DO THE RICH GET RICHER AND THE POOR POORER? EXPERIMENTAL TESTS OF A MODEL OF POWER

Yvonne Durham
University of Arkansas

Jack Hirshleifer
University of California at Los Angeles

Vernon L. Smith
University of Arizona

Working Paper #770
Department of Economics
University of California, Los Angeles
Bunche 2263
Los Angeles, CA 90095-1477
May 1997

* To appear in American Economic Review. This Working Paper revises and supersedes Working Paper #741A, "Experimental Tests of the Paradox of Power" (December 1996).

DO THE RICH GET RICHER AND THE POOR POORER? EXPERIMENTAL TESTS OF A MODEL OF POWER

Abstract

Individuals, groups, or nations -- if rational and self-interested -- will equalize the marginal returns of two main ways of generating income: (1) production combined with mutually advantageous exchange, versus (2) political or military distributive struggles. In such conflicts it might be expected that initially stronger or richer contenders would grow ever stronger and richer still, but the reverse often occurs (the 'Paradox of Power'). The experiments reported here confirm the theoretical prediction that the Paradox of Power holds when m, the <u>decisiveness</u> of fighting effort, is low but not when m is high. In addition, subjects preponderantly arrived at the Nash rather than the cooperative solution -- though with some slippage toward cooperation in experimental treatments that fostered 'learning to cooperate'.

DO THE RICH GET RICHER AND THE POOR POORER? EXPERIMENTAL TESTS OF A MODEL OF POWER

By YVONNE DURHAM, JACK HIRSHLEIFER, AND VERNON L. SMITH*

In a market economy, there is no clear implication as to whether economic activities will tend to reduce or else to widen initial wealth disparities. When it comes to political or military struggles, in contrast, it might be expected that initially stronger or richer contenders would grow ever stronger and richer still. What has been termed "the Paradox of Power" (Jack Hirshleifer 1991) is the observation that very often the reverse occurs: poorer or weaker contestants improve their position relative to richer or stronger opponents. In warfare, small nations have often defeated larger ones, as notably occurred in Vietnam. Or consider political clashes over income redistribution. Although citizens in the upper half of the income spectrum surely have more political strength than those in the lower half, modern governments have systematically been transferring income from the former (stronger) to the latter (weaker) group.[1]

Individuals, groups, or nations -- if rational and self-interested -- will equalize the marginal returns of two main ways of generating income: (1) production and mutually advantageous exchange, versus (2) 'appropriative' efforts designed to redistribute income or capture resources previously controlled by other parties (or to defend against the latter's attempts to do the same). Management and labor jointly generate the aggregate output of the firm, for example, yet at the same time contend with one another over distribution of the proceeds.

As for the 'Paradox of Power', the theoretical explanation is that initially weaker or poorer contenders are typically motivated <u>to fight harder</u>, that is, to devote relatively more effort to appropriative (conflictual) effort. Put another way, the marginal payoff of appropriative effort relative to productive effort is typically greater at low levels of income. (When agricultural prices fell to extraordinarily low levels in the great Depression of 1929-33, Kansas farmers were urged by their leaders to "raise less corn and raise more hell".) Looking at it the other way, while the rich may have the capability of exploiting the poor, it might not pay them to do so.

Nevertheless, in some social contexts, initially richer and more powerful contestants do exploit weaker rivals. Affluent aristocracies often use their power to extort even more resources from the lower classes. So the question is, when does and when does not the Paradox of Power hold? In the model, the governing factor is a parameter m reflecting the decisiveness of conflictual effort. When decisiveness is low, the rich are content to concentrate upon producing a larger social pie of income even though the poor will be gaining an improved share thereof. But when conflictual preponderance makes a sufficiently weighty difference for achieved income -- at the extreme, when the battle is "winner take all" -- the rich cannot afford to let the poor win the contest over distributive shares.

The balance between production and struggle, as two ways of making a living, has been examined in a number of theoretical studies, among them Trygve Haavelmo (1954), Goran Skogh and Charles Stuart (1982), Jack Hirshleifer (1989, 1991), Stergios Skaperdas (1992), and Herschel I. Grossman and Minseong Kim (1994). But how decision-makers choose between productive and conflictual activities has not heretofore, so far as we could determine, been addressed experimentally. That was the first object of the study reported in this paper.

A second aim was to consider the degree to which subjects ended up at the theoretical non-cooperative Nash solution, as opposed to more cooperative outcomes generating larger income for the group as a whole. In the experimental literature the extent of cooperation has been found to be sensitive to, among other things, the number of iterations of the game and whether partners are held fixed or else varied from round to round. Our experimental investigation was designed to address these questions as well.

Section I below outlines the analytic model. Section II explains our implementation of tests of the model. Section III describes the experimental procedures and outcomes. Section IV discusses the results and summarizes.

I. The Model

Each of two contenders $i = 1, 2$ must divide his/her exogenously given resource endowment R_i between productive effort E_i and appropriative ('fighting') effort F_i:

$$(1) \qquad \begin{aligned} E_1 + F_1 &= R_1 \\ E_2 + F_2 &= R_2 \end{aligned}$$

The E_i efforts are inputs to a joint production function. A convenient form for this function, characterized by constant returns to scale and constant elasticity of substitution, is:

$$(2) \qquad I = A(E_1^{1/s} + E_2^{1/s})^s$$

where A is an index of total productivity and s is an index of complementarity between E_1 and E_2.[2] However, for utmost simplicity here we have assumed $A = s = 1$, so that (2) reduces to the simple additive equation:

$$(2a) \qquad I = E_1 + E_2$$

Thus, the parties can cooperate by combining their productive efforts so as to generate a common pool of income available to the two of them jointly. But the respective shares p_1 and p_2 (where $p_1 + p_2 = 1$) are determined in a conflictual process. In particular, the Contest Success Function (CSF) takes the fighting efforts F_i as inputs, yielding the distributive shares as outputs:

$$(3) \qquad \begin{aligned} p_1 &= F_1^m/(F_1^m + F_2^m) \\ p_2 &= F_2^m/(F_1^m + F_2^m) \end{aligned}$$

Here m is a 'decisiveness parameter' controlling the mapping of the input ratio F_1/F_2 into the success ratio p_1/p_2. For $m \leq 1$ the CSF is characterized by diminishing marginal returns as F_1 increases with given F_2, or vice versa. However, for $m > 1$ there will be an initial range of increasing returns before diminishing marginal returns set in.[3] [4]

As a simplifying assumption, we postulate that conflict is non-destructive, i.e., there is no "battle damage". Choosing fighting activity over productive activity involves some opportunity loss of potential output, but the struggle does not itself damage the resource base or otherwise reduce the aggregate of income attainable.

Finally, the incomes accruing to the contestants are:

$$
\begin{aligned}
I_1 &= p_1 I \\
I_2 &= p_2 I
\end{aligned}
\tag{4}
$$

For each level of fighting effort by contender 2, there is a corresponding optimal effort for contender 1 (and vice versa). Thus, 1's optimization problem is to choose $F_1 \geq 0$ so as to solve:

$$\text{Max } I_1 = p_1(F_1|F_2) \times I(E_1|E_2) \text{ subject to } E_1 + F_1 = R_1$$

and similarly for side 2.

Assuming neither party's resource constraint is binding, and using the simplified production function (2a), the Nash-Cournot reaction functions are:

$$
\begin{aligned}
\frac{F_1}{F_2^m} &= \frac{m(E_1 + E_2)}{F_1^m + F_2^m} \\
\frac{F_2}{F_1^m} &= \frac{m(E_1 + E_2)}{F_1^m + F_2^m}
\end{aligned}
\tag{5}
$$

The right-hand sides being identical, $F_1 = F_2$ is always a solution of these equations. That is, the reaction curves intersect along the 45° line between the F_1 and F_2 axes. In fact, this is the sole intersection in the positive quadrant.

If however the boundary constraint is binding for the poorer side (which we always take to be contender 2), the second equation would be replaced by:

(5a) $$F_2 = R_2$$

In that case, at equilibrium F_1 and F_2 are in general unequal, but the intersection of the reaction functions still determines the Nash-Cournot equilibrium values of the fighting efforts.

As indicated above, the experiments were intended in part to challenge a number of specific predictions derived from the model. In particular:

(i) Fighting Intensities: As the decisiveness parameter m exogenously increases, it pays both sides to 'fight harder', i.e., the equilibrium fighting efforts F_i will rise. (Implying, of course, that the ultimate achieved incomes I_i must fall.)

(ii) Conflict as an Equalizing Process (Paradox of Power), Strong vs. Weak Form: For sufficiently low values of the decisiveness parameter m, disparities in achieved income will -- owing to the 'Paradox of Power' -- be smaller than the initial disparities in resource endowments. Letting contender 1 be the initially better endowed side:

(6) $$R_1/R_2 > I_1/I_2 \geq 1$$

When the equality on the right holds (i.e., when the achieved incomes of the initially richer and initially poorer sides end up exactly equal) we have the 'strong form' of the Paradox of Power. As already noted, for any interior solution (that is, when the poorer side does not run into its resource constraint) we must have $F_1 = F_2$, so that the strong form of the paradox necessarily applies.[5] It can be shown that there will be interior solutions up to some critical value ρ of the resource ratio:

(7) $$\rho = (2 + m)/m$$

Thus specifically, in our experiments employing the low value $m = 1$ for the decisiveness parameter, the prediction is that the strong form of the POP will hold for low resource ratios, specifically for $R_1/R_2 \leq 3$. For resource ratios larger than $\rho = 3$, only the weak form, i.e., the strict inequality on the right of equation (6), is predicted.

(iii) Conflict as an Inequality-Aggravating Process: The model also indicates that for sufficiently high values of the decisiveness coefficient m and the resource ratio R_1/R_2, the Paradox of Power will not apply. The rich would get richer and the poor poorer. Specifically, for our experiments using the high decisiveness coefficient $m = 4$, the critical value τ of the resource ratio for this condition is approximately 2.18.[6] Also, from (7), when $m = 4$ the critical ρ separating the weak from the strong forms of the Paradox of Power equals 1.5. Thus in our experiments using the low resource ratio $25/15 = 1.67$ we expect the weak form of the Paradox of Power to hold, since 1.67 lies between ρ and τ. However, for the experiments with $R_1/R_2 = 32/8 = 4 > \tau = 2.18$, the prediction is that the initially better endowed party will improve its relative position compared to the less well endowed side:

(8) $$I_1/I_2 = (F_1/F_2)^m > R_1/R_2$$

II. Implementing Tests of the Model

Certain game-theoretic and implementational concerns are also addressed in our experimental test of the above model. In the strict game-theoretic sense, the noncooperative equilibrium is about strangers who meet once, interact strategically in their self-interest, and will never meet again. Such conditions control for repeated-game effects, since the antagonists have no history or future. Yet in many contexts individuals interact in repeated games, where they can signal, punish, and build reputations. In the particularly simple version where the one-shot game is iterated with the same payoffs each round, we have a supergame. The study of such games has been motivated by the intuition or "folk theorem" that repetition makes cooperation possible (*** Mertens, 1984). But formal theorems to this effect for finite horizons have not been forthcoming, and interest has settled on experimental studies of both single-play games and supergames, and on variations in the protocol for matching players in repeat play.

Kevin McCabe, Stephen Rassenti, and Vernon L. Smith (1996) studied a class of extensive-form games in which the parties move sequentially in a series of rounds. In any round the first-mover can forward signal the desire to cooperate, but the other player can defect. In one game the first player can punish such defections. In the other he/she has no such recourse. If pairs are matched at random for each play, in a repeated sequence of unknown length, subjects gradually learn to cooperate when the punishment option is available; when this option is not available they tend to play non-cooperatively. If instead the same pairs remain matched for the entire length of the supergame, they tend to achieve cooperation whether or not the opportunity for direct punishment or defection is available.

Consequently, in addition to testing the substantive predictions associated with the Paradox of Power, we will be addressing some of these issues that have arisen in the experimental and game-theoretic traditions. Specifically, we will be comparing the results of experiments in which the partners are randomly varied in each round with experiments in which the partners are fixed throughout the supergame. As suggested by the preceding discussion, we anticipate that the condition of fixed partners will favor somewhat more cooperative behavior. However, we will be implementing a normal form game (simultaneous choice of strategies presented in a payoff matrix each round) rather than an extensive form game (sequential choice by the players each round). McCabe and Smith (1996) show that the extensive form favors cooperation relative to the normal form of two theoretically "equivalent" games. This is because cooperative intentions can be signalled by one player and the second player can reciprocate (not defect) within the same round. Hence, the normal form of the experiments reported below is expected to make cooperation (i.e., reduced levels of fighting) difficult even in repeat interactions.

III. Experimental Procedures and Outcomes

A. Experimental Design

We conducted 24 experiments using a total of 278 subjects. No subject participated in more than one experiment. There were 6 bargaining pairs in each experiment, except for a few cases with only 4 or 5 pairs. Each experiment involved repeated play, the payoffs being constant in each round. Within each round, each subject pair chose simultaneously a (row, column) in a matrix displaying the payoffs of each. Subjects were not informed how many rounds would take place; in fact, in each experiment there were 16 or 17 rounds before termination. Subjects were recruited for two-hour sessions but the experiments took much less time, making credible the condition of an unknown horizon.[7]

In every round each subject allocated his/her initial endowment of tokens between an "Investment Account" (IA) and a "Rationing Account" (RA). (We deliberately avoided using any terminology suggestive of "fighting".)[8] Tokens contributed to the IA corresponded to productive effort E_i in the theoretical model: the paired IA contributions generated an aggregate pool of income (in the form of 'experimental pesos') in accordance with equation (2a) above. Funds put into the RA corresponded to fighting effort F_i and determined the respective distributive shares p_1 and p_2 in accordance with equations (3). For simplicity, only integer choices were permitted. (More precisely, each subject could allocate, within his/her resource constraint, amounts in integral hundreds of tokens to invest in the IA, the remainder, of course, going into the RA.) The totals of pesos ultimately achieved were converted into actual dollars at the end of the experiment, so subjects had a substantial motivation to make self-interested choices. (The payoffs ranged from $.25 to $75.25, not including the $5 show-up fee. The average payoff was $17.66.)

To challenge the implications of the model, we manipulated the resource endowments R_1 and R_2 and also the decisiveness coefficient m. Four experiments were run with each of the three endowment vectors $(R_1,R_2) = (20,20)$, $(25,15)$, and $(32,8)$ -- the first series using a low value $m = 1$ of the decisiveness parameter, and the next using a high value $m = 4$. Thus there were 24 experiments in all.

Also, in view of the McCabe, Rassenti, and Smith (1996) result that cooperation is promoted by repeated play with the same partners, each group of four experiments was further subdivided into alternative matching protocols. In the first ('varying partners') protocol, partners were randomly changed each round. Under the second ('fixed partners') protocol, subjects were randomly paired at the beginning of the experiment but played repeatedly with the same partner throughout.

Overall there were eight experiments under each of the three endowment conditions. Four of the eight involved varying partners, and four fixed partners. There was an analogous subdivision between experiments conducted using $m = 1$ and using $m = 4$. The upshot is that there were exactly two experiments for each of the 12 sets of experimental conditions or "treatments". The treatment design is summarized in Table 1.

B. Results -- Nash versus Cooperative Solutions

The theoretical model described in the previous section derived the Nash-Cournot noncooperative equilibrium. However, the experimental literature has intensively investigated conditions under which subjects might arrive at a more cooperative outcome. This is the first issue addressed:

H_o: the null hypothesis is that $(F_1,F_2) = (C_1,C_2)$

H_a: the alternative hypothesis is that $(F_1,F_2) = (N_1,N_2)$

Here the N_i signify the respective fighting efforts F_i under the Nash solution, while the C_i are the fighting efforts under the cooperative solution.

The theoretical Nash solution is generated by the intersection of the reaction functions of equations (5) above for an interior outcome -- or in the case of a boundary outcome (where the poorer contender 2's resource constraint is binding), substituting (5a) for the second equation (5). The cooperative solution was defined as $(C_1,C_2) = (1,1)$. That is, each side devotes the minimal allowable positive amount to fighting effort.[9] Table 2 is a summary that allows a comparison between the Nash equilibrium and the average of the 6 (or, in a few cases, 4 or 5) observations on round 16 in each of the 24 experiments. Note that almost all the contesting pair's choices are much closer to the Nash prediction than the cooperative (1, 1), but are biased on the low side of Nash.

As an illustration, Figure 1 depicts the first two of the 24 experiments. Each * symbol plots the fighting efforts F_1 and F_2 chosen by one of the 12 bargaining pairs in the sixteenth (the last or next-to-last) round. Also shown are the reaction (step) functions, the computed Nash noncooperative equilibrium at the intersection of these functions, and the postulated Cooperative solution at (1,1). At a glance, the observations tend to fall between the Nash and the Cooperative solutions, but much nearer to the former. This was in fact typical; error deviations from Nash tended to the Cooperative side.

We used the likelihood ratio to test the alternative Nash hypothesis H_a against the cooperative null hypothesis H_o.[10] It was assumed that the observations for the 'fighting efforts' F_i are normally distributed, with mean $\mu = C$ under H_o or mean $\mu = N$ under H_a, and variance $V = S^2$ (the sample variance).[11] Then for any given treatment the likelihood ratio is:

(9a) $$\lambda = \frac{\exp\left[-\frac{1}{2S^2}\sum_{t,i}(F_{it}-C)^2\right]}{\exp\left[-\frac{1}{2S^2}\sum_{t,i}(F_{it}-N)^2\right]} = \exp\left\{\frac{1}{2S^2}\left[2\sum_{t,i}(F_{it}(C-N) + Tn(N^2-C^2)\right]\right\}$$

(9b) $$\ln\lambda = Tn\bar{F}(C-N)/S^2 + (N^2-C^2)(Tn/2S^2)$$

Here the t subscript indexes the rounds from 1 to T, while the i subscript indexes the individual pair observations from 1 to n.

A $\lambda < 1$ would indicate that, for this particular treatment, the observed choices had a higher probability of occurring under the alternative (Nash) hypothesis than under the null (cooperative) hypothesis. The twelve rows of Table 3 list the λ's for all the treatments, expressed for convenience in terms of logs (the log likelihood ratios) as in (9b). Positive entries in the table represent results favoring the null hypothesis while negative entries favor the alternative hypothesis.

The columns toward the left of Table 3 identify the conditions for each of the 12 treatments. The remaining columns show the results for "All Rounds" and also for the "Sixteenth Round" (that is, the last or next-to-last round) separately. (For the "Sixteenth Round" columns, equations (9a) and (9b) are modified by simply dropping the indexing over t.) From the statistical point of view, the "All Rounds" reports provide a larger sample size (though not independent) and thus are less influenced by random fluctuations. On the other hand, the "Sixteenth Round" reports are more likely to isolate the mature behavior of the experimental subjects. Finally, F_1 refers to the subject having the larger, and F_2 the smaller resource endowment. (In the equal-endowment cases, the assignment of F_1 versus F_2 was random.)

The results summarized in Tables 2 and 3 overwhelmingly support the Nash as opposed to the Cooperative solution. In Table 3 the predominantly negative values of the log likelihood ratios (46 of the 48 tabulated entries) correspond of course to likelihood ratios less than unity in equation (9a) above.[12] Using a likelihood ratio test, 45 of the 48 entries in Table 3 unambiguously -- at significance level $\alpha = .005$ -- imply rejecting the Cooperative hypothesis in favor of Nash. Only 1 of the 48 (the single entry marked *) unambiguously does the reverse. For the remaining two entries marked §, significance testing using $\alpha = .001$ indicates that whichever hypothesis is taken to be the null is rejected in favor of the other![13]

Apart from the generally negative signs for the log λ's, two features of Table 3 stand out.

First, in all 24 possible comparisons the log likelihood ratios for the "16th Round" columns are somewhat less negative than in the corresponding "All Rounds" columns. This is in part the consequence of smaller sample size, but that is evidently not the entire story -- since the only two instances of positive values both fall under the "16th Round" headings. So, for these cases there is the suggestion that participants were "learning to cooperate" by the 16th round of interaction. Second, again for all 24 comparisons, the results under the 'fixed partners' (F) condition are noticeably less negative than the corresponding 'varying partners' (V) results. Since the 'fixed partners' condition facilitates the development of mutual understanding, we examine the dynamics of their interaction below for evidence of best-response moves.

It is particularly significant that the only exceptions to the observed tendency to converge to near the Nash equilibrium occur under the treatment in which the Nash equilibrium lies at the boundary of the constraint set for one of the bargainers. As shown in Vernon L. Smith and James M. Walker (1993), in such cases any deviation or slippage from the predicted outcome is necessarily biased, and changes in variance will change the mean. We should also note that, with fixed partners, if bargainers deviate from Nash, either to signal cooperation or to punish failures to reciprocate, a bargainer whose Nash outcome is on a boundary can signal cooperation without constraint, but punishment is asymmetrically restricted.

We can quantify the average percent deviations in the direction of cooperation by defining "slippage fractions" S_1 and S_2:

$$S_i \equiv \frac{N_i - F_i}{N_i - C_i} \tag{10}$$

In Table 4, a positive number in the two right-hand columns indicates slippage in the direction of cooperation. A negative number indicates slippage in the direction of conflict beyond that called for by the Nash solution. As expected, the positive numbers far outweigh the negative numbers, and the positive numbers predominate more under the 'fixed partners' (F) condition. Finally, there is a noticeable <u>positive correlation</u> between the S_1 and S_2 numbers on each row of the Table: when one subject behaves cooperatively, his/her partner is likely to do so as well. Once again, as expected, this positive correlation holds particularly for the 'fixed partners' condition. And in addition, it holds noticeably more strongly for the cases with equal resource endowments $(R_1,R_2) = (20,20)$.

C. Dynamics of Interactions for Fixed Partners

A fuller treatment of how individual pairs interact requires analysis of their interactive choices over time. If each player in period t chooses a profit-maximizing strategy based on the other player's choice in period t-1 (a shortsighted best-reply strategy) then they will converge to Nash. In fact subjects' choices may have some inertia, and may involve cooperative signalling. One way of modelling these dynamic interactions, and obtaining a measure of the propensity to choose best replies, is to estimate the following equation for F_{it} (the fighting effort chosen by subject i in period t):

$$(11) \qquad F_{it} = (1-\delta_i)F_{it-1} + \delta_i F_{it}^* + \varepsilon_{it} = F_{it}^* + (1-\delta_i)(F_{it-1} - F_{it}^*) + \varepsilon_{it}$$

where F_{it}^* is the best reply in period t to i's choice of strategy in period t-1. In this myopic Cournot dynamic, the deterministic component of F_{it} is distributed between an adaptive weight element δ related to i's current best reply F_{it}^* (to the opponent's previous-round choice) and an inertial element $1-\delta$ related to i's previous choice F_{it-1}. Or, in the second form of (11) the choice of F_{it} can be interpreted as a best reply, plus an imperfect adaptive adjustment based on the error difference between last period's choice F_{it-1} and this period's best reply. Figure 2 provides a histogram of the frequency distribution of 140 individual estimates of δ_i over all decision trials for each i. Overwhelmingly, the individual δ_i (and therefore the $1-\delta_i$) values are in the unit interval indicating some weight being given to i's previous choice and some to i's best reply. They are also overwhelmingly significantly different from zero. Note that over half the subjects exhibit values of $(1-\delta_i)$ of at least 0.5, indicating considerable weight being attached to correcting the error deviation $F_{it-1}-F_{it}^*$. For the fixed pairs condition the data substantially support the adaptive best-response Cournot dynamic.

D. Results -- The Paradox of Power

The experiments tested a number of specific predictions of the analytic model.

Prediction 1 Higher values of the decisiveness parameter m will lead to larger fighting efforts on both sides.

So the fighting efforts F_1 and F_2 should both be greater at the higher decisiveness level $m = 4$ than at $m = 1$. The upper half of Table 2 shows the results for $m = 1$, and the lower half for $m = 4$. There are 48 comparisons, of which a remarkable 45 are in the direction predicted.

Prediction 2a At the low value $m = 1$ for the decisiveness parameter, the initially poorer side will always end up improving its position.

At $m = 1$, the attained income ratio I_1/I_2 (which for $m = 1$ simply equals the ratio of fighting efforts F_1/F_2) should exceed the resource ratio R_1/R_2. The requirement of unequal initial endowments limits the relevant data to rows 5 through 12 of Table 2. Here all 8 of the 8 comparisons showed the predicted relative improvement -- that is, $I_1/I_2 < R_1/R_2$ -- and almost always by quite a wide margin.

Prediction 2b For $m = 1$ the poorer side should attain approximate equality of income (strong form of the POP) for initial resource ratios $R_1/R_2 < 3$, but only some relative improvement -- $1 < I_1/I_2 < R_1/R_2$ -- for larger resource ratios (weak form of the POP).

Looking once again only at the unequal endowments cases, rows 5 through 12 of Table 2, the average of the tabulated results is $I_1/I_2 = 1.125$, on the high side of the predicted $I_1/I_2 = 1$. By way of comparison, for rows 9 through 12 where only the weak form $I_1/I_2 > 1$ is predicted, the average outcome is $I_1/I_2 = 1.43$. So, at least relatively, the predicted comparison of the strong form versus weak form predictions is supported.

Prediction 2c At the high value $m = 4$ for the decisiveness coefficient, the Paradox of Power should continue to hold (in its weak form) for $\rho < R_1/R_2 < \tau$, where $\rho = 1.5$ and $\tau = 2.18$. But for higher resource ratios the richer side should end up actually improving on its relative position. That is, in this range $I_1/I_2 = (F_1/F_2)^4$ should exceed R_1/R_2.

For the unequal-endowments rows 17 though 20 of Table 2, the resource ratio is $R_1/R_2 = 25/15 = 1.67$, lying between ρ and τ. So the Paradox of Power is predicted in these cases. However, for rows 21 through 24 the resource ratio is $R_1/R_2 = 32/8 = 4 > 2.18 = \tau$, so we expect the rich to become richer still.

Taking up the latter group first, 3 of the 4 cases support the prediction $I_1/I_2 = (F_1/F_2)^4 > 4$. In fact, the average of the observed results was a much higher $I_1/I_2 = 12.19$. Turning to the first group, however, all 4 cases violate the prediction! Quantitatively, the predicted Nash outcome $(N_1,N_2) = (16,15)$ implies $I_1/I_2 = (16/15)^4 = 1.29 < 1.67$ while the average of the observed results was $I_1/I_2 = 2.32 > 1.67$.

IV. Discussion and Summary

This experimental investigation deals with a mixed-incentive, iterated-play, bilateral interaction. In each of some 16 rounds, paired individuals had to strike a balance between production and appropriation: more explicitly, between investing resources in joint production versus engaging in a distributive struggle over the respective shares.

We tested two main kinds of predictions:

(1) The first group dealt with issues common to much of the game-theoretic and experimental literature. Of these, the major question was the degree to which the experimental outcomes

approximated the non-cooperative Nash solution, as opposed to a more cooperative outcome generating a larger income for the group as a whole. We also compared protocols with randomly varying partners each round as opposed to fixed partners over the entire sequence of play.

(2) The second group of predictions dealt with inferences from the specific model of conflict in Hirshleifer (1991), and specifically those associated with the 'Paradox of Power'. The paradox is that, in many situations, an initially poorer side will end up gaining in relative position in comparison with an initially richer and thus stronger opponent.

With regard to the first group of predictions, the experimental observations overwhelmingly supported the Nash as opposed to the cooperative solution. However, while the Nash solution is much better supported in a dichotomous comparison between the two, the experimental results typically displayed some degree of slippage in the direction of cooperation. The convergence toward Nash was weaker under the fixed-partners as opposed to the varying-partners protocol, and also was weaker in the mature (16th round) choices than the overall behavior. Together with an observed tendency toward positive correlation of the deviations from the Nash equilibrium, these results are consistent with a "learning to cooperate" interpretation. Fixed partners over multiple rounds of interaction favor the development of mutual understanding relative to varying partners. Still, we must re-emphasize, overall the results were dominated by non-cooperative (Nash) behavior. A dynamic analysis of fixed-partner interaction predominantly supported a Cournot myopic adaptive best-reply strategy in which subjects' choices were best replies to their opponent's previous choice but with a positive correction for error in anticipating that previous best reply. This dynamic helps to explain the convergence tendencies to Nash.

With regard to the underlying conflict model, the central prediction (Prediction 1) was that larger fighting efforts would be observed for higher values of the 'decisiveness coefficient' m -- a parameter that indicates the degree to which the fighting efforts as inputs determine the relative shares of incomes attained. Prediction 1 was overwhelmingly confirmed: in 45 of 48 comparisons, when fighting became a more decisive determinant of relative income shares, both sides invested more in the struggle.

The evidence was more mixed concerning when the Paradox of Power -- that the poorer side would improve its relative position -- would hold. For the experiments employing a low value $m = 1$ for the decisiveness coefficient, Prediction 2a was that at least the "weak form" of the paradox should always hold, that is, that $I_1/I_2 > R_1/R_2$. In fact eight of eight possible comparisons confirmed Prediction 2a. Prediction 2b was more stringent, specifying that for the four cases where the initial resource ratio was sufficiently low the "strong form" should hold: $I_2 = I_2$. The observed average income ratio for these cases was $I_1/I_2 = 1.125$, not very far from the prediction.

For the high value $m = 4$ of the decisiveness coefficient, Prediction 2c was that the Paradox of Power would hold in its weak form for the low resource ratio $R_1/R_2 = 1.67$, but should be violated for the high resource ratio $R_1/R_2 = 4$. The latter part of this prediction was substantially confirmed. For an already high resource ratio it was predicted that the rich would get richer, and in fact they did so. But they also did so for the 1.67:1 resource ratio where, according to the theory, the poor should instead have improved their position. From the point of view of the theoretical prediction, the richer contestant might be fighting too hard, or the poorer not hard enough. Inspection of lines 17-20 of Table 4 indicates that the rich are on average close to $F_1 = 16$, the predicted amount of resources devoted to fighting, but the poor are falling short of the predicted $F_2 = 15$. However, there is a boundary problem here: the Nash prediction for the poorer side would require them to devote 100% of their resources ($R_2 = 15$) to fighting. Thus, any error whatsoever on their part must necessarily lead to a deficiency of fighting effort, which at least partially rationalizes the "anomalous" result found.

To sum up: in this experimental context our results support the Nash as opposed to the

Cooperative solution, though with some degree of slippage in the direction of the latter. And the theoretical predictions as to when the 'Paradox of Power' -- that an initially weaker party will improve its position relative to a stronger opponent -- will or will not be observed, are also broadly supported.

REFERENCES

Browning, Edgar F. and Browning, Jacquelene M. Public Finance and the Price System, 4th ed. Englewood Cliffs, NJ: Prentice Hall, 1994.

Dixit, Avinash. "Strategic Behavior in Contests." American Economic Review, December 1987, 77(5), pp. 891-898.

Grossman, Herschel I. and Kim, Minseong 1995. "Swords or Plowshares: A Theory of the Security of Claims to Property." Journal of Political Economy, December 1995, 103(6), pp. 1275-1288.

Haavelmo, Trygve, A Study in the Theory of Economic Evolution. Amsterdam: North-Holland Publishing Co., 1954.

Hirshleifer, Jack. "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success." Public Choice, 1989, 63, pp. 101-112.

____. "The Paradox of Power." Economics and Politics, November 1991, 3(3), pp. 177-200.

Lindgren, B. W. Statistical Theory. New York: Macmillan Co., 1962.

McCabe, Kevin A., Rassenti, Stephen J., and Smith, Vernon L. 1996, "Game Theory and Reciprocity in Some Extensive Form Experimental Games," Proceedings National Academy of Science to appear.

McCabe, Kevin and Smith, Vernon. "Intentionality Signalling: Why Game Form Matters." Economic Science Laboratory, University of Arizona, November, 1996.

Mertens, Jean-Francois. "Repeated games", in John Eatwell, Murray Milgate, and Peter Newman, eds., The New Palgrave: A Dictionary of Economics. London: MacMillan Press, 1987, 4, pp. 151-153.

Skaperdas, Stergios. "Cooperation, Conflict, and Power in the Absence of Property Rights." American Economic Review, September 1992, 82(4), pp. 720-739.

20-__. "Contest Success Functions." Economic Theory, 1996, 7, pp. 283-290.

Skogh, Goran and Stuart, Charles. "A Contractarian Theory of Property Rights and Crime." Scandinavian Journal of Economics, 1982, 84, pp. 27-40.

Smith, Vernon L. and Walker, James M. "Monetary Rewards and Decision Cost in Experimental Economics." Economic Inquiry, April 1993, 31(2), pp. 245-261.

Tullock, Gordon, "Efficient Rent Seeking," in James M. Buchanan, Robert D. Tollison, and Gordon Tullock, eds., Toward a Theory of the Rent-Seeking Society, College Station: Texas A&M University Press, 1980.

FOOTNOTES

* Durham: Dept. of Economics, University of Arkansas, Fayetteville, AR 72703; Hirshleifer: Economics Department, UCLA, Los Angeles, CA 90095; Smith, Economic Science Laboratory, University of Arizona, Tucson, AZ 85721. We thank two anonymous referees of this Journal for extremely valuable comments and suggestions.

1. Edgar F. Browning and Jacquelene M. Browning (1994), pp. 259-261.

2. To allow for differential productivity, E_1 and E_2 could be multiplied by productive 'efficiency coefficients' e_1 and e_2. We do not explore this kind of asymmetry here.

3. To allow for possible differences in conflictual ability, the inputs F_1 and F_2 could be multiplied by 'fighting efficiency coefficients' f_1 and f_2. This type of asymmetry is also ruled out here.

4. Alternative possible forms of what are called here Contest Success Functions are discussed in Tullock (1980), Dixit (1987), Hirshleifer (1989), and Skaperdas (1996).

5. This result can come about only using the simplified production function (2a), where the productive complementarity coefficient is set at $s = 1$. For the more general CES production function (2), with $s > 1$, at equilibrium only the weak form of the paradox holds.

6. The value of τ was obtained by finding the resource ratio where the condition $I_1/I_2 = R_1/R_2$ was met for $m = 4$.

7. In McCabe, Rassenti and Smith (1996) this technique was found to be effective in leading to cooperation, even on the "last" repetition.

8. Copies of the instructions are available upon request.

9. In equations (3), the relative shares p_1 and p_2 are indeterminate when $F_1 = F_2 = 0$. To remedy the indeterminacy, the Profit Table in the Instructions provided for zero payoff to a player

whenever he or she put zero into the RA, i.e., whenever $F_i = 0$ was chosen. So the cooperative combination maximizing the aggregate payoff, under the integer constraint, is $(C_1, C_2) = (1,1)$.

10. Computing the likelihood ratio allows the analysis to include Bayesian updating of prior beliefs as well as traditional significance tests. We consider both avenues, with similar results. The likelihood ratio is particularly convenient for Bayesian conversion of prior beliefs p' into posterior beliefs p'' in light of the experimental evidence. The relevant version of Bayes' Theorem is:

$$\frac{p_o''}{p_a''} = \frac{\text{Likelihood of evidence under } H_o}{\text{Likelihood of evidence under } H_a} \times \frac{p_o'}{p_a'}$$

11. As a technical qualification, a strict Bayesian would want to deal with the fact that the true normal variance V is unknown. In principle one ought to specify prior beliefs about the variance and deal with it as a "nuisance parameter". However, we have taken the liberty of simply employing the observed sample variance S^2 for V. Doing so provides an enormous computational saving without substantially affecting the results.

12. Under a Bayesian interpretation, any observer, regardless of prior beliefs, should revise those beliefs so as to attach greater confidence to the Nash hypothesis.

13. In Bayesian terms, for these two cases the likelihoods are about equal under the null and alternative hypotheses, so no great revision of prior beliefs is indicated. The evidence, while improbable either way, is not much more improbable under one hypothesis than under the other.

TABLE 1

TREATMENTS

Number of experiments (number of subjects)*

Endowments (R_1,R_2)	Decisiveness m	Variable Pairing	Fixed Pairing	Totals
20,20	1	2(24)	2(24)	4(48)
25,15	1	2(24)	2(24)	4(48)
32,8	1	2(22)	2(24)	4(46)
20,20	4	2(22)	2(22)	4(44)
25,15	4	2(20)	2(24)	4(44)
32,8	4	2(24)	2(24)	4(48)
	TOTALS	12(136)	12(142)	24(278)

*Due to some recruting problems, a few experiments were run using only 8 or 10 subjects (4 or 5 pairs). Each experiment was run for either 16 or 17 rounds.

TABLE 2

EXPERIMENTAL RESULTS

Treatment Parameters					Nash solution		Average Results (16th observations)		
Experiment Number	m	Pair-ing	R_1,R_2	R_1/R_2	N_1,N_2	N_1/N_2	F_1,F_2	F_1/F_2	I_1/I_2
1	1	V	20,20	1	10,10	1	7.83,6.83	1.15	1.15
2	1	V	20,20	1	10,10	1	8,9	.89	.89
3	1	F	20,20	1	10,10	1	8.67,6.67	1.30	1.30
4	1	F	20,20	1	10,10	1	4,5	.8	.8
5	1	V	25,15	1.67	10,10	1	10.83,8.5	1.27	1.27
6	1	V	25,15	1.67	10,10	1	9,9.17	.98	.98
7	1	F	25,15	1.67	10,10	1	10.17,9	1.13	1.13
8	1	F	25,15	1.67	10,10	1	7.67,6.83	1.12	1.12
9	1	V	32,8	4	10,8	1.25	11.83,7.67	1.54	1.54
10	1	V	32,8	4	10,8	1.25	10.33,7.5	1.38	1.38
11	1	F	32,8	4	10,8	1.25	5.17,3.17	1.63	1.63
12	1	F	32,8	4	10,8	1.25	5.4,4.6	1.17	1.17
13	4	V	20,20	1	16,16	1	10.33,12.83	.81	.42
14	4	V	20,20	1	16,16	1	14.67,15.33	.96	.84
15	4	F	20,20	1	16,16	1	11.67,13.83	.84	.51
16	4	F	20,20	1	16,16	1	10,9.2	1.09	1.40
17	4	V	25,15	1.67	16,15	1.07	15.5,11.83	1.31	2.95
18	4	V	25,15	1.67	16,15	1.07	16.5,12.5	1.32	3.04
19	4	F	25,15	1.67	16,15	1.07	16.17,13.83	1.17	1.87
20	4	F	25,15	1.67	16,15	1.07	13.5,12.33	1.10	1.44

21	4	V	32,8	4	12,8	1.5	11.67,7.33	1.59	6.42
22	4	V	32,8	4	12,8	1.5	11.67,7	1.67	7.72
23	4	F	32,8	4	12,8	1.5	10.5,7.5	1.4	3.84
24	4	F	32,8	4	12,8	1.5	11,4.67	2.36	30.76

TABLE 3

LOG LIKELIHOOD RATIOS

Nash versus Cooperative Bargaining Solution[a]

Treatment Parameters and Matching Protocol				All Rounds (T·n=198)		16th Round (n=12)	
			Pair-	Types		Types	
R_1	R_2	m	ing	F_1	F_2	F_1	F_2
20	20	1	V	- 873	- 654	- 87	- 30
20	20	1	F	- 545	- 169	- 3.8	- 0.7[†]
25	15	1	V	- 939	-1302	- 31	- 133
25	15	1	F	- 262	- 218	- 20	- 23
32	8	1	V	- 879	-2583	- 31	- 111
32	8	1	F	- 123	- 73	0.8[†]	8.5[*]
20	20	4	V	-1260	-1376	- 114	- 78
20	20	4	F	- 68	- 92	- 6.3	- 10
25	15	4	V	-1475	- 541	-1001	- 115
25	15	4	F	- 860	- 796	- 48	- 69
32	8	4	V	-2756	-1643	-1008	- 114
32	8	4	F	- 373	- 332	- 43	- 23

[a] Lindgren (1962), for example, derives a most powerful test (among the class of tests where α errors are not smaller, none has a larger power, 1-β) for a simple H_o against a simple H_a using the likelihood ratio. the best critical region is $\lambda = N_o(x)/N_a(x) < K$, where N(x) is the normal density evaluated for H_o or H_a, and K is a constant chosen to set the Type I error (α) at the desired level. H_o is then rejected in favor of H_a if $\lambda < K$.

Setting $\alpha = 0.001$, K_E, for each experimental treatment, E, was computed from the following (for the 16[th] round case, T = 1 in (9)):

$$PROB_{\mu = C = 1} \ (\overline{F_E} > K_E) = 1 - N\left(\frac{K_E - C}{S_E/\sqrt{n}}\right) = 0.001,$$

where $\overline{F}_E$ is the sample mean level of observed fighting, and S_E^2 is the variance across all n pairs in treatment E. The results from this likelihood ratio test allow us to reject the hypothesis = C = 1 (cooperation) in all cases except for the * entry. The entries marked § indicate cases where cooperation is rejected in favor of Nash, but when H_o and H_a are interchanged so that cooperation becomes the null hypothesis, Nash is rejected in favor of cooperation. This illustrates the inherent ambiguity of classical tests in which the outcome need not be independent of which hypothesis is chosen as the null!

TABLE 4

SLIPPAGE TOWARD COOPERATION

Treatment Parameters					Nash solution	Average Results		Average Slippage*	
Experiment Number	m	Pair-ing	R_1,R_2	R_1/R_2	N_1,N_2	F_1	F_2	S_1	S_2
1	1	V	20,20	1	10,10	7.83	6.83	.24	.35
2	1	V	20,20	1	10,10	8	9	.22	.11
3	1	F	20,20	1	10,10	8.67	6.67	.15	.37
4	1	F	20,20	1	10,10	4	5	.67	.56
5	1	V	25,15	1.67	10,10	10.83	8.5	-.09	.17
6	1	V	25,15	1.67	10,10	9	9.17	.11	.09
7	1	F	25,15	1.67	10,10	10.17	9	-.02	.11
8	1	F	25,15	1.67	10,10	7.67	6.83	.26	.41
9	1	V	32,8	4	10,8	11.83	7.67	-.20	.05
10	1	V	32,8	4	10,8	10.33	7.5	-.04	.07
11	1	F	32,8	4	10,8	5.17	3.17	.54	.69
12	1	F	32,8	4	10,8	5.4	4.6	.51	.46
13	4	V	20,20	1	16,16	10.33	12.83	.38	.21
14	4	V	20,20	1	16,16	14.67	15.33	.09	.04
15	4	F	20,20	1	16,16	11.67	13.83	.29	.15
16	4	F	20,20	1	16,16	10	9.2	.40	.45
17	4	V	25,15	1.67	16,15	15.5	11.83	.33	.23
18	4	V	25,15	1.67	16,15	16.5	12.5	-.03	.18
19	4	F	25,15	1.67	16,15	16.17	13.83	-.01	.08
20	4	F	25,15	1.67	16,15	13.5	12.33	.17	.19

21	4	V	32,8	4	12,8	11.67	7.33	.03	.10
22	4	V	32,8	4	12,8	11.67	7	.03	.40
23	4	F	32,8	4	12,8	10.5	7.5	.14	.07
24	4	F	32,8	4	12,8	11	4.67	.09	.48

* $S_i = (N_i - F_i)/(N_i - C)$

Figure I

Experiments A1 and B1

$(R_1,R_2) = (20,20)$; m = 1; Varying Partners

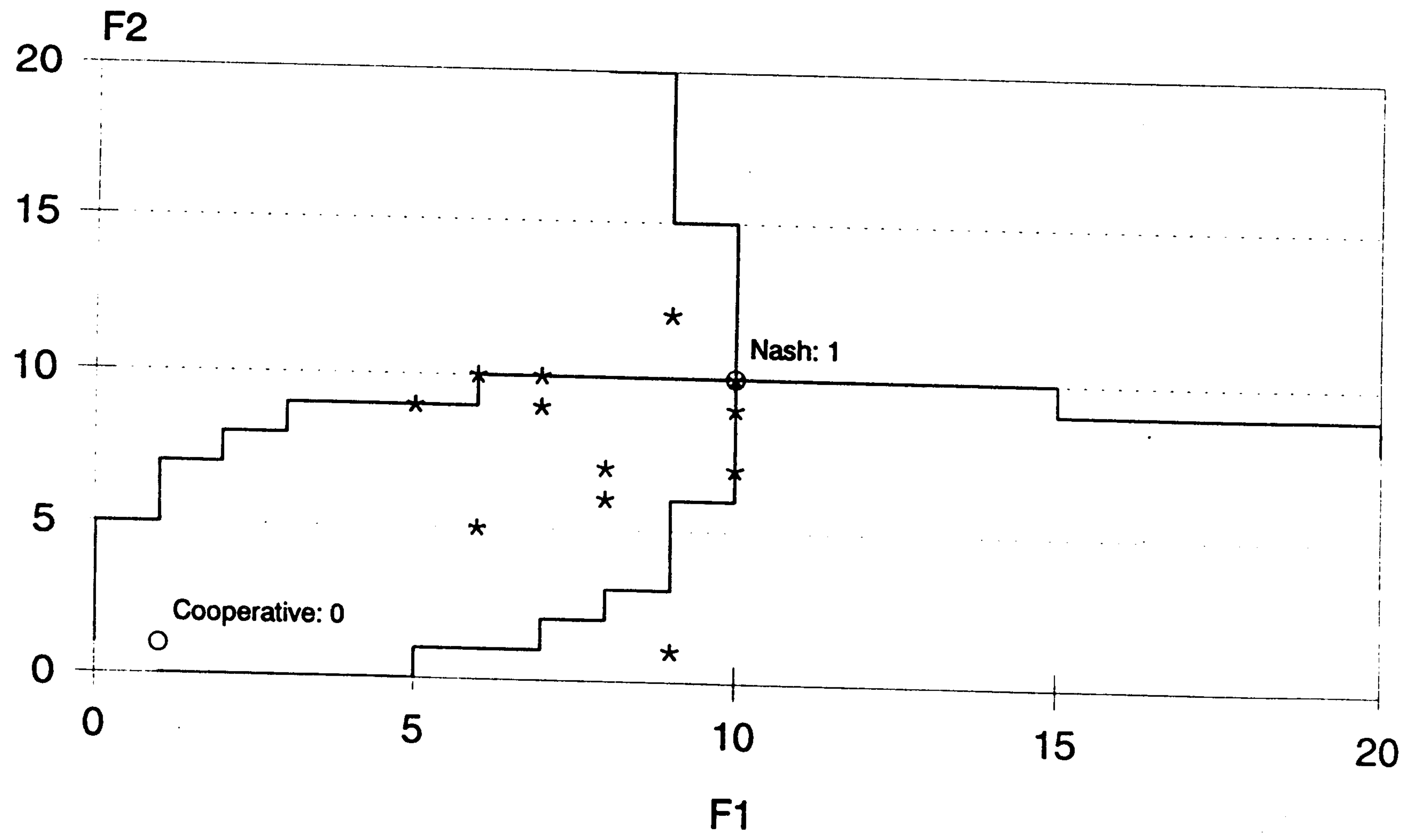

Figure II

Frequencies of Estimated Deltas

N = 140