PRICE EXPECTATIONS IN THE UNITED STATES: 1947-1973

by

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II. THE ADAPTIVE LEARNING MODEL

This section develops an algorithm by which agents might update their expectations about the price level, inflation rate, and higher order rates of change of prices. Consider first the standard adaptive expectations learning rule

$$(1) \quad P^e_{t+1} = P^e_t + \lambda_1(P_t - P^e_t).$$

P_t is the observed price at time t, P^e_t is the forecast of P_t made at time t-1, λ_1 is a constant positive "adaptation coefficient", and P^e_{t+1} is the forecast of P_{t+1} made after observing P_t. Muth (1960) demonstrates that (1) provides a minimum mean squared error forecast, relative to the class of all linear constant coefficient functions of past observations of P, if P_t is generated by the stochastic process

$$(2) \quad \begin{aligned} P_t &= \overline{P}_t + u_t \\ \overline{P}_t &= \overline{P}_{t-1} + v_t \end{aligned}$$

where u_t and v_t are independent random processes with means 0 and respective variances μ and ν. $\overline{P}_t$ is interpreted as the true underlying price level, u_t as a transitory shock to prices (or observation error), and v_t as a permanent shock to the price level.

Persistent trends in the price level are not likely to be generated by the process (2). Yet upward trends in prices clearly prevail during periods of inflation. In the presence of trends in P_t, the simple adaptive rule (1) results in consistent underprediction or overprediction of prices, and would lead a rational individual to reject the working hypothesis that process (2) generates prices in favour of a process in which trends are more likely.

A more general process, which does admit the possibility of trends which change over time, is

$$
\begin{aligned}
&P_t = \overline{P}_t + u_t \\
(3)\qquad &\overline{P}_t = \overline{P}_{t-1} + \overline{\Pi}_t + v_t \\
&\overline{\Pi}_t = \overline{\Pi}_{t-1} + w_t
\end{aligned}
$$

where u_t, v_t and w_t are independent white noise processes with means 0 and respective variances μ, ν, and ω. P_t is still the observed price and $\overline{P}_t$ the true underlying price, but $\overline{\Pi}_t$ is the true underlying inflation rate with v_t being a transitory shock and w_t a permanent shock to the trend in prices.[2] With $\overline{\Pi}_o$ and ω both equal to zero, process (3) reduces to process (2). With ν equal to zero, process (3) becomes that used by Nerlove and Wage (1964) to demonstrate the optimality of adaptive forecasting. With μ equal to zero, and $\Pi_t \equiv P_t - P_{t-1}$ defining the observed inflation rate, process (3) becomes process (2) with observed inflation Π_t replacing observed prices P_t and with the underlying inflation rate $\overline{\Pi}_t$ replacing underlying prices $\overline{P}_t$.

A minimum mean squared error forecast for P_{t+1} when process (3) generates prices is provided by the two level adaptive rule[3]

$$
\begin{aligned}
(4)\qquad &P^e_{t+1} = P^e_t + \lambda_1(P_t - P^e_t) + \Pi^e_{t+1} \\
&\Pi^e_{t+1} = \Pi^e_t + \lambda_2(P_t - P^e_t)\,.
\end{aligned}
$$

The adaptation coefficients are positive constants whose values depend on the relative magnitudes of μ, ν and ω. The variable Π^e_{t+1} is the expected <u>underlying</u> inflation rate over the period from t to t+1. This will differ from the expected <u>observed</u> inflation rate $P^e_{t+1} - P_t$ over the same period by the estimated current transitory shock to prices $P_t - [P^e_t + \lambda_1(P_t - P^e_t)]$. Of course if $\mu = 0$

then $\lambda_1 = 1$, and (4) reduces to the usual adaptive rule on observed inflation rates:[4]

$$\Pi^e_{t+1} = \Pi^e_t + \lambda_2(\Pi_t - \Pi^e_t). \tag{5}$$

Learning rule (4) solves the problem of consistently biased price forecasts in the presence of underlying inflation by having individuals update a forecast of both the price level and its trend. Suppose, for example, that the observed inflation rate jumps from 0 to a higher constant value $\bar{\Pi}$. The individual would initially attribute much of the prediction error to transitory shocks in the price level and underlying inflation rate. This would lead to forecasts P^e_{t+1} which turn out consistently below P_{t+1}, and induce him to revise upward his estimate Π^e of the trend in prices. In general, Π^e overshoots the true trend, either converging to $\bar{\Pi}$ through a damped oscillation or being critically damped to $\bar{\Pi}$.[5]

Periods of persistently rising or falling inflation rates have also been known to occur. How would prediction rule (4) fare in such situations? During the periods of persistent drift in Π rule (4) consistently underpredicts or overpredicts the price level, and would lead a rational individual to reject the hypothesis that (3) represents the process generating prices. A still more general process, which admits the possibility of a persistent drift $\bar{d}_t$ in the underlying inflation rate, is

$$\begin{aligned} (6)\qquad P_t &= P_t + u_t \\ \bar{P}_t &= \bar{P}_{t-1} + \bar{\Pi}_t + v_t \\ \bar{\Pi}_t &= \bar{\Pi}_{t-1} + \bar{d}_t + w_t \\ \bar{d}_t &= \bar{d}_{t-1} + z_t \end{aligned}$$

in which u_t, v_t, w_t and z_t are independent white noise processes. The minimum mean squared error forecast of P_{t+1} for process (6) is generated by the three level adaptive learning rule

$$(7) \quad \begin{aligned} P^e_{t+1} &= P^e_t + \lambda_1(P_t - P^e_t) + \Pi^e_{t+1} \\ \Pi^e_{t+1} &= \Pi^e_t + \lambda_2(P_t - P^e_t) + d^e_{t+1} \\ d^e_{t+1} &= d^e_t + \lambda_3(P_t - P^e_t) \ . \end{aligned}$$

If there are no transistory shocks to the price level ($\mu = 0$) then $\lambda_1 = 1$. The distinction between whether prices P_t or inflation rates Π_t are the primary observation then vanishes since (7) can be rewritten as

$$(8) \quad \begin{aligned} \Pi^e_{t+1} &= \Pi^e_t + \lambda_2(\Pi_t - \Pi^e_t) + d^e_{t+1} \\ d^e_{t+1} &= d^e_t + \lambda_3(\Pi_t - \Pi^e_t) \ . \end{aligned}$$

The three level adaptive learning model (7) is fitted to survey data on expected price changes in the next section. Some of its properties warrant note at this point. The reader may verify that the prediction errors asymptotically approach zero if either the price level, the inflation rate or the rate of change in the inflation rate is constant over time.[6] Thus there is an implicit tendency in the model to eventually extrapolate changes in inflation when $\lambda_3 > 0$. Frenkel (1975), however, suggests that inflation expectations might be regressive over short periods of time, which could explain the rise in real money balances observed in the early phase of monetary expansions. Is (7) consistent with short-run regressivity of inflation expecta tions as well as long-run extrapolativity? Suppose the inflation rate has been zero and that P^e_1, Π^e_1 and d^e_1 are fully adjusted to initial values of P_o, 0, and 0, respectively. Then a higher price level P_1 (equivalently, positive inflation of $\Pi_1 = P_1 - P_o$) is observed. The price level predicted by (7) for the following period is $P_2^e = P_o + (\lambda_1 + \lambda_2 + \lambda_3)(P_1 - P_o)$. The expected <u>observed</u> inflation rate $(P_2^e - P_1)$ is hence $(\lambda_1 + \lambda_2 + \lambda_3 - 1)(P_1 - P_o)$ or $(\lambda_1 + \lambda_2 + \lambda_3 - 1)\Pi_1$. Although higher than expected prices always cause an upward revision in the believed <u>underlying</u> inflation rate, expectations can be regressive in the sense that the expected <u>observed</u> inflation rate initially moves in the opposite direction if $\lambda_1 + \lambda_2 + \lambda_3 < 1$. Short-run regressive and long-run extrapolative forecasting of observed inflation are neither mutually exclusive in nor a priori implied by (7). They correspond to the testable hypotheses that $\lambda_1 + \lambda_2 + \lambda_3 < 1$ and $\lambda_3 > 0$, respectively.[7]

Estimation of the multi-level adaptive expectations model requires the solving of difference equation system (7). Defining the column vectors

$$X_t = \begin{pmatrix} P^e \\ \Pi^e \\ d^e \end{pmatrix}_{t+1} \quad \text{and} \quad Z_t = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 \\ \lambda_2 + \lambda_3 \\ \lambda_3 \end{pmatrix} P_t,$$

the model can be expressed in matrix form:

$$(9) \qquad X_t = AX_{t-1} + Z_t \quad \text{with } A = \begin{bmatrix} 1-\lambda_1-\lambda_2-\lambda_3 & 1.0 & 1.0 \\ -\lambda_2-\lambda_3 & 1.0 & 1.0 \\ -\lambda_3 & 0.0 & 1.0 \end{bmatrix}.$$

For a vector X_o of initial values of P^e, Π^e and d^e, the solution of this system is

$$(10) \qquad X_t = A^t X_o + \sum_{j=o}^{t} A^{t-j} Z_j \quad .$$

Equation (10) provides the model's predicted values of the expectation variables at every point in time as a function of an observed price series and any given set of adaptation parameters and initial conditions. For a given price series, a solution which is equivalent, but easier to implement, can be obtained by generating X_t recursively from equation (9). With given values of λ_1, λ_2, λ_3 and X_o, one can compute X_1 from P_1 using (9). With the value of X_1 so generated and P_2 one then computes X_2 in the same manner. By repeated use of (9) an entire time history of X_t can be simulated. It is this predicted time history of expectations which we compare with reported expectations to assess the explanatory power of the model.

In the course of estimation we consider the possibility of sudden changes in expectations not produced by the prediction errors of equation (7). Such phenomena could result from changes in any variable other than prices which has a direct effect on expectations, such as changes in anticipated monetary or fiscal policy. A jump in expectations at time t* is represented by a discontinuity

$$\Delta X_{t*} = \begin{pmatrix} \Delta P^e \\ \Delta \Pi^e \\ \Delta d^e \end{pmatrix}_{t^*+1}$$

in the vector X_{t*}. For $t < t^*$ the model solution is still given by (10). For $t \geq t^*$ the model solution becomes

$$\text{(11)} \qquad X_t = A^t X_o + A^{t-t*} \Delta X_{t*} + \sum_{j=o}^{t} A^{t-j} Z_j \ .$$

The jump in expectations is equivalent to restarting the recursive algorithm (9) with a new set of initial conditions but the same adaptation coefficients.

$$SSR = \Sigma \left(\frac{\hat{\Pi}^e_t - \Pi^e_{t+2}}{\hat{\sigma}_t} \right)^2 .$$

Since the model's prediction Π^e_{t+2} is nonlinear in the initial conditions and adaptation coefficients, the minimization of SSR requires an iterative algorithm. We used the technique of Marquardt (1963) which involves an interpolation between the Taylor series and gradient methods.

The first row of Table 1 presents the estimation results. Figure 1 depicts the data $\hat{\Pi}^e_t$ and the model predictions Π^e_{t+2}. The initial conditions indicate a large negative expected inflation rate with a significant negative drift. Individuals expected prices to fall in the immediate post-war period. The negative inflation rates expected did not materialize and Π^e adapted to the actual inflation rate. The rapidity of this adjustment is due to the large value of λ_2.[11] The small value of λ_3, however, resulted in very slow adjustment of the expected downward drift in inflation rates--it was still substantial in the late 1950's. For this period individuals apparently expected falling inflation rates and a return to price stability. A significant shift in expectations occurred in 1958 as indicated by the vector ΔX_{t*} estimated at that time. It is the only significant jump in expectations for the entire data set, and marks the end of an expected downward drift in inflation which had existed since the start of the survey. The second row of Table 1 presents an estimate of the model with no shift in expectations. There is very little change in the parameters λ_1, λ_2 and λ_3, but a marked increase in SSR. The appropriate F-test indicates that ΔX_{t*} differs significantly from zero at the 99% significance level.[12]

Turnovsky (1970), in a previous analysis of the Livingston data, concluded that the process of expectations formation was fundamentally different in the 1960's compared with the earlier period. We find no evidence of any

III. EMPIRICAL RESULTS

Testing a theory of expectations formation is difficult because the variables to be predicted are inherently not observable. One approach is to imbed the expectations equations in a model which attempts to explain other phenomena. Testing the model then involves a joint test, and it is impossible to disentangle errors in the expectations equations from errors in the remainder of the model. The Livingston survey data for the United States provide the basis for an alternative approach. Every six months since July of 1947 Joseph A. Livingston conducted a survey of economists asking their forecast of the price level six, twelve and eighteen months into the future. Livingston then published his "consensus" forecast of the price level in the Philadelphia Bulletin. In a recent paper, Paul Wachtel (1977) analyzed the original survey responses for the twelve month price forecast. From these forecasts he computed a mean expected inflation rate over the next twelve months $\hat{\Pi}^e_t$ and a standard deviation over the respondents $\hat{\sigma}_t$. Figure 1 depicts the expectations data generated by Wachtel.

Since the survey data are available every six months, it is taken as the unit of time in equations (7). P_t is the price level observed every six months and Π^e_{t+1} is the forecast inflation rate over the next six months.[8] If the expected drift in inflation is not zero, then the expected inflation rate for more distant periods will differ from Π^e_{t+1}. The forecast average inflation rate over the next twelve months (two periods) is $\Pi^e_{t+2} \equiv \Pi^e_{t+1} + d^e_{t+1}/2$.[9] Given the time history of actual prices, the model solution of equation (11) depends only on the initial conditions and adaptation coefficients. In the estimation process we wish to find those parameter values for which the model solution best fits the data.[10] Our estimation criterion is to minimize the sum of squared weighted residuals change in the process of expectations formation. The model of equations (7) is able to reproduce the survey data from both time periods using the same set of adaptation coefficients. This is illustrated in rows three and four of Table 1. The model was separately estimated for the data from 7/47 to 6/57 and from 1/58 to 12/73. Almost exactly the same coefficients are obtained for both intervals. The F-test supports the hypothesis that λ_1, λ_2 and λ_3 are the same for both time periods.

It is the provision for expectations about the drift in the inflation rate which permits this unified explanation of the entire data set. The importance of this extrapolative element for inflation expectations, despite the small value of λ_3, is indicated by row five of Table 1. Estimating the model of equations (4), which ignores possible persistent drifts in the inflation rate, almost doubles the sum of squared residuals. The F-test rejects the zero drift constraint at the 99% significance level.

Was the provision for beliefs in purely transitory shocks to the price level important? Row six of Table 1 estimates the model of equations (8). That specification assumed that λ_1 equalled one so that individuals essentially formed their expectations on the basis of an observed inflation rate. The F-test rejects that hypothesis against the alternative that λ_1 exceeds one. Moreover the fact that $\lambda_1 + \lambda_2 + \lambda_3$ significantly exceeds one appears to rule out short-run regressivity of expected observed inflation rates, such as that hypothesized by Frenkel and Mussa.

The final row of Table 1 fits the standard adaptive expectations relation, given by equation (5), to the survey data. Simple adaptive expectations is clearly inferior to other specifications of model.

IV. CONCLUSION

The multi-level adaptive expectations scheme replicated the expectation formation process closely. The reported inflation expectations behaved, to a large extent, "as if" formulated on the basis of past observed prices only--albeit with extrapolation of trends in inflation and with transient shocks to prices distinguished from permanent shocks. The "goodness of fit" is all the more surprising since the model's predictions were entirely simulations using only past observed prices as inputs; no data on previous reported expectations, which might have served as proxies for omitted variables, were used. There was evidence that factors other than observed prices influenced expectations: namely, the remaining unexplained residuals and the apparent shift in expectations of 1958. But the adaptation coefficients were remarkably stable over long periods of quite different inflation experience.

A test of the type conducted in this paper cannot resolve the issue of whether or not inflation expectations are generated rationally on the basis of information other than observed prices. What it does indicate is that an individual using a naive algorithm could have generated much the same expectations as those in the survey sample at very little cost.

FOOTNOTES

[1]Alternatively, one might hypothesize that agents form their forecasts rationally on the basis of all economic variables known to them and with full knowledge of the economic process generating inflation. This hypothesis has considerable theoretical appeal as a means of consistently closing a model with endogenous expectations, but has been criticized for ignoring the substantial costs to an individual of acquiring and processing large quantities of data. Naive forecasting rules based only on past observations may be "economically rational" (Feige and Pearce 1976) if the value of improved prediction from more sophisticated rules is outweighed by the cost of collecting and processing the additional information. Moreover, the rational and the seemingly naive forecasting rules may be equivalent in some circumstances (Mussa 1975).

[2]$\overline{\Pi}_t$ is actually the rate of change in the price level $\overline{P}_t$. Only if $\overline{P}_t$ denotes the logarithm of the price level, as it will for estimation purposes later in the paper, does $\overline{\Pi}_t$ represent the inflation rate in the compound growth rate of prices sense.

[3]Jacobs and Jones (1977) derive these equations as the asymptotic form of a Bayesian revision rule.

[4]The first equation of (4) lagged one period becomes $P_t^e = P_{t-1} + \pi_t^e$, which when substituted into the equation for π_{t+1}^e gives (5). The first equation of (4) is no longer needed for the expectation algorithm.

[5]The path of π^e depends on the roots of the characteristic equation $\rho^2 - \rho(2-\lambda_1-\lambda_2) + (1-\lambda_1) = 0$. The model is stable if the roots lie within the unit circle.

[6]This assumes that λ_1, λ_2, λ_3 all exceed 0 and have values such that the model is stable.

[7] Defining $\pi_t \equiv P_t - P_{t-1}$ as the observed inflation rate and $\pi^*_{t+1} \equiv P^e_{t+1} - P_t$ as the expected observed inflation rates, learning rule (4) (no drift) can be expressed as $\pi^*_{t+1} = \pi^*_t - (1-\lambda_1-\lambda_2)(\pi_t - \pi^*_t) + (1-\lambda_1)(\pi_{t-1} - \pi^*_{t-1})$. This is identical to a learning rule derived by Mussa (1975, 436-437) as rational if the money supply is generated by an extrapolative-regressive process, past monetary policy is inferred by individuals, and the demand for real money balances is of the form specified by Cagan (1956). Our estimation of system (4) in the next section can be regarded as a test of Mussa's specification.

[8] To be more exact, P_t is the natural logarithm of the Consumer Price Index. Changes in the price level from period to period are then rates of inflation. There is a slight problem because the Livingston survey dates are sometimes five or seven months apart. We have taken the price index at the time of the survey and ignored the varying time interval between the survey dates.

[9] The units of π^e_{t+2} are rate of change per six months. The Wachtel data are reported as percent/annum. In the estimation process π^e_{t+2} must be multiplied by 200 to maintain compatibility with the data.

[10] We assume that the inflation forecasts $\hat{\pi}^e_t$ are of the average underlying inflation rate π^e_{t+2}, not the expected observed rate.

[11] The model's stability properties depend on the characteristic roots of matrix A of equation (9). The estimated λ_1, λ_2, λ_3 yield characteristic roots of 0.948, -0.813 and 0.832, so the model is stable. All of the estimates reported in Table 1 imply a stable process of expectation formation.

[12] We assume that the test statistic $V = (\frac{T-K}{q})(\frac{SSR^+ - SSR}{SSR})$, where SSR^+ is the constrained SSR, T is the number of observations, K is the number of fitting parameters, and q is the number of constraints, is distributed $F(q, T-K)$. The R^2 is computed in the usual fashion as 1 - SSR/SST in which SST is defined as the minimum sum of squared weighted residuals possible for a constant model prediction.

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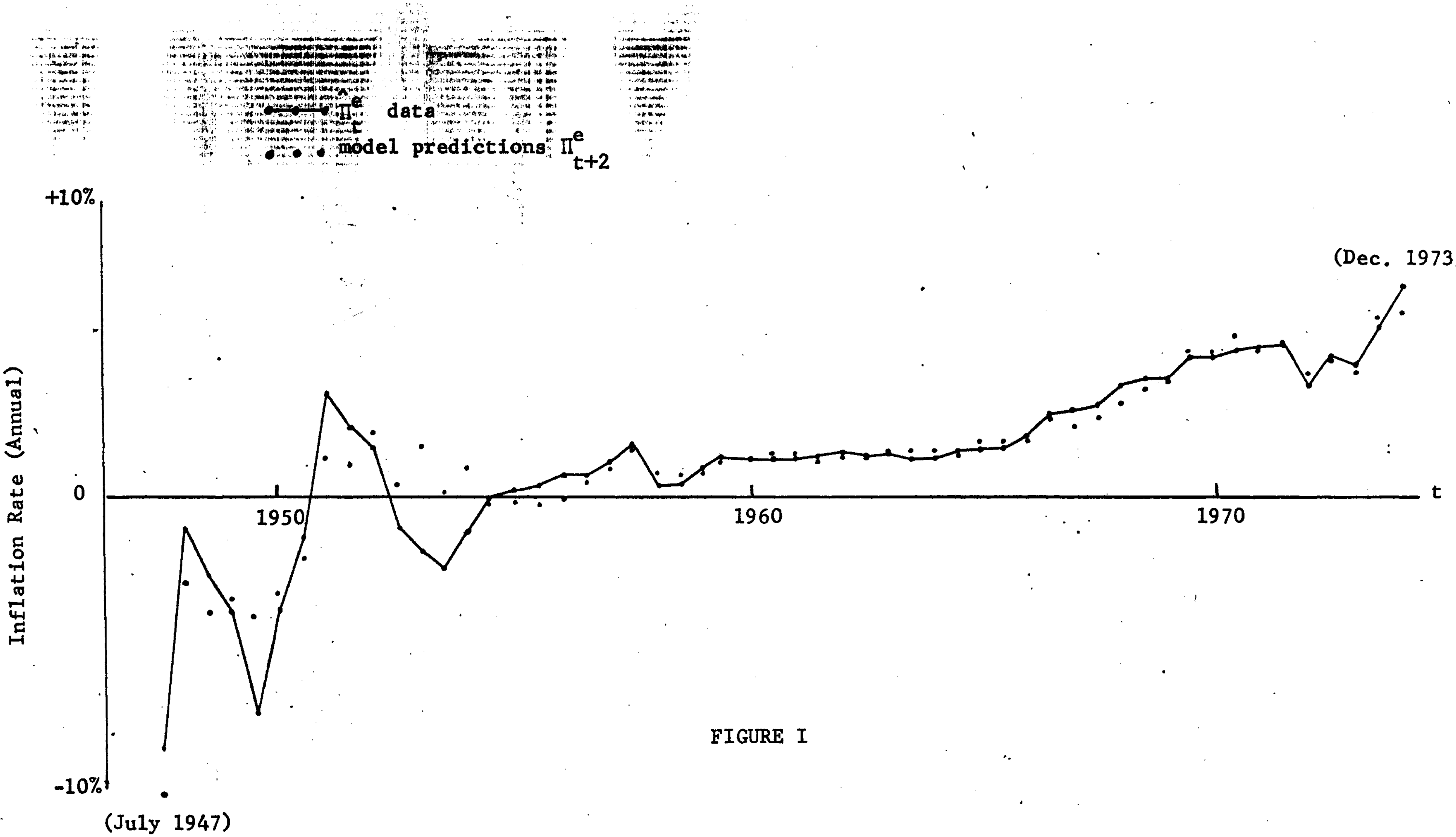
TABLE 1

Estimation Results for Expectations Models

Model	P_o^e	Π_o^e	d_o^e	λ_1	λ_2	λ_3	ΔX_{t*} at 1/58			R^2	SSR	F-stat.
							ΔP^e	$\Delta \Pi^e$	Δd^e			
Equations (7)	4.191	-0.048	-0.003	1.641	0.382	0.011	-0.015	-0.005	0.001	0.906	12.778	-
Equations (7) $\Delta X_{t*} = 0$	4.200	-0.033	-0.004	1.687	0.327	0.022	-	-	-	0.874	17.118	5.095
Equations (7) 7/47 to 6/57	4.206	-0.047	-0.002	1.646	0.345	0.008	-	-	-	0.542	8.822	0.120
Equations (7) 1/58 to 12/73	4.433	0.004	0.001	1.609	0.403	0.009	-	-	-	0.959	3.854	
Equations (4)	4.166	-0.065	-	1.659	0.398	-	-0.011	-0.005	-	0.852	20.060	8.548
Equations (8)	-	-0.049	-0.002	-	0.235	0.003	-	-0.008	0.001	0.875	16.980	14.791
Equations (5)	-	-0.067	-	-	0.254	-	-	-0.008	-	0.819	24.551	10.365



$\hat{\Pi}^e_t$ data
model predictions Π^e_{t+2}
+10%
0
-10%
Inflation Rate (Annual)
1950
1960
1970
t
(July 1947)
(Dec. 1973)

FIGURE I