Probabilistic Business Failure Prediction in Discounted Cash Flow Bond and Equity Valuation

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Abstract

The purpose of the paper is to incorporate probabilistic business failure predictions in discounted cash flow (DCF) models for the valuation of company bonds and owners´ equity. The analysis shows that period-specific probabilities of business failure are instrumental to the assessment of expected values of cash flows in such models. Under somewhat restrictive conditions the failure risk can alternatively be accommodated through an adjustment of the discount rate, i.e. expected values of future cash flows conditioned on business survival can simply be discounted with such a discount rate. The result holds both in bond and equity DCF valuation modelling. In order for the accounting-based residual income valuation model to appropriately capture the failure risk, an additional accounting "failure loss recognition" principle as well as a novel term in the model specification have been identified.

1. Introduction

There is a long research history in the area of accounting and finance of using financial statement information in trying to differentiate between "failure" and "survivor" companies; early attempts go back to Smith & Winakor (1935) and Mervin (1942), with more sophisticated statistical techniques introduced in Beaver (1966), Altman (1968) and Ohlson (1980). Commonly the virtue of being able to distinguish between these two types of business states has been viewed as more or less self-evident, or to quote Wilcox (1971):

"Businessmen have always needed a method of predicting the risk of business failure. Such risks are fundamental variables in decision making." (p. 1)

However, as noted in Ohlson (1980) distinguishing between only two states of business might be a very coarse partition of future outcomes from an investment theory point of view – in general an investment payoff space is much more complex. In this sense, trying to forecast whether a company is about to fail or not might be a somewhat futile exercise. On the other hand, as elaborated in a bank loan decision context in Johansson (1973), knowing the probability of failure (bankruptcy) can be informative in a more partial sense. For example, a bank loan will be fully served (capital plus interest) by a surviving company − i.e. there is only one loan payoff associated with the survival state. Hence, even a coarse distinction between business failure and survival might be useful. The question to be addressed is thus really under what circumstances such a dichotomous partition will have a more profound importance to capital market investors.

Causal observations of professional financial analysts and investment fund managers give a mixed picture of the usefulness of probabilistic business failure predictions. On the one hand there appears to be a wide-spread understanding of the negative stock market impact associated with business failures, but on the other hand there also appears to be wide-spread ignorance about how to more exactly incorporate failure probabilities in valuation modelling.A common notion appears to be that some discretionary addition to the discounting rate of return "in principle" should be able to handle this risk, even if the specification of this addition appears to be vague at best.

The purpose of this paper is to investigate the relevance of probabilistic business failure predictions in discounted cash flow (DCF) valuation modelling.[1] In principle, two more prominent applications of DCF modelling will be analysed − the valuation of company

[1] The potential relevance of probabilistic failure predictions in the context of relative valuation − including option pricing modelling − is not addressed in the paper.

bonds and the valuation of owners´ equity. Within this modelling framework, the possibility of making adjustments to the discounting rate of return to incorporate the risk of business failure will be addressed. In the context of the valuation of owners´ equity, both PVED (present-value-of-expected-dividends) and RIV (residual income valuation) model specifications will be investigated.

The analysis of the paper deals with conceptual and theoretical issues in DCF valuation modelling (with its origins from Fisher, 1906), since this approach is the dominant valuation paradigm in the accounting and finance literature as it relates to the valuation of investment projects, business segments, financial instruments, etc. Throughout the analysis a flat term structure will be presumed, where required rates of return relevant for the discounting of cash flows to capital investors (after company taxes, but before personal taxes) are known. Also, unbiased probabilities of business failure are supposedly exogenously given. Such probability assessments are commonly based on postulated stochastic models of business failure (cf. early references like Wilcox, 1971; and Vinso, 1979) or on statistical analyses of empirical data, ranging from simple calculations of the relative frequency of company failures (cf. Moody's Investor Service, 2005) to more elaborate applications of probit/logit analysis (as for example in Ohlson, 1980; Zavgren, 1985; or Skogsvik, 1990). Questions dealing with methodological issues concerning the estimation of unbiased probabilities of business failure will hence not be addressed in the paper.

The disposition of the paper is as follows. In the next section, the relevance of incorporating probabilities of business failure for the valuation of company bonds is investigated. The importance of incorporating probabilities of business failure in PVED and RIV valuation modelling is analysed in section 3. A summary and concluding remarks are included in section 4 of the paper.

2. Probability of business failure in DCF bond valuation

In this section the relevance of incorporating probabilities of business failure in a DCF bond valuation context will be analysed. The bond valuation model is presented in subsection 2.1. Two basic issues are then addressed - the modelling relevance of a probability measure per se and the potential for accommodating the risk of business failure in the discounting rate of return. A couple of simplified benchmark cases are elaborated in subsection 2.2.

In relation to various applications of option pricing theory to bond valuation − in principle as outlined in the seminal paper by Merton (Merton, 1974) and further developed in, for example, Black & Cox (1976), Longstaff & Schwartz (1995), Leland (1994) and Leland & Toft (1996) − a DCF bond valuation model is less sophisticated in a technical modelling sense.[2] Nevertheless, there are strong operational advantages associated with a DCF bond valuation model. It almost goes without saying, but option pricing modelling is plagued by very complex application and measurement problems. For one thing, option pricing theory hinges on the idea of being able to use arbitrage between the underlying asset, the risk-free interest rate and the option itself, in order to find the correct price of the option. In most real-world cases the underlying asset in a bond valuation context (all operating and financial net assets of the company) cannot be traded at reasonable transaction costs, in turn implying that the generic idea of arbitrage-free pricing is doubtful. Secondly, the estimation of the volatility of the underlying asset is notoriously difficult, even in the presence of historic market prices for this asset. The problem is magnified if the time to maturity for the bond is long (as one obviously cannot rule out). Also, assuming that the asset volatility should be the same disregarding the financial health of the company is dubious.

A virtue of the DCF bond valuation model specified below is obviously its simplicity and operational feasibility. The DCF valuation technique in itself is well-established in the area of accounting and corporate finance and fairly straightforward techniques for estimating probabilistic forecasting models are available. There is also − as indicated in the beginning of the paper − a substantial number of published empirical studies focused on the prediction of business failures, in the main indicating that financial statement numbers in a reliable manner can predict business failures up to about five years before failure (cf. Zavgren, 1995, and Skogsvik, 1990)

2.1. The bond valuation model

In a "net present value" decision context, the difference between the present value of future interest and amortization payments and the amount being lent to a company has to be non-negative in order for a loan to be granted. Hence, from the lender's point of view, we have the following (necessary) condition:

[2] A comprehensive survey of bond valuation modelling − including various applications based on option pricing theory − is found in for example Urig-Homburg (2002).

$$NPV(D)_{j,t} = V(D)_{j,t} - D_{j,t} \geq 0 \qquad \text{(C.1)}$$

where: $NPV(D)_{j,t}$ = net present value at time t of a loan payment equal to D to company j.

$V(D)_{j,t}$ = present value at time t of a loan payment equal to D to company j.

$D_{j,t}$ = loan payment to company j at time t.

The discounted present value $V(D)_{j,t}$ − henceforth simply referred to as the "bond value" − of a loan can in principle always be written as:

$$V(D)_{j,t} = \sum_{\tau=1}^{T} \frac{E_{(t)}(\widetilde{CF}_{D(j,t),t+\tau})}{(1+\rho_{D(j,t)})^{\tau}} \qquad (1)$$

where: $\widetilde{CF}_{D(j,t),t+\tau}$ = cash flow to bondholder at time $t+\tau$, corresponding to a loan of D to company j at time t.

$\rho_{D(j,t)}$ = required expected rate of return by bondholders for a loan of D to company j at time t.

$E_{(t)}(\ldots)$ = expectation operator, based on available information at time t.

Without any loss of generality, it is assumed in (C.1) and (1) that the bond valuation point in time coincides with the point in time when the loan is to be granted. Also, the term structure of the required expected rate of return $\rho_{D(j,t)}$ is assumed to be flat, but the level of the return might depend on the valuation point in time t.[3]

In order to be more specific about the cash flow consequences of the loan, the following simplifying assumptions are introduced:

A-I. Interest coupon payments are due at the end of future years and the loan is to be repaid in full after T years.

[3] Note that the cost of capital $\rho_{D(j,t)}$ refers to the required *expected* rate of return for the bond − not the required *promised* rate of return (cf., for example, Copeland et al, 2000, pp. 209-212).

A-II. If the borrowing company goes bankrupt in some year $t+\tau$, the company cannot fulfil its financial obligations and is liquidated at the end of that year. The bondholders will not receive any coupon or loan repayment in such a liquidation.

A-III. The bond is associated with a risk-free collateral value of $C_{D(j,t)}$, $C_{D(j,t)}$ being less than or equal to the final loan repayment. If the borrowing company is unable to fulfil its financial obligations in some year, the bondholders can take possession of the collateral at the end of the year when the company has failed.

Note that, if the loan agreement does not include any collateral, $C_{D(j,t)}$ can in principle be replaced with the expected value of a cash settlement payment, or the expected market price of the bond immediately after the failure event.[4] Expected values of such terminal payments would then be viewed as exogenously given, but they could of course be correlated with the probability of failure for the borrowing company (as indicated in Altman et al, 2005).

In accordance with the assumptions A-I to A-III above, the underlying cash flows in the valuation model (1) constitute future coupon payments (at the end of years $t+\tau$, $\tau = 1,2,...T$), the final loan repayment (at the end of year t+T) and the collateral security value. These cash flows are now postulated to be a function of the financial health of the borrowing company as follows:

- **Business survival in year $t+\tau$:**

  The cash flow to the bondholders at the end of year $t+\tau$ consists of the annual coupon payment (for $1 \le \tau \le T-1$), or the annual coupon payment plus the final loan amortization (for $\tau = T$).

- **Business failure occurs in year $t+\tau$:**

  The cash flow to the bondholders at the end of year $t+\tau$ is equal to the collateral value, with no additional cash flows to be received after this point in time.

[4] A settlement payment can be estimated using the average bond payout ratio (i.e. final settlement payment in relation to the contractual final loan payment) for an appropriate sample of failure companies. According to Moody´s Special Comment (2005) the mean issuer-weighted recovery rates (measured as bid prices on defaulted debt after the default date, in relation to the contractual loan payment) for unsecured bonds over the period 1982−2003 were 28,9% (junior subordinated), 32,0% (subordinated), 39,1% (senior subordinated) and 44,9% (senior unsecured).

Simplifying the notation by setting t = 0 and suppressing the firm index j, the expected cash flows in (1) can be rewritten as:

$$E_{(0)}(\widetilde{CF}_{D,\tau}) \;=\; (1-P_{surv}^{(\tau-1)})\cdot 0 \;+\; P_{surv}^{(\tau-1)}\cdot(1-p^{*}_{fail,\tau})\cdot r_D\cdot F_{D,T} \;+$$

$$+\; P_{surv}^{(\tau-1)}\cdot p^{*}_{fail,\tau}\cdot C_{D(0)} \qquad (2.a)$$

for $\tau = 1,2,...T-1$, and

$$E_{(0)}(\widetilde{CF}_{D,T}) \;=\; (1-P_{surv}^{(T-1)})\cdot 0 \;+\; P_{surv}^{(T-1)}\cdot(1-p^{*}_{fail,T})\cdot(1+r_D)\cdot F_{D,T} \;+$$

$$+\; P_{surv}^{(T-1)}\cdot p^{*}_{fail,T}\cdot C_{D(0)} \qquad (2.b)$$

where: $P_{surv}^{(\tau-1)}$ = probability of company survival at the end of year $\tau-1$.

$p^{*}_{fail,\tau}$ = probability of business failure to occur in period τ, conditioned on company survival at the end of year $\tau-1$.

r_D = contractual coupon rate for a loan equal to D to the company at t = 0.

$F_{D,T}$ = contractual loan repayment (principal) at time T for a loan equal to D to the company at t = 0.

$C_{D(0)}$ = (risk-free) collateral value associated with a loan equal to D to the company at t = 0.

Note that the probability measure $P_{surv}^{(\tau-1)}$ in (2.a) and (2.b) is the “accumulated” probability of business survival at the end of year $\tau-1$. Presuming that the borrowing company is a "survival company" at time t = 0, it is hence defined as:

$$P_{surv}^{(\tau-1)} \;=\; \prod_{s=1}^{\tau-1}(1-p^{*}_{fail,s}) \qquad (3)$$

for $\tau \;=\; 2,3,\ldots T$.

In accordance with assumption A-II, the company is unable to make any loan service payment in year τ if the company has failed before the beginning of that year. Hence the first term on the RHS (right-hand side) of (2.a) and the first term on the RHS of (2.b) are both equal to 0. This means that the expected value of the loan service payment for each year until T-1 is equal to $P_{surv}^{(\tau-1)}$ multiplied by a weighted average of the contractual coupon payment $(r_D \cdot F_{D,T})$ and the collateral value $(C_{D(0)})$. The weights in this calculation are simply $(1-p^*_{fail,\tau})$ and $p^*_{fail,\tau}$, respectively. In the same manner, the expected value of the loan service payment at the end of year T is equal to $P_{surv}^{(T-1)}$ multiplied by a weighted average of the contractual coupon payment plus repayment of the loan ($r_D \cdot F_{D,T} + F_{D,T} = (1+r_D)F_{D,T}$) and the collateral value $(C_{D(0)})$.

If the expected values in (2.a) and (2.b) and the expression for $P_{surv}^{(\tau-1)}$ in (3) are incorporated in (1), the following valuation model is obtained:

$$V(D)'_0 =$$

$$= \sum_{\tau=1}^{T} \frac{\left[\prod_{s=1}^{\tau-1}(1-p^*_{fail,s})\right] \cdot \left[(1-p^*_{fail,\tau}) \cdot r_D \cdot F_{D,T} + p^*_{fail,\tau} \cdot C_D\right]}{(1+\rho_D)^{\tau}} +$$

$$+ \frac{\left[\prod_{s=1}^{T}(1-p^*_{fail,s})\right] \cdot F_{D,T}}{(1+\rho_D)^{T}} \tag{4}$$

The bond valuation model in (4) illustrates the potential relevance of a dichotomous partitioning ("business survival" versus "business failure") of the unconditional cash flow probability distributions being implied in (1). Note that probabilities of business failure are instrumental to the assessment of expected values of future cash flows in (4).

In principle, the virtue of a dichotomous partitioning of the unconditional probability distributions of future cash flows hinges on a clear-cut association between *at least one* of the two business states and the corresponding cash flow value. Given the above loan contract and assumption about the borrowing company being unable to make any payment if business failure occurs, the two states are actually fully revealing with regard to the future cash flows to the bondholders. All investment risk is thus associated with the risk of business failure. In a bond valuation context in general, one can reasonably presume that there is a clear-cut association between the state of "business survival" and the cash flow payments as long as the company is in a state of "business survival". If business failure occurs, and there is no risk-free collateral that fully compensates for the remaining contractual loan payments, a non-trivial probability distribution is likely to remain for this state and hence the dichotomous partitioning will only be partially revealing.

An interesting issue with regard to the specification of (4) is whether the risk of business failure rather could be taken into account through some adjustment of the discounting rate, instead of having to deal with somewhat complex assessments of future expected values. Previously this problem has been addressed in for example Duffie & Singleton (1999) and valuation approaches of this kind are often suggested in practice (cf. Damodaran, 2002). Adhering to the idea of "keeping it simple", an alternative valuation model with a constant discounting rate ρ_D^x can hence be expressed as in (5).

$$V(D)_0^x = \sum_{\tau=1}^{T} \frac{r_D \cdot F_{D,T}}{(1+\rho_D^x)^\tau} + \frac{F_{D,T}}{(1+\rho_D^x)^T} \tag{5}$$

Provided that $r_D \cdot F_{D,T}$ and $F_{D,T}$ are non-negative, there will always exist a solution for ρ_D^x implying that $V(D)_D^x = V(D)'_0$. However, such a solution would require pre-knowledge of the bond value in accordance with the full-fledged model in (4). The question is rather whether there is some analytical solution for ρ_D^x not requiring $V(D)'_0$ to be known.

In order for ρ_D^x to be invariable over the life of the bond (i.e. for $V(D)_{\tau-1}^x$ to be equal to $V(D)'_{\tau-1}$ for all points in time $\tau = 1,2,...T$), the following conditions have to hold in a solution to $V(D)_D^x = V(D)'_0$:

$$\frac{\left[\prod_{s=1}^{\tau-1}(1-p^*_{fail,s})\right]\cdot\left[(1-p^*_{fail,\tau})\cdot r_D \cdot F_{D,T} + p^*_{fail,\tau}\cdot C_D\right]}{(1+\rho_D)^\tau} = \frac{r_D \cdot F_{D,T}}{(1+\rho_D^x)^\tau} \tag{C.2}$$

for $\tau = 1, 2, \ldots, T$, and

$$\frac{\left[\prod_{s=1}^{T}(1-p^*_{fail,s})\right]\cdot F_{D,T}}{(1+\rho_D)^T} = \frac{F_{D,T}}{(1+\rho_D^x)^T} \tag{C.3}$$

Restricting $p^*_{fail,\tau}$ to be less than 1,00 for $\tau = 1,2,...T-1$, (C.2) can be rewritten as follows:

$$(1+\rho^x_D)^\tau \quad = \quad \frac{(1+\rho_D)^\tau \cdot r_D \cdot F_{D,T}}{\left[\prod_{s=1}^{\tau-1}(1-p^*_{fail,s})\right]\cdot\left[(1-p^*_{fail,\tau})\cdot r_D \cdot F_{D,T} + p^*_{fail,\tau}\cdot C_D\right]} \qquad \text{(C.2')}$$

for $\tau = 1,2,...T$

As ρ_D, ρ^x_D and r_D are constants in the valuation models (4) and (5), condition (C.2') can only hold if $C_D = 0$ and $p^*_{fail,s}$ is constant and less than 1,00 (i.e. $p^*_{fail,\tau} = p^*_{fail} < 1{,}00$ for τ = 1, 2,…T). Given these assumptions, (C.2') can easily be solved for ρ^x_D:

$$\rho^x_D \quad = \quad \frac{\rho_D + p^*_{fail}}{1-p^*_{fail}} \qquad \text{(C.2'')}$$

Having set $p^*_{fail,s}$ to be a constant, the solution for ρ^x_D in (C2") is also consistent with condition (C.3). Intuitively the solution in (C2") makes sense − the higher the risk of business failure, the higher the adjusted discounting rate ρ^x_D. If, for example, ρ_D = 6,00% and the probability of failure p^*_{fail} = 0,05, the failure-adjusted discounting rate would be (0,06 + 0,05)/(1- 0,05) = 11,58%. In order to compensate for the risk of business failure in future years, the adjusted discounting rate ρ^x_D would thus have to be almost twice the expected required rate of return in this example. Also note that, given the assumptions of a constant probability of failure and that the bond holder receives nothing in the event of business failure, ρ^x_D according to (C.2") holds for any time to maturity of the loan.

Viewed somewhat differently, for some promised coupon rate r_D and loan repayment $F_{D,T}$, the bond value according to (5) will unambiguously decrease if the risk of business failure goes up since the adjusted discounting rate ρ^x_D then will increase. A positive correlation between the expected required return ρ_D and the probability of business failure − consistent with the idea that bond holders would require a higher expected risk premium if the failure risk goes up − will obviously strengthen a result of this kind. However, note that a positive correlation of this kind is not a necessary requirement for this to occur.

Adjusting the discounting rate in order to account for the risk of business failure in a bond valuation model is hence not necessarily an erroneous course of action. Given the contextual assumptions in this section, the probability of business failure being constant over time and that there is no collateral, adjusting the discounting rate in accordance with (C2") is consistent with an appropriate application of the DCF bond valuation model.[5]

2.2. Benchmark cases in DCF bond valuation modelling

The valuation model in (4) above can be simplified in order to illustrate a couple of less complex bond valuation problems:

- The case of **"no collateral"**

If there is no collateral associated with the bond, $C_D = 0$ in (4) and we get the following model:

$$V(D)''_0 = \sum_{\tau=1}^{T} \frac{\left[\prod_{s=1}^{\tau}(1 - p^*_{fail,s})\right] \cdot r_D \cdot F_{D,T}}{(1+\rho_D)^\tau} + \frac{\left[\prod_{s=1}^{T}(1 - p^*_{fail,s})\right] \cdot F_{D,T}}{(1+\rho_D)^T} \qquad (6)$$

The value of the loan is now simply the present value of the expected values of future contractual payments.

- The case of **"no collateral and a constant probability of failure"**

In this case, $C_D = 0$ and $p^*_{fail,\tau} = p^*_{fail}$ for $\tau = 1,2,...T$:

$$V(D)'''_0 = \sum_{\tau=1}^{T} \frac{(1-p^*_{fail})^\tau \cdot r_D \cdot F_{D,T}}{(1+\rho_D)^\tau} + \frac{(1-p^*_{fail})^T \cdot F_{D,T}}{(1+\rho_D)^T} =$$

$$= F_{D,T}\left[\frac{r_D \cdot (1-p^*_{fail})\left[(1+\rho_D)^T - (1-p^*_{fail})^T\right]}{(\rho_D + p^*_{fail})(1+\rho_D)^T} + \frac{(1-p^*_{fail})^T}{(1+\rho_D)^T}\right] \qquad (7)$$

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[5] Note that an assessment of p^*_{fail} is still instrumental for the appropriate adjustment of the discounting rate of return.

Assuming that $0 < p^*_{fail} < 1{,}00$ and ρ_D being non-negative, (7) can be further simplified as the number of years to the repayment date T goes towards infinity:

$$V(D)'''_0 \quad = \quad \frac{(1 - p^*_{fail}) \cdot r_D \cdot F_{D,T}}{\rho_D + p^*_{fail}} \qquad \text{as} \quad T \to \infty \tag{7'}$$

Not surprisingly, (7') shows that a higher probability of business failure (ceteris paribus) will reduce the bond value $- p^*_{fail}$ reduces the value of the numerator and increases the value of the denominator on the RHS of (7'). Also note that if p^*_{fail} becomes very small, $V(D)'''_0 \approx r_D \cdot F_{D,T} / \rho_D$. In a situation of this kind there is almost no risk associated with the future coupon payments, implying that ρ_D then presumably would be very close to the risk-free rate of interest.

3. Probability of business failure in DCF equity valuation

The relevance of incorporating probabilities of business failure in equity valuation modelling is addressed in this section. A generic PVED ("present-value-of-expected-dividends") model is elaborated in subsection 3.1, including an analysis of the "Gordon´s constant growth model". The possibility of adjusting the discounting rate of return to recognize the risk of business failure is analyzed in subsection 3.2. As an alternative valuation model − consistent with PVED but specified in terms of accounting measures of income and capital values − a "residual income valuation" (RIV) model that explicitly incorporates probabilities of business failure is derived in subsection 3.3.

3.1. The PVED valuation model

In fundamental valuation analysis, equity investments are based on a comparison between the present value of expected future cash flows to be received by equity investors and the acquisition price of the equity investment. A necessary condition for making an investment can be written as in (C.4).

$$\mathrm{NPV(EQ)}_{j,t} \;=\; \mathrm{V(EQ)}_{j,t} - \mathrm{Ap(EQ)}_{j,t} \;\geq\; 0 \qquad \text{(C.4)}$$

where: $\mathrm{NPV(EQ)}_{j,t}$ = net present value at time t of an equity investment in company j.

$\mathrm{V(EQ)}_{j,t}$ = present value at time t of an equity investment in company j.

$\mathrm{Ap(EQ)}_{j,t}$ = acquisition price at time t of an equity investment in company j.

Presumably the acquisition price of the equity investment $-\mathrm{Ap(EQ)}_{j,t}-$ is exogenous and known to the investor. The investment problem is thus focused on $\mathrm{V(EQ)}_{j,t}$, i.e. the present value of future expected cash flows to the equity investor. Following the analysis of the bond valuation problem in subsection 2.1, this value can in principle be expressed as:

$$\mathrm{V(EQ)}_{j,t} \;=\; \sum_{\tau=1}^{T} \frac{E_{(t)}(\widetilde{CF}_{EQ(j,t),t+\tau})}{(1+\rho_{t,EQ(j,t)})^{\tau}} \qquad (8)$$

where: $\widetilde{CF}_{EQ(j,t),t+\tau}$ = cash flow to equity investor at time $t+\tau$, corresponding to an equity investment at time t of $\mathrm{Ap(EQ)}_{j,t}$ in company j.

$\rho_{j,EQ(j,t)}$ = required expected rate of return for an equity investment at time t in company j.

As in the bond valuation problem, the required return $\rho_{t,EQ(j,t)}$ is allowed to depend on the valuation point in time but is otherwise assumed to be constant over the investment period. Also, note that (8) implies a holding period of the equity investment from time t until (t + T).

In order to be more precise about the specification of (8), future cash flows to the equity investor are henceforth assumed to only consist of annual dividend payments[6] and the selling price of the equity investment at the end of the holding period. Additionally, the

[6] In order not to ignore the possibility of new issues of owners' equity , “dividend payments” should rather be interpreted as “dividends less new contributions of equity capital” (often referred to as “net dividends”).

cash flows are postulated to depend on the financial health of the company in the following way:

- **Business survival in year $\mathbf{t + \tau}$:**
  The cash flow to the equity investor at the end of year $t + \tau$ consists of the annual dividend payment (for $1 \le \tau \le T - 1$), or the annual dividend payment plus the selling price of the equity investment (for $\tau = T$).
- **Business failure occurs in year $\mathbf{t + \tau}$:**
  There is no cash flow from the company to the equity investor at the end of year $t + \tau$, and there will be no cash flows either to or from the equity investor after this point in time.

Simplifying the notation by setting t = 0 and suppressing the company index j, $E_{(0)}(\widetilde{CF}_{EQ,\tau})$ in (8) can now be expressed as:

$$E_{(0)}(\widetilde{CF}_{EQ,\tau}) = \left(1 - P_{surv}^{(\tau-1)}\right) \cdot 0 \; + \\ + \; P_{surv}^{(\tau-1)} \cdot (1 - p^{*}_{fail,\tau}) \cdot E_{(0)}\left(\widetilde{DIV}_{EQ,\tau} \middle| surv(\tau)\right) \; + \; P_{surv}^{(\tau-1)} \cdot p^{*}_{fail,\tau} \cdot 0 \tag{9.a}$$

for $\tau = 1,2,...T - 1$, and

$$E_{(0)}(\widetilde{CF}_{EQ,T}) = \left(1 - P_{surv}^{(T-1)}\right) \cdot 0 \; + \\ + \; P_{surv}^{T-1} \cdot (1 - p^{*}_{fail,T}) \cdot \left[E_{(0)}\left(\widetilde{DIV}_{EQ,T} \middle| surv(T)\right) + E_{(0)}\left(\widetilde{Sp(EQ)}_T \middle| surv(T)\right)\right] + \\ + \; P_{surv}^{T-1} \cdot p^{*}_{fail,T} \cdot 0 \tag{9.b}$$

where: $E_{(0)}\left(\widetilde{DIV}_{EQ,\tau} \middle| surv(\tau)\right)$ = expected value of dividend payment at time τ, conditioned on company survival at the end of year τ.

$E_{(0)}\left(\widetilde{Sp(EQ)}_T \middle| surv(T)\right)$ = expected value of selling price of equity investment (ex dividend) at time T, conditioned on company survival at the end of year T.

In contrast to the bond valuation problem (cf. in particular expressions (2.a) and (2.b)), the cash flow probability distributions conditioned on company survival are certainly not deterministic in the equity valuation model. Just knowing that a company will survive is typically not enough to determine the size of some future dividend payment, or the selling price of the equity investment at the end of the holding period. On the other hand, the cash flow probability distributions are assumed to be deterministic ($\widetilde{CF}_{EQ,\tau} = 0$ for $\tau = 1,2,...T$) if business failure occurs. Obviously the virtue of introducing failure probabilities in an equity DCF valuation model hinges on this deterministic association.

If the expected values of the cash flows in (9.a) and (9.b) are incorporated in (8), we get:

$$V(EQ)'_0 = \sum_{\tau=1}^{T} \frac{\left[\prod_{s=1}^{\tau} (1 - p^*_{fail,s})\right]\left[E_{(0)}\left(D\tilde{I}V_{EQ,\tau} \middle| surv(\tau)\right)\right]}{(1 + \rho_{EQ})^{\tau}} + \\ + \frac{\left[\prod_{s=1}^{T} (1 - p^*_{fail,s})\right]\left[E_{(0)}\left(Sp\tilde{(}EQ)_T \middle| surv(T)\right)\right]}{(1 + \rho_{EQ})^{T}} \qquad (10)$$

(10) illustrates the importance of the risk of business failure in a PVED equity valuation model. Expected values of future dividends and the future selling price of the equity investment conditioned on business survival, have to be multiplied by "accumulated" probabilities of business survival (i.e. $\prod_{s=1}^{\tau} (1 - p^*_{fail,s})$ for s = 1,2,…T) in order to get unconditioned expected values of cash flows. Evidently, this is not a peculiar result – a company has to survive in order to pay dividends or to demand some non-negative price of owners' equity at the end of the holding period.

Also note that the dichotomous partitioning of future business states in particular is helpful in the equity valuation model since cash flows to equity investors are (deterministically) equal to zero if business failure occurs at some future point in time. Hence only cash flow probability distributions conditioned on business survival have to be considered in (10) above.

The importance of business failure in equity valuation models is seldom explicitly recognized in standard textbooks in accounting and corporate finance. The valuation of equity investments is commonly handled in stylized PVED models, at best specified as:[7]

$$V(EQ)_0^{TXTBK} = \sum_{\tau=1}^{T} \frac{E_{(0)}(D\tilde{I}V_{EQ,\tau})}{(1+\rho_{EQ})^{\tau}} + \frac{E_{(0)}\left(Sp\tilde{(}EQ)_T\right)}{(1+\rho_{EQ})^{T}} \tag{11}$$

In order for $V(EQ)_0^{TXTBK}$ to be equal to $V(EQ)'_0$ in (10), it is easily recognized that the numerators on the RHS of (11) have to incorporate probabilities of business failure as follows:

$$E_{(0)}(D\tilde{I}V_{EQ,\tau}) = \left[\prod_{s=1}^{\tau}(1-p^{*}_{fail,s})\right] E_{(0)}\left(D\tilde{I}V_{EQ,\tau} \middle| surv(\tau)\right) \tag{12.a}$$

for $\tau = 1, 2, \ldots T$, and

$$E_{(0)}\left(Sp\tilde{(}EQ)_T\right) = \left[\prod_{s=1}^{T}(1-p^{*}_{fail,s})\right] E_{(0)}\left(Sp\tilde{(}EQ)_T \middle| surv(T)\right) \tag{12.b}$$

It is important to note that the textbook valuation formula is not erroneous per se − if business failure is appropriately recognized in the assessment of expected values of future cash flows, $V(EQ)_0^{TXTBK}$ will be equal to $V(EQ)'_0$.

A notorious special case of the textbook formula is commonly referred to as "Gordon's constant growth model" (Gordon, 1962):

$$V(EQ)_0^{GORDON} = \frac{E_{(0)}(D\tilde{I}V_{EQ,1})}{\rho_{EQ} - g} \tag{13}$$

where: g = relative (yearly) growth of expected values of future company dividends.

[7] Cf., for example, Brealey & Myers (2003), chapter 4; Damodaran (1994), chapter 6; or Penman (2004), chapter 3.

"Gordon's constant growth model" is easily derived from (11) assuming that:

- $E_{(0)}(D\tilde{I}V_{EQ,\tau+1}) = E_{(0)}(D\tilde{I}V_{EQ,\tau}) \cdot (1+g)$, i.e. there is a constant relative growth of expected values of future dividends.
- $g < \rho_{EQ}$, i.e. the relative growth is less than the discounting rate of return.
- $T \rightarrow \infty$ and $E_{(0)}\left(S\tilde{p}(EQ)_T\right)/(1+\rho_{EQ})^T \rightarrow 0$, i.e. there is an infinite holding period and the present value of the expected selling price of the equity investment is close to zero.

Explicitly recognizing the risk of business failure in the first of the above assumptions, we have:

$$E_{(0)}(D\tilde{I}V_{EQ,\tau}) \;=\; \left[\prod_{s=1}^{\tau}(1-p^{*}_{fail,\tau})\right] \cdot E_{(0)}\left(D\tilde{I}V_{EQ,\tau}\middle|surv(\tau)\right) \qquad (14.a)$$

$$E_{(0)}(D\tilde{I}V_{EQ,\tau+1}) \;=\; E_{(0)}(D\tilde{I}V_{EQ,\tau})(1+g^{\circ}_{\tau+1})(1-p^{*}_{fail,\tau+1}) \qquad (14.b)$$

where: g°_{τ} = relative (yearly) growth of the expected value of company dividends in period τ, conditioned on business survival in period τ.

Hence, "Gordon's constant growth model" is consistent with the valuation model in (10) if the following two conditions are fulfilled:

$$E_{(0)}(D\tilde{I}V_{EQ,1}) \;=\; (1-p^{*}_{fail,1}) \cdot E_{(0)}\left(D\tilde{I}V_{EQ,1}\middle|surv(\tau=1)\right) \qquad (C.5)$$

$$g \;=\; g^{\circ}_{\tau} \;-\; p^{*}_{fail,\tau}(1+g^{\circ}_{\tau}) \qquad (C.6)$$

Condition (C.5) simply means that $E_{(0)}(D\tilde{I}V_{EQ,1})$ in the numerator of (13) should be equal to the expected value of the dividend payment at the end of the first year conditioned on company survival, multiplied by the probability of survival at this point in time. (C.6) implies that the unconditioned growth parameter g is a specific function of g°_{τ} and the probability of business failure $p^*_{fail,\tau}$.[8] If the latter two variables are constant over time (typically a reasonable assumption in a "steady-state" setting) and $g^\circ_\tau > -1.00$, failing to recognize a non-zero probability of failure leads to an overstatement of the numerator and an understatement of the denominator in (13). Unequivocally, a mistake of this kind in "Gordon's constant growth model" leads to an overstatement of the value of an equity investment.

Table 1: Numerical example: $V(EQ)_0$ based on "Gordon's constant growth model", assuming that $E_0(D\tilde{I}V_{EQ,1} | surv(\tau = 1)) = 10$, $\rho_{EQ} = 10\%$, and that g°_τ and $p^*_{fail,\tau}$ are constant over time.

	Probability of failure (p^*_{fail}) =					
Growth (g°_τ)	0	0,01	0,02	0,03	0,04	0,05
0	100,0	90,0	81,7	74,6	68,6	63,3
1%	111,1	98,9	88,9	80,6	73,6	67,6
2%	125,0	109,8	97,6	87,7	79,5	72,5
3%	142,9	123,3	108,2	96,1	86,3	78,2
4%	166,7	140,6	121,3	106,4	94,5	84,4
5%	200,0	163,6	138,0	119,0	104,3	92,7
6%	250,0	195,7	160,1	135,1	116,5	102,2
7%	333,3	243,2	190,7	156,2	131,9	113,8
8%	500,0	321,4	235,6	185,1	151,9	128,4

A numerical example illustrating the importance of not neglecting the risk of business failure in "Gordon's constant growth model" is provided in Table 1. The example hinges on an expected value of the company dividend at $\tau = 1$ being equal to 10 conditioned on survival and a required expected return on owners′ equity equal to 10%. Both the growth parameter conditioned on business survival g°_τ and the probability of failure $p^*_{fail,\tau}$ are assumed to be constant over time. The table shows that if, for example, $g^\circ_\tau = 5\%$ and $p^*_{fail} = 0$ − consistent with a "bullet-proof" company, but also the case of erroneously neglecting some positive probability of failure − the value of owners′ equity $V(EQ)_0$ would be equal to 200,0 in "Gordon's constant growth model". Given the same assessment of growth, but with a small positive probability of failure $p^*_{fail} = 0{,}02$, the value of owners′ equity drops by

[8] Note that $g^\circ_\tau - p^*_{fail,\tau}(1 + g^\circ_\tau)$ in (C.6) is equivalent to $(1 + g^\circ_\tau)(1 - p^*_{fail,\tau}) - 1$, i.e. the unconditioned relative growth of expected values of future dividends.

over 40% to 138,0. This is clearly a striking result, indicative of the danger of ignoring the risk of business failure in this model.

Table 1 also shows that the sensitivity of $V(EQ)_0$ to variations in p^*_{fail} increases/decreases when the growth parameter g°_τ increases/decreases. This follows as the denominator in "Gordon's constant growth model" is affected more strongly (in relative terms) than the numerator the higher the growth parameter is, and vice versa. The importance of not ignoring the risk of business failure is consequently to be underscored for companies with strong expected future growth conditioned on survival.

3.2. Adjusting the discounting rate of return to incorporate the risk of business failure in PVED valuation

As in the bond valuation problem, an interesting issue is whether the risk of business failure can be accommodated through some adjustment of the discounting rate of return in a PVED model. A simple model of this kind can be specified as:

$$V(EQ)^x_0 \quad = \quad \sum_{\tau=1}^{T} \frac{E_{(0)}\left(D\tilde{I}V_{EQ,\tau} \middle| surv(\tau)\right)}{(1+\rho^x_{EQ})^\tau} \quad + \quad \frac{E_{(0)}\left(Sp\widetilde{(EQ)}_T \middle| surv(T)\right)}{(1+\rho^x_{EQ})^T} \tag{15}$$

where: ρ^x_{EQ} = failure-adjusted required rate of return for an equity investment (at time t = 0).

An equity valuation model specified as in (15) is obviously tempting from a methodological point of view. Forecasts of company dividends and the selling price of owners´ equity conditioned on business survival are consistent with the idea of the company being a "going concern", an assumption that appears to be implied in much work by professional financial analysts. Replacing ρ_{EQ} with the failure-adjusted rate of return ρ^x_{EQ} would presumably incorporate the valuation impact of the risk of business failure in future years.

The following two conditions are sufficient for $V(EQ)^x_0$ in (15) to be equivalent to $V(EQ)'_0$ in (10):

$$\frac{\left[\prod_{s=1}^{\tau}(1-p^{*}_{fail,s})\right]\left[E_{(0)}\left(D\tilde{I}V_{EQ,\tau}\middle|surv(\tau)\right)\right]}{(1+\rho_{EQ})^{\tau}} = \frac{E_{(0)}\left(D\tilde{I}V_{EQ,\tau}\middle|surv(\tau)\right)}{(1+\rho^{x}_{EQ})^{\tau}} \qquad \text{(C.7)}$$

for $\tau = 1, 2, \ldots T$, and

$$\frac{\left[\prod_{s=1}^{T}(1-p^{*}_{fail,s})\right]\left[E_{(0)}\left(Sp\tilde{(}EQ)_{T}\middle|surv(T)\right)\right]}{(1+\rho_{EQ})^{T}} = \frac{E_{(0)}\left(Sp\tilde{(}EQ)_{T}\middle|surv(T)\right)}{(1+\rho^{x}_{EQ})^{T}} \qquad \text{(C.8)}$$

As both ρ_{EQ} and ρ^{x}_{EQ} have been assumed to be constant over time, condition (C.7) implies that $p^{*}_{fail,s}$ also has to be a constant. Restricting $p^{*}_{fail,s} = p^{*}_{fail}$ to be less than 1,00, we get:

$$\rho^{x}_{EQ} = \frac{\rho_{EQ} + p^{*}_{fail}}{1 - p^{*}_{fail}} \qquad \text{(C.7')}$$

Given that $p^{*}_{fail,s}$ is constant over time and ρ^{x}_{EQ} being assessed as in (C.7′), it is easily verified that condition (C.8) also holds. Hence (C.7′) is a solution for the adjusted discounting return ρ^{x}_{EQ} that – given the relationship between equity cash flows and the financial health of the company as specified previously (cf. section 3.1) and a constant risk of business failure - appropriately will account for the risk of business failure in a PVED model. Also, note that this in principle is equivalent to the result that was obtained for the bond valuation model in section 2.1 above.

3.3. The risk of business failure in the residual income valuation (RIV) model

Given that the "clean surplus relation of accounting" (i.e. that net income, dividends and new issues of equity account for all changes in the book value of owners′ equity) is expected to hold in future financial statements and that market values are used in the accounting for dividends and any new issues, a well-known reformulation of the PVED model is the residual income valuation (RIV) model.[9] Setting the valuation point in time

[9] Cf. Ohlson (1995), or early references such as Preinreich (1938) and Edward & Bell (1961). A straightforward tutorial on residual income valuation is provided in Skogsvik (2002), pp. 1− 14.

$t = 0$, suppressing the firm index j and simplifying the notation in the same manner as previously, the RIV model can be expressed as:

$$V(EQ)_0^{RIV} \;=\; B_0 \;+\; \sum_{\tau=1}^{T} \frac{E_{(0)}(\tilde{I}_\tau - \rho_{EQ} \cdot \tilde{B}_{\tau-1})}{(1+\rho_{EQ})^\tau} \;+\; \frac{E_{(0)}(Sp\tilde{(}EQ)_T - \tilde{B}_T)}{(1+\rho_{EQ})^T} \tag{15}$$

where: I_τ = accounting net income accrued in period τ.

B_τ = book value of owners´ equity, ex dividend and including any new issue of owners´ equity at time τ.

The value of owners´ equity in (15) is decomposed into three terms − the book value of owners´ equity at the valuation point in time (B_0), the present value of future residual income ($\tilde{I}_\tau - \rho_{EQ} \cdot \tilde{B}_{\tau-1}$) and the present value of the residual value of owners´ equity at the horizon point in time ($Sp\tilde{(}EQ)_T - \tilde{B}_T$). In order to be consistent with the assumptions for the PVED model (cf. subsection 3.1), future residual income and the residual value of owners´ equity are postulated to depend on the financial health of the company in the following way:

- **Business survival in year τ:**
  A (positive or negative) value of residual income is realized for the year ($1 \le \tau \le T-1$), or values of residual income and the residual value of owners´ equity are realized ($\tau = T$).
- **Business failure occurs in year τ:**
  The net income for the year is equal to the loss of the opening book value of owners´ equity (i.e. $\tilde{B}_{\tau-1} + \tilde{I}_\tau = 0$) and all future values of net income, residual income, and the residual value of owners´ equity are equal to zero.

If business failure occurs in some year τ, the residual income is hence $\tilde{I}_\tau - \rho_{EQ} \cdot \tilde{B}_{\tau-1} = -(1+\rho_{EQ}) \cdot \tilde{B}_{\tau-1}$. In order for the RIV model to be consistent with the PVED model − where an equity investor did not incur any (positive or negative) cash flow if business failure occurred − this “failure loss recognition principle” is crucial. It can be viewed as a complementary assumption to the “clean surplus relation of accounting” and the “mark-to-market accounting for equity transactions”, stating that the accounting net income has to be equal to the loss of the opening book value of owners´ equity if business failure occurs.[10]

The unconditioned expected values of residual income and the residual value of owners´ equity in (15) can now be expressed as follows:

$$E_{(0)}(\tilde{I}_\tau - \rho_{EQ} \cdot \tilde{B}_{\tau-1}) \;=\; E_{(0)}(R\tilde{I}_\tau) \;=$$
$$= (1 - P_{surv}^{(\tau-1)}) \cdot 0 \;+\; P_{surv}^{(\tau-1)} \cdot (1 - p_{fail,\tau}^{*}) \cdot E_{(0)}\left(R\tilde{I}_\tau \middle| surv(\tau)\right) \;+$$
$$+\; P_{surv}^{(\tau-1)} \cdot p_{fail,\tau}^{*} \cdot E_{(0)}\left[-(1+\rho_{EQ}) \cdot \tilde{B}_{\tau-1} \middle| surv(\tau)\right] \qquad (16.a)$$

for $\tau = 1, 2, \ldots T$, and

$$E_{(0)}\left(Sp\tilde{(}EQ)_T - \tilde{B}_T\right) \;=\; E_{(0)}(R\tilde{V}_T) \;=$$
$$= (1 - P_{surv}^{(T-1)}) \cdot 0 \;+\; P_{surv}^{(T-1)} \cdot (1 - p_{fail,T}^{*}) \cdot E_{(0)}\left(R\tilde{V}_T \middle| surv(T)\right) \;+$$
$$+\; P_{surv}^{(T-1)} \cdot p_{fail,T}^{*} \cdot 0 \qquad (16.b)$$

If the expected values in (16.a) and (16.b) are incorporated in (15) and the "accumulated" probabilities of business failure $P_{surv}^{(\tau)}$ are rewritten in accordance with (3), a new specification of the RIV model is obtained:

$$V(EQ)_0^{RIV(+)} \;=\; B_0 \;+\; \sum_{\tau=1}^{T} \frac{\left[\prod_{s=1}^{\tau}(1 - p_{fail,s}^{*})\right] E_{(0)}\left(R\tilde{I}_\tau \middle| surv(\tau)\right)}{(1+\rho_{EQ})^{\tau}} \;+$$
$$+\; \sum_{\tau=1}^{T} \frac{\left[\prod_{s=1}^{\tau-1}(1 - p_{fail,s}^{*})\right] \cdot p_{fail,\tau}^{*} \cdot E_{(0)}\left[-(1+\rho_{EQ}) \cdot \tilde{B}_{\tau-1} \middle| surv(\tau-1)\right]}{(1+\rho_{EQ})^{\tau}} \;+$$
$$+\; \frac{\left[\prod_{s=1}^{T}(1 - p_{fail,s}^{*})\right] \cdot E_{(0)}\left(R\tilde{V}_T \middle| surv(T)\right)}{(1+\rho_{EQ})^{T}} \qquad (17)$$

[10] Note that the failure loss recognition principle hinges on the assumption that there is no positive liquidation value of owners´ equity if failure occurs, and that equity investors are only exposed to a limited liability bankruptcy risk. If there would be some final non-zero cash flow $\tilde{L}_\tau$ for equity investors associated with business failure, the accounting net income for this year would be equal to $(-\tilde{B}_\tau + \tilde{L}_\tau)$, and consequently $R\tilde{I}_\tau = -(1+\rho_{EQ}) \cdot \tilde{B}_{\tau-1} + \tilde{L}_\tau = (\tilde{L}_\tau - \tilde{B}_{\tau-1}) - \rho_{EQ} \cdot \tilde{B}_{\tau-1}$.

The value of owners´ equity is now the sum of four terms. As in the “standard” formula (15), the first term is the book value of owners´ equity. The second and the fourth term constitute the present value of expected future residual income and the future residual value of owners´ equity conditioned on company survival. The somewhat awkward third term is the present value of negative residual income corresponding to business failure in future years, weighted by the appropriate “accumulated” probabilities of failure.

The RIV model in (17) can be simplified if additional assumptions are specified. As a simple benchmark case, assume that:

- The probability of failure is constant over time, i.e. $p^*_{fail,\tau} = p^*_{fail}$ for $\tau = 1,2,...T$.
- The expected value of residual income conditioned on business survival is constant over time, i.e. $E_{(0)}\left(R\tilde{I}_\tau \middle| surv(\tau)\right) = \overline{RI}_{surv}$ for $\tau = 1,2,...T$. (The assumption is consistent with a full dividend payout policy, in turn implying that both $E_{(0)}\left(\tilde{I}_\tau \middle| surv(\tau)\right)$ and $E_{(0)}\left(\tilde{B}_\tau \middle| surv(\tau)\right)$ are constant over time.)
- The horizon point in time $T \to \infty$ and the present value of the residual value of owners´ equity conditioned on business survival is close to zero.

With the above assumptions, (17) can be rewritten in the following way:

$$V(EQ)_0^{RIV(+simple)} = B_0 + \sum_{\tau=1}^{\infty} \frac{(1-p^*_{fail})^\tau \cdot \overline{RI}_{surv}}{(1+\rho_{EQ})^\tau} +$$

$$+ \sum_{\tau=1}^{\infty} \frac{(1-p^*_{fail})^{\tau-1} \cdot p^*_{fail} \cdot \left[-(1+\rho_{EQ}) \cdot B_0\right]}{(1+\rho_{EQ})^\tau} =$$

$$= B_0 + \frac{(1-p^*_{fail}) \cdot \overline{RI}_{surv}}{\rho_{EQ} + p^*_{fail}} - \frac{p^*_{fail} \cdot (1+\rho_{EQ}) \cdot B_0}{\rho_{EQ} + p^*_{fail}}$$

$$= \left[1 - \frac{p^*_{fail}(1+\rho_{EQ})}{\rho_{EQ} + p^*_{fail}}\right] \cdot B_0 + \left[\frac{1-p^*_{fail}}{\rho_{EQ} + p^*_{fail}}\right] \cdot \overline{RI}_{surv} \qquad (18)$$

Rewriting (18) slightly, it is interesting to note that both the coefficient of the book value of owners´ equity (B_0) and the coefficient of the expected value of residual income in future years ($\overline{RI}_{surv}$) are monotonically decreasing in the failure probability p^*_{fail}. Only for the case of p^*_{fail} = 0 the coefficients of B_0 and $\overline{RI}_{surv}$ would be equal to their highest values of 1,0 and $1/\rho_{EQ}$, respectively.

As the expected value of the residual income in (18) is equal to $(\bar{I}_{surv} - \rho_{EQ} \cdot B_0)$, the simplified model can also be rewritten as:

$$V(EQ)_0^{RIV(+simple)} \quad = \quad \frac{\bar{I}_{surv}(1-p^*_{fail})}{\rho_{EQ} + p^*_{fail}} \qquad (18')$$

The numerator on the RHS of (18´) is equal to the expected value of the company dividend at the end of the first year, and the denominator consists of the discounting rate of return minus the unconditioned expected relative growth of future dividends $(-p^*_{fail})$. Not surprisingly, the RIV model in (18´) is hence recognized as being equivalent to "Gordon´s constant growth model" in a setting where $E_{(0)}(D\tilde{I}V_1) = (1-p^*_{fail,1}) \cdot E_{(0)}\left(D\tilde{I}V_1 \middle| surv(1)\right) = (1-p^*_{fail}) \cdot \bar{I}_{surv}$ and $g = g^\circ_\tau - p^*_{fail,\tau}(1+g^\circ_\tau) = -p^*_{fail}$ (cf. conditions (C.5) and (C.6) previously.)

4. Summary and concluding remarks

Bankruptcy prediction models have been around for decades, presumably providing financial decision makers with important information about business companies. However, the available literature in accounting and corporate finance is surprisingly mute on the issue of how to incorporate the risk of business failure in investment valuation modelling. Casual observations of professional financial analysts and equity investment managers also indicate a feeling of awkwardness with regard to this risk, and one cannot preclude that the risk is treated somewhat ad hoc in much professional work. With the strongly "bearish" development of most major stock markets in the beginning of the 21st century in recent memory, such ignorance can clearly be quite costly to equity investors.

The purpose of the paper has been to investigate how the probability of business failure can be accommodated in discounted cash flow (DCF) valuation of company bonds and owners´ equity. Only conceptual and theoretical issues have been addressed in this context. In addition to an ordinary PVED model for the valuation of owners´ equity, a residual income valuation (RIV) model has also been analysed.

Cash flow valuation modelling hinges on the idea of discounting expected values of future cash flows with appropriately risk-adjusted required rates of return. As a first step in the paper, it has been fairly straightforward to incorporate probabilities of business failure in the assessment of such expected values. Both in the DCF bond and equity valuation models, the probability of business failure in some future period (given survival at the end of the previous period) is valuation relevant. In principle, the usefulness of recognizing the risk of business failure in such models will depend on the cash flow probability distributions conditioned on "business survival" versus "business failure". If at least one of the conditional probability distributions is sufficiently clear-cut, the risk of business failure might encompass a large part of the risk associated with future cash flows. In this respect it was noted that the conditional probability distribution associated with the survival state presumably is more clear-cut in the bond valuation model, while the conditional probability distribution associated with the failure state presumably is more clear-cut in the equity valuation model.

The latter observation opens up for the possibility of tailor-making the concept of "business failure" − depending on the DCF valuation model at hand, the operationalization of "business failure" versus "business survival" could be chosen in order to make one or both of the conditional probability distributions as clear-cut as possible. In this sense it might, for example, be helpful to use a more fatal and/or legal concept of business failure (for example bankruptcy) in a bond valuation context, but a failure state characterized by substantial and persistent losses ("financial distress") in an equity valuation context.

Limiting the analysis to valuation models where the required rate of return is constant over time, the risk of business failure can be accommodated through a rather simple adjustment of the discounting rate under certain conditions. In the bond valuation model, the contractual ("promised") payments can be discounted with such an adjusted rate of return if the probability of business failure is constant over time and there is no collateral value − or other non-zero expected terminal value if failure occurs − associated with the bond. In the equity valuation model, expected values of future cash flows conditioned on company survival can be discounted with an adjusted rate of return if the probability of failure is constant over time and the expected terminal value of owners´ equity is zero if failure occurs.

In order to be able to incorporate the risk of business failure in the RIV model, a complementary accounting principle was required. If business failure occurs, this “failure loss recognition principle” means that the accounting net income for the period is equal to the loss of the opening book value of owners´ equity. Hence a new term (with a negative sign) in the RIV model could be specified, accommodating the expected value of negative residual income associated with business failure in future years.

References

Altman, E. I., 1968, “Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy”, *The Journal of Finance*, No. 4, pp. 589−609.

Altman, E. I., Bradey, B., Resti, A. & Sironi, A., 2005, ”The Link between Default and Recovery Rates: Theory, Empirical Evidence, and Implications”, *Journal of Business*, Vol. 78, No. 6, pp. 2203 − 2227.

Beaver, W. H., 1966, “Financial Ratios as Predictors of Failure”, *Journal of Accounting Research*, Supplement to Volume 4, pp. 71−111.

Black, F. & Cox, J., 1976, “Valuing Corporate Securities: Some Effects of Bond Indenture Provisions”, *The Journal of Finance*, Vol. 31, pp. 351−367.

Brealey, R. A. & Myers, S. C., 2003, *Principles of Corporate Finance* (seventh edition, McGraw-Hill Book Company).

Copeland, T., Koller, T. & Murrin, J., 2000, *Valuation – Measuring and Managing the Value of Companies* (third edition, John Wiley & Sons, Inc.).

Damodaran, A., 1994, *Damodaran on Valuation. Security Analysis for Investment and Corporate Finance* (John Wiley & Sons, Inc.).

Damodaran, A., “Dealing with Distress in Valuation”, 2002, working paper (Stern School of Business).

Duffie, D. & Singleton, K. J., 1999, “Modeling Term Structures of Defaultable Bonds”, *The Review of Financial Studies* Vol. 12, No. 4, pp. 687−720.

Edwards, E. O. & Bell, P. W., 1961, *The Theory and Measurement of Business Income* (Berkeley and Los Angeles, University of California Press).

Fisher, I., 1906, *The nature of Capital and Income*, New York.

Gordon, M. J., 1962, *The Investment, Financing and Valuation of the Corporation* (Richard D. Irwin, Inc.).

Johansson, S-E., 1973, *Ekonomiska bedömningar av företag och investeringar*, Stockholm (Almqvist och Wiksell Förlag).

Leland, H., 1994, ”Corporate Debt Value, Bond Covenants, and Optimal Capital Structure”, *The Journal of Finance*; Vol. 49, pp. 1213−1252.

Leland, H. & Toft, K. B., 1996, “Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads”, *The Journal of Finance*, Vol. 51, pp. 987−1019.

Longstaff, F. & Schwartz, E., 1995, “A Simple Approach to Valuing Risky Fixed and Floating Rate Debt”, *The Journal of Finance*, Vol. 50, pp. 789−819.

Merton, R. C., 1974, ”On the Pricing of Corporate Debt: The Risk structure of Interest Rates”, *The Journal of Finance*, Vol. 29, No. 2, pp. 449−470.

Merwin, C., 1942, *Financing Small Corporations* (National Bureau of Economic Research, New York).

Moody`s Investor Service, 2005, *Special Comment − Default and Recovery Rates of Corporate Bond Issuers, 1920 − 2004* (Moody´s Investor Service, Global credit Research, January 2005).

Ohlson, J. A., 1980, “Financial Ratios and the Probabilistic Prediction of Bankruptcy”, *Journal of Accounting Research* 18 (1), pp. 109−131.

Ohlson, J. A., 1995, “Earnings, Book Values and Dividends in Equity Valuation”, *Contemporary Accounting Research* Vol. 11, pp. 661−687.

Penman, S., 2004, *Financial Statement Analysis and Security Valuation* (second edition, McGraw−Hill/Irwin).

Preinreich, G. A. D., 1938, “Annual Survey of Economic Theory: The Theory of Depreciation”, *Econometrica* , July, pp. 219–241.

Skogsvik, K., 1990, “Current Cost Accounting Ratios as Predictors of Business Failure: The Swedish Case”, *Journal of Business Finance & Accounting* 17 (1), Spring, pp. 137–160.

Skogsvik, K., 2002, “A Tutorial on Residual Income Valuation and Value Added Valuation”, SSE/EFI Working Paper in Business Administration (SWOBA).

Smith, R. F. & Winakor, A. H., 1935, *Changes in the Financial Structure of Unsuccessful Corporations* (University of Illinois, Bureau of Business Research).

Urig-Homburg, M., 2002, “Valuation of Defaultable Claims − A Survey”, *Schmalenbach Business Review* Vol. 54, January, pp. 24 − 57.

Vinso, J. D., “A Determination of the Risk of Ruin”, 1979, *Journal of Financial and Quantitative Analysis* Volume XIV, No. 1, pp. 77−100.

Wilcox, J. W., 1971, "A Gambler's Ruin Prediction of Business Failure Using Accounting Data", *Sloan Management Review* (3), pp. 1−10.

Zavgren, C. V., 1985, "Assessing the Vulnerability to Failure of American Industrial Firms: A Logistic Analysis", *Journal of Business Finance & Accounting*, Spring, pp. 19−45.