SPECULATION AND EQUILIBRIUM:

INFORMATION, RISK, AND MARKETS*

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ABSTRACT

Speculation is commonly regarded as a process for the transfer of price risks. Existing theories all postulate a given probability (belief) distribution of anticipated price changes. But in general equilibrium systems price must always be an endogenous, not an exogenous variable. Prices, and beliefs about prices, must be based on more fundamental determinants. It is the stochastic variability of quantities, i.e., of physical endowments differing according to state of the world, that determines the stochastic variability of prices. It follows that individuals must cope not with price risk alone, but with the interaction of price risk and quantity risk.

Speculation can only occur in an informative situation, when new information (as to which state of the world, with more or less abundant endowment, will obtain) is anticipated to emerge before the close of trading. The emergent information divides trading into an initial (prior) and final (posterior) round. The key question is the dependence of prior prices and trading upon (1) Differences in individuals' risk-tolerance, or (2) Differences in probability beliefs (optimism or pessimism). A closely connected question is the relation between prior and posterior prices.

In this paper a regime of "Semi-complete markets" was assumed, i.e., contingent state-claims to the risky commodity can be traded. With concordant (homogeneous) probability beliefs, there can then be no speculation -- regardless of individual differences in degree of risk-tolerance. Furthermore, prior state-claim prices must then equal the mathematical expectation of posterior prices. Only in the case of differences of belief does speculative or hedging behavior emerge. Hence speculative return is not the reward for bearing price risks, but rather for taking market action in support of better-informed belief.

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A widely accepted view of the nature and function of speculative activity, associated most prominently with the names of J. M. Keynes and J. R. Hicks, underlies and informs most of the professional and theoretical literature on the subject. This standard conception interprets speculation as a process for the transfer of price risks.[1/]

Individuals who anticipate buying or selling at later dates can arrange "futures" contracts -- purchase or sale commitments for later delivery at currently determined prices. But they also have the alternative of waiting and taking their chances with the unknown "spot" prices that will be ruling at the desired delivery dates. To the extent that they adopt the former option they are said to be "hedging", divesting themselves of price risks. Hedgers can either be on the long side of the market for the physical good (commodity suppliers) or on the short side (commodity demanders). The speculators accept futures contracts with long or short hedgers and thus absorb the price risk. Long and short hedgers can also, of course, contract with each other, so that the speculators need only accept the net balance of hedgers' commitments in the future markets.

It used to be assumed that hedgers are mostly long the physical good (i.e., they are suppliers or warehousers of the commodity), and therefore are predominantly short in the futures market. Then the speculators must be net long in futures. This assumption has led to the inference of "normal backwardation" -- that prices of futures contracts of a given delivery date tend on the average to rise as delivery approaches, thus rewarding the speculators for making early purchase commitments and thereby bearing the price risk. The evidence does not conclusively support normal backwardation, however.[2/] Two main explanations for this failure have been proposed: (1) The hedgers may not be predominantly suppliers of the commodity; if they were predominantly demanders instead, normal speculative compensation would dictate a falling rather than a rising price trend over the life of the futures contract.[3/] (2) Or, speculators may not be risk-averse on balance, and so may not require any net compensation.[4/]

An alternative concept of speculation has been put forward by Holbrook Working. In the Keynes-Hicks view the speculators are characterized not by any special knowledge or beliefs but simply by their willingness to tolerate risk. Working argued, in contrast, that both "hedging" and "speculative" commitments depend upon opinions as to price prospects.[5/] Thus, what is commonly called hedging can scarcely be distinguished in fundamental logic and motivation from speculation on anticipated price changes.[6/] Working suggests that the social function of speculation, which for Keynes and Hicks is the shifting of price risks to those less averse to risk-bearing, is rather the improvement in the accuracy with which market prices reflect informed opinions.[7/]

These conflicting theories have never (so far as I am aware) been grounded upon a proper foundation: a general-equilibrium model in which individuals' tastes, endowments, and beliefs in a world of uncertainty interact so as to generate a market equilibrium incorporating both speculative and non-speculative transactions. The more sophisticated theoretical formulations extant[8/] are all partial-equilibrium analyses in that individuals face postulated probability distributions for price changes. But price is an endogenous variable of economic systems: general-equilibrium models must explain prices and price anticipations on the basis of more fundamental determinants. It is the aim of this paper to provide the requisite foundation.

I. PRICE RISK VS. QUANTITY RISK

The key analytical failing of the speculation literature is its preoccupation with price risk while neglecting quantity risk. It is the interaction of these two uncertainties that risk-avoiding individuals must respond to in their hedging/speculative commitments which, in turn, impact upon market prices.

Fig. 1 is a representation of the conventional view, in which only price risk is taken into account. Assuming for simplicity a world of pure exchange with two commodities X and Y, the individual's endowment position is at E. The currently ruling price ratio determines the budget line MM' through E, enabling him to attain a simple consumptive optimum at C*. But let the individual now contemplate the possibility, as an alternative to finalizing his consumptive plan at C*, of holding at some other position with the expectation of re-trading after prices change. In particular, suppose that he attaches some nonzero probability to an upward shift in the price of X (suggested by the steeper dashed market lines), and a complementary probability to a downward shift (suggested by the flatter dotted market lines). This prospect opens up the possibility of his moving along MM' (i.e., while the initial prices are still ruling) not directly to C*, but to a trading position like T.

From T, if the favorable shift of prices occurs, the individual can attain the higher conditional consumptive optimum $\bar{C}'$; with the unfavorable shift, the best he can do is the lower conditional optimum $\bar{C}''$. The trading in the "initial round" (along MM') corresponds, of course, to current dealing in futures contracts; the trading in the "final round" (along one or the other of the conditional posterior market lines TT' or TT") corresponds to dealing at a later date in the spot market.

In this conventional view an individual is said to be speculating if he moves, in the initial round of trading, from the endowment position E to any trading position like T that enlarges his price risk.[9/] Trading in the initial round that moves the individual from E toward C* along MM' is, on the other hand, in the conservative direction and would be called hedging.

But the individual of Fig. 1 is a very special case: his endowment was not a gamble, but rather a vector of X and Y quantities certain. In general, not only prices but endowment quantities vary probabilistically. Indeed, it is the stochastic variation of quantities that generally induces the variability of prices. When the aggregate social total of a commodity X is stochastically large, its price will be low. For an individual with a more or less "representative" endowment gamble holding of X (i.e., his stochastic endowment is positively correlated with economy-wide totals), the price and quantity risks tend therefore to be more or less offsetting!

Before addressing the interaction of price and quantity risks, consider first in Fig. 2 the alternative special case of quantity risk only. Here we have an individual's "cardinal" utility function v(z) defined over a single risky commodity Z.[10/] The concave curvature corresponds to aversion to risk, a property assumed in this paper always to hold. Let us say that an individual finds himself endowed with the gamble $E = (z_a^e, z_b^e)$, where $z_a^e > z_b^e$. This may be interpreted as follows. There are two alternative possible states of the world, a and b.[11/] State-a is a situation of plenty (e.g., a good crop), state-b a situation of dearth. Note that in a two-state world, the endowment position appears as a pair of points on the horizontal axis of Fig. 2. If the individual attaches subjective probability p to state-a and 1-p to state-b, the overall desirability of the endowment position is determined by the expected-utility rule:

$$U(E) = pv(z_a^e) + (1-p)v(z_b^e).$$

Geometrically, in Fig. 2, U(E) is the point on the dashed line connecting $v(z_a^e)$ and $v(z_b^e)$ that divides the horizontal distance in proportion to the probabilities p and 1-p. The endowment gamble may be expressed in "prospect" notation as $E = [z_a^e, z_b^e; p, 1-p]$.

To show the interaction of price risk and quantity risk requires more complicated diagrammatics, since at least one other commodity must be introduced. For simplicity, let us deal with a two-commodity world in which the second commodity N is envisaged as riskless. That is, regardless of which state of the world obtains, each individual's N-endowment remains unaffected. All the quantity risk is therefore associated with commodity Z. Such a situation is pictured in Fig. 3. The individual's endowment vector in prospect notation is now $E = [(n^e, z_a^e), (n^e, z_b^e); p, 1-p]$. Here the first outcome is plotted as the point in the base plane denoted $e_a = (100,200)$ and the second outcome as $e_b = (100,80)$. As before, the endowment position appears as a pair of points in the diagram. The absence of quantity risk with respect to the N-commodity is reflected in the horizontal alignment of the e_a, e_b points along $n = 100$. Since N is assumed riskless throughout, in what follows it will usually be convenient to use a somewhat condensed notation that expresses the endowment position here as $E \equiv (n^e; z_a^e, z_b^e) = (100;200,80)$. Similar notation will be used for trading positions T and consumptive positions C. (At times, however, it will become necessary to revert to the more complete prospect representation of the underlying gambles.)

The vertical axis measures a cardinal preference-scaling function $u(n,z)$ defined over the two commodities N and Z; this function can be arrived at by a natural generalization of the Friedman-Savage one-commodity development.[12/] The expected-utility rule can then be set down in the generalized form:

$$U(n;z_a,z_b) = pu(n,z_a) + (1-p)u(n,z_b)$$

In Fig. 3 we do not see the entire utility surface $u(n,z)$ but only the section of the surface overlying the line $n = 100$ in the base plane. Then the utility of the endowment position is shown by the point $U(E)$ along the line connecting $u(e_a)$ and $u(e_b)$, weighted in accordance with probabilities p and 1-p in complete analogy with the corresponding construction in Fig. 2. But note that the cardinal scale $u(n,z)$ in Fig. 3 indexes the utility levels of entire indifference curves in the N,Z plane — not simply quantities of a single good as in Fig. 2.

It is essential for the analysis to take careful account of the nature of the transactions permitted. It will be assumed throughout that the riskless commodity N is always tradable; N will be taken as numeraire, so that $P_N \equiv 1$ always. As for the risky commodity Z, there are two interesting possibilities: (1) SEMI-COMPLETE MARKETS: Here the <u>conditional</u> state-claims to Z_a and Z_b are separately tradable against units of the riskless commodity N, leading to distinct prices P_{Za} and P_{Zb}. This regime of markets falls short of being fully complete in that conditional state-claims N_a and N_b to the riskless commodity are not assumed tradable. It might be thought that there would never be any requirement for contingent trading of a riskless commodity, but that would not be quite correct. However, under the special assumptions to be made here as to concordant beliefs, there is essentially nothing lost and a very considerable simplification gained from the restriction to Semi-complete Markets. (2) CERTAINTY MARKETS: Here only unconditional (certain) claims even to the risky commodity Z are tradable, so that markets would determine a single price denoted P_Z. Whether the Certainty Markets model or the Semi-complete Markets model represents a better approximation to real world conditions remains arguable.[13/] In this paper, only behavior under the more ample regime of Semi-complete Markets will be studied -- though some comments in the concluding section will indicate the general thrust of the modifications entailed by a more limited market regime.[14/]

In this paper we will be contrasting the implications for speculative-hedging decisions of (1) degree of risk-aversion (as represented by the shape of the u(n,z) function), and (2) probability beliefs. In the world of pure exchange assumed for simplicity here, prices can change only as a result of general shifts in probability beliefs (as to the likelihoods of differently-endowed states of the world).[15/] Such shifts are assumed to be the consequence of the emergence of new information; hence, speculative and/or hedging behavior arise only in an "informative situation" where such emergence is anticipated. Note that in an informative situation there will be two inter-related market equilibria of prices -- one prior to and the second posterior to the shift in probability beliefs.

II. NON-INFORMATIVE EQUILIBRIUM: SIMPLE CONSUMPTIVE GAMBLE

As a base point, consider a non-informative situation -- where no new information about state-probabilities is anticipated before the close of trading. Since there can then be no expectation of price change there will be no speculation or hedging. Rather, each person's decision problem is to choose a simple consumptive gamble. In the regime of Semi-complete Markets assumed throughout this paper, the tradable commodities are: (1) a single riskless good N serving as numeraire, so that $P_N \equiv 1$, and (2) Z_a and Z_b, contingent claims to a risky commodity Z that become valid if and only if the corresponding state of the world obtains.

Each individual will want to trade from his endowment gamble $E = (n^e; z_a^e z_b^e)$ to a preferred consumptive gamble $C^* = (n^*; z_a^*, z_b^*)$ in the light of his utility function. Exchange will take place at market-clearing prices P_{Za} and P_{Zb}. The anticipated sequence of events can be indicated:

Endowment gamble E → Trading → Consumptive gamble C* → Nature's choice of state → Consumption

The individual maximizes his expected utility:

$$(1) \quad U(n; z_a, z_b) = pu(n, z_a) + (1-p)u(n, z_b)$$

Note that there is no need to distinguish between n_a and n_b, which are necessarily equal under the regime of Semi-complete Markets. It will be assumed that the preference-scaling function can be written in additive form:

$$(2) \quad u(n,z) = u_n(n) + u_z(z)$$

At some modest cost in generality, this restriction provides great simplification by clearing away the intricacies of complementarity effects.

Under pure exchange the budget constraint is the wealth-value of the endowment combination, W^e:

(3) $n + P_{Za}z_a + P_{Zb}z_b = n^e + P_{Za}z_a^e + P_{Zb}z_b^e \equiv W^e$

The usual optimization technique leads to:

(4) $p\,\dfrac{\partial u/\partial z_a}{\partial u/\partial n} = P_{Za}$ and $(1-p)\,\dfrac{\partial u/\partial z_b}{\partial u/\partial n} = P_{Zb}$ Consumptive optimality conditions

These conditions, together with the budget constraint (3), suffice to determine the individual's optimum simple consumptive gamble C*. Also determined, of course, are the transactions he must execute to convert his endowment gamble E into C*.

What are the factors governing the market prices P_{Za} and P_{Zb}? Evidently, supply and demand functions could be derived from (3) and (4) whose equality, summed over all individuals, would express the conditions of equilibrium. It will be convenient here, however, to introduce as a heuristic device the idea of a "representative individual". This amounts to assuming that, apart from the possibly deviant trader to whom equations (3) and (4) apply and who as a single person is of negligible weight, everyone else is identical so that any one of them serves as a microcosm of the entire market. (The reader may be reassured that the representative individual, like the Cheshire cat, will in due course disappear -- except possibly for his grin!) In a world of pure exchange the equilibrium prices are then governed by the necessity of sustaining the representative endowment vector; since a representative individual can find no-one to trade with in a closed economy, his endowment vector must be his consumptive optimum.

Let the representative individual's utility function be denoted $\mu(n,z)$, and suppose he assigns probabilities π and $1-\pi$ to states _a_ and _b_, respectively. Then the conditions determining prices are:

$$(5) \quad \pi \frac{\partial\mu/\partial z_a^r}{\partial\mu/\partial n^r} = P_{Za} \text{ and } (1-\pi) \frac{\partial\mu/\partial z_b^r}{\partial\mu/\partial n^r} = P_{Zb}$$

Sustaining prices, representative individual

Here $(n^r;z_a^r,z_b^r) \equiv R$ is both endowment vector and consumptive optimum for the representative individual. Note how the representative probability beliefs π, $1-\pi$ and the representative marginal utilities (involving both the cross-commodity comparison $\partial\mu/\partial z$ versus $\partial\mu/\partial n$, and the intra-commodity comparison $\partial\mu/\partial z_a$ versus $\partial\mu/\partial z_b$ that reflects degree of risk-aversion) enter into the equilibrium price ratios. More generally, of course, the ratios of prices would reflect some <u>average</u> measure of the various individuals' differing probability beliefs and some <u>average</u> measure of their respective marginal utilities.[16/]

NUMERICAL EXAMPLE 1:

Consider an economy consisting of a great number of representative individuals -- plus a single deviant person whose weight is negligible in determining prices. Suppose that all have endowment vectors $E = R = (100;200,80)$ and that all utility functions are $u = \mu = \log_e nz = \log_e n + \log_e z$. Let the representative individual's probability parameter (belief attached to state-<u>a</u>) be $\pi = .6$, and the deviant's be $p = .7$. (Thus, the non-conforming individual here is <u>belief</u>-deviant and not endowment-deviant or utility-deviant.) Then, from equations (5), the prices are determined as $.6\frac{1/200}{1/100} = .3 = P_{Za}$ and $.4\frac{1/80}{1/100} = .5 = P_{Zb}$. It follows that all individuals have endowed-wealths $W^e = 100 + .3(200) + .5(80) = 200$. The budget equation (3) becomes $n + .3z_a + .5z_b = 200$. For the deviant individual the Consumptive Optimality Conditions take the form $.7\frac{1/z_a}{1/n} = .3$ and $.3\frac{1/z_b}{1/n} = .5$. These determine his simple consumptive optimum (see Fig. 4) as the vector $C^* = (100;233\frac{1}{3},60)$.

As for the representative individual, it can be verified [by substituting his belief parameter π for p in equations (4)] that his consumptive optimum is identical with his endowment vector R = (100;200,80).[17]

(END OF NUMERICAL EXAMPLE)

In the numerical example, the deviant individual attaching higher belief to the advent of state-a is willing to take more of a chance on state-a obtaining. But even the representative individual does not move to a riskless situation -- indeed he cannot, because an inescapable quantity risk exists on the social level (e.g., the aggregate crop may be a good one or a poor one). In Fig. 4 the optimal gamble C* for the belief-deviant individual is represented by the point-pair c_a, c_b and the representative individual's gamble R by the point-pair e_a, e_b (same as the endowment gamble). The geometry indicates (in the vertical dimension) how the overall utilities U(C*) and U(R) are determined from the respective probability-weighted outcomes.

Before leaving the non-informative situation, consider the following question: Suppose that after individuals have selected their optimum consumptive gambles, new information is unexpectedly revealed so as to change representative probability beliefs π, $1-\pi$ to π^*, $1-\pi^*$. What will be the effect on prices? Using the device of a fully representative individual, we see immediately from (5) that:

$$(6) \quad \frac{P^*_{Za}}{P_{Za}} = \frac{\pi^*}{\pi} \text{ and } \frac{P^*_{Zb}}{P_{Zb}} = \frac{1-\pi^*}{1-\pi}$$

In order to sustain the representative individual's position, prices will move in simple proportion to changes in probability beliefs. Furthermore, we can let our Cheshire cat, the representative individual assumption, fade away and the proposition still holds true! We must still retain, however, the cat's grin in the form of agreement or concordance of beliefs[18/] -- so that all the personal p's equal π before and π^* after the new information. The validity of (6), given agreed beliefs, is evident from the way in which personal probabilities p, 1-p enter as multiplicative factors in the Consumptive Optimality Conditions (4). The positions attained by individuals of agreed beliefs are sustained when beliefs change, the prices moving in such a way as to keep the gambles optimal given the new probability beliefs.[19/]

III INFORMATIVE EQUILIBRIUM: PRIOR-TRADING OPTIMUM AND COMPOUND CONSUMPTIVE GAMBLE

We are now ready to bring speculation (and/or hedging) into the picture. The traditional literature emphasizes, what is indeed an essential element, the "price risk" faced by transactors. In the model here "quantity risk" always exists in the form of a state-distributed endowment vector. But price risk exists only in an informative situation: where new evidence is expected to emerge, before the close of trading, so as to modify representative beliefs in the market. (The situation could of course be regarded as informative by some traders and as non-informative by others, but we here assume no disagreement on this score.) In an informative situation there will be two distinct rounds of trading, so that the sequence of events is:

Endowment gamble E → Prior trading → Speculative gamble T* → Emergence of information → Posterior trading → Consumptive gamble C**

After traders choose their final consumptive gambles, Nature as before selects the state and then actual consumption takes place.

The individual's decision problem is now more complicated. In the posterior round of trading he will engage in portfolio revisions on the basis of known final prices. In the prior round of trading he will also be facing known prior state-claim prices that we may denote P^o_{Za} and P^o_{Zb}, but he must take some account of the unknown posterior prices $\bar{P}_{Za}$ and $\bar{P}_{Zb}$ that will be ruling in the final round after the new information emerges. Decisions to transact in the initial round may thus turn out to have been misguided, but even standing pat with the endowment gamble leaves the individual exposed to an uncertain price shift.

To carry the analysis further, the nature of the emergent information (as anticipated by traders) has to be specified. It will suffice to assume here the simplest possible information anticipations: posterior unanimity and certainty. Posterior unanimity means that the evidence forthcoming will be so overwhelming that no difference of opinion can afterward persist as to the probabilities of the two states; i.e., concordant and deviant beliefs will no longer diverge. Posterior certainty means that the personal belief parameter p (probability of state-a) will take on only one or the other of the limiting values $p' = \pi' = 1$ or else $p'' = \pi'' = 0$. In short, the information forthcoming is to be absolutely conclusive as to which state will obtain. But of course no-one can in general be certain in advance which way the information will point. Indeed, under the conditions assumed here any trader will have to assign his same personal probability parameter p, that he attached to the likelihood of state-a obtaining, to the likelihood that the conclusive evidence to emerge will convince everyone that state-a is now certain.

The optimizing problem in an informative situation can be formulated as follows:

(7a) $\underset{(T^*)}{\text{Max}}\ U = pu(n',z_a') + (1-p)u(n'',z_b'')$ subject to:

$$(7b)\quad \begin{cases} n' + \bar{P}'_{Za} z'_a = n^t + \bar{P}'_{Za} z^t_a \equiv W' \\ n'' + \bar{P}''_{Zb} z''_b = n^t + \bar{P}''_{Zb} z^t_b \equiv W'' \\ n^t + P^o_{Za} z^t_a + P^o_{Zb} z^t_b = n^e + P^o_{Za} z^e_a + P^o_{Zb} z^e_b \equiv W^e \end{cases}$$

Here n^t, z^t_a, z^t_b are the elements of the optimum trading vector T* to which the individual can move in the initial round. The attainable trading vectors are constrained by endowed wealth W^e, as indicated in the third equation of (7b). The primed symbols are the posterior variables associated with the first information outcome ($\pi' = 1$), and the double-primed symbols are those associated with the other outcome ($\pi'' = 0$). The conditional posterior price $\bar{P}'_{Za}$ is the price for Z_a that would rule given the information result leading to $\pi' = 1$ (certainty of state-a); in this case, of course, the price of state-b claims must necessarily fall to zero and hence no Z_b term enters in the first equation of (7b).[20/] Similarly, $\bar{P}''_{Zb}$ is the price of Z_b claims in the opposite case (certainty of state-b), so that no Z_a term need enter into the second equation of (7b). W' and W'', defined as indicated above, will be called the conditional posterior wealths. The intermediate T* position attained serves in the role of starting-point or endowment position for the posterior trading that leads in the one case to the conditionally optimal n', z'_a point and in the other case to the corresponding n'', z''_b point. The overall choice may be regarded as selection of an optimum compound consumptive gamble that may be denoted in "prospect" form as $C^{**} = [(n', z'_a), (n'', z''_b); p, 1-p]$. Note that the N-consumptions, n' and n'', may differ with two rounds of trading even though N is riskless.

The individual's prior trading decision, as here formulated, involves knowledge of the conditional state-claim prices in both the prior and posterior markets. The prior prices P^o_{Za} and P^o_{Zb} will of course be known to him. But can anything be known about the conditional posterior prices that will help to guide prior trading? The answer is yes![21] In the special case where concordant beliefs represent essentially all the social weight, it will be shown that the posterior and prior prices are related simply in proportion to the posterior and prior concordant probabilities:

$$(8)\quad \frac{\bar{P}'_{Za}}{P^o_{Za}} = \frac{1}{\pi} \text{ and } \frac{\bar{P}''_{Zb}}{P^o_{Zb}} = \frac{1}{1-\pi}$$

Relation between prior and posterior prices

This very satisfying theorem has the possibly surprising corollary:

$$(8a)\quad P^o_{Za} = P_{Za} \text{ and } P^o_{Zb} = P_{Zb}$$

That is, the prior trading prices in an informative situation will (given agreed beliefs) be simply equal to the state-claim prices that would have ruled had the situation been a non-informative one!

The essential idea underlying (8) and (8a) can be appreciated intuitively if we bring back the Cheshire cat, the heuristic device of a fully representative individual. Prices in both prior and posterior markets must be such as to sustain the endowment position of such an individual. To put it another way, prices must be such that for him the simple consumptive gamble C* and the compound consumptive gamble C** are the same and indeed identical with the endowment gamble R. For a representative individual not to find it advantageous to move in the initial round to a position inconsistent with R, equations (5) would have to hold for prior-round trading, i.e., if we substitute P^o_{Za} for P_{Za} and P^o_{Zb} for P_{Zb} as in (8a). And since there is no uncertainty in the posterior markets, to sustain the endowment position there the prices would have to be:

$$\frac{\partial\mu/\partial z^r_a}{\partial\mu/\partial n^r} = \bar{P}'_{Za} \text{ and } \frac{\partial\mu/\partial z^r_b}{\partial\mu/\partial n^r} = \bar{P}''_{Zb}$$

Equations (8) follow directly, and (8a) has already been shown to hold in this representative-individual case.

Dropping the representative-individual assumption, we know that if $P^o_{Za} = P_{Za}$ and $P^o_{Zb} = P_{Zb}$ then in the initial round every trader can attain a T* equal to his simple consumptive optimum C*. If we assume concordant (homogeneous) beliefs, equation (6) holds for any agreed change in the probability parameter π (likelihood of state-a). With conclusive evidence the parameter can only become zero or unity; thus (8) is seen to be a special case of (6). So the C* positions attained in the initial round are sustained -- C** = C*. Equations (8) and (8a) are thus consistent with equilibrium, given only concordant beliefs.[22]

It is very interesting to note that the price-revision relation (8) is a martingale formula. That is, for either state the ratio

$$\frac{\text{Expectation of Posterior Price}}{\text{Prior Price}}$$

is unity. Consider the state-a price. The ratio $\frac{\bar{P}'_{Za}}{P^o_{Za}} = \frac{1}{\pi}$, and this will occur (i.e., the information will point to state-a) with probability π (according to the beliefs of concordant individuals). But with corresponding probability $1-\pi$ the information will conclusively point to state-b and Z_a claims will be worth zero. Hence $E\,\frac{\bar{P}_{Za}}{P^o_{Za}} = 1$. A similar argument holds of course for prior and posterior prices of state-b claims. The relation can be very considerably generalized beyond the special case considered here. But note that what is a martingale according to the beliefs of a concordant individual cannot be one if deviant beliefs are used as probability weights.

The optimizing problem of equations (7a) and (7b) can be reformulated in an illuminating way as a choice between conditional posterior wealths. Using (8), multiply the first equation of (7b) by π and the second by $1-\pi$ to obtain the third. Thus, the three constraints are not independent. The relation between them may be expressed:

$$(9) \quad \pi W' + (1-\pi)W'' = W^e$$

That is, the endowed wealth can be regarded as the mathematical expectation (using the concordant prior probabilities) of the conditional posterior wealths. Then the problem can be expressed as:

$$(10) \quad \underset{(W',W'')}{\text{Max}} \quad U = pu'(W'|\bar{P}') + (1-p)u''(W''|\bar{P}'')$$

subject to (9) as constraint. Here u' and u" are conditional utility elements expressed in "indirect" form as functions of the corresponding wealths, given the posterior price vectors symbolized by $\bar{P}'$ and $\bar{P}''$.[23/] The result is the following condition for optimal prior trading:

$$(11) \quad \frac{p}{1-p} \frac{du'/dW'}{du''/dW''} = \frac{\pi}{1-\pi} \qquad \text{Trading optimality condition}$$

The Trading Optimality Condition (11), together with the constraint (9), determines for each individual the pair of conditional posterior wealths W', W" he must reach in the initial round to obtain the best compound gamble C^{**} within his opportunity set. With W' and W" he would enter equations (7b) to find the elements n^t, z_a^t, z_b^t of the optimal trading vector T^*. But the two wealths are insufficient to determine all three elements of T^*. We therefore arrive at a Principle of Trading Indeterminacy: In a regime of Semi-complete Markets for state-claims, the optimal trading position T^* depends only on attaining the correct conditional posterior wealths. A degree of freedom remains in which any one element of T^* can be arbitrarily selected.

NUMERICAL EXAMPLE 2:

Continuing with the data of Numerical Example 1, consider a belief-deviant individual (of negligible social weight) in an informative situation — where new (and conclusive) evidence is anticipated to emerge, before the final round of trading, as to which state will obtain. With the utility function $u = \log_e nz$, conditional marginal utilities of posterior wealth can be shown to be $\frac{du'}{dW'} = \frac{2}{W'}$ and $\frac{du''}{dW''} = \frac{2}{W''}$. 24/ Then the Trading Optimality Condition (11) takes the convenient form $\frac{p}{1-p}\frac{W''}{W'} = \frac{\pi}{1-\pi}$. Together with (9), this leads to explicit equations for the optimal conditional wealths: $W' = \frac{p}{\pi}W^e$ and $W'' = \frac{1-p}{1-\pi}W^e$. With $p = .7$, $\pi = .6$, and $W^e = 200$, we obtain numerically $W' = 233\frac{1}{3}$ and $W'' = 150$. Fixing $n^t = 100$ (as is possible due to trading indeterminacy), the optimal trading vector is $T^* = (n^t;z_a^t,z_b^t) = (100;266\frac{2}{3},40)$. Using (8) and (8a), the posterior prices are $\bar{P}' \equiv \bar{P}'_{Za} = \frac{P_{Za}}{\pi} = \frac{.3}{.6} = .5$, and $\bar{P}'' \equiv \bar{P}''_{Zb} = \frac{P_{Zb}}{1-\pi} = \frac{.5}{.4} = 1.25$. Then, the individual's conditional posterior optimum positions are found by standard Lagrangian procedures to be $\bar{C}' = (n',z_a') = (116\frac{1}{3},233\frac{2}{3})$ and $\bar{C}'' = (n'',z_b'') = (75,60)$. His compound consumptive gamble can be written in prospect notation as $C^{**} = [(116\frac{1}{3},233\frac{2}{3}), (75,60); .7,.3]$.

Fig. 5 provides a geometrical interpretation of the interaction between price and quantity risk in the example above. The <u>posterior</u> optimization problem takes place under conditions of certainty. Either state-<u>a</u> will be known to obtain, in which case $\bar{P}' = .5$ and the flatter (dashed) price lines in the N,Z plane will be relevant — or state-<u>b</u> will be known to obtain, with $\bar{P}'' = 1.25$ and the steeper (dotted) price lines relevant. There is price risk, just as in Fig. 1, since the individual does not know in advance which of the posterior prices will be ruling in the final round. But the same circumstances that lead to one or other outcome with respect to the posterior prices are also associated with a more or less favorable quantitative situation. Suppose he were to remain with his endowment gamble, pictured as the pair of points e_a, e_b in Fig. 5. Then the advent of state-a would be associated, at one and the same time, with the flatter price lines and the favorable quantity outcome $e_a = (100,200)$ -- and the advent of state-b with the steeper price lines and the unfavorable $e_b = (100,80)$.

In an informative situation the individual will not (in general) stand pat with his endowment gamble in the initial round. The prior trading opportunities are awkward to represent geometrically, since they involve combinations of three types of claims: N, Z_a, and Z_b. However, thanks to the Principle of Trading Indeterminacy we can hold n = 100 and consider only trading between Z_a and Z_b claims. Then trading permits the individual to widen or narrow the gap along n = 100 in comparison with his endowed point-pair e_a, e_b. Here the prior prices are in the ratio $P^o_{Za}/P^o_{Zb} = 3/5$ -- so three units of Z_b can be exchanged in the initial round against 5 units of Z_a. The simple consumptive optimum C* (one round of trading only) for the belief-deviant individual with p = .7 involves a widening of risk as shown by the location of the point-pair c_a, c_b -- where $c_a = (100,233\frac{1}{3})$ and $c_b = (100,60)$. The trading optimum T* (speculative position) involves this individual in still further risk-widening to the point pair t_a, t_b -- where $t_a = (100,266\frac{2}{3})$ and $t_b = (100,40)$.

The final step is the posterior movement to the conditional optimum positions. If state-a obtains, the speculation has succeeded; the trader moves from t_a in the final round along the flatter price line to the indifference-curve tangency $\bar{C}' = (n', z_a') = (116\frac{2}{3}, 233\frac{1}{3})$. If state-b obtains the best attainable position is $\bar{C}'' = (n'', z_b'') = (75, 60)$. The conditional utility levels attainable, $u(\bar{C}')$ and $u(\bar{C}'')$, do not lie over the n = 100 line in the base plane of Fig. 5 — so that the vertical section of the utility surface shown in Fig. 5 is not sliced the same way as the sections shown in Figs. 3 and 4. However, the expected utility $U(C^{**})$ of the compound consumptive gamble is arrived at similarly as the probability-weighted average along the dashed line connecting $u(\bar{C}')$ and $u(\bar{C}'')$.

(END OF NUMERICAL EXAMPLE)

While the Trading Optimality Condition (11) is applicable for any individual, the development leading to (11) postulated that individuals of agreed beliefs constituted essentially all the social weight in the market. For such concordant individuals (11) reduces to the still simpler form:

$$(12) \quad \frac{du'}{dW'} = \frac{du''}{dW''}$$

Using relation (12) it may be verified that <u>individuals with concordant beliefs will choose compound gambles C^{**} identical with their simple consumptive gambles C^*</u>. (In this sense, they will not be <u>speculating</u> — as will be explained below.) <u>This will be so even when utility functions and/or endowment positions diverge</u>. In particular, $n' = n''$ and both are equal to the n^* entering into $C^* = (n^*; z_a^*, z_b^*)$. Similarly, $z_a' = z_a^*$ and $z_b'' = z_b^*$.

To demonstrate this, it will be useful to set down the <u>posterior</u> optimization problems explicitly in Lagrangian form:

$$\text{Max } u(n', z_a') - \lambda'(n' + \bar{P}_{Za}' z_a' - W')$$

$$\text{Max } u(n'', z_b'') - \lambda''(n'' + \bar{P}_{Zb}'' z_b'' - W'')$$

The conditions resulting are:

(13a) $\lambda' = \frac{\partial u}{\partial n'}$ and $\lambda'\bar{P}'_{Za} = \frac{\partial u}{\partial z'_a}$

(13b) $\lambda'' = \frac{\partial u}{\partial u''}$ and $\lambda''\bar{P}''_{Zb} = \frac{\partial u}{\partial z''_b}$

Using (8) and (8a), we can write:

(14) $\pi \frac{\partial u/\partial z'_a}{\partial u/\partial n'} = P_{Za}$ and $(1-\pi) \frac{\partial u/\partial z''_b}{\partial u/\partial n''} = P_{Zb}$

Equations (14), apart from substitution of the concordant belief parameter π for p, look almost the same as the Consumptive Optimality Conditions (4) governing the choice of the individual's simple consumptive gamble. The only difference is the distinction between n' and n" in the denominators of (14). But now note that in (13a) λ' can be identified with du/dW', the marginal utility payoff of relaxing the W' constraint -- and similarly, λ'' can be identified with du/dW". The equality in (12) of the derivatives du'/dW' and du"/dW" in "indirect" form evidently involves the equality of the parametric "direct" derivatives du/dW' and du/dW".[25] Hence $\lambda' = \lambda''$, $\frac{\partial u}{\partial n'} = \frac{\partial u}{\partial n''}$, and n' = n". Thus the conditions (14) are the same as (4), so that C** = C*.[26]

To underline the significance of this result (which has, as we shall see shortly, a direct interpretation in terms of speculation/hedging behavior), note that individuals can differ one from another with regard to beliefs, to endowment scale and composition, and with regard to utility functions (of which risk-aversion is an aspect). The proposition above shows that, in a world where concordant beliefs represent essentially all the social weight, only those deviating in belief will make use of the opportunity provided by an informative situation to choose an optimal gamble C** that diverges from the simple consumptive gamble C* they would have chosen in a non-informative situation. Conversely, contra Keynes and Hicks, differences in utility functions (or in endowments) without deviation from concordant beliefs will not lead to divergences between C* and C**.

IV. CONCLUSION: DETERMINANTS OF SPECULATIVE/HEDGING BEHAVIOR

The results of the analytical development above can now be applied to the key question: Who are the speculators and hedgers, and what factors determine the scale of their respective commitments?

The conventional definition describes hedging as initial-round trading (at the known prior prices) tending to reduce the need for final-round trading (at the unknown posterior prices) -- in short, as behavior tending to reduce exposure to "price risk". Speculation is conventionally defined in the reverse way, as acceptance of price risk. These definitions seemed plausible enough in the situation described in Fig. 1, where no quantity risk was recognized and where only certain (unconditional) claims to commodities could be traded. In the more general situation considered here, however, exclusive concentration upon price risk can easily be misleading: patterns of prior-trading behavior that reduce the scope of posterior trading (exposure to price risk) do not necessarily decrease the riskiness of the overall consumptive gamble attained. We will therefore want to use the more fundamental definition: Speculators and hedgers are those using the prior market, in an informative situation, to achieve a compound consumptive gamble C** that differs from the simple consumptive gamble C* they would have chosen in a non-informative situation. This definition involves the recognition that: (1) Consumptive positions chosen by all individuals will, in a world of uncertainty, generally be gambles ("quantity risk"), and (2) Only individuals who anticipate the emergence of new information tending to modify prices before the close of trading ("price risk") envisage the possibility of using the anticipated price change so as to obtain consumptive gambles otherwise unattainable. The hedgers are then those who employ prior trading to reduce their overall risk (choose a compound gamble C** that is less risky than the simple gamble C* they would otherwise have chosen), while the speculators are of course doing the reverse.

The crucial result attained in the analysis above can be stated: Only those individuals deviating from representative beliefs in the market will hedge or speculate. In particular, contra the Keynes-Hicks or "risk-transfer" theory, differences in degree of risk-aversion alone (i.e., in the absence of deviating beliefs) will not lead to hedging or speculative behavior. For, while relatively risk-averse individuals will tend to select narrower consumptive gambles and relatively risk-tolerant individuals select wider gambles, their choices of compound gambles C** in an informative situation will not differ from the simple gambles C* they would have selected had the situation been non-informative.

Conversely, our results support the Working theory that emphasizes differences of belief as the key to hedging/speculation behavior. Those, and only those, whose beliefs as to what the emergent information will reveal diverge from representative opinion in the market, will regard themselves as able (on the average) to profit from anticipated price change. In an informative situation they will choose compound gambles C** differing from their C* gambles even if their risk-tolerances (and all other aspects of their utility functions and endowment positions) conform with the representative situation in the market.

What about the hedger vs. speculator distinction? It is natural to associate speculation with optimistic opinion and hedging with pessimistic opinion as to the likelihood of more and less favorable states of the world. And indeed it is true that (as in the Numerical Examples above) someone attaching a higher-than-representative probability belief to the more favorable state of the world will choose a relatively risky compound gamble C** (i.e., one with a large gap between the state-dependent results). Note that in a non-informative situation such an individual would also have chosen a simple gamble C* that was relatively risky; however, his C** gamble will be riskier still. For, he expects to profit from the price change consequent upon the emergence of new information tending to validate his own prior beliefs. And it is also true that someone who (over a certain range) is pessimistic tends to hedge -- to trade in the prior round so as to achieve a relatively safe C** gamble, safer still than the conservative C* gamble such an individual would have been inclined to accept in a non-informative situation. However, there is another factor at work that disrupts a simple correlation of the optimism/pessimism parameter with speculation/hedging behavior. As we let the degree of pessimism parametrically increase, the associated belief-deviant behavior will become more conservative only up to a point; pessimism that is sufficiently extreme will actually dictate a widening of risk once again! For, with very extreme pessimistic beliefs the individual will find it attractive to gamble in such a way as to make himself better off should the <u>unfavorable</u> state of the world obtain.

NUMERICAL EXAMPLE 3:

Table 1 below illustrates the effect of differing probability beliefs upon the degree of risks accepted by individuals with common endowment E = (100;200,80) and logarithmic utility function $u = \log_e nz$, as assumed in the earlier numerical examples. As before, the concordant belief parameter (probability attached to state-a) is $\pi = .6$, leading to market equilibrium prices $P^o_{Za} = .3$ and $P^o_{Zb} = .5$ in the initial round — and to the conditional posterior prices $\bar{P}'_{Za} = .5$ or $\bar{P}''_{Zb} = 1.25$ in the final round of trading.

Any belief-deviant individual for whom the probability p attached to the (more favorable) state-a exceeds the representative individual's $\pi = .6$ can be called an optimist. As the Table shows (in the case of an optimist with $p = .7$, the situation of the previous Examples) such a deviant would widen his simple consumptive gamble C* in comparison with the representative individual's. In an informative situation his trading optimum T* (with n^t arbitrarily held at 100 as permitted by the Principle of Trading Indeterminacy) shows a still greater exposure to risk. On the other hand, a deviant with p only slightly below the representative individual's $\pi = .6$ (in the Table, $p = .5$) is a "conservative pessimist". He narrows his simple C* gamble in comparison with the representative individual's and in an informative situation is able to choose a T* so as to reduce still further his risk exposure. Finally, however, the "extreme pessimist" (in the Table, $p = .2$) switches over! His beliefs are so out of line with those determining prices that he is induced to undertake very risky commitments with high payoff to him in the unfavorable state of the world. Note that in this example his z^t_a element is actually negative, i.e., his T* involves a "short" position in Z_a claims.

TABLE 1: Risks Accepted, by Degree of Optimism/Pessimism

BELIEF PARAMETER p =	Optimist .7	Representative beliefs .6	Conservative pessimist .5	Extreme pessimist .2
SIMPLE CONSUMPTIVE GAMBLE $C^* = (n^*; z_a^*, z_b^*)$	$(100; 233\frac{1}{3}, 60)$	(100;200,80)	$(100; 166\frac{2}{3}, 100)$	$(100; 66\frac{2}{3}, 160)$
DESIRED POSTERIOR WEALTHS W', W''	$233\frac{1}{3}, 150$	200,200	$166\frac{2}{3}, 250$	$66\frac{2}{3}, 400$
TRADING POSITION $T^* = (n^t; z_a^t, z_b^t)$	$(100; 266\frac{2}{3}, 40)$	(100;200,80)	$(100; 133\frac{1}{3}, 120)$	$(100; -66\frac{2}{3}, 240)$
ELEMENTS OF COMPOUND COMSUMPTIVE GAMBLES $C^{**} = [\bar{C}', \bar{C}''; p, 1-p]$				
$\bar{C}' = (n', z_a')$	$(116\frac{2}{3}, 233\frac{1}{3})$	(100,200)	$(83\frac{1}{3}, 166\frac{2}{3})$	$(33\frac{1}{3}, 66\frac{2}{3})$
$\bar{C}'' = (n'', z_b'')$	(75,60)	(100,80)	(125,100)	(200,160)

(END OF NUMERICAL EXAMPLE)

In a world of otherwise concordant individuals, we have seen that a degree of belief-deviance is both necessary and sufficient for hedging/speculative behavior in an informative situation. And conversely, differences in degree of risk-tolerance without divergence of belief will not lead to such behavior. Nevertheless, attitudes toward risk do have some effect. For, if the necessary condition of belief-deviance holds it can be shown that the scale of the compound gamble accepted (in an informative situation) will be positively associated with the degree of risk-tolerance that characterizes the individual's utility function. The formal development is straightforward and need not be expounded here.

V. LIMITATIONS AND GENERALIZATIONS

In view of the strength of the main conclusions obtained, it is of some importance to review the presuppositions of the analysis -- in order to estimate the degree to which they may constrain the general acceptability of the results. The assumptions of just two commodities and two states of the world are innocuous simplifications; everything will generalize in these respects. However, the following are not mere simplifications: (1) independence of the commodities in preference (zero complementarity); (2) concordant (homogeneous) prior beliefs constituting essentially all the weight in the market; (3) agreement on informative or non-informative situation; (4) posterior certainty; and (5) Semi-complete Markets.

The main function of the zero-complementarity assumption was to permit the replacement of the regime of fully Complete Markets by the regime of Semi-complete Markets. For, as explained in a footnote above, a crucial step in justifying this simplification was to infer from $\partial u/\partial n_a = \partial u/\partial n_b$ that $n_a = n_b$ -- a condition generally true only with independence in demand. However, it seems reasonable to conclude that since complementarity effects are normally second-order in magnitude (as against comparisons of the direct marginal utilities), the general results here would continue in substance to hold without requiring absolute independence in demand.

The assumption of concordant beliefs (except possibly for deviant individuals of negligible social weight) is, however, a very strong one. As a practical matter, one would want to interpret agreed belief as some kind of average belief (weighted by endowed wealth). Such an interpretation is not strictly permissible, as the formal analysis requires literally agreed beliefs. But there seems little ground for doubting that shifts in average beliefs, for example, would have effects on prices substantially similar to changes in agreed beliefs. On the other hand, one important difference does have to be recognized. In the formal analysis of a world of generally agreed opinion, belief-deviant individuals (the only ones engaging in hedging/speculation behavior) were assumed to be of negligible social weight. But where beliefs generally vary, essentially everyone will be deviant from the average opinion and so would tend to engage in such behavior. Hence, while there might be no clear systematic effect of relaxing the concordance assumption on the nature and direction of price changes between prior and posterior trading rounds, we would tend to expect a significantly greater volume of prior transactions than would be accounted for by the theory above. (On the other hand, transaction costs would tend to counteract this effect.)

What if, in a world of concordant beliefs as to the underlying state-probabilities, there were disagreements as to whether the situation was informative or not? All individuals who regard the situation as non-informative would of course be attempting to move to their simple consumptive optimum positions C^* in the initial round, rather than to trading positions T^*. However, since [from equations (8a)] the prior prices P^o_{Za} and P^o_{Zb} in an informative situation are (respectively) the same as the P_{Za} and P_{Zb} of a non-informative situation, the compound gambles C^{**} being sought by the one concordant group are identical with the simple gambles C^* being sought by the other. So this divergence does not in any way disturb the equilibrium of prices -- prior or posterior.

Relaxing the assumption of posterior certainty raises a number of complex issues. If the anticipated injection of information were to be less than conclusive, posterior trading under either information outcome (i.e., information favoring the likelihood of one state or the other) would still have to allow for desired holdings of both state-claims. Furthermore, we could also then imagine a situation of repeated injections of information, so that there might be several layers of prior and posterior prices -- multiplying in number exponentially. Nevertheless, given concordant beliefs all these prices remain related through a generalized version of equation (8) -- as may be seen by a comparison with equation (6). Hence, while no attempt will be made to develop the generalized system here, the results obtained (as to who hedges or speculates) will remain essentially in harmony with the posterior certainty case.

Finally, as to the assumption of a regime of Semi-complete Markets, the degree to which this is an inessential simplification of a system of fully Complete Markets has already been commented on. What is much more important and interesting, however, is not consideration of a still more ample set of markets but rather comparison with a regime of substantially curtailed trading opportunities in the presence of uncertainty. As indicated initially, the most interesting alternative assumption is to go to the opposite extreme, to a regime of CERTAINTY MARKETS. Here we would be dealing with a world in which individuals were, in general, endowed with gambles over states (quantity risk) -- but in which, nevertheless, market trading was permitted only in certainty claims to commodities, regardless of whether the commodity itself is risky or riskless. In models of such a world, there are some interesting parallels yet significant divergences as to the determinants of speculative/hedging behavior. Unfortunately, space does not permit presentation and interpretation of these results here.[27]

FOOTNOTES

* Professor of Economics, UCLA. Comments and suggestions by Ronald Britto, Harold Demsetz, Jacques Drèze, and Susan Stenger are gratefully acknowledged. Particular thanks are due to Mark Rubinstein, who has cooperated with me in developing some applications of this model [1973] and to John M. Marshall, whose own work contains a number of parallels with the results here reported.

1 Keynes [1930], v. 2, Ch. 29; Hicks [1946], pp. 137-39; Houthakker [1957, 1968]; Cootner [1968], Telser [1959].

2 Compare Houthakker [1968], Rockwell [1967], and Telser [1967].

3 Cootner [1968], p. 119.

4 Friedman [1960(1969)].

5 Working [1953], p. 320.

6 Working [1962], pp. 442-43, 452-53.

7 Ibid. See also Rockwell [1967], pp. 107-10.

8 See Johnson [1960], Feldstein [1968], Houthakker [1968].

9 There are limits to speculative commitments. To take an extreme case, suppose that in the situation of Fig. 1 an individual attaches 100% probability weight to the steeper of the two conditional posterior price ratios. Then he will not be satisfied to choose an interior trading position like T. Instead, he will "plunge" -- buy as much X as the market will let him, thus moving as far as permitted in the southeast direction alomg MM'. He need not be restricted to the positive quadrant, for in a perfect market he can "sell short" commodity Y to finance more purchases of X. The limit is indicated by the position L in Fig. 1. The construction is based on everyone agreeing as to what the posterior conditional prices are, the disagreement being only over the respective probability weights. The plunger then

necessarily has to borrow the units of Y to sell short from someone with different probability beliefs (i.e., someone attaching positive probability to a <u>rise</u> in the price of Y). For, anyone with the same beliefs would be a plunger himself. But beyond the point L on MM' the plunger would be bankrupted in the event that the unfavorable price shift occurred -- he would have negative wealth and thus be unable to repay all the units of Y borrowed. On the basis of these considerations, in this paper we will permit individuals to move to trading positions involving negative holdings of one or more specific commodities, but not to trading positions in which conditional posterior net wealth becomes negative in any state of the world recognized as possible in the market.

10 The possibility of constructing such a function on the basis of the Neumann-Morgenstern postulates of rational choice was shown in Friedman and Savage [1948(1952)], pp. 74-76.

11 In contrast with Johnson [1960] and Houthakker [1968], the <u>state-preference model</u> of uncertainty is adopted here; the ultimate objects of choice are conditional claims defined both as to commodity and state of the world. This model was first proposed by Arrow [1953(1964)] and has been developed and amplified by other authors including Debreu [1959], Borch [1962], Hirshleifer [1965,1966], Radner [1968], and Drèze [1970-71].

12 For an illustration in an intertemporal context (where the two commodities are "present consumption" and "future consumption"), see Drèze and Modigliani [1966] or Hirshleifer [1970], pp. 236-40.

13 The entities exchanged in markets are usually thought of as quantities of commodities certain. But in the securities markets, for example, there exist types of financial instruments -- risk-graded bonds, preferreds,

common shares, warrants, etc. -- that permit investors to take portfolio positions reflecting, as desired, greater or lesser sensitivity of income to more and less prosperous states of the world. In the marketing of physical commodities, also, a farmer might arrange a crop-sharing contract instead of promising to deliver fixed quantities. Percent-of-sales or even more complex contingent arrangements are not uncommon in store rentals and other lease contracts. And, of course, all insurance represents the exchange of contingent claims. So, while a complete set of conditional state-claim markets does not exist in the "real world", neither does the opposite extreme hold true that only certainty claims can be traded.

14 The implications of the regime of Certainty Markets are considered in a forthcoming paper.

15 Holding constant individuals' utility functions and their state-distributed physical endowments of the two commodities.

16 See, for example, Lintner [1969] who develops such average measures in a mean vs. variance model of uncertainty.

17 The fact that the N-elements of the two consumptive optimum gambles are the same is not a general result, but follows here as a consequence of the particular logarithmic utility function employed.

18 The concordant or "homogeneous" beliefs assumption has been found to be a crucially strategic simplification in other areas such as theoretical and empirical studies of security price behavior -- see, for example, Sharpe [1964,1965].

19 We can now see that, insofar as choice of the simple consumptive optimum C^* is concerned, Semi-complete Markets are as satisfactory as a regime of fully Complete Markets in a world of individuals with agreed beliefs. Under

Complete Markets the optimization problem would take the form:

$$\text{Max } U = p(n_a, z_a) + (1-p)u(n_b, z_b) \text{ subject to}$$

$$P_{Na}n_a + P_{Nb}n_b + P_{Za}z_a + P_{Zb}z_b = W^e$$

The usual procedure leads to the following condition (among others):

$$\frac{1-p}{p}\,\frac{\partial u/\partial n_b}{\partial u/\partial n_a} = \frac{P_{Nb}}{P_{Na}}$$

For concordant individuals, $p = \pi$. Now if the price ratio P_{Nb}/P_{Na} were to exceed the probability ratio $(1-\pi)/\pi$, all individuals would attempt to make $\partial n/\partial n_b > \partial u/\partial n_a$, or $n_b < n_a$ (in the absence of complementarity in utility). But the aggregate $\Sigma n_a = \Sigma n_b$, so this is impossible. Hence individuals of agreed beliefs, with endowments and consumptive optima both characterized by $n_a = n_b$, have no need to trade separately in conditional N-claims. Belief-deviant individuals would, on the other hand, have a use for markets in conditional N-claims even though commodity N is riskless in the aggregate.

20 In the notation here, $\bar{P}'_{Zb}$ would be the conditional price of state-b claims given the first information outcome (that state-a will obtain). Evidently, $\bar{P}'_{Zb} = 0 = \bar{P}''_{Za}$, so we will not need to use these symbols.

21 Radner [1968] has emphasized the essentially uncomputable nature of these posterior prices on the basis of the information available at prior dates. The results here are not in conflict, but show that the posterior prices are computable in one important special case.

22 We have not proved uniqueness of this equilibrium. (Indeed, even in riskless pure exchange, uniqueness of the equilibrium price vector cannot generally be proved.) But note that with, for example, $P^o_{Za} < P_{Za}$ there would be unbalanced substitution effects tending to raise the price of Z_a claims again.

23 A similar device is employed in Drèze [1970-71], p. 138.

24 Given the first information outcome (state-a certain), the posterior optimization problem involves the standard tangency condition $\left.\frac{-dn'}{dz'_a}\right|_{u'} \equiv n'/z'_a = \bar{P}'_{Za}$ or, in condensed notation, $\bar{P}'$. The second condition is the budget constraint $n' + \bar{P}'z'_a = W'$. Since $n' = \bar{P}'z'_a = W'/2$, then $u' = \log_e n' + \log_e z'_a = \log_e W'/2 + \log_e W'/2 - \log_e \bar{P}'$. So $\frac{du'}{dW'} = \frac{2}{W'}$. By an analogous development, $\frac{du''}{dW''} = \frac{2}{W''}$.

25 Since u' and u" are nothing but the original u now expressed as functions of the posterior wealth and price parameters. Then du'/dW', for example, is the derivative of u' holding posterior price $\bar{P}_Z = \bar{P}'_{Za}$. The conditional posterior optimization problem that leads to the condition $\lambda' = du/dW'$ also holds $\bar{P}_Z$ constant at $\bar{P}'_{Za}$, hence the derivatives denoted du/dW' and du'/dW' are identical. A similar argument holds of course for du/dW" and du"/dW".

26 The following is an alternative formal development of this result, that achieves greater compactness (with some loss of intuitive appeal) by omitting the intermediate decision variables W', W". Concordant individuals are postulated. (This development is mainly due to Mark Rubinstein.)

A. Simple consumptive gamble

$$\underset{(n,z_a,z_b)}{\text{Max}} \quad \pi u(n,z_a) + (1-\pi)u(n,z_b) - \lambda(n + P_{Za}z_a + P_{Zb}z_b - W^e)$$

First-order conditions: $\frac{\partial u}{\partial n} = \lambda$, $\pi \frac{\partial u}{\partial z_a} = \lambda P_{Za}$, $(1-\pi)\frac{\partial u}{\partial z_b} = \lambda P_{Zb}$

Optimality conditions: $\frac{\partial u/\partial z_a}{\partial u/\partial n} = \frac{P_{Za}}{\pi}$, $\frac{\partial u/\partial z_b}{\partial u/\partial n} = \frac{P_{Zb}}{1-\pi}$

B. Compound consumptive gamble

$$\underset{\substack{(n^t, z_a^t, z_b^t,\\ z_a', z_b'',\\ n', n'')}}{\text{Max}} \quad \pi u(n', z_a') + (1-\pi)u(n'', z_b'') \\ -\pi\lambda'(n' + \bar{P}_{Za}' z_a' - n^t - \bar{P}_{Za}' z_a^t) \\ -(1-\pi)\lambda''(n'' + \bar{P}_{Zb}'' z_b'' - n^t - \bar{P}_{Zb}'' z_b^t) \\ -\lambda^o(n^t + P_{Za}^o z_a^t + P_{Zb}^o z_b^t - W^e)$$

First-order conditions: $\pi\lambda'\bar{P}_{Za}' = \lambda^o P_{Za}^o \qquad (1-\pi)\lambda''\bar{P}_{Zb}'' = \lambda^o P_{Zb}^o$

$$\partial u/\partial z_a' = \lambda'\bar{P}_{Za}' \qquad \partial u/\partial z_b'' = \lambda''\bar{P}_{Zb}''$$

$$\partial u/\partial n' = \lambda' \qquad \partial u/\partial n'' = \lambda''$$

$$\pi\lambda' + (1-\pi)\lambda'' = \lambda^o$$

Optimality conditions: $\dfrac{\partial u/\partial z_a'}{\partial u/\partial n'} = \bar{P}_{Za}' \qquad \dfrac{\partial u/\partial z_b''}{\partial u/\partial n''} = \bar{P}_{Zb}''$

It will be evident that, if $\bar{P}_{Za}' = \dfrac{P_{Za}}{\pi}$ and $\bar{P}_{Zb}'' = \dfrac{P_{Zb}}{1-\pi}$, the conditions under B are identical with those under A, with $n' = n''$. Hence the same solution is an equilibrium for both. Furthermore, $n' = n''$ leads immediately to $\lambda' = \lambda'' = \lambda^o$, from which $P_{Za}^o = P_{Za}$ and $P_{Zb}^o = P_{Zb}$ follows directly.

27 A preliminary discussion appears in Hirshleifer [1972], Secs. IV and V.

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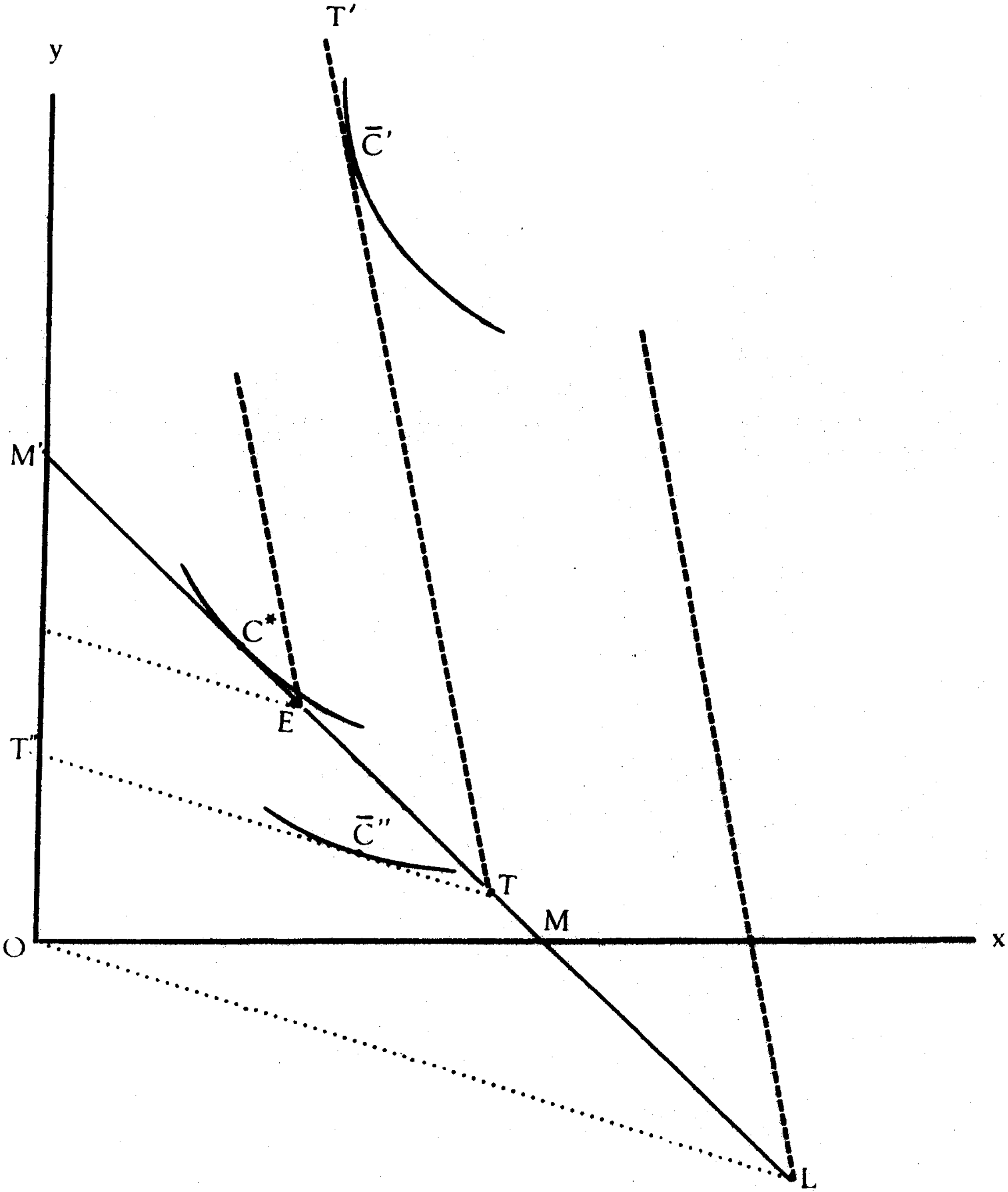

Fig. 1

Simple consumptive plan vs. trading position, pure exchange--price risk only.

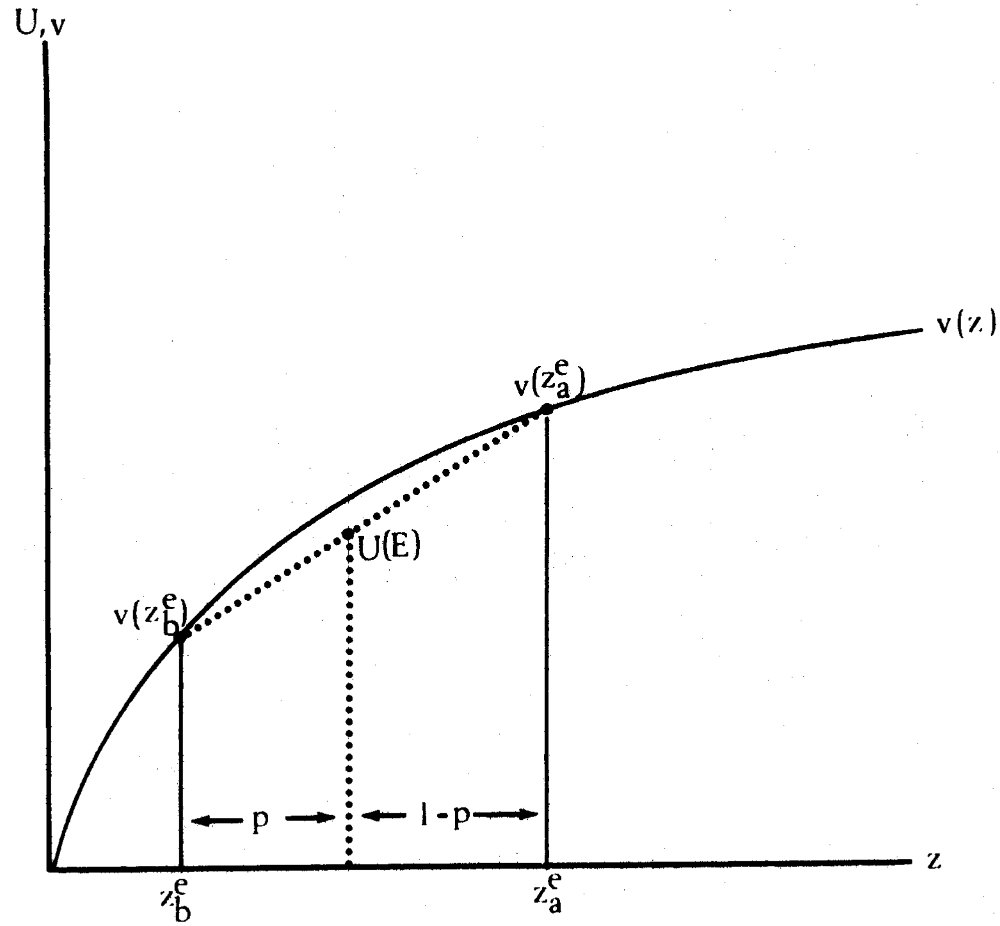

Fig. 2

A representation of state-choice: quantity risk only.

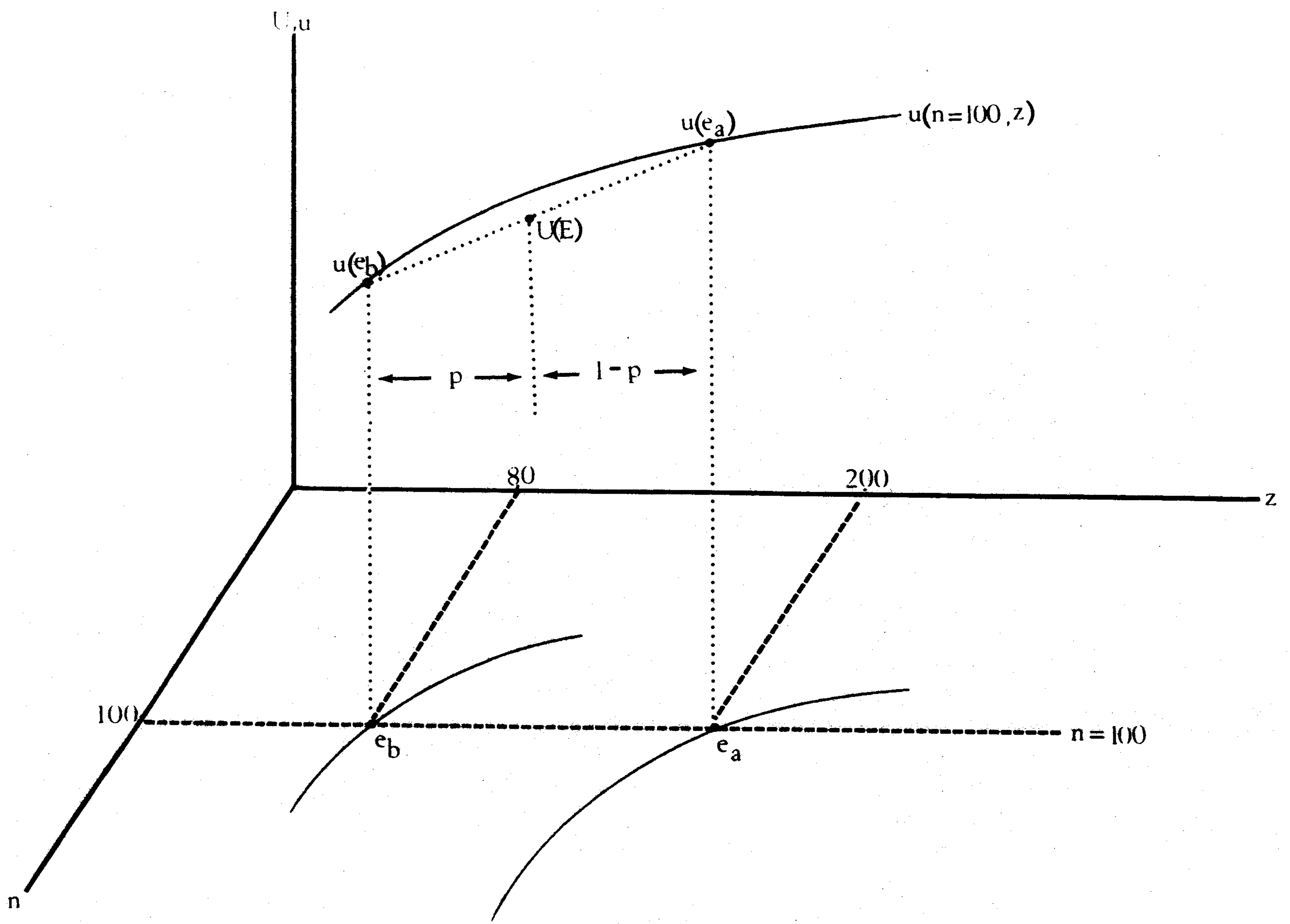

Fig.3

Utility level of endowment position, with two commodities.

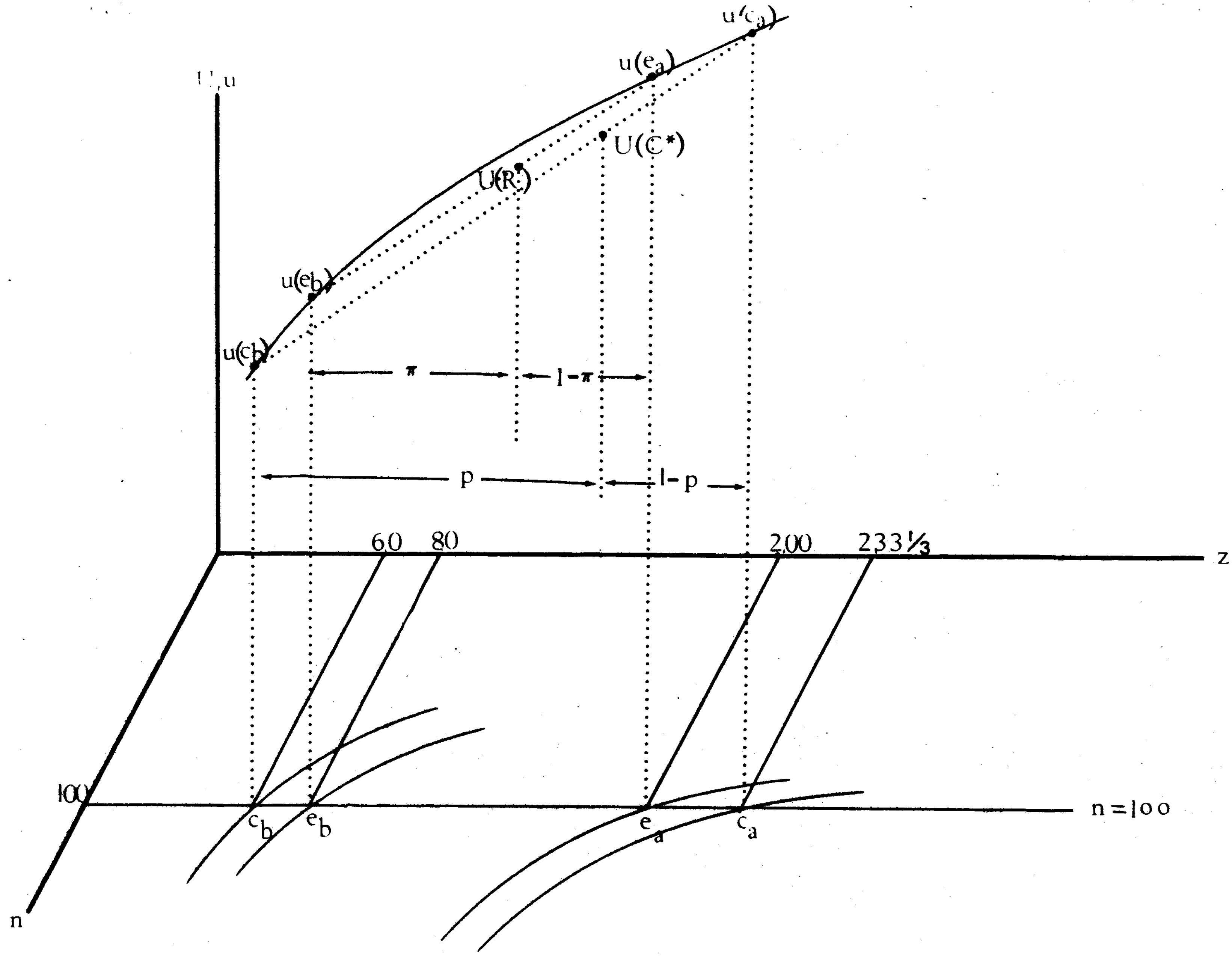

Fig. 4
Simple consumptive solutions--representative individual vs. belief-deviant individual

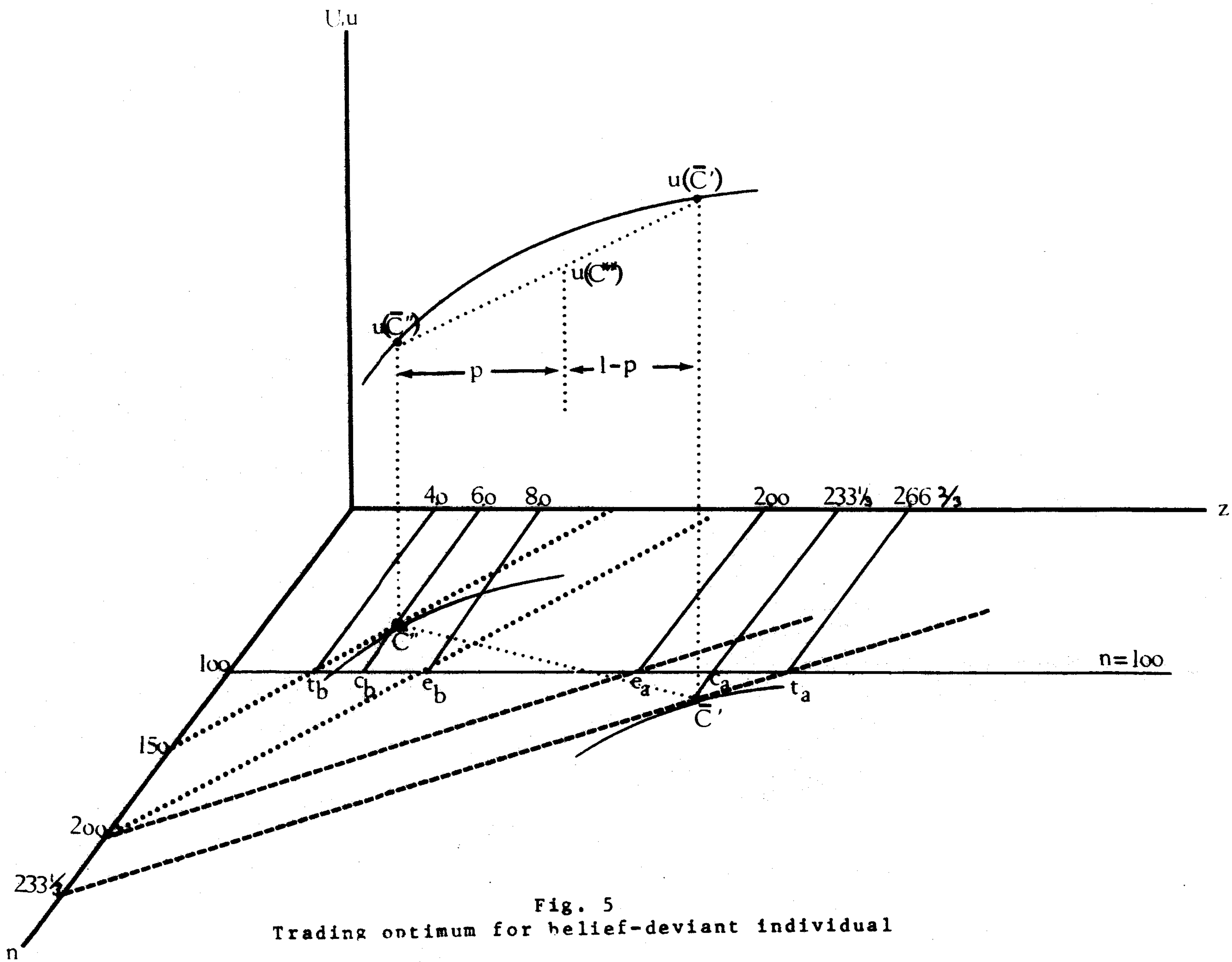

Fig. 5
Trading optimum for belief-deviant individual