

MONETARY BUSINESS CYCLE MODELS: IMPERFECT INFORMATION*

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Business cycle theories based on incomplete information start from the premise that key economic decisions on pricing, investment or production are often made on the basis of incomplete knowledge of constantly changing aggregate economic conditions. As a result, decisions tend to respond slowly to changes in economic fundamentals, and small or temporary economic shocks may have large and long-lasting effects on macroeconomic aggregates.

Incomplete information theories have been popular in particular for explaining sluggish price or wage adjustment in response to monetary shocks. At the heart of this theory lies the assumption that firms or households only pay attention to a relatively small number of indicators regarding conditions in markets relevant to their own activities, but they may not acquire information more broadly about aggregate economic activity. With imprecise information about these aggregate conditions, it takes the firms some time to sort out temporary from permanent changes, or nominal from real disturbances. Prices then respond with a delay to changes in nominal spending, and monetary shocks may have significant effects on real economic activity in the intervening periods - despite the fact that firms have the opportunity to constantly readjust their decisions.

This basic idea was proposed first by Phelps (1970) and formalized by Lucas (1972). In Lucas (1972), economic agents produce in localized markets, in which they observe the market-clearing price at which they can sell their output. This price is affected both by aggregate spending shocks and by market-specific supply shocks. Under perfect information, quantities adjust in response

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to local supply shocks, but not prices, and prices respond to aggregate spending shocks, but not quantities. With imperfect information, agents are unable to filter out the magnitudes of the aggregate and market-specific shocks from the observed prices in the short-run. Output then responds positively to price changes and spending shocks in the short run, but not in the long run, once agents have been able to sort out the spending shocks from the market-specific supply shocks.

Lucas (1972) formulated this idea in a rational expectations market equilibrium model, in which agents' expectations are fully Bayesian, and the resulting output responses are optimal. His model also includes stark assumptions about the nature of local vs. aggregate market interactions, as well as the nature of shocks (monetary vs. real, demand vs. supply, aggregate vs. market-specific) and the information to which firms have access.

Importantly, the model lacks a natural internal amplification mechanism: the extent of incomplete nominal adjustment depends almost entirely on the degree of informational incompleteness. Subsequent work has tried to address these issues, for example by introducing richer information structures. Townsend (1983) considers an investment model, in which firms get to observe how much some of the other firms invest. Therefore, they need to form forecasts about each others' beliefs - forecasting the forecasts of others. This leads to a complicated infinite regress problem, whereby a firm's current investment level depends on its observation of other firms' past investment, which in turn depended on observations about past investment... Townsend showed that this type of problem does not admit a simple finite-dimensional recursive structure. As a result, firms must draw inference about all past realizations of shocks simultaneously, leading to an infinite-dimensional filtering and fixed point problem, with no easily characterized solution.

These and other important technical and computational hurdles effectively imposed limitations on the complexity and economic realism of the early incomplete information models. Moreover, the model is open to the criticism that if incomplete information is a major source of business cycle fluctuations, then there seems to be an important societal benefit to making the relevant information publicly available to everyone. In part because of these difficulties, economists have, from the mid-eighties, turned their attention to New Keynesian sticky price theories that emphasize the role of adjustment and coordination frictions in price-setting.¹

Recently, the incomplete information theories have made a comeback, which can be traced to two factors. First, technological progress has made models such as Townsend (1983) computationally tractable. Second, new game-theoretic results regarding equilibrium analysis with a lack of common knowledge and heterogeneity in beliefs, as well as insights borrowed from the sticky

¹among others, see Calvo (1983), Blanchard and Kiyotaki (1989).

price literature regarding the role of real rigidities and pricing complementarities (Ball and Romer (1990) have enabled us to paint a much richer picture of the adjustment dynamics resulting from incomplete information models. The empirical performance of these new incomplete information models, however, still remains to be seen.

In the remainder of this essay I provide a unified exposition of the main ideas behind the incomplete information theories, from the original contributions to the more recent renewal. I also attempt to chart out some of the challenges that lie ahead. This is a lively and active area of research, with many open questions and few definite answers.

1 A canonical framework

Consider the following model, which is based on the New Keynesian models of monopolistic competition. There is a large number of firms, indexed by $i \in [0, 1]$. In each period, each firm sets its (log-)price $p_t(i)$ equal to its expectation of a target price p_t^* , $p_t(i) = E(p_t^* | \mathcal{I}_t^i)$, where \mathcal{I}_t^i denotes the information set of firm i at date t , i.e. all signals on which it can condition its pricing decision. p_t^* is characterized as

$$p_t^* = ky_t + p_t, \tag{1}$$

where $p_t = \int p_t(i) di$ denotes the average of the firms' pricing decisions, y_t denotes the aggregate real output in period t , relative to its trend level that would prevail with complete information, and $k > 0$ measures the response of optimal pricing decisions to real output. A firm's ideal relative price $p_t^* - p_t$ is determined by real output deviations from trend.

We augment this pricing rule by a quantity equation, $y_t + p_t = m_t$, where m_t denotes nominal spending. Combining the two, we find

$$p_t^* = km_t + (1 - k)p_t \tag{2}$$

Nominal spending m_t is driven by exogenous shocks, for simplicity, assume that $m_t = m_{t-1} + \varepsilon_t$, where $\{\varepsilon_t\}$ is iid white noise.

Each firm's target price is therefore a linear combination of the exogenous shocks and the prices set by the other firms. If $k \in (0, 1)$, prices are complementary, i.e. an increase in the average price level implies that each firm has an incentive to raise its own price. The parameter value of k depends on the substitution elasticity between the firms' products, the firms' returns to scale parameter in the technology, and the Frisch elasticity of labor supply.

To complete the model description, we need to specify each firm's information set \mathcal{I}_t^i - this is where different incomplete information theories vary. An equilibrium of this model requires that

prices satisfy the optimality condition $p_t(i) = E(p_t^* | \mathcal{I}_t^i)$, taking into account that p_t^* itself depends on the aggregate price level.

Common Information: Suppose first that all firms have identical information sets - $\mathcal{I}_t^i = \mathcal{I}_t$. Then, they will set identical prices, equal to $p_t(i) = p_t = E(m_t | \mathcal{I}_t)$. This reflects the implications of the original Lucas model that prices adjust to the common expectation of the underlying shocks. When information is incomplete, firms will only learn gradually about m_t , prices adjust slowly, and monetary surprises have real effects: y_t is determined directly by the discrepancy between the realized and the expected value of m_t . However, if the available information on which these expectations are based is sufficiently precise, then $E(m_t | \mathcal{I}_t)$ cannot be far from the true value of m_t . As discussed above, the real effects of monetary shocks are bounded by the degree of informational incompleteness - as firms have better information, their prices track m_t more closely, and monetary shocks have smaller real effects.

Heterogeneous beliefs, but independent strategies: A similar conclusion emerges when firms have different information sets, but their target prices do not respond to the other firms' decisions ($k = 1$). Each firm's price is set equal to its expectation of the spending shock $p_t(i) = E(m_t | \mathcal{I}_t^i)$, and the average price adjusts according to the average expectation $p_t = \bar{E}(m_t) = \int E(m_t | \mathcal{I}_t^i) di$ of the spending shock. Once again, if firms are sufficiently well informed, their pricing decisions will on average not be far from the nominal spending shock, which implies little delay in price adjustment and only small real output effects.

Heterogeneous beliefs and complementary strategies: Suppose now that instead $k \in (0, 1)$, so that there are complementarities in pricing decisions. Averaging the pricing equation, and substituting forward, firm i 's equilibrium price is given by

$$p_t(i) = k \sum_{s=0}^{\infty} (1-k)^s E \left[\bar{E}^{(s)}(m_t) | \mathcal{I}_t^i \right] \quad (3)$$

where $\bar{E}^{(s)}(m_t)$ denotes the s -order average expectation of m_t , or the average expectation of the average expectation of ... (repeat s times) ... of m_t . A firm's optimal price is therefore given as a geometrically weighted average of higher-order expectations - a firm needs to forecast not only the realized shock, but also the other firms' expectations of the shock, the other firms' expectations of the other firms' expectations of the shock, and so on.

If the firms all had identical information, the Law of Iterated Expectations would simply collapse the right-hand side above into the common first-order expectation of m_t . The model thus derives

its interest from the fact that with heterogeneous information, higher-order expectations respond differently to new information than first-order expectations about m_t .

The following example illustrates this point and serves also to derive the main results of this model. Suppose that all firms observe m_{t-1} exactly, but only a fraction λ (the *informed*) gets to observe m_t . Then, $\overline{E}(m_t) = \lambda m_t + (1 - \lambda) m_{t-1}$, but the second order average expectation is $\overline{E}^{(2)}(m_t) = \lambda [\lambda m_t + (1 - \lambda) m_{t-1}] + (1 - \lambda) m_{t-1} = \lambda^2 m_t + (1 - \lambda^2) m_{t-1}$. By iteration, the s -order average expectation of m_t is $\overline{E}^{(s)}(m_t) = \lambda^s m_t + (1 - \lambda^s) m_{t-1}$. The average price is

$$p_t = k \sum_{s=0}^{\infty} (1 - k)^s \overline{E}^{(s+1)}(m_t) = m_{t-1} + \frac{k\lambda}{1 - (1 - k)\lambda} (m_t - m_{t-1}). \quad (4)$$

Two important conclusions emerge. First, note that $\frac{k\lambda}{1 - (1 - k)\lambda} < \lambda$. The informed firms whose prices may react to m_t take into account that the uninformed firms won't respond, which in turn reduces their incentives to adjust prices. Therefore, while incomplete information serves as the initial source of sluggish price adjustment, the complementarity and the heterogeneity in beliefs dampen the response of prices far beyond what the initial degree of informational incompleteness would suggest. To illustrate the strength of this amplification effect, consider the following numerical example: suppose that $k = 0.15$ (as in standard parametrizations of New Keynesian sticky price models), and that half the firms are informed. Then, the contemporaneous response of average prices is $\frac{k\lambda}{1 - (1 - k)\lambda} \approx 0.13$, i.e. a 1% increase in nominal spending leads to only a 0.13% increase in prices, and a 0.87% increase in real output - despite the fact that half of the firms actually observe the increase in nominal spending and are hence able to respond to it!

Second, this amplification can be large, even if the degree of informational incompleteness is small. If λ is close to 1, almost all firms exactly observe the current realization m_t . Nevertheless, if k is close to 0, i.e. if there is a strong pricing complementarity, they still won't respond to the monetary shock. The presence of only a few uninformed firms is therefore enough to radically overturn the conclusions of the complete information model.

These two observations apply quite generally, once firms have heterogeneous beliefs. They form the central insight of the new incomplete information theories. In Mankiw and Reis (2002), heterogeneous beliefs result because in any given period, only a fraction of firms observe new information. This generalizes the above example to allow for richer adjustment dynamics. In Woodford (2002), all firms observe a conditionally independent idiosyncratic signal x_t^i of the current realization of m_t in each period. The resulting inference problem is more complicated but can be solved numerically. Again, the response of prices to monetary shocks is significantly dampened by the fact that firms do not share in common information, yet their pricing decisions are complementary.

The role of public information: Hellwig (2002) provides a simplified version of Woodford (2002), providing closed-form solutions to a general class of information structures. This simplified model also accommodates the presence of additional public sources of information such as central bank announcements. Besides dampening the response to idiosyncratic private signals, the complementarity in prices generates overreaction to public news. Public announcements thus speed up price adjustment and reduce the real effects of monetary shocks, but the noise in public news creates an additional source of volatility, which in some cases may increase rather than decrease real output fluctuations.²

2 Looking ahead

These new contributions have provided promising insights into the amplification and propagation mechanisms of incomplete information models. But they also abstract from important modeling issues that need to be addressed before a comprehensive quantitative evaluation becomes possible.

So far, much of the analysis is based on a stylized price-setting model that captures the essence of pricing complementarities as described above, without deriving them within a fully specified dynamic general equilibrium model. This short-cut is not without problems. First, the lack of a proper context of markets makes it difficult to interpret these propagation results. Presumably in a market firms obtain some information about price and quantity variables - so far, this is not formally modelled.

Second, the assumption that firms are heterogeneously informed implies that other frictions must be present - in particular, the extent to which information about fundamental shocks can be inferred from publicly observable prices must be limited, implying that the asset market must be incomplete. But then, one faces the problem of isolating the effects of informational heterogeneity from the effects of other market imperfections.³

Third, there is an issue of interpretation. At this point, there exist several different interpretations regarding the source of the differences in beliefs across firms, and they may lead to radically different model conclusions. In Mankiw and Reis (2002), firms update their information only infrequently, and in the intervening periods set prices on the basis of outdated information; Reis (2006) further develops this idea on the basis of menu costs in updating decisions. Woodford (2002)

²Similar results are derived by Amato and Shin (2003) for Woodford's model, and by Ui (2003) in the original Lucas island model.

³In Lorenzoni (2005), for instance, heterogeneous information coexists with market incompleteness, which creates an additional precautionary savings motive.

instead bases his model on the notion of ‘rational inattention’, developed by Sims (2003, 2006). Sims argues that decision makers only have a finite capacity to process new information, which constrains the quality of the signals they observe in any given period. Heterogeneity in beliefs then arises naturally through the idiosyncratic noise in each individual’s information processing channel. A third interpretation suggests that individuals are Bayesian, but access to information is limited - for example, firms observe the demand for their own products, but not the demand for competitors’ products. If each firm is subject to idiosyncratic, as well as common shocks, then an information structure much like the above with idiosyncratic private signals emerges. On the other hand, firms also observe market prices, which generates a source of common information.

Finally, all these models treat the information structure as an exogenous primitive. In reality, firms and households have access to overwhelming amounts of information, and information processing becomes a matter of choice, given the existing constraints and trade-offs. By and large, the effects of information costs and choices and the strategic interaction that results from these choices remains unexplored.⁴

In summary, the most important issue that remains to be resolved is the grounding of new incomplete information theories within a fully specified model of goods and asset markets, with special emphasis on the origins of the informational frictions. Beyond that, the new incomplete information theories raise many intriguing questions, which merit further attention, or have already been addressed to some extent: for example, Ball, Mankiw and Reis (2005) reconsider the role of monetary policy, and Morris and Shin (2002), Hellwig (2005) and Angeletos and Pavan (2004, 2005) discuss the welfare effects of information disclosures. Finally, the combination of new evidence on the cross-sectional and business cycle properties of expectations (Mankiw, Reis and Wolfers, 2004) and new micro-level data on price adjustments (Bils and Klenow, 2004) promises to provide an interesting avenue for evaluating the empirical performance of the model’s cross-sectional and business cycle implications.

⁴For some preliminary developments in this direction, see Mackowiak and Wiederholt (2005), and Hellwig and Veldkamp (2005). In Mackowiak and Wiederholt, firms need to allocate a fixed processing capacity between idiosyncratic and aggregate variables. Hellwig and Veldkamp explore how the pricing complementarities that drive business cycle implications lead to coordination motives and multiple equilibria in information acquisition.

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