# Exploring the Link between Housing and the Value Premium

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## Abstract

This paper shows that an equilibrium model in which heterogeneous households face housing collateral constraints can quantitatively replicate the cross-sectional variation in risk premia on stock portfolios sorted by book-to-market value. A value premium arises because (1) cash flows to growth stocks are situated farther into the future than the cash flows on value stocks, and (2) claims to farther-out cash flows are less risky because they are only subject to low-frequency housing collateral shocks and not to temporary consumption growth shocks. In contrast to many other equilibrium asset pricing models, our model endogenously generates a downward sloping term structure of equity risk premia; a necessary condition for a value premium (Lettau and Wachter, 2006). Our calibration shows that we not only generate the right sign, but also the right magnitude for the returns spread between value and growth stocks.

# Introduction

The canonical consumption-based asset pricing model of Breeden (1979) and Lucas (1978) implies small and roughly constant equity risk premia over time and little or no risk premium variation in the cross-section. Yet, recent research in empirical asset pricing has documented striking differences in risk premia between equity and bonds, between equity at different points in time, and between portfolios formed by sorting equities on their book-to-market ratio. According to Fama and French (1992), value stocks earn returns that are on average six percent higher than growth stocks; this premium is of the same size as the equity risk premium itself.

The two most common approaches to tackling the shortcomings of the Lucas-Breeden model are changing the preferences[1], or changing the dynamics of the aggregate consumption process.[2] While the canonical external habit model is successful at generating an unconditional equity premium and time-variation in conditional asset pricing moments, it generates a negative value premium in the cross-section (Lettau and Wachter (2006)). The reason is that a negative aggregate consumption growth shock depresses the price of long-lived assets by more because the shock affects future marginal utility terms through the surplus consumption ratio dynamics. In other words, the term structure of consumption strip risk premia is upward sloping instead of downward sloping. To generate a positive value premium in the habit model, Santos and Veronesi (2006) model growth and value stocks as having substantially different cash-flow properties. The heterogeneity in correlations between dividend growth and consumption growth needed to generate a six percent value premium is larger than in the data. As for the second approach, Bansal, Dittmar and Lundblad (2001) use the Bansal-Yaron model to price portfolios with different exposure to the small, predictable component in consumption growth. This exposure is estimated to be different for

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[1]Habit style preferences are most commonly used, see Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Abel (1999), Campbell and Cochrane (1999), and Menzly, Santos and Veronesi (2004) for early contributions. Another approach is to model non-separable preferences over a second good, such as housing (Flavin (2001) and Piazzesi, Schneider and Tuzel (2006)) or durables (Dunn and Singleton (1986), Eichenbaum and Hansen (1990), and Yogo (2006))

[2]Bansal and Yaron (2004) introduce a small but very persistent component in aggregate consumption and dividend growth. Bekaert, Engstrom and Grenadier (2004) combine this specification with habit style preferences.

value and growth stocks. Hansen, Heaton and Li (2005) use a similar approach to price the cross-section. The difficulty with this approach is that it is hard to estimate the correlation of dividend growth with the persistent component in consumption growth. The latter is proxied by long-horizon consumption growth, on which we have few observations.

Instead of staying within the representative agent framework, we introduce heterogeneity among agents. Our focus is on the impact of time variation in risk sharing on asset prices. In the model, households differ only by their income histories. They share income risk by trading contingent claims, but they cannot borrow more than the value of their house. When housing collateral is scarce, collateral constraints constrain risk sharing more, and, as a result, risk premia are higher. Thus, risk premia vary over time and with the housing collateral ratio. This modest friction is a realistic one for an advanced economy like the US.[3] The main contribution of this paper is to demonstrate that the endogenous time variation in the amount of housing collateral can *quantitatively* account for the differences in expected returns between value and growth portfolios.

A large literature links the value premium to production-side factors or technological change (Gomes, Kogan and Zhang (2003b), Gomes, Yaron and Zhang (2003a), Zhang (2005), Galla (2005), Gourio (2005), and Panageas and Yu (2006)). Our approach is complementary; its goal is to investigate how much of that value premium can be accounted for by incomplete risk-sharing and housing collateral constraints in an otherwise standard consumption-based asset pricing model.

We start by briefly setting up the model (section 1) and focus on its asset pricing implications. The households trade a complete menu of assets, as in Lucas (1978), but they face endogenous solvency constraints because they can repudiate their debts. When a household chooses to repudiate its debts, it loses all its housing wealth but its labor income is protected from creditors. The household is not excluded from trading.[4] We carefully cal-

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[3]Our emphasis on housing, rather than financial assets, reflects three features of the US economy: the participation rate in housing markets is very high (2/3 of households own their home), the value of the residential real estate makes up over seventy-five percent of total assets for the median household (Survey of Consumer Finances, 2001), and housing is a prime source of collateral (75 percent of household borrowing in the data is collateralized by housing wealth, US Flow of Funds, 2003). To keep the model exposition simple, we abstract from financial assets or other kinds of capital (such as cars) that households may use to collateralize loans. However, in the calibration we explore the effects of using a broader measure of collateral.

[4]In Kehoe and Levine (1993), Krueger (2000), Kehoe and Perri (2002), and Krueger and Perri (2005),

ibrate the model in section 2. The model and calibration are identical to Lustig and Van Nieuwerburgh (2006b), and are repeated for convenience.

Value stocks earn returns that are on average six percent higher than growth stocks and they have higher Sharpe ratios. Our model replicates these features of the data (section 3). Figure 1 shows that return spreads on book-to-market sorted portfolios predicted by the model line up nicely with the same spreads in the data. Our model endogenously generates a positive value premium when value stocks are short-duration assets. The reason lies in the term structure of consumption risk premia it generates. In a recent paper, Lettau and Wachter (2006) point out that, if value stocks are short-duration stocks and growth stocks long-duration stocks, a positive value premium requires the term structure of consumption risk premia to be downward sloping. Yet, the habit formation model of Campbell and Cochrane (1999) generates an upward sloping term structure of consumption risk premia: Since a bad consumption shock increases discount rates almost permanently, the price of long-maturity consumption claims would fall by more. In other words, growth stocks would earn a larger risk premium. In contrast, a bad consumption shock in our model increases discount rates temporarily. It does not affect the collateral ratio, which governs discount rates in the long run. As a result, the price of consumption strips of longer maturity is insulated from bad consumption shocks today. This generates lower expected returns on growth stocks than value stocks.

[Figure 1 about here.]

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limited commitment is also the source of incomplete risk-sharing. But the outside option upon default is exclusion from all future risk sharing arrangements. Alvarez and Jermann (2000) show how to decentralize these Kehoe and Levine (1993) equilibria with sequential trade. Geanakoplos and Zame (2000) and Kubler and Schmedders (2003) consider a different environment in which individual assets collateralize individual promises in a standard incomplete markets economy. We model the outside option as bankruptcy with loss of all collateral assets; all promises are backed by all collateral assets.

# 1 Model

## 1.1 Environment

**Uncertainty**  The economy is populated by a continuum of infinitely lived households. The structure of uncertainty is twofold: $s = (y, z)$ is an event that consists of a household-specific component $y \in Y$ and an aggregate component $z \in Z$. These events take on values on a discrete grid $S = Y \times Z$. We use $s^t = (y^t, z^t)$ to denote the history of events. $S^t$ denotes the set of possible histories up until time $t$. The state $s$ follows a Markov process with transition probabilities $\pi$ that obey:

$$\pi(z'|z) = \sum_{y' \in Y} \pi(y', z'|y, z) \; \forall z \in Z, y \in Y.$$

Because of the law of large numbers, $\pi_z(y)$ denotes both the fraction of households drawing $y$ when the aggregate event is $z$ and the probability that a given household is in state $y$ when the aggregate state is $z$.[5]

**Preferences**  We use $\{x\}$ to denote an infinite stream $\{x_t(s^t)\}_{t=0}^{\infty}$. There are two types of commodities in this economy: a consumption good $c$ and housing services $h$. These commodities cannot be stored. The households rank consumption streams according to the criterion:

$$U\left(\{c\}, \{h\}\right) = \sum_{s^t|s_0} \sum_{t=0}^{\infty} \delta^t \pi(s^t|s_0) u\left(c_t(s^t), h_t(s^t)\right), \tag{1}$$

where $\delta$ is the time discount factor. The households have power utility over a CES-composite consumption good:

$$u(c_t, h_t) = \frac{\left[c_t^{\frac{\varepsilon-1}{\varepsilon}} + \psi h_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}}}{1 - \gamma}.$$

The parameter $\psi > 0$ converts the housing stock into a service flow, $\gamma$ governs the degree of relative risk aversion, and $\varepsilon$ is the intratemporal elasticity of substitution between non-

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[5]The usual caveat applies when applying the law or large numbers.

durable consumption and housing services.[6]

**Endowments**  The aggregate endowment of the non-durable consumption good is denoted $\{c^a\}$. The growth rate of the aggregate endowment depends only on the current aggregate state: $c_{t+1}^a(z^{t+1}) = \lambda(z_{t+1})c_t^a(z^t)$. Each household is endowed with a labor income stream $\{\eta\}$. The labor income share $\hat{\eta}(y_t, z_t) = \eta(y_t, z^t)/c^a(z^t)$, only depends on the current state of nature. Since the aggregate endowment is the sum of the individual endowments,

$$\sum_{y' \in Y} \pi_z(y')\hat{\eta}_t(y', z) = 1, \ \forall z, t \geq 0.$$

The aggregate endowment of housing services is denoted $\{h^a\}$ and $\rho(z^t)$ denotes the relative price of a unit of housing services. The calibration specifies a process for the ratio of non-housing expenditures and housing services expenditures $\{r\}$, $r(z^t) = \frac{c^a(z^t)}{\rho(z^t)h^a(z^t)}$, rather than for $\{h^a\}$ directly.

**Trading**  Each household is assigned a label $(\ell, s_0)$, where $\ell$ denotes the time-zero collateral wealth of this household. The cross-sectional distribution of initial non-labor wealth and income states $(\ell, s_0)$ is denoted $\mathcal{L}_0$.[7] We let $\{c(\ell, s_0)\}$ denote the stream of consumption and we let $\{h(\ell, s_0)\}$ denote the stream of housing services of a household of type $(\ell, s_0)$. The financial markets are complete: households trade a complete set of contingent claims $a$ in forward markets, where $-a_t(\ell, s^t, s')$ is a promise made by agent $(\ell, s_0)$ to deliver one unit of the consumption good if event $s'$ is realized in the next period. These claims are in zero net supply, and trade at prices $q_t(s^t, s')$.[8]  All prices are quoted in units of the non-durable consumption good. There are frictionless rental markets and markets for home ownership; ownership and housing consumption are separated. The rental price is $\rho_t(z^t)$; $p_t^h(z^t)$ denotes the (asset) price of the housing stock. Because of the law of large numbers, these prices only

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[6]The preferences belong to the class of homothetic power utility functions of Eichenbaum and Hansen (1990). Special cases are separability ($\varepsilon = \gamma^{-1}$) and Cobb-Douglas preferences ($\varepsilon = 1$).

[7]So, $\ell$ denotes the value of the initial claim to housing wealth as well as any financial wealth that is in zero net aggregate supply. In the model there is no financial wealth in positive net supply, but in the calibration we consider augmenting the collateral stock to include realistic values of other financial wealth.

[8]This setup is equivalent to having financial intermediaries trade in state contingent claims and provide insurance to the households (Atkeson and Lucas (1993)).

depend on aggregate histories.

At the start of each period, the household purchases non-housing consumption in the spot market $c_t(\ell, s^t)$, housing services in the rental market $h_t^r(\ell, s^t)$, contingent claims in the financial market and ownership shares in the housing stock $h_{t+1}^o(\ell, s^t)$ subject to a wealth constraint:

$$c_t(\ell, s^t) + \rho_t(z^t) h_t^r(\ell, s^t) + \sum_{s'} q_t(s^t, s') a_t(\ell, s^t, s') + p_t^h(z^t) h_{t+1}^o(\ell, s^t) \leq W_t(\ell, s^t). \quad (2)$$

Next period wealth is labor income, plus assets, plus the cum-dividend value of owned housing:

$$W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h_{t+1}^o(\ell, s^t) \left[ p_{t+1}^h(z^t, z') + \rho_{t+1}(z^t, z') \right]. \quad (3)$$

**Collateral Constraints**  Households can default on their debts. When the household defaults, it keeps its labor income in all future periods. The household is not excluded from trading, even in the same period. However, all collateral wealth is taken away. As a result, the markets impose a solvency constraint that keeps the households from defaulting: all of a household's state-contingent promises must be backed by the cum-dividend value of its housing owned at the end of period $t$, $h_{t+1}^o$. In each node $s^t$, households face a separate *collateral constraint* for each future event $s'$:

$$-a_t(\ell, s^t, s') \leq h_{t+1}^o(\ell, s^t) \left[ p_{t+1}^h(z^{t+1}) + \rho_{t+1}(z^{t+1}) \right], \text{ for all } s^t, s'. \quad (4)$$

As in Alvarez and Jermann (2000), these constraints are not too tight: they allow for the maximal degree of risk sharing, given that agents cannot be excluded from trading, while preventing default.

## 1.2   Equilibrium Asset Prices

**Competitive Equilibrium.** *Given a distribution over initial non-labor wealth and initial states $\mathcal{L}_0$, a competitive equilibrium is a feasible allocation $\{c(\ell, s^t), h^r(\ell, s^t), a(\ell, s^t), h^o(\ell, s^t)\}$*

*and prices $\{q, p^h, \rho\}$ such that (i) for given prices and initial wealth, the allocation solves each household's maximization problem (1) s.t. (2), (3) and (4), and (ii) the markets for the consumption good, the housing services, the contingent claims and housing ownership shares clear.*

As in other endogenously incomplete markets models, assets are priced by the unconstrained agents at every date and state (Alvarez and Jermann (2000)). These unconstrained households have the highest intertemporal marginal rate of substitution (IMRS), equal to the stochastic discount factor (SDF) $m$:

$$m_{t+1} = max_{i \in [0,1]} \left\{ \delta \frac{u_c(c_{t+1}^i, h_{t+1}^i)}{u_c(c_t^i, h_t^i)} \right\} = max_{i \in [0,1]} \left\{ \delta \left( \frac{c_{t+1}^i}{c_t^i} \right)^{-\gamma} \left( \frac{1 + r_{t+1}^{-1}}{1 + r_t^{-1}} \right)^{\frac{1-\varepsilon\gamma}{\varepsilon-1}} \right\}. \quad (5)$$

The second equality follows from the form of the utility function, the definition of the expenditure ratio $r = \frac{c^a}{\rho h^a}$, and market clearing in the housing market.[9] No arbitrage implies that the return on any security $j$, $R_{t+1}^j$, satisfies the standard Euler equation $E_t[m_{t+1} R_{t+1}^j] = 1$.

We detail the equilibrium consumption dynamics in section 2 of Lustig and Van Nieuwerburgh (2006b) and show there that they can be used to restate the SDF into the product of three risk factors:

$$m_{t+1} = \delta \left( \frac{c_{t+1}^a}{c_t^a} \right)^{-\gamma} \left( \frac{1 + r_{t+1}^{-1}}{1 + r_t^{-1}} \right)^{\frac{1-\varepsilon\gamma}{\varepsilon-1}} g_{t+1}^{\gamma}. \quad (6)$$

The first two factors arise in the representative agent (or perfect risk-sharing) version of our model, whereas the third term is new and reflects the risk of binding collateral constraints. The quantity $g_{t+1}$ measures the increase in the extent to which the housing collateral constraints bind in the aggregate. The dynamics of $g$ are responsible for generating the value premium in this model.

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[9]The equilibrium rental price is $\rho_t = u_h(c_t^i, h_t^i)/u_c(c_t^i, h_t^i) = \psi(h_t^i/c_t^i)^{-\frac{1}{\varepsilon}}, \forall i$. Since there is one economy-wide rental market, the rental price only depends on aggregate quantities: $\rho_t(z^t) = \psi(h_t^a(z^t)/c_t^a(z^t))^{-\frac{1}{\varepsilon}}$. Consequently, all households equate their non-housing to housing consumption ratios $r(z^t)$.

## 1.3 Two Driving Forces

To build intuition for the asset pricing results, we first explain the two main driving forces of the model: shocks to the wealth distribution, operating at business cycle frequencies, and variation in the housing collateral ratio, operating at low frequencies. Both of these forces affect the SDF $m_{t+1}$ in (6) through its third term $g_{t+1}^\gamma$, which we label the *aggregate weight shock*.

**Shocks to the Wealth Distribution**   Because risk sharing is imperfect, the higher cross-sectional income dispersion in a low aggregate consumption growth state results in more wealth and consumption dispersion. First, the household cutoff levels (at which the collateral constraints hold with equality) are higher in low aggregate consumption growth states, and this makes the consumption increase for households that switch to a state with a binding constraint larger. Second, low aggregate consumption growth states are short-lived in our model and agents are more constrained in these states as a result, because of their desire to smooth out its effect on their consumption. As the combined result of these two forces, the size of the aggregate weight shock increases more in low aggregate consumption growth states $(g_{t+1}(z^t, re) > g_{t+1}(z^t, ex))$. However, after a low consumption growth shock accompanied by a large aggregate weight shock $g_{t+1}$, the left tail of the wealth distribution is cleansed, and subsequent aggregate weight shocks are much smaller. This *cleansing* mechanism lowers the conditional market price of risk $\sigma_t[m_{t+1}/E_t[m_{t+1}]]$ and increases the interest rate after a bad shock. These wealth distribution dynamics operate at business cycle frequencies and are also present in Lustig (2003). They are a first source of heteroscedasticity in the SDF, and will allow the model to match year-to-year variation in stock returns.

**Housing Collateral Mechanism**   There is another source of heteroscedasticity: low frequency changes in the housing collateral ratio. This paper's novel feature are movements in the housing collateral ratio that come from exogenous movement in the non-housing expenditure ratio $r$ together with endogenous movements in the SDF. It is these low frequency movements in the housing collateral ratio that will allow the model the match asset prices at low frequencies.

Figure 2 illustrates the collateral mechanism for a typical two hundred period simulation of the benchmark model. The calibration is in section 2 below. Panel 1 plots the housing collateral ratio $my$ (bold, right axis) together with the expenditure ratio $r$ (single line, left axis). It shows that the housing collateral ratio increases when households spend a larger share of income on housing. The persistence of $my$ comes from this relationship. Panel 2 plots the cross-sectional consumption growth dispersion (single line, left axis) against the housing collateral ratio $my$ (bold line, right axis). It summarizes the risk sharing dynamics in the model. When collateral is scarce, more households run into binding collateral constraints. To prevent default, the consumption share of the constrained households increases. At the same time, the unconstrained households' consumption share decreases precipitously. As a result, the cross-sectional standard deviation of consumption growth increases, evidence of less risk-sharing. For example, in a period of collateral abundance (period 126), $\sigma_t[\Delta \log c_{t+1}]$ is 8.1%, whereas in a period of collateral scarcity (period 174), it is only 0.9%.[10] The aggregate weight shock $g_{t+1}$, plotted in panel 3, measures the economy-wide extent to which the solvency constraints bind. It also governs the new component to the SDF $g_{t+1}^\gamma$. The panel illustrates that when collateral is scarce, constraints bind more frequently and more severely and this is reflected in a large aggregate weight shock. For example, in period 126 the liquidity shock is 1.07, whereas in period 174 it is only 1.01. The SDF is higher and more volatile in such periods of collateral scarcity, and quite different from the representative agent SDF. The last three panels illustrate how this impacts asset prices: the equity premium is lower and the conditional equity volatility and the conditional Sharpe ratio are higher when collateral is scarce. Households demand a larger compensation for risk when it is hard to insure income shocks. As we show below, there is an intimate link between this time-series variation in the conditional market equity premium and the cross-sectional variation in risk premia.

[Figure 2 about here.]

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[10]As an aside, even though the consumption shares change in important ways when collateral constraints bind, the *unconditional* volatility of consumption growth for an individual household is moderate. In our benchmark model it is less than 10% of the unconditional volatility of individual income growth. There is still a considerable amount of risk-sharing.

## 2   Calibration

There are two driving forces in the model: the income process and the non-housing expenditure ratio.

**Income Process**   The first driving force in the model is the Markov process for the nondurable endowment process. It has an aggregate and an idiosyncratic component. The aggregate endowment growth process is taken from Mehra and Prescott (1985) and replicates the moments of aggregate consumption growth in the 1871-1979 data. Aggregate endowment growth, $\lambda$, follows an autoregressive process:

$$\lambda_t(z_t) = \rho_\lambda \lambda_{t-1}(z_{t-1}) + \varepsilon_t,$$

with $\rho_\lambda = -.14, E(\lambda) = .0183$ and $\sigma(\lambda) = .0357$. We discretize the AR(1) process with two aggregate growth states $z = (ex, re) = [1.04, .96]$ (for expansion and recession) and an aggregate state transition matrix $[.83, .17; .69, .31]$. The implied ratio of the probability of a high aggregate endowment growth state to the probability of a low aggregate endowment growth state is 2.65. The unconditional probability of a low endowment growth state is 27.4%. This matches the observed frequency of recessions.

The idiosyncratic labor income volatility in the US increases in recessions (Storesletten, Telmer and Yaron (2004)). Our calibrated labor income process shares this feature. Following Alvarez and Jermann (2001), log labor income shares follow an AR(1) process with autocorrelation of .92, and a conditional variance of .181 in low and .0467 in high aggregate endowment growth states. Discretization into a four-state Markov chain results in individual income states $(\eta^1(hi, ex), \eta^1(lo, ex)) = [.6578, .3422]$ in the high and $(\eta^1(hi, re), \eta^1(lo, re)) = [.7952, .2048]$ in the low aggregate endowment growth state.[11] We refer to the counter-cyclical labor income share dispersion as the Mankiw (1986) effect.

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[11]The one difference with the Storesletten et al. (2004) calibration is that recessions are shorter in our calibration. In their paper the economy is in the low aggregate endowment growth state half of the time. That implies that the unconditional variance of our labor income process is lower.

**Expenditure Ratio**   The second driving force in the model is the process for the ratio of non-housing to housing expenditures $\{r\}$. Calibrating the expenditure ratio is equivalent to calibrating the evolution of the aggregate housing stock $\{h\}$ and imposing the intra-temporal optimality condition. Following Piazzesi et al. (2006), we specify an autoregressive process which also depends on aggregate endowment growth $\lambda$:

$$\log r_{t+1} = \bar{r} + \rho_r \log r_t + b_r \lambda_{t+1} + \sigma_r \nu_{t+1}, \tag{7}$$

where $\nu_{t+1}$ is an i.i.d. standard normal process with mean zero, orthogonal to $\lambda_{t+1}$. In our benchmark calibration we set $\rho_r = .96$, $b_r = .93$ and $\sigma_r = .03$. The parameter values come from estimating equation (7) on US data.[12] We discretize the process for $\log(r)$ as a five-state Markov process. A second calibration switches off the effect of consumption growth by setting $b_r = 0$. Both calibrations fix $\sigma_r = .03$. We choose the constant $\bar{r}$ to match the average housing expenditure share of 19% in the data (NIPA, 1929 to 2004).

**Average Housing Collateral Ratio**   A key quantitative question is whether collateral is sufficiently scarce for our borrowing constraints to have a large effect. Because this question is an important one, we consider two measures to calibrate the average ratio of collateral wealth to total wealth. The first measure focuses on housing collateral, the second measure includes non-housing sources of collateral.

We measure factor payments to housing wealth as total US rental income and factor payments to human wealth as labor income (compensation of employees). NIPA data show that rental income was 3.4% of rental income plus labor income in 1946-2002 and 4.3% in 1929-2002. Because the factor payments ratio maps directly into the housing collateral ratio, the data suggest a housing collateral ratio less than 5%.[13]

To be on the safe side, our second estimate is a broad collateral measure. It includes financial wealth, the market value of the non-farm non-financial corporate sector in the US. We add interest payments and dividend payments to the income stream from collateralizable

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[12]Table 1 in a separate appendix shows regression estimates for $\rho_r$ and $b_r$.

[13]If $r$ is constant, the housing collateral ratio or the ratio of housing wealth to total wealth is $\frac{1/r}{1+1/r} = 1/(1+r)$. This is a very good approximation for the average collateral ratio in the model with stochastic $r$.

wealth and we add proprietary income to the income stream from non-collateralizable wealth. The factor payment ratio increases to 8.6% in the post-war sample and 9.4% in the full sample (row 2), suggesting a housing collateral ratio less than 10%.

An alternative approach is to compare the collateralizable wealth to income ratio in model and data. Assuming that the expected return on total collateralizable assets is 9% and the expected dividend growth rate is 3%, then a collateral ratio of 5% implies a collateral wealth-to-income ratio of 85% according to Gordon's growth formula: $.85 = .05/(.09 - .03)$. Likewise, the implied wealth-to-income ratio is 150% when the collateral ratio is 10%. In US data, the 1929-2004 average ratio of mortgages to income is 55%. If we include financial wealth, that ratio increases to 155%. This approach also points towards a housing collateral ratio of 5% and a broad collateral ratio of 10%.[14]

Finally, Jorgenson and Fraumeni (1989) estimate human wealth to be 93% of total wealth, implying a collateral ratio of 7%.

We take the model with a 5% collateral ratio as our benchmark and consider the economy with a 10% collateral ratio as an alternative. To simultaneously match the average expenditure share of housing services ($\bar{r}$) of 19% and the average ratio of housing wealth to total wealth ($my$) of 5% or 10%, we scale up the aggregate non-housing endowment.

**Preference Parameters** In the benchmark calibration, we use additive utility with discount rate $\delta = .95$, coefficient of relative risk aversion $\gamma = 8$, and intratemporal elasticity of substitution between non-housing and housing consumption $\varepsilon = .05$. We fix the relative weight on housing in the utility function $\psi = 1$ throughout.[15] Because our goal is to explain conditional moments of the market return, we choose the parameter $\gamma$ to match the unconditional market risk premium. We also compute the model for $\gamma \in \{2, 5, 10\}$ and $\varepsilon \in \{.15, .75\}$.

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[14]The Gordon growth model is an approximation. Appendix C of Lustig and Van Nieuwerburgh (2006b) provides a detailed analysis of this asset value approach to calibrating the collateral share. It reports that the benchmark calibration ($my = 0.05$) produces a collateral wealth-to-income ratio of 96%. If the average $my$ were to be calibrated higher, there would have to be *a lot more* tradeable wealth in the US economy.

[15]The Arrow-Pratt measure of relative risk aversion $\frac{-cu_{cc}}{u_c} = \left(\frac{r_t}{1+r_t}\right)\gamma + \left(\frac{1}{1+r_t}\right)\varepsilon^{-1}$ is a linear combination of $\gamma$ and $\varepsilon$ with weights depending on the non-durable expenditure ratio $r_t$. In the simulations $r = 4.26$ on average, so that the weight on $\gamma$ is .81 on average. Because $r_t$ is not very volatile, neither is the degree of risk aversion.

A choice for the parameter $\varepsilon$ implies a choice for the volatility of rental prices:

$$\sigma(\Delta \log \rho_{t+1}) = \left| \frac{1}{\varepsilon - 1} \right| \sigma(\Delta \log r_{t+1}). \tag{8}$$

In NIPA data (1930-2004), the left hand side of (8) is .046 and the right-hand side is .041. The implied $\varepsilon$ is .098. By choosing a low $\varepsilon$, we impose that rental prices are consistent with the expenditure ratio. A choice for $\varepsilon$ closer to one helps to generate a higher average equity premium and lower risk-free rate, but implies excessive rental price volatility.

**Stock Market Return**  We define the stock market return as the return on a *leveraged* claim to the aggregate consumption process $\{c_t^a\}$ and denote it by $R^l$. In the data, dividends are more volatile than aggregate consumption. We choose leverage parameter $\kappa = 3$, where $\sigma(\Delta \log d_{t+1}) = \kappa \sigma(\Delta \log c_{t+1}^a)$.[16] We also price a non-levered claim on the aggregate consumption stream, denoted $R^c$. The excess returns, in excess of a risk-free rate, are denoted $R^{l,e}$ and $R^{c,e}$. Table 1 summarizes the benchmark parametrization and the other values we consider in the sensitivity analysis.

[Table 1 about here.]

**Computation**  Our computational strategy is to keep track of cross-sectional distributions over wealth and endowments that change over time. Appendix B of Lustig and Van Nieuwerburgh (2006b) provides the algorithm.

**Data**  We use two distinct measures of the housing collateral stock: the value of outstanding home mortgages ($MO$) and the market value of residential real estate wealth ($RW$). These time series are from the Historical Statistics for the US (Bureau of the Census) for the period 1889-1945 and from the Flow of Funds (Federal Board of Governors) for 1945-2001. We use both the value of mortgages and the total value of residential wealth to allow for changes in the extent to which housing can be used as a collateral asset. National income is labor

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[16]For the period 1930-2004, the volatility of annual nominal dividend growth is 14.8%, whereas the volatility of annual nominal consumption growth (non-durables and services excluding housing services) is 5.6%, a ratio of 2.6.

income plus net transfer income from the Historical Statistics of the US for 1926-1930 and from the National Income and Product Accounts for 1930-2001. The housing collateral ratio $my_t$ is estimated as the residual from a cointegration relationship between $MO$ or $RW$ and $Y$, and is therefore a stationary variable. Details are provided in Lustig and Van Nieuwerburgh (2005) and the data are downloadable from the authors' web sites. Collateral is scarcer when $my_t$ is lower. For convenience, we introduce a measure of collateral scarcity that is always between 0 and 1: $\widetilde{my}_t = \frac{\max(my_t) - my_t}{\max(my_t) - \min(my_t)}$, where $\max(my_t)$ and $\min(my_t)$ are the sample maximum and minimum of $\{my_t\}$.

# 3  Cross-sectional Variation in Risk Premia

Firms with a high ratio of book value to market value of equity (value firms) historically have higher returns than those with a low book-to-market ratio (growth firms). Panel 1 of Table 2 reports sample means, volatilities, and Sharpe ratios for the excess returns on ten book-to-market deciles. The annual excess return on a zero-cost investment strategy that goes long in the highest book-to-market decile and short in the lowest decile is 6.8% for 1930-2003 and 6.5% for 1945-2003. The Sharpe ratios for the highest and lowest decile portfolios are .56 and .37 for 1945-2003 and .42 and .32 for 1930-2003.[17]

The paper's main result is that the collateral model can endogenously generate this value premium. We perform two exercises to substantiate this claim. In the first exercise we generate excess returns on value decile portfolios from an empirically plausible factor model. In a second exercise, we impose a specific timing on the cash flows of value and growth portfolios and compute returns on these portfolios.

[Table 2 about here.]

---

[17]Similar value premia are found for monthly and quarterly returns and for quintile instead of decile portfolios. Using quarterly data for 1951-2002, unconditional Sharpe ratios for value stocks (.64) are twice as large as for growth stocks (.32) (Lettau and Wachter (2006)).

## 3.1 Plugging the Empirical Betas into Model

Value stocks command higher expected returns because their returns co-vary more strongly with aggregate consumption growth when collateral is scarce (Lustig and Van Nieuwerburgh (2005)). In a first step, we use data on the decile value portfolio returns to describe the return-generating process for each of the book-to-market decile portfolios. We then price these returns inside the model and show that returns on high book-to-market portfolios carry a higher risk premium than returns on low book-to-market portfolios. The return spread in the model matches the spread in the data.

**Decile Return Processes in Data**  To estimate the relationship between excess returns on book-to-market decile portfolios and the model's state variables $(c, r, \widetilde{my})$, we use data on aggregate consumption growth, expenditure share growth, and the housing collateral ratio and estimate the betas in

$$
\begin{aligned}
R_{t+1}^{e,j} \ = \ & \beta_0^j + \beta_{my}^j \widetilde{my}_{t+1} + \beta_c^j \Delta \log c_{t+1}^a + \beta_{c,my}^j \widetilde{my}_{t+1} \Delta \log c_{t+1}^a + \\
& \beta_r^j \Delta \log r_{t+1} + \beta_{r,my}^j \widetilde{my}_{t+1} \Delta \log r_{t+1} + \nu_{t+1}^j,
\end{aligned}
\tag{9}
$$

by OLS. These are the five risk factors in the collateral model. The estimates are reported in Table 3.

[Table 3 about here.]

When collateral is abundant $(\widetilde{my}_t = 0)$, the sensitivity of excess returns to aggregate consumption growth is $\beta_c$ (Figure 3, left column). The returns on value stocks (decile 10) are high in recessions while growth stocks (decile 1) are much less sensitive to aggregate consumption growth; $|\beta_c|$ increases monotonically from decile 1 to decile 10. When collateral is scarce $(\widetilde{my}_t = 1)$, the consumption beta is $\beta_c + \beta_{c,my}$ (right column). Value stocks are more sensitive to consumption growth shocks when collateral is scarce: $\beta_{c,my} > 0$ is much higher for the tenth decile than for the first decile portfolio. This sensitivity pattern results in higher expected returns for value stocks than for growth stocks. This effect is reinforced because value stocks are also more sensitive to aggregate expenditure ratio shocks; $\beta_r$ increases monotonically for both collateral measures (not shown).

**Decile Return Processes in Model** In a second step, we generate ten excess return processes as the product of the previously estimated factor loadings $\left(\beta_{my}^j, \beta_c^j, \beta_{c,my}^j, \beta_r^j, \beta_{r,my}^j\right)$, and simulated model state variables. For each excess return, the intercept $\beta_0^j$ is chosen to make the Euler equation hold: $E_t[m_{t+1}R_{t+1}^{j,e}] = 0$. This ensures that the model SDF prices the book-to-market decile returns correctly on average. We then simulate the model for 10,000 periods and compute unconditional means and standard deviations of each decile portfolio return.

The second panel of table 2 reports the excess returns on the ten value portfolios predicted by the collateral model, ordered from growth (B1) to value (B10) for the benchmark parametrization. We use two sets of empirical factor loadings corresponding to different housing collateral measures. For the mortgage-based collateral measure, the value spread is 6.8%, matching the data. For the residential wealth measure, the value spread is even larger: 8.4%. Furthermore, the model predicts that the Sharpe ratio of the tenth decile (value) is double that of the first decile (growth), similar to the post-war data. Figure 1 (in the introduction) plots the return difference between deciles 2-9 and the lowest book-to-market portfolio for the model and for the data. The model does quite well in reproducing these spreads; if anything, the model's spreads are too large.

**Representative Agent Model** In the representative agent economy, there are no collateral constraints and perfect risk sharing obtains. This amounts to setting $\widetilde{my} = 0$ in equation (9). The estimated consumption growth and expenditure growth betas exhibit little variation across decile portfolios. The simulated returns that these betas imply, do not generate a value premium, and lower Sharpe ratios on value stocks than on growth stocks, all at odds with the data.[18]

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[18]Detailed results available upon request.

## 3.2 Pricing Stocks with Different Duration

Growth stocks have been described as assets with longer maturities than value stocks (Dechow, Sloan and Soliman (2002) and Lettau and Wachter (2006)). The second approach models growth stocks (value stocks) as a basket of consumption strips that is weighted towards longer (shorter) maturities. A period-$k$ consumption strip is a claim to aggregate consumption $c_{t+k}$, $k$ years from the current period $t$.

The multiplicative (one year) equity premium $E_0 R_{0,1}^e[\{c_k\}]$, the expected return on a non-levered claim to the entire stream of aggregate consumption $\{c_k\}_{k=1}^{\infty}$ divided by the risk-free rate, is a value-weighted sum of expected excess returns on consumption strips:

$$1 + E_0[R_{0,1}^e[\{c_k\}]] = \sum_{k=1}^{\infty} \omega_k \frac{E_0 R_{0,1}[c_k]}{R_{0,1}[1]}, \text{ with weights } \omega_k = \frac{E_0 M_k c_k}{\sum_{l=1}^{\infty} E_0 M_l c_l}. \tag{10}$$

The second term in the sum is the (gross) expected return on a period $k$ consumption strip $E_0 R_{0,1}[c_k]$ divided by the (gross) risk-free rate $R_{0,1}[1]$. The weight $\omega_k$ represents the value of the period $k$ consumption strip relative to the total value of all consumption strips. $M_k$ is the pricing kernel in period $k$. It is linked to the stochastic discount factor $m$ by $M_k = m_1 \times \cdots \times m_k$. (See appendix A for the derivation of equation 10).

We think of value stocks as a claim to a differently weighted stream of aggregate consumption $\{f^v(k) c_k\}_{k=1}^{\infty}$, where the function $f(\cdot)$ puts more weight on the consumption realizations in the near future. For example, $f^v(k) = Ce^{ak}$, where $a$ is a negative number and $C$ is a normalization constant, $C = \frac{\sum_{k=1}^{\infty} c_k}{\sum_{k=1}^{\infty} e^{ak} c_k}$. Likewise, growth stocks are a claim to a weighted stream of aggregate consumption $\{f^g(k) c_k\}_{k=1}^{\infty}$, where $f(\cdot)$ puts more weight on the consumption realizations in the far future. For example, $f^g(k) = Ce^{ak}$, where $a$ is a positive number. The multiplicative equity premium $\widetilde{\nu}_0$ on such a basket of consumption strips with weights $f(k)$ is:

$$1 + \widetilde{\nu}_0 = \sum_{k=1}^{\infty} \tilde{\omega}_k \frac{E_0 R_{0,1}[c_k]}{R_{0,1}[1]}, \text{ with modified weights } \tilde{\omega}_k = \frac{f(k) E_0 M_k c_k}{\sum_{l=1}^{\infty} f(l) E_0 M_l c_l}.$$

The following proposition shows that the properties of the pricing kernel determine the sign of the value spread. Appendix A proves that, if the pricing kernel has no permanent

component, then the highest risk premium is the one on the farthest out consumption strip $(k \to \infty)$. The model generates a *growth premium*.

**Proposition 1.** *If* $\gamma > 1$, $f(k) = Ce^{ak}$ *with* $a > 0$ *and if* $\lim_{k \to \infty} \frac{E_1 M_k c_k}{E_0 M_k c_k} = 1$, *then* $\lim_{a \to \infty} 1 + \widetilde{\nu}_0 = \lim_{k \to \infty} R^e_{0,1}[c_k] \geq 1 + \nu_0$, *for any sequence of weights* $\{\omega_k\}$ *in the definition of the multiplicative risk premium* $\nu_0$.

The proof draws on insights from Alvarez and Jermann (2005). The pricing kernel in our model contains a permanent component stemming from the risk of binding solvency constraints, even in the absence of the consumption growth shocks.[19] Such a permanent component is a necessary condition for generating a value premium.

**Representative Agent Economy**   In the representative agent economy, the equity premia on consumption strips do not change with the horizon. This is easy to show for additive preferences that are separable in both commodities and aggregate endowment growth that is i.i.d with mean $\bar{\lambda}$.[20] The pricing kernel is simply a function of the aggregate consumption growth rate between period 1 and period $k$: $M_k = \lambda_k^{-\gamma} \lambda_{k-1}^{-\gamma} \cdots \lambda_1^{-\gamma}$. Because the aggregate endowment grows every period at the rate $\lambda$, $M_1 c_1 = \lambda_1^{-\gamma} \lambda_1 c_0$. For the period $k$ strip, $M_k c_k = \lambda_k^{1-\gamma} \lambda_{k-1}^{1-\gamma} \cdots \lambda_1^{1-\gamma} c_0$. Hence, the expected return on a period $k$ strip is:

$$E_0 R_{0,1}[c_k] = \frac{E_1 (M_k c_k)}{E_0 (M_k c_k)} = \frac{E_1 \left( \lambda_k^{1-\gamma} \lambda_{k-1}^{1-\gamma} \cdots \lambda_1^{1-\gamma} c_0 \right)}{E_0 \left( \lambda_k^{1-\gamma} \lambda_{k-1}^{1-\gamma} \cdots \lambda_1^{1-\gamma} c_0 \right)} = \left( \frac{\lambda_1}{\bar{\lambda}} \right)^{1-\gamma}$$

The expression does not depend on the horizon $k$, meaning that strip risk premia are constant across horizons. Since the term structure of consumption risk premia is flat, the value premium is zero.

**Term Structure of Strip Premia and Time-Varying Value Premia**   Our model behaves like a representative agent economy when housing collateral is abundant. These are times when the term structure of risk premia is nearly flat. However, collateral scarcity

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[19]Section **??** explains that the aggregate weight shock, which measures the extent of binding solvency constraints and enters as a multiplicative term in the SDF, is a non-decreasing stochastic process.

[20]A similar result obtains if preferences are non-separable and aggregate expenditure share growth is i.i.d., even when aggregate expenditure share growth is correlated with aggregate consumption growth.

generates a downward sloping term structure of risk premia. Since the model oscillates between abundant collateral (flat term structure) and scarce collateral (downward sloping term structure), it generates a downward slope on average. This generates a value premium, because value stocks load more heavily on short duration consumption strips. Put differently, because short term assets are more risky than long term assets, the expected return and Sharpe ratio are higher for value stocks than growth stocks.

The downward sloping term structure relies crucially on having two model driving forces that operate at different horizons: changes in aggregate consumption growth, which cause shocks to the wealth distribution, at business cycle frequencies and changes to the collateral ratio at low frequencies. Risk premia on long-maturity assets respond mostly to low-frequency collateral changes, whereas short-maturity assets respond to both shocks. Suppose that collateral is scarce and there is a negative aggregate consumption growth shock (a recession). This shock resolves risk, because it lowers the likelihood of being constrained in the immediate future (the left tail of the wealth distribution is 'cleansed'). The risk premium decreases. But the risk-free rate increases by more so that the discount rate increases. Higher discount rates imply lower prices. But because the aggregate consumption growth shock is temporary, it only lowers the price of short-maturity assets and not the price of long-maturity assets.

Prices and expected returns are inversely related, so that the term structure of expected returns is downward sloping. Put differently, expected returns on value stocks increase because short-duration assets have higher consumption growth risk when collateral is scarce. For long-duration assets such as growth stocks, only long-term discount rates matter. These are governed by the housing collateral ratio and not by aggregate consumption growth.

If there is only one driving force, as in the habit formation model, a fall in aggregate consumption increases marginal utility growth not just today, but persistently into the future. Therefore, it generates a bigger price decline of long-duration consumption strips than of short-duration strips. The term structure of risk premia is upward sloping. The habit model predicts a growth premium (Lettau and Wachter (2006)).

Panels 1 and 2 of Figure 4 plot the expected return and Sharpe ratio on consumption strips of horizons of 1 to 45 years for the benchmark model. Long-maturity strips have risk

premia and Sharpe ratios that are lower than those for short-maturity strips. This pattern can be traced back to their differential consumption risk.

[Figure 4 about here.]

Finally, panel 3 of Table 2 reports equity premia on claims to $\{Ce^{ak}c_k\}$. These are baskets of consumption strips of different maturities, where the constant $a$ governs the duration of the basket. The leverage parameter $\kappa$ is 2. We vary $a$ from .5 to -.5. The corresponding baskets have a duration between 43 years and 2 years (row 1). We think of the baskets with long-duration as the growth portfolios and the baskets with short-duration as the value portfolios (Dechow et al. (2002)). The benchmark model generates a maximum value spread of 5.2% between the 8-year and the 43-year portfolios, close to the value spread in the data. In addition, the Sharpe ratios on the value portfolios are much higher than the Sharpe ratios on the growth portfolios. The bottom row of Figure 4 confirms that long-duration assets (growth stocks) have lower risk premia and lower Sharpe ratios.

# 4  Conclusion

This paper shows how endogenous, state-contingent borrowing constraints interact with shocks in the housing market to deliver plausible asset pricing predictions. Equilibrium changes in the value of the housing stock change the degree to which risk sharing takes place, and modify households' ability to commit to allocations and prices. The model matches the cross-sectional variation in risk premia on book-to-market sorted stocks.

In a recent paper, Daniel and Titman (2005) question the success of a string of recent (conditional) CCAPM papers in explaining the cross-section of size and value returns. Their point is that one only needs two factors to explain returns along the two risk dimensions size and value. What is needed then to tell the models apart is more over-identifying restrictions (more asset pricing moments), and other (non-asset pricing) evidence. We have taken the second route in other work. In this paper, we chose the first route. We have tried to match not only the cross-section of risk premia, but also the time-variation in returns on equity and debt, not only at business-cycle frequencies, but also at lower frequencies. Since most

models in this class have two factors, they argue, it is impossible to judge their relative success. Indeed, the stochastic discount factors in these models all feature consumption growth and a second factor with similar time-series properties (see discussion in Guvenen (2003)).

What is needed then to tell the models apart is (1) more over-identifying restrictions (more asset pricing moments), and (2) other (non-asset pricing) evidence. First, we have tried to match not only at the cross-section of risk premia, but also conditional and unconditional equity premia. Lustig and Van Nieuwerburgh (2006b) shows that this model also accounts for the time-variation in conditional asset pricing moments and that it can generate a large unconditional equity premium and a low risk-free rate. There, we have studied not only business-cycle frequency changes in the equity premium and the risk-free rate, but also low-frequency changes. We have insisted on a serious calibration the model, and have fed the observed shocks into the model. Second, in Lustig and Van Nieuwerburgh (2006a), we use quantity rather than price data to test the model. Focussing on US metropolitan areas, we find that the degree of risk-sharing between them decreases when the housing collateral ratio is low. This finding offers direct support for the collateral mechanism.

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# Figure 1: Return Spreads in Book-to-Market Portfolios

This graph plots returns of the 9 highest book-to-market decile portfolios in excess of the return on the first decile portfolio. It plots the model-implied return spreads on the horizontal axis against the return spreads observed in the data (vertical axis). The stock returns on the book-to-market decile portfolios are from Kenneth French's web site. the data are annual for the period 1945-2003. The first panel shows model-generated spreads computed using a mortgage-based collateral measure. The second panel uses a residential wealth-based collateral measure. The model simulation uses the benchmark calibration, discussed in section 2.2, and the computation is detailed in section 4.2.
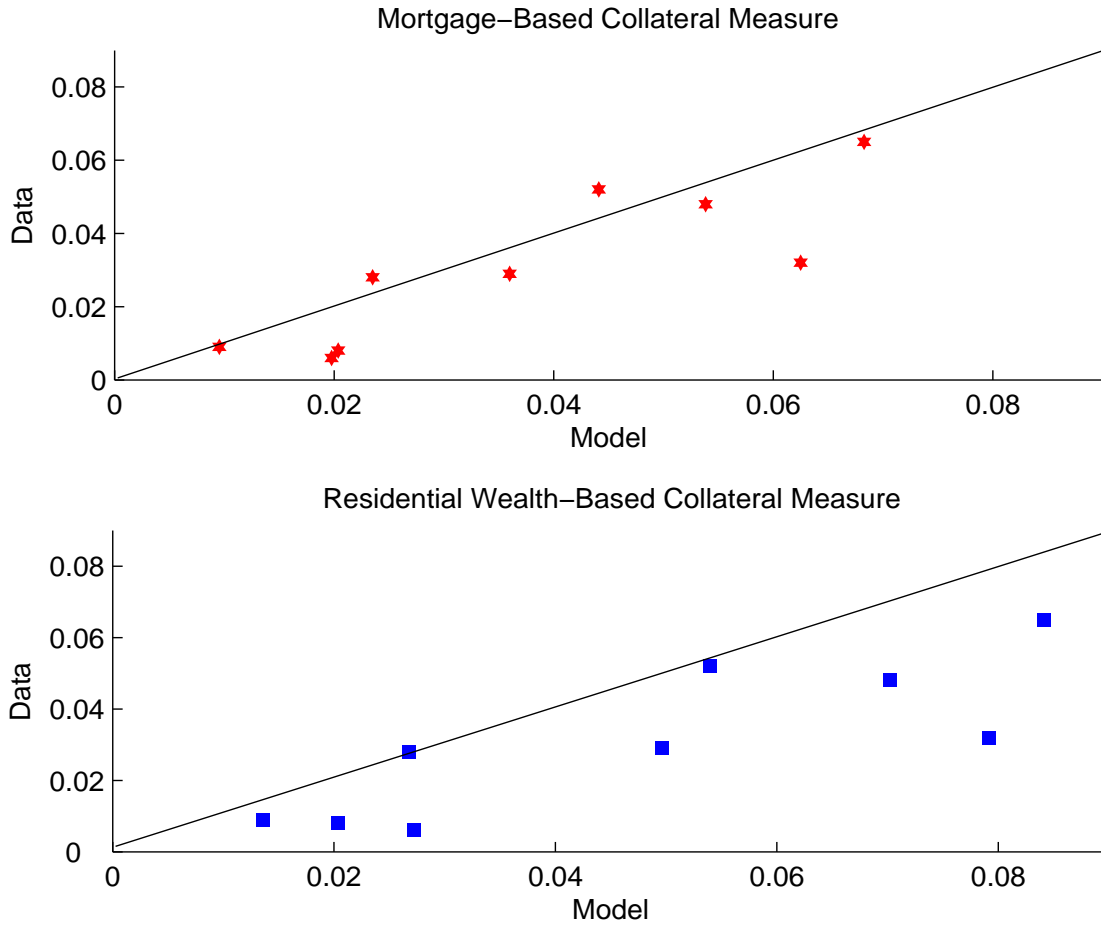
## Figure 2: Risk Sharing, Conditional Asset Pricing Moments and Collateral Ratio

The graphs display a two hundred period model simulation under the benchmark parametrization (see Table 1). The shocks are the same in each panel. The first panel plots the non-housing expenditure ratio $r$. The second panel plots the cross-sectional standard deviation of consumption growth across households ($\sigma_t[\Delta \log c_{t+1}]$). The third panel is the aggregate weight shock $g_{t+1}$. The fourth panel plots the equity premium predicted by the model, i.e. the expected excess return on a non-levered claim to aggregate consumption $E_t\left[R_{t+1}^{c,e}\right]$. The fifth panel is the conditional standard deviation of this excess return $\sigma_t\left[R_{t+1}^{c,e}\right]$. The sixth panel is the conditional Sharpe ratio $E_t\left[R_{t+1}^{c,e}\right]/\sigma_t\left[R_{t+1}^{c,e}\right]$. Each of these series are measured against the left axis and plotted in a single blue line. The housing collateral ratio $my$ is measured against the right axis and plotted in a bold red line.
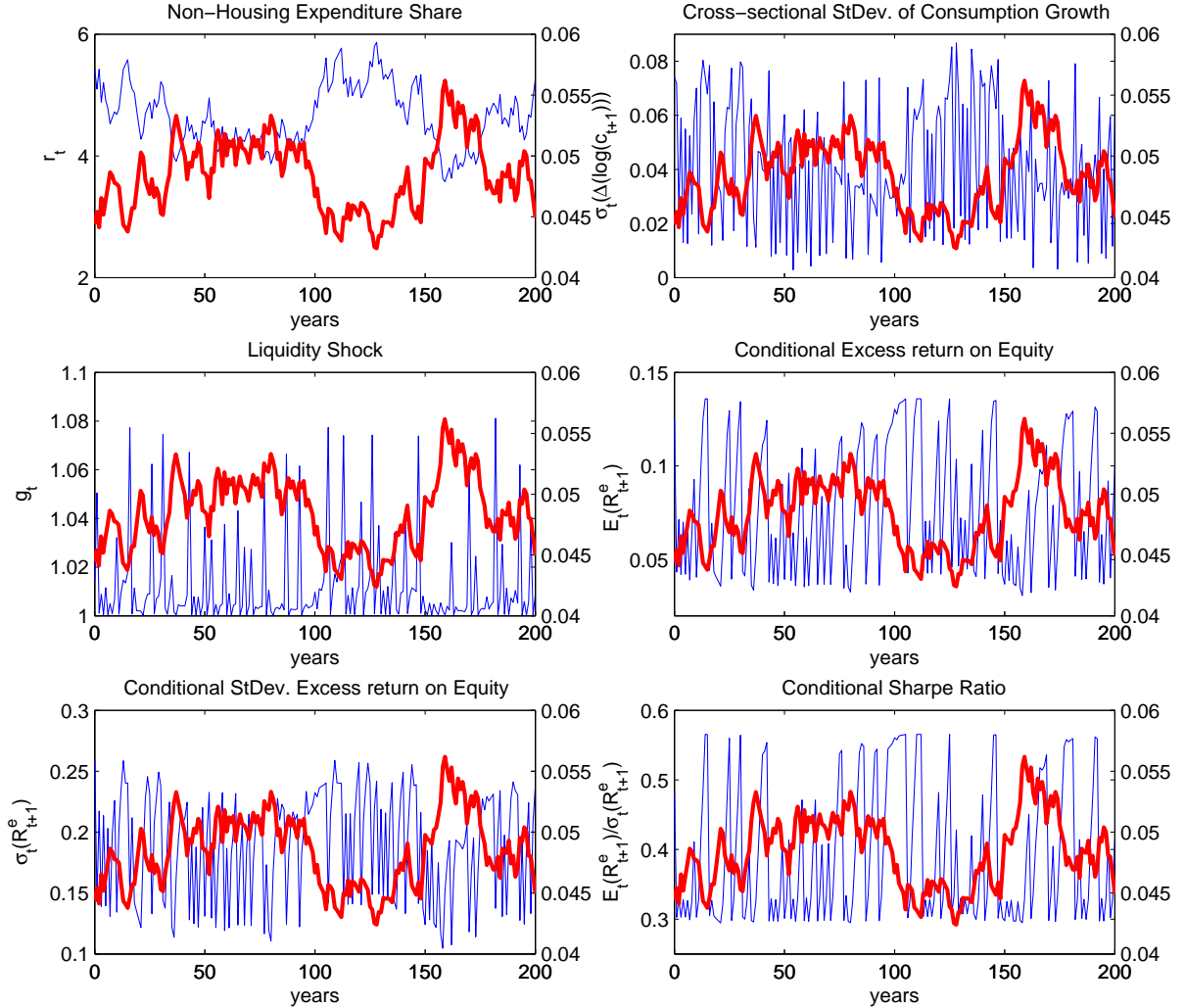
Figure 3: Beta Estimates for Book-to-Market Decile Returns in Data.

This figure plots consumption betas when collateral is abundant (left panels) and when collateral is scarce (right panels) for ten book-to-market decile portfolios. The portfolios are organized from the lowest book-to-market decile (growth) on the left to the highest book-to-market decile (value) on the right of each horizontal axis. The betas are estimated from OLS regression of excess returns of the 10 book-to-market deciles on a constant, the collateral scarcity measure $\widetilde{my}_t$, the aggregate consumption growth rate $\Delta \log c_{t+1}^a$, the interaction term $\widetilde{my}_t \Delta \log c_{t+1}^a$, the aggregate expenditure ratio growth rate $\Delta \log r_{t+1}$, and the interaction term $\widetilde{my}_t \Delta \log r_{t+1}$. These are the five risk factors in the collateral model. In the first panel the housing collateral ratio is based on the value of outstanding mortgages; in the second panel the housing collateral ratio is based on the value of residential real estate wealth. The data are annual for the period 1930-2003.



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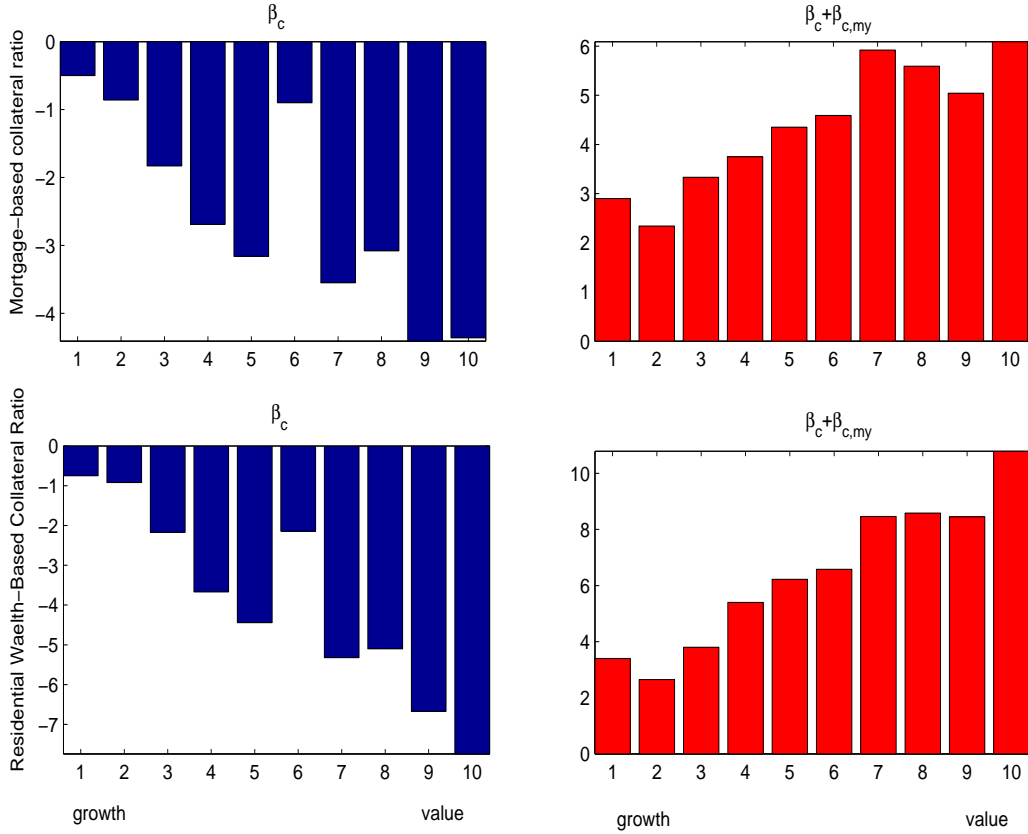
## Figure 4: Term Structure of Consumption Strips.

The first panel plots the conditional expected excess return on a levered claim to aggregate consumption $k$ periods from now, where $k = 2, 3, ..., 45$. The second panel shows the corresponding Sharpe ratios. Panel 3 plots the risk premium for the duration-based portfolios against duration. Panel 4 plots the Sharpe ratio for the duration-based portfolios against duration. The leverage parameter $\kappa$ is 2.
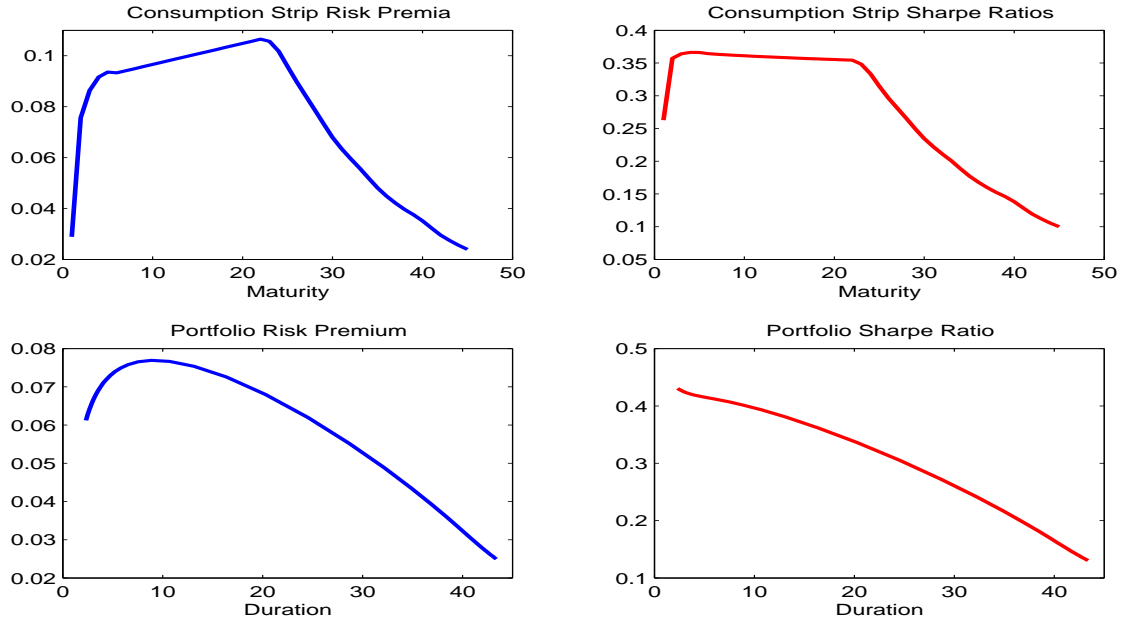
Table 1: Parameter Calibration

| Parameter | Benchmark | Sensitivity Analysis |
|---|---|---|
| $\gamma$ | 8 | [2,5,10] |
| $\varepsilon$ | .05 | [.15,.75] |
| $\psi$ | 1 | . |
| $\bar{r}$ | 4.26 | . |
| $\rho_r$ | .96 | . |
| $b_r$ | .93 | 0 |
| $\sigma_r$ | .03 | . |
| $E[my]$ | .05 | .10 |
| $\lambda$ | [1.04,.96] | . |
| $\eta$ | [.6578,.7952,.3422,.2048] | [.6935,.6935,.3065,.3065] |
| $\kappa$ | 1 | 3 |

## Table 2: Value Premium in Data and in the Collateral Model

Panel 1 reports moments of book-to-market decile portfolio returns in excess of a risk-free rate: the sample mean, the sample volatility and the Sharpe ratio. The risk-free rate is the annual return on a 3-month T-Bill. The value-weighted stock returns on the book-to-market deciles are annual for 1930-2003 and 1945-2003, and the source is Kenneth French's web site. Panel 2 reports the decile returns generated by the model as described in section 5.1. The panel reports expected returns, standard deviation and Sharpe ratio on an artificial asset generated with a set of betas listed in Table 2 of the separate appendix, but with intercept $\beta_0^j$ chosen so that the Euler equation is satisfied for this asset in the model. The parametrization is the benchmark one. The first block corresponds to the betas obtained using the mortgage-based measure; the second block uses the residential wealth-based measure. Panel 3 reports the results of the duration-based asset pricing exercise of section 5.2. It reports equity premia (second row), their volatilities (third row), and Sharpe ratios (fourth row) for 10 portfolios of different duration (first row). The leverage parameter $\kappa$ is 2.

**Panel 1: US Data**

| Decile | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | *Sample 1930-2003* | | | | | |
| $E(R^e)$ | 0.071 | 0.084 | 0.080 | 0.081 | 0.100 | 0.099 | 0.108 | 0.127 | 0.131 | 0.139 |
| $\sigma(R^e)$ | 0.222 | 0.194 | 0.196 | 0.229 | 0.228 | 0.238 | 0.250 | 0.274 | 0.291 | 0.332 |
| $E(R^e)/\sigma(R^e)$ | 0.321 | 0.431 | 0.411 | 0.355 | 0.437 | 0.417 | 0.433 | 0.464 | 0.449 | 0.419 |
| | | | | | *Sample 1945-2003* | | | | | |
| $E(R^e)$ | 0.078 | 0.087 | 0.086 | 0.084 | 0.106 | 0.107 | 0.110 | 0.130 | 0.126 | 0.143 |
| $\sigma(R^e)$ | 0.209 | 0.175 | 0.175 | 0.178 | 0.182 | 0.178 | 0.194 | 0.214 | 0.212 | 0.257 |
| $E(R^e)/\sigma(R^e)$ | 0.372 | 0.497 | 0.492 | 0.473 | 0.580 | 0.599 | 0.566 | 0.604 | 0.592 | 0.558 |

**Panel 2: Model with Empirical Betas**

| Decile | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *Mortgage-Based Collateral Measure* | | | | | | |
| $E(R^e)$ | $-0.002$ | 0.007 | 0.018 | 0.018 | 0.021 | 0.034 | 0.060 | 0.042 | 0.052 | 0.066 |
| $\sigma(R^e)$ | 0.104 | 0.065 | 0.081 | 0.093 | 0.107 | 0.110 | 0.145 | 0.130 | 0.123 | 0.153 |
| $E(R^e)/\sigma(R^e)$ | $-0.020$ | 0.115 | 0.226 | 0.190 | 0.200 | 0.309 | 0.418 | 0.322 | 0.422 | 0.433 |
| | | | | *Residential Wealth-Based Collateral Measure* | | | | | | |
| $E(R^e)$ | 0.001 | 0.015 | 0.022 | 0.029 | 0.028 | 0.051 | 0.080 | 0.055 | 0.072 | 0.085 |
| $\sigma(R^e)$ | 0.116 | 0.062 | 0.076 | 0.088 | 0.106 | 0.131 | 0.178 | 0.149 | 0.159 | 0.191 |
| $E(R^e)/\sigma(R^e)$ | 0.011 | 0.241 | 0.286 | 0.323 | 0.266 | 0.387 | 0.453 | 0.370 | 0.451 | 0.448 |

**Panel 3: Model with Duration-Based Pricing**

| Duration | 43.387 | 42.529 | 39.476 | 32.166 | 20.274 | 13.145 | 8.870 | 4.759 | 3.040 | 2.328 |
|---|---|---|---|---|---|---|---|---|---|---|
| $E[R^e]$ | 0.025 | 0.027 | 0.034 | 0.049 | 0.068 | 0.075 | 0.077 | 0.073 | 0.067 | 0.061 |
| $\sigma[R^e]$ | 0.191 | 0.193 | 0.196 | 0.202 | 0.202 | 0.198 | 0.192 | 0.175 | 0.157 | 0.142 |
| $E[R^e]/\sigma[R^e]$ | 0.130 | 0.138 | 0.171 | 0.242 | 0.336 | 0.381 | 0.402 | 0.416 | 0.424 | 0.431 |

## Table 3: Beta Estimates for Book-to-Market Decile Returns in Data - 5 Factors.

OLS regression of excess returns of the 10 book-to-market deciles on a constant, a scaled version of the collateral measure $\widetilde{my}_t$, the aggregate consumption growth rate $\Delta(\log(c_{t+1}))$, the interaction term $\widetilde{my}_t\Delta(\log(c_{t+1}))$, the aggregate expenditure share growth rate $\Delta\log(\alpha_{t+1})$, and the interaction term $\widetilde{my}_t\Delta(\log(c_{t+1}))$. These are the five risk factors in the collateral model. The first line of each panel is for the lowest book-to-market decile (growth), the last line for the highest book-to-market decile (value). The number is brackets are OLS t-statistics. In the first panel the housing collateral ratio is based on the value of outstanding mortgages and in the second panel, the housing collateral ratio is based on the value of residential real estate wealth. The data are annual for the period 1930-2003.

| | $\beta_0$ | $\beta_{my}$ | $\beta_c$ | $\beta_{c,my}$ | $\beta_\alpha$ | $\beta_{\alpha,my}$ |
|---|---|---|---|---|---|---|
| | **Panel 1: Mortgage-Based Collateral** | | | | | |
| 1 | 0.61 | 12.94 | −0.50 | 3.40 | 15.63 | −34.80 |
| | [5.44] | [12.38] | [1.70] | [3.89] | [6.16] | [15.09] |
| 2 | 3.59 | 10.09 | −0.86 | 3.20 | 10.85 | −22.54 |
| | [4.90] | [11.16] | [1.53 | [3.51] | [5.55] | [13.60] |
| 3 | 4.74 | 6.76 | −1.83 | 5.16 | 13.62 | −26.97 |
| | [4.89] | [11.13] | [1.52] | [3.50] | [5.54] | [13.56] |
| 4 | 3.64 | 10.73 | −2.69 | 6.44 | 24.46 | −40.95 |
| | [5.35] | [12.18] | [1.67] | [3.83] | [6.06] | [14.85] |
| 5 | 3.00 | 16.99 | −3.16 | 7.51 | 22.36 | −40.71 |
| | [5.41] | [12.31] | [1.69] | [3.87] | [6.12] | [15.00] |
| 6 | 0.32 | 18.41 | −0.90 | 5.49 | 21.52 | −37.33 |
| | [5.45] | [12.41] | [1.70] | [3.90] | [6.18] | [15.12] |
| 7 | 6.22 | 9.06 | −3.55 | 9.47 | 20.90 | −33.64 |
| | [6.03] | [13.72] | [1.88] | [4.31] | [6.82] | [16.71] |
| 8 | 3.90 | 19.38 | −3.08 | 8.67 | 24.21 | −42.34 |
| | [6.55] | [14.92] | [2.04] | [4.69] | [7.42] | [18.18] |
| 9 | 5.94 | 18.50 | −4.41 | 9.45 | 23.25 | −33.85 |
| | [7.07] | [16.10] | [2.21] | [5.06] | [8.01] | [19.62] |
| 10 | 4.78 | 21.63 | −4.36 | 10.45 | 25.72 | −37.27 |
| | [8.04] | [18.29] | [2.51] | [5.75] | [9.10] | [22.29] |
| | **Panel 2: Residential Wealth-Based Collateral** | | | | | |
| 1 | 1.24 | 11.30 | −0.78 | 4.34 | 15.88 | −36.85 |
| | [5.64] | [12.75] | [1.84] | [4.17] | [6.23] | [15.75] |
| 2 | 6.05 | 3.78 | −0.89 | 3.42 | 9.61 | −19.25 |
| | [5.12] | [11.57] | [1.67] | [3.79] | [5.65] | [14.30] |
| 3 | 6.77 | 1.92 | −1.76 | 4.95 | 12.23 | −23.24 |
| | [5.11] | [11.55] | [1.67] | [3.78] | [5.64] | [14.27] |
| 4 | 7.22 | 1.93 | −2.65 | 6.43 | 22.09 | −34.30 |
| | [5.65] | [12.76] | [1.84] | [4.18] | [6.24] | [15.77] |
| 5 | 5.70 | 10.28 | −3.23 | 7.71 | 20.58 | −36.54 |
| | [5.69] | [12.86] | [1.86] | [4.21] | [6.28] | [15.89] |
| 6 | 2.66 | 12.08 | −1.28 | 6.62 | 19.70 | −32.80 |
| | [5.72] | [12.93] | [1.87] | [4.23] | [6.32] | [15.98] |
| 7 | 9.74 | 0.18 | −3.99 | 10.43 | 18.55 | −26.68 |
| | [6.27] | [14.18] | [2.05] | [4.64] | [6.93] | [17.52] |
| 8 | 5.72 | 14.60 | −3.54 | 9.79 | 22.61 | −38.87 |
| | [6.82] | [15.41] | [2.23] | [5.05] | [7.53] | [19.04] |
| 9 | 8.86 | 10.78 | −4.89 | 10.54 | 20.70 | −26.95 |
| | [7.35] | [16.62] | [2.40] | [5.44] | [8.12] | [20.53] |
| 10 | 7.85 | 13.46 | −4.94 | 11.81 | 23.54 | −31.84 |
| | [8.34] | [18.85] | [2.72] | [6.17] | [9.21] | [23.30] |

# A    Technical Appendix

This section contains the proofs of the propositions in the main text. For more details on the model (definition of the cumulative multipliers, derivation and optimality of the risk sharing rule and the optimality of the law of motion for the cumulative multipliers), we refer the reader to section 2 of the separate appendix to this paper, available on our web sites.

**Proof of Proposition 1**    Following the definition of Alvarez and Jermann (2005), the pricing kernel $M$ has no permanent component if

$$\lim_{k \to \infty} \frac{E_{t+1} M_{t+k}}{E_t M_{t+k}} = 1.$$

We focus on a slightly different condition:

$$\lim_{k \to \infty} \frac{E_{t+1} M_{t+k} c_{t+k}}{E_t M_{t+k} c_{t+k}} = 1.$$

Let the one period holding return on a period-k consumption strip be given by:

$$R_{t+1,k}^c = \frac{M_t}{M_{t+1}} \frac{E_{t+1} M_{t+k} c_{t+k}}{E_t M_{t+k} c_{t+k}},$$

then we know, from the derivation above, that

$$\lim_{k \to \infty} R_{t+1,k}^c = \frac{M_t}{M_{t+1}}.$$

Furthermore, for any return $E_t[\frac{M_{t+1}}{M_t} R_{t+1}] = 1$, we know that $E_t[\log(\frac{M_{t+1}}{M_t} R_{t+1})] \leq \log E_t[\frac{M_{t+1}}{M_t} R_{t+1}] = 0$ by Jensen's inequality. This implies that $E_t \log(\frac{M_t}{M_{t+1}}) \geq E_t \log(R_{t+1})$ or

$$E_t \log \lim_{k \to \infty} R_{t+1,k}^c = \log \frac{M_t}{M_{t+1}} \geq E_t \log(R_{t+1}) \text{ for any asset return } R_{t+1}$$

This implies that the expected log excess return exceeds that any other asset:

$$E_t \log \lim_{k \to \infty} \frac{R_{t+1,k}^c}{R_{t+1,1}} \geq E_t \log \frac{(R_{t+1})}{R_{t+1,1}}$$

Let $f(k) = Ce^{ak}$ with $a > 0$ for growth stocks. In the absence of a permanent component in the pricing kernel:

$$\lim_{a \to \infty} 1 + \widetilde{\nu}_0 = \lim_{k \to \infty} R_{t+1,k}^c \geq 1 + \nu_0 \text{ for any other sequence of weights } \{\omega_k\}$$

This implies that the highest equity premium is the one on the farthest out consumption strip. In the absence of a permanent component in the pricing kernel, there is a growth premium. $\square$

**Derivation of Value Premium** The multiplicative risk premium on an (un-levered) consumption strip is derived as follows:

$$
\begin{aligned}
1 + \nu_0 = 1 + E_0[R_{0,1}^e[\{c_k\}]] \ &= \ E_0 M_1 E_0 \left( \frac{\sum_{k=1}^{\infty} E_1 M_k c_k}{\sum_{k=1}^{\infty} E_0 M_k c_k} \right) = \sum_{k=1}^{\infty} \frac{E_0 M_k c_k}{\sum_{k=1}^{\infty} E_0 M_k c_k} \frac{\frac{E_1 M_k c_k}{E_0 M_k c_k}}{\frac{1}{E_0 M_1}} \\
&= \ \sum_{k=1}^{\infty} \omega_k \frac{E_0 R_{0,1}[c_k]}{R_{0,1}[1]} = \sum_{k=1}^{\infty} \omega_k E_0 R_{0,1}^e[c_k],
\end{aligned}
$$

with weights

$$
\omega_k = \frac{E_0 M_k c_k}{\sum_{k=1}^{\infty} E_0 M_k c_k}.
$$