# Information Processing and Learning: Testing the Analogy-based Expectation Approach

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**Abstract**

Mental contents come to mind more or less easily depending on their 'accessibility' (Higgins, 1996). We consider how the pattern of learning is affected by the framing of feedback about opponents' play when subjects are engaged in several interactions. Specifically, we examine a setting where the feedback about opponents' play made available to subjects along the learning process is identical across two treatments, but the presentation differs to make it accessible game by game in one treatment but not in the other. Behavior differs strongly between the two treatments and we show that the difference in behaviors can be explained by using the concept of analogy-based expectation equilibrium (Jehiel 2005). Two further experimental treatments were designed to examine behavior when information is restricted to match the theoretical mental representations, and behavior in these treatments strongly confirms the hypothesis. More generally, the experiment suggests the need to incorporate framing considerations into equilibrium analysis.

Keywords: Analogy-based expectation; information processing; accessibility; interactive learning.

JEL Classification: C72; D82.

## 1 Introduction

Information comes to mind more or less easily depending on its *accessibility* (Higgins, 1996). For the sake of illustration, consider Figures 1a and 1b.[1] As we look at the object of Figure 1a, we have an immediate impression of the height of the tower, but not of the total area that its building blocks would cover if the tower were dismantled (the total area can be estimated by a deliberate procedure, such as multiplying the base area of a block by the number of blocks, but, admittedly, it is less immediately accessible). The situation is reversed in Figure 1b in which the total area is immediately accessible, but the height of the tower that could be constructed with these building blocks is not.

The accessibility dimension of information may have important implications for the understanding of how information is being processed in a variety of applications. In this paper, we consider the question of how information is being processed in an interactive learning situation in which players

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[1] This example is taken from Kahneman (2003).

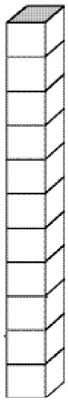


Figure 1a

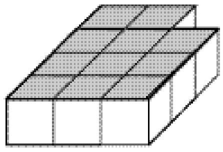


Figure 1b

try to learn the behaviors of other players in various games so as to make the best choice of strategy in each game. Our main insight concerns the comparison of play in two treatments of the experiment in which the same *objective* feedback about opponents' past play is given to subjects—but with various degrees of *accessibility*. Subjects play two different games about which they only know the structure of their own payoffs. In each round and prior to the action choice, it is announced whether subjects play game $A$ or game $B$. In addition, subjects receive some information about the past play of their opponents in both games. In principle, the information is precise in that it details past play for both games separately. However, as it turns out the precise information is relatively accessible in the first treatment but not in the second. As a consequence, subjects in the second treatment use the precise information as if it were much coarser. In addition, we run two control treatments, one where accessibility is not an issue because the information is precisely spelled out for subjects and one where the information is exogenously coarse. We observe that the difference in accessibility yields very different patterns of behaviors even after 60 rounds of play.

We will explain the difference of behaviors in the two treatments by relying on the recent approach of analogy-based expectation equilibrium (Jehiel 2005). This approach relaxes the traditional Nash equilibrium concept by assuming that players may have a *partial* rather than *total* understanding of their opponents' strategies, and it parameterizes players by how precisely they understand the strategy of their opponents. Specifically, players bundle situations or games into *analogy classes*, and they form expectations only about the aggregate behavior in each analogy class. Players are assumed to best-respond against these beliefs and, in equilibrium, these beliefs coincide with the aggregate play in each class. In general, the equilibrium that is so obtained is sensitive to the analogy partitions used by the players. The analogy-based expectation equilibrium is viewed as the limiting outcome of a learning process in which players would only keep track of the aggregate statistics of the opponents' average play in each analogy class; it is not viewed as the result of introspective reasoning (see Jehiel 2005).

In our experiment, there were two underlying normal form games $A$ and $B$, both involving a Column Player and a Row player. In both $A$ and $B$, the Row player had to choose between three rows $\alpha, \beta$, or $\gamma$, and the Column player had to choose between five columns $a, b, c, d$, or $e$. There were *a priori* two possible analogy grouping for each player: the fine analogy grouping (the two games are treated separately) and the coarse analogy grouping (the two games are bundled into one class).

We have conducted several experiments in which subjects were assigned either to the role of the Row player or to the role of the Column player for the entire experiment. Each subject played an equal number of games $A$ and $B$ over 60 rounds. Subjects were informed of the action space structure of the two players and of their *own* payoff structure, but not of the payoff structure of their opponent.[2] In each round, subjects of each population of players were randomly matched to subjects of the other population. The various treatments differed in the feedback given to subjects after each round of the experiment. In all cases, the feedback was about the past play of the subjects assigned to the role of the other player in the last five rounds. Note that subjects received no feedback about their own performance until the end of the experiment at which time they were informed how much they had earned over the 60 rounds.

Intuitively, when in the experiment players have access to the behavior of their opponents game by game this corresponds to the fine analogy grouping and when they have access to the aggregate behavior only this corresponds to the coarse analogy grouping. In all treatments, Column players had access to the behaviors of Row players game by game. Specifically, as a Column player was to play game $\omega = A$ or $B$ he or she was being informed of the distribution of play of Row players (in the entire population) over the last five rounds in game $\omega$. So the corresponding analogy grouping of Column players was always the fine grouping.

Treatments differed in the accessibility dimension of the feedback given to Row players. In the first benchmark treatment, FINE, Row players received feedback about Column players' past behavior in the same way as Column players. In the second benchmark treatment, COARSE, they only received aggregate feedback, that is, they were told how often Column players had chosen each of the two actions in *both* games without being told how the behavior was distributed between game $A$ and $B$.

In addition, we have two treatments, our main treatments, where, in principle, the (relevant) information provided is identical as in treatment FINE but not quite as immediately accessible.[3] In treatment FINEACCESS, this feedback was given in a matrix form as depicted in Figure 2 (where the letter inside a square indicates the game and the color of the square indicates the action played by the Column player in that specific interaction—actions appeared as colors also when playing the game; grey squares indicate that not enough rounds had yet been played to fill the grid).

In treatment COARSEACCESS, the feedback was given in two *consecutive* screens, depicted in Figures 3a and 3b where the first screen informed subjects about the distribution of actions (in color form) that had been played in the last five rounds and the second screen gives a string of letters indicating the games in which these actions had been taken. It is worth emphasizing that, even

[2] We adopted this framework so as to make introspective reasoning about how to play the game inaccessible. This also avoided the difficulty pointed out by Ehrblatt et al. (2005) that the belief formation may be affected by the knowledge of the opponent's payoff.

[3] Admittedly, in treatment FINE, row players received only information about the past play in the game they were playing unlike in treatment FINEACCESS and COARSEACCESS but as this is the only relevant information we don't consider the information to be practically different.

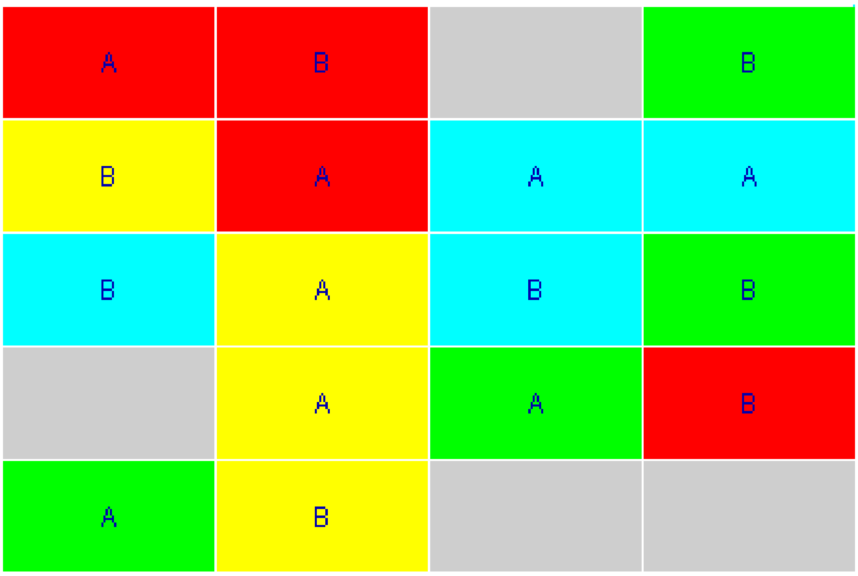



Figure 2

though the two screens appeared in different formats, subjects were informed about the correspondence between the two[4]. Thus, feedback in the two treatments was objectively the same (and the same as in treatment Fine), even though accessibility was not (the distribution of play in each game $A$ and $B$ was obviously made more accessible in FineAccess than in CoarseAccess).

The payoffs in games $A$ and $B$ were chosen so that the prediction of the analogy-based expectation equilibrium approach is markedly different depending on whether the Row player uses the fine or the coarse analogy grouping (assuming the Column player uses the fine analogy grouping)—with a unique equilibrium employing pure strategies in each case (see Section 2). In regard of the analogy-based expectation approach, we ask three questions:

**Question 1** Do the observed distributions of play in treatments FineAccess and CoarseAccess stabilize?

**Question 2** Do the long run behaviors in FineAccess and CoarseAccess correspond to the behaviors in Fine and Coarse, respectively?

**Question 3** Do the long run behaviors in Fine and FineAccess on the one hand, and Coarse and CoarseAccess on the other correspond to the prediction of the analogy-based expectation equilibrium approach?

[4]The order is as follows: the first element of the string is the top left box of the colored grid and the sequence continues down the first column, then the second etc, until the last element which corresponds to the bottom right box in the colored grid. The instruction sheet handed out to explain this is included in the appendix. Note also that figures 2 and 3 show the same information.

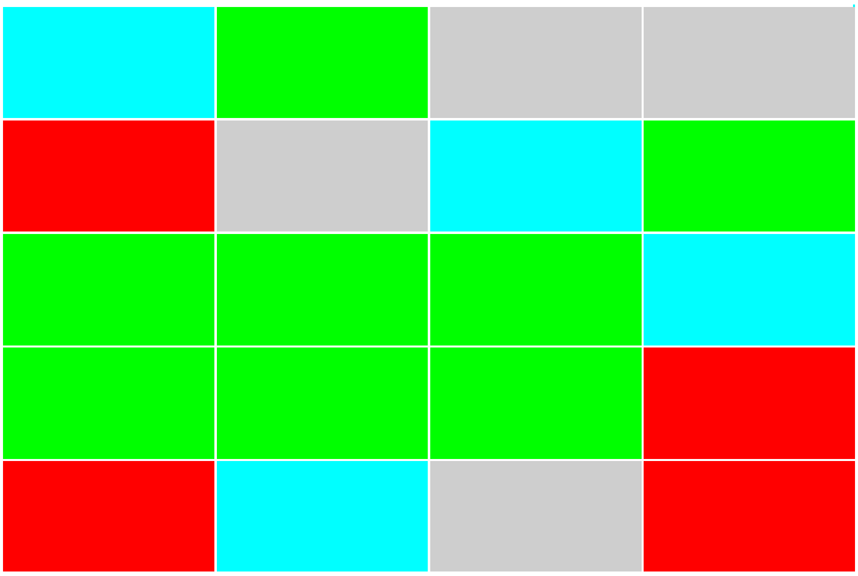


Figure 3a

AABBAB?BBA?ABB??BAAA

Figure 3b

With the required caveats, we obtained positive answers to all three questions. The behavior does stabilize; treatments FINE and FINEACCESS are similar to each other and so are COARSE and COARSEACCESS;[5] and all observations are remarkably well organized by the theory.

The remainder of the paper is organized as follows. In Section 2 we describe more formally the analogy-based expectation approach and we apply it to the environment considered in our experiment. In Section 3 we present more precisely the experimental design. In Section 4 we present the results of the experiments. Section 5 puts this work in a broader perspective stressing the importance of incorporating framing considerations into equilibrium analysis.

## 2 Background and Theory

Consider a family of normal form games denoted by $\omega \in \Omega$. Each game has two players $i$ and $j$. For each $\omega$, the action space of player $i$ is $A_i$ and the action space of player $j$ is $A_j$. Action spaces $A_i$ and $A_j$ are finite. The payoff obtained by player $i$ in game $\omega$ when $(a_i, a_j) \in A_i \times A_j$ is played is denoted by $u_i(a_i, a_j; \omega)$. The probability of game $\omega$ is denoted by $p(\omega)$. We assume that each player $i$ knows

[5] Specifically, we compared FINE and FINEACCESS in terms of the frequency with which Row players played best-responses to the distribution of actions game by game as resulting from the feedback they received. Similarly, we compared COARSE and COARSEACCESS in terms of the frequency with which Row players played best-responses to the aggregate distribution of actions over $A$ and $B$ as resulting from the feedback they received. We did the same comparisons in terms of the distribution of the payoff losses vis a vis these two theories.

which game $\omega \in \Omega$ he is playing.

A strategy of player $i$ is a mapping $\sigma_i : \Omega \to \Delta A_i$ where $\sigma_i(a_i \mid \omega)$ denotes the probability with which action $a_i \in A_i$ is chosen by player $i$ in game $\omega$.

Each player $i$ is endowed with an analogy partition $\mathcal{A}_i$ over $\Omega$. The element of $\mathcal{A}_i$ containing $\omega$ is denoted by $\alpha_i(\omega)$ and called the analogy class of player $i$ at $\omega$. Player $i$ is assumed to understand only the aggregate behavior of player $j$ in every analogy class in $\mathcal{A}_i$. Formally, given the strategy $\sigma_j$ of player $j$, the strategy of player $j$ *perceived* by player $i$ (given $\mathcal{A}_i$) is defined by the function $\overline{\sigma}_j : \Omega \to \Delta A_j$ such that for all $\omega \in \Omega$ and $a_j \in A_j$

$$\overline{\sigma}_j(a_j \mid \omega) = \frac{\sum_{\omega' \in \Omega} p(\omega')\sigma_j(\omega')}{\sum_{\omega' \in \Omega} p(\omega')} = \sum_{\omega' \in \Omega} p(\omega' \mid \alpha_i(\omega))\sigma_j(a_j \mid \omega') \tag{1}$$

That is, given the strategy $\sigma_j$ of player $j$, player $i$ perceives only the average behavior of player $j$ in each analogy class where the weight assigned to a specific game $\omega'$ of an analogy class is proportional to $p(\omega')$.

**Definition** A strategy profile $\sigma = (\sigma_1, \sigma_2)$ is an analogy-based expectation equilibrium (ABEE) given the analogy partitions $\mathcal{A}_1$, $\mathcal{A}_2$ if for all $i$, $\omega \in \Omega$ and $a_i^* \in Supp[\sigma_i(\omega)]$:

$$a_i^* \in \arg\max_{a_i \in A_i} \sum_{\omega' \in \Omega} \overline{\sigma}_j(a_j \mid \omega) u_i(a_i, a_j; \omega),$$

where $\overline{\sigma}_j(a_j \mid \omega)$ is given by (1).

In other words, in equilibrium each player $i$ plays a best-response to the belief that player $j$ behaves in each game $\omega$ according to the aggregate behavior in $\alpha_i(\omega)$, i.e. $\overline{\sigma}_j$.

Jehiel (2005) (see also Jehiel and Koessler, 2006) motivates the analogy-based expectation approach by a learning story. More precisely, Jehiel (2005) interprets the ABEE as the limiting outcome of a learning process in which each player $i$ would base his strategy on the sole feedback about the aggregate play of player $j$ in the various analogy classes of $\mathcal{A}_i$ . One of the objectives of this paper is to test that view.

In our experiment, we considered two games $\omega = A$ and $B$ whose payoff matrices for the Row ($i = 1$) and Column player ($i = 2$) are depicted in the following tables:

| A | a | b | c | d | e |
|---|---|---|---|---|---|
| $\alpha$ | 25,10 | 0,10 | 10,20 | 0,0 | 0,0 |
| $\beta$ | 20,15 | 15,0 | 5,0 | 10,0 | 0,0 |
| $\gamma$ | 15,0 | 10,0 | 0,0 | 5,25 | 25,0 |

| B | a | b | c | d | e |
|---|---|---|---|---|---|
| $\alpha$ | 15,0 | 20,10 | 10,0 | 5,5 | 0,0 |
| $\beta$ | 0,10 | 25,10 | 0,0 | 10,20 | 0,0 |
| $\gamma$ | 10,0 | 15,0 | 5,25 | 0,0 | 25,0 |

The two games $A$ and $B$ were played with the same frequency so that $p(A) = p(B) = \frac{1}{2}$. Given the feedback given to Column player (to be described more precisely in the next Section), the relevant analogy partition for Column player was the fine partition $\mathcal{A}_2^f = \{\{A\}, \{B\}\}$. For the Row player, two analogy partitions can be considered: either the fine partition, $\mathcal{A}_1^f = \{\{A\}, \{B\}\}$, or the coarse partition, $\mathcal{A}_1^c = \{\{A, B\}\}$.

**When the Row player's analogy partition is fine:**

When both players use the fine analogy partition, ABEE coincides with Nash equilibrium. In game $A$ the only Nash equilibrium requires that the Row player plays $\alpha$, $\sigma_1(A) = \alpha$, and the Column player plays $c$, $\sigma_2(A) = c$. In game $B$ the only Nash equilibrium requires that the Row player plays $\beta$, $\sigma_1(B) = \beta$, and the Column player plays $d$, $\sigma_2(B) = d$. This is easily seen as the iterated elimination of strictly dominated strategies single out a unique pair of actions in each game.[6]

**When the Row player' analogy partition is coarse:**

When $\mathcal{A}_1 = \mathcal{A}_1^c$ and $\mathcal{A}_2 = \mathcal{A}_2^f$, the only (analogy-based expectation) equilibrium is $(\sigma_1, \sigma_2)$ where $\sigma_1(A) = \beta$, $\sigma_1(B) = \alpha$ and $\sigma_2(A) = a$, $\sigma_2(B) = b$. That is, the Row player plays $\beta$ in game $A$ and $\alpha$ in game $B$; the Column player plays $a$ in game $A$ and $b$ in game $B$. Thus, the strategies are markedly different from the Nash equilibrium strategies.

It is easy to understand why this strategy profile defines an analogy-based expectation equilibrium. The Column player plays $a$ in $A$ because this is the best-response to $\beta$; she plays $b$ in $B$ because this is the best-response to $\alpha$. The aggregate behavior of the Column player is a balanced mix of $a$ and $b$ (remember that $p(A) = p(B) = \frac{1}{2}$). Thus, $\overline{\sigma}_2(a \mid \omega) = \overline{\sigma}_2(b \mid \omega) = \frac{1}{2}$ for $\omega = A$ and $B$. Given the expectation that the Column player plays $a$ and $b$ with an equal frequency, the Row player finds it optimal to play $\beta$ in game $A$ (because $\frac{20+15}{2} > \frac{\max(20+0,15+10)}{2}$) and $\alpha$ in game $B$ (because $\frac{15+20}{2} > \frac{\max(0+25,10+15)}{2}$). It is a routine exercise to check that there is no other equilibrium in this case.

# 3 Experimental Design

The experiments were computerized using Tomlinson's (2005) Expecon software. They were conducted at the University of London between February and December 2005. Upon arrival at the lab, subjects sat down at a computer terminal to start the experiment. Experiment instructions were presented on the computer screen and a written summary of the experiment instructions was also handed out. At all times, subjects were invited to raise their hands to ask questions which would be answered privately.

The experiment consisted of four treatments which varied in the accessibility of the information available to subjects about the actions of others. Each session involved eight subjects and four sessions were run for each treatment. In total 128 subjects participated in the experiment, drawn from the student population at UCL. Their subjects of study included a cross section of arts, humanities, science

[6] In both games $A$ and $B$ action $e$ is strictly dominated (by a mixture over $a$ and $d$) for the Column player. After eliminating action $e$ action $\gamma$ is strictly dominated for the Row player (by action $\beta$ in $A$ and action $\alpha$ in $B$). Following these eliminations, in game $A$, actions $d$ and $b$ are strictly dominated by $a$ and a mixture of $a$ and $c$, respectively. In game $B$, actions $c$ and $a$ are strictly dominated by $b$ and a mixture of $b$ and $d$, respectively. Finally, with the remaining actions, $\alpha$ in $A$ and $\beta$ in $B$ strictly dominate $\beta$ and $\alpha$ respectively, and we can conclude.

and medical subjects. Subjects were paid a turn-up fee of £5 and in addition to this were given £0.05 per point won during the experiment. The average payment was around £13 per subject, including the turn-up fee. All of the sessions lasted between 45 minutes and 1 hour, with the CoarseAccess treatments taking the longest. And subjects took longer to consider their choices at the start of the experiment: generally, over our sessions of 60 rounds, the first 20 rounds took a similar length of time as the last 40 rounds.

In all treatments, subjects were split up equally into two roles, Row and Column. Each session consisted of sixty rounds where Row and Column subjects were randomly matched into four pairs to make a choice in one of two normal form games, the Row subject choosing the row in the game matrix and the Column subject choosing the column. The two normal form games chosen were detailed in section 2. In each round, two pairs were allocated to "situation A" and two to "situation B", and this information was common knowledge to both subjects in each pair. Subjects could only see their own payments in each situation, and were given information about the choices made by the subjects in the other role in the previous five rounds, and the situations in which these choices were made. The treatments differed only in how this feedback was presented to subjects with role Row.

In all treatments the Column subjects were presented in every round with the number of times each row had been chosen, in the current situation, over the last five rounds. The number was shown against the row on their payoff matrix, and the experiment instructions explained the meaning of the numbers and that they were being provided "to help you make your decision." Column subjects were never given any feedback about play in the situation not currently seen. For example, if a Column subject was in situation A in round 25, she only saw the distribution of choices for situation A on the screen. In a later round, she may have been in situation B, and only feedback for situation B would have been seen. Appendix A shows the instruction sheets handed out at the start of the experiment, which also show screenshots from the experiment software.

The two main treatments, CoarseAccess and FineAccess were designed to examine how the accessibility (or *salience*[7]) of feedback information affects how the information is used. In both treatments Row subjects were given full information about the distribution of columns chosen in the last five rounds, and the situations in which the choices were made. For the Row subjects, each column was given a color, and these colors were used to indicate the choices. In the FineAccess treatment, before seeing the current situation and being able to make a choice, each row player was shown a screen containing a grid of squares. Each square was given the color of a previous choice, and contained the letter of the situation it was made in. The ordering of this grid was randomized independently each round, and subjects were informed of this. Row subjects were allowed to consider this information for as long as they wanted, and then continue to the next screen to see the situation to which they had been allocated and to make a choice. The choice screen contained no information about previous choices—it was necessary for subjects to choose how to interpret the feedback grid and remember what they thought relevant before going on to make a choice.

The CoarseAccess treatment made it harder to connect the situation to the choices. In this

[7]Higgins (1996) gives a good overview of the concepts of accessibility and salience.

treatment, Row subjects were shown a similar grid of colored squares, but the situation letters in each square were removed. They were presented on a following screen as a string of As and Bs. The first letter corresponded to the top left in the grid, and it worked down each column in turn, to the last letter giving the situation for the choice indicated in the bottom right of the grid. The instruction sheet handed out at the start of the experiment gave a clear indication of this ordering. Again, screenshots are included in Appendix A. Subjects were free to consider each screen for as long as desired before continuing to the next, but could not go back once they had moved on. Clearly now, although the same information is being presented, it is cognitively much more difficult to create two separate distributions of play for each situation. Effectively, the whole grid of colored squares needs to be remembered when connecting the distribution of moves to the situations. The situation information is thus less *accessible* in the CoarseAccess treatment, even though the subjects are presented with screens giving identical information content.[8]

The two control treatments Coarse and Fine were simpler. Here the Row subjects were given similar information to the Column subjects. Every subject was told the number of times each choice had been made by subjects in the other role, over the last five rounds. In the Fine treatment the Row subjects saw this information just for the current situation, identically to the information given to Column subjects. However, in the Coarse treatment the row players saw the total frequency of each column choice aggregated over the two situations, and were not given any information about the situation in which the choices were made.

These two treatments establish an obvious benchmark as accessibility is trivial in both cases. From the viewpoint of ABEE these treatments correspond to different exogenous analogy classes that are directly induced. Thus, the prediction for the Fine treatment is given by ABEE with fine analogy classes (i.e. Nash equilibrium), while the prediction for Coarse corresponds to the ABEE with coarse analogy classes (see section 2).

# 4 Results

## 4.1 Aggregate data

A first set of summary statistics is given in Table 1. They show for all four treatments the frequencies of each choice in each of the two situations for row and column players. A first observation is that there is a significant difference of behaviors across treatments. Second the modal behaviors of Row players and Column players in Fine and Coarse coincide with the behaviors arising in the Nash equilibrium and the analogy-based expectation equilibrium with coarse grouping, respectively. Third, the distributions of behaviors in Fine and FineAccess on the one hand and Coarse and CoarseAccess on the other are similar, thereby confirming our intuition that the feedback about opponents' play is accessible game by game in FineAccess but only in aggregate over the two games in CoarseAccess.

[8] This isn't necessarily as difficult as it sounds as, near equilibrium, only two colors would be seen, and would be partitioned identically to the situations, making it much easier to match the two parts of the feedback.

| Row Player Situation A | Fine | FineAccess | CoarseAccess | Coarse |
|---|---|---|---|---|
| $\alpha$ (Nash Equilibrium) | 73% | 80% | 24% | 24% |
| $\beta$ (ABE equilibrium) | 18% | 19% | 64% | 66% |
| $\gamma$ (Level 1 reasoning) | 9% | 2% | 12% | 10% |
| **Row Player Situation B** | **Fine** | **FineAccess** | **CoarseAccess** | **Coarse** |
| $\alpha$ (ABE Equilibrium) | 16% | 23% | 62% | 73% |
| $\beta$ (Nash equilibrium) | 76% | 73% | 21% | 17% |
| $\gamma$ (Level 1 reasoning) | 8% | 4% | 17% | 10% |
| **Col Player Situation A** | **Fine** | **FineAccess** | **CoarseAccess** | **Coarse** |
| $a$ (ABE Equilibrium) | 43% | 33% | 66% | 75% |
| $b$ | 0% | 1% | 0% | 0% |
| $c$ (Nash equilibrium) | 47% | 60% | 8% | 12% |
| $d$ | 10% | 5% | 25% | 13% |
| $e$ | 0% | 0% | 0% | 0% |
| **Col Player Situation B** | **Fine** | **FineAccess** | **CoarseAccess** | **Coarse** |
| $a$ | 0% | 0% | 0% | 0% |
| $b$ (ABE Equilibrium) | 15% | 29% | 42% | 51% |
| $c$ | 8% | 7% | 30% | 16% |
| $d$ (Nash equilibrium) | 77% | 64% | 28% | 33% |
| $e$ | 0% | 0% | 0% | 0% |

Table 1: Summary of choices made

Observe that there are still some systematic deviations from equilibrium play, such as column players choosing $a$ in situation A in the Fine and FineAccess treatments. However, in all cases these deviations are the same between the Fine and FineAccess treatments and the Coarse and CoarseAccess treatments.

The results, in particular the comparative statics, appear to be very encouraging for the theoretical model. In particular, the table suggests that the answer to Question 2 is yes. Behavior in FineAccess appears to be very similar to behavior in Fine and behavior in CoarseAccess very similar to behavior in Coarse. There is also some support for an affirmative answer to Question 3. The theoretical predictions are modal (though not met all the time).

The key question is, of course, whether the aggregate frequencies are coincidental or can be traced to individual behavior. In order to investigate this question we examine whether individual decisions are best responses to the information provided. Of course, in doing so we shall distinguish between fine and coarse information, taking into account that in some instances best replies to both types of information may coincide. We consider three different beliefs that the row players might hold about the strategy of the column players. The first two are constructed from the empirical distributions of past play over the previous 5 rounds. The Fine beliefs use the fine feedback, thereby considering only previous play of the Column players for the situation observed by the row player. The Coarse beliefs use the Coarse feedback, e.g. the total frequencies of past play of the Column players over both situations. To these beliefs we add a third, the Uniform beliefs, which assumes a uniform strategy of the column players. This can be interpreted as the row players ignoring the feedback and taking the row with the highest average payoff. The Uniform beliefs correspond to the "Level 1" reasoning of

| Row | | | | | |
|---|---|---|---|---|---|
| | None | Uniform | Coarse | Fine | Coarse & Fine[9] |
| Fine | 5% | 8% | 17% | 76% | 7% |
| FineAccess | 2% | 3% | 21% | 80% | 6% |
| CoarseAccess | 8% | 15% | 60% | 43% | 26% |
| Coarse | 5% | 10% | 68% | 28% | 11% |
| Column | | | | | |
| | None | Uniform | | Fine | Uniform & Fine[10] |
| Fine | 9% | 69% | | 68% | 46% |
| FineAccess | 13% | 55% | | 71% | 38% |
| CoarseAccess | 11% | 75% | | 62% | 48% |
| Coarse | 11% | 68% | | 66% | 45% |

Table 2: Best response frequencies

Camerer, Ho and Chong (2004), with Level 0 meaning a uniform strategy. Higher level beliefs in this model would require knowledge of the payoffs of the opponent, which are not available to subjects. So we don't consider them here. Having constructed these beliefs from the information made available to row players we are able to calculate the expected profit for choosing each row. Table 2 details the frequencies of best responses for each method of forming beliefs and also shows the best response frequency for column players to the Fine and Uniform beliefs which are constructed similarly to those for the row players.

The table is very suggestive. In fact, the table lends even stronger support to our theoretical reasoning than the previous table that summarized outcomes. Both, in Fine and FineAccess row subjects best respond to fine information around 80% of the time while in Coarse and CoarseAccess row subjects best respond to coarse information around 65% of the time. It is maybe not surprising that the numbers for the coarse treatments are lower than those for the fine treatments, since after all fine information is more reliable than coarse information. Moreover, the number is lowest in CoarseAccess, which may be attributed to the fact that in this treatment even accessing the coarse information requires some substantial cognitive effort and is much harder than in the case where the coarse information is simply given exogenously.

Column players receive only the Fine feedback in all treatments, and through the same screens. It is still interesting to observe that the distribution of best-responses to the Uniform beliefs and the Fine beliefs are very similar in all treatments in which different behaviors were observed. Specifically, Column players best respond to the Fine feedback between 62% and 71% of the time. However, following the Uniform beliefs also seems popular to them. This could be understood as a response to varying risk attitudes amongst the column players. Notably, in the FineAccess treatment where the fewest number of row players deviate from best-responding to the Fine beliefs, we see the highest level of column players following the Fine beliefs as well. For now, though, it is enough that we note that the variation between treatments comes mostly from the choices of beliefs of the row players, that is, the Fine beliefs in treatments Fine and FineAccess and the Coarse belief in treatments Coarse and CoarseAccess.

In a further test of the similarity or difference between the treatments we consider directly the distribution of expected losses with respect to the fine and coarse models of opponents' play. We calculated the theoretical expected losses for the choices taken, if the row players were to treat the two kinds of feedback as the true distribution of column play. Figure 4 shows a histogram giving the empirical distribution of expected losses of the choices made by the Row players, when using the Fine beliefs, i.e the past distribution of Column players' actions in the current situation. It shows a clear similarity between the Fine and FineAccess treatments on the one hand, and the Coarse and CoarseAccess treatments on the other. The next graph, Figure 5 shows the expected loss of the choice taken when using the Coarse beliefs, i.e. the past distribution of Column players' actions over both situations $A$ and $B$. Again, this clearly shows the similarity between the Fine and FineAccess treatments on the one hand, and the Coarse and CoarseAccess treatments on the other. Finally, Figure 6 shows the distribution of expected losses generated from the uniform beliefs.

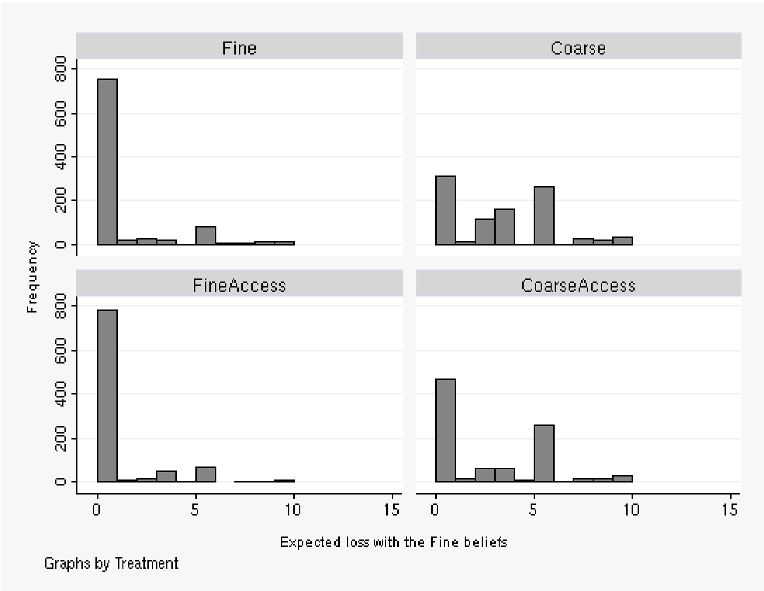



Figure 4: Expected loss using the Fine beliefs

The results strongly support the hypothesis that the accessibility of the information determines how the information is used when making decisions. The FineAccess treatment shows that when the situation information is made accessible, it is used and a separate distribution of opponents' actions is considered for each situation, similar to that shown in the Fine treatment. The Coarse Access treatment makes the situation information considerably less accessible and thus subjects behave very similarly to the Coarse control treatment where the situation information is deliberately removed.

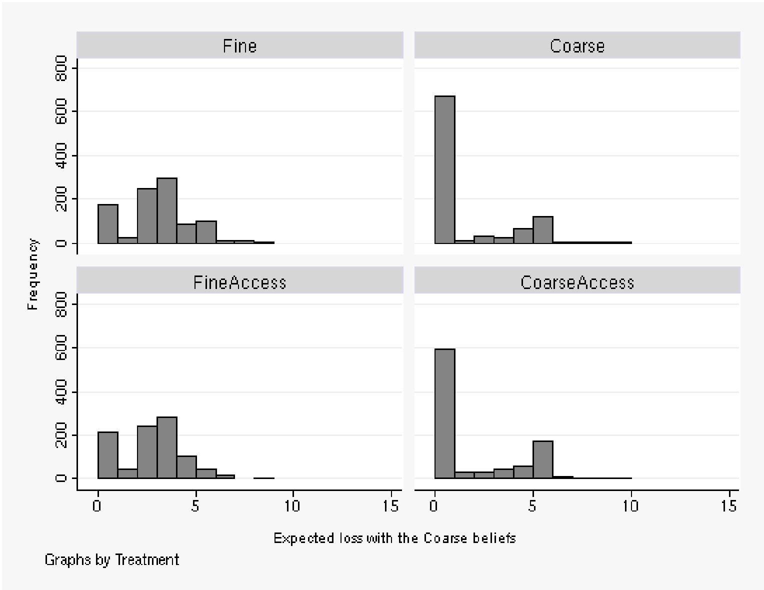



Figure 5: Expected loss using the Coarse beliefs

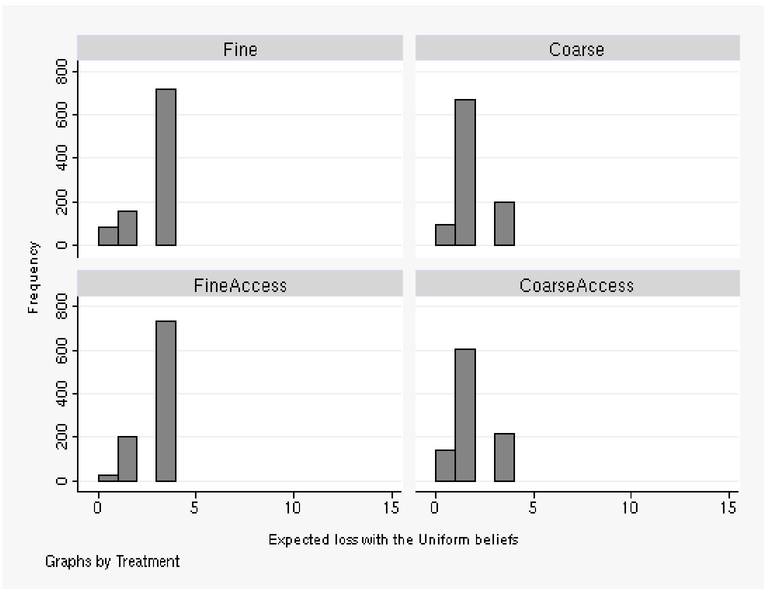



Figure 6: Expected loss using the Uniform beliefs

## 4.2 Individual data

Next we consider individual behavior for the row subjects who are at the centre of our attention (because the form of the feedback was varied only for Row players). Our objective is to see whether we can identify different *types* of subjects, where types are defined by their "typical" best response behavior. Subjects who "typically" (to be defined later) best respond to Fine and Coarse beliefs shall be called "Nash" players and "ABEE" players, respectively. Row players who typically best respond to the uniform beliefs over the 60 rounds will be called "Level 1" players. Finally, we add a fourth type where we cannot say that any of the beliefs are followed. This type we call "Level 0", with the assumption that this subject uses none of the information provided in choosing which row to click on.[11] The difference with Subsection 4.1 is that we follow each individual Row player across all 60 rounds, and we ask ourselves whether his use of feedback is consistent with either the Fine, the Coarse or the Uniform belief throughout the duration of the experiment.

The use of the word "typical" in the above paragraph might sound ambiguous but in what follows we shall explain precisely our procedures. The key idea is that for each type of belief we calculate a variable that indicates the realized gains or losses from the actual choice against the hypothetical best response. More specifically, when a player does not best respond to a particular belief this variable is equal to the difference between the expected profit from taking the optimal choice, minus that of the choice taken. (This is the expected loss that we have also used above in the analysis of the aggregate data.) When a player does best respond the variable is set equal to the difference between the second best choice and the best that he chose, which is thus negative. (One might refer to this as the avoided expected loss.) Thus, the variable is positive when subjects do not best respond, and negative when they do. Let's call this variable the subject's *incentive* with respect to a particular belief. $I_F$, $I_C$ and $I_U$ give the incentives with respect to Fine, Coarse and Uniform beliefs respectively. For each subject we, thus, have a sequence of 59 values (one for each round other than round 1) for both $I_F$ and $I_C$.

If a player typically best responds to a given belief, then we can expect two things. First, the median of the observed incentives will be negative. This is straightforward — we are just saying that the player best responds more often than not and thus that there are more negative values than positive.[12] The second aspect is slightly more subtle. In line with standard models of noisy decision making we should expect that subjects who in principle want to follow a particular best response mode, but are prone to mistakes, would more often deviate if the incentives were small than if they were large. Hence, for a player who "typically" best responds against a certain belief we should expect the observed distribution of incentives to be skewed towards lager negative and smaller positive values.[13]

[11] This is justified as in almost all rounds the choices made by the first 3 types span the strategy set for row players.

[12] As we are proceeding with a hypothesis testing method, what we really want to define is what we wouldn't see if the player was not using these incentives. The uniform strategy is such a strategy and here we would expect that each row would be chosen with a probability of one third, which would lead to a strictly postive expected median. However, we also have to exclude the possibility that any two of the strategies are mixed. In using the relatively weaker (more difficult to reject) null hypothesis of a median of 0, which implies that this strategy be chosen with an empirical probability of greater than one half, we aim to ask whether or not *this* incentive is used, rather than whether *any* incentive is used.

[13] We are slightly mis-using the concept of the skew of a sample in considering the relative sizes of samples on each side of the median, rather than the mean, as is more conventional. However, this interpretation is consistent with our test procedure, and all mention of the 'skew' of a distribution should be interpreted as using the median, rather than

Following this line of reasoning we define the following null hypotheses:

1. Median($I$) $\geq 0$ and
2. If Median($I$) $= 0$ then Skew($I$) $\geq 0$

For each method of forming beliefs, we have different bounds for the incentives and different distributions under both the null hypothesis, and under the different equilibrium outcomes. We therefore require a test or tests for our null hypotheses that are invariant to the scale and the particular shape of the distribution of the incentives, apart from the median and the skewness. The first part is easily tested using the sign test, which employs a simple binomial test on the number of positive and negative values. Gibbons (2003) suggests that the Wilcoxon matched-pairs rank-sum test can be used as a test for distributional symmetry (assuming a known median).[14] We proceed by using the sign test to try to reject the hypothesis that the player best responds on average less than half of the time. If we are able to reject this hypothesis, then we allocate the player to the corresponding type.[15] If we are unable to reject the null, then we test for the positive skewness in the distribution, while assuming that the median is zero. If we are able to reject this hypothesis then again we allocate the player to the appropriate type.

The Wilcoxon test proceeds by ordering the observed incentives by absolute magnitude, ignoring the sign. The smallest incentive (perhaps 0) would then be given a rank of 1 continuing to the largest incentive, which is given the rank of 59. Then the ranks corresponding to positive incentives are summed and normalised using Wilcoxon's formulae to give a test statistic which is asymptotically normal with mean 0 and variance 1. We use a one sided test against the Normal distribution at our significance level to test our null hypothesis. It is important to note here that the test statistic is increasing in the median (i.e. the number of positive incentive values) and the skew (i.e. the relative magnitude of the postive incentive values compared to the negative values). Hence if we can reject the hypothesis that both the median and the skew are 0, we can also reject that they are both non-negative.

For each of the methods of forming beliefs, we test our hypothesis against the corresponding incentives using the combination of the sign and Wilcoxon tests described with a significance level of 5%. If a null hypothesis is rejected, then we consider that the player is close to following the incentives, and is likely to have formed beliefs of opponents play using the corresponding method. If none of the null hypotheses are rejected then we allocate that player to the "Level 0" type as we are unable to say that any of the incentives are followed. In essence, we are allocating players to a type only if we can reject that their incentive values could be generated by a player who doesn't use them to make decisions. This may seem to be a test of relatively low strength, and it may seem likely that there

the mean as the location measure in it's calculation.

[14] In fact, the Wilcoxon test can also be used as a test of Location (when assuming a symmetric distribution). This would open up the possibility of combining both parts of the test by using just the Wilcoxon test. However, we are reluctant to follow this path as it is not clear how the skew and location effects are compensated in the test. This would make the results ambiguous if the skew and median of a sequence of incentives had different signs. Also, we specifically want to ignore the effect of the skewness if the player best responds most of the time.

[15] It would not be possible to allocate a player to more than one type here, unless the best responses coincided very often, as they would need to best respond more than half the time to be able to reject the null.

| Type | Fine | FineAccess | CoarseAccess | Coarse |
|---|---|---|---|---|
| Nash player | 12 | 14 | 3 | 3 |
| ABEE player | 1 | 1 | 7 | 12 |
| Level 1 player | 1 | 0 | 0 | 1 |
| Level 0 player | 2 | 1 | 6 | 0 |

Table 3: Type allocation over all rounds

would be high numbers of Level 0 players, but this can only work in favour of any postive results. We are relatively cautious about putting players into the Nash, ABEE and Level 1 types, so we can be relatively confident of these allocations. Finally, we are presented with a problem if more than one of our null hypotheses are rejected at each stage. Thankfully, this happens in only two cases out of all 64 row players—in one particular session in the CoarseAccess treatment where the best responses to the Fine and Coarse feedback coincided often. For these two cases, we allocated the subjects to the type where the incentive gave the lowest $p$-value in our test (one was allocated to the Nash and one to the ABEE type). Similarly, only three type allocations were changed by the test of skewness. The results are detailed in table 3.

In Fine and FineAccess the vast majority of subjects can indeed be classified as Nash players. In Coarse and CoarseAccess the majorities are smaller but the most frequent type is indeed the ABEE player. We also observe that the CoarseAccess treatment has the greatest number of subjects who were classified as Level 0. There are a number of possible reasons for this. It seems that the increased complexity of the feedback means that subjects experiment more in their strategies and are thus harder to categorize.[16]

Another reason for the difficulty in identifying types in the CoarseAccess treatment is that a player's type might not be constant throughout the experiment. Perhaps some of the players who start out ignoring the less accessible feedback are able to interpret it later on in the session. Or perhaps some players are confused about how to interpret the information at all, and require a few rounds to understand how to use the available information to make decisions. Our previous method of distinguishing types takes no notice of the dynamics, so we were unable to detect this. In our final test, we consider the possibility of such learning during the experiment, and that each player's method of forming beliefs (as well as the information contained in those beliefs) might change during the experiment. We split the experiment into four phases of 15 rounds each and we repeat the same tests using the sign and Wilcoxon tests, with the same 5% significance level for each stage of the experiment and each row player. The results are given in table 4.

Initially, we find a huge share of Level 0 types but after the first quarter of the experiment there is a huge jump in sophistication. From the second quarter onwards most subjects are either Nash or ABEE players and then remain so for the rest of the experiment. This gives further weight to the idea that players learn during the experiment both in terms of the information revealed about opponents play, but also in terms of how to interpret that information. However, it seems that, once subjects

[16] Incidentally, this leads to column players changing their strategies such that the best responses to the fine and coarse feedbacks coincide for the row players. Therefore, there are less rounds where the strategies can be distinguished which makes it even more difficult to reject the null.

| Rounds 1 to 15 | | | | |
|---|---|---|---|---|
| Type | Fine | FineAccess | CoarseAccess | Coarse |
| Nash player | 9 | 9 | 1 | 1 |
| ABEE player | 1 | 0 | 3 | 4 |
| Level 1 player | 0 | 0 | 0 | 0 |
| Level 0 player | 6 | 7 | 12 | 11 |
| Rounds 16 to 30 | | | | |
| Type | Fine | FineAccess | CoarseAccess | Coarse |
| Nash player | 12 | 12 | 3 | 2 |
| ABEE player | 0 | 0 | 5 | 7 |
| Level 1 player | 1 | 0 | 0 | 1 |
| Level 0 player | 3 | 4 | 8 | 6 |
| Rounds 31 to 45 | | | | |
| Type | Fine | FineAccess | CoarseAccess | Coarse |
| Nash player | 12 | 13 | 3 | 2 |
| ABEE player | 1 | 0 | 8 | 10 |
| Level 1 player | 0 | 0 | 0 | 1 |
| Level 0 player | 3 | 3 | 5 | 3 |
| Rounds 46 to 60 | | | | |
| Type | Fine | FineAccess | CoarseAccess | Coarse |
| Nash player | 12 | 12 | 3 | 1 |
| ABEE player | 2 | 0 | 9 | 13 |
| Level 1 player | 0 | 0 | 0 | 1 |
| Level 0 player | 2 | 4 | 4 | 1 |

Table 4: Type allocation over different phases

have settled on a way of accessing and interpreting the given information, they rarely change this interpretation later in the experiment. Once they have found a method of forming beliefs, their type is largely fixed. We note that the latter finding goes against the view (at least in the present context) that subjects process information in a finer way as more experience accumulates.

# 5 Conclusion

In this paper we have shown conclusive evidence for the role of accessibility of information in decision making. When information is presented in a complex manner it would be very naive to assume that subjects can process it with full accuracy. Our experiments confirm this very intuitive statement. But our study also shows that accessibility (or more generally, framing) issues can be modelled—and in quite familiar ways. Thus, acknowledging the role of framing effects does not imply giving up the apparatus of game theoretic modeling and equilibrium analysis.

More specifically, we find that when we uncouple quantitative information about choice frequencies from information about in which context these choices were made subjects do what appears to be very natural: They bundle the frequency information, irrespective of context. This is very similar to the analogy-based expectation equilibrium approach as considered in Jehiel (2005). The logic behind this is surely that subjects are hesitant to throw away what is obviously useful information and decide, in face of its complexity, to simplify its content. They may or may not know how this distorts their actions from the optimal choice. Unconstrained optimization is simply not an issue for them—when the presentation gets too difficult they simply cannot process it correctly. Of course, for some this may not be surprising. The more important aspect of our result is, therefore, that subjects do process the information nevertheless and in a systematic way.

From the viewpoint of the analogy-based expectation equilibrium approach, our study is suggestive that the analogy partitions considered in that approach should not necessarily be viewed as deriving solely from the characteristics of the interaction. Here, with an identical underlying interaction, we were able to generate different grouping simply by playing with the accessibility of the feedback given to subjects. Thus, our approach suggests to include as part of the description of the interaction the type of feedback given to subjects and how accessible it is. That is, it suggests including the analogy partitions used by the players as part of the description of the environment.

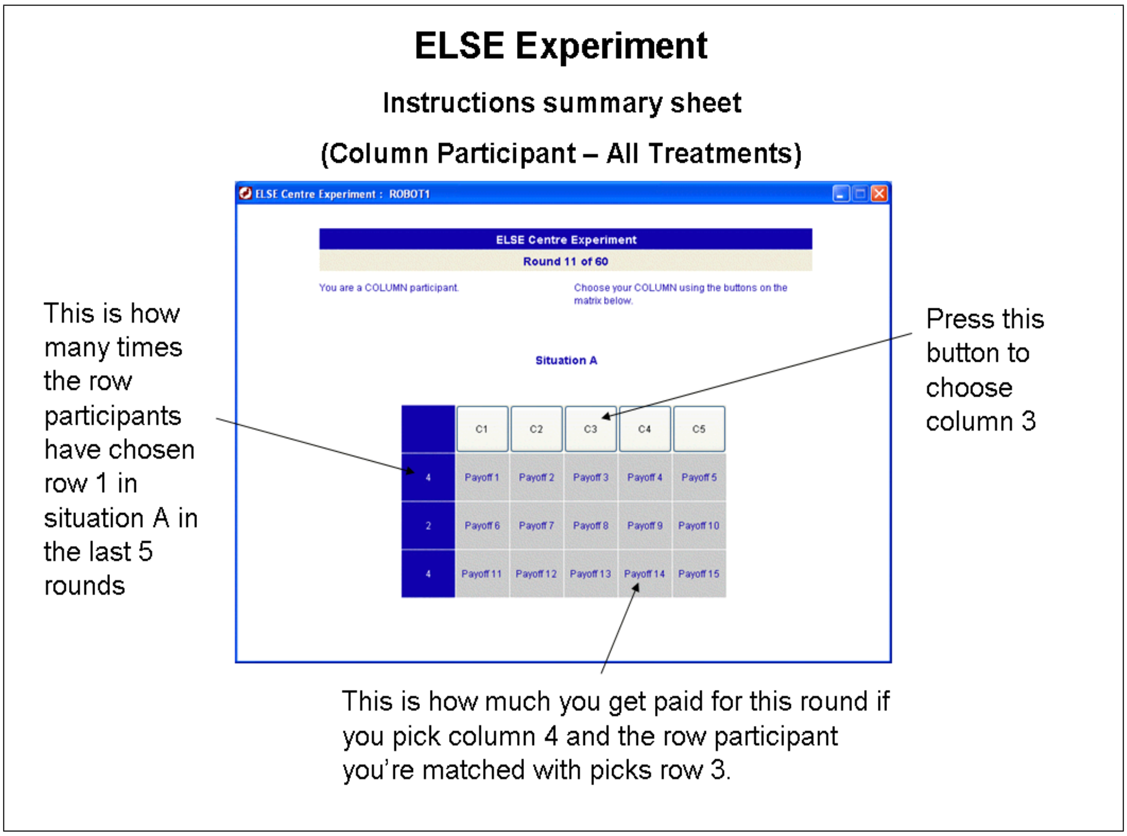
From a broader perspective, it seems legitimate to say that in the real world, we are faced with myriads of information, and one could even argue that information is more and more symmetrically shared as permitted by the internet technology. Yet, this view misses the point that the processing of information is not free of cost as usually assumed in economic theory. In order to make use of the information one has to simplify it. That is, one has to throw away many of the dimensions of the real world problem in order to identify regularities[17] and eventually reach a decision. Our experiment shows that the information processing of agents is likely to affect the behaviors of the agents even after these behaviors have stabilized (i.e., after one has reached an equilibrium). It is thus of primary

[17] As recently shown by Aragones et al. (2005), this is a hard problem in a computer science sense.

importance to understand how agents process information by focussing on some aspects of the data rather than others in order to inform theorists which behaviors should be considered as stable and in which contexts.[18] We view this experiment as a preliminary step toward this end.[19]

# A Experiment Instructions

There follows the instruction sheets handed out at the start of the experiment. Step-by-step instructions were presented on the computer screen at the start of each session, and these sheets were intended to be used as a reference during the experiment. They have all been amended to show the name of the treatment they apply to, but are otherwise identical to those handed out.



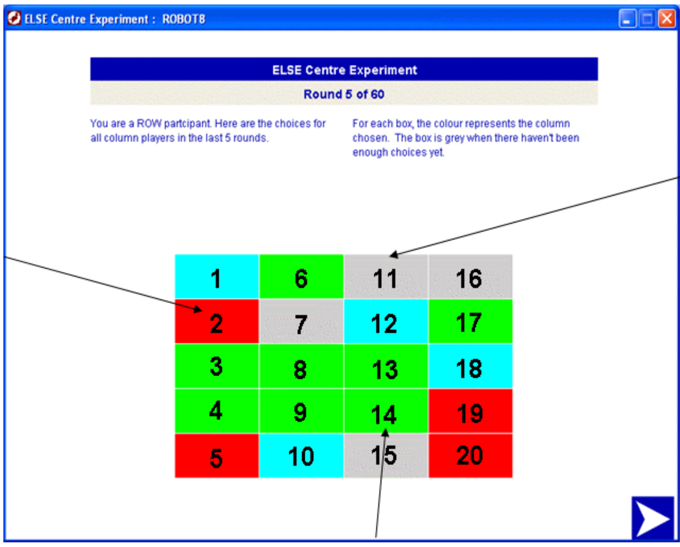

[18] The identification of equilibria (or stable behaviors) is also necessary for the understanding of incentives in given environments, which is required to design well performing institutions.

[19] The views that theories are simplifications of the world and that experiments may help design better theories agree with the recent paper by Samuelson (2005).

# ELSE Experiment

**Instructions summary sheet**

**(Row Participant – CoarseAccess Treatment)**




A column participant chose the red column in the last 5 rounds. The situation was the 2nd in the sequence on the next screen.

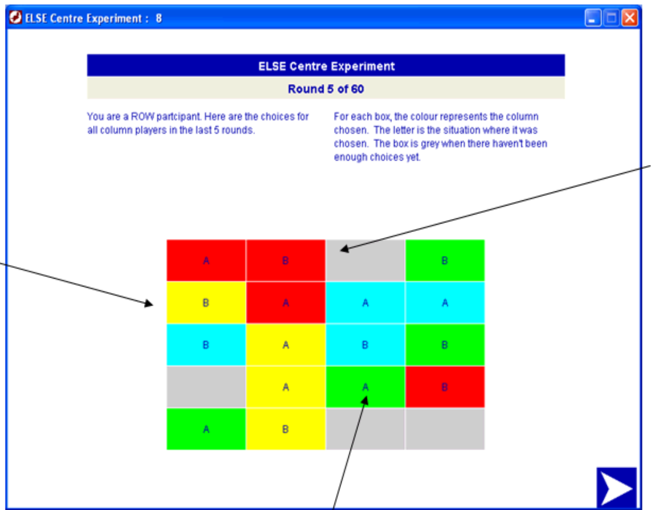
Grey boxes mean not enough choices have been made yet.

A column participant chose the green column in the last 5 rounds. The situation was the 14th in the sequence on the next screen.

# ELSE Experiment

**Instructions summary sheet**

**(Row Participant – FineAccess Treatment)**




A column participant chose the yellow column in situation B in the last 5 rounds.

Grey boxes mean not enough choices have been made.

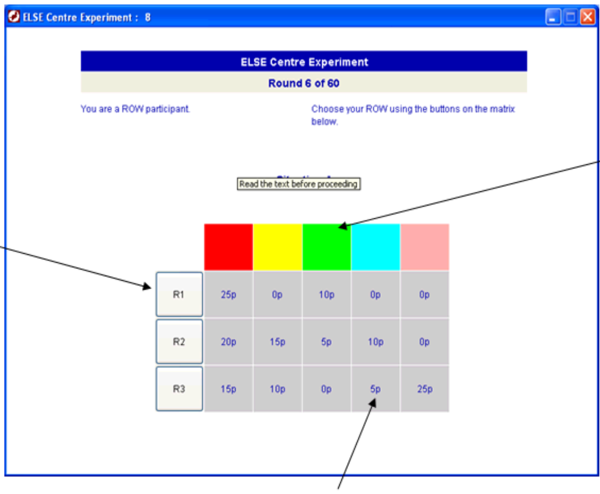
A column participant chose the green column in situation A in the last 5 rounds.

# ELSE Experiment

**Instructions summary sheet**

**(Row Participant – FineAccess and CoarseAccess Treatments)**

Press this button to choose row 1




Each choice for the column participant has a different colour

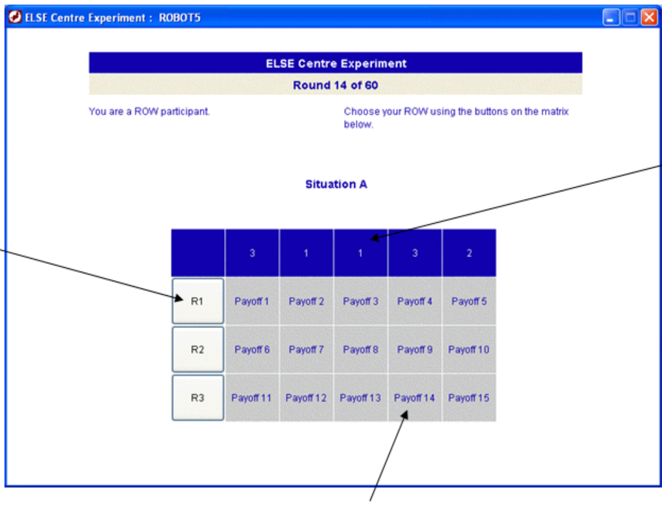
This is how much you get paid for this round if you pick row 3 and the column participant you're matched with picks column 4.

# ELSE Experiment

**Instructions summary sheet**

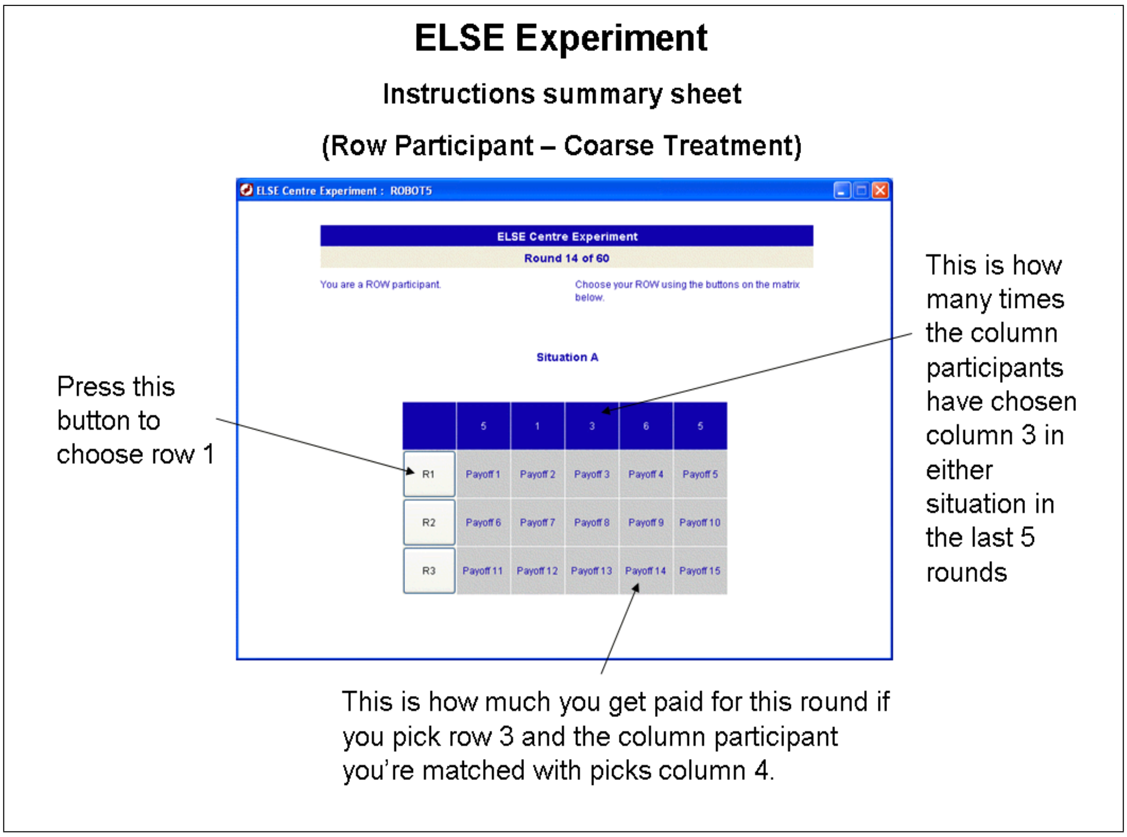
**(Row Participant – Fine Treatment)**

Press this button to choose row 1




This is how many times the column participants have chosen column 3 in situation A in the last 5 rounds

This is how much you get paid for this round if you pick row 3 and the column participant you're matched with picks column 4.

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