

# Balance Sheet Effects, Bailout Guarantees and Financial Crises

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This paper provides a model of boom-bust episodes in middle income countries. It features balance-of-payments crises that are preceded by lending booms and real appreciation, and followed by recessions and sharp contractions of credit. As in the data, the nontradables sector accounts for most of the volatility in output and credit. The model is based on sectoral asymmetries in corporate finance. Currency mismatch and borrowing constraints arise endogenously. Their interaction gives rise to self-fulfilling crises.

*Query 1*

## 1. INTRODUCTION

In the last two decades, many middle-income countries have experienced boom-bust episodes centered around balance-of-payments crises. There is now a well-known set of stylized facts. The typical episode began with a lending boom and an appreciation of the real exchange rate. In the crisis that eventually ended the boom, a real depreciation coincided with widespread defaults by the domestic private sector on unhedged foreign-currency-denominated debt. The typical crisis came as a surprise to financial markets, and with hindsight it is not possible to pinpoint a large “fundamental” shock as an obvious trigger. After the crisis, foreign lenders were often bailed out. However, domestic credit fell dramatically and recovered much more slowly than output.

This paper proposes a theory of boom-bust episodes that emphasizes sectoral asymmetries in corporate finance. It is motivated by an additional set of facts that has received little attention in the literature: the tradables (T-) and nontradables (N-) sectors fared quite differently in most boom-bust episodes. While the N-sector was typically growing faster than the T-sector during a boom, it fell harder during the crisis and took longer to recover afterwards. Moreover, most of the guaranteed credit extended during the boom went to the N-sector, and most bad debt later surfaced there.

Our analysis is based on two key assumptions that are motivated by the institutional environment of middle income countries. First, N-sector firms are run by managers who issue debt, but cannot commit to repay. In contrast, T-sector firms have access to perfect financial markets. Second, there are *systemic bailout guarantees*: lenders are bailed out if a critical mass of borrowers defaults. The first part of the paper derives optimal investment and financing choices for the N-sector when these imperfections are present. We show that both borrowing constraints and a risky currency mismatch of assets and liabilities arise in equilibrium. Moreover, even in a world with no exogenous shocks, self-fulfilling crises can occur. The second part of the paper considers a dynamic small open economy where our two assumptions hold, and it provides an account of a complete boom-bust episode.

The emergence of both real exchange rate risk and systemic credit risk in our model is self-fulfilling. On the one hand, if N-sector managers expect the real exchange rate to fluctuate, they find it optimal to create a “*currency mismatch*” and thereby risk going bankrupt. Indeed, suppose managers believe that a real depreciation could occur. With guarantees in place, it makes sense for them to coordinate exposure to the real exchange rate by denominating debt in T-goods. This creates a currency mismatch: while firms’ revenue depends on the real exchange rate, their debt obligations do not. However, if *all* firms go bankrupt in case of a depreciation, they trigger a bailout and shift debt repayment to the taxpayer. This increases *ex ante* profits.

On the other hand, once there is a currency mismatch, a *balance sheet effect* validates the initial expectations of real exchange rate fluctuations. This effect is due to the managerial commitment problem. Indeed, since the guarantees are systemic, they do not insure lenders against idiosyncratic default by an individual firm. To credibly abstain from stealing, an individual manager must therefore respect a borrowing constraint.<sup>1</sup> As a result, N-sector investment depends on N-sector cash flow—a balance sheet effect. The currency mismatch actually relaxes the borrowing constraint and amplifies the balance sheet effect: since the government commits to a bailout, the manager can commit to pay lenders at least in states where he defaults and a bailout takes place. Guarantees thus permit more leverage.

A self-fulfilling crisis occurs as follows. Suppose the price of N-goods falls. N-sector cash flow also falls and there are widespread defaults. Due to the balance sheet effect, investment demand by the N-sector collapses. If the N-sector is an important enough buyer in its own market, the price must indeed fall to clear the market. It follows that managers were correct all along in expecting the possibility of a depreciation. Of course, the mechanism can only work if the risk of crisis is not so high that managers’ plans become unprofitable: self-fulfilling crises must be rare events if they are to occur in equilibrium.<sup>2</sup>

The second part of the paper characterizes the dynamics of boom-bust episodes. Here we make a third important assumption: the demand for N-goods by other sectors is expected to increase at some point in the future. These expectations could arise, for example, because of a reform. They kick off a boom during which the size of the N-sector and the relative price of its output rise hand-in-hand. Indeed, an increase in output not only increases supply of N-goods, but also N-sector cash flow. As long as prices are expected to rise further, N-sector investment is profitable and the increase in cash flow stimulates investment demand for N-goods. Future prices are in turn high because higher investment increases future output and cash flow and so on. This “self-feeding” investment boom is sustainable because the eventual increase in demand from other sectors ensures that the N-sector can repay debt accumulated along the way.

Since bailout guarantees amplify the balance sheet effect, they strengthen the boom. The use of T-denominated debt that is effectively subsidized by taxpayers allows managers to borrow more. Bailout guarantees thus counteract the underinvestment problem otherwise faced by the borrowing-constrained N-sector. However, faster growth comes at a cost: the economy becomes vulnerable to crises. Once the N-sector is large enough, the self-fulfilling meltdown described above can occur. The main result of the paper is the existence of a sunspot equilibrium in which lending booms end-rarely-in crises that involve a real depreciation and widespread defaults. These crises have persistent effects on N-output: it subsequently takes time for the N-sector to accumulate internal funds.

1. For this result, it also matters that agents cannot write contingent contracts that promise large payments in bailout states only.

2. The explicit treatment of risk is crucial here. If crises were *unanticipated*, firms would be indifferent between T- and N-debt. We would thus need to *assume* the currency mismatch. Only if crises are anticipated can we rationalize the endogenous emergence of risk.

According to our model, the “fundamentals” responsible for recent crises are found in financial markets and their regulation. Crises need not be triggered by exogenous shocks, such as productivity shocks. They are also “real”: the nature of the nominal exchange rate regime is irrelevant. Instead, *both* systemic guarantees and borrowing constraints in the N-sector play an essential role. If there were only guarantees, self-fulfilling crises could not occur because managers could easily borrow in case of a real depreciation. Conversely, if there were no guarantees, managers would have no incentive to create the currency mismatch that is required for crises to occur. Indeed, in the presence of bankruptcy costs, they would instead prefer to hedge real exchange rate risk by denominating their debt in nontradables.

Our paper is related to a number of other recent “third generation” models of financial crises. These models share the feature that financial market distortions faced by the private sector play an important role. In particular, some existing models contain either bailout guarantees or a managerial commitment problem. To our knowledge, this paper is the first to analyse both distortions in an explicit microeconomic setting and to show that their interaction is nontrivial and important for understanding crises. Moreover, our focus on sectoral asymmetries leads us to provide a new account of the dynamics of boom-bust episodes. The related literature is discussed in more detail in Section 8.

The paper proceeds as follows. Section 2 provides an overview of the stylized facts. Section 3 introduces the model. Section 4 characterizes the financial structure of the economy. Section 5 considers the crisis mechanism. Section 6 characterizes equilibrium dynamics. Section 7 relates the times series generated by the model to the stylized facts. Section 8 discusses related literature. Section 9 concludes. All proofs are collected in an Appendix.

## 2. STYLIZED FACTS

The stylized facts on boom-bust episodes described in the introduction have been documented in several papers.<sup>3</sup> Here we illustrate them through an event study on a set of eleven frequently studied countries over the period 1980–1999. The countries are Argentina, Brazil, Chile, Finland, Indonesia, Korea, Malaysia, Mexico, Philippines, Sweden, and Thailand.

Figure 1 shows the typical behaviour of several macroeconomic variables around the crises. The middle line in each panel represents the average deviation, relative to tranquil times, for the variable considered. It is apparent that the typical crisis was preceded by a real appreciation and a lending boom during which bank credit to the private sector was growing unusually fast. In addition, the N-sector was growing faster than the T-sector.

During the typical crisis, the real exchange rate depreciates and real credit growth falls drastically, reflecting problems in the banking system. The recession following the crisis was usually short-lived, while the slump in credit was more drawn out. In Figure 1, GDP growth recovers to its tranquil time mean by period  $t + 3$ , that is, three years after the crisis. In contrast, real credit growth at  $t + 3$  is significantly lower than during tranquil times, and the gap seems to be widening. Sectoral asymmetries are also apparent during and after the crisis: the N-sector falls harder than the T-sector during the crisis and recuperates more sluggishly afterwards. The N-to-T output ratio actually falls monotonically during the whole bust phase.

Interestingly, along a boom-bust episode investment exhibits quite large (and statistically significant) deviations from tranquil times, while consumption deviations are very mild and insignificant. Figure 1 also shows that there is no significant deterioration in the terms of trade in

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3. See Sachs, Tornell and Velasco (1996), Corsetti, Pesenti and Roubini (1999), Kaminski and Reinhart (1999), Krueger and Tornell (2000) and Gourinchas, Landerretche and Valdes (2001).

## The Boom-Bust Cycle in Middle Income Countries

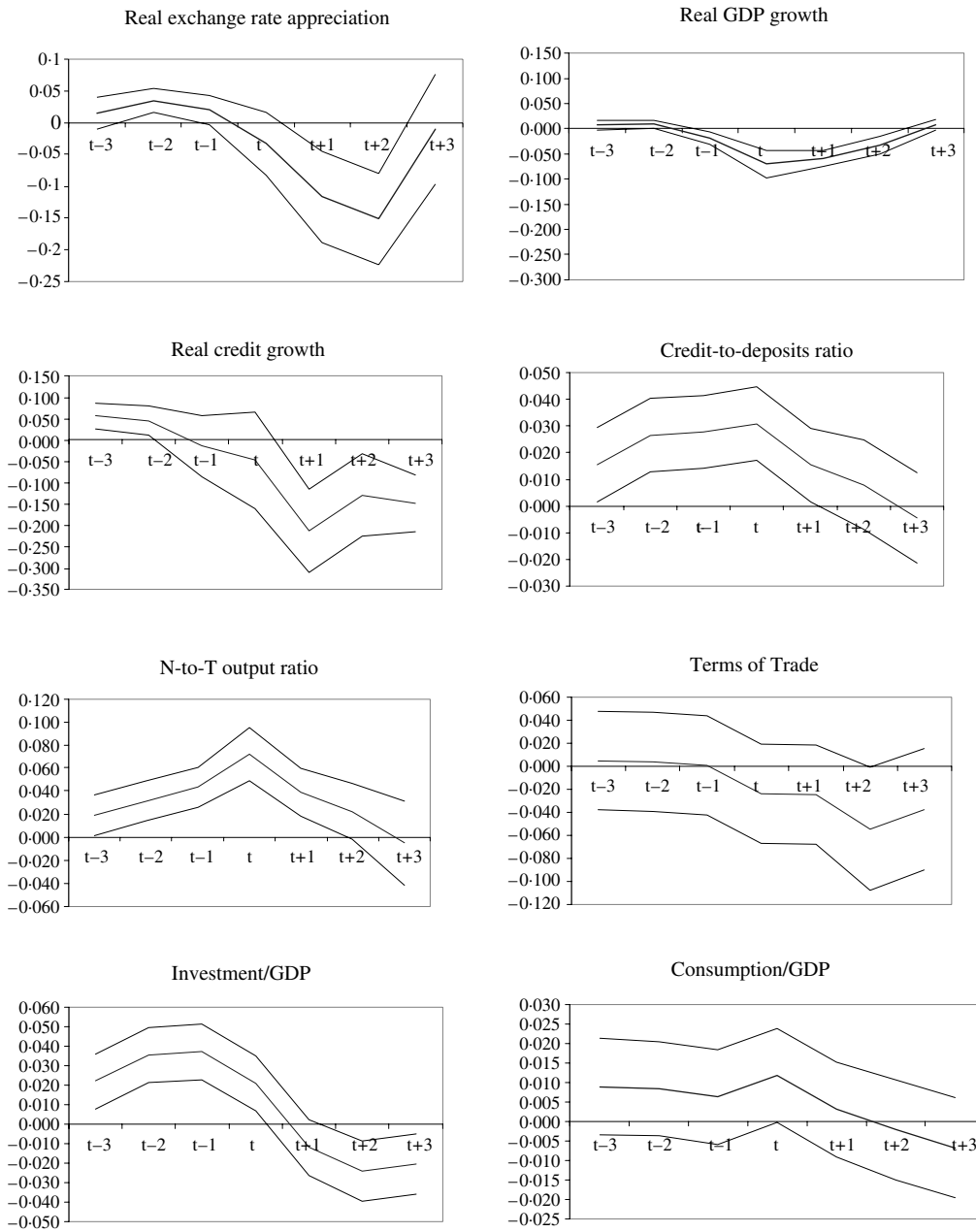


FIGURE 1

the year prior to a crisis. This fact suggests that among the usual suspects there is no evidence of a large exogenous shock that rocks the boat and generates a crisis.

We would like to emphasize that although almost every crisis has been preceded by a lending

boom, the converse is not true. Gourinchas *et al.* (2001) find that the typical lending boom does not end in crisis, but with a soft landing. That is, *crises are rare events*. Finally, the properties of a boom-bust episode do not depend on a particular exchange rate regime. In particular, Tornell and Westermann (2002) find that the macroeconomic patterns along a boom-bust episode under fixed exchange rates are not significantly different from the patterns under non-fixed regimes.

### 3. THE MODEL

We consider a small open economy which exists for  $T$  periods. There are two goods: an internationally tradable (T) good, which is the numeraire, and a nontradable (N) good. The only source of uncertainty is a sunspot variable  $\sigma_t$  which is i.i.d. and takes values in  $\{good, bad\}$ . The “good” state occurs with probability  $\alpha$ . We denote the inverse of the real exchange rate by  $p_t = \frac{p_{N,t}}{p_{T,t}}$ , and by  $\bar{p}_{t+1}$  and  $\underline{p}_{t+1}$  the values that  $p_{t+1}$  is expected to take on in the good and bad state, respectively.<sup>4</sup> There are three types of agents: foreign investors, N-sector managers and consumers.

#### 3.1. Foreign investors

Foreign investors are risk neutral and have “deep pockets”: they lend any amount of funds as long as they are promised the riskless world interest rate in expected value. They also issue default-free bonds: an N-bond and a T-bond. The T-bond pays one unit of the T-good next period and trades today at a price  $\beta := \frac{1}{1+r}$ , where  $r$  is the constant world interest rate. The N-bond pays  $p_{t+1}$  units of the T-good next period, and trades today at a price  $\frac{1}{1+r_t^n}$ . Since the sunspot takes only two values, the two bonds complete the market. In addition, the existence of risk neutral deep pocketed investors will imply that uncovered interest parity holds in equilibrium:

$$(1 + r_t^n) p_{t+1}^e = 1 + r, \quad \text{where } p_{t+1}^e := \alpha \bar{p}_{t+1} + (1 - \alpha) \underline{p}_{t+1}. \quad (3.1)$$

More generally, the price of every payoff stream will simply be its discounted expected value.

#### 3.2. Nontradables sector

There is a continuum of N-sector firms of measure one. Since we will impose symmetry, we consider a representative firm. It has access to a linear technology that produces N-goods at time  $t + 1$  by investing N-goods at  $t$ :

$$q_{t+1} = \theta I_t.$$

**Financing.** To finance investment, the firm can use internal funds,  $w_t$ , or issue standard debt. Debt is short-term and may be denominated in T-goods or in N-goods. If the firm issues T-bonds (N-bonds) worth a total of  $b_t (b_t^n)$  units of T-goods at time  $t$ , it promises to repay  $(1 + \rho_t) b_t$  ( $p_{t+1} (1 + \rho_t^n) b_t^n$ ) units of T-goods at time  $t + 1$ . The firm may also purchase T- and N-default-free bonds ( $s_t, s_t^n$ ) that will repay  $(1 + r) s_t$  and  $p_{t+1} (1 + r_t^n) s_t^n$  at  $t + 1$ .<sup>5</sup>

The firm’s budget constraint at time  $t$ , measured in T-goods, is

$$p_t I_t + s_t + s_t^n = w_t + b_t + b_t^n. \quad (3.2)$$

4. That is,  $\bar{p}_{t+1} = E[p_{t+1} \mid \mathcal{I}_t, \{\sigma_{t+1} = good\}]$  and  $\underline{p}_{t+1} = E[p_{t+1} \mid \mathcal{I}_t, \{\sigma_{t+1} = bad\}]$ , where  $\mathcal{I}_t$  is the information available at  $t$ .

5. Using these instruments, the firm can generate any state-contingent payoff. In particular, it can choose to hedge all real exchange rate risk by denominating all debt in N-goods ( $b = 0$ ). In contrast, a “risky currency mismatch” can be created by setting  $b > s$  and  $b^n = s^n = 0$ .

At time  $t + 1$  profits  $\pi_{t+1}$  are equal to gross returns  $G_{t+1}$  minus the debt burden:

$$\begin{aligned} G_{t+1} &:= p_{t+1}\theta I_t + (1+r)s_t + p_{t+1}(1+r_t^n)s_t^n, \\ \pi_{t+1}(p_{t+1}) &:= G_{t+1} - (1+\rho_t)b_t - p_{t+1}(1+\rho_t^n)b_t^n. \end{aligned}$$

A firm is *insolvent* if profits are negative ( $\pi_{t+1} < 0$ ). In this case all of the firm's gross returns  $G_{t+1}$  are dissipated.

**Diversion.** The firm's investment and financing decision are made by overlapping generations of managers. The time  $t$  manager inherits a cash position  $w_t$  from his predecessor. He then issues bonds and invests. In addition, he can make arrangements to *divert* the returns of the firm. Setting up a diversion scheme requires a non-pecuniary diversion cost  $h < 1 + r$  per unit of the firm's assets. Once the scheme is in place, the manager can divert the gross returns at date  $t + 1$ , provided that the firm is solvent. The goal of every manager is to maximize next period's expected profits net of *ex ante* diversion costs.<sup>6</sup> Finally, if profits are positive and there is no diversion, the old manager pays out a fraction  $c$  of the profits as dividends to himself and passes on the rest,  $w_{t+1}$ , to his successor.

**Bailout guarantees.** A firm is in default if it is either insolvent or the manager diverts the gross returns. Lenders are given a "systemic" bailout guarantee: if a critical mass of firms (for concreteness, 50%) is in default, the government steps in and ensures that lenders walk away with what they were owed. The guarantee covers both T- and N-debt. In addition to paying lenders, the government recapitalizes firms in default by giving a small endowment  $e$  of T-goods to the new managers. In other words, for insolvent firms,  $w_{t+1} = e$ . The total cost of a bailout is thus  $f_{t+1} = (1+\rho_t)b_t + p_{t+1}(1+\rho_t^n)b_t^n + e$ . It is financed by a lump sum tax on consumers.

### 3.3. Consumers

The representative consumer derives utility from tradable ( $c_t^T$ ) and nontradable goods ( $c_t^N$ ):

$$E \sum_{t=0}^T \beta^t [c_t^T + d_t \cdot \log(c_t^N)],$$

where  $(d_t)_{t=0}^T$  is a deterministic sequence. Every period, the consumer receives an endowment of  $y_t = y_0 \lambda^t$  ( $\lambda > 1$ ) units of the tradable good. In light of (3.1) and complete markets, her budget constraint is

$$E \sum_{t=0}^T \beta^t [c_t^T + p_t c_t^N + f_t - y_t] \leq 0.$$

If  $y_0$  is large enough, which we assume is the case, the consumer's demand for N-goods is

$$D_t(p_t) = \frac{d_t}{p_t}.$$

### 3.4. Equilibrium

Every period, investment and financing decisions are determined by managers' interaction with financial markets. Since the occurrence of a bailout depends on *how many* firms default, managers' decisions are interdependent. Formally, we define a "credit market game" that captures competitive bond pricing with a large number of risk neutral lenders, while allowing for strategic interaction of bond issuers.

6. The parameter  $h$  can be interpreted as a measure of the severity of the enforceability problem, with a low  $h$  representing lax contract enforcement. If  $h \geq 1 + r$ , diversion is always more expensive than repayment of debt and the diversion cost has no effect.

**The credit market game.** The order of moves is as follows. At the beginning of period  $t$ , each new manager is assigned two risk-neutral lenders, one of whom invests in T-bonds only, while the other invests in N-bonds only. Given internal funds  $w_t$ , all managers simultaneously announce a plan that satisfies budget constraint (3.2). All lenders then simultaneously decide whether or not to fund the plan proposed to them (*i.e.* to purchase the bonds) or not. Subsequently, those managers whose plans have been funded decide whether or not to incur the diversion cost.<sup>7</sup>

Payoffs are determined during the following period. If the firm is solvent, the manager receives the profit  $\pi_{t+1}(p_{t+1})$  if he has not set up a diversion scheme, or the gross returns minus the diversion cost,  $G_{t+1} - h(w_t + b_t + b_t^n)$ , otherwise. If the firm is insolvent, the manager's payoff is zero if he has not set up a diversion scheme, or  $-h(w_t + b_t + b_t^n)$  otherwise. T-lenders (N-lenders) receive payoffs  $(\rho_t - r)b_t((p_{t+1}\rho_t^n - r)b_t^n)$  if *either* the firm they lend to is solvent *or* that firm is in default together with more than 50% of all firms, and bailout guarantees are in place. In all other cases, T-lenders (N-lenders) receive payoffs of  $-(1+r)b_t(-(1+r)b_t^n)$ .

**Equilibrium concept.** The following definition integrates the credit market game with the rest of the economy. To start off the economy, we assume that in period 0 there is both a cohort of initial incumbent managers who have an amount  $q_0$  of nontradables to sell and a cohort of new managers who have an endowment  $e_0$  of tradables.

*Definition.* A symmetric equilibrium is a collection of stochastic processes  $\{I_t, s_t, s_t^n, b_t, b_t^n, \rho, \rho_t^n, p_t, w_t\}$  such that:

1. Given internal funds  $w_t$ , current prices  $p_t$  and the distribution of next period's prices, the plan  $\{I_t, s_t, s_t^n, b_t, b_t^n, \rho, \rho_t^n\}$  is part of a symmetric subgame perfect equilibrium of the credit market game.
2. The market for nontradables clears:

$$\frac{d_t}{p_t} + I_t = \theta I_{t-1}.$$

3. Internal funds evolve according to

$$w_{t+1} = \begin{cases} (1-c)\pi(p_t) & \text{if } \pi(p_t) > 0 \\ e & \text{otherwise.} \end{cases}$$

### 3.5. Discussion of the assumptions

We now relate the setup of our model to the institutional environment we want to capture.

**A. The credit market.** Three main results of the credit market analysis in Section 4 are crucial for the macroeconomic dynamics in Section 6: (i) firms face binding borrowing constraints that make investment depend on cash flow, (ii) bailout guarantees increase the multiplier linking investment to cash flow and (iii) bailout guarantees encourage firms to denominate debt in T-goods. Result (i) arises in many models in which insiders offer contracts to lenders that do not share their objectives. However, the result is not obvious when contracts are *guaranteed*. Lenders need not care about insiders' objectives as long as the government commits to pay them off. This weakens the mechanisms that give rise to borrowing constraints in common contracting models. More generally, any model of guaranteed contracts must explain why insiders cannot run "scams" that exploit the guarantee at an arbitrary scale by promising

7. For completeness, if a plan is not funded, managers and lenders receive a payoff of zero. This is not restrictive, since plans can involve no borrowing.

huge interest rates. Here, we provide a concrete setup that is in line with the institutional environment of middle income countries and in which the above issues can be made explicit.

*Diversion.* The diversion technology provides a basic conflict of interest between insiders (managers) and lenders. Diversion may be thought of as a transfer of the firm's assets to another company indirectly owned by the manager. It is reasonable that such transfers are easier when the firm is solvent—creditors typically monitor managers of bankrupt firms much more closely. Our assumption that diversion from insolvent firms is impossible thus captures increased creditor scrutiny in a stark way.

The essential feature of the diversion technology is that the cost of diversion is proportional to total assets, while the benefit increases with the debt burden. A firm can thus commit to repay as long as the debt burden is not too large relative to the size of the firm. This gives rise to a borrowing constraint and a multiplier linking investment to cash flow (result (i) above). The effective debt burden to the firm, and hence the benefit of diversion, is lower when a bailout occurs—this leads to result (ii). The timing of the diversion decision (that is, it is taken before uncertainty is resolved) is convenient because it implies that the investment multiplier is independent of future prices. Finally, the restriction to standard debt and the fact that managers cannot steal from insolvent firms are both important in ruling out scams. This is explained further below.

*Standard debt.* Standard debt is the typical instrument accessible to firms in middle income countries. In addition, bailout guarantees are usually defined over gross debt positions, rather than, say, the net financial asset positions of the firm. By imposing standard, noncontingent, debt, we can naturally explore the former type of guarantees. We also emphasize that, in our two-state environment, a restriction to standard debt does not limit firms' ability to hedge real exchange rate risk: issuing N-debt provides a perfect hedge. Allowing for derivatives would not improve hedging, it would only facilitate risk taking. Given our interest in equilibria that exhibit crises, the assumption thus appears conservative; it ensures that our model also applies to countries with less developed derivatives markets.

*Systemic guarantees.* The assumption seems realistic: a bailout does not occur if just any individual firm is insolvent, especially not if the firm is small. Instead, bailouts occur when there is a critical mass of insolvencies.<sup>8</sup> The role of guarantees in the model is to induce managers to take on insolvency risk (result (iii) above). The systemic nature is crucial: if any insolvency resulted in a bailout, diversion would be costless and firms would not face borrowing constraints. Systemic guarantees also help explain why risk taking is coordinated through debt denomination. Simply investing in projects subject to large idiosyncratic risk is not a profitable way to exploit these guarantees.

*Bankruptcy costs.* As usual, bankruptcy costs capture deadweight costs of the bankruptcy process. In our analysis, they matter only when a bailout is not expected. In that case, they provide a reason for managers to actively avoid insolvency, as in Froot, Scharfstein and Stein (1993). This means that managers strictly prefer to hedge real exchange rate risk, by denominating debt in N-goods. In contrast, if a bailout is expected, bankruptcy costs are borne by the government and their size is irrelevant for contracting between managers and lenders.

8. For example, consider the Mexican experience. In the aftermath of the 1994 crisis, the entire financial system became insolvent. In order to ensure that all debt obligations were met, the U.S. Treasury and international organizations provided a generous bailout. In 1999, however, a big Mexican firm, GAN, announced the suspension of the service of its debt (which stood at more than one billion U.S. dollars). The Mexican government did not provide a bailout.



*No scams.* In our model, the nature of debt contracts, the diversion technology, and the guarantees all combine to rule out scams in which managers promise huge interest rates and have their plans financed by a bailout. To clarify the role of the various assumptions, it is helpful to informally preview the argument of Section 4.1. Suppose a bailout occurs in only one of the two states. A lender will then only fund a plan that entails diversion if the interest rate in the bailout state is large enough to compensate the loss in the other state. With standard debt, this will drive the firm into insolvency in both states, if the bailout state is unlikely enough. Since the manager cannot steal from an insolvent firm, diversion becomes unprofitable. The restriction to standard debt is important here: guaranteed contingent claims would allow managers to shift all payments to the taxpayer by issuing claims that pay out only in the bailout state.

*Extreme parameter values.* Throughout the paper we make stark assumptions on the parameters of the contracting problem. For example, we have assumed that managers can divert all gross assets, that all assets are dissipated in bankruptcy and that no diversion whatsoever can take place in insolvency. These assumptions are not essential for the qualitative effects stressed in the analysis of the credit market game. However, they greatly simplify our dynamic analysis in Section 6 below, since they guarantee that the investment multipliers are independent of future equilibrium prices.

### B. Other elements of the economy.

*The tradables sector.* We have represented the tradables sector by a deterministic endowment. This may be viewed as a reduced form of the following more elaborate setup. Suppose that there are competitive, financially unconstrained, firms that produce T-goods from labour ( $l_t$ ) and T-goods ( $k_t$ ) with a constant returns technology

$$q_t^T = g_t l_t^{1-\varepsilon} k_t^\varepsilon \quad g_t = g_0(1+g)^t, \varepsilon \in [0, 1),$$

where  $g$  is the rate of technological progress. In addition, suppose that the consumer is endowed with one unit of labour, supplied inelastically and that there is full depreciation. Since the rental rate of capital is the gross world interest rate  $1+r$ , equilibrium T-production and consumer's income are, respectively

$$q_t^T = g_t^{\frac{1}{1-\varepsilon}} \left[ \frac{\varepsilon}{1+r} \right]^{\frac{\varepsilon}{1-\varepsilon}}, \quad y_t = [1-\varepsilon]q_t^T.$$

The endowment  $y_t$  in the consumer's budget constraint may thus be viewed as wage income.

*Sectoral asymmetries.* The assumption that T-sector firms have access to perfect capital markets, whereas the N-sector faces credit market imperfections is motivated by two institutional features of middle income countries. First, bank credit is the major source of external finance for N-sector firms. In contrast, many T-sector firms have access to international capital markets because they can pledge export receivables as collateral to foreign lenders. Banks in turn are strongly exposed to the N-sector and do not hedge real exchange rate risk.<sup>9</sup> Second, systemic bailout guarantees apply to bank debt.

The assumption that the N-sector uses its own product as an input is important to allow the N-to-T output ratio to grow during booms and also to generate crises associated with a self-fulfilling collapse in demand for N goods. The constant-returns, single-input technology simplifies the analysis considerably since the contracting problem becomes linear.

9. Even when banks denominate loans in foreign currency, they face the risk that domestic firms will not be able to repay in the event of a real depreciation.

*Consumers.* The role of consumers is to provide a demand for N-goods. What is key for our argument is that this demand for N-goods (i) is downward-sloping, and (ii) is expected to shift outward at some point in the future. Requirement (i) is necessary for multiple prices to exist, while (ii) sets the stage for the expansion of the N-sector. In our dynamic analysis below, we will assume that the consumer's demand curve  $d_t/p_t$  shifts out at time  $T$ .<sup>10</sup>

Our model permits perfect intertemporal consumption smoothing and hence does not emphasize the role of consumption fluctuations in explaining boom-bust cycles. Instead, we focus on investment. This is in line with the stylized fact that consumption fluctuates much less than investment over the boom-bust cycle.

*Managers.* We have assumed that N-sector firms are run by overlapping generations of managers with short horizons. This assumption makes the model tractable, since the credit market game can be solved period by period. At the same time, a long-lived manager would typically find insolvency more costly if he stood to lose a "franchise value" arising from the future operation of the firm. This would provide another reason to actively avoid insolvency and could make him less willing to take on risk than in our setting. However, the qualitative tradeoffs should remain the same in a more complicated model with long-lived managers.<sup>11</sup>

### 3.6. Road map

We characterize equilibria in two steps. In Section 4 we examine the credit market game for given prices. We show that (i) in a world without guarantees N-sector managers hedge insolvency risk by financing their firms with N-debt, and (ii) if there are guarantees and "enough" real exchange rate risk, it is optimal for all N-sector managers to coordinate on T-debt financing. This latter case generates a fragile capital structure for which a real depreciation induces widespread insolvencies in the N-sector. We also show that in either case binding borrowing constraints arise in equilibrium.

In Section 5 we characterize the mechanism behind self-fulfilling crises. Taking as given the capital structure and expected future prices, we show that if N-sector firms hold enough T-debt and there is a strong balance sheet effect, there are two market clearing prices. The high price leaves firms solvent, while the low price bankrupts them and triggers a bailout.

In Section 6 we characterize equilibrium dynamics. In Section 6.1 we show that a world without guarantees gives rise to "safe lending booms," financed by N-debt, in which real appreciation coexists with an increase in both the credit-to-GDP and the N-to-T ratio. Then, in Section 6.2, we establish the existence of sunspot equilibria which feature similar (but stronger) lending booms, financed by T-debt. These booms may be punctuated by self-fulfilling crises or may end in a soft landing. Finally, in Section 7 we relate the time series generated by the model to the stylized facts.

10. Although "good news" about the future value of N-goods is a crucial element to produce a boom, it is not essential for creating fragility at a point in time. In particular, the "crisis mechanism" that arises from the interaction of the two distortions could be activated in any environment where the N-sector starts with a sufficiently large debt burden. For instance, this debt burden could be inherited from a formerly state-owned banking system.

11. See Schneider (1999) for a model of banking with long-lived managers and deposit insurance in which the franchise value plays a key role.

## 4. FINANCIAL STRUCTURE

In this section, we take prices as given and derive equilibria of the credit market game. We assume throughout that there is “enough” real exchange rate variability:<sup>12</sup>

$$\frac{\bar{p}_{t+1}\theta}{p_t} > 1 + r > h > \frac{p_{t+1}\theta}{p_t}. \quad (4.1)$$

## 4.1. Investment and financing without guarantees

For an economy without bailout guarantees, we establish existence of a *safe credit market equilibrium*. Firms hedge insolvency risk to avoid bankruptcy costs and face binding borrowing constraints. As a result, investment is proportional to internal funds.

**Proposition 4.1.** *Suppose there are no bailout guarantees. If the probability of the good state  $\alpha$  is close to or equal to one, there exists a symmetric SPE of the credit market game such that (i) firms never become insolvent, (ii) managers do not divert, (iii) all debt is N-debt ( $b_t = 0$ ) with  $1 + \rho_t^n = \frac{1+r}{p_{t+1}^e}$ , (iv) firms do not purchase default-free bonds ( $s_t = s_t^n = 0$ ), and (v) physical investment expenditure is proportional to internal funds:*

$$p_t I_t = \frac{w_t}{1 - \beta h} = m^s w_t. \quad (4.2)$$

A formal proof is provided in the Appendix; here we sketch the intuition. In the absence of guarantees, no manager will propose a plan that entails diversion: lenders would reject it since they could not break even.<sup>13</sup> It follows that, under any proposed plan, the cost of diverting all assets must be larger than the manager’s benefit of diversion, which equals expected debt payments. Regardless of what type of debt is issued, managers have no incentive to pay lenders more than their opportunity cost  $1 + r$ . The no-diversion condition is thus

$$h(w_t + b_t + b_t^n) \geq (1 + r)(b_t + b_t^n). \quad (4.3)$$

Since  $h < 1 + r$ , this inequality is an endogenous borrowing constraint. Importantly, the debt capacity of the firm does not depend on the denomination of debt.

If  $\alpha$  is large enough, condition (4.1) implies that physical investment has a positive NPV. It is thus optimal to invest as much as the borrowing constraint allows and to not hold any default-free bonds. The bound on investment then follows by substituting for debt from the budget constraint (3.2):

$$hp_t I_t \geq (1 + r)(p_t I_t - w_t). \quad (4.4)$$

Since the maximal investment level is the same whether or not the firm always remains solvent, the optimal choice of solvency depends on the expected profit per unit of investment. The lender always breaks even in expected value. Therefore, the manager of a solvent firm captures the whole NPV of investment, while the manager of a firm that is insolvent, say, in the bad state, must bear bankruptcy costs. His payoff is then only

$$\begin{aligned} E(\pi_{t+1}) &= \{p_{t+1}^e \theta I_t - (1 + r)(p_t I_t - w_t)\} - (1 - \alpha)p_{t+1} \theta I_t \\ &= \{\text{NPV of the project}\} - \{\text{expected bankruptcy cost}\}. \end{aligned} \quad (4.5)$$

12. The equilibrium price process derived in Section 6 will satisfy this condition.

13. At the time lenders decide whether to buy bonds, the manager’s diversion decision is perfectly foreseen. In addition, without guarantees, there is no interdependence among firms.

Bankruptcy costs thus provide an incentive to hedge against a real depreciation and to remain solvent in both states. A perfect hedge is to denominate all debt in N-goods. Indeed, managers want their firms to remain solvent in both states:

$$\begin{aligned}\pi(\bar{p}_{t+1}) &= \bar{p}_{t+1}\theta I_t - (1+r)b_t - (1+r_t^n)\bar{p}_{t+1}b_t^n \geq 0, \\ \pi(\underline{p}_{t+1}) &= \underline{p}_{t+1}\theta I_t - (1+r)b_t - (1+r_t^n)\underline{p}_{t+1}b_t^n \geq 0.\end{aligned}$$

Since investment has positive NPV and interest rate parity holds, we have  $\theta > 1 + r_t^n$ . Holding fixed total debt payments  $(1+r)(b_t + b_t^n)$ , the solvency constraints can be made nonbinding by setting  $b_t = 0$ . It is thus always (at least, weakly) better to finance a safe plan with N-debt.

#### 4.2. Investment and financing with guarantees

For an economy with guarantees, the safe credit market equilibrium discussed above continues to be an equilibrium. If everybody else chooses to hedge, there is no bailout. Thus, contracting between any individual manager and his creditors works exactly as before. However, we now establish existence of a second *risky credit market equilibrium*, in which firms finance investment with T-debt and become insolvent in the bad state.

**Proposition 4.2.** *Suppose bailout guarantees are present. If  $\alpha$  is close to (but less than) one, there exists a symmetric SPE of the credit market game such that the equilibrium plan  $(I_t, s_t, s_t^n, b_t, b_t^n, \rho_t, \rho_t^n)$  satisfies: (i) firms become insolvent in the bad state; (ii) managers do not divert; (iii) all debt is T-debt ( $b_t^n = 0$ ) with  $1 + \rho_t = 1 + r$ ; (iv) firms do not purchase default-free bonds ( $s_t = s_t^n = 0$ ); and (v) physical investment expenditure is proportional to internal funds*

$$p_t I_t = \frac{w_t}{1 - \alpha^{-1}\beta h} = m^r w_t, \quad (4.6)$$

where the multiplier  $m^r$  is strictly larger than the multiplier in the absence of guarantees.

In the Appendix, we prove that, given other managers' strategies, it is optimal for any individual manager to select a plan satisfying (i)–(v). The argument parallels that of the safe case. First, it is established that an endogenous borrowing constraint arises. This is no longer obvious, because managers might promise huge payments and then divert the firm's assets. Lenders should be happy to fund such diversion schemes at any scale as long as they are bailed out. Three assumptions are critical to rule out such scams: (a) bailout guarantees are systemic, (b) debt is noncontingent, and (c) a manager cannot divert if his firm is insolvent.

Suppose all other managers default only in the bad state. By (a), a bailout only occurs in the bad state. Thus, under any diversion scheme proposed by an individual manager, lenders will receive nothing in the good state. To induce them to fund the diversion scheme, the manager must promise very high interest rates: the bailout, which occurs only in the highly unlikely crisis state, must compensate lenders for the loss to be incurred in the good state. However, if interest rates are too high, then, by (b), the firm becomes insolvent in both states. By (c), diversion in the good state becomes impossible, and the diversion scheme is not profitable.

Any proposed plan must therefore imply diversion costs that exceed expected debt payments. This gives rise to a borrowing constraint, as in the case without guarantees. What is new is that guarantees lower expected debt payments and thereby raise the borrowing limit. Indeed, lenders are happy to fund a no-diversion plan at the riskless rate  $r$ . A firm that issues T-debt and defaults in the bad state thus expects to repay only  $\alpha(1+r)b_t$ . The borrowing constraint

becomes

$$hp_t I_t \geq \alpha(1+r)b = \alpha(1+r)(p_t I_t - w_t). \quad (4.7)$$

This implies an investment limit  $m^r w_t$  that is strictly greater than the limit faced by a firm that always remains solvent. The latter expects to repay  $(1+r)(b_t + b_t^n)$  and hence faces the same investment limit as a firm in a world with no guarantees:  $m^s w_t$ .

The guarantee not only raises debt capacity, it also increases the per unit benefit for a firm that defaults in the bad state. For such a firm, we now have

$$E_t(\pi_{t+1}) = \{p_{t+1}^e \theta I_t - (1+r)(p_t I_t - w_t)\} - (1-\alpha)\underline{p}_{t+1} \theta I_t + (1-\alpha)(1+r)(p_t I_t - w_t).$$

Here, the third term is the subsidy implicit in the guarantee. The manager trades off the gains from a risky plan against the bankruptcy costs. We show in the appendix that for  $\alpha$  close to 1, defaulting in the bad state is preferred to always remaining solvent.

Finally, consider optimal debt denomination. The manager desires insolvency in the bad state only, that is,  $\pi(\bar{p}_{t+1}) \geq 0 > \pi(\underline{p}_{t+1})$ . For a given amount of borrowing,  $b_t + b_t^n$ , this is easier to achieve by increasing the share of T-debt because uncovered interest parity implies  $(1+r_t^n)\underline{p}_{t+1} < 1+r < (1+r_t^n)\bar{p}_{t+1}$ . At the same time, expected profits increase the more of the firm's borrowing is done with T-debt. It is therefore optimal to set  $b_t^n = 0$ .

## 5. SELF-FULFILLING CRISES

We now provide intuition for the crisis mechanism in a typical period  $t < T$ . In particular, we show that given future prices and the outcome of the credit market game in period  $t-1$ , there can be multiple market clearing prices in period  $t$ . This is important for the existence of sunspot equilibria in the next section. Suppose that some credit market equilibrium was played in period  $t-1$ . Incumbent managers enter the current period with a supply of nontradables  $q_t$ , no bond holdings and debt burden

$$(1+\rho_t)b_t + p_t(1+\rho_t^n)b_t^n =: L_t + p_t L_t^n.$$

The new cohort of managers takes as given future prices as well as internal funds inherited from incumbents. If firms are solvent, new managers start out with internal funds

$$w_t = (1-c)(p_t q_t - L_t - p_t L_t^n). \quad (5.1)$$

In contrast, if the bad state is realized and firms become insolvent,  $w_t = e$ . Investment is determined in an equilibrium of the credit market game, *i.e.*  $p_t I_t = m_t w_t$ , where  $m_t$  can be  $m^s$  or  $m^r$ , depending on whether the equilibrium is safe or risky. The real exchange rate adjusts to equalize the supply and demand of nontradables:

$$q_t = \begin{cases} \frac{d_t}{p_t} + \eta_t \left[ q_t - L_t^n - \frac{L_t}{p_t} \right] & \text{if } p_t q_t > L_t + p_t L_t^n \\ \frac{d_t + m_t e}{p_t} & \text{otherwise,} \end{cases} \quad (5.2)$$

where  $\eta_t = (1-c)m_t$  is the *cash flow multiplier* of the N-sector.

Suppose the credit market equilibrium played in period  $t-1$  was safe. Incumbent managers then have only N-debt ( $L_t = 0$ ). As a result, demand slopes downward and there is a unique equilibrium price. In contrast, suppose a risky equilibrium was played and managers have only T-debt ( $L_t^n = 0$ ). In this case, price movements affect revenues, but leave the debt burden unchanged. Demand can become backward bending, as in Figure 2, and there can be multiple equilibria.

Indeed, for prices below the cutoff price  $p_t^c = L_t/q_t$ , all N-firms are insolvent. Total demand in this range is driven by consumers and is thus downward-sloping. In contrast, for

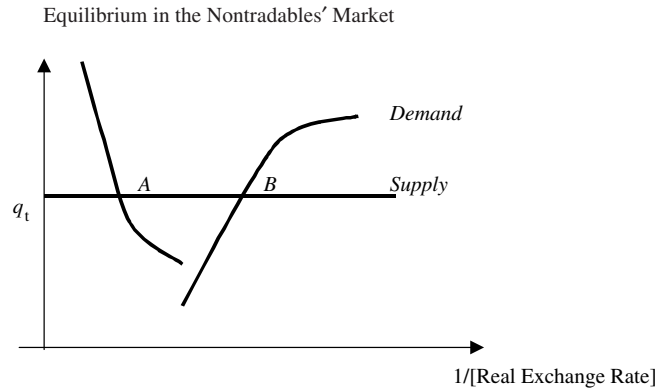


FIGURE 2

prices above  $p_t^c$ , an increase in the price is accompanied by an increase in cash flow. Since there are binding borrowing constraints at the firm level, aggregate investment demand is increasing in the price of N-goods. The next proposition states that multiple equilibria exist if and only if there is enough T-debt and there is a “strong” balance sheet effect:  $\eta_t > 1$ .

**Proposition 5.1 (Self-fulfilling Crises).** *Multiple market clearing prices (that is, solutions to (5.2)) exist if and only if*

- (i) *the level of T-debt in the N-sector is high enough:  $L_t > d + m_t e$ ,*
- (ii) *the cash flow multiplier  $\eta_t$  is larger than one.*

The first condition says that the N-sector cannot be too small. For a small debt burden  $L_t$ , aggregate demand mainly consists of downward sloping consumer demand. The consumer thus “stabilizes” the price. As  $L_t$  grows, N-sector expenditure on N-goods rises relative to consumption expenditure. The second condition requires that the balance sheet effect is strong enough, so that a change in the price of N-goods induces a more than proportional change in the N-sector’s expenditure on its own goods. This occurs when  $\eta_t > 1$  because investment expenditure equals  $\eta_t(p_t q_t - L_t)$ . A high  $\eta_t$  requires a low dividend payout rate  $c$  and low enforceability problems (large  $h$ , permitting high leverage).<sup>14</sup>

With identical fundamentals, in terms of supply and debt, the market may clear in one of two equilibria. In a “solvent” equilibrium (point B in Figure 2), the price is high, inflating away enough of N-firms’ debt (measured in nontradables) to allow them to bid away a large share of output from the T-sector. In contrast, in the “crisis” equilibrium (point A), the price is low to allow the T-sector and bankrupt N-firms with little internal funds to absorb the supply of N-goods. Which of these two points is reached depends on expectations. This does not mean that fundamentals are irrelevant. They determine whether the environment is fragile enough to allow two equilibria.

14. Proposition 5.1 does not depend on the fact that the aid payment  $e$  is a constant. For example, it also holds if aid is a fraction of output,  $e_t = \omega p_t q_t$ .

## 6. EQUILIBRIUM DYNAMICS

In this section, we characterize the dynamics of the economy. We begin with an environment with enforceability problems, but no bailout guarantees. We show that if the future looks brighter than the present and if there are strong balance sheet effects, there can be “safe lending boom” equilibria, characterized by increasing credit-to-GDP and N-to-T output ratios, and by real appreciation. We then introduce bailout guarantees and characterize a “risky lending boom equilibrium” where the N-sector grows faster, but the economy is vulnerable to self-fulfilling crises. Both types of equilibria are consistent with the stylized fact that many of lending booms do not end in crisis, but in a soft landing.

Lending booms typically take place at times when the local economy is expected to expand, which will increase the demand for local (nontradable) goods, such as construction and services. High expectations might come about because of a reform (such as a liberalization of trade) or discovery of a natural resource (oil). In our model, we capture this “fundamental” reason for a lending boom by an anticipated outward shift in the demand for N-goods by consumers. The anticipated shift is represented by the preference parameter  $d_t$ , which satisfies

$$d_t = \begin{cases} d & \text{if } t < T \\ \hat{d} \geq d & \text{if } t = T. \end{cases} \quad (6.1)$$

## 6.1. Safe lending boom equilibria

In our model, the only source of uncertainty is a sunspot. Multiple market clearing prices, which are crucial for a sunspot to matter, exist only if there is a large amount of T-debt. Since managers will choose a risky debt structure only if there are bailout guarantees, it follows that in their absence there cannot be an equilibrium in which prices depend on the sunspot. Instead, in economies without guarantees, safe credit market equilibria obtain every period and firms are always solvent.

We now characterize a safe equilibrium. Market clearing for nontradables requires that consumption and investment expenditures sum to the value of output:  $d_t + m^s w_t = p_t q_t$ . From Proposition 4.1, we have that internal funds evolve according to  $w_t = [1 - c][p_t q_t - p_t L_t^n] = [1 - c][p_t q_t - h m^s w_{t-1}]$ . Since output is proportional to internal funds in the previous period:  $q_t = \theta I_{t-1} = \frac{\theta m^s w_{t-1}}{p_{t-1}}$ , it follows that any equilibrium path of output and internal funds  $(q_t, w_t)$  must be a solution to

$$q_t = \theta \frac{m^s w_{t-1}}{m^s w_{t-1} + d} q_{t-1}, \quad t \leq T \quad (6.2)$$

$$\frac{1 - \eta^s}{1 - c} w_t = d - h m^s w_{t-1}, \quad t < T \quad (6.3)$$

$$w_T = \hat{d} - h m^s w_{T-1}, \quad (6.4)$$

with initial conditions  $q_0$  and  $w_0 = e_0$ , and where  $\eta^s = (1 - c)m^s$  is the “safe” cash flow multiplier. A solution to (6.2)–(6.4) is an equilibrium if the implied price path given by

$$p_t = \begin{cases} \frac{d + m^s w_t}{q_t} & t < T \\ \frac{\hat{d}}{q_T} & t = T \end{cases} \quad (6.5)$$

is steep enough to make the technology a non-negative NPV undertaking (that is,  $\theta p_{t+1}/p_t \geq 1 + r$  for all  $t < T$ ).

Equation (6.2) states that the fraction of nontradables production that is invested depends on the financial strength of the N-sector. If internal funds are low, N-firms can borrow very little.

Holding supply fixed, weak investment demand implies that the price is low and that consumers absorb a larger fraction of the available supply. On the other hand, a strong N-sector (high  $w_{t-1}$ ) can expand and will be able to bid more resources away from consumers at time  $t$ .

Equation (6.3) provides a “flow of funds” account for the “consolidated” N-sector, putting both cohorts of managers together. The R.H.S. is “consolidated cash flow”: sales to the household sector minus repayment of debt to foreigners. The L.H.S. includes the “net new funds raised:” new debt issued ( $b_t + b_t^n = w_t(m^s - 1)$ ) minus dividends paid out ( $c\pi_t = \frac{c}{1-c}w_t$ ).

We are interested in lending boom equilibria in which the real exchange rate appreciates, and N-output ( $q_t$ ) as well as the credit-to-GDP ratio ( $(b + b^n)/(p_t q_t + y_t)$ ) and the N-to-T ratio ( $p_t q_t / y_t$ ) increase over time. The existence of such equilibria is established in the next proposition. The proposition makes clear that lending booms need not only reflect excessive risk-taking and corruption, and that they need not end in crashes. Lending booms can be episodes during which credit constrained sectors grow faster because the future looks brighter than the present. Furthermore, they can end in a soft landing.

**Proposition 6.1 (Safe Lending Booms).** *Suppose that there is a strong balance sheet effect ( $\eta^s > 1$ ).*

1. *If the N-sector’s initial funds satisfy  $e_0 > \underline{e}$  and the future shift in T-sector’s demand for N-goods is large enough  $\hat{d} > \underline{d}(e_0)$ , there exists a safe symmetric equilibrium in which the N-sector’s internal funds increase over time.*
2. *For a large enough terminal time  $T$  there is a  $\tau < T - 1$ , such that if, in addition:*
  - (i)  $\theta \in (1, \frac{\eta^s h}{\eta^s - 1})$ , *then the real exchange rate appreciates and the output of N-goods increases from  $\tau$  to  $T - 1$ ;*
  - (ii)  $\lambda < \frac{\eta^s h}{\eta^s - 1}$ , *then the credit-to-GDP and N-to-T ratios increase from  $\tau$  to  $T - 1$ .*

Three conditions must be met to ensure growth of the N-sector over time. First, there must be a strong balance sheet effect ( $\eta^s > 1$ ). If instead  $\eta^s < 1$ , the N-sector could not expand over time, and could not run a deficit in anticipation of strong demand in the future.<sup>15</sup> Second, the accumulated debt must be repaid in the final period. Thus, the preference shift  $\hat{d} - d$  must be sufficiently large. Third, for the return on investment  $R_{t+1} = p_{t+1}\theta/p_t$  to be high enough in periods  $t < T - 1$ , investment demand must grow fast enough in relation to the supply of N-goods. Since supply at time  $t$  is proportional to internal funds available at  $t - 1$  and demand is proportional to internal funds available at  $t$ , it follows that internal funds must grow fast enough. It is apparent from (6.3) that if  $w_t$  is increasing over time, it will do so at an *increasing* rate. Thus, if  $w_0$  is above a certain threshold, investment will always have a positive NPV provided that the demand shift at  $T$  is sufficiently large.

In order to match the observation that increasing N-output coincides with a real appreciation, we need to determine how the rise in the value of N-output ( $p_t q_t$ ) translates into changes in prices and quantities. If the technology parameter  $\theta$  were very high, supply would outpace demand. As a result the price would fall over time, while investment would rise. At the other extreme, if  $\theta$  were small, we could have an equilibrium along which nontradables become increasingly scarce, with firms chasing the returns offered by rising prices, but being able to

15. If  $\eta^s < 1$ , the size of the N-sector measured, for example, by the value of assets converges to a steady state value (note that (6.3) may be rewritten as  $w_t = \frac{\eta^s h}{\eta^s - 1} w_{t-1} - \frac{(1-c)d}{\eta^s - 1}$ , for  $t < T$ ). In this type of equilibrium, the N-sector makes a profit every period and firms’ behaviour is independent of the demand shift occurring at date  $T$ . It would be the natural case to consider if we were interested in long run issues.



afford less and less investment. Thus, to match the stylized facts,  $\theta$  must take the intermediate values specified in Proposition 6.1.

Finally, the model matches the stylized fact that along a lending boom there is an increase in both the credit-to-GDP and the N-to-T ratios  $((b + b^n)/(p_t q_t + y\lambda^t)$  and  $p_t q_t / y\lambda^t$ ). The N-sector's internal funds not only grow, but do so at an *increasing* rate (which converges to  $\frac{\eta^s h}{\eta^s - 1} - 1$ ). Thus, the credit constrained N-sector can eventually grow faster than the unconstrained T-sector if the horizon is long enough and T-sector growth is not too high ( $\lambda < \frac{\eta^s h}{\eta^s - 1}$ ).

## 6.2. Sunspot equilibria (SE)

Proposition 6.1 has established that systemic guarantees are not necessary to generate lending booms. We now show that when guarantees are present, economies which would otherwise exhibit safe lending booms can now exhibit risky lending booms which allow faster growth (financed by cheap T-debt), but which may end in self-fulfilling crises. We begin with a preliminary question: is there a possibility of *unanticipated* self-fulfilling crises along a safe lending boom equilibrium? We then establish the existence of *sunspot equilibria*. These equilibria exhibit three phases. Initially, the economy travels along a safe path. Then, when the N-sector becomes large enough there is a switch to a fragile phase in which the risky credit market equilibria of Section 4 are played every period. If the boom lasts long enough, the economy must switch back to a safe path before terminal time (that is, there is a soft landing).

**Safe equilibria and unanticipated crises.** The safe lending boom equilibria continue to be equilibria in an economy with bailout guarantees. Suppose every manager is convinced that the safe equilibrium price will be realized in the next period. No bailouts will be granted. Therefore, all managers will simply play the best safe plan and the price evolves exactly as in a safe equilibrium.

To think about unanticipated crises, we use the fact that managers are *indifferent* between T and N-debt if prices are *deterministic*.<sup>16</sup> Suppose that all debt is actually denominated in tradables. Unanticipated crises can now occur during a *sufficiently long* safe lending boom. We know from Proposition 5.1 that multiple market clearing prices exist provided the amount of T-debt to be repaid in period  $t$  is large enough ( $L_t \geq d + m^s e$ ):

$$hm^s w_{t-1} > d + m^s e. \quad (6.6)$$

We call a state  $(q_t, w_{t-1})$  *fragile* if (6.6) holds. If the economy is in a fragile state, then the outstanding stock of T-debt is so large that it cannot be repaid by selling output to the T-sector and new managers with internal funds  $e$ . It follows that there is a market clearing price at which all firms default.

Fragility need not be present at all times along a safe lending boom equilibrium. In particular, if it is difficult to enforce contracts (low  $h$ ), and  $e_0$  is low, the initial phase of a boom need not be fragile. However, by Proposition 6.1, the debt burden  $hm^s w_{t-1}$  is increasing over time. The economy must thus enter into a “fragile region” if the boom continues long enough.

**Anticipated crises.** We now ask whether crises can actually occur with positive probability along the equilibrium path. We know from Section 4 that under exchange rate risk and bailout guarantees, managers may create credit risk from real exchange rate risk by financing investment with T-debt. This requires that there is “sufficient real exchange rate risk” in the sense

16. See Lemma A2 in the Appendix for a formal argument.

of condition (4.1). In particular, firms must expect (i) a sufficiently high return on investment in the absence of a depreciation, and (ii) a sufficiently low return after a depreciation, so that it is possible to claim the bailout subsidy by defaulting. In addition, the probability of a crisis must be sufficiently low to ensure that the *ex ante* expected return is high enough and borrowing constraints arise in equilibrium.<sup>17</sup> Section 4 has also shown that if there is enough T debt, there are two market clearing prices, the lower of which bankrupts firms, and hence triggers a bailout. The next proposition shows that these two effects can be elements of one consistent story.

**Proposition 6.2 (Sunspot Equilibria).** *Suppose bailout guarantees are in place and there is a strong balance sheet effect ( $\eta^r > 1$ ). There exists a region*

$$S = \{(e_0, \alpha, T, d) : e_0 > \underline{e}, \alpha > \underline{\alpha}(e_0), T > \underline{T}(e_0, \alpha), \hat{d} > \underline{d}(e_0, \alpha, T)\}$$

*for the N-sector's initial funds, the probability of the bad sunspot state, the terminal time, and the consumer's demand shift, such that for all economies with  $(e_0, \alpha, T, \hat{d}) \in S$ , there is a sunspot equilibrium with the following properties:*

1. *There is a "fragile phase"  $[\underline{\tau}, \bar{\tau}]$ , with  $\bar{\tau} \leq T - 1$ , during which a risky credit market equilibrium is played as long as the good sunspot state is realized.*
2. *For any period during the fragile phase ( $t \in [\underline{\tau}, \bar{\tau}]$ ), a crisis in which all N-sector firms default occurs the following period if the bad sunspot state is realized.*
3. *A safe credit market equilibrium is played outside the fragile phase and after a crisis has occurred.*
4. *There is a  $\tau \in [\underline{\tau}, T)$  such that, along the "lucky path" on which no crisis occurs, (i) the real exchange rate appreciates between  $\tau$  and  $T - 1$ , and the output of nontradables increases from  $\tau$  on, if  $\theta \in (1, \frac{\eta^r h \alpha^{-1}}{\eta^r - 1})$ , and (ii) both the credit-to-GDP and the N-to-T ratios increase from  $\tau$  to  $T - 1$ , if  $\lambda < \frac{\eta^r h \alpha^{-1}}{\eta^r - 1}$ .*
5. *If a crisis occurs at  $t$ , (i) investment and credit are lower in  $t$  than in  $t - 1$ , (ii) there is a real depreciation from  $t - 1$  to  $t$ , and (iii) if  $e < \frac{d}{(\theta - 1)m^s}$ , N-output falls between  $t$  and  $t + 1$ .*

This proposition states that if initial internal funds ( $e_0$ ) are small, the first phase of a SE must be safe because the debt burden is too small and crises cannot occur. Thus, managers adopt safe plans. Since wealth and the debt burden follow increasing paths along this safe phase, there is a time, say  $\tau - 1$ , at which a fragile state is reached (*i.e.* (6.6) holds at  $(q_{\tau-1}, w_{\tau-1})$ ). Starting at time  $\tau - 1$  there can be a *switch* to a risky phase provided agents believe that there will be a crisis with probability  $1 - \alpha$  during the next period. Of course, this risky phase cannot last until terminal time because the economy cannot be in a fragile state at  $T - 1$ : there cannot be a crisis in the final period because firms do not reinvest at  $T$ . Thus, there must be a *switch* to a third safe phase no later than  $T - 1$ .

In order to characterize the risky phase consider a typical period  $t - 1$ . Suppose agents believe that there will be a crisis with probability  $1 - \alpha$  in period  $t$ , and that this risk induces them to issue T-debt. Firms are solvent in the good state in period  $t$ . Thus, internal funds evolve according to

$$\bar{w}_t = (1 - c)(\bar{p}_t q_t - L_t) = (1 - c)(\bar{p}_t q_t - \alpha^{-1} h m^r w_{t-1}). \quad (6.7)$$

17. Recall that Proposition 4.2 required that  $\alpha$  is close enough to one.

Still conditioning on the good state being realized, the market clearing and output equations are:

$$d + m^r \bar{w}_t = \bar{p}_t q_t, \quad q_t = \frac{\theta m^r w_{t-1}}{p_{t-1}}. \quad (6.8)$$

The solution to (6.7)–(6.8) determines the “lucky path,” along which no crisis occurs. This path is part of an SE provided that two conditions are satisfied. First, the expected price path is steep enough to ensure that investment in N-goods has a non-negative NPV ( $\theta p_{t+1}^e \geq (1+r)p_t$ ). Second, the “crisis return” must be sufficiently low: if the bad state were to be realized, the price would be low enough to bankrupt firms ( $\pi(\underline{p}_t) < 0$ ).

If the initial level of internal funds is high enough,  $w_t$  will increase along the lucky path provided there is a strong balance sheet effect. Moreover, if crises are rare events, the expected return will be high enough to make investment profitable. Since during a crisis internal funds of the new cohort are  $\underline{w}_t = e$ , the second condition is satisfied provided that  $\underline{p}_t q_t = d + m^s e < \alpha^{-1} h m^r w_{t-1}$ . Clearly, for  $\alpha$  close to one this condition is implied by the condition for a fragile state (6.6). It follows that there is a range of crisis probabilities,  $(1-\alpha) \in (0, 1-\underline{\alpha})$ , such that if the economy is in a fragile state in period  $t-1$ , a crisis can occur in period  $t$  with conditional probability  $1-\alpha$ . Since a fragile state is only reached if the N-sector is large enough, this explains why it may take time for the economy to reach the fragile phase.

Since the economy cannot be in a fragile state at  $T-1$ , there must be a *switch* to a safe credit market equilibrium no later than  $T-1$ . Finally, the demand shift at terminal time  $T$  (i.e.  $\hat{d}$ ) must be large enough to ensure that the N-sector can repay its accumulated deficits. If  $\hat{d}$  were not high enough, there would be no investment at  $T-1$ , and, by backward induction, there would be no investment throughout. We conclude that *if the N-sector has enough time to grow*, the sunspot can eventually matter and self-fulfilling crises can be anticipated.

### 6.3. Pareto optimal production

To highlight the role of bailout guarantees, we compare safe and risky equilibria. Consider two economies A and B with parameters in the set  $S$  (defined in Proposition 6.2). The only difference between these economies is that A has systemic bailout guarantees. Then, there is an sunspot equilibrium where A and B behave identically up to time  $\underline{t}-2$ , after which the N-sector in economy A grows faster and exhibits higher leverage along the lucky path, as long as a crisis does not occur. However, A experiences a crisis and subsequent credit crunch with positive probability while B does not. This argument implies that systemic bailout guarantees might induce faster economic growth by easing borrowing constraints. Thus, it is not obvious that eliminating them is desirable under all circumstances. To illustrate this point, it is useful to characterize the set of Pareto optima. The allocation problem that has to be solved in our economy is (i) to distribute the available amount of nontradables among consumers and managers and (ii) to efficiently accumulate nontradables to equate the marginal rates of substitution and transformation. It follows that the Pareto optimal production of N-goods can be characterized by the following law of motion:<sup>18</sup>

$$q_t = \left( 1 - \frac{1-\beta}{1 + \beta^{T-t} \left( \frac{\hat{d}(1-\beta)}{d} - 1 \right)} \right) \theta q_{t-1}; \quad t = 1, \dots, T. \quad (6.9)$$

18. The key here is that only nontradables are used to produce nontradables, and only the consumer enjoys nontradables. This means that the Pareto optimal law of motion for nontradables can be derived independently of managerial preferences and welfare weights of different agents.

The fraction of output that should be devoted to investment is thus increasing over time, and depends positively on the anticipated preference shift  $\hat{d}/d$ . Comparing (6.9) and (6.2), there is no reason that the use of nontradables in a no-bailout regime should be Pareto optimal. There could be too little or too much investment, depending on the financial position of the N-sector. Since guarantees may induce a faster growth rate of output (compare (6.2) and (6.8)), we have that:

**Corollary 6.3 (Effects of Bailout Guarantees).** *If the future is much brighter than the present ( $\hat{d} \gg d$ ), systemic bailout guarantees might bring the path of N-goods output nearer to the Pareto optimal path.*

## 7. AN ACCOUNT OF THE STYLIZED FACTS

In this section we relate the equilibrium time series described by Propositions 6.1 and 6.2 to the stylized facts of Section 2. We also discuss when an economy is vulnerable to boom-bust episodes. Figure 3 depicts the paths that an economy follows in risky and safe equilibria. The boom starts because the financially constrained N-sector anticipates a favourable demand shift in the future. This encourages the N-sector to run a deficit to build up productive capacity. Deficits are financed by borrowing from abroad. Growth is gradual, as borrowing constraints are relaxed only through the reinvestment of profits. The price of N-goods rises throughout the boom: a real appreciation leads to more demand by the N-sector for its own goods, which in turn leads to greater appreciation and so on. In contrast, the T-sector does not face financing constraints: its growth is determined by investment opportunities (that grow at a constant rate). This *asymmetry* in the growth patterns of the N- and T-sectors generates three regularities associated with lending booms: an appreciating real exchange rate, as well as increasing credit-to-GDP and N-to-T output ratios.

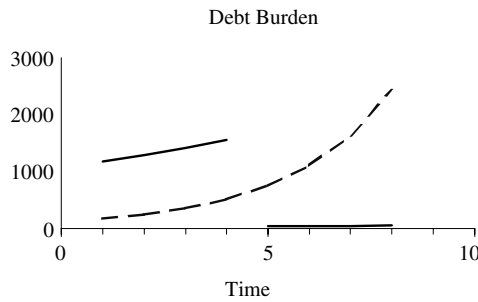
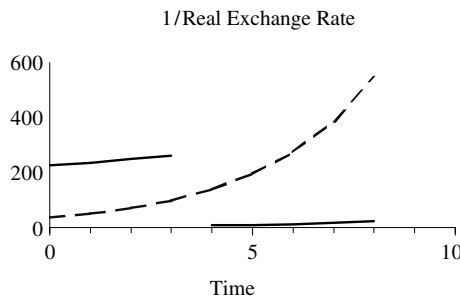
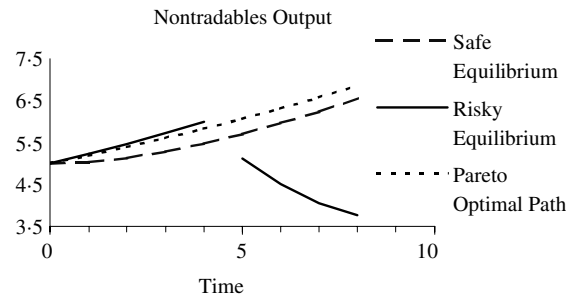
*Query 5*

Absent bailout guarantees and adverse exogenous shocks, this transition period will simply see the fast growth of the N-sector. If guarantees are present, their interaction with balance sheet effects both strengthens the boom and creates endogenous risk. On the one hand, guarantees alleviate the underinvestment problem usually associated with constrained agents. They permit high leverage with debt denominated in T-goods (a currency mismatch), and hence faster credit and output growth. On the other hand, the stock of T-debt eventually becomes high enough so as to permit self-fulfilling crises. Crises entail both real depreciation and widespread defaults in the N-sector; they trigger a bailout of lenders to that sector. Importantly, crises are not merely financial, but have substantial output costs: in the crisis period internal funds and investment demand collapse. Subsequently, balance sheet effects permit only a slow recovery of credit and a decline of credit-to-GDP and N-to-T output ratios. This provides an account of a complete boom-bust episode.

**Who is to blame (1): guarantees and contract enforceability.** A key finding of this paper is that the *interaction* of contract enforceability problems and systemic guarantees creates the fragility required for self-fulfilling crises. If there were no guarantees, firms would not be willing to take on price risk. Costly enforceability of contracts would still imply that the N-sector can grow only gradually and balance sheet effects could play a role during the lending boom. However, there would be no force that makes a boom end in a crisis. In contrast, if there were only guarantees but no enforceability problems, then there would not be any balance sheet effects that make demand backward-bending, a necessary condition for a sunspot to matter.

There is an interesting *nonlinearity* in the relationship between the parameter  $h$ , which measures the contract enforceability problems, and the fragility of the economy. On the one

The Model Economy



Note: The parameters used are  $e_0=8.1417$ ,  $q_0=5$ ,  $e=1.6417$ ,  $\alpha=0.9598$ ,  $\theta=1.0527$ ,  $d=8$ ,  $g=0.7783$ ,  $\beta=0.9434$ ,  $h=1.01$ ,  $T=8$ ,  $\hat{d}=d \cdot (1+g)^T$ .

FIGURE 3

hand, if the diversion cost is too large, risky equilibria do not exist. In this case, at the level of the individual firm, a credit constraint would not arise. Balance sheet effects are then absent and crises cannot occur. In other words, our crisis mechanism cannot work in countries where there are no asymmetries in financing opportunities and the N-sector is well integrated into international credit markets. On the other hand, if  $h$  is very small, balance sheet effects are not strong enough ( $\eta_t < 1$ ). This precludes the existence of lending boom equilibria, whether safe or

risky. Obviously, countries in which agents have very little access to international credit markets should not be expected to exhibit booms, and are therefore immune to crises.

**Who is to blame (2): financial liberalization and a rosy future.** In our model, booms cannot occur in just any economy with bailout guarantees and enforceability problems. It is also necessary to have “good news” about the future value of N-goods. Otherwise the N-sector would not be able to repay the accumulated deficits it runs during the lending boom. Backward induction then dictates that the sequence of returns that supports the lending boom would collapse. This suggests that booms are likely to occur during a transition period following a reform or a financial liberalization. During the boom the N-sector expands in anticipation of a future increase in demand.

Note that even if the reform also increases anticipated T-sector productivity, we should still expect an increase in the N-to-T output ratio. Financially unconstrained T-firms will not increase investment until the productivity increase materializes, whereas constrained N-firms begin investing immediately. This is because N-sector firms can only grow by gradually building debt capacity.<sup>19</sup> Note also that even if the reform leads to an ongoing productivity growth in the T-sector, the N-to-T output ratio may still increase. Suppose T-sector productivity  $g_t$  grows at rate  $\lambda$ . The unconstrained T-sector’s investment and output then also grow at rate  $\lambda$ . In contrast, the constrained N-sector’s investment and output grow at an *increasing* rate. This implies that as long as  $\lambda$  is not too large, the N-to-T output ratio will eventually increase if the boom goes on long enough.

**No exogenous shocks and no money.** We consider an economy with no exogenous uncertainty. This is in contrast to explanations of crises that are based on the premise that emerging economies suffer from larger exogenous shocks than other countries. Moreover, we consider a non-monetary economy. This clarifies that neither money nor nominal rigidities are necessary for crises to occur. Furthermore, a real model fits well with the fact that the boom-bust cycle does not differ significantly under fixed and non-fixed exchange rate regimes.

**How likely is a crisis?** Our model implies that even during a transitional period the likelihood of a self-fulfilling crisis is not a free parameter. If crises were not rare events, the production of N-goods would not be a positive NPV undertaking. This implies that an equilibrium with binding borrowing constraints would not exist. If internal funds are initially low, crises cannot occur during the initial phase of a boom. It is only after the boom has gone long enough that fragility arises. The size of the possible downturn becomes more severe, as the anticipated event that triggered the boom draws near.

In our model lending booms need *not* end in crisis. Instead a boom can end in a soft landing. In a safe equilibrium the financial structure is not fragile. Meanwhile, in a sunspot equilibrium there must be a switch from a risky to a safe phase before terminal time. Thus, if the boom lasts long enough there is a soft landing.

## 8. LITERATURE REVIEW

There are a number of “third generation” crisis models that have invoked financial market imperfections to explain crises.<sup>20</sup> These models are typically based on *one of two* distortions:

19. If there were time-to-build constraints or investment adjustment costs in the T-sector, this result would be less stark. However, it would still be valid if these adjustment costs were smaller than the financial adjustment costs faced by the N-sector.

20. The Mundell–Fleming framework and traditional BoP crisis models are not appropriate for explaining these new boom-bust episodes, because credit plays no essential role in these models. In the standard Mundell–Fleming model,

either “bad markets”, in the form of an imperfection that generates borrowing constraints, or “bad policy”, in the form of bailout guarantees. To our knowledge, this paper is the first to rationalize a complete boom-bust episode, and to formally study bailout guarantees and contract enforceability problems in a unified framework.<sup>21</sup>

Beginning with Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) balance sheet effects have been at the heart of a large literature in macroeconomics.<sup>22</sup> Recent applications in a two sector, open economy context include Aghion, Bachetta and Banerjee (2000a) and Caballero and Krishnamurthy (2001). In Aghion, Bolton and Dewatripont, T-goods are produced using a country-specific factor, which is nontradable. In their setup it is the T-sector the one that is constrained by net worth, and there are no bailout guarantees. An increase in T-sector net worth relaxes borrowing constraints, and drives up the input price. As T-sector wealth builds up, the second effect gains strength. Thus, there comes a time when the real appreciation spell comes to an end and there is a drastic real depreciation. Caballero and Krishnamurthy (2001) consider a three-period, two-sector economy with credit constraints. They also single out the N-sector as having more difficulties in obtaining external finance. They show that N-sector firms do not have incentives to hedge against future uncertainty and that in a crisis, shocks can get propagated across sectors and amplified through collateral prices. In contrast to our story, exogenous shocks are essential for crises to occur.

Query 6

Bailout guarantees have been prominent in discussions of the Asian crisis. Corsetti *et al.* (1999); Krugman (1998) and McKinnon and Pill (1987) emphasize the role of guarantees for over-investment and the behaviour of asset prices. Burnside, Eichenbaum and Rebelo (2001) consider a one-sector economy and show that guarantees discourage agents from hedging their foreign currency exposure. Self-fulfilling devaluations are possible because a devaluation transforms government’s contingent liabilities into actual liabilities and depletes government reserves.

In terms of the “crisis mechanism”, the papers most related to ours are Calvo (1998) and Krugman (1999). They also argue that, with risky debt denomination, balance sheet effects can be responsible for self-fulfilling meltdowns. In contrast to our work, they simply assume the existence of foreign currency denominated debt and credit constraints.

Following Obstfeld (1986), a number of papers have described crises in models with multiple equilibria. Chang and Velasco (1998) and Cole and Kehoe (1997), emphasize coordination problems among lenders in the presence of short term debt. In these models, lenders refuse to roll-over debt because they fear others may also refuse to do so. Although this coordination failure can also occur in our model, it is distinct from the self-fulfilling real depreciations we emphasize in this paper.

There are no banks in our model. The credit chain is subsumed in a single borrower–lender relationship. Holmstrom and Tirole (1997) have modelled how the capital of the banking system constrains lending and hence spills over to bank-dependent firms and constrains their investment. The role of banks in the spread of crises has been analysed by Diamond and Rajan (2000), and Aghion *et al.* (2000b). In the first paper enforceability problems imply that the only way illiquid investments can be financed is through domestic banks, which in turn must borrow short-term. A crisis occurs when an exogenous productivity shock forces early liquidation. This precipitates a

when there is a capital outflow the needed improvement in the current account can be attained with a real depreciation and with no output costs. According to this view, a depreciation induces a shift of resources from the nontradable to the tradable sector, and makes the economy more competitive in world markets. As a result growth resumes fast after the depreciation.

21. In Schneider and Tornell (1999) we consider an economy with these two distortions and exogenous shocks, and focus on the behaviour of prices of fixed-supplied assets (*i.e.* real estate).

22. For a survey see Bernanke, Gertler and Gilchrist (2000).

meltdown of the banking sector and generates a credit crunch. The second paper shows that while the elimination of systemic guarantees reduces moral hazard and thus the likelihood of individual bank insolvencies, it amplifies the effect of a systemic shock through contagious bank runs.

## 9. CONCLUSION

In the late 1980s several middle income countries implemented far reaching reforms that reduced the size of the government sector and promoted the rapid growth of the private sector. These reforms were associated with large increases in credit to the private sector. As a result, the well-known balance of payments (BoP) crises associated with fiscal and monetary imbalances were superseded by “new” boom-bust cycles in which credit, currency mismatch, and bailout guarantees took center stage.

In this paper we have shown that boom-bust cycles can be generated by the interaction of two characteristics of financing typical of middle income countries. First, there is an *asymmetry in financing opportunities* across sectors. While the T-sector has access to several sources of external finance, the N-sector faces enforceability problems. Second, lenders enjoy *systemic bailout guarantees*. One distinctive feature of this paper is that excessive risk taking and credit constraints arise simultaneously in equilibrium. This is because in our setup systemic bailout guarantees do not neutralize the contract enforceability problem.

A second distinctive feature is that both credit risk and real exchange rate risk arise endogenously. We do not assume real exchange rate risk by introducing shocks to fundamentals, and we do not assume credit risk by imposing currency mismatch. Instead, in our model, real exchange rate risk is translated into credit risk by the optimal currency mismatch chosen by the N-sector. In turn, credit risk translates into real exchange rate risk because self-fulfilling crises can occur if there is currency mismatch. Thus, the existence of both credit risk and real exchange rate risk is self-reinforcing.

A third distinctive feature is that bailout guarantees are not just a nuisance. In our model eliminating them does not unambiguously lead to an improvement in economic outcomes. Since there is a financial friction, there is an interesting trade-off: guarantees could increase the growth rate, but they make the economy vulnerable to self-fulfilling crises. Under some circumstances bailout guarantees bring the resource allocation nearer to the Pareto optimal one. This reflects the fact that in the model lending booms reflect not only excessive risk taking. Instead, booms are episodes during which borrowing constraints are eased following events that make the future look brighter than the present.

A fourth distinctive feature is that the results were obtained in a real model. This shows that the particulars of the nominal exchange rate regime are irrelevant to explain the characteristics of a boom-bust cycle. It might well be the case, for example, that a specific nominal exchange rate regime blocks certain transmission mechanisms. However, this does not mean that the candidate regime has gotten rid of the boom-bust cycle: the cycle might simply appear under a different guise. This result fits well with the fact that the boom-bust cycle does not differ significantly under fixed and non-fixed exchange rate regimes.

Finally, faster productivity growth in the T-sector is often invoked to rationalize the long-run real appreciation observed in countries that have experienced substantial income growth (the Balassa–Samuelson effect). This effect does not explain the coexistence of real appreciation and an increasing N-to-T output ratio that is typically observed during lending booms. Our model can explain this stylized fact because there is a self-reinforcing mechanism in the N-sector.



## APPENDIX

## A1. The credit market game

We establish two versions of Propositions 4.1 and 4.2. The versions in the text assume that prices are given and  $\alpha$  goes to one for given prices. To prove the results in Section 6, we will need two more general Propositions 4.1\* and 4.2\* that assume a set of conditions involving both  $\alpha$  and prices. The link between the two sets of propositions is provided by the following (obvious) lemma.

**Lemma A1.** *If condition (4.1) is satisfied, and  $\alpha$  is close to one, the following conditions hold:*

$$\text{Positive net present value: } R^e \equiv p^e \theta / p \geq (1 + r) \quad (\text{A.1})$$

$$\text{Bound on the low return: } \underline{R} \equiv \underline{p} \theta / p < h / \alpha \quad (\text{A.2})$$

$$\text{Bound on the high return: } \bar{R} \equiv \bar{p} \theta / p < h / \alpha (1 - \alpha) \quad (\text{A.3})$$

$$\text{Parameters' restrictions: } \alpha > \max\{\beta h, 1 - \beta h\} \quad (\text{A.4})$$

$$\text{Positive net return in the high price state: } \bar{R} \geq [1 + r] / \alpha. \quad (\text{A.5})$$

As a preparation for the proof of Propositions 4.1\* and 4.2\*, we now consider the problem faced by an individual manager, given other managers' strategies. It is helpful to introduce three random variables. Let  $\zeta_{t+1} = 1$  indicate whether the firm is solvent ( $\pi_{t+1} \geq 0$ ), and let  $\delta_t = 1$  indicate whether there is *no* diversion. Other managers' strategies enter only through their effect on bailouts. We assume that a bailout occurs in the bad state only. Let  $\phi_{t+1}$  denote a random variable that is equal to  $F$  in the bad state and equal to zero in the good state, where  $F \in \{0, 1\}$  is the fraction of claims paid to lenders in a bailout. The manager's problem, for given  $F$ , can now be written as (we will drop time indices and use hyphens to refer to the next period):

*Problem P(F).* Given prices  $(p, \underline{p}, \bar{p})$ , choose a plan  $(I, s^n, s, b^n, b, \rho^n, \rho, \delta, \zeta')$  that maximizes  $\Pi = E(\delta \zeta' \hat{\pi} + [1 - \delta][G' \zeta' - h[w + b + b^n]])$  subject to the budget constraint (3.2), and the lenders' break-even constraints

$$(1 + \rho_t) E_t[\delta_t \zeta_{t+1} + (1 - \delta_t \zeta_{t+1}) \phi_{t+1}] \geq 1 + r,$$

$$(1 + \rho_t^n) E_t[p_{t+1}(\delta_t \zeta_{t+1} + (1 - \delta_t \zeta_{t+1}) \phi_{t+1})] \geq 1 + r.$$

Next period's profit in a state with price  $p'$  is given by

$$\hat{\pi}(p') := p' \theta I + (1 + r)s + (1 + r^n)p' s^n - (1 + \rho^n)p' b^n - (1 + \rho)b. \quad (\text{A.6})$$

We solve this problem in three steps. First, Lemma A2 determines the best "safe plan", that is, the best plan such that the firm is solvent in both states of the world ( $\zeta' \equiv 0$ ). Second, Lemma A3 determines the best "risky plan", that is, the best plan such that the firm is solvent in the good state only. Lemma A4 then determines the optimal plan over all.

**Lemma A2 (Best Safe Plan).** *Suppose  $\alpha \leq 1$ . If conditions (A.1) and (A.3) hold, the best safe plan does not lead to diversion. It satisfies  $s = s^n = 0$  and  $pI = m^s w := \frac{1}{1 - \beta h} w$ . Optimal debt denomination keeps the share of tradables debt  $\gamma = \frac{b}{b + b^n}$  below a threshold. In particular, it is always weakly optimal to denominate all debt in nontradables. The interest rates satisfy:  $\rho = r$  and  $1 + \rho^n = [1 + r] / p^e$ .*

*Proof.* The best safe fundable plan  $(I, s, s^n, b, b^n, \rho, \rho^n)$  that does not lead to diversion maximizes

$$\begin{aligned} \Pi^{s,nd} = & \alpha(\bar{p}\theta I + (1 + r)s + (1 + r)\bar{p}(p^e)^{-1}s^n - (1 + \rho)b - (1 + \rho^n)\bar{p}b^n) \\ & + (1 - \alpha)(\underline{p}\theta I + (1 + r)s + (1 + r)\underline{p}(p^e)^{-1}s^n - (1 + \rho)b - (1 + \rho^n)\underline{p}b^n), \end{aligned}$$

subject to the budget constraint (3.2), the borrowing constraint

$$(1 + \rho^n)p^e b^n + (1 + \rho)b \leq h[pI + s + s^n], \quad (\text{A.7})$$

the solvency constraints  $\hat{\pi}(\bar{p}) \geq 0$ ,  $\hat{\pi}(\underline{p}) \geq 0$ , and the lenders' break-even constraints

$$1 + \rho \geq 1 + r, \quad (1 + \rho^n)p^e \geq 1 + r. \quad (\text{A.8})$$

It is clearly optimal to set  $\rho = r$  and  $1 + \rho^n = \frac{1+r}{p^e}$ . Condition (A.1) implies that  $s^n > 0$  cannot be optimal, since N-bonds are dominated in rate of return by investment. Similarly, we cannot have  $\theta I < (1 + \rho^n)b^n$  at the optimum. If this was the case, both  $I$  and  $b^n$  could be reduced, strictly increasing profits and relaxing all constraints. But if

$\theta I \geq (1 + \rho^n)b^n$ , the solvency constraint for the high price state is implied by that for the low price state, which is given by  $\hat{\pi}(\underline{p}) = \underline{p}\theta I + (1+r)s + (1+r)\underline{p}(p^e)^{-1}s^n - (1+\rho)b - (1+\rho^n)\underline{p}b^n \geq 0$ . It then follows that  $b = 0$  is optimal: the two types of debt and the two types of storage are exchangeable in the objective and in all constraints except for the low price state solvency constraint, where T-debt is more expensive. It is thus weakly better to replace all  $b$  with  $b^n$ . We can now reformulate the problem as choosing  $(I, s, b^n)$  to maximize  $(\frac{p^e\theta}{p} - (1+r))pI + (1+r)w$ , subject to the borrowing and solvency constraints

$$\left(1 - \frac{h}{1+r}\right)(pI + s) \leq w, \quad \left(1 - \frac{p^e\theta}{p(1+r)}\right)pI + \left(1 - \frac{p^e}{p}\right)s \leq w.$$

Since  $p^e\theta \geq (1+r)p$  (by (A.1)), the solvency constraint is irrelevant and optimal investment is  $pI = \frac{w}{1-\beta h}$ . This yields a payoff equal to  $\bar{\Pi}^{s,nd} = \frac{p^e\theta p^{-1}-h}{1-\beta h}w$ .

We now show that the best safe, fundable plan that leads to diversion cannot yield a profit higher than  $\bar{\Pi}^{s,nd}$ . This plan maximizes profits subject to the requirement that the no-diversion constraint (A.7) does not hold, and  $\hat{\pi}(\bar{p}), \hat{\pi}(\underline{p}) \geq 0$ . Since we just want to bound the payoff, we ignore (A.7). Imposing it cannot increase payoffs. We then know that the interest rates should be set as low as possible to relax the solvency constraints, which can now be written as

$$\begin{aligned} \hat{\pi}(\bar{p}) &= \bar{p}\theta I + (1+r)s + (1+r)\bar{p}(p^e)^{-1}s^n - \frac{1+r}{(1-\alpha)\phi}b - \frac{1+r}{(1-\alpha)\phi}\frac{\bar{p}}{p}b^n \geq 0, \\ \hat{\pi}(\underline{p}) &= \underline{p}\theta I + (1+r)s + (1+r)\underline{p}(p^e)^{-1}s^n - \frac{1+r}{(1-\alpha)\phi}b - \frac{1+r}{(1-\alpha)\phi}b^n \geq 0. \end{aligned}$$

Without loss of generality, we can set  $b^n = 0$ . Moreover, the constraint for the bad state implies the one for the good state. We thus reformulate the problem as maximizing  $(p^e p^{-1}\theta - h)pI + (1+r-h)s + (1+r-h)s^n$  subject to

$$(1 - \beta p p^{-1}\theta(1-\alpha)\phi)pI + (1 - (1-\alpha)\phi)s + (1 - \underline{p}(p^e)^{-1}(1-\alpha)\phi)s^n \leq w.$$

The shadow costs multiplying the portfolio choices in the constraint are positive. For the shadow cost of investment, this follows from (A.3), and it is obvious for T-bonds. In addition, (A.1) implies that the shadow cost of N-bonds is higher than that of investment and so the former must be positive also.

Since investment has a higher marginal benefit (but a possibly higher shadow cost than T-bonds), it is not clear what the preferred instrument is. We can concentrate on plans that satisfy the constraint with equality and bound the payoff by showing that for both the plan with  $I = s^n = 0$  and that with  $s^n = s = 0$ , the payoff is lower than  $\bar{\Pi}^{s,nd}$ . For the former plan we have  $\bar{\Pi}^{s,d} \leq \frac{1+r-h}{1-(1-\alpha)\phi}w < (1+r)w \leq \bar{\Pi}^{s,nd}$ . The second inequality uses the fact that  $\alpha > 1 - \beta h$  and the third inequality follows because setting  $s = w$  is a fundable safe plan. For the plan with  $s^n = s = 0$  we have  $\bar{\Pi}^{s,d} \leq \frac{p^e p^{-1}\theta - h}{1 - \beta p p^{-1}\theta(1-\alpha)\phi}w$ , which is less than  $\bar{\Pi}^{s,nd}$  by (A.3). Finally, we consider the optimal debt policy. We already know that any plan with  $b^n = pI - w$  ( $\gamma = 0$ ) is optimal. The remaining question is whether there are other plans with  $b > 0$  and  $b + b^n = pI - w$ . We need only check whether these plans satisfy the solvency constraint  $\hat{\pi}(\underline{p}) = \underline{p}\theta - (1+r)\gamma(pI - w) - \underline{p}\frac{1+r}{p^e}(1-\gamma)(pI - w) \geq 0$ . This is equivalent to  $\gamma < \bar{\gamma} = \min\left\{\left(\frac{p^e}{p}\theta - h\right)\left(\frac{p^e}{p}\frac{h}{p} - h\right)^{-1}, 1\right\}$ .  $\parallel$

**Lemma A3 (Best Risky Plan).** *Suppose  $\alpha < 1$ . If conditions (A.1)–(A.4) hold, then*

1. *If (A.5) holds, there exists a best risky plan. In this plan there is no diversion,  $s = s^n = 0$  and  $pI = \frac{w}{1-\alpha^{-1}\beta h}$ .*
2. *If (A.5) does not hold, the profit from risky plans is bounded above by  $(1+r)w$ .*
3. *If (A.5) holds and bailout guarantees are present ( $F > 0$ ), all debt is T-debt.*

*Proof. Parts 1 and 2.* As a first step, we determine the best risky fundable plan that does not lead to diversion. In the second step we will show that the best risky fundable plan with diversion is less profitable.

**Step 1.** The best risky fundable plan with *no diversion* maximizes  $\alpha\hat{\pi}(\bar{p}_{t+1})$  subject to the budget constraint (3.2), the lenders' break-even constraints, as well as the riskiness requirement,

$$1+r \leq (1+\rho)(\alpha + (1-\alpha)F), \quad 1+r \leq (1+\rho^n)(\alpha\bar{p} + (1-\alpha)F\underline{p}), \quad (\text{A.9})$$

$$\hat{\pi}(\bar{p}) = \bar{p}\theta I + (1+r)s + (1+r)\bar{p}(p^e)^{-1}s^n - (1+\rho)b - (1+\rho^n)\bar{p}b^n \geq 0, \quad (\text{A.10})$$

$$\hat{\pi}(\underline{p}) = \underline{p}\theta I + (1+r)s + (1+r)\underline{p}(p^e)^{-1}s^n - (1+\rho)b - (1+\rho^n)\underline{p}b^n < 0. \quad (\text{A.11})$$

Without loss of generality, the break-even constraints can be taken to be binding. Suppose one of them was slack at the optimal plan. We could then reduce the interest rate: profits are not decreased (they go up if debt is positive) and all constraints would still hold. Interest rates are thus

$$1 + \rho^n = \frac{1 + r}{\alpha \bar{p} + (1 - \alpha) F \underline{p}}, \quad 1 + \rho = \frac{1 + r}{\alpha + (1 - \alpha) F}. \quad (\text{A.12})$$

We first show that  $b^n > 0$  cannot be optimal. For any policy involving  $b^n > 0$ , we can construct an alternative policy by increasing  $b$  slightly and decreasing  $b^n$  by the same amount. Since  $[1 + \rho^n] \underline{p} < 1 + \rho < [1 + \rho^n] \bar{p}$ , the alternative policy must yield higher expected profits and still satisfy all the constraints. Note also that the profit is strictly increased if  $F > 0$ . Therefore,  $F > 0$  implies  $b^n = 0$ .

A risky plan with  $s > 0$  can never be optimal. Suppose to the contrary that  $s > 0$  is optimal. Then we can reduce slightly  $s$  and  $b$  by the same amount, strictly increasing the expected payoff (note that  $b > 0$  must hold at the initial plan by the insolvency constraint). This leads to strictly higher profits (since  $\rho \geq r$ ). The borrowing constraint is unaffected. The insolvency constraint still holds if the change is small enough, since it held with strict inequality at the original plan. This is a contradiction.

Setting  $b^n = s = 0$ , we can reformulate the problem as that of choosing  $(I, s^n)$  to maximize

$$\alpha \left( \frac{\bar{p}^\theta}{p} - (1 + \rho) \right) pI + \alpha \left( \frac{\bar{p}(1 + r)}{p^e} - (1 + \rho) \right) s^n + \alpha(1 + \rho) w, \quad (\text{A.13})$$

subject to the borrowing and insolvency constraints

$$\left( 1 - \frac{1}{\alpha(1 + \rho)} h \right) (pI + s^n) \leq w, \quad (\text{A.14})$$

$$\left( 1 - \frac{\underline{p}^\theta}{p(1 + \rho)} \right) pI + \left( 1 - \frac{\underline{p}(1 + r)}{p^e(1 + \rho)} \right) s^n > w. \quad (\text{A.15})$$

By (A.1), the net return on physical investment in the objective is higher for funds invested in the technology than for investment in N-bonds. In addition, by (A.4), (A.14) imposes an upper bound on total funds invested.

Suppose that (A.5) does not hold. Then the net return on both investment opportunities is negative. It follows that there does not exist a best risky plan that does not lead to diversion and that the profit from all risky plans is bounded above by  $(1 + r)w$ . Otherwise, if (A.5) holds, the investment expenditure that maximizes (A.13) subject to (A.14) *only* is  $pI = \frac{w}{1 - \frac{1}{\alpha(1 + \rho)} h} = \frac{w}{1 - \beta h \left( 1 + \frac{1 - \alpha}{\alpha} F \right)}$ . Finally, (A.2) implies that this solution also satisfies the second constraint. We

have found the optimal no-diversion-plan under (A.5). The expected payoff of this plan is  $\bar{\Pi}^{r,nd} = \frac{\alpha \theta \bar{p} p^{-1} - h}{1 - \frac{h}{\alpha(1 + \rho)}} \cdot w$ .

**Step 2.** Consider now the best risky, fundable, diversion plan. Such plans maximize  $\Pi^{r,d} = \alpha[\bar{p}^\theta I + (1 + r)s + (1 + r^n)\bar{p}s^n] - h(pI + s + s^n)$ , subject to budget constraint (3.2), the riskiness requirement (A.10)–(A.11), the requirement that the no-diversion constraint does *not* hold, and the break-even constraints,

$$(1 - \alpha) \phi_{t+1}(1 + \rho_t) \geq 1 + r, \quad (1 - \alpha) \phi_{t+1}(1 + \rho_t^n) \underline{p}_{t+1} \geq 1 + r. \quad (\text{A.16})$$

The debt choices enter only through the solvency and budget constraints. By a similar argument to that above, we can set  $b^n = 0$  without loss of generality. Our goal is to bound the payoff under a diversion plan. We thus ignore the no diversion condition and (A.11). Imposing them will, if anything, make this payoff even lower. It is then optimal to set the interest rate as low as possible and to use the break-even constraint for T-debt holding with equality. We solve the problem of choosing  $(I, s, s^n)$  to maximize

$$[\alpha \bar{p} p^{-1} \theta - h] pI + [\alpha(1 + r) - h] s + [\alpha(1 + r) \bar{p} (p^e)^{-1} - h] s^n,$$

subject to (3.2), (A.16) and the insolvency constraint:

$$(1 - \beta \bar{p} p^{-1} \theta (1 - \alpha) F) pI + (1 - (1 - \alpha) F) s + (1 - \bar{p} (p^e)^{-1} (1 - \alpha) F) s^n \leq w.$$

This problem has a solution because the three terms (shadow costs) multiplying the portfolio choices in the constraint are positive. For the shadow cost of investment, this follows from (A.3),  $\alpha > \beta h$  and  $F \leq 1$ . It follows trivially for T-bonds. Finally, (A.1) implies that the shadow cost of N-bonds is higher than that of investment, so that this shadow cost must be positive also.

By (A.1), the marginal benefit of investment is higher than that of either type of bond, and the shadow cost of investment is lower than that of T-bonds. The best plan must thus involve as much investment as possible, such that the insolvency constraint binds. This yields an upper bound on the payoff of  $\bar{\Pi}^{r,d} \leq \frac{\alpha \theta \bar{p} p^{-1} - h}{1 - \beta \bar{p} p^{-1} \theta (1 - \alpha)} \cdot w$ . It then follows

that if (A.5) does not hold, then  $\bar{\Pi}^{r,d} < (1+r)w$ . In contrast, if (A.5) holds, then a best risky no diversion plan exists and (A.3) implies  $\bar{\Pi}^{r,d} < \bar{\Pi}^{r,nd}$ .

**Part 3.** Consider the optimal debt policy. We have already shown in Step 1 that under the optimal non-diversion plan it is *strictly* optimal to have  $b^n = 0$  if  $F = 1$ . Since the optimal non-diversion plan is the overall optimal plan, this proves Part 3.  $\parallel$

**Lemma A4 (Optimal Plan).** *If conditions (A.1)–(A.4) hold, then:*

1. *If  $F = 1$ , the optimal plan (i.e. the solution to the managerial decision problem  $P(F)$ ) is the best risky fundable plan (characterized in Lemma A3).*
2. *If  $F = 0$ , the optimal plan is the best safe fundable plan (characterized in Lemma A2).*

*Proof.* If  $\alpha = 1$ , then the concept of a risky plan is not defined. It follows that the best safe plan is the optimal plan. Suppose instead that  $\alpha < 1$ . Consider first the case  $F = 0$ . If (A.5) does not hold, then the optimal plan cannot be a risky plan, since, by Lemma A3, the best risky plan yields less than  $(1+r)w$ . If (A.5) holds, we know from Lemmas A2 and A3 that a best safe plan and a best risky plan exist. From the proofs of these lemmas, payoffs are  $\Pi^s := \frac{p^e \theta p^{-1} - h}{1 - \beta h} w$  and  $\Pi^r = \frac{\alpha \bar{p} \theta p^{-1} - h}{1 - \beta h (1 + \frac{1-\alpha}{\alpha} F)} w$ , respectively. It is clear that a safe plan is preferred if  $F = 0$ .

For  $F = 1$ , (A.5) is implied by (A.1). Hence, both a best safe plan and a best risky plan exist. Using the definition of  $p^e$ , we obtain after some algebra that  $\Pi^r > \Pi^s$  if and only if  $\bar{p} \theta p^{-1} < \alpha^{-1} h \frac{p^e \theta p^{-1} - h}{1+r-h}$ . We know that  $\bar{p} \theta p^{-1} < \alpha^{-1} h$  by (A.2), and the fraction on the R.H.S. is greater than one, by (A.1).

We have shown that a safe plan dominates if  $F = 0$  or  $\alpha = 1$ , while a risky plan dominates if  $\alpha < 1$  and  $F = 1$ . The only task left is to rule out plans that are neither safe nor risky. Plans that lead to insolvency in *both* states yield zero profit and are obviously inferior. Consider a plan that leads to solvency in the bad state and insolvency in the good state. For such a plan, positive profits accrue only in the bad state and the plan satisfies at least the budget constraint and  $\hat{\pi}(\bar{p}) < 0$ ,  $\hat{\pi}(p) \geq 0$ . Since we want to bound the payoff, we ignore constraints related to diversion. It cannot be optimal to have either  $I$ ,  $s^n$  or  $b$  positive (since investment and N-bonds are dominated by T-bonds in return in the low price state, while N-debt is strictly more expensive than T-debt in that state).

Since lenders have to break even,  $(1 + \rho^n)(1 - \alpha)pF \geq 1 + r$ . Also, T-bond holdings have to satisfy  $s(1 - (1 - \alpha)F) \leq w$ . Profit are bounded by gross returns on T-bonds, which are in turn bounded above by  $(1 - \alpha) \frac{1+r}{1-(1-\alpha)F} w$ . With  $F = 0$ , this is clearly inferior to the best safe plan. The same is true for  $F = 1$ , because  $\alpha > \frac{1}{2}$  by (A.4). This concludes the proof  $\parallel$

Having solved the problem  $\mathbf{P}(F)$  for an individual manager, we are now ready to prove the main propositions.

**Proposition 4.1\*.** *If conditions (A.1)–(A.4) hold, then the implications of Proposition 4.1 are true.*

*Proof.* In a world without guarantees, managers' choices are not interdependent. Every manager simply solves the problem  $\mathbf{P}(0)$  defined at the beginning of this Appendix. Lemma A4, part 2 says that the solution to this problem is the best safe fundable plan, which is characterized in Lemma A2.

*Proof of Proposition 4.1.* By Lemma A1, conditions (A.1)–(A.4) hold for  $\alpha$  close enough to one. Thus proposition 4.1\* applies  $\parallel$

For risky equilibria, we proceed as in the safe case. We state a proposition for general  $\alpha$ , and then invoke Lemma A1.

**Proposition 4.2\*.** *If conditions (A.1)–(A.4) hold, then the implications of Proposition 4.2 are true.*

For  $F > 0$ , the feasible plans in problem  $\mathbf{P}(F)$  are exactly those that are fundable *if everybody else chooses a risky plan*. A risky plan is part of a symmetric equilibrium if and only if it solves  $\mathbf{P}(F)$ . Indeed, any risky plan that is feasible in  $\mathbf{P}(F)$  but is not a maximizer can never be part of a symmetric equilibrium. Suppose it was, then a bailout would be expected in the good state, so an individual entrepreneur could choose any plan from the feasible set and have it funded. He could thus simply pick the maximizer. Conversely, we can construct an equilibrium from any risky maximizer of  $\mathbf{P}(F)$ . Now since conditions (A.1)–(A.4) hold, Proposition A4 implies that under the maintained assumptions, there is a risky plan that solves  $\mathbf{P}(F)$  if and only if  $F = 1$ . We can read off the properties of such a plan from Lemma A3.  $\parallel$

*Proof of Proposition 4.2.* By Lemma A1, conditions (A.1)–(A.4) hold for  $\alpha$  close enough to one. Thus Proposition 4.2\* applies.  $\parallel$

#### A2. Dynamics

*Proof of Proposition 6.1. Part 1.* We need to show that, under the conditions of the proposition, there is a solution  $(q_t, w_t)$  to (6.2)–(6.4) which satisfies the positive NPV condition on prices. It is clear that there is a solution for every set of initial conditions. Along this solution,  $p_t \theta > (1+r)p_{t-1}$  for all  $t < T$  and  $p_T \theta > (1+r)p_{T-1}$  if and only if  $w_{t-1} > \frac{\beta d}{cm^s} =: \underline{e}_1$  for  $t < T-1$  and  $w_{T-1} < \beta(1-\beta h)\hat{d}$ . Moreover, for  $t < T$ ,  $w_t > w_{t-1}$  if and only if  $w_{t-1} > \left(\frac{1}{1-c} - \frac{1-h}{1-\beta h}\right)^{-1} d =: \underline{e}_2$ . We have  $\underline{e}_2 > 0$  because  $\eta^s > 1$  implies  $c < \beta h$ . Let

$$\underline{e} = \max \left\{ \frac{\beta}{cm^s}, \left( \frac{1}{1-c} - \frac{1-h}{1-\beta h} \right)^{-1} \right\} \cdot d, \quad (\text{A.17})$$

and let  $w_t(e_0)$  denote the solution of an economy starting at  $w_0 = e_0 > \underline{e}$ . This solution increases over time. Finally, to ensure  $w_{T-1} < \beta(1-\beta h)\hat{d}$ , let  $\hat{d} > \underline{d}(e_0, T) = \beta^{-1}(1-\beta h)^{-1}w_{T-1}(e_0)$ .

**Part 2.** We show that it holds at  $t$  if and only if  $w_{t-1} > w_p$ ,  $w_{t-1} > w_q$ , and  $w_{t-1} > w_c$  for appropriately chosen lower bounds  $w_p$ ,  $w_q$  and  $w_c$ . Since  $w_t$  is increasing over time (by Part 1), this implies Part 2. For prices, we have that  $p_t > p_{t-1}$  for  $t < T$  along a solution if and only if  $m^s(\eta^s h - \theta(\eta^s - 1))w_{t-1} > d$ . Condition  $\theta < \eta^s h / (\eta^s - 1)$  implies that the term multiplying  $w_{t-1}$  is positive. Thus, the lower bound is  $w_p := d[m^s \eta^s h - m^s \theta(\eta^s - 1)]^{-1}$ . For output, we have that N-output is increasing ( $q_t > q_{t-1}$ ) when  $w_{t-1}[m^s(\theta - 1)] > d$ . Thus,  $q_t > q_{t-1}$  if and only if  $\theta > 1$  and  $w_{t-1} > w_q := [m^s(\theta - 1)]^{-1}d$ . For the N-to-T output ratio, since T-output grows at rate  $\lambda$  and  $p_t q_t = d + m^s w_t$ , it follows that

$$\frac{p_t q_t}{y_t} > \frac{p_{t-1} q_{t-1}}{y_{t-1}} \iff \frac{w_t}{w_{t-1}} > \lambda + d \cdot \frac{\lambda - 1}{m^s w_{t-1}}. \quad (\text{A.18})$$

Equation (6.3) implies that (A.18) holds only if  $\lambda < \eta^s h / (\eta^s - 1)$ . Since  $\eta^s > 1$ , we know from (6.3) that  $w_t/w_{t-1}$  is increasing in  $w_{t-1}$ . Thus, there is a lower bound  $w_c$  such that the inequality in (A.18) holds if and only if  $w_{t-1} > w_c$ . Finally, for the credit-to-GDP ratio, note that, along a solution, credit is  $b_t + b_t^n = m^s h w_t$  and  $GDP_t = y_t + p_t q_t$ . Using the market clearing condition and  $p_t I_t = m^s w_t$ ,  $\frac{b_t + b_t^n}{GDP_t} > \frac{b_{t-1} + b_{t-1}^n}{GDP_{t-1}} \iff \frac{[m^s - 1]w_t}{[m^s - 1]w_{t-1}} > \frac{y_t + d + m^s w_t}{y_{t-1} + d + m^s w_{t-1}} \iff \frac{w_t}{w_{t-1}} > \frac{d + y_t}{d + y_{t-1}}$ . Since the R.H.S. is lower than  $\frac{y_t}{y_{t-1}} = \lambda$ , the last inequality holds if (A.18) holds and  $w_{t-1} > w_c$ .  $\parallel$

*Proof of Proposition 6.2. Parts 1 and 2.* We begin by constructing a candidate “lucky path” on which no crisis occurs. Pick an  $e_0 > \underline{e}$ , where  $\underline{e}$  is given by (A.17), and let  $w_t(e_0)$  denote the solution to the safe equilibrium difference equation (6.3). Select  $\underline{t}$  such that  $\underline{t} - 1$  is the smallest  $t$  such that  $w_t(e_0)$  satisfies (6.6). We will construct a path where  $(q_t, w_t)$  evolve according to a safe equilibrium until time  $\underline{t} - 2$ , and then according to a risky equilibrium until  $T - 1$ . The last step, from  $T - 1$  to  $T$ , is again according to a safe equilibrium. For any  $T$ , we can define a lucky path as the unique solution, for given  $w_0 = e_0$  and  $q_0$ , to

$$q_t = \theta \frac{m_{t-1} w_{t-1}}{m_{t-1} w_{t-1} + d} q_{t-1} \quad \text{for } t \leq T,$$

$$\frac{1 - \eta_t}{1 - c} w_t = d - \xi_{t-1} m_{t-1} w_{t-1} \quad \text{for } t < T, \quad \text{and} \quad w_T = \hat{d} - h m^s w_{T-1}.$$

where for  $t \leq \underline{t} - 2$ , we have  $m_t = m^s$ ,  $\eta_t = \eta^s$  and  $\xi_t = h$ . Meanwhile, for  $t = \{\underline{t} - 1, \dots, T - 2\}$ ,  $m_t = m^r$ ,  $\eta_t = \eta^r$  and  $\xi_t = \alpha^{-1} h$ . To prove that this path is an equilibrium, we need to show that for  $t = \{\underline{t} - 1, \dots, T - 2\}$ , conditions (A.1)–(A.3) hold with  $p = p_t$ ,  $\bar{p} = \frac{d + m_{t+1} w_{t+1}}{q_t}$  and  $\underline{p} = \frac{d + e m^s}{q_t}$ . If this is the case, the transition from  $t$  to  $t + 1$  is indeed consistent with a risky equilibrium. Similarly, we have to show that for  $t \leq \underline{t} - 2$  and  $t = T - 1$  (A.1) holds, but for  $\alpha = 1$ . Some algebra reveals that (A.1) holds at time  $t$  if and only if

$$\left( \alpha \xi_{t-1} \frac{\eta_t}{\eta_t - 1} - \beta^{-1} \right) m_{t-1} w_{t-1} \geq \alpha \frac{d(1-c)}{\eta_t - 1} - (1-\alpha) \frac{d}{1 - m_t \mu_e}. \quad (\text{A.19})$$

Conditions (A.2) and (A.3) hold along the candidate path if and only if

$$\xi_{t-1} m_{t-1} (1 - m_t \mu_e) w_{t-1} \geq d, \quad \text{and} \quad (\text{A.20})$$

$$\left( \frac{\alpha^{-1} h}{1 - \alpha} m_{t-1} - \frac{\xi_{t-1} \eta_{t-1}}{\eta_t - 1} m_t \right) w_{t-1} \geq \frac{-d}{\eta_t - 1}. \quad (\text{A.21})$$

We also have that  $w_t > w_{t-1}$  along the candidate path if and only if

$$(1 - \eta_t + \xi_{t-1}\eta_{t-1})w_{t-1} > (1 - c)d. \quad (\text{A.22})$$

If  $\alpha \rightarrow 1$ , all lower bounds for  $w_{t-1}$  required by (A.22) converge to a number smaller or equal than  $\underline{e}$  (in (A.17)). Since  $w_0 > \underline{e}$ , it follows that, for  $\alpha$  close enough to one,  $w_t$  is increasing over time. Similarly, the lower bounds required by (A.19) converge to a number smaller or equal than  $\underline{e}$ . It follows that (A.19) holds for every  $t$  if  $\alpha$  is close to one. Moreover, (A.20) holds at time  $\underline{\tau}$  for  $\alpha$  close to one, because  $w_{\underline{\tau}-1}(e_0)$  satisfies (6.6) by construction. Since  $w_t$  is increasing, (A.20) then holds for all  $t$ , up to  $T - 1$ . If  $\alpha$  is close enough to one, the term multiplying  $w_{t-1}$  on the L.H.S. of (A.21) is positive. This implies that (A.21) holds for all  $t$ . We have thus ensured that the candidate solution is consistent with a risky equilibrium in  $t = \{\underline{\tau} - 1, \dots, T - 2\}$ . To ensure that the path is consistent with a safe equilibrium in period  $T - 1$ , we pick  $\hat{d}$  large enough such that  $\hat{d} > \underline{d}(e_0, \alpha, T)$ .

**Part 3.** We follow the same steps as in the proof of Proposition 6.1. During any time  $\tau < t < T - 1$ , the economy is in a risky equilibrium. Thus, we have that  $p_t > p_{t-1}$  if and only if  $w_{t-1} > w_p(\alpha) := d/m^r[\eta^r h\alpha^{-1} - \theta(\eta^r - 1)]$  and  $\theta < \eta^r h/\alpha(\eta^r - 1)$ . Second, for N-output we have that  $q_t > q_{t-1}$  if and only if  $\theta > 1$  and  $w_{t-1} > w_q(\alpha) := d/m^r(\theta - 1)$ . Third, the N-to-T ratio increases if and only if  $\lambda < \eta^r h/\alpha(\eta^r - 1)$  and  $w_t/w_{t-1} > \lambda + d[\lambda - 1]/m^r w_{t-1}$ . These conditions imply that the credit-to-GDP ratio increases, like in the proof of Proposition 6.1. Since  $w_t$  and  $w_t/w_{t-1}$  are increasing in the safe phase, they will continue to be so in the risky phase because  $\eta^r > \eta^s$  and  $m^r > m^s$ . It follows that Part 3 is established by picking a sufficiently big lower bound on  $T$  (i.e.  $\underline{T}(e_0, \alpha)$ ).

**Part 4.** Consider the case where  $t \in \{\tau, \dots, T - 1\}$  is a crisis period. When a crisis occurs, internal funds revert to  $w_t = e < w_{t-1}$ . Since  $q_t > q_{t-1}$  for  $t \geq \tau$ , we have  $p_t \leq \frac{d+m^s e}{q_t} < \frac{d+m^r w_{t-1}}{q_{t-1}} = p_{t-1}$ , so that a crisis involves a real depreciation. Finally, we know from Proposition 6.1 that, if  $e > \underline{e}$ , a safe equilibrium with positive investment exists. In this equilibrium  $\frac{q_{t+1}}{q_t} = \theta \frac{m^s e}{d+m^s e} < 1$  under the stated condition.  $\parallel$

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