## Job Search in More Than One Labor Market

by

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## Section 1: Introduction

A person looking for a new job faces a choice, not only of which jobs to accept, but where to search for a job. His choice affects the duration of his unemployment and the type of job he eventually takes. In a changing economy with the demand for labor shifting between sectors, this choice has important implications for the adaptability of the economy as a whole.

This paper analyzes the optimal search behavior, the speed of reemployment, and the intersectoral mobility of unemployed workers who may search in two distinct labor markets or sectors of the economy. A worker has alternatives in searching for a new job: he may search in either or both of the two sectors. The optimal strategy for searching for a new job in one sector will be affected by the possibility of locating a job in another sector with different attributes.

The two sectors, A and B, are each characterized by a layoff rate, an offer-arrival rate, and a wage-offer distribution. The worker may not search while employed. There is no possibility of being recalled to a previous job. The worker maximizes his or her expected wealth in a stationary environment over an infinite time horizon. 1

We shall see that an improvement in the prospects for employment in one sector induce a worker following the optimal search strategy to increase the intensity of his or her search for

<sup>&</sup>lt;sup>1</sup>Many generalizations and specializations of the basic models of optimal job search in one sector have been developed and analyzed. My purpose is not to repeat the discoveries about learning, nonstationarity, risk aversion, etc. which have already been made. It is to investigate the essential character of job search in two sectors as opposed to one. Therefore I omit many of the complications which have been the focus of much of the search literature.

a job in that sector while decreasing it in the other sector, but to increase his reservation wages in both sectors. The reservation wages associated with the two sectors of the economy are equal, despite differences in layoff rates. However, a worker will search more intensively in the sector with a lower layoff rate. The analysis can facilitate empirical analysis of the search behavior and mobility of unemployed workers faced with more than one labor market in which to search (See Fallick (1988)).

## Section 2: The Model

Job search and work take place in continuous time. At any time, the worker may be either unemployed, employed in sector A at some wage rate, w, or employed in sector B at some wage rate. (The basic model generalizes easily to N sectors, and contains the standard search paradigm as a special case.) The number of hours of work per period associated with a job is fixed, so that w is the rate of current compensation. If and only if a worker is unemployed, he or she may search for a job in either or both of the two sectors simultaneously. He pays search costs depending upon how much he searches in total.

At any time, the worker may move from unemployment to employment in either sector by receiving and accepting a job-offer from that sector. Job offers expire immediately if not accepted. If employed, the worker may move from employment in either sector to unemployment by being laid off. Every job in a sector has the same layoff rate. It is assumed that an employed worker never quits (It would never by optimal to quit anyway).

The worker is assumed to maximize his or her expected income over an infinite horizon. Since the environment is stationary, the elements of the optimal strategy as well as the value

functions defined below will be stationary.<sup>2</sup>

Since a job is characterized by its sector and wage, a job-offer from sector j can be described as being acceptable if and only if the the wage associated with the job belongs to an acceptance set. It will be obvious that the optimal strategy exhibits the reservation wage property. That is, for each sector there is a threshold wage below which the job is not be acceptable and above which it is acceptable. Denote the reservation wage for jobs in sector j by  $\mathbf{w}_{j}^{r}$ . Assume that the wage rates associated with job offers from sector j are random variables drawn independently from an exogenous distribution with cumulative probability distribution function  $\mathbf{F}^{j}(\mathbf{w})$  which is constant over time.

Assume that the instantaneous probability of receiving a job offer from sector j has two multiplicative components: the exogenous instantaneous probability that an offer from sector j potentially arrives during the period, which is assumed to be constant over time, is called an "offer arrival rate", and will be denoted  $\alpha_j$ , and a function of the "intensity" with which the worker searches for a job in sector j if he or she is unemployed,  $\sigma(s_j)$ . Search intensity, s, may refer to time, financial resources, effort, or other resources devoted to search. There is some total amount of search intensity available to the worker at any one time. Normalize this amount to unity, so that  $s_A + s_B \le 1$ . If the worker is employed, it is assumed that working takes up all of the available "intensity" so that there is none left for searching for a different job. Assume that  $\sigma(.)$  is an increasing concave function with  $\sigma(0)=0$  and  $\sigma(1)=1$ .

See Beckman (1972) for example.

Assume that job offers are generated by a Poisson process, so that the probability of receiving n offers from sector j during a  $-\alpha_j\sigma(s_j)\times (\alpha_j\sigma(s_j)\times)^n/n!$  period of unemployment of length x is e  $(\alpha_j\sigma(s_j)\times)^n/n!$  . The probability of being laid off from a job in sector j during any period of length x is e  $\lambda_j\times$ , which is assumed to be constant over time. As x+0, the (instantaneous) probability of receiving more than one offer vanishes, and the "offer-reception" rate and the layoff rate for sector j go to  $\alpha_j\sigma(s_j)$  and  $\lambda_j$ , respectively. The instantaneous probability of receiving an offer from both sectors,  $\lim_{x\to 0} \left[\prod_j e^{-\alpha_j\sigma(s_j)\times} \alpha_j\sigma(s_j)\times\right]/x$  also goes to zero.

Given all of this, the hazard rate for transitions from unemployment to employment in sector j at time t is  $\alpha_j\sigma(s_j)\ [1-F^j(w_j^r)]\ .$  The worker affects this rate by choosing  $s_j$  and  $w_j^r$ . A set of these elements constitute a search strategy, S.

Assume that the worker chooses a search strategy so as to maximize the expected discounted value of his current and future income, with discount rate r. Beginning from a state of unemployment, the worker maximizes  $\mathbf{E} \int_0^\infty e^{-rt} [\mathbf{w}(t) - (\mathbf{s}_A(t) + \mathbf{s}_B(t)) c]$ , where  $\mathbf{s}_j(t) = \mathbf{s}_j$  if he is unemployed,  $\mathbf{s}_j(t) = 0$  if he is employed, and  $\mathbf{w}(t) = 0$  if he is unemployed at time t, and c is the constant marginal cost of searching. The worker chooses a search strategy subject to the constraints that  $\mathbf{s}_A \geq 0$ ,  $\mathbf{s}_B \geq 0$ ,  $\mathbf{s}_A + \mathbf{s}_B \leq 1$ , and the transition rates outlined above.

Let  $V^j(w)$  be the expected present discounted value of current and future income as viewed from time 0 if the worker were employed in sector j at wage w at time 0 and follows the optimal search strategy, j=A,B and let  $V^U$  be the analogous quantity for

unemployment at time 0. According to Bellman's Principle of Optimality, and explicitly incorporating the transition probabilities.

(1) 
$$V^{u} = \max_{s_{A}, s_{B}, w_{A}, w_{B}} - (s_{A} + s_{B})cx + e^{-rx} V^{u}$$

$$+ \sum_{j} e^{-\alpha_{j} \sigma(s_{j})x} \alpha_{j} \sigma(s_{j}) x e^{-rx} \int_{w_{i}^{r}} [V^{j}(w) - V^{u}] dF^{j}(w) + O(x)$$

s.t the feasibility constraints, and

 $V^{j}(w) = wx + e^{-\lambda} \int_{\lambda_{j}}^{x} xe^{-rx} V^{u} + e^{-\lambda} \int_{0}^{x} e^{-rx} V^{j}(w) + O(x)$ , where O(x) denotes a function such that  $\lim_{x\to 0} \frac{O(x)}{x} = 0$ , representing the possibility of more than one event occurring in an interval of length x, which is negligible when x is small.

At the optimal strategy, dividing each equation by x and taking the limit as  $x \to 0$ , (1) can be written

(2a) 
$$rV^{u} = -\left(s_{A} + s_{B}\right)c + \sum_{j} \alpha_{j} \sigma(s_{j}) \int_{w_{j}}^{\infty} [V^{j}(w) - V^{u}] dF^{j}(w)$$

(3) 
$$(r+\lambda_j)V^j(w) = w + \lambda_jV^{ll}$$
  $j=A,B$ 

From (2a) it is clear that the reservation wages are those which satisfy

$$(4a) V^{j}(w_{j}^{\Gamma}) = V^{L} \quad j=A,B \quad .$$

Assume that  $\sigma'(s) \to \infty$  as  $s \to 0$ , which ensures that  $s_A, s_B>0$ . The three first order Kuhn-Tucker conditions for the search intensities are

(5a) 
$$-c + \alpha_{j}\sigma'(s_{j}) \int_{w_{j}}^{\infty} \left[ V^{j}(w) - V^{u} \right] dF^{j}(w) - \mu = 0, \quad j=A,B.$$

(6a) 
$$1-s_{A}-s_{B} \ge 0$$
, = 0 if  $\mu > 0$ ,

where  $\mu$  is the Kuhn-Tucker multiplier on the constraint that  $\mathbf{S_A}^{+\mathbf{S_B}} \leq 1.$ 

## Section 3: Reservation Wages

Combining (3) with (4a) yields

(4b) 
$$V^{u} = \frac{w_{j}^{r} + \lambda_{j}V^{u}}{r + \lambda_{j}} \Rightarrow w_{j}^{r} = rV^{u} \text{ for } j=A,B$$

Result: 
$$w_{A}^{\Gamma} = w_{B}^{\Gamma} = w^{\Gamma} = rV^{U}$$
.

The reservation wages for jobs in the two sectors are equal, despite the fact that the layoff rates, offer-arrival rates, and offer distributions differ between the sectors. Changes in  $\alpha$ , q, and  $\lambda$  in one sector will not affect the reservation wage for that sector differently than they affects the reservation wage for the other.

This result is surprising, especially since it holds when the layoff rates differ. Looking at equations (4a), in order to affect the reservation wages in the two sectors differently, a change would have to affect the  $V^A(w)$  and  $V^B(w)$  differently. The parameters  $\alpha$  and q affect the probabilities associated with obtaining an offer at a particular wage, but do not affect the value of having a job once it is taken, the  $V^J(w)$ , except through the possibility that one will be searching in the future, i.e., that one will be laid off. Once laid off, it does not matter where you were employed, so if the layoff rates were equal, there would be no reason for the reservation wages to react differently between the sectors. Although the layoff rates may differ, this makes no difference.

It seems intuitively reasonable that a higher probabality of being laid off, less job security, in a sector would make many

See Wright(1987) and the references cited there for similar results. In Wright's model, each job offer is characterized by w and  $\lambda$ . He demonstrates that, under certain assumptions similar to those made here, w does not vary from job to job. Here we see that this result generalizes to a situation in which "jobs" differ not only by w and  $\lambda$  but also by  $\alpha$  and F(w).

jobs in that sector less attractive to a prospective employee and therefore raise the reservations wage for that sector relative to the other sector. However, this is not the case. The important point is that at the reservation wage, the worker is indifferent between remaining employed at that job and being laid off. (Note that in this model there are no fixed costs to being laid off or to becoming employed. See Wright (1987) for further discussion.) Section 4: Comparative Statics

We are now prepared to examine how the optimal search strategy changes as the opportunities for employment in the two sectors and other elements of the environment vary. I am particularly interested in seeing whether or not changes in the relative prospects for employment in the two sectors change the worker's optimal search strategy with respect to the two sectors differently, in particular, whether an improvement in the prospects for employment in one sector induce the optimally searching worker to concentrate more on that sector at the expense of the other. We have already seen that there are no different effects on the reservation wages, but, as we shall see, the same does not hold true for search intensity.

## Part A: The Reservation Wage

Substituting the value functions out of (2a) using (3) and (4b) yields the fundamental reservation wage equation

(2b) 
$$w^r = -(s_2 + s_3)c + \sum_{j} \frac{\alpha_j \sigma(s_j)}{r + \lambda_j} \int_{w^r}^{\infty} (w - w^r) dF^j(w)$$

Equation (2b) is a generalization of the standard reservation wage equation, e.g., Mortenson (1986), as is clear if  $\alpha_{\rm g}=0$ .

Differentiating (2b)<sup>4</sup>,  $\frac{dw^{\Gamma}}{d\alpha_{j}} = -\frac{\sigma(s_{j})}{r+\lambda_{j}} \int_{w^{\Gamma}}^{\infty} (w-w^{\Gamma}) dF^{j}(w)/\phi ,$  where  $\phi = 1 + \sum_{j} \frac{\alpha_{j}\sigma(s_{j})}{r+\lambda_{j}} (1-F^{j}(w^{\Gamma})) > 0$ .

Therefore,

Result:  $\frac{dw^T}{d\alpha_j} > 0$  j=A,B. An increase in the offer-arrival rate in either sector increases the common reservation wage of the optimal search strategy. Similarly, it is easy to show the following:

Result:  $\frac{dw^{\Gamma}}{d\lambda_{j}} < 0$  j=A,B. An increase in the layoff rate in either sector decreases the common reservation wage. Note the difference between this and the earlier result that  $w^{\Gamma}$  for each sector is invariant to a difference between  $\lambda_{A}$  and  $\lambda_{B}$ . Here a change in the *levels* of the layoff rates affects the common reservation wage.

Result:  $\frac{dw}{dc}$  < 0 j=A,B . An increase in the variable costs of searching for a job decreases the reservation wage.

In order to better compare the wage-offer distributions in the two sectors, let  $F^{A}(w)$  and  $F^{B}(w)$  belong to the same class of probability distributions, the class being as general as you like. Let the members of the class be indexed by the parameter q, and normalize it by letting  $F^{B}(w) \equiv F(w;0)$  and  $F^{A}(w) \equiv F(w;q)$ . Varying q represents a change in the wage-offer distribution in sector A relative to that in sector B.

 $\frac{dw^{\Gamma}}{dq}$  r $\frac{dV^{U}}{dq}$ , from (4b). A change in the wage-offer distribution in sector A (or, symmetrically, in sector B) which

 $<sup>^4</sup>$  If (1) is rewritten as  $v^u=\max_{A,S_B}\max_{w,w}\ldots$  then the envelope satisfies theorem can be employed to justify ommitting terms in  $\frac{ds_k}{d\alpha_j}$  .

increases the present discounted value of the worker's expected stream of current and future income increases the common reservation wage.

More generally, 
$$\frac{dw^{\Gamma}}{dq} = \frac{\alpha_{A}\sigma(s_{A})}{r+\lambda_{A}} \int_{w^{\Gamma}}^{\infty} (w-w^{\Gamma})dF_{2}(w;q) /\phi = \frac{\alpha_{A}\sigma(s_{A})}{r+\lambda} \int_{w^{\Gamma}}^{\infty} F_{2}(w;q) /\phi$$
. Therefore, for example, if dq represents a first-order stochastic improvement,  $\frac{dw^{\Gamma}}{dq} > 0$ , and a mean-preserving spread also increases the reservation wage.

To sum up the results on the reservation wage, most changes in the environment affect the reservation wage in the directions which we would intuitively expect. However, the reservation wages for jobs in the two sectors are always identical, despite the fact that the layoff rates (as well as the offer-arrival rates and wage distributions) associated with jobs in the two sectors may differ. However, it is important to note that higher layoff rates in the economy overall lower the common reservation wage. In brief,

Proposition: 
$$w_A^{\Gamma} = w_B^{\Gamma} = w^{\Gamma}$$

$$\frac{dw^{\Gamma}}{d\alpha_i} > 0 \qquad \frac{dw^{\Gamma}}{d\lambda_i} < 0 \qquad \frac{dw^{\Gamma}}{dc} < 0 \qquad \frac{dw^{\Gamma}}{dq} = r\frac{dV^{U}}{dq} \qquad j=A,B$$

#### Part B: Search Intensities

We must distinguish between cases wherein the constraint that  $s_A + s_B \le 1$  is and is not binding. The static analysis assumes that we do not jump from one case to the other.

Notice that if the constraint is not binding, then neither the value of current income, the marginal "productivity" of search in corralling offers,  $\sigma'(s_k)$ , nor the marginal cost of searching, c, depends upon the total amount of search undertaken at any one time,  $s_2 + s_3$  (or over an interval of time). When the constraint is not binding, therefore, in this model the total amount of search intensity per se undertaken does not affect the decision of how intensively to search in each sector. There is, for example, no issue of increasing marginal utility of leisure or income as the sum of the search intensities increases. In this sense the model does not present a traditional problem of allocating a limited amount of resources to alternative uses, in this case a limited amount of "intensity" to searching in alternative sectors. Nevertheless, search in the two sectors will be seen to be related through the dynamics of the search problem.

## Case 1: s\_+s\_<1 (Interior Solution)

In this case, in which it would be feasible to search more overall, the first order conditions become, after eliminating the value functions,  $^{5}$ 

$$(5a.1) \quad \frac{\alpha_{j}\sigma'(s_{j})}{r+\lambda_{j}} \int_{w}^{\infty} (w-w'') dF^{j}(w) = c , j=A,B.$$

$$^{5}\text{since }V^{j}(w)-V^{u}=\frac{w-w}{r+\lambda}_{j}^{r}$$

The Offer Arrival Rates:

Differentiating (5a.1) with respect to a and minipulating yields

$$(7) \operatorname{sgn}(\frac{ds_{\mathbf{A}}}{d\alpha_{\mathbf{A}}}) = \operatorname{sgn}\left\{\frac{\sigma'(s_{\mathbf{A}})}{r+\lambda_{\mathbf{A}}}\int_{\mathbf{W}}^{\infty} (\mathbf{W}-\mathbf{W}^{\Gamma})dF^{\mathbf{A}}(\mathbf{W}) - \frac{\alpha_{\mathbf{A}}\sigma'(s_{\mathbf{A}})}{r+\lambda_{\mathbf{A}}}\frac{d\mathbf{W}^{\Gamma}}{d\alpha_{\mathbf{A}}}\left[1-F^{\mathbf{A}}(\mathbf{W}^{\Gamma})\right]\right\}$$

$$= sgn\left\{\frac{\sigma'(s_A)}{r+\lambda_A}\int_{w}^{\infty}(w-w^F)dF^A(w)\left[\frac{1+\frac{\alpha_B\sigma(s_B)}{r+\lambda_B}[1-F^B(w^F)]}{\phi}\right]\right\} > 0,$$

and

(8) 
$$\operatorname{sgn}(\frac{ds}{d\alpha_{\mathbf{A}}}) = \operatorname{sgn}\left\{-\frac{\alpha_{\mathbf{B}}\sigma'(s_{\mathbf{B}})}{r+\lambda_{\mathbf{B}}}\frac{d\mathbf{w}^{\mathsf{T}}}{d\alpha_{\mathbf{A}}}\left[1-\mathsf{F}^{\mathbf{B}}(\mathbf{w}^{\mathsf{T}})\right\}\right\} < 0.$$

An increase in  $\alpha_{\bf A}$  holding  ${\bf s}_{\bf A}$  constant has two effects on the marginal productivity of searching in sector A. It increases the the marginal probability of receiving an offer, which effect is represented by the first term in (7). It also decreases the expected gain (relative to continued search) from receiving an offer from sector A, which effect is reflected in the second term in (7). The first effect is stronger. Only the one corresponding to the second term appears in (8) since  $\alpha_{\bf A}$  does not directly affect the probability, marginal or otherwise, of receiving an offer from sector B.

Intuitively, since  $\alpha_{\bf A}$  affects the two value functions,  $V^{\bf A}({\bf w})$  and  $V^{\bf u}$ , only through its effect on the productivity of future search, which ultimately rests upon its effect upon the marginal probability of receiving an offer during future search, then the effect of  $\alpha_{\bf A}$  on the marginal probability of receiving an offer during current search should be dominant. In a sense, the pull

away from more current search rests upon the pull towards more future search. In the future, this latter pull will exactly resemble the current pull towards more current search. If the pull towards more future search were to win now, then it would win at all future times as well, which would put the worker in the paradoxical position of forever delaying current search in favor of greater future search which he never undertakes.

The derivatives with respect to  $\alpha_n$  are symmetric with these.

Result: 
$$\frac{ds_{A}}{d\alpha_{A}} > 0$$
 and  $\frac{ds_{B}}{d\alpha_{B}} > 0$ .
$$\frac{ds_{A}}{d\alpha_{B}} < 0 \text{ and } \frac{ds_{B}}{d\alpha_{A}} < 0$$
.

The Wage Offer Distribution:

Differentiating (5a.1) with respect to q and manipulating yields

$$\begin{split} & \operatorname{sgn}\left(\frac{d}{dq}s_{\mathbf{A}}\right) = -\operatorname{sgn}\left\{ \begin{array}{l} \left[\frac{\alpha_{\mathbf{A}}\sigma(s_{\mathbf{A}})}{r+\lambda_{\mathbf{A}}}[1-F(w^{\Gamma};q)] - 1\right] \int_{w^{\Gamma}}^{\infty}(w-w^{\Gamma})dF_{2}(w;q) \right\} \\ & \text{while } \operatorname{sgn}\left(\frac{d}{dq}s_{\mathbf{B}}\right) = -\operatorname{sgn}\left\{ \frac{\alpha_{\mathbf{A}}\sigma(s_{\mathbf{A}})}{r+\lambda_{\mathbf{A}}} \int_{w^{\Gamma}}^{\infty}(w-w^{\Gamma})dF_{2}(w;q) \right\} \,. \\ & \text{Therefore, } \operatorname{sgn}\left(\frac{d}{dq}s_{\mathbf{A}}\right) = -\operatorname{sgn}\left(\frac{d}{dq}s_{\mathbf{B}}\right) = \operatorname{sgn}\left(\int_{w^{\Gamma}}^{\infty}(w-w^{\Gamma})dF_{2}(w;q)\right) \\ & = \operatorname{sgn}\left(-\int_{w^{\Gamma}}^{\infty}F_{2}(w;q)dw\right). \end{split}$$

This expression makes it pretty clear that any change which we would be inclined to call an improvement in  $F^A(w)$  within the relevant range, i.e. above the reservation wage, will result in an increase in  $s_A$  and a decrease in  $s_B$ . A first-order stochastic improvement or a mean-preserving spread are two such changes.

The Layoff Rates:

Differentiating (5a.1) by 
$$\lambda_A$$
 yields  $sgn\left(\frac{d}{d\lambda_A}s_A\right) =$ 

$$-\operatorname{sgn}\left[\left[\frac{\alpha_{\mathbf{A}}^{\sigma^{\dagger}(\mathbf{S}_{\mathbf{A}})}}{(\mathbf{r}+\lambda_{\mathbf{A}})^{2}}\int_{\mathbf{W}^{\Gamma}}^{\infty}(\mathbf{w}-\mathbf{w}^{\Gamma})d\mathbf{F}^{\mathbf{A}}(\mathbf{w})\right]\left[1-\frac{1-\mathbf{F}^{\mathbf{A}}(\mathbf{w}^{\Gamma})}{\phi}\frac{\alpha_{\mathbf{A}}^{\sigma(\mathbf{S}_{\mathbf{A}})}}{\mathbf{r}+\lambda_{\mathbf{A}}}\right]\right]<0$$

and sgn 
$$\left(\frac{d}{d\lambda_2}s_3\right) =$$

$$-\text{sgn}\left[\int_{\mathbf{W}^{\Gamma}}^{\infty} (\mathbf{W} - \mathbf{W}^{\Gamma}) dF^{\mathbf{A}}(\mathbf{W}) \left[ -\frac{1 - F^{\mathbf{B}}(\mathbf{W}^{\Gamma})}{\phi} \frac{\alpha_{\mathbf{A}} \sigma(s_{\mathbf{A}})}{(r + \lambda_{\mathbf{A}})^{2}} \frac{\alpha_{\mathbf{3}} \sigma'(s_{\mathbf{3}})}{r + \lambda_{\mathbf{B}}} \right] \right] > 0 \text{ . The}$$

expression for  $\frac{d}{d\lambda}_{A}$  was missing a component representing the contribution of the increased chance of being laid off from the sector should one become employed there.

The derivatives with respect to  $\lambda_{\mbox{\scriptsize B}}$  can be signed in a symmetric fashion, so

Result:  $\frac{d}{d\lambda_A}$  and  $\frac{d}{d\lambda_B}$  and  $\frac{d}{d\lambda_B}$  and  $\frac{d}{d\lambda_B}$  and  $\frac{d}{d\lambda_A}$  > 0 . An increase in the layoff rate in either sector decreases the optimal amount of search in that sector and increases the optimal amount of search in the other sector. The relative search intensities do adjust to changes in the layoff rates, in contrast to the reservation wage which remains the same for the two sectors regardless of the layoff rates.

#### The Cost of Searching:

Differentiating (5a.1) with respect to to c yields, for j=A,B,  $sgn\left(\frac{d}{dc}s_j\right) = -sgn\left(1 - \frac{\alpha_j\sigma'(s_j)}{r+\lambda_j}\left[1-F^j(w^\Gamma)\right] - \frac{(s_A+s_B)}{\phi}\right)$  which is ambiguous. If  $\sigma'(s_j) \leq 1$ , then  $\frac{d}{dc}s_j < 0$ .

# Case 2: $s_p + s_3 = 1$ (Corner Solution)

In this case, the constraint that the total amount of search intensity be no greater than the total amount available is binding. In a sense, scale effects are absent, and the total amount of search undertaken matters. In response to any small change in the environment the worker must reallocate a "fixed"

amount of search intensity between the two sectors, if there is to be any change in search intensities at all.

The equations in (5a) hold with equality, but since in this case  $\mu>0$  they reduce to a single condition:

$$(5a.2) \frac{\alpha_{A}\sigma'(s_{A})}{r+\lambda_{A}} \int_{\omega}^{\infty} (w-w^{\Gamma})dF^{A}(w) = \frac{\alpha_{B}\sigma'(1-s_{A})}{r+\lambda_{B}} \int_{\omega}^{\infty} (w-w^{\Gamma})dF^{B}(w)$$

where  $1-s_A$  replaces  $s_B$  since  $s_A+s_B=1$ . (5a.2) says that search intensity should be allocated between the two sectors so that the marginal "productivities" of searching in each sector, that is the marginal increase in the expected gain from searching in that sector, are equal.

## The Offer Arrival Rates:

In Case 1, when the constraint on the total amount of search was not binding, we found that an increase in the offer-arrival rate in one sector increased the optimal search intensity in that sector while decreasing it elsewhere. A qualitative result of this sort is not inconsistent with the total amount of search intensity remaining the same, so we might expect that the same qualitative result will hold in this case, when the constraint is binding. It does.

Differentiating (5a.2) with respect to  $\alpha_{\underline{A}}$  yields

An increase in the offer-arrival rate in one sector increases the optimal search intensity in that sector while decreasing the optimal search intensity in the other sector.

In the same way, the analyses of the other derivatives follow from the analyses presented in Case 1. Any change in the wage-offer distribution in sector A which we would be wont to call an improvement will cause a reallocation of search intensity away from sector B toward sector A. An increase in the layoff rate in one sector will result in an decrease in search intensity in that sector and an increase in search intensity in the other sector. The effects of an increase in the cost of searching on the search intensities is ambiguous.

## Section 5: Conclusion

The purpose of this paper has been to discover what can be learned about job search and the mobility of unemployed workers across sectors of the economy by recognizing that workers search for jobs in more than one labor market. To this end I have developed a model of job search in two sectors. The model provides a theoretical basis for the common-sense notion that the relative prospects for reemployment in different sectors of the economy will affect the allocation of search effort between the

sectors.6

In many cases the directions in which various labor market conditions affect search efforts and reservation wages are intuitively clear, so that the model provides formal expression of these intuitions and the avenues through which they are effected. An improvement in the prospects for employment in one sector, being an increase in the offer-arrival rate, an improvement in the wage-offer distribution, or a decrease in the lay-off rate in that sector, induce a worker following the optimal search strategy to increase the intensity of his or her search for a job in that sector while decreasing it in the other sector. An improvement in the prospects for employment in either sector, in the form of an increase in the offer-arrival rate or the wage-offer distribution, raise the reservation wages for a job in both sectors, since such an improvement makes declining a job in order to search further a better deal.

In two areas in particular the formal analysis provides guidance where intuition is lacking.

The first is in understanding that the prospects for employment in one sector can affect search intensity in the other sector via the dynamic nature of the problem alone. It is not

A one-sector version of this model is equivalent to assuming that one of the offer-arrival rates, say  $\alpha_3$ , is equal to zero. Since the worker is assumed to begin from a state of unemployment, this means that the worker will never find himself employed in sector 3. The optimal intensity of search in that sector will obviously be zero, and the reservation wage for jobs in that sector will be irrelevant. All of the results concerning the signs on the derivatives of  $s_2$  and w with respect to  $\alpha_2$ , q,  $\lambda_2$ , q, and c hold in a one-sector version of this search model.

necessary for the total amount of search to affect the current marginal benefits or marginal costs of searching, nor for the total amount of search to be constrained. For example, suppose that the model had assumed that workers maximized the present value of expected utility over income and leisure, rather than the present value of expected income alone. The model would then bear out the intuitively plausible notion that an improvement in the prospects for employment in one sector would induce the worker to search more intensively there, raising the marginal utility of leisure which would induce the worker to search less intensively in the other sector. The present model does not include any such channel for the amount of search in one sector to affect the current marginal "cost" of searching in the other sector, yet an improvement in the prospects for employment in one sector does reduce the search intensity in the other sector even when the constraint on the total amount of search is not binding. The reason is that better prospects for employment in one sector raises the marginal benefit to future search, making continued unemployment more attractive relative to finding a job in the other sector.

The second area in which the formal analysis fills a substantial gap in intuition is the effects of layoff rates on search behavior. One might expect that since job security is a valued characteristic of a job, the sector offering greater job security (i.e., a lower layoff rate) would be associated with a lower reservation wage. In the absence of costs to making transition between employment and unemployment (as opposed to costs to finding employment) this is not the case. The reservation wages associated with the two sectors of the economy

are equal. A difference in layoff rates is not sufficient to cause a divergence in reservation wages.

The same is not true of search intensities. All else equal, a worker will search more intensively in the sector with a lower layoff rate.

While the relative layoff rates do not affect the relative reservation wages, the levels of the layoff rates do affect the level of the common reservation wage. If one takes the perspective that a worker would require a compensating differential in the wage rate in order to be willing to take a job with lesser job security, then one would expect higher layoff rates to lead to higher reservation wages. The opposite is the case. It is less worthwhile to wait for a higher paying job if that job is likely to end shortly, so higher layoff rates lead to lower reservation wages in the search model.

In addition to these insights, this analysis of optimal search behavior will facilitate empirical analysis of the search behavior and mobility of unemployed workers faced with more than one labor market in which to search. The results presented here, when combined with those of richer one-sector search models should provide us with a solid basis for making predictions about and interpreting actual search behavior when the differences between sectors are important.

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