A Note on Budget Balance under Interim Participation Constraints: The Case of Independent Types[*]

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Abstract

We provide a simple proof of the equivalence between ex ante and ex post budget balance constraints in Bayesian mechanism design with independent types when participation decisions are made at the interim stage. The result is given an interpretation in terms of efficient allocation of risk.

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1 Introduction

In many applications of mechanism design, a key aspect of the implementation problem is to ensure that the budget is balanced. For example, when mechanisms are designed for the provision of public goods, the contributions need to cover the costs of producing the good. In bilateral trade, the buyer's payment needs to equal the seller's revenue. In the dissolution of partnerships, the transfers must add up to zero.

If there is a risk neutral lender with sufficient liquidity available, who does not otherwise participate in the mechanism, budget balance is most naturally imposed as an ex ante constraint. If resources are balanced in expectation, the outside lender is willing and able to offer an insurance contract to the mechanism designer that guarantees a zero deficit in every state of the world. However, in some applications, such as the bilateral trade example, the assumption of a risk neutral outsider seems to run counter to the spirit of the model. In such cases it appears more natural to require that the budget is balanced ex post.

Ex post budget balance tends to be analytically less tractable than ex ante budget balance. Fortunately, in many settings one can show that the restrictions are equivalent in the sense that an ex ante balanced mechanism can be transformed into an ex post balanced mechanism without upsetting the incentive constraints, and without changing the allocation rule. That is, payments are rearranged, but "real" decisions are unaffected. These sort of results appear in two different versions. In one version participation constraints are either absent, or they are imposed ex ante, so that agents must choose whether to participate behind the veil of ignorance. In another version, participation constraints are interim, so that agents can still opt out after learning their types.

The literature that does not impose interim participation constraints goes back to the seminal contributions by d'Aspremont and Gérard-Varet (1979) and Arrow (1979). A concise summary of this work can be found in d'Aspremont, Crémer and Gérard-Varet (2004). This literature provides a more or less complete characterization for when it is possible to implement an *efficient* decision rule and when it possible to implement *any* decision rule, while maintaining a balanced budget in any state of the world.

For the case of interim participation constraints a result that guarantees the existence of an equivalent ex post budget balanced mechanism for any ex ante budget balanced mechanism has

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been obtained in some special contexts, such as bilateral trade (Myerson and Satterthwaite's, 1983, Theorem 1), resolution of partnerships (Cramton, Gibbons and Klemperer's, 1987, Lemma 4) and public goods (Mailath and Postlewaite, 1990, Theorem 1). All these special results concern the case of independent types. For the case with correlated types, Fudenberg, Levine and Maskin (1995) and Kosenok and Severinov (2002) have provided a general result that holds under generic conditions on the type distribution.

This focus of this paper is on the case of independent types. The purpose of the paper is twofold. Firstly, we point out that independence of types guarantees equivalence between ex ante and ex post budget balance with interim participation constraints not just in the special cases covered by the literature so far, but in general. Secondly, our proof provides a simple interpretation of this result in terms of efficient, incentive compatible, and interim individually rational allocation of risk. That is, we begin the analysis by asking when a risk averse agent and a set of partially informed risk neutral agents can agree on an efficient risk sharing agreement. Whenever this is possible, it follows more or less immediately that, regardless of the details of the implementation problem, ex ante and ex post budget balance are equivalent.

To obtain some intuition for our result, note first that we assume, as most of the literature does, that agents have additively separable preferences, are risk neutral in transfers, and have no liquidity constraints. Thus, in principle, the agents are able and willing to provide insurance to the mechanism designer just as an outside lender would be. However, the insurance scheme is agreed upon after agents observe their private information, implying that it must give all agents' incentives to participate and to report their private observations truthfully. These problems can be resolved if agents' private signals are independent. The scheme used in the proof designates one of the agents, the "primary insurer", to provide insurance for the budget deficit in as far as it is not predictable on the basis of this agent's own signal. Some other agent, the "secondary insurer", provides insurance for the part of the budget deficit that can be predicted using the first agent's signal. The role of the independence assumption is to ensure that truth-telling is incentive compatible for the primary insurer and that the participation constraints hold for the secondary insurer.

We also show that the same idea can be used to obtain a weaker result in the case when there exist two types with *conditionally* independent signals, a condition that first appeared in Crémer and Riordan (1985). Then, we can guarantee that participation constraints are satisfied for all but

a single agent.[1]

Our paper is most closely related to d'Aspremont, Crémer and Gérard-Varet (2004). While they focus on first best efficient mechanisms and do not consider the implications of interim participation constraints, the insurance scheme that we consider is the same as a construction used in one of their proofs.[2] Our contribution may thus be viewed as clarifying that, in a slightly more general setting than theirs, this construction preserves interim incentives to participate, and to provide an explanation based on elementary insurance theory.

The remainder of this paper is organized as follows. In Section 2 we consider an insurance problem with privately informed insurance providers. We then use the analysis of the insurance problem to prove our main result in Section 3 for the case that agents' signals are independent. In Section 4 we prove the extension with the weakened independence assumption. Section 5 contains some further discussion of the literature.

2   The Insurance Problem

Consider a risk-averse agent facing some uncertain expenditure (or income) that depends on N random variables $\tilde{s}_i$ where $i \in I = \{1, 2, ..., N\}$. For $i \in I$ denote by $s_i \in S_i$ a generic realization of the i-th random variable, and define $s \equiv (s_1, s_2, ..., s_N)$ and $S \equiv \times_{i=1}^{N} S_i$. The expenditure that the risk averse agent faces is a function $x : S \to R$.

From textbook economic theory we know that if there exists a risk neutral agent who is not privately informed about s, this agent is willing to fully insure the risk averse agent at an actuarially fair rate. This would generate an ex ante Pareto efficient allocation. No such *uninformed* risk neutral agent is available in our setup. Instead, there are N *partially informed* risk neutral agents, where for each positive integer $i \leq N$, the risk neutral agent i knows the realization s_i at the time at which the risk averse agent seeks to purchase insurance.[3]

Definition 1 *An insurance contract is a pair* $\langle p, m \rangle$, *where* $p \in R^N$ *and* $m : S \to R^N$.

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[1] This result is a slight generalization of the main result in Crémer and Riordan (1985), where the added generality is that it applies to arbitrary allocation rules rather than the surplus maximizing rule.

[2] See part (i) of the proof of Theorem 2 d'Aspremont, Crémer and Gérard-Varet (2004).

[3] It is without loss of generality to assume away any "residual uncertainty" in addition to s, since such uncertainty can be insured away as a first step of the analysis.

We write $p = (p_1, p_2, ..., p_N)$ for what can be interpreted as up-front payments and adopt the convention that a positive p_i is a transfer to agent i. Similarly, we write $m(s) = (m_1(s), ..., m_N(s))$ and treat a positive value $m_i(s)$ as a transfer from i to the agent that seeks insurance.

An insurance contract provides *full insurance* if the payments by the N agents cover the uncertain expenditure. It is *actuarially fair* if the up-front payments equal the expected expenditure. We consider two incentive constraints. The first is *non-manipulability*. This requires that truthful reporting of the signals by the N agents is a Bayesian equilibrium. The second condition is *interim individual rationality*. We imagine that each agent i observes her signal s_i, but not that of any other agent, *before* signing the insurance contract, and that agents cannot be forced to sign the contract. All agents must therefore earn a weakly positive expected profit in order to be willing to accept the contract.

To formalize the four properties listed in the previous paragraph, we will assume that all relevant expectations are well-defined and let $E[z|y]$ denote the expected value of z conditional on y.

Definition 2 *An insurance contract* $\langle p, m \rangle$

- *provides* full insurance *if*

$$\sum_{i=1}^{N} m_i(s) = x(s)$$

  for every $s \in S$;

- *is* actuarially fair *if*

$$\sum_{i=1}^{N} p_i = E\left[\sum_{i=1}^{N} m_i(\tilde{s})\right];$$

- *is* non-manipulable *if*

$$E[m_i(s_i, \tilde{s}_{-i})|s_i] \geq E[m_i(\hat{s}_i, \tilde{s}_{-i})|s_i]$$

  for each $i \in I$ *and all* $s_i, \hat{s}_i \in S_i$;

- *is* interim individually rational *if*

$$p_i \geq E[m_i(\tilde{s})|s_i]$$

  for each $i \in I$ *and* $s_i \in S_i$.

If the set of signals S_i is discrete, non-manipulability is a consequence of interim individually rationality and actuarial fairness.[4] In general, individual rationality and actuarial fairness guarantee that non-manipulability is satisfied for almost all pairs $(s_i, \hat{s}_i)$, but may fail on a set of measure zero. Since non-manipulability fails if just a single type earns a higher payoff than all others, we need the definition in general.

Proposition 1 relies on the following condition.

(IND)   *The random variables $\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_N$ are independent.*

We are now ready to state the result:

Proposition 1 *If (IND) holds and $N \geq 2$, then there exist insurance contracts that are interim individually rational, provide full insurance, are actuarially fair, and are non-manipulable. Moreover, for any $i, j \in I$ with $i \neq j$, there is some such insurance contracts that has the property that agent j's payment does not depend on his type, and that agents other than agents i and j make zero payments in all states.*

Proof: Take as the starting point an insurance scheme where $p_i = \mathrm{E}[x(\tilde{s})]$, $m_i(s) = x(s)$, and where $p_k = m_k(s) = 0$ for all $k \neq i$ and $s \in S$. That is, agent i provides full insurance at an actuarially fair rate and all other agents are inactive. The problem with this scheme is that it will in general fail individually rationality and non-manipulability, since $x(s)$ in general varies in s_i. But, since s_i is independent from s_{-i}, the obvious solution to this problem is to pick some other agent, agent j in the statement of Proposition 1, to insure agent i against the variability in s_i.

To make this idea formal, we consider the following decomposition of $x(s)$.

$$x(s) \quad = \quad \underbrace{\mathrm{E}[x(\tilde{s})]}_{\text{Unconditional expected value}} \quad +$$

$$\underbrace{\mathrm{E}[x(s_i, \tilde{s}_{-i}) \mid s_i] - \mathrm{E}[x(\tilde{s})]}_{\text{Deviation of conditional expected value from unconditional expected value}} \quad +$$

$$\underbrace{x(s) - \mathrm{E}[x(s_i, \tilde{s}_{-i}) \mid s_i]}_{\text{Deviation of realization from conditional expected value}}$$

The idea of the insurance contract constructed below is that one agent, whom we label agent 1, pays for the first and third term in the above decomposition. The second term is paid by someone

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[4]This is seen by using the law of iterated expectations in the definition of actuarial fairness.

else, whom we label as agent 2. These two agents are compensated by appropriate up-front lump sum payments, and all other agents pay nothing. Formally, let $\langle p^*, m^* \rangle$ be given by,

$$
\begin{aligned}
p_1^* &= \mathrm{E}[x(\tilde{s})] \\
m_1^*(s) &= \mathrm{E}[x(\tilde{s})] + x(s) - \mathrm{E}[x(s_1, \tilde{s}_{-1})|s_1] \\
p_2^* &= 0 \\
m_2^*(s) &= \mathrm{E}[x(s_1, \tilde{s}_{-1})|s_1] - \mathrm{E}[x(\tilde{s})] \\
p_k^* &= m_k^*(s) = 0
\end{aligned}
$$

for every $k \geq 3$ and every $s \in S$.

It is obvious by construction that $\langle p^*, m^* \rangle$ provides full insurance, is actuarially fair, and that non-manipulability is trivially satisfied for each agent $k \geq 3$. Moreover, agent 2's payment is independent of his report, so non-manipulability needs only to be checked for is agent 1. If agent 1 observes signal realization s_1 but reports $\hat{s}_1$, her expected payment is:

$$
\mathrm{E}[x(\tilde{s})] + \mathrm{E}[x(\hat{s}_1, \tilde{s}_{-1})|s_1] - \mathrm{E}[x(\hat{s}_1, \tilde{s}_{-1})|\hat{s}_1] = \mathrm{E}[x(\tilde{s})],
$$

where the equality follows from the independence assumption (IND). Since the expected payment, $\mathrm{E}[x(\tilde{s})]$, is independent of the report $\hat{s}_1$, there is no incentive for 1 to manipulate the contract.

It remains to check interim individual rationality. This holds trivially for agents $k \geq 3$, and the above calculation demonstrates that it is satisfied for agent 1. Agent 2's interim expected payment is:

$$
\mathrm{E}\left[\mathrm{E}[x(\tilde{s}_1, \tilde{s}_{-1})|\tilde{s}_1]|s_2]\right] - \mathrm{E}[\tilde{x}(s)].
$$

By (IND), the first and the second term are identical. The interim expected payment for agent 2 is thus zero, implying that the individual rationality condition is satisfied. The claim in the second sentence of Proposition 2 follows by relabeling of the agents.

Q.E.D.

The proof of Proposition 1 uses an insurance contract where one actor, agent 1, acts as a "primary insurer", and a second actor, agent 2, insures the primary insurer against the variation that can be predicted by agent 1. One can construct alternative contracts in which the role of the primary insurer is divided arbitrarily between the N agents, and in which the role to provide

secondary insurance for i is divided arbitrarily among the remaining $N - 1$ agents. Formally, let $\alpha \in R^N$ be a vector which may be thought of as "primary insurance weights" in that we require that $\sum_i \alpha_i = 1$, but where a particular α_i need not be nonnegative. In the same way, for each $i \in \{1, 2, ..., N\}$, let $\beta_i \in R^{N-1}$ be a vector of "secondary insurance weights", that is the shares in the insurance against the variation that can be predicted by i. Again, $\sum_{j \neq i} \beta_{ij} = 1$, but a particular β_{ij} need not be nonnegative. Now, for each $i \in I$ and every $s \in S$ let:

$$
\begin{aligned}
p_i^* &= \alpha_i \mathrm{E}[x(\tilde{s})] \\
m_i^*(s) &= \alpha_i \left(\mathrm{E}[x(\tilde{s})] + x(s) - \mathrm{E}[x(s_i, \tilde{s}_{-i}) \mid s_i] \right) \\
&\quad + \sum_{j \neq i} \beta_{ji} \alpha_j \left(\mathrm{E}[x(s_j, \tilde{s}_{-j}) \mid s_j] - \mathrm{E}[x(\tilde{s})] \right).
\end{aligned}
$$

We leave it to the reader to verify that an insurance scheme of this form provides full insurance, and is actuarially fair, non-manipulable, and interim individually rational. However, it obviously does not have the properties referred to in the second sentence of Proposition 1.

3   Ex Ante and Ex Post Budget Balance

Assume again that there are N agents $i \in I = \{1, 2, ..., N\}$. They have to choose one decision a from a set A of possible collective decisions. Each agent i privately receives a signal $\tilde{s}_i$ with realizations in S_i. We now interpret $\tilde{s}_i$ as agent i's type. Preferences are defined over A and a numéraire good called "money". Let $t_i \in R$ be the transfer of money from agent i. Each agent i has a quasi-linear von Neumann Morgenstern utility given by

$$u_i(a, s) - t_i$$

where $u_i : A \times S \to R$. As in the previous section, each agent observes s_i, but remains uninformed about s_{-i}. Finally, there is a resource constraint: implementing decision $a \in A$ costs $r(a, s) \in R$ units of the numéraire good.

Note the generality of our model. Each agent i's signal potentially affects agent i's and other agents' preferences as well as the resource requirements.

Definition 3 *A mechanism is a pair $\langle f, t \rangle$, where*

- $f : S \to A$ is the allocation rule. *For every $s \in S$ the decision $f(s)$ is implemented when s is announced.*

- $t : S \to R^N$ is the payment rule. *We define $t(s) = (t_1(s), t_2(s), ..., t_N(s))$, and for every $i \in I$ and $s \in S$ the value $t_i(s)$ is the transfer from agent i when s is announced.*

By the revelation principle it is without loss of generality that we restrict attention to direct mechanisms in which agents announce their types. For simplicity of notation, we only consider *pure direct revelation mechanisms*, that is, mechanisms that pick some alternative in A with probability 1, conditional on the agents' announcements, but our argument is easily extendable to the case of random decisions.

Definition 4 *A mechanism $\langle f, t \rangle$ is*

- ex post budget balanced *if, for each $s \in S$ we have:*

$$\sum_{i=1}^{N} t_i(s) = r(f(s), s);$$

- ex ante budget balanced *if*

$$E\left[\sum_{i=1}^{N} t_i(s)\right] = E[r(f(s), s)];$$

It now follows almost immediately from Proposition 1 that the set of allocation rules that can be implemented under ex post budget balance and under ex ante budget balance are related in the way described in Proposition 2 below.

Proposition 2 *Suppose (IND) holds and $N \geq 2$. For every ex ante budget balanced mechanism $\langle f, t \rangle$, and for any two agents $i, j \in I$ with $i \neq j$, there is an ex post budget balanced mechanism $\langle \hat{f}, \hat{t} \rangle$ such that:*

- *The allocation rule is unchanged:*
  $\hat{f}(s) = f(s)$ *for every $s \in S$;*

- *The interim expected payments by agents i and j are unchanged:*
  $E[\hat{t}_k(s_k, \tilde{s}_{-k}) \mid s_k] = E[t_k(s_k, \tilde{s}_{-k}) \mid s_k]$ *for every $k \in \{i, j\}$ and every $s_k \in S_k$;*

- *The change in agent j's payment does not depend on agent j's signal:*

  $\hat{t}_j(s_j, s_{-j}) - t_j(s_j, s_{-j}) = \hat{t}_j(s'_j, s_{-j}) - t_j(s'_j, s_{-j})$ *for any* $s_j, s'_j \in S_j$;

- *The payment rule for agents other than i and j is unchanged:*

  $\hat{t}_k(s) = t(s)$ *for every* $k \in I$ *with* $k \neq i, j$ *and for every* $s \in S$.

Proof: Consider a mechanism $\langle f, t \rangle$ that is ex ante budget balanced. For every state $s \in S$ define

$$x(s) = r(f(s), s) - \sum_{i=1}^{N} t_i(s).$$

That is, $x(s)$ is the ex post deficit under the proposed mechanism. We know from Proposition 1 that there exist insurance contracts that provide full insurance for the deficit x, and have the properties listed in Proposition 1. Let $\langle p, m \rangle$ be one such contract. Non-manipulability implies that for every $i \in I$ there exists some real number k_i such that $p_i - \mathrm{E}[m_i(s_i, \tilde{s}_{-i}) \mid s_i] = k_i$ for every $s_i \in S_i$. To satisfy interim individual rationality, k_i must be weakly positive, and if any k_i were strictly larger than 0 a violation of actuarial fairness would be implied. We conclude that $p_i - \mathrm{E}_{-i}[m_i(s) \mid s_i] = 0$ for every $i \in \{1, 2, ..., N\}$ and every $s_i \in S_i$.

Now consider the transfer scheme $\hat{t}$ where

$$\hat{t}_i(s) = t_i(s) - p_i + m_i(s)$$

for every $i \in I$ and $s \in S$. Interim expected payments are unchanged for every $i \in S_i$ by the earlier calculation. As the scheme provides full insurance, the mechanism with allocation rule f and transfer rule $\hat{t}$ is ex post budget balanced. The last two bullet points in Proposition 2 follow from the fact that $\langle p, m \rangle$ has the properties listed in the second sentence of Proposition 1.

Q.E.D.

Proposition 2 does not require either incentive compatibility or individual rationality. However, if truth telling is a Bayesian equilibrium in the original mechanism (with interim participation constraints imposed), the same holds true in the ex post budget balanced mechanism. Moreover, if for any of the agents other than agent i, truth telling was a dominant strategy in the original mechanism, the same will be true in the ex post budget balanced mechanism. Finally, if the original mechanism satisfied an interim individual rationality constraint for any agent, then the same will be true in the ex post budget balanced mechanism.

4   An Extension

We now relax the independence condition (IND). Unfortunately, the conclusion that we then obtain is slightly weaker than the conclusion of Proposition 1, as one of the agents' interim individual rationality constraint may be violated for some realizations of this agent's signal. The weaker condition that we are considering is called "Condition S" in Crémer and Riordan (1985).[5] We call it (CIND).

(CIND)   *There are two agents $i, j \in I$, where $i \neq j$, such that conditional on every realization of the other agents' signals, the signals $\tilde{s}_i$ and $\tilde{s}_j$ of agents i and j are independent.*

Condition (CIND) implies that $N \geq 2$, so Proposition 3, unlike Proposition 1, does not explicitly mention this condition.

Proposition 3 *Let i and j be two agents for whom condition (CIND) holds. Then there are insurance contracts that provide full insurance, are actuarially fair, are non-manipulable, and that satisfy the interim individually rationality condition except possibly for agent j. Moreover, agent j's payment does not depend on his type, and all agents other than agents i and j make zero payments in all states.*

Proof: Without loss of generality, assume that agents $1, 2$ are two agents for whom (CIND) is satisfied. We write s_{-12} for realizations of the random variable $\tilde{s}_{-12} \equiv (\tilde{s}_k)_{k=3}^N$ and observe that.

$$
x(s) \quad = \quad \underbrace{\mathrm{E}[x(\tilde{s})]}_{\text{Unconditional expected value}} \quad +
$$

$$
\underbrace{\mathrm{E}[x(s_1, \tilde{s}_2, s_{-12})|s_1, s_{-12}] - \mathrm{E}[x(\tilde{s})]}_{\text{Deviation of conditional expected value from unconditional expected value}} \quad +
$$

$$
\underbrace{x(s) - \mathrm{E}[x(s_1, \tilde{s}_2, s_{-12})|s_1, s_{-12}]}_{\text{Deviation of realization from conditional expected value}}
$$

As in the proof of Proposition 1, the idea of the insurance contract $\langle p^*, m^* \rangle$ that we construct is that the first and the third component in the above decomposition will be agent i's payment. The second component will be agent j's payment. Agent 1 is compensated by an up-front payment.

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[5] We comment below on the relation between our results in this section and Crémer and Riordan's main result.

Agent 2's expected payment will be shown to be zero, and therefore he needs no up-front payment. All other agents' payments equal zero in all states, and they receive no up-front payment. That is,

$$
\begin{aligned}
p_1^* &= \mathrm{E}[x(\tilde{s})] \\
m_1^*(s) &= \mathrm{E}[x(\tilde{s})] + x(s) - \mathrm{E}[x(s_1, \tilde{s}_2, s_{-12})|s_1, s_{-12}] \\
p_2^* &= 0 \\
m_2^*(s) &= \mathrm{E}[x(s_1, \tilde{s}_2, s_{-12}) \mid s_1, s_{-12}] - \mathrm{E}[x(\tilde{s})] \\
p_k^* &= m_k^*(s) = 0
\end{aligned}
$$

for all $k \geq 3$ and all $s \in S$.

This contract obviously provides full insurance and is actuarially fair. Non-manipulability is trivially satisfied for each agent $K \geq 3$, and since the payment is independent of his report, non-manipulability is also satisfied for agent 2. It remains to check it for agent 1. Suppose agent 1 has observed signal realization s_1, and reports signal realization $\hat{s}_1$. Then his expected payment is:

$$
\mathrm{E}[x(\tilde{s})] + \mathrm{E}[x(\hat{s}_1, \tilde{s}_2, \tilde{s}_{-12})|s_1] - \mathrm{E}[\mathrm{E}[x(\hat{s}_1, \tilde{s}_2, \tilde{s}_{-12})|\hat{s}_1, \tilde{s}_{-12}] \mid s_1].
$$

By the law of iterated expectations, the second term can be re-written as

$$
\begin{aligned}
\mathrm{E}[x(\hat{s}_1, \tilde{s}_2, \tilde{s}_{-12})|s_1] &= \mathrm{E}[\mathrm{E}[x(\hat{s}_1, \tilde{s}_2, \tilde{s}_{-12})|s_1, \tilde{s}_{-12}]|s_1] \\
&= \mathrm{E}[\mathrm{E}[x(\hat{s}_1, \tilde{s}_2, \tilde{s}_{-12})|\hat{s}_1, \tilde{s}_{-12}]|s_1],
\end{aligned}
$$

where the second inequality follows from the conditional independence assumption (CIND). Thus, the second and third term in 1's expected payment cancel each other out, implying that the expected payment is $\mathrm{E}[x(\tilde{s})]$ for any report $\hat{s}_1 \in S_1$. Hence, non-manipulability is satisfied for agent 1 as well.

Individual rationality obviously holds for agents other than agents 1 and 2. It also holds for agent 1, by the above calculation. Finally, the insurance contract obviously has the properties referred to in the second sentence of Proposition 3.

Q.E.D.

In the same way in which Proposition 2 followed from Proposition 1, we can now deduce from Proposition 3:

Proposition 4 *Let i and j be two agents for whom condition (CIND) holds. For every ex ante budget balanced mechanism $\langle f, t \rangle$ there is an ex post budget balanced mechanism $\langle \hat{f}, \hat{t} \rangle$ such that:*

- *The allocation rule is unchanged:*

  $\hat{f}(s) = f(s)$ *for every* $s \in S$;

- *The interim expected payment by agent i is unchanged:*

  $E[\hat{t}_i(s_i, \tilde{s}_{-i}) \mid s_i] = E[t_i(s_i, \tilde{s}_{-i}) \mid s_i]$ *for every* $s_i \in S_i$;

- *The change in agent j's payment does not depend on agent j's signal:*

  $\hat{t}_j(s_j, s_{-j}) - t_j(s_j, s_{-j}) = \hat{t}_j(s'_j, s_{-j}) - t_j(s'_j, s_{-j})$ *for any* $s_j, s'_j \in S_j$;

- *The payment rule for agents other than i and j is unchanged:*

  $\hat{t}_k(s) = t(s)$ *for every* $k \in I$ *with* $k \notin \{i, j\}$ *and for every* $s \in S$.

Like Proposition 2, also Proposition 4 is true regardless of whether incentive compatibility or individual rationality are satisfied or not. However, if truth telling is a Bayesian equilibrium in the original mechanism, then the same will be true in the ex post budget balanced mechanism. Moreover, if for any of the agents other than agent i, truth telling was a dominant strategy in the original mechanism, the same will be true in the ex post budget balanced mechanism. Finally, if the original mechanism satisfied an interim individual rationality constraint for any agent, then the same will be true in the ex post budget balanced mechanism, except possibly for agent j.

An implication of Proposition 4 is as follows. Consider the case that there are private values, i.e. every agent's utility depends only on his own type, but not on the other agents' types. Suppose also that the resource costs are a linear function of agents types: $r(a, s) = \bar{r} + \sum_{i=1}^{N} r_i(a, s_i)$. Let f^* be an allocation rule that maximizes social surplus, i.e. the sum of utilities minus resource costs, in every state. A Vickrey-Clarke-Groves payment rule will ensure that truthful reporting of type is a dominant strategy. By adding appropriate constants, we can ensure that the mechanism is ex ante budget balanced. By Proposition 4 we can then construct another payment scheme that is ex post budget balanced, that makes for $N - 1$ agents truth telling a dominant strategy, and that has the property that for the remaining agent truth telling is expected utility maximizing provided that all other agents tell the truth. This is Theorem 2 in Crémer and Riordan (1985). Proposition 4 above strengthens Crémer and Riordan's result because it applies not only to surplus maximizing allocation rules but to other allocation rules as well.

5 Discussion

The paper that is closest to ours is d'Aspremont, Crémer and Gérard-Varet (2004). They prove that, in the absence of interim participation constraints, efficient decision rules can always be implemented if, for every real-valued function $r\left(s\right)$, there exists a transfer rule t such that $\sum_i t_i\left(s\right) = r\left(s\right)$ for every s and

$$\mathrm{E}\left[t_i\left(s_{-i}, s_i\right)|s_i\right] \geq \mathrm{E}\left[t_i\left(s_{-i}, s_i'\right)|s_i\right]$$

for every agent i and every pair $s_i, s_i' \in S_i$. In our language this condition is simply saying that for every uncertain expenditure that the mechanism designer might have there exists an incentive compatible full insurance agreement between the agents and the mechanism designer. While the condition it is not expressed in terms of primitives, there are several known sufficient conditions, one being stochastic independence (see d'Aspremont, Crémer and Gérard-Varet, 2004).

For the case when types are correlated, Fudenberg, Levine and Maskin (1995) and Kosenok and Severinov (2002) have shown that large sets of allocation rules can be implemented by interim individually rational and ex post budget balanced mechanisms under very general conditions. These results rely, though, on the logic of the full surplus extraction results in Crémer and McLean (1985, 1988) and McAffee and Reny (1992). As has been pointed out by Neeman (2004), such results hinge crucially on preferences being uniquely determined by beliefs, an assumption which is possible to relax while still allowing for correlations. Qualitatively, breaking the one-to-one correspondence between beliefs and preferences results in a model more similar to the (non-generic) case with stochastic independence in the sense that informational asymmetries become relevant. Our approach to ex ante versus ex post budget balance may therefore be useful in this case.

References

[1] Arrow, K.J. (1979) The Property Rights Doctrine and Demand Revelation Under Incomplete Information, in M.J. Boskin Edited *Economics and Human Welfare: Essays in Honor of Tibor Scitovsky*, Academic Press, New York.

[2] d'Aspremont, C. J. Crémer and L.-A. Gérard-Varet (2004), Balanced Bayesian Mechanisms, *Journal of Economic Theory* 115, 385-396.

[3] d'Aspremont, C. J and L.-A. Gérard-Varet (1979), Incentives and Incomplete Information, *Journal of Public Economics* 11, 25-45.

[4] Cramton, P., R. Gibbons, and P. Klemperer (1987), Dissolving a Partnership Efficiently, *Econometrica* 55, 615-632.

[5] Crémer, J. and R. McLean (1985), Optimal Selling Strategies Under Uncertainty for a Discriminating Monopolist When Demands are Interdependent, *Econometrica* 53, 345-361.

[6] Crémer, J. and R. McLean (1988), Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions, *Econometrica* 56 (1988), 1247-1258.

[7] Crémer, J., and M. Riordan, (1985), A Sequential Solution to the Public Goods Problem, *Econometrica* 53, 77-84.

[8] Fudenberg, D., D. K. Levine, and E. Maskin (1995) Balanced-Budget Mechanisms with Incomplete Information, mimeo, Harvard and UCLA.

[9] Kosenok, G. and S. Severinov (2002), Individually-Rational, Budget-Balanced Bayesian Mechanisms and the Informed Principal Problem, mimeo., New Economic School (Moscow) and Fuqua School of Business (Duke University), revised 2004.

[10] Mailath, G. and A. Postlewaite (1990), Asymmetric Information Bargaining Problems with Many Agents, *Review of Economic Studies* 57, 351-368.

[11] McAfee, R. P. and P. J. Reny (1992), Correlated Information and Mechanism Design, *Econometrica* 60, 395-421.

[12] Myerson, R., and M. Satterthwaite (1983), Efficient Mechanisms for Bilateral Trading, *Journal of Economic Theory* 29, 265-281.

[13] Neeman, Z. (2004) The Relevance of Private Information in Mechanism Design, *Journal of Economic Theory* 117, 55-77.