AN ALTERNATIVE TEST OF THE HEDONIC THEORY OF HOUSING MARKETS*

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Discussion Paper Number 104
November, 1977

*Research for this paper was supported by National Science Foundation Grant SOC74-23109. The assistance of Barry Fishman and Peter A. Morrison is gratefully acknowledged. I would like to thank Jerome Rothenberg and Dennis Smallwood for many helpful comments on an earlier draft.

1. INTRODUCTION

In this paper the hedonic theory of housing markets is used to generate a conditional logit model of household behavior in an urban housing market. Application of hedonic theory to housing markets is by now fairly familiar (see, e.g., Quigley [16] or Straszheim [19]) and a link to conditional logit has also been established (see Friedman [4] or Quigley [16]). However, by emphasizing more heavily the bid price interpretation of hedonic theory, in this paper we develop a new connection to econometric estimation that essentially involves running the usual logit equations in reverse.

Letting t index the type of household (categorized, e.g., by income or race) and z the vector of housing characteristics, the usual approach is to estimate conditional probability functions of the form $p(z|t)$. We estimate instead functions of the form $p(t|z)$, giving the conditional probability that a house with characteristics z will be occupied by a household of type t. The advantage of this approach is that the coefficients appearing in the logit equation represent coefficients of bid price functions, and this in turn permits the empirical results to be given an extremely clear interpretation. The general methodological approach should prove useful in other contexts where hedonic price theory is deemed appropriate.

Section 2 presents a brief recapitulation of the hedonic theory of housing markets. Section 3 establishes the connection between this theory and the econometric model to be estimated. In Section 4 the model is estimated using data from the San Francisco metropolitan area.

2. THE HEDONIC THEORY OF URBAN HOUSING MARKETS

HEDONIC PRICES AND RESIDENTIAL CHOICE

Housing markets are complex phenomena, not at all well suited to an application of the standard tools of price theory. Housing is not a homogeneous commodity, but rather a label for a collection of commodities that are all distinct to some degree. Houses exhibit substantial variation in structural features, lot size, characteristics of the surrounding neighborhood, and quality of local public services. Indeed, if we simply adopt the usual approach of indexing all commodities by location, it is clear that no two houses can be exactly alike. Housing is also not a divisible commodity. A consumer either chooses to reside in a particular dwelling or he does not. Thus, housing violates two of the most basic requirements for the application of standard price theory, the homogeneity and divisibility of commodities in a given market.

It should come as no surprise, therefore, that the economic theory of urban housing markets developed over the last two decades is not conventional. The essential break with tradition that made this development possible was a shift in focus from the housing commodities themselves, inherently indivisible and distinct, to their underlying characteristics. Consumer choice is assumed to depend solely on these characteristics, and, in this way, an infinite dimensional problem is reduced to one of manageable size and one that is amenable to the use of calculus.

Assume that every household in a particular housing market has tastes that can be described by a utility function

$$U_n(x_n,z_n), \qquad n \in N, \tag{1}$$

where x_n is an r-dimensional vector of private goods, z_n is an s-dimensional vector of housing attributes, and N indexes the set of households. The utility function is assumed to be quasi-concave and twice continuously differentiable in x_n when the vector z_n is held fixed. We will avoid for the moment a discussion of the behavior of utility relative to the vector of housing attributes.

The budget constraint for consumer n is assumed to be

$$p_x x_n + h(z_n) + T_n(z_{n1}) = y_n, \tag{2}$$

where p_x is an r-dimensional vector of private good prices; $h(z_n)$ is the hedonic price function relating the price of a dwelling to its characteristics; $T_n(z_{n1})$ is the transportation cost function for household n (assumed, for simplicity, to depend only on the first characteristic, which should be interpreted as the distance to the central business district); and y_n is the income of consumer n.

For fixed z_n, the consumer is assumed to maximize his utility function (1) subject to the budget constraint (2). Under the assumptions we have introduced so far, the solution to this problem of constrained maximization can be represented by an indirect utility function

$$\phi_n[p_x, z_n, y_n - h(z_n) - T_n(z_{n1})] \tag{3}$$

giving the maximum utility that the consumer can achieve at prices p_x if he is residing in a dwelling with characteristics z_n, costing an amount $h(z_n)$, and implying transportation costs equal to $T_n(z_{n1})$.[1] In other words, we assume that, based on his choice of a house, the consumer's behavior relative to the consumption of commodities other than housing can be characterized by an indirect utility function of the prices of these commodities and his income net of the costs associated with this housing choice. The indirect utility function has all

[1] Indirect utility functions have been employed in related contexts by Ellickson [3], Solow [18], and Polinsky and Shavell [13].

of the usual properties that it is assumed to have in the standard theory of competitive behavior. The second stage of the process involves the choice of a house with characteristics that will maximize the indirect utility function (3). Assuming that the available characteristics can be restricted to some compact set K, the consumer's choice is described by the solution to the following maximization problem:

$$\max_{z_n \varepsilon K} \phi_n[p_x, z_n, y_n - h(z_n) - T_n(z_{n1})].$$

Assuming that the indirect utility function has continuous first partial derivatives relative to the elements of z_n, the first-order conditions for a maximum are[2]

$$\frac{\partial\phi_n}{\partial z_{n1}} = \frac{\partial\phi_n}{\partial y}\left(\frac{\partial h}{\partial z_{n1}} + \frac{dT_n}{dz_{n1}}\right), \tag{4}$$

$$\frac{\partial\phi_n}{\partial z_{nj}} = \frac{\partial\phi_n}{\partial y}\frac{\partial h}{\partial z_{nj}}, \qquad j = 2, \ldots, s. \tag{5}$$

The primary advantage of this reformulation of the problem of consumer choice is that it permits a direct connection to be made with the theory of hedonic prices as formulated by Rosen [17] and Mas-Colell [6]. To make this connection, we first derive a generalized version of bid price functions. We define a bid price function as a function giving the price for housing with characteristics z_n that will yield utility of level U_n for consumer n. In terms of the indirect utility function (3), this bid price is the solution to the equation

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[2]Some readers may wonder why accessibility is not treated symmetrically with the other housing characteristics. We have chosen this approach to clarify the relationship between the theory presented here and the traditional theory of urban housing markets. However, the natural symmetry among characteristics will be imposed shortly.

$$\phi_n[p_x,z_n,y_n-V_n-T_n(z_{n1})] = U_n, \tag{6}$$

where V_n denotes the bid price. Assuming consumers are not satiated, U_n will be a monotonically increasing function of income net of housing and transportation costs, $y_n - V_n - T_n(z_{n1})$; so for a vector of housing characteristics z_n and income y_n, it will be a monotonically decreasing function of the bid price V_n. Thus, we can invert function (6) to obtain the bid price function:

$$V_n = \psi_n^*[p_x,z_n,y_n-T(z_{n1}),U_n] = \psi_n(p_x,z_n,y_n,U_n), \tag{7}$$

where by the second equality we have chosen to absorb the transportation cost function into the function $\psi_n(\cdot)$.

By applying the implicit function theorem repeatedly to Eq. (7), we obtain as expressions for the partial derivatives of the bid price function

$$\frac{\partial\psi_n}{\partial z_{n1}} = \frac{\frac{\partial\phi_n}{\partial z_{n1}} - \frac{\partial\phi_n}{\partial y_n}\frac{dT_n}{dz_{n1}}}{\frac{\partial\phi_n}{\partial y_n}}, \tag{8}$$

$$\frac{\partial\psi_n}{\partial z_{nj}} = \frac{\frac{\partial\phi_n}{\partial z_{nj}}}{\frac{\partial\phi_n}{\partial y_n}}, \qquad j = 2, \ldots, s. \tag{9}$$

Thus, we obtain a simple interpretation for the first-order conditions (4) and (5). They express tangency conditions between the bid price function and the hedonic price function for each of the housing characteristics $j = 1, \ldots, s$.

Figure 1a illustrates the tangency condition for a "desirable" characteristic (e.g., lot size, number of rooms, or quality of the neighborhood), and Fig. 1b illustrates this condition for an "undesirable"

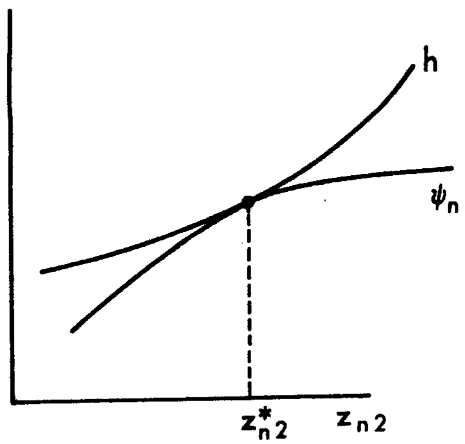

Fig. 1a — Equilibrium condition for a desirable characteristic

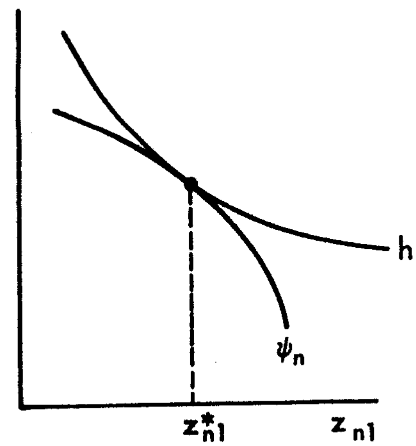

Fig. 1b — Equilibrium condition for an undesirable characteristic

characteristic (e.g., distance to the central business district or age of the dwelling unit). Once we recognize that higher levels of utility correspond to lower bid price curves, these tangency conditions are as easy to interpret as the familiar tangency between an indifference contour and a budget plane. The consumer chooses from among the opportunities available in the market (represented by the hedonic price function, the curve labeled h) that house which places him on the bid price curve corresponding to the highest attainable level of utility (the curve labeled ψ_n); equivalently, the consumer selects a house with characteristics z^*_n for which his marginal willingness to pay for more of each characteristic is equated to the marginal cost of obtaining this characteristic in the market.

This diagrammatic interpretation of the hedonic theory provides a convenient bridge to existing work on residential choice. Much of the literature can be interpreted as arguing that as income increases, the slopes of the bid price curves will increase for desirable characteristics and decrease for undesirable characteristics. This leads to the conclusion that consumer income will be positively correlated with size of house and lot, quality of neighborhood, and of local public services and negatively correlated with structure age and accessibility (assuming that the value attached to commuting time increases with income). Whites are assumed willing to outbid blacks for houses in white neighborhoods, with segregation the result.

However, although these hypotheses about bid price curves seem reasonable, treating characteristics one at a time does not do justice to the complexity of the housing market. In the San Francisco metropolitan area, the setting for the empirical part of this paper, simple correlations between household income and housing characteristics are not particularly strong.[3] It is not difficult to see why this should be so. High-income households may opt for less accessible locations where land is inexpensive or for older housing when the neighborhood is well-maintained and the houses are large or otherwise of high quality; attractive housing in the central city may become available to low-income consumers if schools and other public services are poor.

The diagrammatic interpretation of the hedonic theory, by focusing attention on one characteristic at a time, obscures a primary virtue of the hedonic approach--its ability to treat housing characteristics simultaneously. The standard hypotheses of urban economics, translated into statements about bid price curves, seem most reasonable when interpreted *ceteris paribus*. Holding other characteristics fixed, it does seem plausible that willingness to pay for more rooms, a larger lot, a newer house, and so on will increase as income increases. When hedonic theory is used to treat housing attributes simultaneously, as we shall see later, the data support the standard hypotheses about bid price functions. Holding other characteristics fixed, high-income households are willing to pay more for larger houses and lots, better neighborhoods and schools, newer houses, and more accessible locations. When all components of housing bundles are taken into account, the model is able to make considerable sense out of the demographic patterns observed in the housing market.

EXISTENCE OF EQUILIBRIUM: THE CONTINUUM OF AGENTS APPROACH

Up to this point, the work of Rosen on hedonic prices has been the main point of reference; it is the formulation of Mas-Colell [6],

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[3] See the first two rows of Table 9 on p. 31

however, that provides the most profound insight into the workings of models of this sort. What he has done is to demonstrate the existence of competitive equilibrium and the equality of core and competitive allocations (within a continuum of agents context) where some commodities are indivisible. If the number of indivisible commodities is finite, these results go through with some modification of the standard assumptions (essentially intended to guarantee that all consumers use some amount of the divisible commodities). But if the number of indivisible commodities is infinite, the existence and equivalence theorems will generally fail. Heuristically, the size of the commodity space is too large relative to the size of the economy, so that markets are not thick enough to sustain competitive behavior. Suppose, however, that each type of indivisible commodity can be identified with a point z in some compact set of characteristics K. What Mas-Colell has shown is that if all consumers regard commodities with similar characteristics as close substitutes, the existence and equivalence theorems will be valid.

Mas-Colell's proof of these results, while mathematically complex, is intuitively quite clear. Consider, for example, an economy $\mathcal{E}$ in which the indivisible commodity (housing) can be described by a single characteristic with the compact set K identified with an interval on the real line. Suppose now that we construct a sequence of economies $\mathcal{E}_\nu$ where each economy in the sequence has only a finite number of commodities, selected by choosing a finite number of points in the interval K. As ν increases, we choose more and more points on the interval K. For each economy $\mathcal{E}_\nu$ with a finite number of commodities, the existence and equivalence theorems for competitive equilibrium are valid. The fact that consumers treat commodities with similar characteristics as close substitutes allows passage to the limit, providing existence and equivalence for the economy $\mathcal{E}$. The important point to note is that nowhere in the argument are characteristics interpreted as commodities. By capturing the notion that similar commodities will be good substitutes, the use of characteristics to describe commodities introduces an essential ingredient of homogeneity into a market for heterogeneous commodities.

In articulating the theory in this way, Mas-Colell provides a powerful new insight into the role that characteristics play in the theory of hedonic prices. One of the misconceptions that plagued early uses of hedonic price functions was the interpretation of characteristics as commodities, leading to the implication that hedonic functions should be linear in attributes. Rosen's arguments against this view apply with full force to the assertion that housing prices should be a linear function of lot size (or quantity of housing services), an assumption adopted universally in the theoretical literature on the "new urban economics." Mas-Colell's analysis leads to an even more startling conclusion: A given characteristic may be a "good" for one consumer and a "bad" for another; in fact, a particular characteristic may be neither a good nor a bad for some consumers. Thus, by stripping characteristics of the last vestiges of a "commodity" interpretation, we obtain great flexibility in the way attributes can affect utility.[4] One consumer may value houses closer to the central business district more highly, while another prefers to live in the suburbs; whites may prefer white neighborhoods, while blacks may prefer black neighborhoods. For a given consumer, utility need not vary monotonically as a function of an attribute. Moderate-size lots may be preferred to those that are large (with a big lawn to mow) or small; moderate terrain may be preferred to lots that are steep and to those that are flat; and blacks (or liberal whites) may like integrated neighborhoods best of all. All that is required for the analysis is that all consumers regard commodities with similar attributes as essentially equivalent (in the sense that they are nearly perfect substitutes).

[4] In particular, this theory stands in sharp contrast with the "commodity hierarchy" model developed by Sweeney [20], where all consumers are assumed to rank houses in the same way with respect to a quality index. With a single quality index, Sweeney's assumption might seem tenable (although even in that context, it seems quite possible that some consumers would prefer old houses). But with multiple attributes, the approach is not satisfactory. It seems intuitively plausible that different types of households will weigh various attributes differently, a supposition that receives ample support in the empirical work reported in the following section.

The results obtained by Mas-Colell provide an elegant foundation for the theory of hedonic prices. It is the hypothesis that commodities with similar characteristics will be close substitutes that gives the hedonic approach its power and enables the economist to capture the intuitive notion that a market such as housing, despite the heterogeneity of the commodities involved, may be competitive. The relevance of hedonic price theory clearly extends far beyond its application to housing markets. Analysis of consumer behavior in terms of hedonic and bid price functions should become as much a staple of the standard price theory course as the conventional model using indifference contours and budget planes. The constructs introduced in the early work on residential location appear not as devices peculiar to urban economics but rather as a prototype for a much more general form of economic analysis.

Full development of the hedonic approach to the theory of housing markets will require an effort that goes far beyond the scope of this paper. We have sketched the main outlines of the theory with no attention to the specifics of how the transportation system, the production technology for housing, and the aging of existing stock affect market equilibrium. Of particular importance to the interpretation of the empirical results that follow, we take for granted the possibility of integrating this model of residential choice with a model of competitive equilibrium with local public goods. In our empirical analysis, we include not only characteristics of the house in which a consumer resides but also attributes of the surrounding neighborhood and public schools. These characteristics are local public goods, entering the utility functions of all consumers in a particular neighborhood or school attendance area.

The key to integrating the notion of a market for local public goods with the theory of residential choice developed in this paper is the demonstration in Ellickson [1] that local public goods can be regarded as indivisible private goods where the indivisibility reflects the fact that a consumer either resides in a particular neighborhood (political jurisdiction) or he does not. Because public goods are by definition produced subject to increasing returns to scale, a competitive market may fail to exist. However, if the optimal size of a neighborhood or political jurisdiction is small (in an appropriate sense) compared with the size of the economy, a competitive equilibrium can exist.[5] Although the hypothesis that local public goods are competitively produced must be regarded as very much an open question, a model of the housing market, competitive with respect to all aspects of the choice of residence, seems a reasonable way to begin. Therefore, at this juncture we leave these matters of theory and turn to another question of equal importance: What relevance does this theory have for the empirical analysis of housing markets?

3. EMPIRICAL IMPLEMENTATION OF THE THEORY

On one level the relevance is quite clear. Much of the recent empirical research on housing markets has involved the estimation of hedonic price functions. But although hedonic price functions have, of course, played a central role in our analysis, the theory also implies that these functions convey little information about the effect of the housing market on consumers' choice of location. Hedonic price functions are simply an infinite dimensional analogue of a finite vector of equilibrium prices. Clearly, we need to go beyond the estimation of hedonic functions and ask how consumers will react to these prices.

One way to proceed would be to specify a form for consumer utility functions and the hedonic price function, using this information to derive consumers' demand for characteristics. This approach is not very satisfactory because it relies too heavily on specifying the correct form for the hedonic price function. Straszheim's [19] results for the San Francisco Bay Area indicate that hedonic relationships are complex, with coefficients varying from neighborhood to neighborhood (presumably reflecting interactions between attributes of housing structures and neighborhood characteristics). There is another more basic reason to reject this approach, however: It is not well suited to testing the most interesting propositions that have appeared in the literature regarding urban housing markets.

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[5] For a proof of this result, which represents a formalization of earlier work of Tiebout [21] and McGuire [10], see Ellickson [1].

As indicated above, much of the existing literature on residential choice can be interpreted in terms of bid price and hedonic price functions. The most natural way to interpret such models is in terms of a prediction of what sort of household is most likely to occupy a house with a specified set of characteristics. The house will be occupied by the household offering the highest bid price. Thus, the traditional accessibility model predicts that houses located far away from the central business district will be occupied by households with low marginal commuting costs and relatively high demand for housing space. The filtering model predicts that newer housing will be occupied by wealthier households. And the Tiebout models predict that houses in communities offering higher quality public services will be occupied by households with high income or a strong preference for public services.

Suppose we classify households into types indexed by a set T. Equilibrium in the housing market can be regarded as inducing a joint probability density $p(t,z)$ over household types $t \in T$ and housing characteristics $z \in K$. Models of the demand for characteristics focus on the conditional probability $p(z|t)$. What we are asserting is that most hypotheses about housing markets can be more naturally interpreted in terms of the conditional probability $p(t|z)$, the probability that a house with characteristics z will be occupied by a household of type t. It is remarkable that, despite its natural link to existing theory, the latter approach has been entirely neglected in empirical studies of the housing market with the notable exception of the work by Mayo [7].[6]

The hedonic theory we have presented implies that a house with characteristics z will be occupied by the household n whose bid price function evaluated at z,

$$V_n = \psi_n(p_x, z, y_n, U_n) = \tilde{\Psi}_n(z), \tag{10}$$

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[6] The empirical model we will present is very close in spirit to that of Mayo. The major differences are that he estimates $p(t|z)$ with linear probability functions, his empirical model is only loosely related to a theory of the housing market, and his data set contains no information on the characteristics of individual structures.

is higher than that of all other households, where by the last equality we have suppressed the price vector p_x (assumed invariant throughout the market), the household income y_n, and the equilibrium level of utility U_n. If all members of group to have the same income and preferences, if the characteristics represented by z capture all of the aspects of a house relevant to consumers and if there are no information costs associated with search in the housing market, then all households of type t would offer the same price $\tilde{\Psi}_t(z)$ for a dwelling unit with characteristics z. In any empirical application, of course, none of these conditions will be met. Therefore, we replace the bid price function $\tilde{\Psi}_t(z)$ with a *stochastic bid price function*

$$V_t = \tilde{\psi}_t(z) + \varepsilon_t, \tag{11}$$

where ε_t is a random disturbance term reflecting differences in tastes and income among households in group t and unmeasured characteristics of the dwelling unit. Then the deterministic proposition that a house with characteristics z will be occupied with probability one by a particular household type is replaced by the probabilistic statement that

$$p(t|z) = \text{prob}\left\{\tilde{\psi}_t(z)+\varepsilon_t>\tilde{\psi}_{t'}(z)+\varepsilon_{t'},\ t' \neq t\right\}. \tag{12}$$

Readers familiar with McFadden's [8] approach to the analysis of qualitative choice will recognize this formulation. If, following McFadden, we assume that the disturbance terms are independently and identically distributed Weibull, Eq. (12) takes the form

$$p(t|z) = \frac{\exp[\tilde{\psi}_t(z)]}{\sum_{t\in T} \exp[\tilde{\psi}_t(z)]}. \tag{13}$$

Assuming that the bid price functions are linear in the parameters, we obtain

$$p(t|z) = \frac{\exp(\alpha_t z)}{\sum_{t\in T} \exp(\alpha_t z)}, \tag{14}$$

a conditional logit model that is identical in form with McFadden's except that bid price functions replace the utility functions for the representative consumer. The parameters of this model can be estimated through maximum likelihood in exactly the same way that McFadden estimates his model where the parameters are now interpreted as the coefficients of the nonstochastic part of the bid price function for each type of household.

There is an alternative way to arrive at Eq. (14), which is not only interesting in its own right but also provides some justification for estimating the model in a less costly fashion than the maximum likelihood procedure. Returning to the deterministic formulation of the hedonic model, suppose that household n chooses a house with characteristics z_n. Assume again that households are classified into a number of groups indexed by the set T. To translate the model into stochastic form, suppose that for households of type t the most preferred vector of housing characteristics is distributed with probability density

$$f(z|t) = (2\pi)^{-s/2}|\Sigma_t|^{-1/2} \exp\left\{\left[-(z-\mu_t)' \ \Sigma_t^{-1} \ (z-\mu_t)\right]/2\right\}, \tag{15}$$

a normal density with mean μ_t and covariance matrix Σ_t. If we let p(t) represent the prior probability that an observation will be a household of type t and assume $\Sigma_t = \Sigma$ for all t, then Bayes' theorem implies that

$$p(t|z) = \frac{p(t)f(z|t)}{\sum\limits_{t\epsilon T} p(t)f(z|t)}$$

$$= \frac{\exp\left[\log p(t) - \frac{1}{2}\mu_t'\Sigma^{-1}\mu_t + \mu_t'\Sigma^{-1}z\right]}{\sum\limits_{t\epsilon T} \exp\left[\log p(t) - \frac{1}{2}\mu_t'\Sigma^{-1}\mu_t + \mu_t'\Sigma^{-1}z\right]}, \tag{16}$$

where $\log p(t) - \frac{1}{2}\mu_t'\Sigma^{-1}\mu_t + \mu_t'\Sigma^{-1}z$ is the linear discriminant score for households of type t.[7] Comparing Eqs. (14) and (16), we see that the assumptions used to justify linear discriminant analysis lead quite naturally to a conditional probability function equivalent to the logit.

Of course, the conditional logit model may be valid in cases where the linear discriminant model is not, and we have chosen not to take a firm position regarding their relative merit. Either approach seems satisfactory as a means of translating the deterministic hedonic model into stochastic form. For our purposes, the primary value of the second approach is that it provides a computationally much less expensive method of estimating the parameters of the logit model. We used linear discriminant analysis for the bulk of our exploratory research, while the final results, as reported in the following section, are the maximum likelihood estimates of the conditional logit model. In practice, the two approaches yielded very similar parameter estimates in most cases.

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[7] The prior probability p(t) can be estimated using the proportion of households of type t in the sample. This use of Bayes' theorem to establish the connection between linear discriminant analysis and conditional logit apparently has a long history in the statistics literature. Nerlove and Press [12] cite Truett, Cornfield, and Kannel [22] as a source. For a good discussion of the procedure, see Warner [23].

4. RESIDENTIAL CHOICE IN THE SAN FRANCISCO BAY AREA

THE DATA BASE AND DEFINITION OF VARIABLES

The data we will use to estimate the model proposed in the preceding section are drawn from a sample survey of 28,000 households in the San Francisco Bay Area conducted by the Bay Area Transportation Study Commission (BATSC) in 1965. In searching for data appropriate for our purposes, one of the features we were looking for was the inclusion of information not only on characteristics of individual housing structures but also on the attributes of the surrounding neighborhood, including the quality of local public services. Although the BATSC survey itself does not include such information, it does provide a location code for each dwelling unit down to the level of a census block. Using this location code, we were able to create a file matching the BATSC data with information from the 1960 U.S. Census of Population and Housing on census tract characteristics and with information on characteristics of public schools disaggregated to the level of individual elementary, junior, and senior high schools.

The major cost in creating this file involved collecting the data on schools, a process that required both contacting each school district to obtain maps of attendance areas in 1965 and assigning each observation in the BATSC sample to the appropriate elementary, junior, and senior high school. To create such a file for the entire BATSC sample, which covers a nine-county area stretching from the wine country of Napa and Sonoma in the north to Santa Clara in the south, would have been prohibitively expensive. Thus, we chose to concentrate our attention on two counties, San Francisco (which coincides with the city) and Alameda (which covers what is commonly known as the East Bay).

To obtain our final sample, we eliminated observations from the BATSC file for San Francisco and Alameda if (1) reported race was neither white nor black (the remaining category, Oriental, contained too few observations to justify a separate classification), (2) appropriate information on schools was unavailable (which was the case for a few outlying areas in Alameda County), or (3) information on household income or housing characteristics was missing. Finally, we used a regression of the log of household income on housing characteristics to identify gross outliers in order to reject observations from the sample when either household income or housing characteristics appeared to be grossly miscoded. In this way we arrived at a final sample consisting of 2314 white homeowners, 2301 white renters, 214 black owners, and 323 black renters.

The housing characteristics we will use are listed in Table 1.

Table 1

HOUSING CHARACTERISTICS USED IN THE EMPIRICAL ANALYSIS

Z1	Log (travel time to San Francisco in minutes)
Z2	Log (age of dwelling unit in years)
Z3	Log (lot size in square feet)
Z4	Log (number of rooms)
Z5	Log (median tract income in 1960)
Z6	Log (elementary median income)
Z7	Percent of students in elementary school who are black
Z8	Percent of students in junior high school who are black
Z9	Percent of households in census tract in 1960 who are black
Z10	Hedonic residual

All of the characteristics except the hedonic residual and the housing attributes pertaining to racial composition of the schools or the census tract were transformed by taking natural logarithms. We adopted this procedure in order to make the assumption of normality underlying the discriminant approach more plausible (since plots on probability paper indicate that at least the marginal distributions of the transformed variables are approximately normal). The first characteristic measures accessibility in terms of the log of one-way commuting time in minutes. To obtain a measure of the age of a structure, we converted the BATSC four-category code into a scalar index by taking the midpoint of each interval. In a similar fashion, we converted the six-category BATSC code for lot size into a scalar index (this characteristic is used only in the analysis of homeowners because the BATSC survey did not report lot size for renters). Number of rooms in the dwelling unit, also obtained from the BATSC file, is self-explanatory. Median tract income, used as a crude proxy for neighborhood quality, was obtained from the 1960 census. Median elementary school income requires some explanation. We originally intended to use achievement scores as an index of school quality, but these are available only starting with 1968. Berkeley totally desegregated its schools in the fall of 1968 through an extensive busing program, which invalidates the use of 1968 school scores as an index of school quality in 1965, the date when the BATSC survey was conducted. To retain Berkeley in our sample (and, therefore, to avoid a drastic reduction in our already small sample of black households), we used the 1960 census tract data to estimate the median household income in each elementary school attendance area.[8] The percent of students in the elementary and junior high schools who are black was obtained from the Office of Education of the State of California. The percent of households who are black in the census tract was derived from the 1960 census.

The final characteristic, the hedonic residual, requires a more complete explanation. In our initial estimates, we used only the first nine housing characteristics, obtaining results very similar to those reported below. But although the estimated coefficients conform strongly to our prior hypotheses, the model leaves much of the variation in location behavior unexplained. Presumably, this unexplained variation reflects to some extent housing characteristics left out of the model, and this poses a dilemma. Although it is obvious that the characteristics we have included do not do justice to the nuances of structure quality, we have exhausted all that the BATSC file has to offer and supplemental data are unobtainable. Finer measures of neighborhood quality might be available, but adding indexes for the presence of

[8]We used census block data to estimate the fraction of households in each tract residing within a given elementary school attendance area and then estimated the median income for the school as a weighted average of census tract median incomes. As the results will show, the variable thus constructed has an effect independent of median tract income. Furthermore, we found that median school income and achievement scores have independent effects, in the expected direction, in samples where the households in Berkeley are excluded.

amenities of various units would put an intolerable strain on a model that already approaches the limits of computational feasibility. To surmount this problem, we adopted the following strategy: For the observations in our sample, we estimated hedonic equations of the form

$$H = \sum_{j=1}^{9} \beta_j z_j + \varepsilon, \tag{17}$$

where the z_j are the characteristics listed in Table 1 and H is the natural logarithm of housing value or rent (the equation was estimated separately for owners and renters). For any given structure, the residual from Eq. (12) can then be regarded as an index of those aspects of housing quality not captured by the vector of characteristics.

Of course, any procedure of this sort is crude and has to be regarded with considerable caution. For this reason, in all cases we estimated our models with the hedonic residual both included and excluded. In every case, the inclusion of the residual had negligible effects on the estimates of the coefficients for the other characteristics, but its inclusion markedly improves goodness of fit of the models, and its coefficient always has the expected sign.[9] Thus, we have chosen to include it in our report of final results.

To classify households into types, we stratify along four dimensions: income, tenure, race, and family composition. Although it is not difficult to imagine more detailed classifications that would be desirable, the scheme we have adopted already presses against the limits imposed by computational feasibility and the need to maintain adequate cell sizes. And, in fact, when black households are included in the analysis, the categories have to be collapsed because of the small sample size.

The classification we will use groups households by race (black versus white), tenure (owner versus renter), family composition (presence or absence of children 18 years of age or younger), and income class (a three-category classification). Ideally, all of the results we are interested in could be obtained by a single estimate of the model, a conditional logit analysis involving 24 household types

[9]For a report of these results, see Appendix C of Ellickson [2].

with a separate vector of 11 coefficients to be estimated for each household type. The computational cost of such an estimate, however, would be astronomical. Fortunately, it is possible to estimate the model for subsets of the categories of household type, holding the remaining categories fixed, without losing the desirable properties of the maximum likelihood estimator.[10] We will begin with an analysis of white households where the sample size permits a relatively detailed classification of household types. To accommodate the small sample size for black households, we will then treat the joint classification of households by income class and race by collapsing the income classification into two categories and by dropping the classification according to family composition.

We have made no attempt to use housing characteristics to classify households by tenure, despite the fact that this is an aspect of the housing market clearly worth study. Classifying households by tenure does not raise conceptual problems for the theory, but in practice there are two major sources of difficulty: (1) The income categories we are using are not comparable between renters and owners because the BATSC file reports only current annual income, neglecting any imputation of the income derived from home ownership; and (2) lot size is available on the BATSC file only for owners so that it would function as a "generic" attribute predicting tenure perfectly in an entirely spurious manner. Thus, in all that follows, owners and renters are treated as two separate subsamples.

The three income categories we will use in the analysis of white households are described in Table 2, where the income figures refer to current annual income as reported in the BATSC file. The table also gives the number of households in the sample who are owners and renters further subdivided into those with children 18 or younger (C) and those without children of that age (NC).

The classification of households by presence or absence of children 18 years of age or less represents a modest effort to capture some of

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[10] See McFadden [9], pp. 25-27.

Table 2

CELL SIZES FOR WHITE HOUSEHOLDS

(Three-income category classification)

Group and Income	Number of Observations			
	Owners		Renters	
	C	NC	C	NC
Y1 Under $7,000	236	180	386	580
Y2 $7,000-$9,999	498	236	305	385
Y3 $10,000 or more	698	466	262	383

the effects of life cycle on housing choice.[11] Defining the categories in this way reflects a hypothesis that households with children will exhibit a stronger relative preference for housing space, for higher quality schools, and, in the case of whites, for schools with a high proportion of white students. Although we made an effort to disaggregate the category of households without children by identifying elderly homeowners (whose reported income seems likely to seriously understate their true income because of the equity in their home), the attempt was abandoned because resulting cell sizes were too small to yield meaningful results.

WHITE HOMEOWNERS AND RENTERS

Parameter Estimates

The maximum likelihood estimates for white homeowners and renters are reported in Tables 3 and 4. To facilitate assessment of the relative effect of each characteristic on household location, the parameters are presented in normalized form with each characteristic measured in

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[11]The information required to make this classification was not available on the standard BATSC household file. We would like to thank Patrick Hackett of the Metropolitan Transportation Commission for allowing us to process a special file to obtain these data.

Table 3

LOGIT ESTIMATES: WHITE OWNERS

Group	Parameter Estimates										
	Constant	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10
Y2,C	-11.1	-.16	-.08	.19	.25	.14	.08	.09	-.28	-.10	.36
Y3,C	-35.5	-.49	-.37	.35	.71	.41	.34	-.03	-.14	-.05	.73
Y1,NC	5.0	-.46	.57	.22	-.26	-.43	.43	.23	-.11	-.18	.47
Y2,NC	-9.4	-.38	.25	.28	-.32	-.08	.38	.09	-.12	-.01	.50
Y3,NC	-33.5	-.84	-.16	.43	.14	.40	.48	.05	-.07	0	.68

Group i	t Statistics (Group i Relative to Group j)[a]											Group j
	Constant	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	
Y3,C	-6.40**	-4.06**	-3.75**	3.50**	7.93**	2.73**	2.13*	-.22	-1.24	-.43	7.54**	Y1,C
Y3,NC	-6.23**	-3.19**	-4.73**	2.03*	4.37**	4.55**	.27	-1.38	.42	1.71	2.02*	Y1,NC
Y1,C	-.68	3.23**	-3.45**	-1.82	2.52*	2.11*	-2.12*	-1.59	.88	1.70	-3.90**	Y1,NC
Y2,C	-.31	1.84	-2.90**	-.92	6.52**	1.34	-1.81	0	-1.32	-.89	-1.51	Y2,NC
Y3,C	-.59	4.40**	-2.83**	-1.23	8.58**	.11	-1.37	-.75	-.87	-.48	.98	Y3,NC

[a]A single asterisk indicates significance (two-tailed) at the .05 level; a double asterisk, at the .01 level.

Table 4

LOGIT ESTIMATES: WHITE RENTERS

Group	Parameter Estimates									
	Constant	Z1	Z2	Z4	Z5	Z6	Z7	Z8	Z9	Z10
Y2,C	-6.0	-.34	-.23	.45	.11	.07	-.39	.33	.03	.37
Y3,C	-31.3	-.55	-.56	.82	.49	.25	-.21	.36	.06	.78
Y1,NC	-4.7	-.63	.05	-1.11	0	.36	.01	.22	-.01	-.05
Y2,NC	-22.6	-.65	-.17	-.63	.28	.44	.03	.34	-.13	.27
Y3,NC	-23.2	-1.08	-.59	-.20	.34	.47	-.06	.43	-.10	.81

Group i	t Statistics (Group i Relative to Group j)[a]										Group j
	Constant	Z1	Z2	Z4	Z5	Z6	Z7	Z8	Z9	Z10	
Y3,C	-5.42**	-4.95**	-5.72**	7.69**	3.41**	1.67	-1.36	3.06**	.48	8.58**	Y1,C
Y3,NC	-4.30**	-5.59**	-8.09**	11.69**	3.25**	1.00	-.60	2.35*	-.89	10.43**	Y1,NC
Y1,C	.97	6.84**	-.59	13.34**	0	-2.87**	-.08	-2.11*	.06	.61	Y1,NC
Y2,C	3.15**	3.12**	-.66	11.43**	-1.30	-2.75**	-2.96**	-.18	1.28	1.10	Y2,NC
Y3,C	-1.53	5.35**	.34	9.58**	1.18	-1.72	-.98	-.65	1.22	-.34	Y3,NC

[a]A single asterisk indicates significance (two-tailed) at the .05 level; a double asterisk, at the .01 level.

units of one standard deviation. The results for owners and for renters were normalized separately, using the standard deviations given in Table 5.[12]

Table 5

STANDARD DEVIATIONS USED TO NORMALIZE COEFFICIENTS

Group	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10
Owners	.39545	.78676	.72274	.21851	.18434	.19259	.18783	.24782	.08017	.77803
Renters	.52322	.99736	--	.29932	.18898	.19267	.24241	.28561	.11980	.84200

Readers familiar with conditional logit will realize that an additional normalization is required. When the model is specified in the form given by Eq. (14), the α_t parameter vectors are not identifiable. This is easily seen if we multiply the numerator and denominator of Eq. (17) by $\exp(-\alpha_1 z)$, where α_1 is the vector of coefficients for the first group, which yields

$$p(t|z) = \frac{\exp[(\alpha_t-\alpha_1)z]}{\sum_{t\varepsilon T} \exp[(\alpha_t-\alpha_1)z]} . \tag{18}$$

All that can be determined empirically are the slopes of the bid price functions relative to the first (reference) group. In Tables 3 and 4 the household type labeled Y1,C (the lowest income group with children) is chosen as the reference group, so the vector of coefficients for that type is identically zero (and hence is not reported in the

[12]These standard deviations were actually obtained as part of the output of the linear discriminant analysis routine. Recall from our earlier discussion of linear discriminant analysis that the vector of housing characteristics z_t chosen by households of type t is assumed to be normally distributed with mean μ_t and covariance matrix Σ; the latter is assumed identical for all groups. The standard errors reported in Table 5 are the square roots of the estimates of the diagonal elements of Σ for the case of six household categories (three income categories, each subdivided according to whether there are children present or not), corresponding to the analysis presented in Tables 3 and 4.

tables). Accompanying each table of parameter estimates is a set of t statistics used to test the significance of differences in coefficients between household groups. The first two rows compare the highest to the lowest income groups for households with and without children, respectively, while the last three rows compare households with and without children for each of the three income categories.

The results presented in Tables 3 and 4 provide strong confirmation of several hypotheses that have appeared in the housing market literature. To interpret these results, we begin with a comparison across income classes, family composition held constant. With only a few minor exceptions, the coefficients of the first six characteristics and Z10 (the hedonic residual) exhibit the pattern one expects. The coefficients of Z1 (commuting time to San Francisco) tend to become increasingly negative as income increases, indicating a stronger relative preference for central locations on the part of higher income households--precisely the result one expects if higher income households attach a higher value to commuting time.[13] Higher income households also prefer newer housing (characteristic Z2), larger lots (Z3), more rooms (Z4), a better neighborhood (as represented by Z5, median census tract income in 1960), and those aspects of housing quality captured by the hedonic residual (Z10).[14] As household income increases, owners and renters with children attach more value to housing within the attendance area of elementary schools drawing from a higher income population (characteristic Z6), while households without children do not.[15] Income differences appear to have no effect on the reaction of white households to racial composition of the schools (Z7 and Z8) or

[13]The difference between the low- and the middle-income groups is not significant for owners and renters without children, with the pattern reversed in the case of owners.

[14]A statistically insignificant exception to these patterns occurs with the coefficients for number of rooms (Z4) for income classes Y1 and Y2 in the case of owners without children.

[15]The difference in coefficients between groups Y1 and Y3 is significant only at the 10 percent level in the case of renters with children.

of the census tract (Z9) except for the stronger aversion of low-income renters to junior high schools with a higher proportion of black students. The absence of differences among whites may simply reflect the high degree of segregation in the housing market, a conjecture that receives strong support when we turn to the classification of blacks and whites.

The difference in parameter estimates for households with and without children, income held constant, is also reasonable with one major exception. Owners and renters with children put much more weight on number of rooms and, in the case of owners, newer houses at the expense of accessibility to the center of San Francisco. There is some evidence, particularly with renters, that households with children are more reluctant to live in areas served by schools with a higher proportion of black students, but the effect is not always statistically significant. The most disturbing result, as far as validity of the model is concerned, involves the coefficients on Z6: Owners and renters without children attach a higher value to areas served by elementary schools with higher median income than do their counterparts with school age children, and in half of the cases the difference in coefficients is statistically significant at the 5 percent level or better. It is not difficult to explain this anomaly. Households without children may anticipate their arrival or, in the case of older households, their housing choice may reflect the presence of children earlier in the life cycle. One explanation that apparently will not suffice is that median school income may be a poor proxy for school quality. In discriminant analyses with Berkeley excluded, where it was possible to add elementary third-grade reading scores to the model, both Z6 and the reading score exhibited this anomalous behavior. Rather than trying to explain away the result, we simply note that this is the one significant instance where the model failed to perform as expected.

To obtain some sense of the relative importance of the various characteristics in determining location by income class, note that for any pair of household types t and t' Eq. (14) implies that

$$\frac{p(t'|z)}{p(t|z)} = \exp[(\alpha_{t'}-\alpha_t)z] \tag{19}$$

or

$$\log[p(t'|z)/p(t|z)] = (\alpha_{t'}-\alpha_t)z. \tag{20}$$

If we represent a change in the vector of characteristics by Δz, the percentage change in the odds ratio $p(t'|z)/p(t|z)$ will be given by $100\{\exp[(\alpha_{t'}-\alpha_t)\ \Delta z] - 1\}$. Letting t be group Y1 and t' be group Y3, the results given in Tables 3 and 4 imply that a one standard deviation change in each characteristic will have the effects described in Table 6.

Table 6

PERCENTAGE CHANGE IN p(Y3|z)/p(Y1|z) FOR A CHANGE IN Zi OF ONE STANDARD DEVIATION[a]

Group	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10
Owners, C	-38.7	-30.9	41.9	103.4	50.7	40.5	*	*	*	107.5
Owners, NC	-31.6	-51.8	23.4	49.2	129.3	*	*	*	*	23.4
Renters, C	-42.3	-42.9	--	127.0	63.2	*	*	43.3	*	118.1
Renters, NC	-36.2	-47.3	--	148.4	40.5	*	*	23.4	*	136.3

[a]An asterisk indicates that the difference in coefficients was not significant at the 5 percent level.

The location behavior of owners and renters with children and renters without children is dominated by number of rooms (Z4) and the hedonic residual (Z10). Owners with no children are most heavily influenced by median tract income (Z5). But probably the most important implication of these results is that no single factor can adequately account for location by income class. Accessibility, filtering, neighborhood effects, and the demand for space all work in the expected direction and all have a substantial effect in sorting out households by income class.

Goodness of Fit and Linear Discriminant Analysis

As noted in the second section, linear discriminant analysis provides an alternative way to estimate the coefficients of the model we have proposed. The results presented in Appendix A of Ellickson [2] demonstrates that the linear discriminant analysis corresponding to Tables 3 and 4 produces essentially the same parameter estimates. The discriminant analysis also provides a useful by-product, a means of assessing goodness of fit of our models by examining how well they succeed in classifying observations according to the type of household occupying the dwelling unit. Each observation is assigned to the household type t with the highest discriminant score (i.e., the highest bid price under the conditional logit interpretation). The results are summarized by the normalized "confusion matrices" presented in Table 7. The first row of each confusion matrix represents those observations corresponding to houses actually occupied by a household of type Y1,C (low income with children), and the entries along the row give the fraction of those observations classified into

Table 7

NORMALIZED CONFUSION MATRICES FOR WHITE HOUSEHOLDS

Group	Y1,C	Y2,C	Y3,C	Y1,NC	Y2,NC	Y3,NC
			Owners			
Y1,C	.17	.33	.08	.18	.13	.10
Y2,C	.09	.42	.15	.12	.10	.12
Y3,C	.05	.26	.40	.05	.05	.19
Y1,NC	.10	.16	.04	.39	.19	.13
Y2,NC	.11	.22	.04	.26	.18	.18
Y3,NC	.08	.14	.21	.14	.10	.33
			Renters			
Y1,C	.42	.16	.12	.13	.07	.09
Y2,C	.28	.23	.24	.05	.08	.12
Y3,C	.16	.17	.45	.04	.05	.13
Y1,NC	.16	.05	.03	.50	.17	.10
Y2,NC	.18	.06	.10	.25	.20	.20
Y3,NC	.10	.08	.16	.14	.12	.40

each of the six household categories. The remaining rows are interpreted accordingly, so that the diagonal gives the fraction of each income group classified correctly. Table 8 presents some summary statistics for the conditional logit and discriminant analysis: the log likelihood for the logit and for the discriminant analysis, the

Table 8

SUMMARY STATISTICS FOR THE LOGIT AND DISCRIMINANT MODELS FOR WHITE HOUSEHOLDS

Category	Owners	Renters
Log likelihood (logit)	-3478.1	-3423.5
Log likelihood (discriminant):		
Equal prior probabilities	-3709.2	-3491.4
Sample priors	-3491.6	-3433.9
Percent correctly classified	34.5	38.1
χ^2	531.6	762.1

percent correctly classified for each confusion matrix, and a corresponding χ^2 statistic testing the significance of the classification against the null hypothesis of a random assignment.[16]

Comparison of the values of the log likelihood function provides additional evidence that the logit and discriminant analyses are producing similar results. When sample frequencies are used for the prior probabilities, the log likelihood for the discriminant analysis is within .4 percent of that for the conditional logit for both owners and renters. The critical value for the χ^2 statistic with 1 degree of

[16] For a discussion of linear discriminant analysis, confusion matrices, and the χ^2 statistic employed here, see Press [16], pp. 369-386. Table 8 gives the value of the log likelihood for the discriminant model both for the case of equal prior probabilities and for the case where p(t) is set equal to the fraction of households in the sample of type t. The classification used to generate the confusion matrices employed equal prior probabilities, despite the fact that a higher percentage of observations is correctly classified when sample frequencies are used, because the results seem intuitively to yield more information about how well the model classifies the lowest and highest income groups.

freedom is 10.8 at the .001 level, so the classifications are highly significant.

Accessibility and Location by Income Class

One of the questions often asked in studies of this sort is why higher income households tend to live in the suburbs while lower income households choose more central locations. In the standard accessibility model, the choice of how far to reside from the central business district is determined by a balancing of commuting costs and the costs of obtaining housing space. It is frequently conjectured that as income increases, the point of balance shifts to more outlying locations. However, in his analysis of census tract data, Muth [11] rejects this conclusion, finding that the positive correlation between median tract income and distance from the business district disappears when age of the housing stock is held constant. Thus, he concludes that it is the demand for new housing, and not for reduced cost of housing space, that attracts high-income households to the suburbs.

The model we have presented lends itself easily to an analysis of the various factors influencing the mix of income classes as one travels out from the central business district. Suppose for a specified value of Z1 (commuting time to the center of San Francisco) we rewrite Eq. (23) in the form

$$\log[p(t'|Z1,\overline{Z2},\ldots,\overline{Zs})/p(t|Z1,\overline{Z2},\ldots,\overline{Zs})]$$

$$= (\alpha_{t'}-\alpha_t)(Z1,\overline{Z2},\ldots,\overline{Zs})', \qquad (21)$$

where $\overline{Z2}$, ..., $\overline{Zs}$ are the mean values of housing characteristics conditional on Z1. Then, given empirical estimates of α_t and $\alpha_{t'}$, Eq. (24) yields an estimate of the ratio of type t' to type t households residing in an average dwelling unit when commuting time is Z1. Differentiating Eq. (21) with respect to Z1,[17] we get

[17] By definition, $\overline{Z1}$ = Z1.

$$\frac{d}{dZ1} \log[p(t'|Z1,\overline{Z2},\ldots,\overline{Zs})/p(t|Z1,\overline{Z2},\ldots,\overline{Zs})]$$

$$= \sum_{j=1}^{s} (\alpha_{t'j}-\alpha_{tj}) \frac{d\overline{Zj}}{dZ1} . \qquad (22)$$

The left-hand side of Eq. (22) gives the percentage rate of change in the ratio of the two household types with respect to Z1. The derivatives $d\overline{Z}j/dZ1$ appearing on the right-hand side give the rate of change in the average housing characteristics relative to Z1. To estimate these rates of change, we can regress each characteristic on Z1 for our sample. But because of the normalization we have adopted for characteristics, the slopes of these regressions are the simple correlation coefficients between Z2, ..., Zs, and Z1. These correlation coefficients for all white owners and all white renters in our sample are presented in the first two rows of Table 9. Again taking income group Y1 for type t and Y3 for type t', the remaining four rows of Table 9 give the value for each of the terms on the right-hand side of Eq. (22) and the total rate of change for each of the four white household groups, owners with and without children and renters with and without children.[18]

As one would expect, age of the structure exhibits a strong negative correlation with commuting time to San Francisco, while lot size (for owners) has a strong positive correlation with Z1. Number of rooms and commuting time have nearly a zero correlation for owners, but show a modest positive correlation for renters. As a consequence, the strong relative preference for locations near to San Francisco by the upper income group is offset to some extent by the desire for newer housing and large lots or more rooms. But the net effect is negative for all groups except owners without children, leading to a smaller proportion of high-income households as one moves away from the center of San Francisco. The strong attraction of the central city in this

[18]The terms corresponding to the three racial characteristics (Z7, Z8, and Z9) are not presented because the coefficients are significantly different from zero in only two cases, and in those cases the contribution to the percentage rate of change in $p(Y3|\overline{Z})/p(Y1|\overline{Z})$ is negligible.

Table 9

DECOMPOSITION OF THE PERCENTAGE RATE OF CHANGE OF $p(Y3|\overline{Z})/p(Y1|\overline{Z})$ RELATIVE TO Z1

Simple Correlation:	Z1	Z2	Z3	Z4	Z5	Z6	Z10	Total Rate of Change
Owner		-.512	.458	.008	.025	-.032	.064	
Renter		-.380	--	.114	.087	-.018	.021	
Owner,C	-49	19	16	1	1	-1	5	-8
Owner,NC	-38	37	10	0	2	(a)	1	12
Renter,C	-55	21	--	9	4	(a)	2	-19
Renter,NC	-45	24	--	10	3	(a)	2	-6

[a]The difference in coefficients for the two income groups was not significant at the 5 percent level.

case stands in sharp contrast to what is supposed to be the norm, but these results are consistent with the *negative* correlation between household income and commuting time in our sample: -0.069 for owners, -0.039 for renters.

Jurisdictional Boundaries and Housing Choice

It seems clear that the attraction of San Francisco for high-income households reflects more than a mundane concern for economizing on commuting costs. Whatever the reason, high-income households had not, at least by 1965, abandoned the city for the suburbs. More surprisingly, perhaps, Oakland, which shows more signs of a city in decline, also remained an attractive location for a substantial number of high-income households. San Francisco, Oakland, and Berkeley resemble the typical central city in many ways. The three communities hold a near monopoly on the ghetto, with 75 percent of the black population of the Standard Metropolitan Statistical Area (SMSA) within their borders in 1960. All three contain substantial concentrations of low-income households, with the percentage of families with incomes less than $4000 in 1960 ranging from a low of 21.1 percent for San Francisco to a high of 24.8 percent for Oakland, as contrasted with 18.1 percent for the SMSA as a whole. But through 1960 all three cities were also able to retain substantial numbers of high-income households. While 7.5 percent of families in the SMSA had incomes of $15,000 or more, the corresponding percentage in Berkeley was 9.4 percent and in San Francisco 7.2 percent. Even in Oakland 5.8 percent of the families had incomes of at least $15,000, with most of these households located in the hills rimming the eastern border of the city.

Jurisdictional boundaries do not seem to play much of a role in determining residential choice in the area covered by this study. In particular, there is little evidence of the stratification by income class one expects when households are able to choose among political jurisdictions offering different menus of public services. Apart from one affluent suburb (Piedmont), median incomes of the cities in Alameda County varied from a low of $6000 for Emoryville to a high of $7900 for Castro Valley in 1960, with Berkeley and Oakland (as well as San Francisco) occupying positions near the bottom of the range. When median community income from the 1960 census was added to the discriminant analyses corresponding to Tables 3 and 4, the effect on the coefficients of the housing characteristics introduced earlier was negligible, the coefficients on community income exhibited no coherent pattern, and the effect on log likelihood was not statistically significant.[19]

One could object that two of the housing characteristics already introduced, median tract income (Z5) and median elementary school income (Z6), are already picking up the effects of stratification by political jurisdiction. But when median community income was used in place of these characteristics, its coefficients exhibited the expected pattern (increasing with income of the household type) only for owners and renters with children 18 or under, and log likelihood was reduced substantially; in all four cases, reintroducing characteristics Z5 and Z6 resulted in a highly significant increase in log likelihood. One could also object that if political jurisdictions vary the mix as well

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[19]To test for the significance of the change in log likelihood, we rely on the fact that the log likelihood for the discriminant analysis, with sample frequencies as prior probabilities, is close to the log likelihood for conditional logit; twice the change in log likelihood is then distributed χ^2 (with 5 degrees of freedom in the present instance). See McFadden [8], pp. 120-121.

as the quality of local public services, the use of median community income does not provide an adequate test for stratification along jurisdictional lines. But even when a separate dummy variable was introduced for every political jurisdiction included in the sample, there was no significant effect on log likelihood and a negligible effect on the estimated coefficients for the other housing characteristics.

Thus, there is little evidence that households pay much attention to the characteristics of political jurisdictions in choosing where to live, at least in any way that can distinguish among different income classes. In a sense, this result should come as no surprise. For a process of the Tiebout sort to operate, political jurisdiction should be small in size with boundaries changing over time to accommodate variations in demand in an optimal fashion. But there are only 11 jurisdictions in the area covered by this study (excluding some unincorporated regions), serving a population over 1.5 million, and their boundaries have remained fixed over long periods of time.

These negative results do not imply, however, that public goods have no effect on residential location. Neighborhood quality can be regarded as a local public good, and our results indicate that median tract income does exert a substantial influence on residential choice. Quality of the public schools, disaggregated to the individual attendance area rather than the school district, also has an effect (though modest). Finally, one can view racial characteristics of the schools and the neighborhood as entering utility functions as a collective good. Although such racial characteristics appear to have little effect in comparisons among subgroups of whites, we will see shortly that they have a dramatic effect in distinguishing between whites and blacks.

Thus, in its narrow form, where public goods are supplied by political jurisdictions, the Tiebout hypothesis does not fare too well. But with a broader interpretation focusing on the provision of collective goods at the more local level of a neighborhood or an individual school, the hypothesis receives much stronger support.

BLACK AND WHITE HOMEOWNERS AND RENTERS

The results presented so far have dealt only with white households. Unfortunately, the number of black households in the sample is not adequate to permit an analysis of their housing choice behavior in comparable detail. Although the representation of blacks in the BATSC sample accords quite closely with their proportion in Alameda and San Francisco counties, and although these counties contain 75 percent of all black households in the SMSA, in absolute terms the number of observations is very small: 214 owners and 323 renters. To make some sort of analysis possible, we dispense with the distinction between households with and without children 18 or under and collapse the income classification into two categories. The definition of the low (YL) and high (YH) income categories is described in Table 10; note in particular that the dividing line between income categories is different for owners and renters. The table also gives the number of white (W) and black (B) households falling into each cell.

Table 10

CELL SIZES FOR BLACK AND WHITE HOUSEHOLDS
(Two-income category classification)

	Owners	W	B	Renters	W	B
YL	Under $9000	887	142	Under $7000	966	204
YH	$9000 or more	1427	72	$7000 or more	1335	119

The maximum likelihood estimates for owners and renters are given in Tables 11 and 12, respectively. The normalization procedure described earlier has been applied here as well, with low-income whites serving as the reference group and characteristics measured in terms of the sample standard deviations given in Table 5. In the table of t statistics accompanying each set of parameter estimates, the first two rows compare coefficients between income groups with race held constant, and the last two rows compare coefficients between blacks and whites with income held constant.

The difference in coefficients between low- and high-income whites follows the pattern we observed before. High-income whites exhibit a

Table 11

LOGIT ESTIMATES: BLACK AND WHITE OWNERS

Group	Constant	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	
	Parameter Estimates											
YH,W	-29.3	-.36	-.32	.17	.40	.43	.23	0	-.07	.04	.33	
YL,B	-8.5	-.11	-.13	-.07	.17	-.61	.75	.88	.23	.17	-.30	
YH,B	-30.0	-.41	-.40	.27	.29	.01	.58	.74	.40	.23	-.21	
	t Statistics (Group i Relative to Group j)[a]											
Group i	Constant	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Group j
YH,W	-9.40**	-5.64**	-5.45**	3.19**	8.04**	4.72**	2.52**	.06	-1.17	.60	6.25**	YL,W
YH,B	-2.10*	-1.46	-1.37	2.07*	.87	2.11*	-.54	-.93	1.12	.92	.52	YL,B
YL,B	-1.04	-.65	-.75	-.56	1.54	-2.80**	3.46**	7.36**	1.83	2.77**	-2.26**	YL,W
YH,B	-.08	-.28	-.52	.64	-.79	-1.64	1.33	5.26**	3.46**	2.46**	-3.61**	YH,W

[a]A single asterisk indicates significance (two-tailed) at the .05 level; a double asterisk, at the .01 level.

Table 12

LOGIT ESTIMATES: BLACK AND WHITE RENTERS

Group	Constant	Z1	Z2	Z4	Z5	Z6	Z7	Z8	Z9	Z10	
	Parameter Estimates										
YH,W	-17.0	-.31	-.39	.58	.29	.13	-.12	.24	-.07	.55	
YL,B	7.9	.16	-.10	.14	-.29	-.02	.93	.06	.26	-.32	
YH,B	-23.6	-.15	-.55	.64	-.06	.48	.82	.26	.42	.12	
	t Statistics (Group i Relative to Group j)[a]										
Group i	Constant	Z1	Z2	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Group j
YH,W	-5.92**	-5.85**	-7.62**	12.79**	3.91**	1.75	-1.52	4.07**	-.96	10.24**	YL,W
YH,B	-3.77**	-1.90	-3.22**	4.02**	1.07	2.14*	-.61	1.28	1.98	2.80**	YL,B
YL,B	1.19	1.28	-.86	1.44	-1.67	-.09	6.96**	.50	3.54**	-2.40**	YL,W
YH,B	-.84	1.08	-1.25	.53	-1.70	1.70	5.76**	.09	5.40**	-3.12**	YH,W

[a]A single asterisk indicates significance (two-tailed) at the .05 level; a double asterisk, at the .01 level.

stronger preference for more accessible locations, newer houses, larger lots, more rooms, higher tract and elementary school income, and a higher value for the hedonic residual, and except in the case of elementary school income all of these differences in coefficients are statistically significant at the .01 level or better. However, there are no significant differences in response to the three characteristics measuring racial composition of the schools or the census tract except for the greater aversion of low-income whites to housing served by junior high schools with a high ratio of black students. A similar pattern is observed in distinguishing between low- and high-income black households although the effects are not as strong. Thus, blacks and whites seem to respond to housing characteristics in much the same way. Turning to a comparison of coefficients between blacks and whites, income held constant, there are no significant differences as far as commuting time, age of the housing structure, lot size, or number of rooms are concerned. There is some evidence that whites attach a higher value to neighborhood income and to those aspects of housing quality captured by the hedonic residual, while blacks weigh school income more heavily. By far the most striking results, however, concern the coefficients on those characteristics measuring racial composition of the schools and of the census tract. Although differences within racial groups are almost never significant, the differences between races are highly significant and in the expected direction except in the case of the racial composition of junior high schools (and even there the difference in coefficients is highly significant for high-income households and significant at the 10 percent level for low-income households). These results provide strong evidence for the proposition that housing markets are highly segmented along racial lines.

The models of black and white households were also estimated using linear discriminant analysis and, in contrast with our earlier results, the estimated coefficients differed markedly. Although for most characteristics the estimates of coefficients were about the same, discriminant analysis strongly accentuates the difference between blacks and whites with regard to the three racial characteristics.[20] The normalized confusion matrices for owners and renters are presented in Table 13 and summary statistics for the conditional logit and discriminant analysis in Table 14.

Table 13

NORMALIZED CONFUSION MATRICES FOR BLACK AND WHITE HOUSEHOLDS

Group	YL,W	YH,W	YL,B	YH,B
Owners				
YL,W	.64	.29	.03	.04
YH,W	.38	.59	.01	.03
YL,B	.16	.04	.55	.25
YH,B	.18	.14	.36	.32
Renters				
YL,W	.59	.31	.07	.03
YH,W	.32	.63	.03	.03
YL,B	.16	.07	.46	.31
YH,B	.13	.12	.23	.52

Consistent with the discrepancy in coefficients, the value of log likelihood for the discriminant model deviates markedly from the maximum, by 17 percent for owners and by 11 percent for renters when sample frequencies are used for the prior probabilities. But the models classify very well, with nearly 60 percent correctly classified for both owners and renters, and the χ^2 statistic indicates that the classification is highly significant.

[20] These results are not particularly surprising. Although for the other characteristics the assumption of normality is relatively plausible (and appeared to be satisfied when we examined the data using plots on probability paper), this is clearly not the case for the variables defined as percentage black in the schools or the census tract.

Table 14

SUMMARY STATISTICS FOR BLACK AND WHITE HOUSEHOLDS

Category	Owners	Renters
Log likelihood (logit)	-1860.9	-2065.0
Log likelihood (discriminant):		
Equal prior probabilities	-2317.0	-2422.1
Sample priors	-2171.8	-2294.6
Percent correctly classified	59.7	59.2
χ^2	1615.2	1657.3

5. CONCLUSION

One of the major flaws in existing approaches to hedonic price theory is that estimation of hedonic price functions provides little information about consumer behavior. Much of the confusion in the early hedonic literature reflects a misguided attempt to interpret the estimated coefficients as somehow representing either demand or supply, an approach that has been effectively vitiated by the recent work of Rosen [17] and Mas-Colell [6). To surmount this problem, we have translated the hedonic theory of housing markets into a statement about the conditional probability that a dwelling unit will be occupied by a household of a particular type given the characteristics of the housing structure.

The form that is taken by these probability functions is that of conditional logit. A word should be said at this point about the relationship between this approach and that popularized by McFadden [8] and applied to housing markets by Friedman [4] and Quigley [16]. McFadden's model of quantal choice behavior is essentially the approach we have taken in reverse: Within the context of housing markets, it predicts the type of house that will be chosen by a consumer with given characteristics rather than the type of consumer who will reside in a dwelling unit with given characteristics. One of the problems with McFadden's model when applied to residential choice is that it is unclear which criteria should be used to classify dwelling units into types, and the effect of such aggregation on parameter estimates is unknown. In the approach we have taken, however, aggregation is quite natural. Any grouping of households that is of interest can be accommodated by the theory. Furthermore, the coefficients that are estimated have an obvious interpretation in terms of traditional location theory, as parameters of bid price functions, and comparison of these coefficients among groups provides a direct way of testing various propositions in the urban economics literature regarding the effect of such factors as accessibility, filtering, racial discrimination, or jurisdictional fragmentation on where households of different types will choose to locate.

The empirical results presented in Section 4 provide strong confirmation of the theory. All of the housing characteristics introduced have the effect predicted by the theory except for the stronger preference for higher income schools exhibited by households without schoolage children. Number of rooms, the hedonic residual, and median tract income have the greatest effects in discriminating between the residential choices of low- and high-income consumers. But perhaps the most important implication of these results is that no one or two characteristics dominate to the exclusion of the rest. Single factor theories cannot do justice to the complexity of urban housing markets.

Observed patterns in the estimated coefficients proved robust to alteration of the scheme for classifying households into types as we collapsed income categories from three to two; the results presented in Appendix B of Ellickson [2] exhibit similar robustness when income categories are expanded from three to five.[21] The models fit the data as well as one could expect, particularly in view of the crudeness of the income measure, the small sample sizes at the extremes of the income distribution, and the absence of detailed data on housing characteristics.[22]

It will be necessary to apply this model to other data sets before one can generalize from the results we have found with any degree of confidence. Much more attention needs to be given to the supply side of the market and to the decision to own or to rent. But it seems safe to conclude that the model we have proposed provides a means of analyzing housing markets that holds great promise.

[21] Considerations of computational cost forced us to analyze the five-income category classification with family composition held constant, which is why the three-income category classification is presented here.

[22] The recent work of King [5] estimating hedonic functions for New Haven provides a convincing demonstration of the return to having more detailed information on housing characteristics.

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