Groves Mechanisms in Continuum Economies: Characterization and Existence *

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Abstract

The equivalence in the finite agent case between the families of efficient dominant strategy and Groves mechanisms is extended to continuum economies. The concept of an individual's marginal product is used to link the two families of mechanisms when agents are non-atomic.

Unlike the finite agent case, feasible and efficient dominant strategy mechanims exist in the continuum, but these mechanisms do not guarantee individual rationality. For the latter condition to hold, the environment must satisfy an adding-up condition: each individual receives a payoff exactly equal to his marginal product, which we also characterize as equivalent to the condition that each individual creates no external effects. Environments and examples are given that exhibit or fail to exhibit adding-up.

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1 Introduction

To appreciate the role of numbers of individuals in fostering incentives, it frequently suffices to recognize that prices cannot be manipulated when there are large numbers of buyers and sellers, at least when they are trading private goods. What if public goods are included? Or, what if allocations are not market price-guided, but are directed by the more flexible procedures of abstract mechanism design? What difference do numbers make then? The results of this paper show that for the *characterization* of efficient dominant strategy (DS) mechanisms in models with quasi-linear preferences, numbers make no difference. As in the finite numbers case (Vickrey [1961], Groves and Loeb [1975], Green and Laffont [1977], Walker [1978], Holmstrom [1979]), we show that such DS mechanisms are characterized by the extension to nonatomic models of Groves' demand-revealing mechanisms.

On the question of the *existence* of feasible mechanisms, however, there is a considerable difference. For a Groves mechanism to be feasible it is necessary and sufficient that money transfers sum to zero, i.e., exhibit money balance. But Green and Laffont [1977], Laffont and Maskin [1980], Guesnerie and Laffont [1982], and especially Walker [1980] and Hurwicz and Walker [1984] have demonstrated that this money balance property is almost always impossible to achieve with a finite number of individuals. In contrast, it will be easy to see from our characterization that in the nonatomic setting the DS and money balance properties are possible.

Because the large numbers environment is the most likely setting to provide the right incentives, what better place to find out how to solve incentive problems than by studying models with a continuum of individuals? One difficulty with this suggestion is that answers to incentive questions in the continuum can be too immediate to be informative. For example, on the question why is price-taking rational in the continuum, the obvious answer – an individual who is of null measure cannot possible effect price – does little to explain the general principles of incentive compatibility. Perhaps for this reason the bold moves by Hurwicz, Vickrey, Clarke, Groves and others were more informative about general principles because they studied incentive problems in finite agent models where they could not rely on market-guided allocations by price-taking individuals. We follow their lead, extending their program by avoiding any explicit reference to prices. By this route we seek to refine the principles established for finite agent models by highlighting a principle of incentive compatibility that is applicable not only to finite agent but also to continuum models. We shall show that continuum environments are the most likely settings to solve incentive problems not for the "obvious" reasons (which may be false), but for a deeper reason: in continuum environments a principle is feasible that is already known to be necessary and sufficient for incentive compatibility in the finite individual model.

What is this principle of mechanism design that goes "behind prices" and is applicable to large and small numbers? In a previous paper (Makowski and Ostroy [1987a]) we exploited the fact that in a finite individual model Groves mechanisms are equivalent to ones satisfying the *marginal productivity principle:* give each individual a utility equal to his marginal product (MP) plus perhaps a lump sum. This equivalence will be the basis for our nonatomic extension of finite agent demand-revealing mechanisms. We will show that the marginal productivity principle characterizes efficient DS mechanisms irrespective of numbers.

An individual's MP can be decomposed into the sum of two terms: the utility an individual receives by, as it were, joining the economy and the external effects that are imposed on the utility of others to "make room" for the individual. [N.B.: Our use of the term "external effects" is only intuitively related rather than logically related to the same term as it is applied to market relations.] Therefore, in a model with quasi-linear preferences where the utility an individual receives is equal to the sum of the utilities from non-money goods plus "money", the above MP principle can be restated by the following *internalization principle:* the money allocation (positive or negative) to each person must equal the external effects attributable to that person plus perhaps a lump sum.

To summarize, modulo a lump sum, efficient DS mechanisms require that individuals receive their MP's; and with quasi-linear preferences, this is equivalent to the prescription that money payments be set so as to internalize one's external effects. Appropriating one's MP and internalizing one's externalities are two sides of the same coin describing the operating principle behind the DS notion of efficient mechanism design.

The observations so far have been limited to characterization and have not taken account of existence, i.e., feasibility. The money balance condition required for existence translates into an aggregate consistency condition on the internalization principle which we call *feasible internalization* of external effects. (The corresponding consistency condition for the marginal productivity condition we call *feasible appropriation.*) In nonatomic models with quasi-linear preferences, it is a short step from the internalization/marginal product principle to feasible internalization/feasible appropriation, provided the mechanism satisfies certain smoothness assumptions.

It is useful to make a further distinction between feasible internalization and the more demanding *exact internalization* of external effects. This occurs when the sum of the individuals' external effects is itself zero. In this case feasible internalization will imply that each individual's money payment must exactly equal his external effects, i.e., each individual's lump sum must be zero. In MP terms exact internalization corresponds to *exact appropriation:* where each individual receives a utility exactly equal to his MP (again the lump sum is zero) and the sum of all individuals' marginal products adds up to the maximum total gains from the participation of all individuals.

We shall show that exact internalization is necessary and sufficient for the existence of an efficient DS mechanism (1) whenever the underlying environment is "homogeneous" or (2) whenever an individual rationality condition is imposed on the mechanism. Joining these two conclusions points to a limitation on the existence of incentive compatible mechanisms in nonatomic models: inspite of the above general existence results, efficient DS mechanisms satisfying individual rationality can only be found in homogeneous environments.

Though we have followed the methodology of abstract mechanism design, eschewing any explicit reliance on price-guided allocations, this is not for lack of interest. Quite the opposite. We want to apply these results to understand why price-guided allocations work so well in certain environments but not in others. For example, consider the finding that in nonatomic models with private goods the Walrasian mechanism has the DS property; and, more importantly, it is the only one to have this property while also satisfying efficiency (Roberts and Postlewaite [1976], Hammond [1979], Champsaur and Laroque [1982], McLennan [1982], and Mas-Colell [1983]). Obviously, it is not merely the fact that the mechanism is Walrasian that accounts for its success, otherwise it would also be successful with a finite number of individuals. What is it about Walrasian allocations in the continuum that is different from Walrasian allocations with finite numbers? Private goods models are instances of homogeneous environments; therefore exact internalization/exact appropriation is required for incentive compatibility. So the Walrasian mechanism enjoys its distinguished position because it is the only one that exactly internalizes external effects in this nonatomic environment. By contrast, with finite numbers there is no mechanism that can exactly internalize external effects.

We may apply the same kind of reasoning to public goods. When and why does the presence of public goods create incentive problems? As is well-known, this can be regarded as a cost-sharing problem. If there are no costs of production to be shared, the underlying environment is again homogeneous and again exact internalization is possible in the nonatomic case, although not when there are finite numbers. (See Green and Laffont [1979], Rob [1982] and Mitsui [1983] for asymptotic results.) When there are costs, they may be allocated so as to satisfy feasible internalization and therefore achieve an efficient DS result (Hammond [1979], Groves and Ledyard [1986]). However, because public goods models with costs are not homogeneous environments, incentive compatible cost-sharing is generally incompatible with individual rationality (Roberts [1976]).

As the above illustrates, the marginal product concept and characterization results based on it give a general framework in which many, apparently unrelated, results from mechanism theory can be synthesized. Elsewhere (Makowski and Ostroy [1987b]) we demonstrate how the results of this paper for nonatomic models with quasi-linear preferences can be extended to nonatomic NTU models.

The sequel is organized as follows. Section 2 presents our model, a continuum0of-agents extension of a standard demand-revealing model, applicable to private or public good environments. Section 3 discuss some exigencies of non-atomic models and introduces regularity conditions on continuum mechanisms, to meet these exigencies. Section 4 develops the general equilibrium extension of the marginal product concept that is the key to successful extension of Groves finite mechanisms to the continuum. Section 5 then displays our continuum extension of the finite characterization results for Groves mechanisms. Section 6 displays a general existence result for such mechanisms in the continuum. It also displays some special characterization and existence results for individually rational mechanisms and for mechanisms on homogeneous environments. Section 7 discusses related large economy results in the literature. The proofs of all our results are collected in the final section, Section 8.

2 The Model

The function $u : \mathbf{R}^\ell \times V \to \mathbf{R} \cup (-\infty)$ defines tastes and trade/production possibilities: $u(y, v)$ is the utility an individual of type v receives from the allocation y. Extreme disutility attaches to an infeasible allocation; and $Y_v = \{y : u(y, v) > -\infty\}$, assumed to be non-empty, closed and convex, is the set of v's feasible allocations.

The parameterization of individual characteristics is such that V is compact and if $v_n \to v$, the Hausdorf distance between Y_{v_n} and Y_v goes to zero. Therefore, the set $X = \{(y, v) : y \in Y_v\}$ of allocations and types that are jointly feasible is closed in $\mathbf{R}^\ell \times V$.

The parameterization also satisfies the condition that $\partial_y^2 u$ is continuous on X and is negative definite so that $u(\cdot, v)$ is concave. One might think of this in the following way: there is a function $\tilde{u} : \mathbf{R}^\ell \times V \to \mathbf{R}$ with $\partial_y^2 \tilde{u}$ continuous and negative definite throughout its domain. Then $\partial_y^2 u(y, v) = \partial_y^2 \tilde{u}(y, v)$ whenever $(y, v) \in X$.

An *economy* will be described by (positive Borel) measure μ on V. The set of all such economies is $M[V]$.

Commodities are divided into two categories: those which are the arguments of $u(\cdot, v)$, referred to as y-commodities ($y \in \mathbf{R}^\ell$), and the money commodity, denoted by m. An individual with characteristics v will evaluate $(y, m) \in \mathbf{R}^\ell \times \mathbf{R}$ according to the quasi-linear utility function

$$U(y, m; v) = u(y, v) + m.$$

An *allocation* for the economy μ is a specification of the y-commodities and money that each type receives. Let $(y(\mu, v))$, where $v \in \text{supp}\ \mu$, be an allocation of the y-commodities. This allocation is *individually* feasible for v if $y(\mu, v) \in Y_v$. It is *aggregately feasible* if $y(\mu, v) \in Y_v, \mu$ - almost everywhere and $(y(\mu, v)) \in Y(\mu)$.

The set $Y(\mu)$ will determine the nature of the economic environment. For example, a private goods exchange economy would be described by

$$Y(\mu) = \left\{ (y(\mu, v)) = \int y(\mu))d\mu(v) = 0 \right\}.$$

Alternatively, a public goods environment, without any costs of production, could be described by

$$Y(\mu) = \{(y(\mu, v)) = y(\mu) : y(\mu) \in C\}.$$

Environments involving costly production, with or without public goods, could similarly be defined. In the spirit of mechanism theory, we shall suppress the differences to look for principles in common.

About the money component of the allocation, $(m(\mu, v))$, there are no restrictions on individual feasibility, e.g., an individual can deliver any quantity of money. Aggregate feasibility of the money allocation requires

$$\int m(\mu, v)d\mu(v) = 0.$$

However, unless otherwise stated, we shall make no aggregate feasibility restrictions on the money allocation.

A *Y-optimal* allocation or PO_Y allocation for μ is a y-allocation achieving

$$g(\mu) = max(\int u(y(\mu, v), v)d\mu(v) : (y(\mu, v)) \in Y(\mu)).$$

The maximum is assumed to exist. Note that it is taken subject to the constraint that all individuals of the same type receive the same allocation. The assumed concavity of $u(\cdot, v)$ along with a convexity assumption on the set of aggregately feasible trades would ensure that such a constraint was not binding.

The function g can be interpreted as a gains-from-trade function; alternatively, it can be thought of as a production function whose inputs are distributions of agents characteristics and whose output is total gains from trade, measured in utility.

A *Pareto-optimal* or PO allocation for μ is a pair $(y(\mu, v), m(\mu, v))$ in which $(y(\mu, v))$ is PO_Y and $\int m(\mu, v)d\mu = 0$.

3 The Dominant Strategy Property and Regular PO_Y Mechanisms

A *mechanism* is a mapping $f: M[V] \times V \rightarrow \mathbf{R}^\ell \times \mathbf{R}$ where $f(\mu, w) = (y(\mu, w), m(\mu, w))$ and $(y(\mu, v))_{v \in \text{supp } \mu} \in Y(\mu)$; i.e., there is a restriction on the feasibility of the y-allocation but not necessarily on the m allocation.

A mechanism f is PO_Y if for all $\mu \in M[V], (y(\mu, v))$ satisfies

$$g(\mu) = \int u(y(\mu, v), v)d\mu;$$

and it is PO if in addition for all μ, $(m(\mu, v))$ satisfies

$$\int m(\mu, v)d\mu = 0.$$

Note that for both PO and PO_Y, the mechanism f need only be defined for pairs (μ, v) in

$$\Delta = \{(\mu, v) : \mu \in M[V], v \in \text{supp } \mu\},$$

a subset of $M[V] \times V$.

A mechanism f exhibits the *dominant strategy* (DS) *property* at μ if $\forall\, v \in \text{supp } \mu, \forall\, w \in V$

$$U(f(\mu, v), v) \geq U(f(\mu, w), v).$$

A mechanism f exhibits the DS property on $N \subset M[V]$ if it exhibits DS at each $\mu \in N$.

Recalling the definition of U, this says that the utility an individual receives from the allocation mechanism by reporting his characteristics truthfully, $u(y(\mu, v), v) + m(\mu, v)$, is at least as large as the utility an agent of that type could obtain by reporting other characteristics, $u(y(\mu, w), v) + m(\mu, w)$. Notice if f satisfies DS then v has no incentive to misrepresent himself as a w, even for $w \notin \text{supp } \mu$.

It will be assumed throughout that the y-allocation satisfies PO_Y. As remarked above, this prescribes the behavior of the y-allocation only on Δ, whereas if a mechanism is DS on N the y-allocation must be defined on $N \times V$. Consistency requires that the behavior of the mechanism on $N \times V$ must fit together with its behavior on Δ. To achieve this consistency we shall impose certain smoothness properties of the PO_Y allocation on Δ which will provide the base for a consistent extension to $(N \times V)\backslash\Delta$.

To characterize DS mechanisms, it will suffice for our formal analysis to concentrate upon populations μ in a neighborhood N of some fixed population $\mu^o \in M[V]$. While our entire analysis could be carried out globally on $M[V]$ rather than locally on N – and all of our results are valid if N is replaced by $M[V]$ – the approach we adopt makes the results more useful for applications. (For example, regular economies typically only exist locally, not on a universal domain; see Remark 2 below for the connection between regular economies and regular mechanisms.)

To proceed, let N be the intersection of a norm closed and bounded neighborhood of μ^o with $M[V]$. Define

$$\Delta(\mu^o) = \{(\mu, v) \in \Delta : \mu \in N\}$$

and

$$\Gamma(\mu^o) = \{(\mu, v, w) : \mu \in N,\ v, w \in \text{supp } \mu\}$$

Let $Dy : \Gamma(\mu^o) \to \mathbf{R}^\ell$ be defined by

$$Dy(\mu, v; w) = \lim_{t \to 0_+} \frac{y(\mu + t\delta_w, v) - y(\mu, v)}{t},$$

the directional derivative of $y(\mu, v)$ in the direction w. (δ_w is the Dirac measure with unit mass concentrated at w.) The vector $Dy(\mu, v; w)$ is the infinitesimal effect on v's allocation in the population μ of the addition of an infinitesimal individual of type w.

Our main restriction on the PO_Y allocation is

(R.1) Dy exists and is continuous on $\Gamma(\mu^o) \subset N \times V \times V$ when N is given the weak-star topology.

For a PO_Y mechanism there are no restrictions on $m(\mu, v)$. However, to achieve consistency of the sort described for the y-allocation we shall assume

(R.2) m is continuous on $\Delta(\mu^o) \subset N \times V$ when N is given the weak-star topology.

The consequences of (R.1) and (R.2) are fairly immediate. $\Gamma(\mu^o)$ is dense in the compact set $N \times V \times V$ and $\Delta(\mu^o)$ is dense in the compact set $N \times V$. Since Dy and m are each continous on a dense set whose closure is compact, there exists a unique continuous extension of both Dy and m to the closure of their domains. The uniqueness of these continuous extensions suggests that this is "the" way that allocations should be defined on pairs $(\mu, w) \in (N \times V)\backslash\Delta$.

Note that the existence of the continuous extension for Dy implies that there exists a unique, continuous $y: N \times V \to \mathbf{R}^\ell$.

We shall require one more assumption on y-optimal allocations.

(R.3) If $y(\mu_n, w_n) \notin Y_v$, $\mu_n \to \mu$ and $w_n \to w \neq v$, then $y(\mu, w) \notin Y_v$.

This condition says that all y-allocations to individuals that the mechanism calls for and that are not in v's effective domain are away from the boundary of v's effective domain. This rules out the possibility of $u(y(\mu, w), v)$ being discontinuous (relative to the mechanism) for allocations approaching the boundary of v's effective domain. It is a strong assumption. But it is only a sufficient condition to prove our results for general DS mechanisms; it is not required to prove any of our results for (less general) demand-revealing mechanisms, where $Y_v = Y_w$ for all v and w.

A PO_Y mechanism $f = (y, m)$ on $\Delta(\mu^o)$ satisfying (R.1-3) will be called a *regular* PO_Y mechanism. We have seen that such a mechanism can be smoothly extended to $N \times V$, and from now on we shall not distinguish f from its extension.

REMARK 1 (Fair Allocations): An allocation $f(\mu, v)$ is said to be *fair* at μ if it is Pareto-optimal and for all $v, w \in \text{supp}\ \mu$

$$U(f(\mu, v), v) \geq U(f(\mu, w), v).$$

(See Schmeidler and Vind [1972], Varian [1976], Champsaur and Laroque [1981] and others. We are ignoring a μ-almost everywhere qualification.)

Interpreting this condition in the language of misrepresentation, it says that an allocation is fair if no individual in supp μ would prefer to represent himself as any other individual in supp μ. By contrast an allocation is DS at μ if no individual in supp μ would prefer to represent himself as any individual in V. Evidently a DS allocation is fair, but the converse need not hold. However, the two definitions can lead to quite similar conclusions provided supp μ is a connected set. (See Section 7.) Characterization of the DS property requires a similar connectedness assumption but on V rather than supp μ (see Holmstrom [1979] and below); and the results of this paper could, with straightforward modifications, be applied to show that the marginal productivity/internalization principle underlies fair allocations.

Despite the important similarities between the fair and DS definitions of misrepresentations in nonatomic models, for our purposes the differences are significant. For example, since connectedness of supp μ is crucial for the fair definition of misrepresentation to narrow down the class of possible allocations, there is only a very loose connection between fair allocations and the DS property in finite agent models. Unless supp μ is a singleton it is necessarily disconnected in finite individual models. But finiteness of the actual types does not preclude connectedness of V, the set of potential types, even in finite individual models; and this is what permits a single characterization of DS mechanisms applicable to finite and nonatomic models. This is one reason for regarding the DS version of misrepresentation as the one to emphasize when our primary focus is on strategic rather than ethical behavior.

If we look only at the limit, the fair definition of strategic misrepresentation — confining misrepresentations of characteristics at μ to those in supp μ — might appear to be the natural one because the announcement of any $w \notin$ supp μ is obviously a misrepresentation which could be discouraged by a penalty function that assigned large negative utility to any characteristic not in supp μ. The difficulty with this proposal is that individuals must know supp μ *beforehand*, an assumption that dilutes its appeal as a substitute for the DS property where it is an essential point that such knowledge is unnecessary.

REMARK 2 (Regular Private Goods Economies and Regular PO_Y Mechanisms): We outline an argument that in a private goods exchange economy the "regularity" of the economy will ensure that a y-optimal mechanism satisfies (R.1).

Let $e(p, v)$, where $p \in \mathbf{R}^{\ell}$, be the vector of y-commodities that maximizes

$u(y,v) + m$ subject to the constraint $py + m = \alpha$ (the price of m is unity). Because u is strictly concave $e(p,v)$ is unique, and because utility is quasi-linear $e(p,v)$ is independent of α.

If $(y(\mu,v))$ is y-optimal for μ and utility functions are monotone there is a $p \in \mathbf{R}^{\ell}_{+}$ such that

$$y(\mu,v) = e(p,v).$$

Feasibility of net trades implies $\int y(\mu,v)d\mu = 0$; therefore

$$E(p,\mu) = \int e(p,v)d\mu = 0.$$

Thus, p is an "equilibrium" price vector for the ℓ y-commodities. Note: $E(p,\mu) = 0$ *and* $pe(p,v) = m(\mu,v)$ would constitute a Walrasian equilibrium for the full $(\ell+1)$-commodity model.

The economy μ is said to be *regular* (Debreu [1970]) if $\partial_p E(p,\mu)$ is non-singular. In this case we can apply the Implicit Function Theorem to obtain

$$Dp(\mu;w) = [\partial_p E(p,\mu)]^{-1} DE(p,\mu;w),$$

where $Dp(\mu;w)$ and $DE(p,\mu;w)$ are the directional derivatives in the direction w of the equilibrium price mapping $p(\mu)$ and the excess demand function $E(p,\mu)$, respectively.

A simple calculation shows that

$$DE(p,\mu;w) = -e(p,w).$$

Therefore, the formula for $Dy(\mu,v;w)$ is

$$Dy(\mu,v;w) = \partial_p e(p,v) Dp(\mu;w) = -\partial_p e(p,v)[\partial_p E(p,\mu)]^{-1} e(p,w).$$

The regularity of μ plus the hypothesis that $\partial_y^2 u(y,v)$ is jointly continuous implies that Dy satisfies (R.1).

4 The Marginal Products of Individuals and Their External Effects

The key to our characterization of $DSPO_Y$ mechanisms is the concept of an individual's marginal product. This is no less true in the finite numbers model than in the continuum (see Makowski and Ostroy [1987a]), but in the continuum the infinitesimal scale of each agent is ideally suited for the application of the calculus.

Define the marginal product of w in the population μ, $MP(\mu, w)$, as

$$Dg(\mu; w) = \lim_{t \to 0_+} \frac{g(\mu + t\delta_w) - g(\mu)}{t}$$

Substituting the definitions of $g(\mu + t\delta_w)$ and $g(\mu)$, we obtain

$$\begin{aligned} MP(\mu, w) &= \lim_{t \to 0_+} t^{-1} \cdot [\int u(y(\mu + t\delta_w, v), v) d\mu + tu(y(\mu + t\delta_w, w), w) \\ &\quad - \int u(y(\mu, v), v) d\mu] \\ &= \int \partial_y u(y(\mu, v), v) Dy(\mu, v; w) d\mu + u(y(\mu, w), w) \\ &= \xi(\mu, w) + u(y(\mu, w), w). \end{aligned}$$

The rate at which the total gains function g changes as an infinitesimal individual with characteristics w is added to μ, $Dg(\mu; w)$, consists of two parts: (a) the sum of the "external effects" that the very presence of w creates for all the other agents in $\mu, \xi(\mu, w)$, plus (b) the utility that w enjoys in this y-optimal allocation.

To elaborate on the externality component of w's MP, notice that the external effect on any one agent of type $v \in$ supp μ caused by type w is the infinitesimal change in v's utility from his y-allocation, $\partial_y u(y(\mu, v), v)$, evaluated according to the directional derivative of $y(\mu, v)$ in the direction w, i.e., $Dy(\mu, v; w)$. The magnitude of this effect will be insignificant compared to the total utility of agent v, but the cumulative sum of these external effects of the presence of w on the entire population μ, $\xi(\mu, w)$, can be of the same order of magnitude as an individual's total utility. (N.B.: Even if $\xi(\mu, w) \neq 0$, w's "externalities" may still be internalized by w. See the definition of *exact internalization,* below.)

The following result summarizes the implications of a regular mechanism for the marginal product of an individual.

Lemma 1 *Let f be a regular PO_Y mechanism on $N \times V$. Then*

$$MP(\mu, w) = \xi(\mu, w) + u(y(\mu, w), w)$$

Moreover, $\xi(\mu, w)$ and $MP(\mu, w)$ are continuous on $N \times V$.

4.1 The MP of an Individual Who Misrepresents His Type

We shall show that any mechanism f is $DSPO_Y$ if and only if it always rewards all types with their MP's, plus perhaps a lump sum. Since any type $v \in \text{supp}\,\mu$ may claim he is really some other type $w \in V$, as a final preliminary we need to define not only v's MP when he is truthful, $MP(\mu, v)$, but also his MP to society when he announces some other type w, $MP(\mu, w; v)$.

Just as $MP(\mu, v)$ is defined by taking limits, so

$$MP(\mu, w; v) = \lim_{t \to 0_+} \frac{g(\mu + t\delta_w; \mu + t\delta_v) - g(\mu)}{t},$$

where

$$g(\mu + t\delta_w; \mu + t\delta_v) = \int u(y(\mu + t\delta_w, z), z)d\mu(z) + tu(y(\mu + t\delta_w, w), v),$$

is the total gains in the economy μ when t agents of type v are added to the population but announce characteristics w. Notice that for some v and w, v may be called upon to deliver a y-optimal allocation that is infeasible, i.e., $u(y(\mu, w), v) = -\infty$. Certainly it is not in v's interest to make such an announcement; in terms of the above formula it leads to an $MP(\mu, w; v) = -\infty$.

The implications of a regular mechanism for the MP of an individual who misrepresents his type are given by

Lemma 2 *Let f be a regular PO_Y mechanism on $N \times V$. Then,*

$$MP(\mu, w; v) = MP(\mu, w) - u(y(\mu, w), w) + u(y(\mu, w), v) = \xi(\mu, w) + u(y(\mu, w), v).$$

Moreover, $MP(\mu, w; v)$ is continuous on $\{(\mu, w, v) : y(\mu, w) \in Y_v\} \subset N \times V \times V$.

5 Characterization of $DSPO_Y$

5.1 The Marginal Product/Internalization Principle as a Sufficient Condition

The payoff in a regular PO_Y mechanism can always be written as

$$U(f(\mu, w), v) = u(y(\mu, w), v) + m(\mu, w) = MP(\mu, w; v) - H(\mu, w),$$

where $H(\mu, w)$ is simply the residual establishing the equality.

The marginal productivity reward principle has a built-in dominant strategy property.

Lemma 3 $\max_w MP(\mu, w; v) = MP(\mu, v; v) \equiv MP(\mu, v)$.

Say that $H : N \times V \to \mathbf{R}$ is a *lump sum* function if there is an $h: N \to \mathbf{R}$ such that

$$h(\mu) = H(\mu, w).$$

This might be better termed an anonymous lump sum function in contrast with the lump sum function described for finite agent models (e.g., see Groves and Loeb [1975]). In the latter, the lump sum is invariant to the individual's characteristics but may vary with the individual's "name". Of course the distribution approach taken here builds in anonymity.

The DS property of the MP reward principle with lump-sums follows immediately from Lemma 3.

Theorem 1 *Let f be a regular PO_Y mechanism on $N \times V$. If*

$$U(f(\mu, w), v) = MP(\mu, w; v) - h(\mu),$$

then f is a DS mechanism.

Rearranging the terms in the total payoff

$$U(f(\mu, w), v) = u(y(\mu, w), v) + m(\mu, w) = MP(\mu, w; v) - h(\mu),$$

and using Lemma 2 yields,

$$\begin{aligned} m(\mu, w) &= MP(\mu, w; v) - u(y(\mu, w), v) - h(\mu) \\ &= \xi(\mu, w) + u(y(\mu, w), v) - u(y(\mu, w), v) - h(\mu) \\ &= \xi(\mu, w) - h(\mu). \end{aligned}$$

Hence, the MP reward principle may be equivalently described as giving an individual of type v who announces w a y-allocation based on his announced type to satisfy y-optimality, and then guaranteeing that the money allocation, $m(\mu, w)$, will equal (ignoring $h(\mu)$) the external effects associated with the type he announces, $\xi(\mu, w)$; i.e., external effects are internalized.

Let us restate Theorem 1 in terms of external effects.

Theorem 1' *Let $f = (y, m)$ satisfy the hypothesis of Theorem 1. If*

$$m(\mu, w) = \xi(\mu, w) - h(\mu),$$

then f is a DS mechanism.

Theorem 1' says that when external effects are internalized, it pays to tell the truth. Note, however, that this internalization is from the individual point of view but not necessarily from the point of view of the economy as a whole. For that we would also need the budget balancing condition $\int m(\mu, v)d\mu(v) = 0$.

5.2 The Marginal Product/Internalization Principle as a Necessary Condition

There remains the converse, that to achieve the DS property a regular PO_Y mechanism must be specified as in Theorem 1 (or Theorem 1'). Based on the preparations given above and those to follow, we shall show that Holmstrom's [1979] demonstration of necessity for the finite agent model can be "lifted" to the nonatomic case.

For the sufficient conditions on $DSPO_Y$ to become necessary it is well-known that V must exhibit a certain amount of variety. A simple method of insuring enough variety is to assume that *V is a convex set.*

The role of convexity will be to ensure that for any $v, w \in V$ such that $u(y(\mu, w), v) > -\infty$ and $u(y(\mu, v), w) > -\infty$ (i.e., the y-optimal allocation to w is feasible for v and vice-versa), the environment will contain the parameterization $v_\alpha = \alpha v + (1 - \alpha)w, \alpha \in [0, 1]$, connecting v and w.

Recalling that Y_v (resp. Y_w) equals the effective domain of v (resp. w), note that if $v_\alpha = \alpha v + (1 - \alpha)w$, then $Y_{v_\alpha} = Y_v \cap Y_w$, provided $0 < \alpha < 1$. However, $Y_{v_0} = Y_v$ and $Y_{v_1} = Y_w$ may differ from $Y_v \cap Y_w$, and therefore this parameterization need not be "smooth". Before dealing with this problem, we consider a simpler one.

It is common in mechanism theory to assume $Y_w = Y_v$ for all w and v. First we shall prove a converse to Theorem 1 for this special case, where only tastes may be misrepresented. Call a mechanism f *demand-revealing* (DR) if it is DS and $Y_w = Y_v$ for all w and v. A DR mechanism is a special case of a DS mechanism in which, as it were, information about feasible net trades of individuals is always common knowledge.

Theorem 2 *Let V be a convex set and let f be a regular PO_Y mechanism* *on $N \times V$. If f is DR, then*

$$U(f(\mu, w), v) = MP(\mu, w; v) - h(\mu).$$

Equivalently, if $f = (y, m)$ is DR, then

$$m(\mu, w) = \xi(\mu, w) - h(\mu).$$

In some settings, such as models of exchange economies, we must deal with the fact that individual characteristics include, besides variations in tastes, variations in what is individually feasible. The following assumption, by providing for sufficient variation in what is individually feasible in V, allows for a more complete converse to Theorem 1.

V is feasibly connected: $\forall \mu \ \forall v, w$ there exists z such that

(1) z could have delivered $y(\mu, v)$ or $y(\mu, w) : y(\mu, v), y(\mu, w) \in Y_z$,

(2) v and w could have delivered $y(\mu, z) \in Y_v \cap Y_w$.

In the above, $y(\mu, \cdot)$ is the y-optimal allocation in f.

To illustrate feasible connectedness consider a two-commodity-plus-money exchange economy in which $\forall v, v(0,0) > -\infty$, i.e., it is individually feasible for any agent not to trade. Suppose $y(\mu, v) = (1, -1)$ and $y(\mu, w) = (-1, 1)$. This assumption requires, for example, that there is a z which can feasibly make the trade $(1, -1)$ or $(-1, 1)$ but is called upon in an optimal allocation to deliver $y(\mu, z) = (0, 0)$.

COROLLARY: *Let f be a regular PO_Y mechanism, and let V be convex and feasibly connected. If f is DS, then*

$$U(f(\mu, w), v) = MP(\mu, w; v) - h(\mu).$$

Equivalently, if $f = (y, m)$ is DS, then

$$m(\mu, w) = \xi(\mu, w) - h(\mu).$$

REMARK 3: A weaker assumption would suffice. It is enough to postulate that for $\forall \mu \ \forall v \in \text{supp}\ \mu \ \forall w$, there exists a finite sequence $(z_o, z_1, \ldots, z_n)$ with $z_o = v$ and $z_n = w$ such that for all $i = 1, \ldots, n-1$, z_i could have delivered $y(\mu, z_{i-1})$ or $y(\mu, z_{i+1})$ and z_{i-1} and z_{i+1} could have delivered $y(\mu, z_i)$. In Holmstrom's terminology this, along with convexity, would make V into a piecewise smoothly connected domain.

6 Existence Theorems for *DSPO* and *DSPOIR* Mechanisms

6.1 A Possibility Theorem for *DSPO* Mechanisms: Feasible Internalization/Appropriation

Recall that a *DSPO* mechanism is a $DSPO_Y$ mechanism in which the sum of money transfers, $\int m(\mu, v)d\mu(v)$, is always zero. If that sum were positive, the allocation of the money commodity would not be feasible for the participants in the economy and the balance would have to be made up by some outside authority; or if it were negative, the sum would represent the departure from full utility maximization and Pareto-optimality. While the results for $DRPO_Y$ mechanisms in nonatomic models completely parallel the finite agent mechanism results (the literature concentrates on *DR*, rather than the more general *DS* mechanisms), the situation for *DRPO* is quite the opposite. Instead of the impossibility results for *DRPO* cited above for finite agent models, there is always possibility—even for *DSPO* mechanisms.

Since $MP(\mu, v) = u(y(\mu, v), v) + \xi(\mu, v)$ and $g(\mu) = \int u(y(\mu, v), v)du(v)$,

$$\int \xi(\mu, v)du(v) = \int MP(\mu, v)d\mu(v) - g(\mu);$$

i.e., the sum of the external effects is the difference between the sum of the marginal products and the total gains from trade for the economy μ.

From Theorems 1 and 2 and the Corollary we know that a $DSPO_Y$ mechanism implies that

$$m(\mu, v) = \xi(\mu, v) - h(\mu).$$

Suppose the lump sum term to each agent, $h(\mu)$, just equalled the average external effect; i.e.,

$$h(\mu) = \int \xi/\overline{\mu},$$

where $\overline{\mu} = \int d\mu(v)$ is the size of the economy μ. Then $m(\mu, v) = \xi(\mu, v) - \int \xi/\overline{\mu}$.

Summing, we evidently have,

$$\int m(\mu, v)d\mu(v) = \int \left[\xi(\mu, v) - \int \xi/\overline{\mu}\right] d\mu(v) = 0.$$

We are led immediately to the following conclusion.

Theorem 3 *Let f be a regular PO_Y mechanism, and let V be convex and feasibly connected. Then f satisfies $DSPO$ if and only if $\forall\, \mu\, \forall\, v$,*

$$m(\mu, v) = \xi(\mu, v) - \int \xi/\overline{\mu}.$$

Equivalently, f is $DSPOifandonlyif$ $\forall\, \mu$, $\forall\, w$, $\forall\, v$

$$U(f(\mu, w), v) = MP(\mu, w; v) - \int \xi/\overline{\mu}.$$

REMARK 4: Obviously, the conclusions of Theorem 3 also hold for $DRPO$ mechanisms, without the assumption that V is feasibly connected.

Assuming the mechanism is PO_Y, the unique method to obtain $DSPO$ is: set the money payment for any announcement v, whether or not $v \in$ supp μ, equal to the external effect that announcement would create for others, $\xi(\mu, v)$, minus the average external effect in the population μ, $\int \xi/\overline{\mu}$.

With a finite number of individuals this method of strategically internalizing external effects fails because each individual announcement typically changes the average so that it cannot act as a lump sum. This observation agrees with — but does not, of course, demonstrate — the conclusion that $DSPO$ mechanisms typically do not exist in finite individual models. However, as the number of individuals increases, each individual external effect will influence the average less and less, and with a continuum of individuals the influence will be nil. (This conclusion requires certain smoothness assumptions as well as large numbers.)

6.2 A Characterization of Individually Rational $DSPO$ Mechanisms (DSPOIR Mechanisms)

There is an interesting qualification to Theorem 3, one that highlights the role of the money commodity in quasi-linear preferences as a built-in medium for making lump sum transfers. The qualification involves individual rationality.

No matter what the value of $\int \xi$, a regular mechanism can reward each agent with his/her MP — thus ensuring $DSPO_Y$ — and then, by requiring each agent to make a lump sum payment in the money commodity of $\int \xi/\overline{\mu}$, the mechanism can ensure the PO property. It is the ability to break down the construction of a $DSPO$ mechanism into the separate problems of (1) $DSPO_Y$ and then (2) PO, which we shall call the "separation phenomenon", that permits Theorem 3 to apply to a wide range of nonatomic economic environments.

In this section, we show that even within the class of models with quasi-linear preferences, there is a way to "undermine" the separation phenomenon through the introduction of a voluntary participation, or individual rationality, restriction. It is as if the degree of freedom on making lump sum transfers provided by quasi-linearity is removed once this added restriction is imposed. The argument will require further definitions and assumptions.

Attention is confined to environments satisfying the following conditions:

(E.1) (Non-decreasing returns) $\forall\, \mu, \int \xi = \int MP - g(\mu) \geq 0$.

(E.2) (Characteristics are benign) $\forall\, \mu\, \forall\, w, MP(\mu, w) \geq 0$.

(E.3) (Existence of "dummies") $\forall\, \mu\, \exists\, v^o(\mu) \in V$ such that $MP(\mu, v^o(\mu)) = 0$.

Were we to formulate more explicitly a particular model of an economy with private or public goods of the kinds referred to above, assumptions (E.1) and (E.2) could be derived as conclusions. Here we simply assert that these conditions do not go beyond conventional restrictions. (See Section 7, below, for partial confirmation and also for an illustration of a model in which $\int \xi < 0$.)

(E.3) postulates the existence of individuals having no effect on the gains from trade. For example, in a private goods exchange economy if $p(\mu)$ were the efficiency price vector corresponding to the y-optimal allocation in the population μ, then $v^o(\mu)$ could be taken to be those preferences for which the hyperplane $\{x \in \mathbf{R}^\ell : p(\mu)x = 0\}$ is tangent to the indifference curve of v^o passing through the origin of $\mathbf{R}^\ell$; with public goods, v_o would be the tastes of someone entirely indifferent to public goods and who, furthermore, has no resources that contribute toward their production.

Lastly, we assume the existence of a ***status quo allocation***, a $y^o \in \mathbf{R}^\ell$ such that

(E.4) $\forall\, v, u(y^o, v) = 0$, and $\forall\, \mu\, \forall\, v \in \operatorname{supp} \mu$,

$$\text{if } y(\mu, v) = y^o \text{ then } (y(\mu, v)) \in Y(\mu).$$

Such a y-allocation is both individually feasible and aggregately feasible, independent of the characteristics of the individual and the composition of the population. For environments in which allocations can be described by net trades (with or without public goods), y^o would be the null trade; and for environments in which $Y_v = Y$ is a fixed class of public projects, y^o would represent the status quo project. The utility functions are scaled so that $u(y^o, v) = 0$.

Assuming (E.4), say the mechanism f satisfies ***individual rationality*** if

(IR) $\forall\, \mu\, \forall\, v \in \text{supp}\ \mu, U(f(\mu, v), v) \geq u(y^o, v) + 0 = 0.$

There is no "rationality" behind this inequality unless the mechanism gives each individual the choice of whether or not to depart from the status quo. Where the status quo is the null trade, the IR condition can be interpreted as a modification of the DS property: it gives each individual the right to receive the null trade, not only in y-commodities but also in money, whenever the "null" characteristics $v^o(\mu)$ are announced. (To verify this interpretation, the reader must work through the proof of theorem 4 below.)

With the above assumptions, the following result is a simple corollary of Theorem 3.

Theorem 4 *Assume (E.1-4). Then under the hypotheses of Theorem 3, f is a $DSPOIR$ mechanism if and only if $\forall\, \mu$, $\int \xi = 0$.*

The interpretation and meaning of this characterization result will form the subject matter of the next section. The existence of $DSPOIR$ mechanisms will also be discussed there; we will see that they exist on homogeneous environments.

6.3 A Characterization of *DSPO* Mechanisms in Homogeneous Environments: Exact Internalization / Appropriation

We shall conclude this investigation into dominant strategy mechanisms in nonatomic economies by pointing out the connections between the condition $\int \xi = 0$ and the century-old problem of "adding-up" in the marginal productivity theory of distribution.

To motivate the discussion recall that since

$$MP(\mu, v) = u(y(u, v), v) + \xi(u, v),$$

and $\int u(y(\mu, v), v)du = g(u)$, then $\int \xi = \int MP - g$. So, $\int \xi = 0$ is ***equivalent*** to the adding-up condition that the sum of all agent's MP's should equal the total gains in the economy as a whole.

We shall say that a mechanism f satisfies *exact appropriation* if (1) individuals always receive their MP's and (2) the adding-up condition ($\int \xi = \int MP - g = 0$) is satisfied. There is exact appropriation in the sense that each individual exactly appropriates in utility the benefits that his presence confers on the rest of the economy. Or, alternatively put, others neither gain nor lose from the presence of any individual.

There is another way to describe exact appropriation. Recalling that an individual's total utility is equal to $u(y(\mu, v), v) + m(\mu, v)$, if each individual's total utility equals his MP, then $\xi(\mu, v) = m(\mu, v)$. Thus, if $\xi(\mu, v) = m(\mu, v)$, each individual's money payment (positive or negative) exactly measures the external effects the person contributes to others, so that on balance each person exactly internalizes the utility effects (positive or negative) that he/she confers on others.

Say that there is *exact internalization* when (1) $m(\mu, v) = \xi(\mu, v)$ and (2) $\int \xi = 0$. This is simply another version of the exact appropriation conditions.

Having interpreted the condition $\int \xi = \int MP - g = 0$ and its role in $DSPO$ mechanisms (Theorems 3 and 4), we look for conditions under which it will exist. (Note: Theorem 3 inplies that if adding-up does occur, then a $DSPO$ mechanism must give each person their MP, while Theorem 4 implies that a $DSPOIR$ mechanism is possible only when there is adding-up.) Traditional MP theory suggests that adding-up will require constant returns in the function g, and this is indeed the case.

To prepare the argument, we expand the domain of g from the set $N \subset M[V]$ to the smallest positive cone containing N. This will allow the comparison of $g(\mu)$ with $g(t\mu)$, $t > 0$.

We also make the following adding-up assumption on the directional derivatives of g. Namely,

(R.4) $\quad \int Dg(\mu; v) d\mu(v) = Dg(\mu; \mu)$

There is an abuse of notation here. Above, Dg was defined on $N \times V$ rather than $N \times M[V]$ because we preferred to write the directional derivative of $g(\mu)$ in the direction δ_v as $Dg(\mu; v)$ rather than as it should have been, $Dg(\mu; \delta_v)$. In the more consistent notation, (R.4) says that the sum of the individual MP's, $\int Dg(\mu; \delta_v) d\mu(v)$, equals the MP of the sum, $Dg(\mu; \int \delta_v d\mu)$, where $\int \delta_v d\mu(v) = \mu$.

The adding-up condition on the directional derivatives of g should not be confused with the adding-up condition on g itself; which is $\int \xi = \int MP - g = 0$ or $\int Dg(\mu; v) du = g(\mu)$.

The following is a straightforward infinite-dimensional version of Euler's Theorem for linearly homogeneous functions.

Theorem 5 *If C is a positive cone in $M[V]$ and $\forall\ \mu \in C$, $g{:}C \to \mathbf{R}$ satisfies (R.4), there is adding-up if and only if $\forall\ t > 0, \forall\ \mu \in C, g(t\mu) = tg(\mu)$.*

Some of the familiar qualifications to Euler's Theorem in finite dimensions also apply to Theorem 5. For example, if g is homogeneous but not differentiable (because (R.1) and (R.4) are not satisfied), the condition $\int \xi = \int MP - g = 0$ may fail. Also, $\int \xi = 0$ may hold for a particular μ^o even though g fails to be homogeneous. In that case g would exhibit constant returns only locally near μ^o. By insisting that exact internalization must hold everywhere on C we preclude this possibility.

To emphasize the independence of (R.4) from the above conclusion, we point out that under the hypotheses of Theorem 5 the following generalization can be obtained: for all $\mu \in C$ and $t > 1$,

$$\int Dg(\mu; v)du - g(\mu) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} = 0 \quad \text{if and only if} \quad g(t\mu) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} tg(\mu).$$

That is, increasing [decreasing] returns in the sense of $g(t\mu) > tg(\mu)$ [$g(t\mu) < tg(\mu)$], $t > 1$, are reflected by the property that the sum of the individual MP's more than [less than] exhaust the total gains from trade.

We have not traced the returns to scale property of g back to the underlying conditions on the allocation y. This is because, in the final analysis, it is the results of the y-allocation on utility that matters. Nevertheless, the function g is derived from PO_Y allocations and we shall comment briefly on the implications for g of some relevant properties of y. For example, condition (R.4) must ultimately be derived from a condition on y. That condition is

$$\int Dy(\mu, v; w)d\mu(w) = Dy(\mu, v; \mu).$$

It says that the effects on the allocation to an individual from adding-up the separate effects on $Dy(\mu, v; w)$ over $w \in$ supp μ is equal to the effects on $Dy(\mu, v; \cdot)$ of an infinitesimal change in the scale of the population.

Also, the constant returns property $g(t\mu) = tg(\mu)$ will derive from the condition on y that $y(t\mu, v) = y(\mu, v)$, i.e., constant returns to g are due to the fact that scale changes in the population cause no changes in *per capita* allocations.

REMARK 5 (The Value): Readers familiar with the *value* of a nonatomic game (Aumann and Shapley [1974]) will recognize important similarities between the formulas for the value and for $DSPO$ mechanisms. This is a good point at which to make some comparisons.

Let $I = [0,1]$ be the players in a nonatomic game and $e : I \to M[V]$ be a function describing each player's characteristics with the restriction that $e(i) = \delta_v$ so that each player is endowed with a pure characteristic.

Denote $\mu = \int e d\lambda$, where λ is Lebesgue measure, as the total of all players' characteristics in the game and let $\mu_S = \int_S e d\lambda$ be the characteristics of the players in $S \subset I$.

Ignoring how the construction is obtained let $g(\mu_S)$ be the worth of coalition S. (This is an infinite-dimensional version of Aumann and Shapley's finite-dimensional vector measure game.) The *value* assigned to an individual of type v in a game g where the total of all players' characteristics is μ is a utility $\phi(\mu, v)$ given by the "diagonal formula,"

$$\phi(\mu, v) = \int_0^1 Dg(t\mu; v)dt = \int_0^1 MP(t\mu, v)dt.$$

The formula for the utility in a $DSPO$ mechanism is

$$\Phi(\mu, v) = Dg(\mu; v) - h(\mu) = MP(\mu, v) - h(\mu),$$

where $h(\mu) = \int \xi/\overline{\mu}$.

If the formulas do not coincide, i.e., $\phi(\mu, v) \neq \Phi(\mu, v)$ for some μ-non-null set, then the value allocation as a prescription for a mechanism cannot be DS because $\Phi(\mu, v)$ is *the* method of achieving $DSPO$. Alternatively put, if the two formulas differ then $[\phi(\mu, v) - \Phi(\mu, v)]$ is not a lump sum.

The one environment on which the two payoffs agree is the homogeneous one. With homogeneity, $Dg(t\mu; v) = Dg(\mu; v)$ whenever $t > 0$, from which it readily follows that $\phi = \Phi$.

Homogeneity is well-known to be important for the Value Equivalence Theorems. For example, Aumann and Shapley [1974] demonstrate that a class of homogeneous games is derived from nonatomic exchange economies and for these games/economies they show that the core, the value and Walrasian equilibrium coincide. Regarding the value as a mechanism yielding utilities given by the formula $\phi(\mu, v)$, we are led to the following conclusion based on Theorem 3 and 5: *Assuming (R.4), the value is a DSPO mechanism if an only if the environment is homogeneous.*

7 The Work of Others

To conclude our analysis, we comment briefly on some of the connections between the results of this paper and the work of others mentioned in the Introduction. We wish to show that our general equilibrium extension of the marginal product concept and the characterization results derived using it give a framework in which many, apparently unrelated, results from mechanism theory can be synthesized. Our focus is on results for models with large numbers or a continuum of individuals but it should be re-emphasized that our characterization results depend upon constructs also applicable to finite individual models. In particular, the formula for a $DSPO_Y$ mechanism — give each individual his marginal product plus a lump sum — is equivalent to the necessary and sufficient for a $DSPO_Y$ mechanism in finite individual models (see Makowski-Ostroy [1987a]). So the framework also allows one to readily analyze the similarities and differences between large and small economy results; e.g., why the Walrasian mechanism is incentive compatible with large but not with small numbers.

Two further preliminary remarks are in order. First, we shall not distinguish between results quoted below that apply to models with quasi-linear utility and those that apply to more general models without quasi- linearity. Second, in keeping with the mechanism approach and the emphasis of this paper in which explicit reliance on price-guided allocations is minimized, we shall not elaborate upon the pricing interpretations of the results stated below. Demonstrations that the findings of this paper for quasi-linear utility models can be extended to models without quasi-linearity as well as elaborations upon pricing interpretations of DS mechanisms are the subject of Makowski-Ostroy [1987b].

We divide the literature on DS mechanisms with large numbers of individuals according to returns-to-scale properties of the models and then remark on the link with finite individual models.

1. Constant Returns (Homogeneous Environments)

1A. Private Goods. Private goods economies have a built-in homogeneity: doubling the number of each type clearly doubles the total gains, i.e., $g(2\mu) = 2g(\mu)$. Therefore, a $DSPO$ mechanism must reward each individual with an allocation the utility of which is exactly equal to his marginal product. Using the equivalence of the no-surplus (Makowski [1980], Ostroy [1980]) and marginal product characterizations of an allocation, it may be shown that such an exact appropriation allocation is necessarily a Walrasian equilibrium allocation. This confirms the findings of Roberts and Postlewaite [1976], and Hammond [1979] that the Walrasian mechanism has the DS property and it is the only one to have this property. Champsaur and Laroque [1981], McLennan [1982] and Mas-Colell [1983] give versions of this result under the hypothesis that the net trades in a given economy must be "fair." (See Remark 1, above).

1B. Costless Public Goods. In a model with a fixed set of *costless public goods* projects among which only one will be selected, the environment is also homogeneous: if $y(\mu)$ is the project chosen to maximize total utility when the population is μ, then $y(2\mu) = y(\mu)$ will be chosen when the population is 2μ, so $g(2\mu) = 2g(\mu)$. (This model does not capture the distinguishing property of pure public goods. See case 3, below.)

Since the environment is homogeneous, we have $\int \xi = 0$. But the costless public goods model can be shown to have the stronger property that *each* $\xi(\mu, v) = 0$. Thus, in a $DSPO$ mechanism, $m(\mu, v) = 0$. Asymptotic versions of this result are demonstrated by Tideman and Tullock [1976], Green and Laffont [1979], Rob [1982], and Mitsui [1983]; they show that the *per capita* tax imposed by the "pivot" version of a Groves mechanism (our MP mechanism with zero lump sums) converges to zero.

2. Decreasing Returns Consider a model of private goods without private property where individuals "own" their tastes but total resources *are fixed* and under the control of the mechanism. Because an individual's characteristics include only his tastes (representable by a concave utility function) and *not* resources, when the population doubles total utility less than doubles because the same resources must be shared among twice as large a population. The decreasing returns property $g(2\mu) < 2g(\mu)$ is equivalent to $\int \xi = \int MP - g < 0$. Therefore, $DSPO$ requires that the utility of each individual's allocation equal his MP plus a lump sum *subsidy* equal to $(-\int \xi / \overline{\mu})$ to make up for the difference between the sum of the MP's and the total gains, g.

Varian [1976], Kleinberg [1980], McLennan [1982] and Champsaur and Laroque [1982] use this model to study fair allocations. Their findings established that the only fair allocations are Walrasian equilibria arising from an initial allocation in which each individual has an equal-valued share of total resources. We note that such an allocation is the only way to realize the formula for a $DSPO$ mechanism in this model of private goods without private property.

3. Increasing Returns Consider a nonatomic model with public goods produced using private goods as inputs, e.g., Meunch [1972]. Two identical populations μ, each producing the same optimal quantities of public goods with the same resources — so producing total utility $2g(\mu)$ — could simply by combining to form one economy, halve the *per capita* resources contributed and maintain the same total quantity of public goods, and therefore produce total gains for the population 2μ such that $g(2\mu) > 2g(\mu)$.

In this situation, $\int \xi = \int MP - g > 0$. Here a *DSPO* mechanism gives each individual his *MP* and then imposes a uniform lump sum *tax* of $(-\int \xi/\overline{\mu})$. Hammond [1979] has given a price characterization of a *DSPO* mechanism with public goods. It can be shown (Makowski-Ostroy [1987b]) that his "privately fair Lindahl allocation" is equivalent to the above *MP* mechanism plus lump sum.

Does a *DSPOIR* mechanism exist for models with costly public goods? (Clearly, they do exist for private goods, while *IR* is not applicable in case 2.) A model of costly public goods satisfies the hypotheses of Theorem 4. Therefore, a *DSPOIR* mechanism exists if and only if $\int \xi = 0$. But $\int \xi = 0$, the homogeneity condition, contradicts the distinguishing feature of public goods models, namely the cost sharing and consequent increasing returns property, $\int \xi = \int MP - g > 0$. So, we can conclude that a *DSPOIR* mechanism cannot possibly exist when there are (costly) public goods. This agrees with the finding of Roberts [1976].

4. Finite Numbers: Indivisibilities We have used the *MP* theme to provide an interpretative survey of the various results for *DS* mechanisms in continuum economies. But what is the connection with finite agent models, to which the literature on *DS* mechanisms is overwhelmingly devoted? In making this connection, we will be implicitly shifting the focal point of the literature from models with small numbers of individuals to models with large numbers. Rather than viewing the large numbers case as an extension of the finite agent model, we shall regard the continuum model as the focal point and the finite individual model as a "special case". This change in perspective is suggested by the parallels with traditional *MP* theory.

It suffices to confine attention to constant returns models — cases 1A and 1B, above. While constant returns environments are the ideal setting for *DSPO* mechanisms in nonatomic models, how to explain that *DSPO* mechanisms do not generally exist when the number of individuals is finite, i.e., when the space of agents contains a finite number of atoms?

Consider the analogy with the *MP* theory of distribution. In a continuum model, we have shown that in constant returns environments, the necessary and sufficient condition for $DSPO$ is to pay individuals exactly their MP's. Such a condition is not automatically guaranteed by constant returns; it also requires the differentiability condition (R.4). (Recall the similar requirement in the conventional statement of Euler's Theorem for homogeneous functions.) Going behind the g function to the economic environment from which it is derived, it can be demonstrated that while (R.4) need not always obtain, it will hold generically for the kinds of economies to which we have referred. Thus, in constant returns environments, when each individual is infinitesmial it is typically possible to pay each one his MP and therefore to demonstrate that a $DSPO$ mechanism is feasible.

Now make the following modification: while continuing to assume a constant returns environment such as would come from a private goods or costless public goods economy where doubling the number of each type of individual doubles the total gains, assume each individual is an atom. The fact that individuals are no longer infinitesimal is similar to the hypothesis in the theory of production that even though there is constant returns, if factors of production are *indivisible*, then it will typically be impossible to pay each one its MP. A similar interpretation appears to lie behind the non-existence results for $DSPO$ mechanisms in finite individual economies. There is firm support for this interpretation in the case of costless public goods. Laffont and Maskin [1979] have shown that among all the $DSPO_Y$ mechanisms, there is none that dominates the pivot mechanism in minimizing the absolute value of the sum of monetary transfers. (Recall that for $DSPO$, the sum must be zero.) Since the pivot mechanism rewards individuals with their MP's, we can trace the non-existence of $DSPO$ mechanisms to the failure to obtain adding-up, which in turn can be traced to the fact that the "factors of production" in the gains function g, i.e., the individuals, are indivisible.

8 Proofs

Lemma 1 Holding μ and w fixed, let

$$\begin{aligned} K(t) &= g(\mu + t\delta_w) \\ &= \int u(y(\mu + t\delta_w, v)d\mu + tu(y(\mu + t\delta_w, w), w) \\ &= \int k(t, v)d\mu + tk(t, w). \end{aligned}$$

It is well-known that

$$K'(t) = \int \partial_t k(t,v)d\mu + \partial_t[tk(t,w)],$$

provided $\partial_t k(t,v)$ and $\partial_t[tk(t,w)]$ are t-continuous. From the definitions of $k(t,v)$ and $k(t,w)$,

$$\begin{aligned}
\partial_t k(t,v) &= \partial_y u(y(\mu + t\delta_w, v), v)Dy(\mu + t\delta_w, v; w) \\
\partial_t[tk(t,w)] &= \partial_y u(y(\mu + t\delta_w, w), w)Dy(\mu + t\delta_w, w; w) + u(y(\mu + t\delta_w, w), w).
\end{aligned}$$

From (R.1), both y and Dy are t-continuous (since they are μ-continuous) and by the differentiability hypothesis on $v, w \in V$, $\partial_y u$ are y-continuous. Also, Y_v is closed, $v \in V$. Therefore, $\partial_t k(t,v)$ and $\partial_t[tk(t,w)]$ are continuous.

It is also well-known that $K'(t)$ is continuous on $[0,a]$ provided $\partial_t k$ is jointly continuous on $[0,a] \times V$. Appeal to (R.1) and the properties of $v, w \in V$ guarantees this joint continuity. Thus, $K'(t) \to K'(0)$ as $t \to 0$.

Now,

$$\begin{aligned}
Dg(\mu; w) &= \lim_{t \to 0_+} \frac{K(t) - K(0)}{t} = K'(0) \\
&= \int \partial_t k(0,v)d\mu + k(0,w) \\
&= \int \partial_y u(y(\mu,v),v)Dy(\mu,v;w)d\mu + u(y(\mu,w),w) \\
&= \xi(\mu,w) + u(y(\mu,w),w),
\end{aligned}$$

where the third equality follows from the second after substituting the definitions of $k(0,w)$ and $\partial_t k(0,v)$.

The continuity of $MP(\mu,w) = Dg(\mu;w)$ on $N \times V$ will be established through the continuity of $\xi(\mu,w)$ since by the hypothesis on V and (R.1), $u(y(\mu,w),w)$ is continuous on $N \times V$.

Let

$$\xi(\mu,w) = \int h(\mu,w,v)d\mu(v),$$

where $h(\mu,w,v) = \partial_y u(y(\mu,v),v)Dy(\mu,v;w)$; i.e., $h(\mu,w,v) = \partial_t k(0,v;\mu,w)$ shows the explicit dependence of $\partial_t k$ on μ and w. The continuity of ξ on $N \times V$ follows from the continuity of h on $N \times V \times V$ which is readily found to follow from (R.1) and the assumptions on V.

Lemma 2 Whenever $y(\mu,w) \notin Y_v$ define $MP(\mu,w;v) = -\infty$.

Alternatively, fix a μ, w and v such that $y(\mu, w) \in Y_v$. We show that there exists an $a > 0$ such that for $t \in [0, a]$, $y(\mu + t\delta_w, w) \in Y_v$.

Suppose not; then there exists a sequence $\{t_n\} \to 0$ such that $y_n = y(\mu + t_n\delta_w, w) \notin Y_v$. By (R.1), $y(\cdot, w)$ is μ-continuous so $y_n \to y(\mu, w)$. But by (R.3), $y_n \notin Y_v, y_n \to y(\mu, w)$ implies $y(\mu, w) \notin Y_v$ contradicting the original hypothesis.

Proceeding along lines similar to Lemma 1, let

$$\begin{aligned} H(t) &= g(\mu + t\delta_w; \mu + t\delta_v) = \int u(y(\mu + t\delta_w, z)d\mu(z) + tu(y(\mu + t\delta_w), v) \\ &= \int h(t, z)d\mu + th(t, v). \end{aligned}$$

Provided $\partial_t h(t, z)$ and $\partial_t[th(t, v)]$ are t-continuous,

$$H'(t) = \int \partial_t h(t, z)d\mu + \partial_t[th(t, v)].$$

Note that for $z \neq v$, $h(t, z) = k(t, z)$ given in Lemma 1. The continuity argument there applies here to $\partial_t h(t, z)$.

By definition and differentiation,

$$\begin{aligned} \partial_t[th(t, v)] &= t\partial_t h(t, v) + h(t, v) \\ &= t\partial_y u(y(\mu + t\delta_w, w), v)Dy(\mu + t\delta_w; w) + u(y(\mu + t\delta_w, w), v). \end{aligned}$$

Because $y(\mu + t\delta_w, w) \in Y_v, u(y(\mu + t\delta_w, w), v)$ is t-continuous and the same agrument as in Lemma 1 applies to show that $H'(t) \to H'(0)$ as $t \to 0$.

Thus

$$\begin{aligned} MP(\mu, w; v) &= \lim_{t \to 0_+} \frac{H(t) - H(0)}{t} = H'(0) \\ &= \int \partial_t h(0, z)d\mu + h(0, v) \\ &= \int \partial_t k(0, z)d\mu + u(y(\mu, w), v) \\ &= \xi(\mu, w) + u(y(\mu, w), v). \end{aligned}$$

The continuity of $MP(\mu, w; v)$ on the set $\{(\mu, w, v) : y(\mu, w) \in Y_v\}$ follows first from the continuity of $\xi(\mu, w)$ in Lemma 1. For the remainder of the argument suppose $\{(\mu_n, w_n, v_n)\} \to (\mu, w, v)$, and $y(\mu_n, w_n) \in Y_{v_n}$. Then, by the fact noted in Section 2, $X = \{(y, v) : y \in Y_v\}$ is closed in $\mathbf{R}^\ell \times V$, and by the assumption (R.1) that y is continuous on

$N \times V, y(\mu_n, w_n, v_n) \to y(\mu, v) \in Y_v$. The hypotheses about the parameterization of V imply that $u(y, v)$ is jointly continuous on X and therefore $u(y(\mu_n, w_n), v_n) \to u(y(\mu, w), v)$. This completes the argument for the continuity of $MP(\mu, w, v)$ on $\{(\mu, w, v) : y(\mu, w) \in Y_v\}$.

Lemma 3 It is evident from the definition of g that for all $t > 0$ and v,

$$g(\mu + t\delta_w; \mu + t\delta_v) \leq g(\mu + t\delta_v; \mu + t\delta_v) = g(\mu + t\delta_v),$$

i.e., the *total* gains from trade cannot possibly be increased through misrepresentation. Therefore,

$$\begin{aligned} MP(\mu, w; v) &= \lim_{t \to 0_+} \frac{g(\mu + t\delta_w; \mu + t\delta_v; \mu + t\delta_v) - g(\mu)}{t} \\ &\leq \lim_{t \to 0_+} \frac{g(\mu + t\delta_w; \mu + t\delta_v) - g(\mu)}{t} \\ &= \lim_{t \to 0_+} \frac{g(\mu + t\delta_v) - g(\mu)}{t} \\ &= MP(\mu, v). \end{aligned}$$

Theorem 1 If $U(f(\mu, w), v) = MP(\mu, w; v) - h(\mu)$, then by Lemma 3, $U(f(\mu, v), v) - U(f(\mu, w), v) = MP(\mu, v; v) - MP(\mu, w; v) \geq 0$.

Theorem 2 Fix $(\mu, v, w) \in N \times V \times V$. By convexity, for $\alpha, \beta \in [0, 1]$ there exists $v_\alpha = \alpha v + (1 - \alpha)w$ and $v_\beta = \beta v + (1 - \beta)w$.

By Lemma 2,

$$MP(\mu, v_\beta; v_\alpha) = u(y(\mu, v_\beta), v_\alpha) + \xi(\mu, v_\beta). \tag{1}$$

Therefore, we may write $U(f(\mu, v_\beta), v_\alpha) = u(y(\mu, v_\beta), v_\alpha) + m(\mu, v_\beta)$ as

$$\begin{aligned} U(f(\mu, v_\beta), v_\alpha) &= MP(\mu, v_\beta; v_\alpha) - H(\mu, v_\beta) \\ &= \psi(\beta, \alpha) - k(\beta), \end{aligned}$$

taking advantage of the fact that μ, v and w are fixed.

From the hypothesis that f is a regular DR mechanism, we have that for all $\alpha, \beta \in [0, 1]$

$$\alpha \in \arg_\beta \max \psi(\beta, \alpha) - k(\beta).$$

From Lemma 3,

$$\alpha \in \arg_\beta \max \psi(\beta, \alpha).$$

By (1) and (2),

$$\psi(\beta,\alpha) = u(y(\mu, v_\beta), v_\alpha) + \xi(\mu, v_\beta),$$

where $u(y(\mu, v_\beta), v_\alpha) = \alpha u(y(\mu, v_\beta), v) + (1-\alpha)u(y(\mu, v_\beta), w)$

Differentiating ψ with respect to α,

$$\begin{aligned}\frac{\partial\psi(\beta,\alpha)}{\partial\alpha} &= \partial_\alpha u(y(\mu, v_\beta), v_\alpha) \\ &= u(y(\mu, v_\beta), v) - u(y(\mu, v_\beta), w).\end{aligned}$$

Let $Q = \{y(\mu, v_\beta) : \beta \in [0,1]\}$. Now, because $y(\mu,\cdot)$ is continuous in v_β and $\{v_\beta : \beta \in [0,1]\}$ is compact, Q is compact. Therefore

$$sup_{\beta,\alpha}|\partial_\alpha\psi(\beta,\alpha)| = sup\{|u(y,v) - u(y,w)| : y \in Q\} < -\infty.$$

Having established (a) – (c), now apply the following basic result proved in Holmstrom [1979],

Lemma Let $\psi\colon [0,1]\times[0,1] \to \mathbf{R}$ and $k\colon [0,1] \to \mathbf{R}$ satisfy (a), (b) and (c), then k is constant.

Therefore, there is an h such that $h(\mu) = H(\mu, v) = H(\mu, w)$, as was to be demonstrated.

Corollary Note that what is required to prove Theorem 2 is that for all $\beta \in [0,1], y(\mu, v_\beta) \in Y_v \cap Y_w$, hence ψ is real-valued. The feasible connectedness assumption says that if this does not hold there is a z such that for all $\beta \in [0,1]$, (1) $y(\mu, v_\beta) \in Y_v \cap Y_z$ when $v_\beta = \beta v + (1-\beta)z$, and (2) $y(\mu, z_\beta) \in Y_z \cap Y_w$, where $z_\beta = \beta z + (1-\beta)w$. Apply the conclusions of Theorem 2 to (1) to obtain $H(\mu, v) = H(\mu, z)$ and to (2) to obtain $H(\mu, z) = H(\mu, w)$, leading to the same final conclusion as Theorem 2 that $h(\mu) = H(\mu, v) = H(\mu, w)$.

Theorem 3 By Theorem 1′, if f is a regular PO_Y mechanism and $m(\mu, v) = \xi(\mu, v) - h(\mu)$, then f is $DSPO_Y$. If $h(\mu) = \int \xi/\overline{\mu}$, then $\int m = 0$ and therefore f is $DSPO$.

Conversely, if f is $DSPO$ then by Theorem 2 and its Corollary $m(\mu, v) = \xi(\mu, v) - h(\mu)$ and $\int m = 0$. Thus $\int \xi - h(\mu)\overline{\mu} = 0$, or $h(\mu) = \int \xi/\overline{\mu}$.

Theorem 4 From Theorem 3, if f is $DSPO$ and $h(\mu) = \int \xi/\overline{\mu} = 0$, then $m(\mu, v) = \xi(\mu, v)$ and therefore $U(f(\mu, v), v) = \xi(\mu, v) + u(y(\mu, v), v) = MP(\mu, v)$. But by (E.2), $MP(\mu, v) \geq 0$, so f satisfies IR.

To demonstrate that $DSPOIR$ implies $\int \xi = 0$, suppose the contrary. Then, by (E.1) there is a μ' such that $\int \xi(\mu', v)d\mu' > 0$. Let $v^o = v^o(\mu')$

(recall E.3), and let $\mu = \mu' + t\delta_{v^o}$. Notice by the continuity of $\sigma(\mu) = \int \xi(\mu, v)d\mu$ (see Lemma 2) and the continuity of $MP(\cdot, v^o)$, that as $t \to 0$, $\sigma(\mu)/\overline{\mu} \to \sigma(\mu')/\overline{\mu'} > 0$ and $MP(\mu, v^o) \to MP(\mu', v^o) = 0$. Hence, $\exists\ t > 0$ such that $MP(\mu, v^o) - \sigma(\mu)/\overline{\mu} < 0$. But by Theorem 3, *DSPO* implies

$$U(f(\mu, v^o), v^o) = MP(\mu, v^o) - \sigma(\mu)/\overline{\mu}.$$

The RHS, we have verified is negative for some μ, contradicting *IR*.

Theorem 5 This is a straightforward extension of the finite-dimensional version of Euler's Theorem on functions homogeneous of the first degree.

If $g(t\mu) = t(\mu), t > 0$, then

$$\begin{aligned} Dg(\mu;\mu) &= \lim \frac{g(\mu + t\mu) - g(\mu)}{t} = \lim \frac{(1+t)g(\mu) - g(\mu)}{t} \\ &= g(\mu). \end{aligned}$$

By (R.4), $\int Dg(\mu; v)d\mu = Dg(\mu;\mu)$, so it and first degree homogeneity imply adding-up.

Conversely, if there is adding-up then $\int Dg(\mu; v)d\mu = g(\mu)$ and by (R.4), $\int Dg(\mu; v)d\mu = Dg(\mu;\mu)$, so $Dg(\mu;\mu) = g(\mu)$. Fixing μ, let

$$L(t) = g(t\mu) = Dg(t\mu; t\mu)$$

Therefore,

$$L'(t) = Dg(t\mu;\mu) = t^{-1}L(t)$$

where the first equality follows from the definition of $L'(t)$ as

$$\lim_{t\to 0+} t^{-1}[g([t+h]\mu) - g(t\mu)]$$

and the second from the linear homogeneity of the directional derivative, i.e., $Dg(t\mu; a\mu) = aDg(t\mu;\mu), a > 0$.

The equation $L(t) = tL'(t)$ is well-known to have the solution $L(t) = ct$ and putting $t = 1, c = L(1) = g(\mu)$. Therefore, $g(t,\mu) = L(t) = tL'(t) = tc = tg(\mu)$.

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