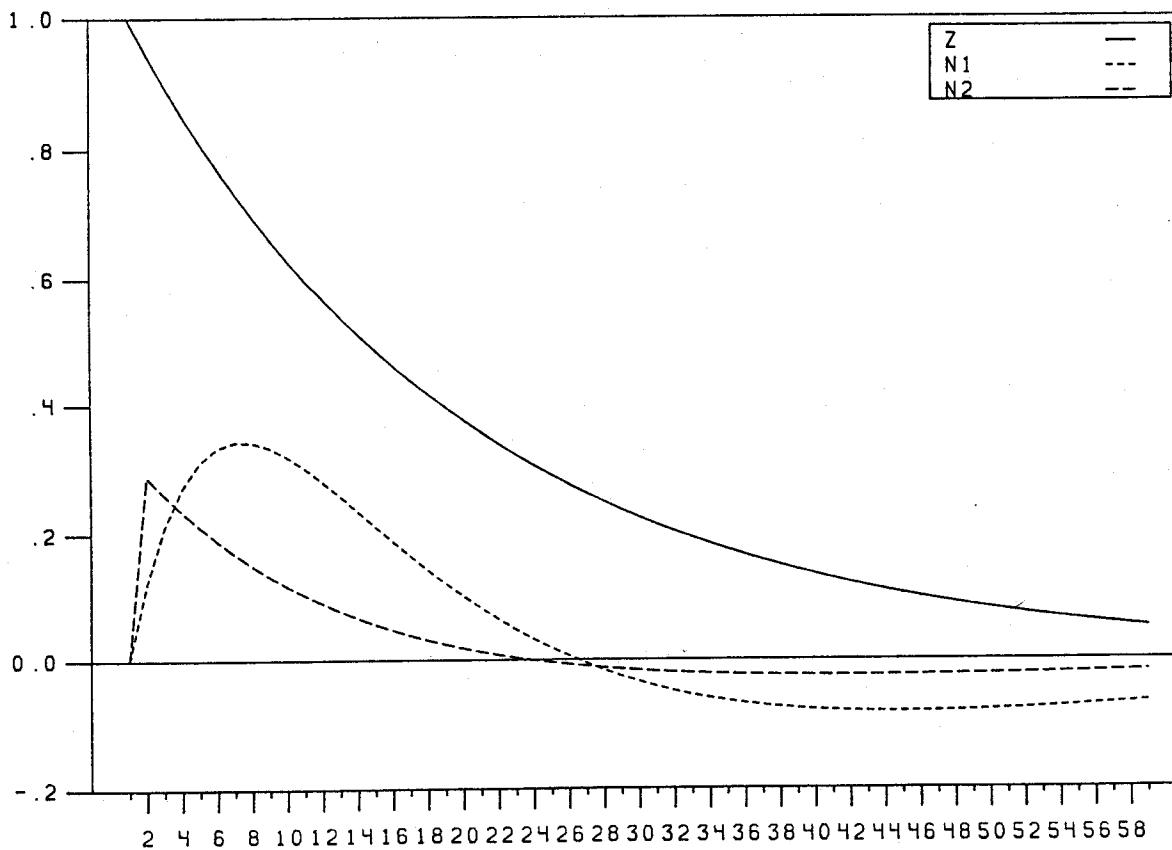


FIGURE 5

OUTPUT, CONSUMPTION, AND INVESTMENT

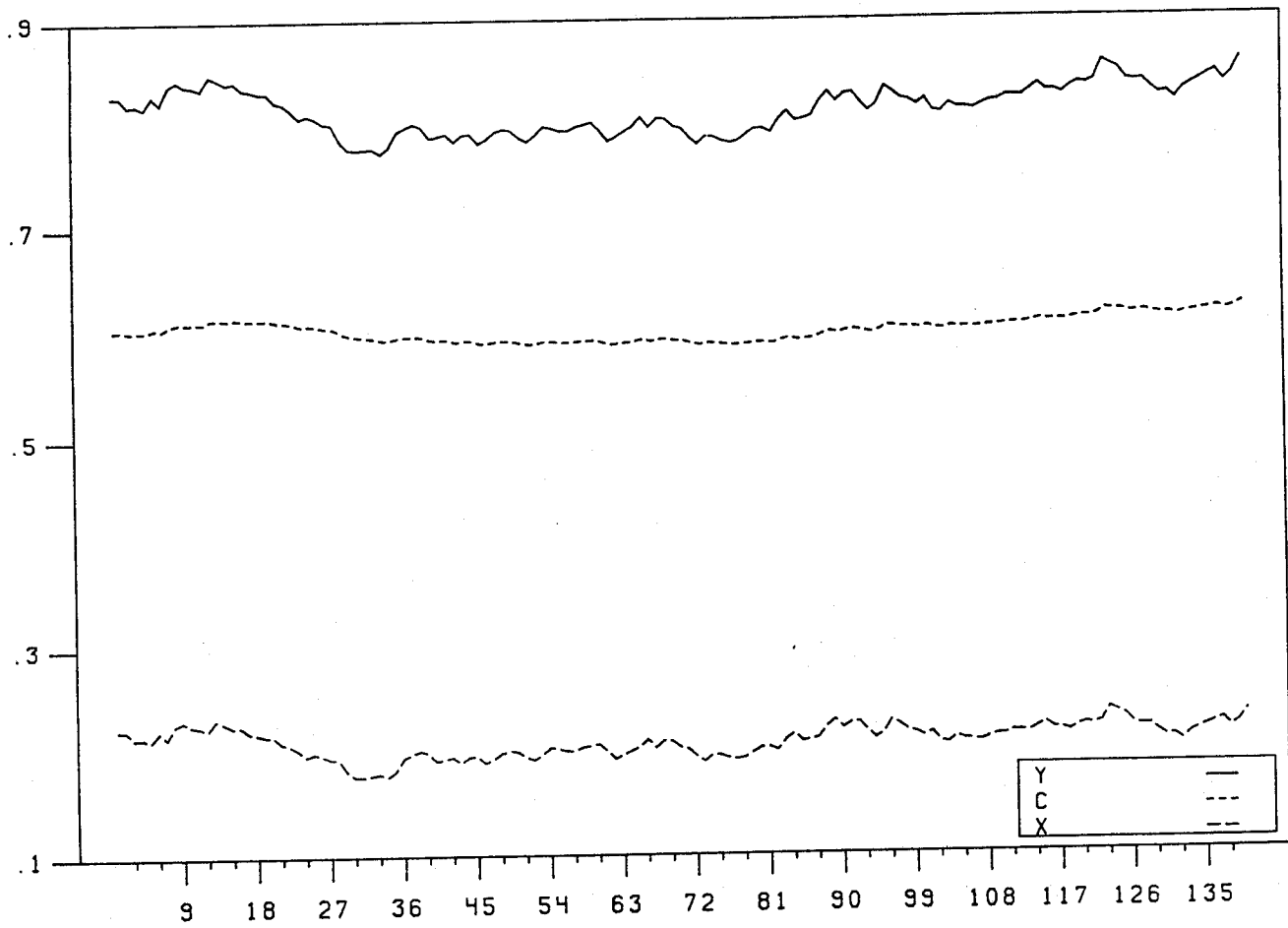


FIGURE 6

N1 AND N2

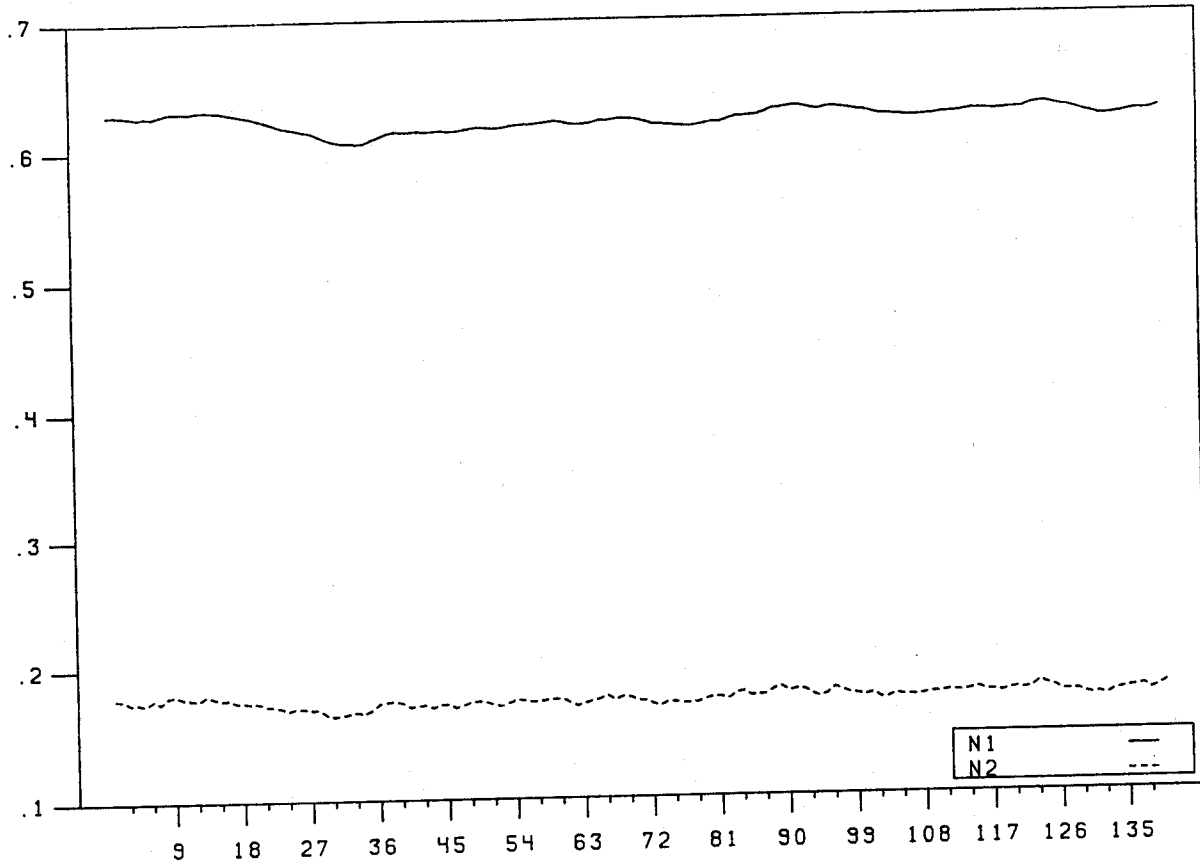


FIGURE 7

W1 AND W2

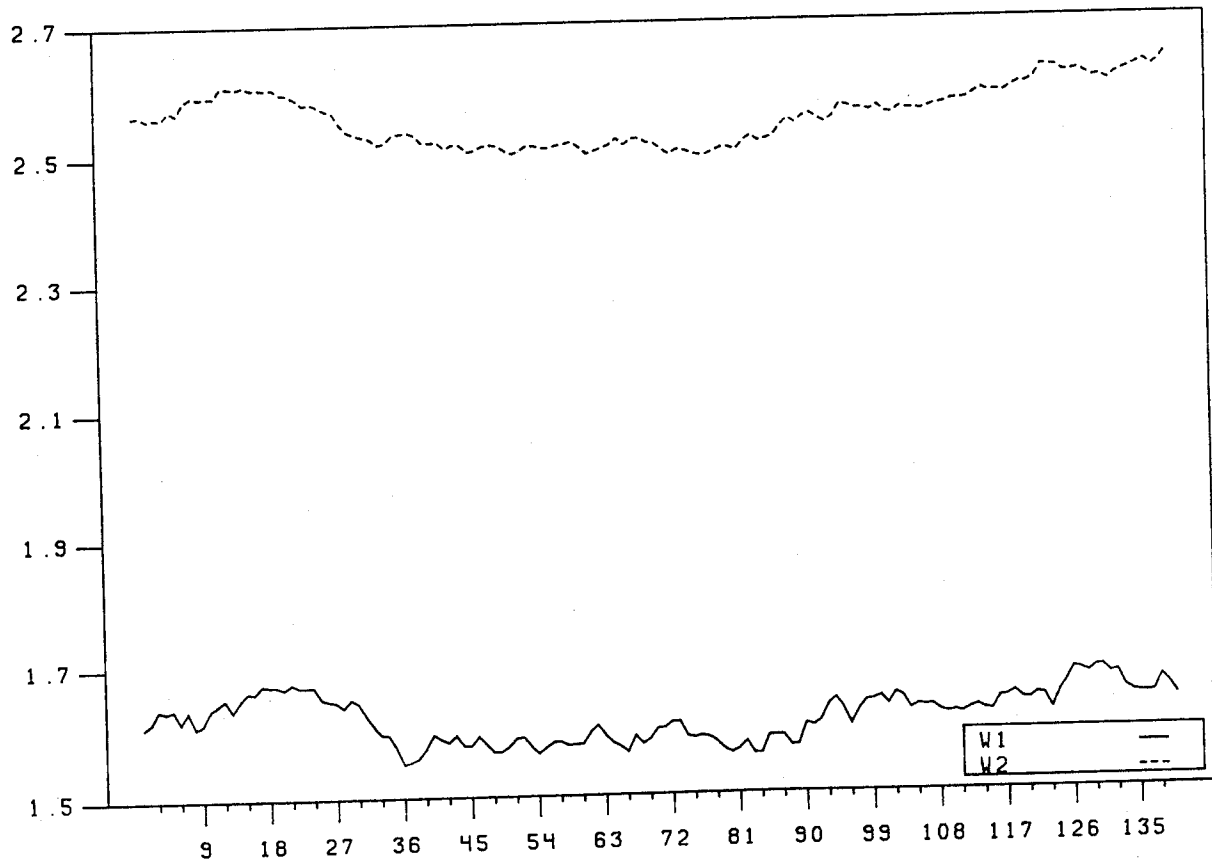


FIGURE 8

ERROR RIDDEN OUTPUT, CONSUMPTION, AND INVESTMENT

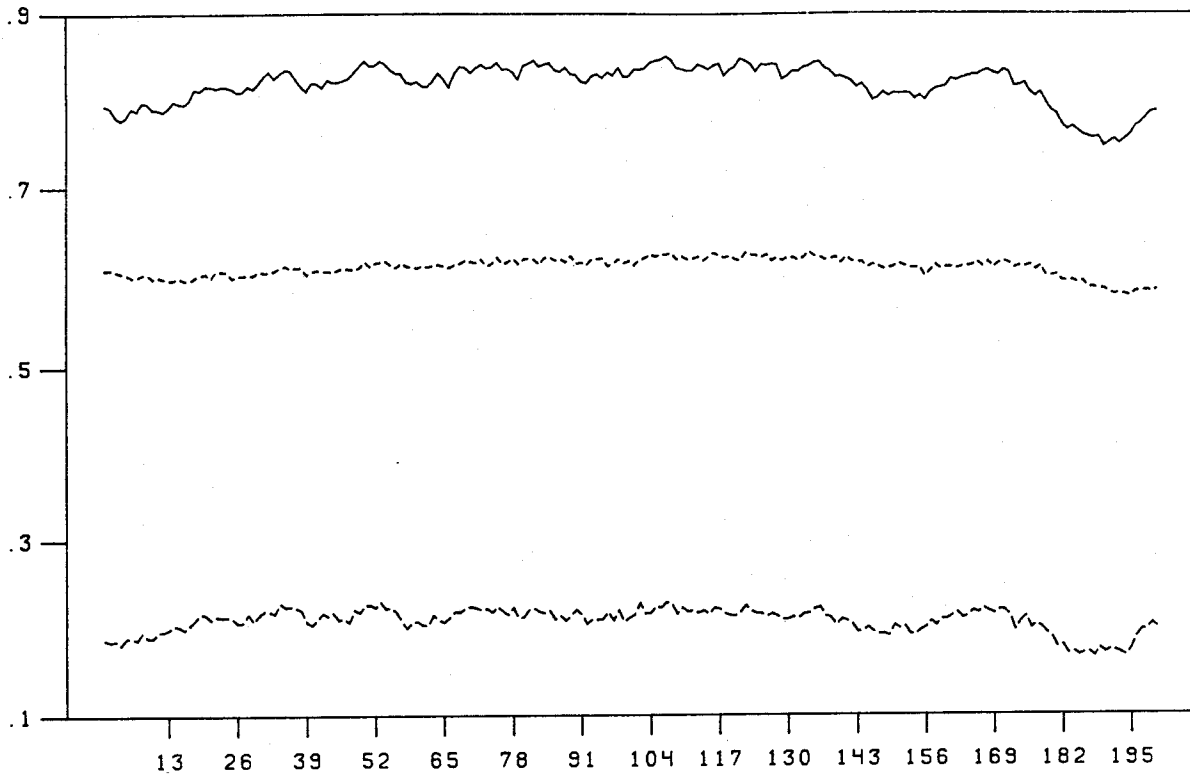


FIGURE 9



RESPONSE TO INNOVATION IN MEASURED OUTPUT

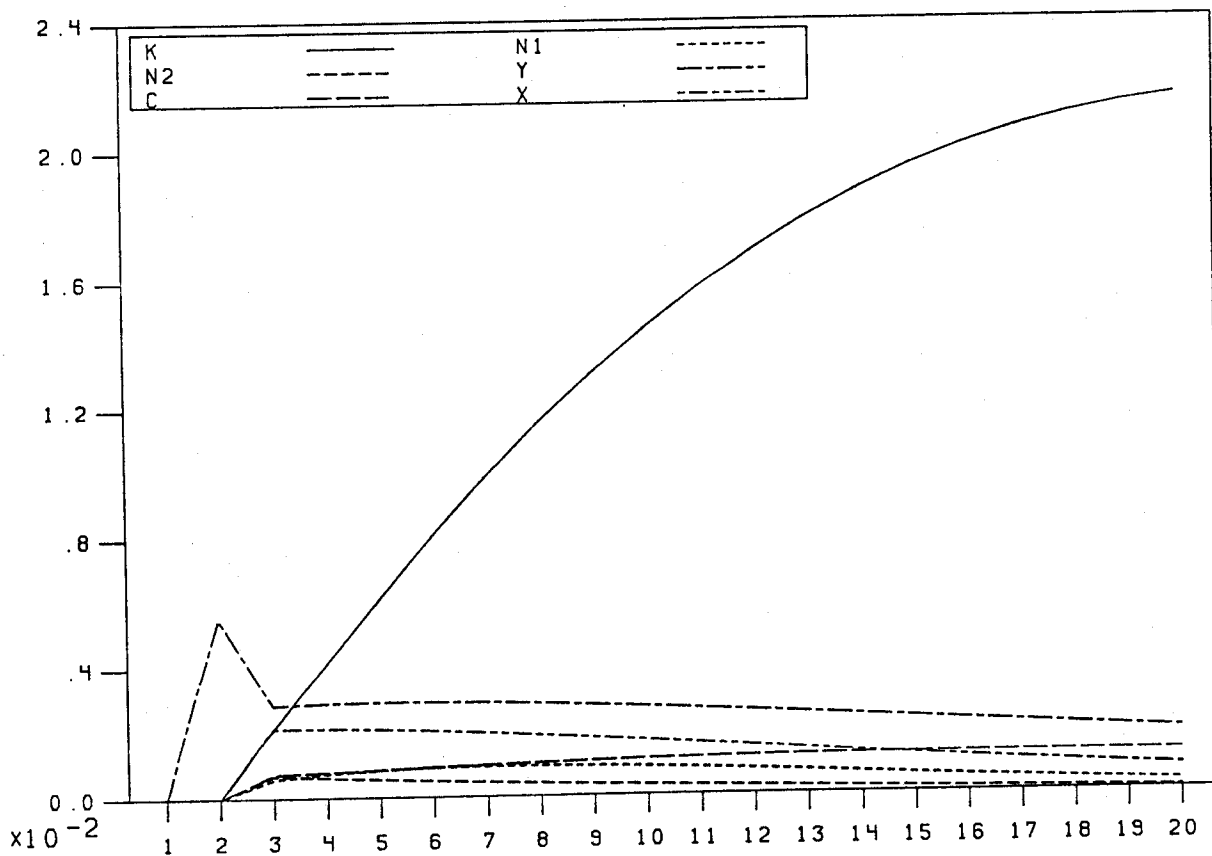


FIGURE 10

RESPONSE TO INNOVATION IN MEASURED INVESTMENT

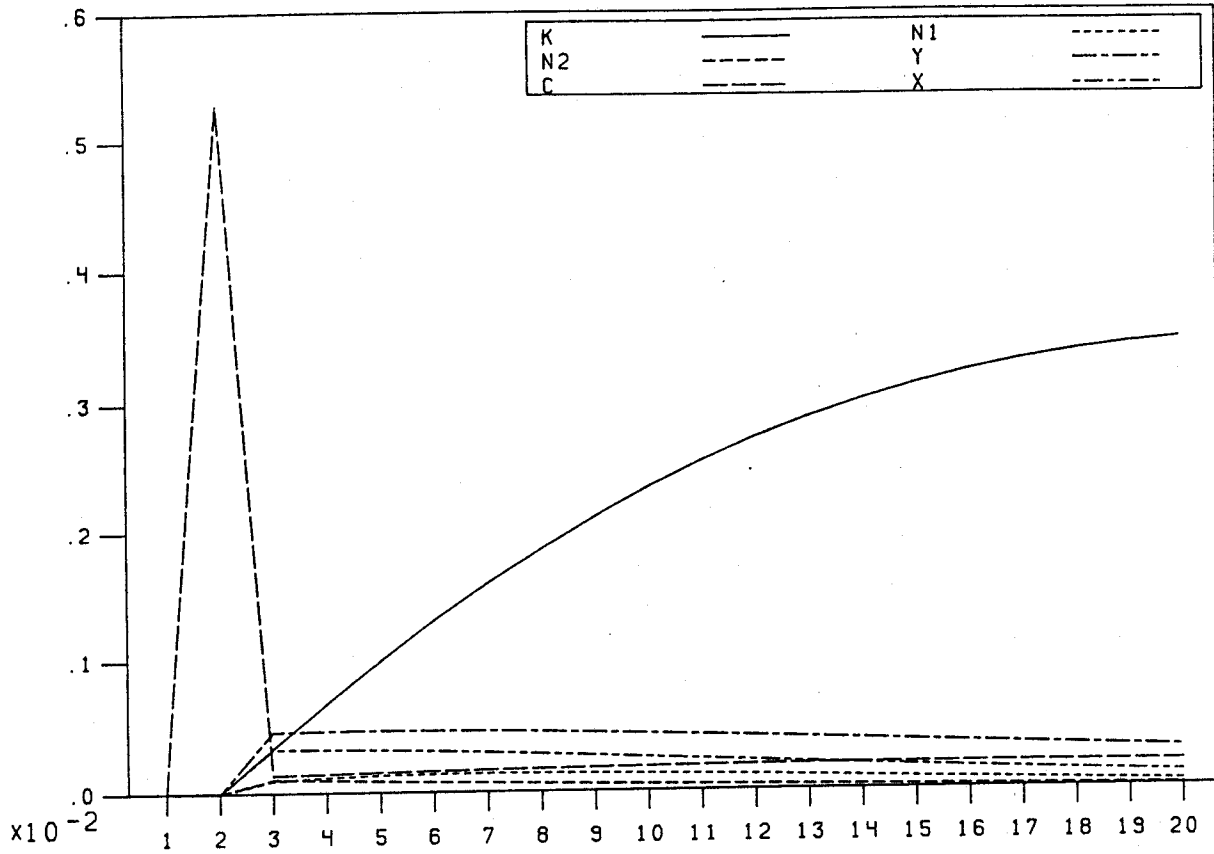


FIGURE 11

RESPONSE TO INNOVATION IN MEASURED CONSUMPTION

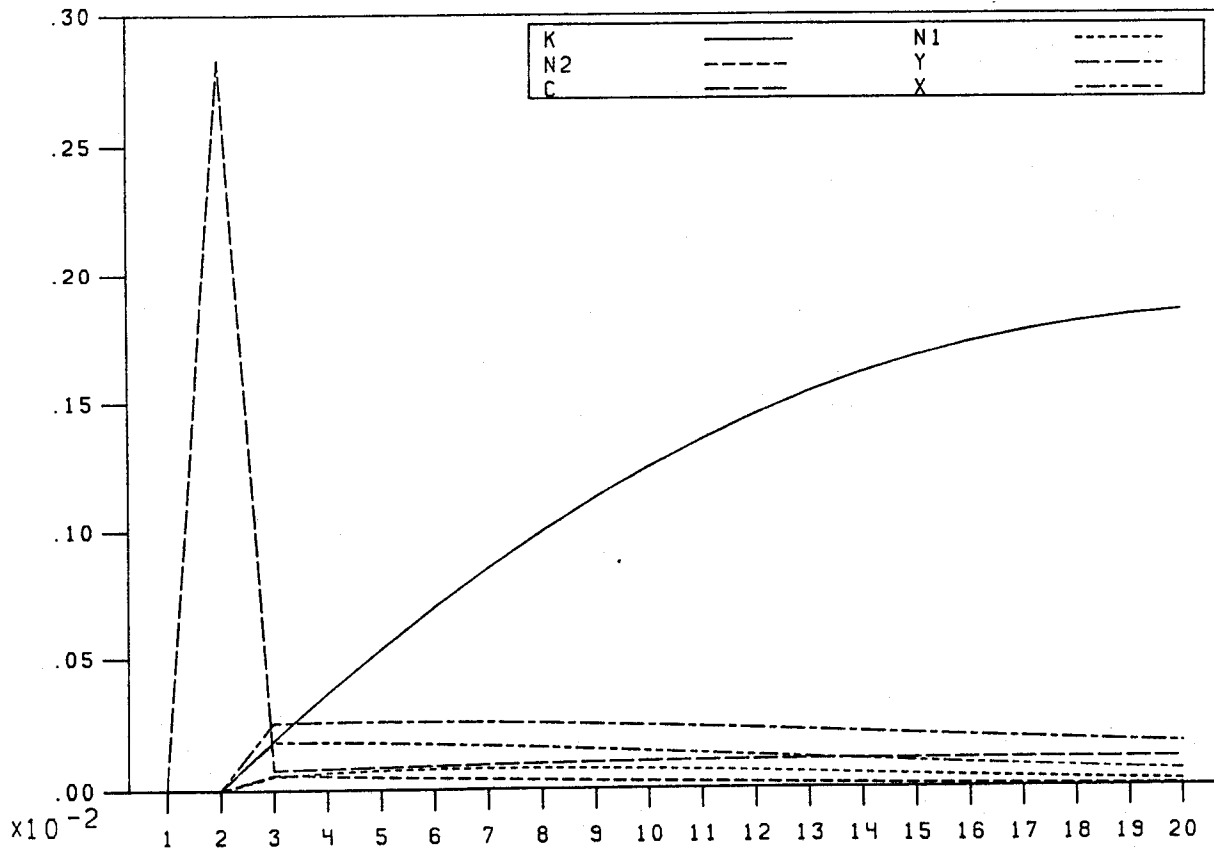


FIGURE 12

Captions for Figures 1 to 12

- Figure 1 Number of workers employed during the straight time shift (N1) and overtime shift (N2) based on quarterly U.S. time series from 47,1 to 85,2. The series N1 is equal to the reported series on total number at work in all industries, and $N2_t$ is equal to $N1_t \cdot (AVRS_t - 37) / 10$, where AVRS is average hours at work in all industries.
- Figure 2 The response of capital (CAP) and output (GNP) to an innovation in output. The impulse response function was computed from an estimated vector autoregression consisting of a trend and eight lags of real GNP, straight time employment, overtime employment, and capital. The quarterly U.S. time series used is from the period 47,1 to 85,2.
- Figure 3 The response of output divided by 10 (GNP/10), straight time employment (N1), and overtime employment (N2) to an innovation in output. The impulse response function was computed from a vector autoregression consisting of a trend and eight lags of real GNP, straight time employment, overtime employment, and capital. The quarterly U.S. time series used is from the period 47,1 to 85,2.
- Figure 4 The response of the technology shock (Z) and capital (K) to an innovation in Z. The responses are the appropriate coefficients of the moving average representation computed in section 5 for our example equilibrium without measurement error.
- Figure 5 The response of the technology shock (Z), straight time employment (N1), and overtime employment (N2) to an innovation in Z. The responses are the appropriate coefficients of the moving average representation computed in section 5 for our example equilibrium without measurement error.
- Figure 6 Simulated data on output (Y), consumption (C), and investment (X) based on the example equilibrium in section 5 without measurement error.
- Figure 7 Simulated data on straight time employment (N1) and overtime employment (N2) based on the example equilibrium in section 5 without measurement error.
- Figure 8 Simulated data on the straight time wage rate (W1) and the overtime wage rate (W2) based on the example equilibrium in section 5 without measurement error.

Figure 9 Simulated data on output (Y), consumption (C), and investment (X) based the example equilibrium in section 5 with measurement error.

Figure 10 The response of capital (K), straight time employment (N1), and overtime employment (N2), output (Y), consumption (C), and investment (X) to an innovation in Y. The responses are the appropriate coefficients of the moving average representation computed in section 5 (equation 24) for our example equilibrium with measurement error.

Figure 11 The response of capital (K), straight time employment (N1), and overtime employment (N2), output (Y), consumption (C), and investment (X) to an innovation in X. The responses are the appropriate coefficients of the moving average representation computed in section 5 (equation 24) for our example equilibrium with measurement error.

Figure 12 The response of capital (K), straight time employment (N1), and overtime employment (N2), output (Y), consumption (C), and investment (X) to an innovation in C. The responses are the appropriate coefficients of the moving average representation computed in section 5 (equation 24) for our example equilibrium with measurement error.