The Margin of Appropriation and an Extension of the First Theorem of Welfare Economics*

Louis Makowski[†] Joseph M. Ostroy[‡]

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Abstract

The standard First Theorem of Welfare Economics rests on two assumptions, price-taking behavior and complete markets. Thus, individuals have neither price-*making* nor market-*making* capacities. We offer an extension of the First Theorem in which individuals have such capacities. Two noteworthy features of the extension are its emphasis on aligning private rewards with social contributions at the "individual margin" as the key to market efficiency and, relatedly, its emphasis on pecuniary externalities as an important potential source of market failure.

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[†]Department of Economics, UC Davis

[‡]Department of Economics, UCLA

We take another look at the First Theorem of Welfare Economics with a view toward building some bridges to the more recent literature on mechanism design (e.g., Vickrey [1961], Hurwicz [1973], Groves [1973]), on commodity innovation (Hart [1980], Makowski [1980b]), and on perfect competition (Ostroy [1980], Makowski [1980a], Makowski and Ostroy [1987]). In this literature, as in much of modern economics, strategic behavior and incentives figure more prominently than in the Walrasian model. Our main result is an extension of the First Theorem which treats these strategic/incentive issues more explicitly. Two noteworthy features of the extension are its emphasis on appropriation at the "individual margin"—viewing the whole individual, with his entire package of demands and supplies, as the margin of analysis—as the key to market efficiency and, relatedly, its emphasis on pecuniary externalities as an important potential source of market failure.

Since Pigou's classic, *The Economics of Welfare*, our intuitive understanding of market failure has rested largely on the divergence between private and social benefit, that is on failures of appropriation. For example, we say that a commodity is undersupplied because the supplier is unable to appropriate the full social benefit of his actions. Conversely, we also have come to appreciate the importance of appropriation for market success. In the First Theorem of Welfare Economics, market success depends on two conditions:

- price-taking behavior, and
- complete markets.

The assumption of complete markets ensures that all potential benefits are priced, so that in a Walrasian equilibrium marginal private and social benefits will coincide at all relevant commodity margins, e.g., no real externalities (among others, Arrow [1969], emphasizes this role of complete markets).

Two central incentive questions are begged by the above assumptions, namely, the incentives for price-*making* and market-*making*. Thus, the reasons why competition prevents individuals from exerting monopoly power in their price-making and why it might efficiently guide innovators in their market-making occur outside the model and are therefore not illuminated by the standard First Theorem. We offer an extension of the First Theorem to a model in which individuals have both a price-making and a market-making capacity. In such a setting, "appropriation logic"— that is, the alignment of private and social benefits in order to give individuals good incentives—will be seen to play an even bigger role in ensuring market success than the standard First Theorem suggests.

A summary of the difference in our perspective as to the sources of market success/failure is the emphasis we place on the "individual margin of appropriation" rather than the "commodity margin of appropriation". To illustrate the distinction, compare the following two statements. A commodity will be efficiently supplied if:

(1) the producer is able to appropriate the full social benefit of the marginal unit he supplies

(2) the producer is able to appropriate the full social benefit of all the units he supplies.

The first is about the *commodity* margin of appropriation, the second about the *individual* margin. Obviously the two margins are related. If the producer is able to appropriate the full social benefit of all his units [(2)], this implies appropriation with respect to the marginal unit [(1)]; hence the individual margin is more inclusive. As already noted, it is appropriation at the commodity margin which underlies the standard First Theorem. In any Walrasian equilibrium (1) will be satisfied; but (2) may not be (see Theorem 1). The implication is that under price-taking and complete markets, appropriation at the individual margin is superfluous for market efficiency. However, when individuals are given a price-making or market-making role, the individual margin can no longer be ignored. This is the message of our extension of the First Theorem.

To make room for the extension, we work with a generalization of a Walrasian model in which the price-taking and complete markets assumptions are relaxed. We call it a model of *occupational choice*. The formulation is related to models of mechanism design in that the market outcome can be described as a Walrasian mechanism in which prices as well as marketed commodities respond to individuals' occupational choices. Thus, in the model of occupational choice there is

- price-making: individuals may be able to influence market-clearing prices by their choice of occupations, and
- market-making: individuals determine the set of available markets by their choice of occupations.[1]

To illustrate its workings, production at different scales can be modeled as the choice of different occupations, so the producer may be able to exert monopoly power by choosing to operate at a smaller scale. Or to illustrate market-making, different occupations may involve the introduction of different *new* commodities (in the model, markets are open only for commodities that can be actually supplied given individuals' occupational choices; hence the choice of occupations has a market-making role). An equilibrium in the model is called an *occupational equilibrium*.

Our main result identifies conditions under which occupational equilibria will be efficient, in spite of the greater scope for self-interested behavior. We show that if

(full appropriation) each individual's private benefit from any occupational choice coincides with his/her social contribution in that occupation, and

(non-complementarity) a non-complementarity condition is satisfied among occupational choices made by different individuals

then the allocation of resources will be Pareto efficient (Theorem 4).

The central condition, full appropriation, represents an extension of Pigovian appropriation logic to the more inclusive individual margin of analysis. Its role is to give individuals

[1]The model of occupational choice collapses to a standard Walrasian model when each individual has only one occupational choice, i.e., no choice at all.

good incentives in their occupational choices, hence in both their price-making and market-making. The (only) role of the second condition, non-complementarity, is to help ensure efficiency in market-making.

Even with full appropriation, interesting *coordination failures* in innovation can occur in the model of occupational choice, namely, under-innovation traps similar to those pointed out by Scitovsky [1954]. The non-complementarity condition rules these cases out. It is restrictive, but we do identify conditions under which it will hold. Therefore, a special case of the extension is an optimality theorem which does not require the complete markets hypothesis (Corollary 2). This result is based on conditions identified by Hart [1980] and Makowski [1980b] as sufficient for efficient product innovation. While both these studies contain heuristics pointing to the importance of appropriation, their primary focus is on a particular application rather than on incorporating their findings into standard welfare economics, i.e., the First Theorem.

How do the assumptions of the extension compare with those of the standard First Theorem? The answer is that the former either are (i) weaker than or else (ii) considerably stronger than the price-taking and complete markets assumptions of the standard Theorem: it depends on how one interprets the price-taking assumption. If one interprets Walrasian equilibrium as a model of a *perfectly competitive economy*, where individuals act as price-takers because no single individual could influence market-clearing prices even if he/she were to try, then full appropriation at the individual margin will be satisfied in any Walrasian equilibrium. Schematically,

$$\text{perfect competition} \Rightarrow \text{full appropriation}$$

(Ostroy [1980], Makowski [1980a], or in an occupational choice setting, Theorem 3 below). Further, perfect competition plus complete markets imply non-complementarity (Theorem 5). Thus, insofar as price-taking is a shorthand for perfect competition, the assumptions of the extension are implied by those of the standard First Theorem. But not the converse; so the former are weaker than the latter. Alternatively expressed, insofar as price-taking is simply a *veil* for perfect competition, the full appropriation condition lifts the veil, allowing for the stronger conclusions that market efficiency can occur without complete markets.

While price-taking originated as a veil, repetition has led to the conclusion that perfect competition *means* price-taking. Under this interpretation, where the hypothesis of price-taking acts as a *substitute* and not just as a veil for perfect competition, full appropriation is considerably stronger condition than Walrasian equilibrium (see Theorem 1). To illustrate the difference, the moral of the standard First Theorem is that *if* individuals act as price-takers then *only* the commodity margin of appropriation matters for efficiency, the individual margin is irrelevant; whereas the moral of the extended First Theorem is that when price-taking is an unsupported behavioral assumption, individuals will *not* act as price-takers and consequently inefficiencies will follow (see Example 1).

Staying within the boundaries of the First Theorem, we shall ignore questions of the existence of equilibria satisfying full appropriation. Given its connection with perfect competition, the obvious setting to examine the existence question involves a continuum of individuals and, to achieve a level of generality useful for several applications, an infinite number of commodities. (See Ostroy [1984], Ostroy and Zame [1988], Makowski and Ostroy [1991]). This would take us well beyond the technical range of this paper. Fortunately even in a finite setting, where it is only with care that examples of perfect competition are constructed, the theoretical principles of market success and failure can be formulated that carry over to the continuum, where the existence question is more naturally posed.

In this paper we do not strive for the utmost generality, preferring to emphasize principles. One simplifying assumption deserves special mention. We shall assume that individuals have quasi-linear preferences, hence constant marginal utility from income or "transferable utility". This allows for a cardinal measure of individuals' private rewards and of their social marginal products; hence, it greatly facilitates emphasizing the appropriability theme. From our work on the *no-surplus* approach to perfect competition, the current paper being a continuation on this line, we strongly surmise that there are ordinal analogs of our current results, albeit less concrete versions, just as there is both an ordinal and cardinal version of the no-surplus condition. Nevertheless, the ordinal extensions of the current results remain an open question for research. There is also a simplified treatment of firms in this paper, relative to the Arrow-Debreu version of the Walrasian model. While the "individuals" in the model of occupational choice may possess production possibilities and may be interpreted as single proprietary firms, the model does not include firms with multiple shareholders. This is just to avoid the notational complications involved in including shareholdings and the required redistributions of profits, unenlightening complications that would distract from our main goal: an alternative presentation of the First Theorem.

The contents of the rest of the paper are as follows. The model of occupational choice is described in Section 1. Section 2 proves that rewarding individuals with their social marginal products (full appropriation), is good for incentives: it leads to efficient occupational choices, excepting perhaps for some coordination problems. Section 3 proves that when the changes in the gains from trade are subadditive (the non-complementarity condition), no coordination problems will arise. Section 4 provides an interpretation of our results in terms of the standard First Theorem. It also makes the connection with the literature on underdevelopment traps (à la Scitovsky [1954]) and innovation under perfect competition.

The goal of any formalization of an Invisible Hand Theorem is to guide our understanding of market success/failure. Thus in the concluding section, 5, we discuss that taxonomy of market failure suggested by the extended First Theorem. Compared to the traditional taxonomy, pecuniary externalities play a much more vital role. As an historical supplement, Appendix A contains an analysis of Pigou's famous *misapplication* of appropriation logic, which led to the longtime expulsion of pecuniary externalities from welfare economics. Appendix B contains a proof of Lemma 1.

1. An Alternative Approach to Competitive Equilibrium: The Allocation of Resources as a Problem of Occupational Choice

Although the Walrasian model permits a broad range of possible interpretations, the Walrasian conception of the coordination of economic activity fosters a certain point of view that might be termed a "thick markets mentality". According to this vision, the world is described by a fixed set of commodity markets as the paved highways of economic travel. In contrast to this, we shall take a "thin markets" approach. What we mean by this is that we shall try to avoid the fixed set of roads upon which individuals travel. The aim is to portray a world in which economic actors are connected not by several main highways but by a myriad of individual byways of their own construction. It is this alternative vision that underlies the following approach.

We pose the problem of the coordination of economic activity by supposing the each individual can be one of several different types. Call these types the possible "occupations" for the individual. More formally, there are n individuals, indexed by i. For each individual there is a given set of possible occupations V_i from which individual i must choose exactly one. An *assignment* of individuals to occupations is a $v = (v_1, \ldots, v_i, \ldots, v_n) \in \times_i V_i$. Let $V \equiv \times_i V_i$ represent the set of all possible assignments.

To include both pure exchange and production-and-exchange economies, it shall be simpler to work in *trade space*. Thus we leave implicit i's consumption and production decisions, which are his private information, to focus on what is essential for the model, his trade relationships. A trade for individual i is a point $z_i \in \Re^\ell$ with the sign convention that positive (negative) components of z_i represent his purchases (sales). When i chooses an occupation $v_i \in V_i$, he chooses both a trading possibility set Z_i and preferences over the possible trades in Z_i.[2] We can compactly describe both these elements of i's occuational choice by letting v_i be an *extended* real-valued function from $\Re^\ell$ to $\Re \cup \{-\infty\}$. Then v_i can do double duty: Individual i's trading possibility set in occupation v_i is given by the effective domain of v_i, that is, $Z_i = \text{dom } v_i$ where

$$\text{dom } v_i = \{z_i : v_i(z_i) > -\infty\}.$$

And the values of v_i on dom v_i describe i's preferences over the trades in his trading possibility set. Observe that i's preferences over trades will generally change when his occupation changes even if his consumption tastes remain constant; e.g., if he becomes a baker then he will value the purchase of 1,000 bushels of wheat more than if he becomes a candlestick maker. Several examples illustrating the flexibility of the set-up will be given. Notice since we are in trade space, the zero vector in $\Re^\ell$ corresponds to no trade, which we shall assume is always an option. More specifically, we assume

- for all i and all $v_i \in V_i$, v_i is continuous on dom v_i, $0 \in \text{dom } v_i$, and $v_i(0) = 0$ (a normalization).

[2] To illustrate the sign conventions, in the case of pure exchange where choosing an occupation just amounts to choosing an endowment, Z_i would equals $\{z_i \geq -\omega_i\}$ if ω_i were i's endowment.

In addition to trade in the ℓ commodities, there is also a money commodity that the individual can use to establish quid pro quo in exchange. The utility from these $(\ell + 1)$ commodities depends only on i's characteristics v_i because all individuals have *quasi-linear* utility functions with respect to the money commodity. That is, i's utility from $(z_i, m_i) \in \Re^\ell \times \Re$ when he is in occupation v_i is given by

$$u(z_i, m_i; v_i) = v_i(z_i) + m_i.$$

To preserve the quasi-linearity of the model we put no limitation on the amount of money i can supply (the spirit is that he never hits the boundary of his money endowment).

The set of commodities that can be potentially supplied in the economy is restricted by individuals' occupational choices. Let

$$H(v) = \{h : z_{ih} < 0 \text{ for some } i \text{ and some } z_i \text{ in dom } v_i\}$$

represent the set of commodities that can be potentially supplied in v, where h indexes commodities, $h = 1, \ldots, \ell$. We shall make the harmless assumption that all commodities can be potentially supplied:

- $\cup_{v \in V} H(v) = \{1, \ldots, \ell\}$.

Define the subspace

$$\Re^{\ell(v)} = \{x \in \Re^\ell : x_h = 0 \text{ for all } h \notin H(v)\}.$$

We shall suppose that given any assignment v to occupations, markets are incomplete in the sense that trading is restricted to $\Re^{\ell(v)}$. A trade $z = (z_i)$ is *feasible for v* if each $z_i \in \text{dom } v_i \cap \Re^{\ell(v)}$ and $\sum z_i = 0$. Let $Z(v)$ be the set of all such trades. Notice that $Z(v) \neq \emptyset$ since $0 \in Z(v)$. We shall also assume

- For all $v \in V$, $Z(v)$ is compact.

DEFINITION: Given an assignment v, a *Walrasian equilibrium for v* is a pair (z, p) such that z is feasible for v, $p \in \Re^\ell$, and for all i

$$v_i(z_i) - pz_i \geq v_i(z_i') - pz_i' \text{ for all } z_i' \in \Re^{\ell(v)}.$$

That is, i maximizes $v_i(z_i') + m_i'$ subject to the (trading) budget constraint $pz_i' + m_i' = 0$.

Exploiting the quasi-linearity of the model, define the maximum potential *gains from trade* in v as

$$g(v) = \sup\{\sum v_i(z_i) : z \in Z(v)\}.$$

Observe that this sup is attained by a trade in $Z(v)$ since $\sum v_i$ is continuous and $Z(v)$ is compact, nonempty.

DEFINITION: The trade z is *efficient for* v (synonymously, "efficient relative to v") if z is feasible for v and $\sum v_i(z_i) = g(v)$. An allocation (v, z) is *(globally) Pareto efficient* if z is efficient for v and $g(v) \geq g(v')$ for all $v' \in V$.

As an application of the standard First Theorem of Welfare Economics, we have

Proposition 1 *If (z, p) is a Walrasian equilibrium for v then z is efficient for v.*

Proof: Let z' be any other feasible allocation for v. Then from the condition for Walrasian equilibrium, summing over the i and recalling $\sum z_i' = 0$ since z' is feasible:

$$\sum v_i(z_i) \geq \sum v_i(z_i') \text{ for all feasible } z',$$

that is, $\sum v_i(z_i) = g(v)$. □

Nevertheless, a Walrasian equilibrium for v can evidently be very inefficient—not globally Pareto efficient— since the set of feasible trades may be restricted to a very inefficient subset of commodities: people may be in the wrong occupations. We will be interested in how the Invisible Hand may be able to lead the economy to a Pareto efficient outcome.

Suppose occupational choice is the Nash equilibrium outcome of a game, in which people hold rational conjectures about how Walrasian prices will change when they change occupations. Let $\wp : V \to \Re^{\ell}$ be a Walrasian price selection in the sense that for each $v \in V$, there are trades z such that $(z, \wp(v))$ is a Walrasian equilibrium for v; and let

$$\pi_i(v) = \max\{v_i(z_i) - \wp(v)z_i : z_i \in \Re^{\ell(v)}\}$$

represent i's payoff (synonymously, "profit" or "utility") in the assignment v under prices $\wp(v)$.

DEFINITION: An *occupational equilibrium* (OE) is a triple $(\wp, v, z)$ such that $(z, \wp(v))$ is a Walrasian equilibrium for v, and for all i and all $v_i' \in V_i$

$$\pi_i(v) \geq \pi_i(v_i', v^i),$$

where $v^i \equiv (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ is the assignment v with individual i omitted; and consequently, (v_i', v^i) represents the assignment v with only i's occupation changed from v_i to v_i'.

The displayed condition expresses the idea that v is a Nash equilibrium in occupational choice. In terms of traditional economics, it picks up the idea of *resources flowing into their* (privately) *most profitable uses*. We will be interested in identifying conditions under which OE are Pareto efficient. Note that if $(\wp, v, z)$ is an occupational equilibrium, then $\pi_i(v) = v_i(z_i) - \wp(v)z_i$.

In an occupational equilibrium, the market outcome for v is obtained from a predetermined selection among the Walrasian, and therefore price-taking, equilibria for v. This should be regarded as a convenient simplification in which we ignore the monopoly problems in a given v to focus on the monopoly issues across V. Note, however, that the more variation there is in the choice of "occupations", the closer this fiction will come to mimicking conventional monopoly. For example, suppose a seller with occupations/activities that distinguish between different quantities of the same good supplied. Then, the seller can observe the Walrasian outcome from selling one unit, from selling two units, etc., i.e., the seller can observe the aggregate demand schedule just as a simple monopolist would do. If buyers are permitted to have similar quantity-varying "occupations", they will attempt to exercise their monopsony power. An illustration along these lines follows.

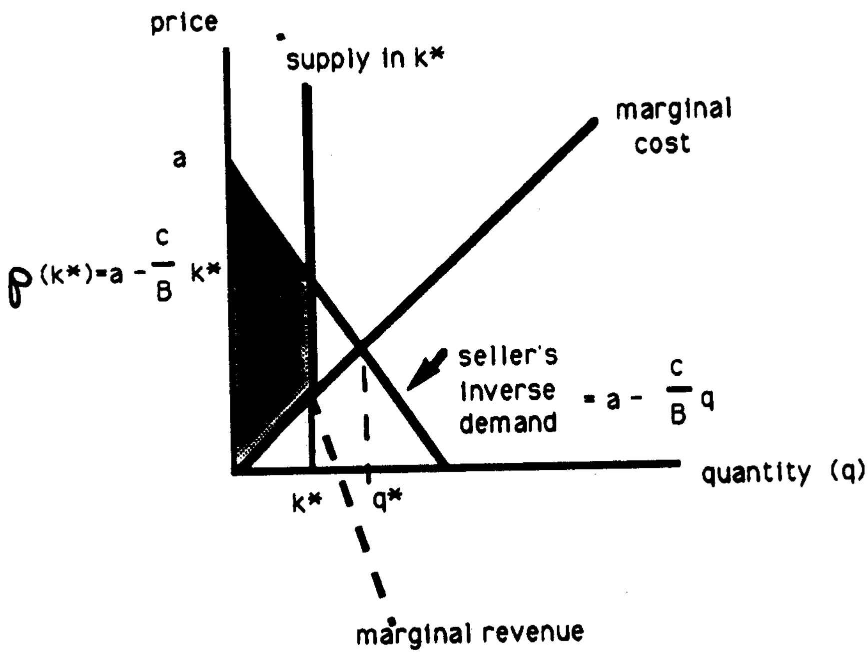

Figure 1: The occupational equilibrium in Example 1

EXAMPLE 1 (*simple monopoly as an occupational equilibrium*): Let $\ell = 1$ and partition individuals into one seller, s, and $B \equiv n - 1$ buyers indexed by b. The seller only likes money. His possible occupations are parameterized by $k \in [0, K]$; when in occupation k, he can supply up to k units of the commodity at a cost of $\frac{1}{2}q^2$ for any $q \in [0, k]$. Thus, $V_s = \{v_s^k : k \in [0, K]\}$ where

$$v_s^k(z_s) = \begin{cases} -\frac{1}{2}z_s^2 & \text{if } z_s \in [-k, 0] \\ -\infty & \text{otherwise.} \end{cases}$$

Buyers are identical. Each buyer has no initial endowment of the commodity and values its consumption according to a quadratic utility function; further, we view buyers as passive here, so model them with only one occupation. Thus, for each buyer b, $V_b = \{v_b\}$, where

$$v_b(z_b) = \begin{cases} az_b - \frac{1}{2}cz_b^2 & \text{if } z_b \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

and where a and c are positive constants.

Let $q*$ be the output where the seller's inverse demand curve intersects his marginal cost curve (see Figure 1), and let us assume $K > a$. Writing $p(k)$ for $p(v_s^k, v^s)$, it is easy to check that $\wp(k)$ is unique and given by

$$\wp(k) = \begin{cases} a - \frac{c}{B}k & \text{if } k < q* \\ a - \frac{c}{B}q* & \text{otherwise.} \end{cases}$$

For efficiency, we want the seller to produce $q*$ and thus to choose an occupation $k \geq q*$. But the seller's profit, $\pi(\cdot)$, is maximized in occupation

$$k* = \frac{aB}{2c+B},$$

where his marginal revenue equals his marginal cost, again see the Figure. His equilibrium occupational choice exhibits the usual inefficiency associated with simple monopoly: he can influence market clearing prices $\wp(k)$ by his choice of occupation (quantity), hence he enters the wrong occupation (undersupplies).

2. Giving Individuals Their Marginal Products is Good for Incentives: A Partial Extension of the First Theorem

2.1 Monopoly power leads to a divergence between private profit and social benefit

The market failure that Example 1 illustrates may be explained in terms of a failure of appropriation at the individual margin, i.e., as arising from a divergence between the seller's private reward and his social marginal product. To see this, we shall need some new terminology.

As a preliminary observe that, for any individual i and any assignment to occupations v, the maximum potential ***gains from trade in v without i*** is given by

$$g^i(v^i) = \sup\{\sum_{j \neq i} v_j(z_j) : \sum_{j \neq i} z_j = 0\},$$

where recall $v^i \equiv (v_1, \ldots, v_{i-1}, v_{1+1}, \ldots, v_n)$, represents the occupations of all individuals except i. Thus, individual i's contribution to society is naturally defined as the difference between the gains from trade with him and without him.

DEFINITION: The ***(social) marginal product*** of individual i in occupation v_i when others are in occupations v^i is given by

$$MP_i(v) = g(v) - g^i(v^i).$$

By contrast, the ***private marginal product*** of individual i in occupation v_i when others are in occupations v^i is given by

$$PMP_i(v) = \pi_i(v).$$

In an occupational equilibrium $(\wp, v, z)$, since $(z, \wp(v))$ is Walrasian for v, z is efficient relative to v, i.e., $\sum v_i(z_i) = g(v)$. Thus, $\sum PMP_i(v) = g(v)$. Further, we have

Theorem 1 (Inappropriability Theorem) *If* $(z, \wp(v))$ *is a Walrasian equilibrium for* v *then, for each individual* i,

$$PMP_i(v) \leq MP_i(v).$$

Thus, $\sum MP_i(v) \geq g(v)$.

Proof: Let z' be any set of trades that are feasible without i, i.e., that satisfy $\sum_{j \neq i} z'_j = 0$. Then as in the proof of Proposition 1, from the definition of a Walrasian equilibrium for v,

$$\sum_{j \neq i} v_j(z_j) - \sum_{j \neq i} \wp(v) z_j \geq \sum_{j \neq i} v_j(z'_j).$$

So, $\sum_{j \neq i} v_j(z_j) - \sum_{j \neq i} \wp(v) z_j \geq g^i(v^i)$. Multiplying both sides of this inequality by -1 and adding $\sum_{j=1}^{n} v_j(z_j)$ to both sides shows

$$\sum_{j=1}^{n} v_j(z_j) - \sum_{j \neq i} v_j(z_j) + \sum_{j \neq i} \wp(v) z_j \leq g(v) - g^i(v^i).$$

But recalling the feasibility of z, the LHS just equals $v_i(z_i) - \wp(v) z_i$, i.e., $PMP_i(v)$; while the RHS equals $MP_i(v)$. Hence, $PMP_i(v) \leq MP_i(v)$, as claimed. The second assertion of the Theorem now follows immediately from the fact that $\sum PMP_i(v) = g(v)$. □

So "at best" in a OE, everyone will be rewarded with their full social marginal product. We call the result the "Inappropriability Theorem" to emphasize that usually some individuals will be rewarded with strictly less than their MP's. This was illustrated in Example 1. In this example, for any assignment v, the seller's social marginal product in v is the *whole* gains from trade in v since no one else has any of the commodity to trade; e.g., when $k = k*$ then the seller's MP is the entire shaded area in Figure 1. But his PMP, his profit, is just a fraction of $g(v)$ since he faces a downward sloping demand curve and, so, must give up some of $g(v)$ to the buyers as consumer surplus; e.g., when $k = k*$ then he must give the darker shaded consumer surplus triangle in Figure 1. Thus, in the example, the seller appropriates less than his MP. This explains why he undersupplies in the OE: beyond $k*$, the change in his PMP is negative, even though the change in his social marginal product is still positive.

While the undersupply equilibrium in Example 1 is bad, things could get worse: the unique seller may not want to produce at all. Specifically, consider the variant of Example 1 in which the seller, in addition to his marginal cost, has a fixed cost C that he must suffer if he enters any occupation $k > 0$. Suppose this fixed cost *exceeds* his equilibrium profit in Example 1; that is, his (now) U-shaped average cost curve lies strictly above his downward sloping inverse demand curve (see Figure 2). Thus, while $\wp(k)$ remains unchanged from Example 1, the unique occupational equilibrium now involves autarky: the seller does not produce any of the commodity.[3] But also suppose the sum of producer and consumer surplus would be strictly positive for some output levels (i.e., the area of the dark shaded rectangle in Figure 2—his losses in occupation q*—is smaller than the shaded consumer

[3]The reader may have expected a non-existence problem. Indeed, such a problem does occur in the Walrasian version of this variant because of the discontinuity in the firm's supply curve caused by the fixed

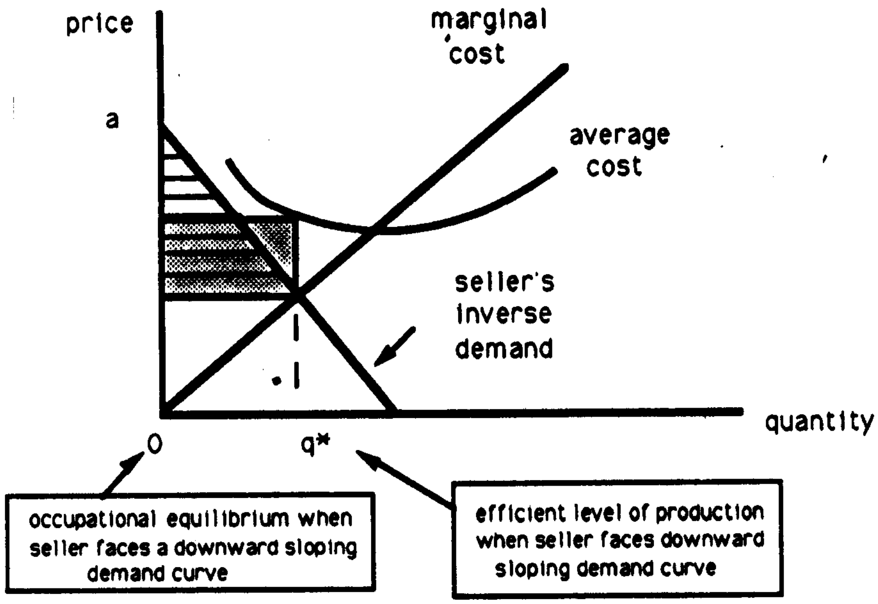

Figure 2: The variant of Examples 1 with a fixed cost

surplus triangle in the Figure), so the no production equilibrium is Pareto inefficient. In accord with traditional teaching, the source of the inefficiency is that the seller cannot *appropriate* the consumer surplus his commodity would produce. Or, in our language, he would not get the full social marginal product of his commodity.

If the buyers in this variant were producers rather than final consumers, the seller's entry would translate into lower factor prices and (perhaps) higher profits for others. Along these lines, the recognition that appropriability issues are central for welgre economics long precedes even Pigou, whose great work in large part was a synthesis of a longer tradition. Consider the example illustrated in Figure 2 in the context of the following interesting citation from J. B. Clark [1892, pp. 215-217]:

> In the case of railroads the inappropriable utilities are so great as almost to overbalance those which can be retained by the owners. The railroad creates a value far in excess of that which its projectors can realize; and this distributes itself among the adjacent population, and appears in the enhanced values of lands and the increased rewards of general industry. It has often happened that a railroad which enriched the population of the section which it traversed,

cost. (In the Walrasian version the firm's occupational choices are trivial, say $V_s = \{v'_s\}$, where

$$v'_s(z_s) = \begin{cases} -\frac{1}{2}z_s^2 - C & \text{if } z_s \in [-K, 0) \\ 0 & \text{if } z_s = 0 \\ -\infty & \text{otherwise.)} \end{cases}$$

There is no non-existence problem in the occupational choice version because the firm takes into account that the equilibrium price will change when it changes occupations (quantity); and when it enters any occupation $k > 0$, the fixed cost C is a bygone cost for the firm, so its supply curve in any *given* occupation k is continuous. Nevertheless, the occupational choice model does not guarantee existence of equilibria even if each v_i is concave; see Roberts and Sonnenschein [1979].

rendered its projectors bankrupt.

...Much of the utility created by the building and operation of the railroad remains inappropriable. The important fact is, that this portion becomes a matter of indifference to the corporation. Benefits which the railroad company confers, but for which it can secure no reward, are of no consequence to it; they may, therefore, be sacrificed with impunity. Through the working of this principle of inappropriable utilities, much of the welfare of large populations is intrusted to corporations having no interest in maintaining it. It will be subserved as long as the company has nothing to gain by sacrificing it, not longer.

It is also interesting to observe that Clark is fully aware that the problem of "inappropriable utilities" is intimately connected with the absence of perfect competition: the above quotation comes from the chapter in his book entitled "Non-Competitive Economics".

REMARK 1 (*imperfect competition and appropriation logic*) As is well-known, the market failures illustrated in Figures 1 and 2 would disappear if we allowed the seller to act as a perfectly discriminating monopolist, not just as a simple monopolist. But this extension of appropriation logic to imperfect competition is somewhat misleading. Apart from the well-known informational demands confronting the perfectly discriminating monopolist, the Inappropriability Theorem can be used to show a fundamental difficulty. To illustrate, consider the case of bilateral monopoly. While each of the two parties could appropriate all the surplus from the other, certainly ***both*** could not simultaneously appropriate. That is, the sum of their MP's is ***strictly*** greater than the total gains from trade between them—there just is not enough surplus to go around. This is always the case when there is imperfect competition; see Makowski and Ostroy [1987].

2.2 GIVING INDIVIDUALS THEIR MARGINAL PRODUCTS IS GOOD FOR INCENTIVES

Traditional appropriation logic, as amended here to emphasize individuals rather than commodities, says that any discrepancy between private and social marginal products will typically be accompanied by market inefficiency, as illustrated by Example 1 and its variant. But it also says that if there is no such discrepancy, private initiative leads to socially efficient allocations. Let us now formally examine this second assertion, that giving individuals their marginal products is good for incentives. Accordingly, let us suppose that in an OE $(\wp, v, z)$, private and social marginal products coincide at v in the sense that

Full Appropriation (FA) For every individual i such that $V_i \neq \{v_i\}$ and every $v_i' \in V_i$,

$$PMP_i(v_i', v^i) = MP_i(v_i', v^i).$$

(Notice that any individual for whom $V_i = \{v_i\}$, a singleton, cannot influence prices by his occupational choice since his choice set is trivial; hence, we need not worry about his incentives.)

Introduce the following suggestive notation. Let $\Delta v_i \equiv v_i' \in V_i$, let

$$\frac{\Delta PMP_i(v)}{\Delta v_i} = PMP_i(v_i', v^i) - PMP_i(v),$$

and let

$$\frac{\Delta MP_i(v)}{\Delta v_i} = MP_i(v_i', v^i) - MP_i(v).$$

In this notation, **FA** implies in any OE

$$\frac{\Delta PMP_i(v)}{\Delta v_i} = \frac{\Delta MP_i(v)}{\Delta v_i} \quad \text{for all } \Delta v_i.$$

But notice that

$$\begin{aligned}\frac{\Delta MP_i(v)}{\Delta v_i} &\equiv MP_i(v_i', v^i) - MP_i(v) \\ &= [g(v_i', v^i) - g^i(v^i)] - [g(v) - g^i(v^i)] \\ &= g(v_i', v^i) - g(v) \equiv \frac{\Delta g(v)}{\Delta v_i}.\end{aligned}$$

Hence, **FA** implies in any OE

$$\frac{\Delta PMP_i(v)}{\Delta v_i} = \frac{\Delta g(v)}{\Delta v_i} \quad \text{for all } \Delta v_i.$$

But in any OE, individuals choose their occupations to maximize their private payoffs; hence in any OE satisfying **FA**

$$\frac{\Delta PMP_i(v)}{\Delta v_i} = \frac{\Delta g(v)}{\Delta v_i} \leq 0 \quad \text{for all } \Delta v_i.$$

That is, the assignment v is not Pareto dominated by any other assignment v' involving a change in a "pure direction" Δv_i. Stated as a theorem, we have proved

Theorem 2 (Partial Optimality) *If $(\wp, v, z)$ is an occupational equilibrium satisfying* **FA**, *then for all i and all $\Delta v_i \in V_i$,*

$$g(v) \geq g(\Delta v_i, v^i).$$

The word "partial" in the name of the theorem is to suggest two ideas. First and most obvious, the theorem is only a "partial" optimality result in that it does not claim that v is globally Pareto efficient. Second, the word "partial" suggests in what sense v *is* efficient; here the word is intended to be suggestive of partial derivatives: The assignment v cannot be Pareto dominated by any changes in the "pure directions", i.e., by any $v' = (\Delta v_i, v^i)$; but it may be Pareto dominated by changes in the "diagonal directions", i.e., by some $v' = (\Delta v_1, \dots, \Delta v_n)$ that involves several individuals changing occupations simultaneously. Thus the theorem does not preclude the possibility of *coordination failures* in an OE, even when everyone is rewarded with their social marginal product. We will examine this possibility in Section 4 of the paper.

REMARK 2 (*the mechanism design connection*) Readers familiar with Vickrey-Clarke-Groves mechanisms from the theory of mechanism design will see an intimate connection between the proof of Theorem 2 and the proof that Vickrey-Clarke-Groves mechanisms efficiently solve the revelation problem. This is no accident; e.g., see Makowski and Ostroy [1987, 1990] for an interpretation of these mechanisms as mimicking the logic of the perfectly competitive market. The difference is that, while in this mechanism literature there is a central allocator who can costlessly find an efficient allocation once it knows the true types of individuals, here individuals must find such an allocation on their own. Thus there is the possibility of coordination failures, to be discussed in the next section

Combining Proposition 1 and Theorem 2, we immediately have

Corollary 1 (A Partial Extension of the First Theorem) *Suppose only one individual has a non-trivial occupational choice, i.e., $V_i = \{v_i\}$, a singleton, for all individuals except one. Then, any occupational equilibrium satisfying* **FA** *is Pareto efficient.*

The corollary implies that we can construct an example of a Pareto efficient OE by modifying Example 1 so that the seller always earns his social marginal product. Since the discrepancy between his PMP and his MP resulted from his facing a downward sloping demand curve, hence having to give up a part of $g(v)$ to the buyers as consumer surplus, it should suffice if we modify the example so that the seller faces a perfectly elastic demand for his product.

EXAMPLE 2 (*an efficient occupational equilibrium*): This example is the same as Example 1 except that each buyer's preferences now exhibit a constant marginal utility from consuming the commodity equal to a for the first $\frac{d}{B}$ units, where $d > K \equiv$ the seller's maximum potential supply. That is now $V_b = \{v_b'\}$, where

$$v_b'(z_b) = \begin{cases} az_b & \text{if } z_b \in [0, \frac{d}{B}] \\ a(z_b - \frac{d}{B}) - \frac{1}{2}c(z_b - \frac{d}{B})^2 & \text{if } z_b > \frac{d}{B} \\ -\infty & \text{otherwise.} \end{cases}$$

Since $d > K$, the seller's inverse demand curve is now perfectly elastic in his operating range, i.e.,

$$\wp(k) = a \quad \text{for all } k \in [0, K].$$

Hence, the seller's profit $\pi_s(v_s^k, v^s) \equiv \pi_s(k)$ is maximized by choosing any occupation $k \in [\hat{q}, K]$ and producing where his marginal cost curve intersects the perfectly elastic portion of his demand curve (see Figure 3). So, in accord with Corollary 1, the equilibrium is efficient. Notice that **FA** is satisfied since for any occupation k he may choose

$$\pi_s(k) = g(v_s^k, v^s) = MP_s(v_s^k, v^s).$$

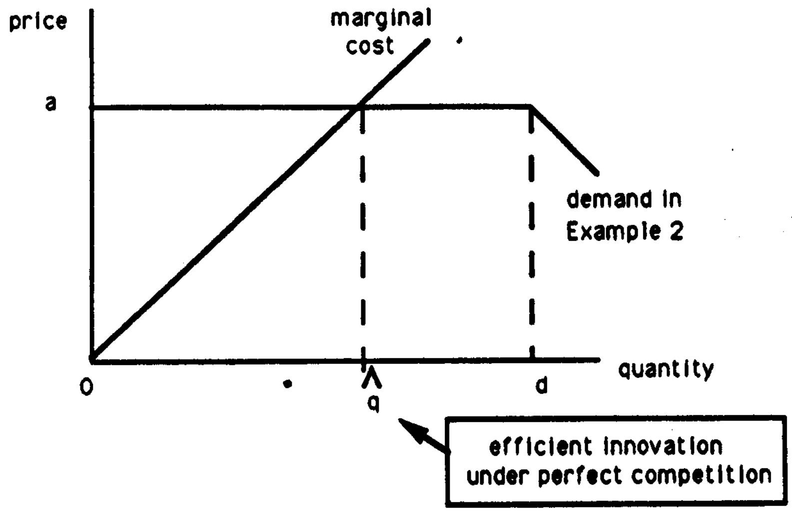

Figure 3: The occupational equilibrium in Example 2

It is interesting to observe that this efficient outcome is the limiting outcome of the occupational equilibria in Example 1 as one replicates the number of buyers. As B increases, the inverse demand curve in Figure 1 rotates around point a on the vertical axis, becoming more and more elastic. Hence asymptotically the seller's profits would equal his full social marginal product (the entire shaded area in Figure 1). Given this context, one can regard Example 2 as a finite "magnification" of the limiting economy (notice the length of the flat segment in any buyer's utility function, d/B, goes to zero as B approaches infinity; hence, v_b' approaches v_b, the preferences of the buyers in Example 1, as B approaches infinity). A similar, but asymptotic, example appears in Hart [1979].

Either the finite "magnification" or the asymptotic version of the example tells an interesting moral: A unique seller of a product may still be a perfect competitor—in the sense of facing a perfectly elastic demand for his product in the relevant region—provided his desired supply is less than the demands of the highest valuing buyers. Viewing the seller as innovating a new commodity, the example illustrates that the phrase "a perfectly competitive innovator" is not an oxymoron. This is the lesson of the literature on product innovation under perfect competition, e.g., Hart [1979, 1980], Makowski [1980b, 1983].

Since the model of occupational choice collapses to a standard Walrasian model when everyone's occupational choice is trivial, i.e., when $V_i = \{v_i\}$ for all i — in which case FA is not binding, — Corollary 1 is a generalization of the standard First Theorem. Note it does not assume complete markets! Indeed, it implies that when there is only one economic agent with a nontrivial occupational choice and he is a perfect competitor, the Invisible Hand will always efficiently guide his market-making. To illustrate, consider the variant of Example 2 in which the seller, in addition to his marginal cost, has a fixed cost of production/innovation. The efficiency result would remain intact: If his minimum average cost were less than a (see Figure 3) then he would still produce/innovate his product, producing it at the efficient level $\hat{q}$. If, on the other hand, his average cost always exceeds

a then he would not innovate his product and the resulting occupational equilibrium would exhibit incomplete markets ($H(v)$ would be empty), but this would be efficient.

Alternatively, consider the following richer example. Suppose there is only one individual i with a non-trivial occupational choice; he must choose one of K occupations. In any given occupation he can innovate exactly one of K new commodities. Then, Corollary 1 says that if he is always rewarded with his marginal product, no matter which occupation he chooses, he will innovate efficiently. That is, the resulting OE will be globally Pareto efficient, although it will necessarily involve at least $K-1$ missing markets.

Comparing these two examples with the variant of Example 1 with a fixed cost, we see that innovation under perfect competition is much more likely to turn out efficient than innovation when there is monopoly power, in accord with appropriation logic and Theorem 2.

2.3 The perfect competition connection

Both the Invisible Hand idea and the traditional formalization of it involve claims about perfect competition. Similarly, Theorem 2 above and the extended First Theorem in the next section are about perfect competition. Specifically, the key hypothesis required for efficiency, **FA**, results from perfect competition. This was illustrated in Example 2; we now show it is a general principle.

The following definition captures the idea that there is perfect competition in an occupational equilibrium, in the sense that no individual can influence prices by his/her choice of occupation.

DEFINITION: All individuals face *perfectly elastic demands* in the occupational equilibrium $(\wp, v, z)$ if

(PED) $\quad \wp(v_i', v^i) = \wp(v)$ for all i and all $v_i' \in V_i$.

Recall in the model of occupational choice individuals, by their choice of occupations, may be able to exercise initiative in price-making. Given this context, the assumption of **PED** in the model of occupational choice is the analogue, in spirit, of the price-taking assumption in the Walrasian model. Given **PED**, price-taking is a valid assumption: If any individual i switched to any other occupation v_i', then he indeed would not be able to change market clearing prices. Hence, he might as well act as a price-taker.

To establish the link from perfect competition to **FA**, define the concept of a dummy occupation.

DEFINITION: Individual i can choose a *dummy occupation* when others are in occupations v^i if there exists a $v_i{}^0 \in V_i$ such that

$$MP_i(v_i{}^0, v^i) = 0.$$

The occupational equilibrium $(\wp, v, z)$ is a *regular* occupational equilibrium if for each individual i with a non-trivial occupational choice (i.e, $V_i \neq \{v_i\}$), i can choose a dummy occupation when others are in v^i.

Since $MP_i(v_i{}^0, v^i) \equiv g(v_i{}^0, v^i) - g^i(v^i)$, a dummy occupation for i is one in which i contributes nothing to the potential gains from trade. To illustrate, in Example 2 the occupation v_s^0, where the seller could not supply any of the commodity, was a dummy occupation for the seller.

Theorem 3 *For any regular occupational equilibrium* $(\wp, v, z)$,

$$\textbf{PED} \Rightarrow \textbf{FA}.$$

Proof: Suppose $V_i \neq \{v_i\}$ and choose an arbitrary $v_i{}' \in V_i$. Let $v' = (v_i{}', v^i)$, $v^0 = (v_i{}^0, v^i)$, $p' = \wp(v_i{}', v^i)$, and $p^0 = \wp(v_i{}^0, v^i)$. Also, let (z', p') be Walrasian for v' and (z^0, p^0) be Walrasian for v^0 (such pairs exist since $\wp$ is defined as a Walrasian price selection). Notice that since $0 \in \Re^{\ell(v^0)}$, Theorem 1 implies $v_i{}^0(z_i{}^0) - p^0 z_i{}^0 = MP_i(v^0) = 0$. Hence,

$$\sum_{j \neq i} [v_j(z_j^0) - p^0 z_j^0] = g(v_i{}^0, v^i) = g^i(v^i).$$

Now given **PED**, we can let $p = p' = p^0$. Since both z_j^0 and z_j' are optimal for each individual $j \neq i$ under prices p,

$$\sum_{j \neq i} [v_j(z_j') - pz_j'] = \sum_{j \neq i} [v_j(z_j^0) - pz_j^0] = g^i(v^i).$$

But $\sum_{j=1}^{n} [v_j(z_j') - pz_j'] = g(v_i{}', v^i)$. Hence,

$$\begin{aligned} PMP_i(v_i{}', v^i) &\equiv v_i(z_i{}') - pz_i{}' \\ &= g(v_i{}', v^i) - g^i(v^i) \\ &= MP_i(v_i{}', v^i). \end{aligned}$$

□

Theorem 3, that **FA** is assured under perfect competition, is an important fact for competitive analysis. Its significance relates to the "meaning" of prices under perfect competition.

REMARK 3 (*the Invisible Hand, the meaning of competitive prices, and the two margins of analysis*) When economic agents possess monopoly power, levels of prices reflect the complex interplay of relative bargaining strengths; they may only dimly reflect the true social values of commodities. Under perfect competition, the situation is different. This is incorporated into our traditional teachings, where it is emphasized that under perfect competition *prices reflect the social values of resources*. That is the beauty of the competitive price system. Thus each seller of a commodity "sees" (appropriates) the social value of what he is producing; and each buyer of a commodity only has to pay the social opportunity cost of his obtaining the object.

Not only do prices have a meaning under perfect competition, but as a corollary, the level of profit also has a meaning: it reflects a firm's social contribution. Specifically, by Theorem 3

$$\textbf{PED} \Rightarrow \textbf{FA} \Rightarrow \pi_i(\cdot) = MP_i(\cdot).^4$$

Thus, Adam Smith's injunction to "profit maximize"—i.e., to seek to maximize your selfish interests—is in full harmony with the injunction to "maximize your contribution to society". From this perspective, the price system is viewed as the means for rewarding all individuals with their full social marginal products: to coin a phrase, "appropriation via prices". That such a reward system has remarkable allocative consequences, is shown by Theorem 2 and our extended First Theorem of Welfare Economics to follow.

In the absence of perfect competition in the sense of **PED**, Walrasian prices do not have such a distinct meaning. They reflect appropriation logic at the "commodity margin". But they may only dimly reflect it at the "individual margin"; this is the message of the Inappropriability Theorem, Theorem 1. To illustrate, consider the Walrasian equilibrium for any *efficient* assignment in Example 1, say $v*$. (That is, any assignment in which the seller enters some occupation $k \in [q*, K]$ and consequently produces $q*$ units. See Figure 1.) The seller appropriates only a small fraction of his marginal product in this Walrasian equilibrium, even though he does fully appropriate the contribution of the last infinitesimal unit he supplies in $v*$.

Although the two margins often merge—i.e, when there is perfect competition in the sense of **PED**,— our position is that whenever the two margins do diverge, one should follow the path of the individual margin of analysis. To illustrate, by following the individual margin one sees why the Walrasian equilibrium for $v*$ (above) is no equilibrium at all, since the seller can at least appropriate a larger fraction of his potential social marginal product by switching occupations (read: undersupplying).

Following the individual margin of analysis offers the prospect of further unifying the theories of market success and market failure since it is the individual margin of appropriation that frequently underlies the intuitive description of market failure. For example, we say that a desirable new bridge will not be supplied without government intervention because any private supplier could only appropriate a small fraction of the bridge's *total* social value. This may be illustrated as in Figure 2, with the average cost curve declining throughout its domain, rather than U-shaped. Notice the fixed cost (indivisibility) of the bridge causes the two margins to diverge.[5] Also notice that the issue here is not the absence of markets (read: real externalities)—one can charge a toll for crossing the bridge,—but whether the market price will be sufficiently high to adequately reflect the bridge's true social value or will have to be so low as to only create large beneficial pecuniary externalities—in Clark's language, "inappropriable utilities"—for its users.

[4] A partial converse to Theorem 3 can also be given, see Ostroy [1980] or Makowski [1980b]. But in the current context, the important fact is that **FA** is assured under perfect competition.

[5] Similarly, in Example 1, it is the "largeness" of the monopolist's potential supply relative to the demands of the highest valuing buyers that causes the two margins to diverge.

3. The Coordination Problem and a Complete Extension of the First Theorem

3.1 The coordination problem resulting from the superadditivity of changes in the gains from trade

While giving individuals their marginal products is good for incentives, **FA** does not suffice to guarantee that all occupational equilibria will be globally Pareto efficient. Continuing with our suggestive notation, let $\Delta v = (\Delta v_1, \Delta v_2, \ldots, \Delta v_n) \in V$ and $\frac{\Delta g(v)}{\Delta v} = g(\Delta v) - g(v)$. Then, if v is an equilibrium assignment, **FA** ensures

$$\frac{\Delta g(v)}{\Delta v_i} \leq 0 \text{ for all } \Delta v_i;$$

but it does not ensure

$$\frac{\Delta g(v)}{\Delta v} \leq 0 \text{ for all } \Delta v,$$

i.e., including Δv involving multi-agent changes of occupation. The possibility of coordination failures can be illustrated by framing Edgeworth's famous master-servant example as a problem of occupational choice. An economically more interesting coordination failure appears in Section 5.2; here our purpose is to show that **FA** may not suffice for global efficiency.

Example 3 (*the occupational equilibria for a master-servant example*): Suppose $\ell = 1$; call this commodity "servant services". Partition the set of individuals into B buyers of the services (called "masters"and indexed by b) and S sellers of the services (called "servants" and indexed by s); suppose they are of equal number, $B = S = n/2$. Each servant must choose one of $K+1$ occupations: $V_s = \{v_s^k : k = 0, 1, \ldots, K\}$. In occupation k he can supply up to k units of services at a marginal cost of c each, hence

$$v_s^k(z_s) = \begin{cases} cz_s & \text{if } z_s \in [-k, 0] \\ -\infty & \text{otherwise.}^6 \end{cases}$$

Similarly, each master must choose one of $K+1$ occupations: $V_b = \{v_b^k : k = 0, 1, \ldots, K\}$. No master has any endowment of the services and in occupation k he desires at most k units of the commodity at a willingness-to-pay of w each, hence

$$v_b^k(z_b) = \begin{cases} wz_b & \text{if } z_b \in [0,k] \\ wk & \text{if } z_b > k \\ -\infty & \text{if } z_b < 0. \end{cases}$$

Any assignment v can be simply thought of as an n-vector $a \in \Re^n_+$, where $a_i \in \{0, 1, ..., K\}$ represents the occupational choice of individual i. We assume that $w > c > 0$, hence clearly for efficiency we want $a = a* \equiv (K, K, \ldots, K)$ with each servant selling K units and each master buying K units.

[6] Edgeworth's version assumed servants' services are indivisible. But, contrary to his impression, this assumption was not essential to his moral. Thus, we drop it.

Edgeworth [1881] used the example to show that in some economies, large numbers do not guarantee perfect competition. He restricted himself to the case $K = 1$; but his observation holds for any K. He observed that in the assignment $a*$, any price between c and w is market clearing (see Figure 4). But if one master were to reduce his demands,

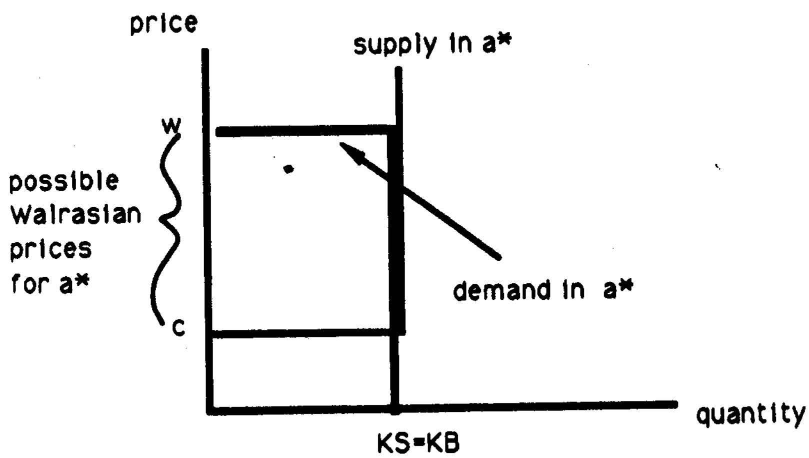

Figure 4: The efficient Walrasian equilibrium in Example 3

then the (*unique* Walrasian) price would fall to c; while if any one servant were to reduce his supply of services, the price would jump to w. This, no matter how large is n. Hence any one master or servant in the efficient Walrasian equilibrium always has monopoly power (in our language, he does not face PED). It is left to the interested reader to check that for any $K \geq 1$,

- in all occupational equilibria, either all servants are selling at most 2 units or all masters are buying at most 2 units.

Thus, the larger is K, the less efficient are the occupational equilibria.[7]

Our current interest in the example is to show that giving individuals their marginal products, FA, may not suffice to guarantee global efficiency. Accordingly, focus on the assignment $a^0 \equiv (0, 0, \ldots, 0)$. This may be thought of as the autarkic assignment since there is only one trade that is feasible in a^0, namely, $z = (0, 0, \ldots, 0)$. Let

$$p(a) = \begin{cases} 0 & \text{if } \sum_b a_b = 0 \text{ or } \sum_s a_s = 0 \\ c & \text{if } \sum_s a_s > \sum_b a_b > 0 \\ \frac{w+c}{2} & \text{if } \sum_s a_s = \sum_b a_b > 0 \\ w & \text{if } \sum_b a_b > \sum_s a_s > 0 \end{cases};$$

let $z = (0 \ldots, 0)$; and let v be the autarkic assignment a^0. Then (p, v, z) is an OE since for each i and each $v_i{}^k \in V_i$

$$PMP_i(v) = PMP_i(v_i{}^k, v^i) = 0.$$

[7] It should now be clear why considering $K > 1$ is more interesting; in Edgeworth's version, even though individuals can influence prices, the efficient assignment $a = (1, 1, \ldots, 1)$ *is* a Nash equilibrium.

Further, this assignment satisfies **FA** since

$$MP_i(v) = MP_i({v_i}^k, v^i) = 0.$$

The reason for these properties of v should be plain: There is no private or social gain from supplying servant services if no master needs any. And given $H(v) = \emptyset$ (there is not market for servant services), masters are indifferent among their possible occupations, including the one they choose in v, the one in which they do not need any servants.[8] Also note that the autarkic equilibrium is perfectly competitive (satisfies **PED**), e.g., servants can sell all the services they want—they face a perfectly elastic demand curve—at a price of zero.

The problem illustrated by the example may be usefully viewed as a coordination failure. Starting from a^0, notice that the social marginal product of either a servant in occupation $k > 0$ or of a master in occupation $k > 0$ is zero; but the social marginal product of a simultaneous switch, where both a servant and a master switch to an occupation $k > 0$, is strictly positive, namely $k(w - c)$. Alternatively expressed, at the autarkic assignment v, for some Δv the changes in the gains from trade are strictly ***superadditive***:

$$\sum_i \frac{\Delta g(v)}{\Delta v_i} < \frac{\Delta g(v)}{\Delta v}.$$

In words, the gains from a simultaneous occupational switch by several individuals exceeds the sum of the gains from the individual switches. Heuristically, the whole is bigger than the sum of its parts. Or in economic terms, there are complementarities between some of the new occupational choices in Δv.

3.2 The main result

Our extension of the First Theorem says that, provided everyone is rewarded with his/her social marginal product, such superadditivity is the *only* possible source of inefficiency. Say that the changes in the gains from trade are ***subadditive at v*** or, synonymously, satisfy the ***non-complementarity*** condition if

Non-Complementarity (NC) For all assignments $\Delta v \in V$,

$$\sum_i \frac{\Delta g(v)}{\Delta v_i} \geq \frac{\Delta g(v)}{\Delta v}.$$

Theorem 4 (An Extension of the First Theorem of Welfare Economics) *Any occupational equilibrium* $(\wp, v, z)$ *satisfying* **FA** *and* **NC** *is globally Pareto efficient.*

[8] Following the literature on innovation under perfect competition, e.g. Hart [1979, 1980], the model of occupational choice has built in the assumption that suppliers of new commodities make the market (recall the definition of $H(v)$); hence masters must passively accept that the market for servant services is missing in v.

Proof:[9] By definition of an OE, for all i and all Δv_i,

$$\frac{\Delta PMP_i(v)}{\Delta v_i} \leq 0.$$

Thus,

$$\sum \frac{\Delta PMP_i(v)}{\Delta v_i} \leq 0.$$

But by **FA**, the $LHS = \sum \frac{\Delta MP_i(v)}{\Delta v_i} = \sum \frac{\Delta g(v)}{\Delta v_i}$. Thus, using **NC**,

$$\frac{\Delta g(v)}{\Delta v} \leq 0,$$

i.e., no assignment v' Pareto dominates v. So, if $\sum v_i(z_i) = g(v)$ then (v, z) is globally Pareto efficient.

But from the definition of an OE, for all i and all $z_i' \in \Re^{\ell(v)}$

$$v_i(z_i) - \wp(v)z_i \geq v_i(z_i') - \wp(v)z_i'.$$

Thus, since z is feasible for v, summing over the i,

$$\sum v_i(z_i) \geq \sum [v_i(z_i') - \wp(v)z_i'].$$

Or, for any z' that is feasible for v,

$$\sum v_i(z_i) \geq \sum v_i(z_i'),$$

i.e., $\sum v_i(z_i) = g(v)$, as required. □

Observe that the last step in the proof—the demonstration that z is efficient relative to v—follows the lines of the standard proof of the First Theorem. But in the current context—where price-taking and market-taking cannot be taken for granted—"efficiency relative to v" is a weak property, not the whole story. To illustrate, recall from Example 1 that the simple monopoly outcome is "efficient" in this sense. Similarly, all OE in the master-servant example are efficient in this sense; but if K is large then all these OE are very far from globally efficient.[10]

To give an example of an efficient OE with several individuals making occupational choices, hence an example in which the coordination problem is non-trivial, consider the following variant of Example 3.

EXAMPLE 4: This example is just like Example 3 except there are now at least two more masters than servants, $B - 1 > S > 0$.

Claim: The following $(\wp, v, z)$ is an efficient OE for the example. Let $\wp$ be defined as in Example 3; let $v_s = v_s^K$ for each servant and $v_b = v_b^K$ for each master; finally, let $z_s = -K$ for each servant, $z_b = \frac{B}{S}K$ for each master. The equilibrium is illustrated in Figure 5. Notice

[9] While parts of the argument have appeared earlier, i.e., in the proofs of Proposition 1 and Theorem 2, to make the proof self-contained we repeat these parts here.

[10] In the degenerate case when $V_i = \{v_i\}$ for each i, the model of occupational choice collapses to a standard Walrasian model. Then **FA** and **NC** are satisfied trivially in any OE, i.e., issues of appropriation become moot. And, as one would expect, our proof of the First Theorem then collapses to the standard proof.

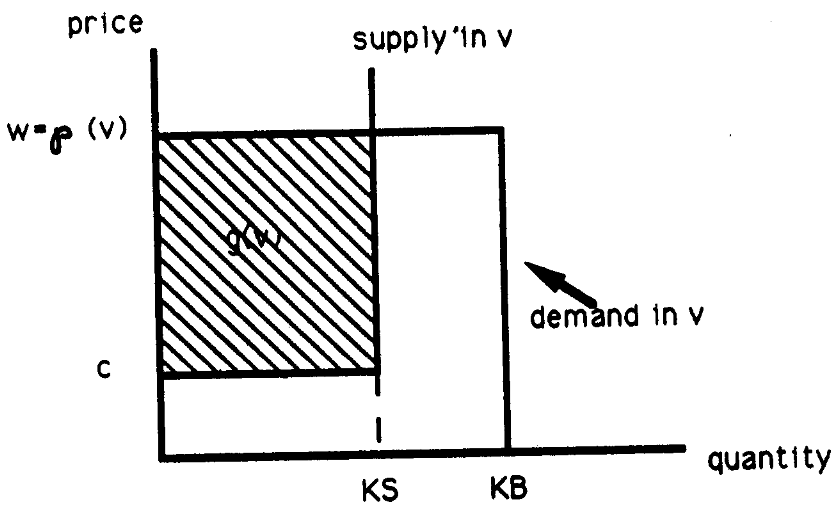

Figure 5: The occupational equilibrium in Example 4

the servants appropriate the whole gains from trade since competition among masters leads to the equilibrium price being bid up to w.

Even though the global efficiency of the equilibrium is obvious in this example, it is instructive to observe that this OE satisfies FA and NC, hence it illustrates Theorem 4. Specifically, it is left to the reader to verify that in the OE, FA is satisfied since for all s and all v_s^k

$$PMP_s(v_s^k, v^s) = MP_s(v_s^k, v^s) = k(w - c),$$

while for all b and all v_b^k

$$PMP_b(v_b^k, v^b) = MP_b(v_b^k, v^b) = 0.$$

It can also be verified that NC is satisfied. This may be less obvious, so the reader may wish to wait for Theorem 5 in the next section.

4. Two Special Cases of the Optimality Theorem

The extended First Theorem depends on two conditions, FA and NC. Full appropriation is implied by the hypothesis of perfect competition (Theorem 3), hence it can be taken for granted in the current context: a formalization of the Invisible Hand idea.[11] But NC is not implied by perfect competition, e.g., the autarkic equilibrium in Example 3 satisfies PED but not NC. Thus the question remains, when will NC be satisfied? This section gives two answers: (i) when markets are complete, and (ii) under some conditions, even when

[11] Of course, the question of the *existence* of perfect competition, hence of FA, is also interesting. While finite examples involving perfect competition can be constructed, as illustrated in this paper, for a general existence result one needs a model with a non-atomic continuum of agents. (This ensures that each individual's demands and supplies will be small relative to the market.) See Makowski and Ostroy [1989] for a continuum version of the model of occupational choice.

they are incomplete. In the process we will build some bridges to the literature. First, we examine the connection between the standard and extended First Theorems. Second, we build a bridge from the extended First Theorem to the literature on underdevelopment traps (à la Scitovsky [1954]) and innovation under perfect competition.

4.1 An optimality theorem in the spirit of Walras

Even given perfect competition, non-complementarity is an additional assumption required to ensure efficiency. Notwithstanding Adam Smith's praises of its powers, in the absence of **NC** the Invisible Hand may fail to efficiently guide entrepreneurs in their market-making.

On the other hand we shall now show, when competition is restricted to coordinating economic activities under *complete markets* then **FA** and **NC** are both assured. Thus the assumptions of the extension do not, in spirit, go beyond those used to prove the traditional First Theorem. (We say "in spirit" for two reasons. First, as already noted, since price-making is permitted in the model of occupational choice, we require price-taking to be validated (**PED**); we cannot just take it for granted. Second, since market-making is also permitted, our definition of complete markets, see below, will be slightly stronger than that in the Walrasian model. The first point is important; the second, merely technical.)

Definition: Markets are *complete* in an occupational equilibrium $(\wp, v, z)$ if

(C) $\quad \Re^{\ell(v)} = \Re^{\ell} = \Re^{\ell(v_i', v^i)}$ for all i and all $v_i' \in V_i$.

Note the definition requires markets to be complete irrespective of any one individual's occupational choice; hence no one individual is responsible for "completing" the set of markets. Thus, to a limited extent, the definition requires that markets are not just complete, but also "thick" (e.g., the efficient occupational equilibrium in Example 2 does not satisfy C since there is only one potential supplier of the commodity).

One would expect that together **PED** and **C** should suffice to prove the global optimality of occupational equilibria since these are the essential ingredients used to prove the traditional First Theorem. This is confirmed by the following result which is—as will be clear from the proof— essentially the Walrasian First Theorem, framed inside the model of occupational choice. It is interesting to observe that once the strong assumption of complete markets is imposed, optimality can be proved along standard lines, which minimize the explicit role of appropriation logic.

Proposition 2 (First Theorem of Welfare Economics in the Spirit of Walras) *Any occupational equilibrium $(\wp, v, z)$ satisfying* **PED** *and* **C** *is globally Pareto efficient.*

Proof: By definition of an OE, for all i, all v_i', and all $z_i' \in \Re^{\ell(v_i', v^i)}$:

$$v_i(z_i) - \wp(v)z_i \geq v_i'(z_i') - \wp(v_i', v^i)z_i'.$$

Given **PED** and **C**, this can be strengthened to, for all i, all v_i', and all $z_i' \in \Re^\ell$:

$$v_i(z_i) - pz_i \geq v_i'(z_i') - pz_i',$$

where $p \equiv \wp(v)$. Summing over the i, for all $v' \equiv (v_i')$ and all $z' \equiv (z_i')$ such that $\sum z_i' = 0$

$$\sum v_i(z_i) \geq \sum v_i'(z_i'),$$

i.e., (v, z) is globally Pareto efficient. □

We can now show the interesting fact mentioned above, that while **NC** goes beyond the mere hypothesis of perfect competition, it does not in spirit go beyond the assumptions needed to prove the traditional First Theorem. So the extension's stronger conclusions about the efficacy of the Invisible Hand are bought at a zero price, in terms of stronger assumptions.

Theorem 5 *In any regular occupational equilibrium* $(\wp, v, z)$,

PED *and* **C** $\Rightarrow$ **FA** *and* **NC**.

Proof: We already know from Theorem 3 that **PED** implies **FA**. To verify **NC**, let $v' \in V$ and let z_i' be i's Walrasian trade in the assignment (v_i', v^i). Using **PED** and **C**

$$v_i'(z_i') - pz_i' \geq v_i'(z_i'') - pz_i'' \quad \text{for all } z_i'' \in \Re^\ell,$$

where $p \equiv \wp(v)$. Note by **FA**, the LHS equals $MP_i(v_i', v^i)$. Summing, for all v' and all z'' such that $\sum z_i'' = 0$,

$$\sum MP_i(v_i', v^i) \geq \sum v_i'(z_i'').$$

Letting z'' be such that $\sum v_i'(z_i'') = g(v')$ shows

$$\sum MP_i(v_i', v^i) \geq g(v').$$

Thus, since $\sum MP_i(v) = g(v)$, we have for all $\Delta v \equiv v'$:

$$\sum \frac{\Delta MP}{\Delta v_i} \geq \frac{\Delta g(v)}{\Delta v}.$$

Since the LHS equals $\sum \frac{\Delta g(v)}{\Delta v_i}$, we have arrived at **NC**. □

REMARK 4 (*"coordination via prices" versus "appropriation via prices"*) This remark continues along the lines begun in Remark 3. Depending on one's interpretation of the Walrasian model, its twin assumptions of price-taking and complete markets function either as a substitute or as a veil for appropriation logic at the individual margin.

Interpreted literally, the twin assumptions are a substitute. To illustrate, consider the Walrasian version of Example 1 in which the firm does not consider shading its production; hence its occupational choices are trivial, $V_s = \{v_s^K\}$. The Walrasian equilibrium for this economy will be efficient—the firm will produce $q*$ units—since it is *required* to act as a price-taker even though it really faces a downward sloping demand curve. Thus, the standard First Theorem "goes through" in spite of the fact that the firm only appropriates a small fraction of its social marginal product when it produces $q*$ in the Walrasian equilibrium. This illustrates that, under the literal interpretation, the Walrasian model is a model of "coordination via prices", but not a model of "appropriation via prices".

Under (what we regard as) a more sympathetic interpretation of the Walrasian model, the twin assumptions act as a veil. In this interpretation it is implicitly understood that the Walrasian model is only meant to be applied to economies in which individuals really face perfectly elastic demands, hence, economies in which the price-taking assumption can be validated. In such economies, we have seen that full appropriation will hold (Theorem 3); but given the twin assumptions of the Walrasian model this fact is not important—it can be overlooked, as it traditionally has been. With complete markets, the First Theorem can be proved without recourse to appropriation logic in any vital sense—without penetrating behind the veil of price-taking to its appropriation realities,— as is illustrated in the proof of Proposition 2 above.

4.2 An optimality theorem in the spirit of Scitovsky

4.2.1 Background

In his well-known article "Two Concepts of External Economies", Scitovsky argues for a broadening of the concept of externalities to include not just "real externalities", but also "pecuniary externalities", i.e., interdependences among economic agents that operate through the market mechanism. He points out that in the development literature (at least of his time) pecuniary externalities figure prominently as a powerful explanatory concept, while real externalities figure hardly at all. Under the umbrella of pecuniary externalities, Scitovsky includes situations where an economic agent possesses monopoly power and consequently can effect the market prices others face (in our language, situations where there is not **PED** as in Example 1 and its variant). But he is most interested in the pecuniary externalities that can arise in a competitive setting.

Scitovsky sketches an example with two industries, A and B, where the output of A is an input used in industry B. If A expanded, commodity A would become cheaper to industry B which in turn would expand. Contrariwise, if B expanded then A's demand would increase and it would have an incentive to expand. But without this push from B, A may remain underdeveloped, and consequently B may also remain underdeveloped for want of cheap inputs from A: the economy may remain stuck in an underdevelopment trap. Applying appropriation logic, he concludes that when production "gives rise to pecuniary external economies its private profitability understates its social desirability" (p. 149). In terms of the model of occupational choice, from this one might be tempted to infer that Scitovsky's example involves a failure of **FA**. But we think it does not (indeed, this would be impossible under perfect competition, recall Theorem 3). Rather, what Scitovsky's interpretation shows is that failures of **NC** also sometimes may be interpreted as failures of appropriability, broadly conceived. To make this point, we sketch a Scitovsky-like example using the model of occupational choice.

The following may be thought of as a variation on Example 3. Suppose in addition to money there are only two commodities, A and B. Partition the set of agents into a set of firms in industry A, a set of firms in industry B, and a set of final consumers who only like commodity B (and, of course, money). Like the servants in Example 3, any firm in industry A can enter one of $K+1$ occupations; in occupation k it can supply up to k units of A at a marginal cost of c each. As in Scitovsky's example, firms in industry B use A as an input. Specifically, any firm in industry B can enter one of $K+1$ occupations, indexed by $k = 0, 1, \ldots, K$; in occupation k it can supply up to k units of B using only A's as inputs: α units of A produce α units of B, for any $\alpha \in [0, k]$. Consumers have only one occupation—they are passive— in which they are willing to consume any amount of B at a price of w per unit (w equals their constant marginal utility from consuming B's), where $w > c > 0$.

Clearly, for efficiency we want both industries to operate up to capacity since $w > c$. But like the master-servant example it is easy to check that autarky, with zero production by both, is also an OE. Specifically, let v be the assignment in which each firm enters its occupation $k = 0$; hence, the assignment in which the set of marketed commodities $H(v) = \emptyset$. This assignment is supported as an OE by a p such that for all i and all (v_i', v^i), the prices of both A and B equal zero. The OE obviously satisfies **PED**, and hence **FA**: In the absence of any demand from industry B, the social and private marginal products of any firm entering industry A (i.e., some occupation $k > 0$) would be zero; similarly, in the absence of any supply of inputs from industy A, the social and private marginal products of any firm entering industry B would be zero. But, as in the master-servant example, this equilibrium involves a failure of **NC**.

Specifically, the market failure can be explained in terms of a failure of "group appropriation". In the above OE there is a group change, $\Delta v \equiv v'$, that would result in a Pareto improvement, $\frac{\Delta g(v)}{\Delta v} > 0$. But the change in the group's private marginal product (resulting from the group change), $\sum \frac{\Delta PMP_i(v)}{\Delta v_i} \equiv \sum[\pi_i(v_i', v^i) - \pi_i(v)]$, is less than the change in the group's social marginal product, $\frac{\Delta g(v)}{\Delta v}$. Specifically, if a group consisting of 1 firm from each industry entered their occupations $k = 1$, the gains from trade would increase by $w - c$. But the change in the group's private marginal product from this pair of occupational switches is 0+0=0, less than the change in the group's social marginal product. Because the private return is zero, the change is not made. Notice in particular that since **PED** $\Rightarrow$ **FA**, the change in the group's private marginal product, $\sum \frac{\Delta PMP_i(v)}{\Delta v_i}$, just equals the sum of the changes in the members' social marginal products, $\sum \frac{\Delta MP_i(v)}{\Delta v_i}$. Hence, in accordance with Theorem 4, in this example **NC** is violated:

$$\sum_i \frac{\Delta MP_i(v)}{\Delta v_i} < \frac{\Delta g(v)}{\Delta v}.$$

Alternatively, the failure can be explained in terms of pecuniary externalities. As above, let v' be the assignment in which all firms except one firm in each industy remain in their v occupations; exactly one firm in each industry switches to its occupation $k = 1$. Hence all commodities are marketed in v', $H(v') = \{1, 2\}$. Let v^A (resp., v^B) be the assignment in which all firms except one are in their v occupations, with exactly one industry A firm (resp., B firm) switching to its occupation $k = 1$. Notice that the set of possible Walrasian prices for commodity A in v' is the interval $[c, w]$, while the price of B must be w since consumers' demand is perfectly elastic at w. Thus in the assignment v', at least one of the two firms in the group would make positive profits (the fact that the sum of their profits would equal $w - c$ implies that the group would earn its social marginal product in switching to v'). Thus, starting from v, at least one of the following is true:

- If an industry A firm entered its occupation $k = 1$—moving the economy to v^A—then an industry B firm would see positive profits to entering its occupation $k = 1$—moving the economy on to v'. That is, the industry A firm could create a positive pecuniary externality for an industry B firm; or,

- If an industry B firm entered its occupation $k = 1$—moving the economy to v^B— then an industry A firm would see positive profits to entering its occupation $k = 1$—moving the economy on to v'. That is, the industry B firm could create a positive pecuniary externality for an industry A firm.

The reason the economy does not move beyond v to v' is that no individual A or B firm takes the favorable pecuniary externalities it can create for others into account in its private optimizing decisions.

To quote again from Scitovsky (p. 149):

> ...what inhibits investment in A is the limitation on the demand for industry A's product imposed by the limited capacity of industry B, the consumer of this product; just as investment in industry B is inhibited by the limited capacity of industry A, the supplier of one of industry B's factors of production. These limitations can be fully removed only by a simultaneous expansion of both industries. We conclude, therefore, that only if expansion in the two industries were integrated and planned together would the profitability of investment in each one of them be a reliable index of its social desirability.

4.2.2 An optimality theorem with incomplete markets

Both Example 3 and the Scitovsky example illustrate that in the absence of complete markets **NC** is not assured, even under perfect competition (unless, of course, there is only one agent with nontrivial occupational choices, in which case **NC** is trivially satisfied and Corollary 1 takes the foreground). One would like conditions under which **NC** is satisfied even when the coordination problem is nontrivial. The literature on product innovation under perfect competition, e.g. Hart [1980], Makowski [1980b], provides sufficient conditions under which perfect competition will lead to efficient innovation. In this subsection we provide a bridge to this "applied" literature by showing that under the conditions identified by this literature, **NC** is satisfied. Hence efficiency in product innovation is implied by the extended First Theorem, Theorem 4.

With intermediate innovations (innovations useful for producing yet other innovations), efficiency is very unlikely as illustrated by the Scitovsky example. Several other examples illustrating this point are given in Makowski [1980b]. Further, even when all innovations are earmarked for final consumers, efficiency may fail if individuals' preferences are non-convex or non-differentiable (perfect complements). See Hart [1980] for several interesting examples illustrating this. Accordingly, in our simple innovation model below, we will exclude these troublesome cases to identify a family of environments in which perfectly competitive innovation will be efficient.

A SIMPLE INNOVATION MODEL. Partition the set of individuals into B, a set of consumers, and F, a set of producers/firms. Suppose only firms can innovate new commodities, so consumers are relatively passive. Specifically, suppose each consumer b has only one occupational choice and has concave, differentiable preferences over trades:

(a.1) The sets B and F partition $\{1,\ldots,n\}$. For each individual $b \in B$, $V_b = \{v_b'\}$ and there is a concave, continuously differentiable function $v_b : \Re^\ell \to \Re$ such that $v_b'(z_b) = v_b(z_b)$ for all $z_b \in \text{dom } v_b$.

Also suppose each consumer b only has a commodity endowment ω_b and no transformation possibilities, hence his trading possibility set is given by

(a.2) for all $b \in B$, dom $v_b' = \{z_b : z_b \geq -\omega_b, \text{ where } \omega_b \in \Re^\ell_+\}$.

Firms only like money, which is not used as an input, hence

(a.3) for all $f \in F$ and all $v_f' \in V_f$:

$$z_f' \in \text{dom } v_f' \Rightarrow v_f'(z_f') = 0.$$[12]

They may have many occupations, which may be interpreted as activities involving innovating different commodities. In the literature above it is typically assumed that a firm can only innovate one commodity from a set of possible commodities. We will not need to assume this, but we will assume that all innovations are earmarked for final consumers:

(a.4) for all $f \in F$ and all v_f' :

$$z_f' \in \text{dom } v_f' \Rightarrow z_f' \in \Re^{\ell(v_f', v^f)}.$$

Hence, to produce z_f', firm f does not require any other firms' potential innovations.

A central result in the competitive innovation literature is what Hart (1980) calls the "simultaneous reservation price property". Under the above conditions, any perfectly competitive, regular OE will exhibit this property.

[12]Interpret $v_f'(z_f') = 0$ to mean z_f' involves a feasible input/output combination for firm f.

Lemma 1 (the simultaneous reservation price property) *In any regular occupational equilibrium* $(\wp, v, z)$ *satisfying* **PED** *and* **a.1-4**, *for all b:*

$$z_b \textit{ maximizes } v_b(z_b') - pz_b' \textit{ over all } z_b' \in \Re^{\ell},$$

where $p \equiv \wp(v)$.

Proof: See Appendix B. □

The result is called the "simultaneous reservation price property" because it says that even if markets were complete and consumers could choose any consumption bundle they liked in $\Re^{\ell}$, not just bundles in $\Re^{\ell(v)}$, the prices p would lead them to still choose their optimal bundles in v.

The reader may surmise that the simultaneous reservation price property has an interpretation as a "no Scitovsky-type pecuniary externalities" property; e.g., see Hart (1980) or Makowski (1980b). This may be explained as follows. Suppose innovating firms in all the different industries were integrated (although at a sufficiently small scale as to preserve the perfect competition between firms assumed so far).[13] Then all commodities for which there do not exist markets in v could be introduced simultaneously by one (integrated) firm—at a scale such that the firm's outputs would be small relative to their market demands. That is, the firm's supplies of new commodities would be sufficiently small so that it would receive the economy's reservation prices for its commodities, like the innovating firm in Example 2. The simultaneous reservation price property says that the prices this firm would receive for its commodities are no different than the prices the non-integrated firms would receive. In terms of the Scitovsky example, when the simultaneous reservation price property holds, no industry A firm could create a positive external pecuniary economy for an industry B firm, or vice versa.

When there is perfect competition and the simultaneous reservation price property holds, optimality is assured by Theorem 4, whether or not markets are complete. This is an immediately corollary of the following result, which may be thought of as an incomplete markets analogue of Theorem 5.

Theorem 6 *In any regular occupational equilibrium* $(\wp, v, z)$,

$$\textbf{PED} \textit{ and } \textbf{a.1-4} \Rightarrow \textbf{FA} \textit{ and } \textbf{NC}.$$

Proof: Since **PED** implies **FA**, we need only verify **NC**. Let $v' \equiv \Delta v \in V$ and let the trade z' be efficient relative to v'.

By the simultaneous reservation property, for each consumer b, for any z_b'

$$v_b(z_b) - pz_b \geq v_b(z_b') - pz_b'.$$

[13] It is interesting to observe that this is precisely the sort of mental experiment that Scitovsky suggests we try, in order to understand the source of the market failure in his example: "[Compare] the situation under consideration with that which would obtain if industries A and B were integrated (although in such a way as to preserve the free competition assumed so far)." (p. 149)

Hence, summing over consumers,

$$\sum v_b(z_b) - \sum pz_b \geq \sum v_b(z_b') - \sum pz_b'.$$

Since firms only like money, the first term on the LHS equals $g(v)$ and the first term on the RHS equals $g(v')$; also since both z and z' are feasible trades we have

$$g(v) + \sum pz_f \geq g(v') + \sum pz_f'.$$

But for each firm f

$$-pz_f = \pi_f(v), \text{ with } \pi_f(v) = MP_f(v) \text{ if } V_f \neq \{v_f\}$$

where the second equality follows from **FA**. Similarly, since z_f' is in f's effective domain when he is in occupation v_f', **a.4** implies for each firm f

$$-pz_f' \leq \pi_f(v_f', v^f), \text{ with } \pi_f(v_f', v^f) = MP_f(v_f', v^f) \text{ if } V_f \neq \{v_f\}$$

where the inequality uses **PED** and the equality follows from **FA**. Substituting these last two displayed results into the immediately preceding displayed result shows,

$$g(v) - \sum MP_f(v) \geq g(v') - \sum MP_f(v_f', v^f),$$

where both summations are taken only over firms f such that $V_f \neq \{v_f\}$. That is, since consumers cannot change occupations,

$$\sum \frac{\Delta MP(v)}{\Delta v_i} \geq \frac{\Delta g(v)}{\Delta v},$$

as was to be shown. □

An immediate corollary from combining Theorems 4 and 6 is an optimality result with incomplete markets.

Corollary 2 (Optimality Theorem in the Spirit of Scitovsky) *Any regular occupational equilibrium* $(\wp, v, z)$ *satisfying* **PED** *and* **a.1-4** *is globally Pareto efficient*

We call the corollary an optimality theorem in the spirit of Scitovsky, not because he conjectured the result, but because it helps delimit the scope for the sort of market failures he introduced into economic theory. Notice the two key assumptions: **PED**, hence the absence of pecuniary externalities arising from individuals having monopoly power, i.e., being able to influence the prices others face for marketed goods, and **a.1-4**, which assures the simultaneous reservation price property, hence the absence of Scitovsky-type pecuniary externalities in the innovation process. Thus, heuristically, the corollary says that in the absence of pecuniary externalities, global efficiency will result—whether or not markets are complete.

5. Pecuniary Externalities as a Source of Market Failure

The goal of any formalization of an Invisible Hand Theorem is to guide our understanding of market success/failure. Thus the full appropriation and non-complementarity conditions of the extended First Theorem define an agenda: to find out when markets will work successfully, find environments under which these conditions hold; conversely, in environments for which these conditions fail, be on the lookout that markets may not work so well. Similarly the price-taking and complete markets conditions define the agenda of the standard First Theorem. How do the two agendas compare? Overall, we shall argue that the full appropriation and non-complementarity conditions gives a more significant role to the elimination of *pecuniary* externalities as a rationale for market success and, conversely, these conditions attach greater emphasis to the presence of pecuniary externalities as a source of market failure.

Perhaps the most basic contrast is the explicit attention paid in the extension to the conditions for perfect competiton, i.e. full appropriaton, while perfect competition is only implicit in the price-taking hypothesis of the standard First Theorem. Links with full appropriation are so loose in the standard Theorem that no general equilibrium explanation is really provided for the inefficiencies associated with monopoly power. Instead, the explanation of market failure due to monopoly power has largely been left to partial equilibrium analysis where the traditional argument is in terms of the commodity margin (price greater than marginal cost) or to partial equilibrium game theoretic models of strategic interaction.[14] By contrast in our extension of the First Theorem the importance of eliminating monopoly power becomes one of the two central conditions to be satisfied. In addition, full appropriation gives a general equilibrium interpretation as to why the presence of monopoly power causes problems: monopoly power leads to the failure of full appropriation at the individual margin, which in turn leads to pecuniary externalities. For example, the supplier who undersupplies because he cannot appropriate the full social benefits of additional units ignores the adverse effect on consumers of the price rise/pecuniary externality his undersupply creates. As with externalities in general, this adverse effect is irrelevant as far as the supplier is concerned since the consumers' surplus is inappropriable.

A central part of the agenda is to identify environments in which full appropriation exists. As stated in the Introduction, a formal examination of this problem would take us beyond the framework of this paper to include models with large numbers of individuals as well as large numbers of commodities. Heuristically, however, the mostly negative conclusions for models with small numbers can be carried over to the continuum setting when, because of increasing returns which are significant relative to the economy as a whole, one or more agents operate at a non-negligible scale. But "atoms" are not the only source of failures of full appropriation. The presence of pure public goods (Makowski and Ostroy [1987,1991])

[14] It can be argued that to some extent monopoly problems are implicitly covered in the standard First Theorem by the *non*-existence of Walrasian equilibrium, e.g., when monopoly power originates in non-convexities precluding existence. But then the importance of full appropriation is obscured by non-existence. Further, as emphasized in Theorem 1, the converse is not true: existence of Walrasian equilibrium does not guarantee the existence of full appropriation.

or monopolistically competitive product differentiation (Ostroy and Zame [1988], Gretsky, Ostroy and Zame [1991]) also preclude full appropriation in nonatomic economies.

The above are familiar instances of market failure. In fact, when to the list (i) large-scale increasing returns, (ii) public goods, and (iii) monopolistic competition, we add (iv) *non-pecuniary* (real) external effects, we have the traditional taxonomy of market failure. While the list is the same, the reasons for their inclusion are not identical. There is a long and mostly informal history pointing to the similarities among (i)–(iv), so what we are about to say is not novel. (See, for example, Head [1962].) Conditions (i)–(iii) are instances of failures of full appropriation: more explicitly, they describe environments where the impossibility of full appropriation unavoidably leads to the presence of pecuniary externalities.[15] Placing (i)–(iii) under the heading of pecuniary failures of full appropriation and (iv) under the heading of non-pecuniary failures, we can give a more parsimonious explanation of the traditional taxonomy of market failure: they come from the failure of full appropriation.

The above discussion did not make use of the second condition of our extension, non-complementarity. Because market-making is not allowed in the standard First Theorem, Scitovsky-type underdevelopment traps in innovation cannot arise. As a consequence, the kind of pecuniary externality/inefficiency associated with the failure of non-complementarity (see Section 5.2) is not a part of the standard taxonomy. For example, in a well-known article Bator (1958) classifies them in a footnote as arising from "disequilibrium dynamics"; similarly, Arrow (1969) only makes a passing nod in Scitovsky's direction.

The non-complementarity condition and the pecuniary externalities that it eliminates are, however, related to some recent developments in macro-economics and industrial organization. For example, Cooper and John (1988), Heller (1990), Vives (1989), and Milgrom and Roberts (1990), among others, study the implications of *strategic complementarities* (Bulow, Geanakoplos, Klemperer [1985]). From our point of view, it is significant that these contributions draw little or no connection to the standard taxonomy of market failure. Briefly, we establish some links between this literature and our extension of the First Theorem.

The typical starting point for the study of strategic complementarities is a setting in which individuals have monopoly power. This contrasts with our application of the non-complementarity condition where individuals do not have monopoly power. Upon closer inspection, however, when the issues are framed in terms of appropriation problems and the resulting pecuniary externalities, the differences are not as large as they first appear. A perfect competitor cannot effect the price of any commodity he might supply or the price of any other currently supplied commodity. The need for the non-complementarity condition arises because this does not rule out the possibility that by innovating commodity A this might raise the potential market price for innovation B and/or vice-versa (see the Scitovsky example in Section 4.2). Unable to appropriate the results of these third-party pecuniary externalities, innovations which are privately unprofitable may nevertheless be so-

[15]Of course, there are qualifications. For example, in (ii) the public goods should be "pure" rather than "local" (see, e.g., Tiebout [1956], Ellickson [1979], Chari and Jones [1991]), just as in (i) it is necessary to emphasize that the increasing returns be large relative to the economy as a whole.

cially desirable. The implications of these perfectly competitive pecuniary externalities are all-or-nothing: either the innovations are efficiently supplied or else they are not supplied at all. Compare this to the monopoly power problem in which pecuniary externalities typically lead to undersupply rather than zero supply. In a market setting, the theory of strategic complementarities can be regarded as a generalization/conjunction of the pecuniary externalities associated with monopoly power and the pecuniary externalities associated with perfectly competitive product innovation.

Recent developments in international trade (e.g., Krugman [1987]) and economic growth (Romer [1987]) are predicated on monopolistically competitive environments. In these models, large numbers of small scale agents are prevented from achieving full appropriation because of a predominating complementarity among heterogeneous commodities. Such an environment can be usefully regarded as a continuum version of the finite agent models studied in the strategic complements literature. Thus, in addition to price-making pecuniary externalities (failure of full appropriation), there are also market-making externalities (failure of non-complementarity).[16]

This summary of strategic complementarities in models with small and large numbers of individuals and commodities is quite incomplete and by its brevity we do not mean to suggest that these results are obvious implications of what will happen when the qualifying conditions of our extension of the First Theorem are not satisfied. Rather, we call attention to these contributions because they demonstrate the variety of important market behavior which can occur when the full appropriation and non-complementarity conditions fail to hold.

[16]The two conditions are hardly independent: failure of full appropriation will almost certainly imply failure of non-complementarity.

Appendix

A. Historical Note on Pecuniary Externalities and Pigou's Mistake

The concept of pecuniary externalities appears only marginally (if at all) in the traditional taxonomy of market failure. It has been largely expelled from the court of welfare economics since the days when Pigou's famous analytical mistake in *Wealth and Welfare*, the original version of *The Economics of Welfare*, was uncovered. The mistake involved a *misapplication* of appropriation logic. The extended First Theorem helps clarify this point, as well as showing that there was, after all, a germ of truth in Pigou's error.

In *Wealth and Welfare* Pigou argued, following in Marshall's footsteps, that an industry with increasing transfer costs (due to the presence of a factor which can be drawn in greater amounts from other industries only by a rise in its price) will tend to overproduce relative to a constant cost industry, even under perfect competition. Using appropriation logic, Pigou's intuition can be explained as follows. As the industry with increasing costs expands, the new entrants impose a negative pecuniary externality on the existing firms in the industry since the latter will experience rising factor costs. But this implies that there is a discrepancy between the new entrants' private gains and the social gains from their entry. There will tend to be too much entry. Note carefully that since firms are assumed to be perfect competitors, any one new entrant cannot effect factor prices; but if a non-negligible set of new entrants were withdrawn from the industry, then factor prices would fall for the remaining firms. Thus the problem Pigou envisioned involves a *coordination failure* among firms. There is a discrepancy between a *group* of new firms' private rewards and the *group's* social marginal product.

The argument certainly had intuitive appeal. Sorting out Pigou's mistake involved a long debate among the giants of the profession, which finally led to the conclusion that pecuniary externalities (as opposed to "real" or "technological" externalities) are largely irrelevant for welfare economics. See Ellis and Fellner [1943] for a survey of the debate; in their language, the increase in factor costs is just a rent, and a rent is not a cost in social resources. Alternatively expressed, the rising transfer cost is desirable and efficient since it correctly signals the increasing social cost of transferring resources to the industry.

Theorem 4, in particular the non-complementarity condition **NC**, illuminates Pigou's error. It is true that if a group of firms exited the increasing cost industry, the remaining firms in the industry would have to pay lower factor prices and consequently earn a larger producers' surplus. Thus, even under perfect competition, not all groups earn their "group's marginal product". But this failure in group appropriation will not lead to a market failure as long as **NC** holds: given **NC** it suffices that all *individuals* earn their marginal products; Pigou did not have to worry about *groups* also earning their marginal products.

In terms of the language of the model, suppose $(\wp, v, z)$ is an occupational equilibrium and $\Delta v \equiv v'$ is an occupational switch for the group of agents $I* \equiv \{i : v_i' \neq v_i\}$. The change in their private rewards that the group members perceive is given by

$$\sum_{i \in I*} \frac{\Delta PMP_i(v)}{\Delta v_i},$$

while the group's social marginal product from making the switch is naturally defined as

$$\frac{\Delta g(v)}{\Delta v}.$$

Under perfect competition, the first expression just equals

$$\sum \frac{\Delta g(v)}{\Delta v_i}$$

(we have used full appropriation; notice that $\frac{\Delta g(v)}{\Delta v_i} = 0$ for all $i \notin I*$). Hence, under perfect competition, **NC** ensures that the change in their private rewards is at least as large as the group's social marginal product from any coordinated change in occupations, Δv. Alternatively expressed, **NC** ensures that individual marginal products are never biased in the "wrong" direction, one that could lead to coordination failures. Specifically, in the case considered by Pigou, if the industry with increasing cost contracted then there would indeed be an increase in profits to the remaining firms, but the gain in their producers' surplus would be *less than* the loss in consumers' surplus resulting from the contraction in output by the industry: $\frac{\Delta g(v)}{\Delta v}$ would be non-positive.

As noted above, recognition of Pigou's mistake led economists to drop pecuniary externalities as a cause of market failure, to focus exclusively instead on real externalities. This to our mind was something like throwing out the "baby" with the "bath water", because there was an important germ of truth in Pigou's mistake. Specifically, while the sort of coordination failure envisioned by Pigou cannot arise when there is perfect competition *and complete markets* —because the non-complementarity condition, **NC**, is always satisfied under these circumstances,—when there is perfect competition *but incomplete markets*, coordination failures may arise in the innovation of new commodities—because **NC** may then fail. In this circumstance just giving individuals their marginal products, but not giving groups of individuals their marginal products, may lead to market failure. This was illustrated in our analysis of Scitovsky's example, which indeed involved a failure of "group appropriation"; here Pigou's intuition does not miss the mark (see the antepenultimate paragraph in Section 5.2.1).

Scitovsky is, of course, the main champion of the importance of pecuniary externalities for welfare economics in a competitive situation. Yet his 1954 appeal for a broadening of the concept of externalities to include not just real but also pecuniary externalities only met with limited success in vanquishing the ghost of Pigou's mistake. In one respect, Scitovsky weakened the case for the acceptance of pecuniary externalities by suggesting they did not fit in with (mainstream) equilibrium analysis, being a dynamic disequilibrium phenomenon. In this regard, we would differ with Scitovsky. Just as any Nash equilibrium may be viewed as the outcome of a perhaps complex dynamic process, so our Nash equilibrium in occupational choice may be viewed as an equilibrium "snapshot" reflecting Scitovsky-type interactions among firms.

B. Proof of Lemma 1

For any firm f, let z^f maximize $\sum_{i \neq f} v_i$ over all trades in $Z^f(v) \equiv Z(v) \cap \{z = (z_i) \in \Re^{\ell n} : z_f = 0\}$. Since $Z(v)$ is compact and nonempty, $Z^f(v)$ is also compact and nonempty. Thus this maximum exists. Clearly, $\sum_{i \neq f} v_i(z_i^f) = g^f(v^f)$.

Claim 1: For all f such that $V_f \neq \{v_f\}$,

$$v_i(z_i{}^f) - pz_i{}^f = v_i(z_i) - pz_i \quad \text{for all } i \neq f.$$

Proof:

$$\begin{aligned} v_f(z_f) - pz_f &= MP_f(v) \equiv g(v) - g^f(v^f) \\ &= \sum (v_i(z_i) - pz_i) - \sum_{i \neq f} (v_i(z_i{}^f) - pz_i{}^f). \end{aligned}$$

Thus,

$$\sum_{i \neq f} (v_i(z_i) - pz_i) = \sum_{i \neq f} (v_i(z_i{}^f) - pz_i{}^f). \tag{1}$$

But since z^f is feasible for v, for all $i \neq f$,

$$v_i(z_i) - pz_i \geq v_i(z_i{}^f) - pz_i{}^f.$$

Thus, (1) implies that we must have a strict equality for all $i \neq f$, as was to be shown. □

Claim 2: For all f such that $V_f \neq \{v_f\}$, for all consumers b,

$$z_b{}^f \text{ maximizes } v_b(z_b') - pz_b' \text{ over all } z_b' \in \cup_{v_f' \in V_f} \Re^{\ell(v_f', v^f)}.$$

That is, $z_b{}^f$ is optimal for b on $\cup_{v_f' \in V_f} H(v_f', v^f)$.

Proof: ¿From Claim 1, $z_b{}^f$ is optimal for b on $H(v)$. Consider next v_f'. Let (z', p) be Walrasian for (v_f', v^f). Then, as in the proof of Step 1, $v_b(z_b{}^f) - pz_b{}^f = v_b(z_b') - pz_b'$. That is, $z_b{}^f$ is also optimal for b on $H(v_f', v^f)$ for any $v_f' \in V_f$.

Thus, it satisfies the first-order conditions, namely,

$$\partial_h v_b(z_b{}^f) \leq p_h \tag{2}$$
$$\partial_h v_b(z_b{}^f) < p_h \Rightarrow z_{b,h} = -\omega_{b,h} \tag{3}$$

for all commodities $h \in H(v_f', v^f)$ and all $v_f' \in V_f$. But these are precisely the first-order conditions for $z_b{}^f$ to be optimal for b on $\cup_{v_f' \in V_f} H(v_f', v^f)$. And given the concavity of v_b, they are also sufficient conditions. □

Claim 3: For all firms f and $\overline{f}$ such that $V_f \neq \{v_f\}$ and $V_{\overline{f}} \neq \{v_{\overline{f}}\}$, for all consumers b,

$$z_b{}^f \text{ maximizes } v_b(z_b') - pz_b' \text{ over all } z_b' \in \cup_{\overline{v}_{\overline{f}} \in V_{\overline{f}}} \Re^{\ell(\overline{v}_{\overline{f}}, v^f)}.$$

That is, $z_b{}^f$ is optimal of b on $\cup_{\overline{v}_{\overline{f}} \in V_{\overline{f}}} H(\overline{v}_{\overline{f}}, v^f)$.

Proof: $v_b(z_b{}^{\overline{f}}) - pz_b{}^{\overline{f}} = v(z_b{}^f) - pz_b{}^f = v(z_b) - pz_b$ and $z_b{}^{\overline{f}}$ is optimal for b on $\cup_{\overline{v}_{\overline{f}} \in V_{\overline{f}}} H(\overline{v}_{\overline{f}}, v^f)$. Thus, since $z_b{}^f$ is feasible for b on $\cup_{\overline{v}_{\overline{f}} \in V_{\overline{f}}} H(\overline{v}_{\overline{f}}, v^f)$, it is also optimal for b on this set of commodities. □

Claim 4 (conclusion of the proof): By Claim 3, the first-order conditions (2)-(3) are satisfied for all commodities $h \in H(\overline{v}_{\overline{f}}, v^f)$, all $\overline{f}$ such that $V_{\overline{f}} \neq \{v_{\overline{f}}\}$, and all $\overline{v}_{\overline{f}} \in V_{\overline{f}}$. Hence, as in the proof of Claim 2, $z_b{}^f$ is optimal for b on

$$\{h \in H(\overline{v}_{\overline{f}}, v^f) : \overline{f} \text{ satisfies } V_{\overline{f}} \neq \{v_{\overline{f}}\} \text{ and } \overline{v}_{\overline{f}} \in V_{\overline{f}}\}.$$

But given the fact that consumers only have one occupational choice, **a.1**, and that all innovations are earmarked for final consumers, **a.4**, the above set just equals $\cup_{v \in V} H(v)$, i.e., $\{h : h = 1, \ldots, \ell\}$. Hence $z_b{}^f$ is optimal for b on $\Re^\ell$. Which in turn, by Claim 1, implies z_b is optimal for b on $\Re^\ell$, as was to be shown. ||

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