§24. Spectrum of the Velocity Distribution Function in the Slab Ion Temperature Gradient Driven Turbulence

Sugama, H., Watanabe, T.-H.

A spectral analysis of the velocity distribution function in the slab ion temperature gradient driven turbulence is done by using high-resolution Eulerian kinetic simulation results [1,2]. It is clarified how the entropy variable associated with the fine-scale structure of the distribution function is produced by the turbulent heat transport in the presence of the temperature gradient, transferred from macro to microscales in the velocity space through phase-mixing processes, and dissipated by collisions.

In order to investigate the velocity-space structure of the distribution function, we expand the fluctuation part of the ion distribution function $\hat{f}_k$ as

$$\hat{f}_k(v) = \sum_{n=0}^{\infty} \hat{f}_{k,n} H_n(v) F_M(v),$$

where $H_n(v)$ is the Hermite polynomial of order $n$. In terms of the coefficient $\hat{f}_{k,n}$ in the Hermite-polynomial expansion, the ion gyrokinetic equation (integrated in $v_\perp$) is written as

$$\frac{\partial \hat{f}_{k,n}}{\partial t} + i k_y \Theta \left[ \hat{f}_{k,n-1} + (n+1) \hat{f}_{k,n+1} \right] + \sum_{k=k'-k''} (k'_x k''_x - k'_y k''_y) \psi_{k'} \hat{f}_{k'',n} = -i k_y \psi_k \left[ \delta_{n,0} \left( 1 - \frac{m}{2} k^2 \right) + \delta_{n,1} \Theta + \delta_{n,2} \frac{m}{2} \right],$$

where $\delta_{n,m} = 1$ for $n = m$ and 0 for $n \neq m$. The phase mixing process associated with the parallel streaming of particles are now represented by the interaction to the adjacent-order $(n-1)$ and $(n+1)$ terms in the Hermite-polynomial expansion of the perturbed distribution function with the same wave number vector $k$ as shown in the second and third terms on the left-hand side. On the other hand, the $E \times B$ convection, which is given by the last term on the left-hand side, involves the distribution functions of only the same order $n$ but with different wave number vectors $k''$. The linear source terms proportional to $\psi_k$ (the gyrophase-averaged potential) on the right-hand side disappear for $n \geq 3$, which is the reason why the Hermite-polynomial expansion is employed here. A clear cutoff of the source like this never occurs if we use the Fourier expansion in terms of $\exp(ilt v_\parallel)$ ($-\infty < l < \infty$) as basis functions.

We define the entropy variable $\delta S$ by

$$\delta S = \sum_k \int dv_\parallel \left( \frac{\hat{f}_{k,n}^2}{2 F_M} \right) \equiv \sum_{n,k} \frac{1}{2} n \hat{f}_{k,n}^2 \equiv \sum_n \delta S_{k,n} \equiv \sum_n \delta S_n,$$

where $\delta S_{k,n} = \frac{1}{2} n \hat{f}_{k,n}^2$ and $\delta S_n = \sum_k \delta S_{k,n}$ represent the entropy spectral functions in the $(k,n)$-space and in the $n$-space, respectively, and $\langle \cdots \rangle$ denotes the ensemble average. Then, we obtain

$$\frac{d}{dt} \delta S_n = \eta_i Q_i (\delta_{n,2} + J_{n-1/2} - J_{n+1/2} - 2 \nu n \delta S_n) \text{ for } n \geq 2,$$

where

$$J_{n-1/2} = \sum_k \Theta_k \psi_n \text{Im} (\langle \hat{f}_{k,n-1} \hat{f}_{k,n} \rangle),$$

$$J_{n+1/2} = \sum_k \Theta_k (n + 1) \text{Im} (\langle \hat{f}_{k,n} \hat{f}_{k,n+1} \rangle),$$

$$Q_i = \frac{1}{2} \sum_k k_y \int dv_\parallel (v_\parallel^2 - 1) \text{Im} (\langle \Psi_k \hat{f}_{k,n=2} \rangle).$$

Here, $Q_i$ denotes the turbulent ion heat flux downward in the temperature gradient and $\eta_i Q_i$ represents the entropy production supplied at $n = 2$ while $-2 \nu n \delta S_n$ is the collisional entropy dissipation term. We also find that $J_{n-1/2}$ ($J_{n+1/2}$) represents the entropy transfer from the $(n-1)$th ($n$th) to the $n$th ($[n+1]$) Hermite-polynomial portion.

The entropy spectral function is analytically derived and compared with the slab ITG turbulence simulation result [see Fig.1]. The entropy spectrum obeys a power law in the range that is free from instability sources and collisional dissipation. The entropy spectrum observed by the simulation can be well described by the analytical expressions as shown in Fig.1.

References