Veto Constraint in Mechanism Design: Inefficiency with Correlated Types*

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Abstract

We consider bargaining problems in which parties have access to outside options, the size of the pie is commonly known and each party privately knows the realization of her outside option. We allow for correlations in the distributions of outside options, which are required to derive from smooth and bounded densities with large supports. Parties are assumed to have a veto right, which allows them to obtain at least their outside option payoff in any event. Besides, agents can receive no subsidy ex post. We show that inefficiencies are inevitable whatever the exact form of correlation. We also illustrate how veto constraints differ from ex post participation constraints in an extension of the model in which the size of the pie may depend on the parties' private information. The same insights apply to the bargaining between a buyer and a seller privately informed of their valuations and to public good problems in which agents are privately informed of their willingness to pay for the public good.

1 Introduction

Private information is well known to be a source of inefficiency in bargaining. For the sake of illustration, consider a seller and a buyer bargaining over the transaction price

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of an indivisible object. Myerson and Satterthwaite (1983) have shown that if the valuations of the seller and the buyer are smoothly and independently distributed and if it is not sure who values the object most, inefficiencies must arise in any bargaining game in which no outside money is given to the bargaining parties.

The strength of Myerson and Satterthwaite's result is that it applies to any bargaining game including protocols in which a broker could help improve the bargaining outcome, as well as protocols allowing for several stages of bargaining. The result is obtained by relying on the "so called" *revelation principle*, which allows to derive constraints that should be satisfied in any Nash-Bayes equilibrium of any game (whether static or dynamic): these constraints are the "so called" *incentive constraints* and the *interim participation constraints*. As shown by Myerson and Satterthwaite, these constraints together with the constraint that the bargaining parties receive no outside money in expectation cannot be simultaneously satisfied, unless there are inefficiencies.

The inefficiency result obtained by Myerson and Satterthwaite is a corner stone of information economics because it illustrates simply the essential role of private information in welfare analysis. The main weakness is that it does not extend to the case of correlated types. Whenever types are correlated (and whatever the exact shape of correlation), the works of Crémer and McLean (1985, 1988) and McAfee and Reny (1992) can be used to show that the first-best can be achieved even without subsidies (in expectation) while satisfying the incentive constraints and the interim participation constraints of the agents.

While the idea that correlation should help is sensible (because the report of agent j can then be used to alleviate the incentive constraints of agent i, $i \neq j$), the conclusion that inefficiencies can be entirely eliminated when there is correlation sounds unintuitive.

In this paper, we consider a bargaining problem between n parties who bargain on the division of a pie of known size V and who are privately informed of their outside options. We allow for any form of correlation in the distribution of outside options, and we allow third parties to help reaching a better bargaining outcome. Up to some re-labelling, the buyer/seller problem (à la Myerson-Satterhwaite) can be cast into our framework as well as public good problems in which agents would privately know their willingness to pay for the public good (see subsection 3.3).

We depart from the usual mechanism design approach by assuming that agents (including third parties) can *quit* the mechanism, thereby enjoying their outside option at any point in time until a complete agreement has been reached. We refer to such situations as *non-binding bargaining protocols.*

Our main result is that in non-binding bargaining protocols, inefficiencies are inevitable, as long as the distribution of outside options has a smooth and bounded density with support $[0, V]^n$.

The restriction to non-binding bargaining protocols imposes additional constraints on what can be achieved through Nash Bayes equilibria as compared to the traditional mechanism design approach. We refer to these extra constraints as the *ex post veto constraints.* These constraints include the familiar *ex post participation constraints* because agreements violating such constraints would be ex post vetoed. They also include the constraint that the bargaining parties should receive no subsidy ex post, as otherwise the third party would have to lose money in some events and she would veto it. But, veto rights impose additional constraints, as they also affect the incentive constraints: when considering a deviation, an agent takes into account the fact that he can opt out if things turn out badly.[1]

It may be worth stressing that our inefficiency result applies to virtually *all* densities of interest, and not merely to densities that are nearly independent, as otherwise (if inefficiencies arose only for nearly independent distributions) our result would follow from continuity considerations (as in Robert (1991) or Kosmopoulou and Williams (1998)).[2]

From a technical viewpoint, our method of proof is different from that of Myerson and Satterthwaite. In the non-correlated case, interim transfers are determined up to a constant by the allocation rule. The constant is then fixed by the participation constraint of an extreme type, here the type with largest outside option. So proving that inefficiencies must arise in the non-correlated case amounts to proving that the transfers so derived from the efficient allocation rule must violate another constraint, the budget-balancedness constraint. In the correlated case, this method is no longer applicable because the same allocation rule can be obtained through different interim

[1] A similar distinction between ex post participation and ex post veto constraints appears in Matthews and Poslewaite (1989) who analyze in double auctions preceded by cheap talk communication the constraints that must apply to equilibrium outcomes. (See also Forges (1999) for similar considerations in an interdependent value setup.)

[2] Indeed, the ex post veto constraints imply that the transfer received by agents cannot exceed V, which guarantee that the space of admissible transfer and allocation rules is compact, hence the continuity result (see Robert (1991) for related considerations in a context with limited liability and/or risk aversion, and Kosmopoulou Williams (1998) in a context with limited liability and/or ex post participation constraints). See also Laffont and Martimort (2000) who observe that transfers should be bounded when agents can collude in reaction to the proposed mechanism.

transfers. (The work of Crémer-McLean offers a simple illustration of this.)

We prove our result by directly showing that, in non-binding bargaining protocols, if efficiency could be achieved, the incentive constraint not to pretend that one has a higher outside option would force every agent i to receive the entire surplus for himself, i.e $V - \sum_{j \neq i} w_j$ where w_j is the outside option of j. Of course, this is not possible as it would violate the ex post no subsidy constraint when the outside options of all agents are low enough (thus showing by contradiction that inefficiencies must arise).

In Section 2 we present the bargaining setup and our main inefficiency result using direct truthful mechanisms. In subsection 3.1 we provide an intuition for the result based on a specific discretization of the type space. We show in subsection 3.2 how to apply the revelation principle, thereby proving that our inefficiency result applies to all non-binding bargaining protocols. In subsection 3.3 we show how to apply our result to other economic situations including buyer/seller problems and public good problems. In Section 4 we discuss the interpretation of the ex post veto constraints, and contrast it with ex post participation constraints. While inefficiency would also prevail if instead of ex post veto constraints we had considered the ex post participation constraints and the ex post no subsidy constraint, we highlight that ex post veto constraints and ex post participation constraints have, in general, different welfare implications. This is illustrated through an extension of the model in which the size of the pie may vary with the signals held by parties. In an example in which parties are not allowed to pretend they have an outside option larger than it is in reality and in which the entire surplus must be distributed between the bargaining parties, we show that the first-best can be achieved when the sole ex post participation constraints are required whereas inefficiencies cannot be avoided under the ex post veto scenario. In Section 5 we suggest several extensions and avenues for future research. These include a more general treatment of pies whose size may depend on the signals held by agents, the relaxation of the ex post no subsidy constraint, and the analysis of the second-best.

2 The Inefficiency Result

We consider the following bargaining problem. There is a pie of size V to be shared between $n+1$ parties $i = 0, 1...n$ where party 0 is an intermediary who may help other parties $i = 1, ...n$. Utilities are transferable between parties and we assume that each party i, $i = 1, ...n$, has an outside option w_i where $w_i \in [0, V]$.[3] That is, if no agreement is reached, party i gets w_i. The values of $w = (w_1, w_2, ...w_n)$ are not commonly known. Party i (but not party j, $j \neq i$) knows the realization of w_i. We let $g(w)$ denote the joint density of w on $[0, V]^n$. That is, we explicitly allow for correlations in the distribution of outside options. We assume that the density $g(w)$ is bounded and positive on its support, and that $g(w)$ is differentiable with bounded derivative. That is, there exists a strictly positive scalars m and M such that for all $w \in [0, V]^n$ and all $i = 1, ...n$, $m \leq g(w)$ and $\left|\frac{\partial g}{\partial w_i}(w)\right| \leq M$.

We are interested in bargaining situations in which all players keep a right to veto any proposal and to leave the bargaining table. Specifically, a *non-binding bargaining protocol* is a (possibly multi-stage) game that generates *proposals* for agreement and where each proposal for agreement is followed by a *ratification* stage in which every one of the parties has the option to veto the proposal and the option to leave the bargaining table.[4]

The ratification stage will generate two types of constraints:

(1) The assumption that each party $i = 1, ...n$ may leave the bargaining table implies that in equilibrium each party i should (ex post) get at least his outside option. From a mechanism design perspective, this assumption should thus be associated with the well-known ex post participation constraints. However, as we will illustrate later on, this assumption will generate a stronger condition, because it allows for the possibility that a party w_i takes his outside option even after a deviation from the assumed equilibrium strategy, thereby leading to a new formulation of the incentive constraints.

(2) The assumption that the intermediary may leave the bargaining table at any point in time implies that no outside money can be given to the bargaining parties in any event. From a mechanism design perspective, this condition will generate an ex post no subsidy constraint.[5]

Rather than analyzing all possible non-binding bargaining protocols, we will follow a mechanism design approach and define below a class of direct truthful mechanisms with veto rights. We will later on show, by applying the revelation principle to the present context, that there is no loss of generality in restricting

[3] Larger outside options would clearly result in no agreement and thus there is no loss of generality in assuming that outside options can be no greater that the size of the pie.

[4] Transfers in case of disagreement should be 0 as any other transfer would be vetoed by at least one party.

[5] This is sometimes referred to as ex post budget balancedness. Note that we make no requirement here as to whether the entire pie should be distributed to parties $i = 1, ...n$.

attention to such mechanisms: if inefficiencies must arise in any direct truthful mechanism with veto rights, then they must also arise in any equilibrium of any non-binding bargaining protocol.

Formally, a *direct mechanism with veto rights* consists of two stages: an *announcement* stage and a *ratification* stage; and it is characterized by the functions $(\phi, t_1, ...t_n)$ where $\phi : [0, V]^n \to [0, 1]$ and $t_i : [0, V]^n \to \mathbb{R}$ specify, as a function of the announcements, a probability that a proposal is made, and transfers to be received by each party, respectively.

Specifically, in the announcement stage, each party i simultaneously makes an announcement $\widehat{w}_i \in [0, V]$. This generates, conditional on the profile of announcement $\widehat{w} = (\widehat{w}_1, ...\widehat{w}_n)$, a probability $\phi(\widehat{w})$ that a proposal for agreement is made, and in the event a proposal is made, monetary transfers $t_i(\widehat{w}_1, ...\widehat{w}_n)$, $i = 1, ..., n$ to be received by player i in case the proposal is ratified in the next stage.[6] The intermediary would then receive $V - \sum_{i=1}^{n} t_i(\widehat{w})$ in this event.

In case a proposal is made, the game moves to the ratification stage in which each party (including the intermediary) simultaneously decides whether to accept or veto the proposal. In case all parties accept, the proposal is implemented, each party i gets $t_i(\widehat{w})$ and the intermediary gets $V - \sum_{i=1}^{n} t_i(\widehat{w})$. In case no proposal has been made, or one (or more) party vetoes the proposal, each party i gets his outside option w_i, and the intermediary gets 0. The key feature of the ratification stage is that a party with outside option w_i can always secure a payoff of w_i whatever the profile $\widehat{w}$ of announcements made at the announcement stage by deciding to reject the agreement at the ratification stage. This will be referred to as the veto right constraint. Similarly, the intermediary must make no losses leading to the ex post no subsidy constraint.

We consider direct mechanisms of the above form in which, it is an equilibrium to report the true private information at the announcement stage, and in which, in equilibrium, proposals are not vetoed. Such direct mechanisms will be called *direct truthful mechanisms with veto rights.*

In a direct truthful mechanism with veto rights, party i with type w_i should report $\widehat{w}_i = w_i$, and for any $w \in [0, V]^n$ such that $\phi(w) > 0$, the intermediary should

[6] We implicitly assume that $t_i(\widehat{w})$ is deterministic but this is without loss of generality for our main result (see subsection 3.2).

get at least 0, and party i should get at least w_i, hence:

$$\begin{aligned} \sum_i t_i(w) &\leq V \text{ and} && (1) \\ t_i(w_1, ...w_n) &\geq w_i && (2) \end{aligned}$$

Constraint (1) will be referred to as the *ex post no subsidy constraint.* Observe that we allow for situations in which the entire pie V is not fully distributed to the agents, i.e. $\sum_i t_i(w) < V$. This allows us to cover applications in which the intermediary may extract some surplus from offering a division of the pie.[7]

Constraint (2) is referred to as the *ex post participation* constraint in the literature. It guarantees that whatever the realization of w_{-i}, party i with type w_i is sure to get at least w_i when she announces her true type $\widehat{w}_i = w_i$.

But, the veto right of party i does not reduce to the ex post participation constraint (2). It also affects party i's incentive constraints, since party i can always exercise her outside option after pretending she is of type $\widehat{w}_i$ if the transfer $t_i(\widehat{w}_i, w_{-i})$ turns out to be less than w_i. Formally, let $U_i(\widehat{w}_i; w_i)$ denote the expected payoff obtained by party i in the above game when party i's outside option is w_i, party i's announcement is $\widehat{w}_i$ and party i expects other parties j, $j \neq i$ to report truthfully $\widehat{w}_j = w_j$. We have that

$$U_i(\widehat{w}_i; w_i) = E_{w_{-i}} \{\max[t_i(\widehat{w}_i; w_{-i}), w_i]\phi(\widehat{w}_i, w_{-i}) + w_i(1 - \phi(\widehat{w}_i, w_{-i})) \mid w_i\} \quad (3)$$

The effect of veto rights is captured in $\max[t_i(\widehat{w}_i; w_{-i}), w_i]$: in any circumstance party i with type w_i should get at least w_i when she announces she is of type $\widehat{w}_i$.

The incentive constraints require that for all $i = 1, ...n$, $w_i \in [0, V]$ and $\widehat{w}_i \in [0, V]$

$$U_i(w_i; w_i) \geq U_i(\widehat{w}_i; w_i). \quad (4)$$

We ask ourselves whether there can be a direct mechanism with veto rights satisfying the above constraints (1)-(2)-(4) and at the same time results in an efficient outcome whatever the realizations $w = (w_i)_{i=1}^{i=n}$ of the outside options. That is,

$$\begin{aligned} \phi(w_i, w_{-i}) &= 1 \text{ if } \sum_i w_i < V \text{ and} \\ \phi(w_i, w_{-i}) &= 0 \text{ if } \sum_i w_i > V. \end{aligned}$$

Our main result is that such a mechanism does not exist, thereby showing the following impossibility result.

[7] Of course, our inefficiency result holds *a fortiori* if we further impose that the surplus should be entirely distributed, i.e. $\sum_i t_i(w) = V$.

Theorem 1: *Inefficiencies must arise in any direct truthful mechanism with veto rights.*

As we will show in Section 3.2, a corollary of Theorem 1 is that the impossibility result must hold for any non-binding bargaining protocol. An intuition for Theorem 1 appears in the next Section. Theorem 1 is formally proven in Appendix. We now put Theorem 1 in perspective.

Myerson and Satterthwaite (1983) considered a bargaining problem between a seller and a buyer who are assumed to know their valuation of the good. Their inefficiency result assumes that valuations are independently distributed between the seller and the buyer, the supports of valuations of the seller and the buyer overlap, and the seller and buyer can receive no subsidy on average.

Our conclusion is similar to that of Myerson and Satterthwaite. Yet, our result differs from that of Myerson and Satterthwaite in several respects. First, we allow for correlations between agents' types. Second, we assume that parties have veto rights (in the buyer/seller problem, the seller should in any event get at least her valuation and the buyer should get at least 0). Third, we impose an ex post rather than ex ante no subsidy constraint.

In the setup studied by Myerson and Satterthwaite, if agents have no veto right, budget must be balanced ex ante and the distributions of valuations are *correlated* in a non-degenerate way, the first-best outcome can be achieved. This can be viewed as a corollary of the works of Crémer and McLean (1986-88) and McAfee and Reny (1992). Their work focuses on the possibility of full rent extraction from a monopoly interested in maximizing profit, but it equally applies to the welfare maximization problem. Basically, their construction allows in the correlated case to set each agent (whatever his type) to his reservation utility while inducing an efficient outcome.[8] Their construction thus guarantees that efficiency can be obtained in the correlated case even without subsidies (on average), as long as agents have no option to quit once they have voluntarily agreed to join the mechanism.

The full rent extraction result of Crémer and McLean is somewhat puzzling because it suggests that in the correlated case (which some people consider to be the generic case, see however Neeman (2004) and Heifetz-Neeman (2006) who challenge this view) private information should have no effect (since the outcome so obtained is the same as the one with complete information).[9]

[8] The reservation utility is from the interim viewpoint in which agents know only their own type.

[9] One should be cautious about how to interpret the work of Neeman (2004). His basic idea is

From this perspective, Theorem 1 shows that there is an effect of private information even in the correlated case whenever agents can quit the mechanism at any point in time

It should be noted that our result is distinct from the observation that in the case of nearly independent distributions some inefficiencies must arise if transfers are bounded (see Robert (1991) and Kosmopoulou and Williams (1998)).[10] In contrast, our result applies to all distributions with bounded and smooth density, and not merely to nearly independent distributions. The reason for our stronger result is that the veto constraints put a lot of additional structure beyond the fact that transfers must be bounded. It is this extra structure together with the no subsidy requirement that allows us to derive Theorem 1.

In Theorem 1, we have assumed that $g(\cdot)$ has full support on $[0,V]^n$.[11] However, for those w such that $\sum_i w_i > V$, a disagreement is inevitable. So, in any truthful mechanism with veto rights, we must have $\phi(\cdot) = 0$ on the set $\Gamma^d \equiv \{w \mid \sum_i w_i > V\}$, and each party must be getting his outside option for that range of signal profiles. It follows that our inefficiency result does not rely on the specification of $g(\cdot)$ on Γ^d. In particular, if $g(\cdot) = 0$ on Γ^d, Theorem 1 still holds, as long as

that it is restrictive to assume that the payoff relevant-type of party i, here his outside option w_i, should also determine i's belief about party j's type, and a more satisfactory formulation should include further determinants in i's belief. A natural way to model this is to assume that party i receices a signal β_i in addition to w_i where β_i together with w_i determines i's belief about j's type (party i's type should now be described as $s_i = (w_i, \beta_i)$). As long as the correlation matrix between all variables $w_i, \beta_i, w_j, \beta_j$ has full rank, the insight of Crémer and McLean applies, and the full rent extraction result holds. We note that such an extension of the model would not invalidate our inefficiency result (because the arguments on i's incentive constraints can be made for each realization of β_i, see Section 3 and appendix).

So Neeman's challenge of the full rent extraction result not only depends on the fact that the payoff-relevant part of party i's type does not fully determine i's belief, but also on a notion of consistency of belief that implies some form of conditional independence (thus challenging the idea that the full rank assumption should be thought of as a generic situation, see Heifetz- Neeman for further elaboration).

[10] This is so because the solution to the mechanism design problem becomes then essentially continuous with respect to the distribution of private information, and by Myerson-Satterthwaite we know that inefficiencies must arise in the independent case. The same conclusion arises in the case of slightly risk averse agents.

[11] We have also assumed that $g(\cdot) > m$ on its support. If there is perfect correlation in the sense that all w_j, $j \neq i$ are determined by w_i, the first-best can be achieved (any deviation can be detected and one can choose to implement the oustide option alternative in such a case, thereby deterring any deviation). If there is almost perfect correlation, the first-best can approximately be obtained by simply ignoring those reports that correspond to non-typical types.

$g(\cdot)$ satisfies the boundedness and smoothness conditions on $\{w \mid \sum_i w_i \leq V\}$ (see the start of section for a formal definition of boundedness and smoothness). In this case, inefficiencies must arise even though it is common knowledge that an agreement is beneficial.[12] The latter insight is a bit reminiscent of Akerlof's (1970) lemon example. Yet, the logic of the two results is quite different as our model is one of private values with multi-sided and correlated private information, whereas Akerlof considers a model with one-sided private information and interdependent values.[13]

3 Insights

We first present a simple intuition for our inefficiency result in the case where outside options take their values on a discrete but fine grid. We next show how to apply the revelation principle to our setup. Finally, we show how the seller/buyer problem studied by Myerson and Satterthwaite and the public good problem such as studied by Clarke and Groves can be cast into our setup.

3.1 Inefficiency in a Finite Grid

The veto right constraint, together with the ex post no subsidy constraint, imply the following set of inequalities on transfers:[14]

$$w_i \leq t_i(w_i, w_{-i}) \leq V - \sum_{j \neq i} w_j. \tag{5}$$

Our approach consists in showing that incentive compatibility conditions require that the second inequality binds, i.e.:

$$t_i(w_i, w_{-i}) = V - \sum_{j \neq i} w_j. \tag{6}$$

[12] Note that correlation accross types is important for this result. The assumption that $g(w) \geq \underline{g} > 0$ for all $w \in \{w \mid \sum_i w_i \leq V\}$ and that $g(w) = 0$ for $w \in \{w \mid \sum_i w_i > V\}$ implies some correlation accross types. Besides, if types were independently distributed, and if it were common knowledge that an agreement is beneficial, then inefficiencies could be avoided.

[13] Mailath and Postlewaite (1990) provide an interesting private (and correlated) value example in which it is common knowledge that the provision of a public good is efficient, and yet, no mechanism with fixed limited liability permits to implement it when the number of agent is large enough (the probability even tends to 0 as the number of agents tend to infinity). By contrast, our result does not rely on the number of agents being large, and the limited liability constraint is replaced by the veto constraint.

[14] The second inequalities follows from $t_i(w) + \sum_{j \neq i} t_j(w) \leq V$ and $t_j(w) \geq w_j$ for all j.

That is, each party i must always get the residual surplus generated by the agreement assuming that all other parties are set to their reservation utility (their outside option payoff). Of course, this cannot be, as such transfer rules would result in the violation of the ex post no subsidy constraint for quite a range of outside option profiles (think of w_j being close to 0 for every j; all transfers t_i should then be close to V, leading to a violation of the no subsidy constraint).

The main task (performed in appendix) consists in showing that incentive compatibility conditions lead to equality (6).

In this subsection, we show why this is true in the case of two players when the distribution of outside options has full support over the discrete but fine grid[15]

$$G = \{(k_1V/N, k_2V/N), k_i \in \{0, ..., N\}\},$$

where N should be thought of as being large. This case is not covered by our main Theorem, but it will permit us to provide a simple intuition as to why our result holds.

Let $w_1 = k_1V/N$ and $w_2 = k_2V/N$. Efficiency requires that an agreement should be reached whenever $k_1 + k_2 \leq N$. We wish to show that in any such event,

$$t_1(w_1, w_2) = V - w_2. \tag{7}$$

When $k_1 = N$ and $k_2 = 0$ (and more generally in any event where $k_1 + k_2 = N$), player 1's outside option w_1 coincides with the residual surplus $V - w_2$, so that there are no other choices than setting the transfer t_1 equal to $V - w_2$.

Now fix $k_1^0 \leq N$, and assume that for all $k_1 \geq k_1^0$ and $k_2 \leq N - k_1$, equality (7) holds. We will show below that equality (7) must also hold for all $k_1 \geq k_1^0 - 1$ and $k_2 \leq N - k_1$, thereby concluding the argument.

Agent 1 with outside option $w_1 = (k_1^0 - 1)V/N$ could consider reporting $\widehat{w}_1 = k_1^0V/N$. For all realizations of w_2 that fall strictly below $V - w_1$ (that is, for all realizations $k_2 \leq N - k_1^0$), the induction hypothesis tells us that player 1 should get $V - w_2$, which is in any case the maximum payoff player 1 can hope to get.

[15] Matsuo (1989) considers a two-type formulation in the buyer/seller problem of Myerson and Satterthwaite (1983) with independent distributions of types across agents. He notes that in contrast to Myerson and Satterthwaite efficiency may sometimes be achieved even when it is not known which alternative is socially efficient. His result hinges not only on the fact that the distribution of types takes his value on a discrete grid, but that the underlying grid is coarse (two types). In a discrete but fine grid, the same inefficieny result as the one in Myerson-Satterthwaite would arise.

Now for the realizations of w_2 that coincide with or exceed $V - w_1$ (that is, when $k_2 \geq N - k_1^0 + 1$), player 1 cannot hope to get more than w_1, whether an agreement is proposed or not.

It follows that the announcement $\widehat{w}_1$ allows player 1 to extract all the residual surplus, hence the only way to provide player 1 with incentives to report w_1 truthfully is to give him that surplus even when he announces w_1, that is, to set the transfer t_1 equal to $V - w_2$ for all realizations of w_2 below or equal to $V - w_1$.

This proof is rather simple, yet it does not easily extend to other forms of discretization nor to the case of distributions with smooth densities.[16] In the appendix we show how to deal with this.

3.2 Using the Revelation Principle

Let us see now why Theorem 1 applies not only to direct truthful mechanisms with veto rights, but also to any non-binding bargaining protocol.

Consider any non-binding bargaining protocol, possibly allowing for multiple stages $k = 1, 2...$, and an equilibrium σ of the game associated with this protocol. Denote by $\sigma_i(w_i)$ the strategy used by party i in equilibrium, when his outside option is w_i. Each strategy profile $(\sigma_i(w_i), \sigma_{-i}(w_{-i}))$ induces a probability $\phi^k(w_i, w_{-i})$ that an agreement is proposed and ratified in stage k, and, conditional on ratification in stage k, a distribution $\widetilde{\mathbf{t}}_i^{k,w_i,w_{-i}}$ over transfers $\widetilde{t}_i^k$. Define $t_i(w_i, w_{-i})$ as the expected stage 1 transfer to player i induced by this strategy profile:

$$t_i(w_i, w_{-i}) = E\widetilde{\mathbf{t}}_i^{1,w_i,w_{-i}}$$

We assume that delay is costly, so that if σ involves no efficiency loss, it should specify that an agreement is reached in stage 1 with probability one whenever $\sum_i w_i < V$, and it should specify that bargaining stops with no agreement reached whenever $\sum_i w_i > V$. Moreover, since the agreement should not be vetoed in equilibrium, for all w such that $\sum_i w_i < V$, all transfer realizations $\widetilde{t}_i^1$ in the support of $\widetilde{\mathbf{t}}_i^{k,w_i,w_{-i}}$ should be such that $\widetilde{t}_i^1 \geq w_i$, implying that:

$$t_i(w_i, w_{-i}) \geq w_i.$$

[16]A common feature between the proof in the simple case and the general proof, though, is that we only use i's incentive constraints to deviate upwards from $\widehat{w}_i = w_i$ to $w_i + \varepsilon > w_i$. This feature will be referred to in Section 4.

Similarly, the approval of the third party requires that $\sum_i \widetilde{t}_i^1 \leq V$, yielding:

$$\sum_i t_i(w_i, w_{-i}) \leq V.$$

Now define $U_i(\widehat{w}_i; w_i)$ as in (3) (see Section 2), using the transfers $t_i(w)$ and the probabilities $\phi^1(w)$:

$$U_i(\widehat{w}_i; w_i) = E_{w_{-i}} \left\{ \max[t_i(\widehat{w}_i; w_{-i}), w_i]\phi^1(\widehat{w}_i, w_{-i}) + w_i(1 - \phi^1(\widehat{w}_i, w_{-i})) \mid w_i \right\}$$

We show below that the incentive constraints $U_i(w_i; w_i) \geq U_i(\widehat{w}_i; w_i)$ must hold for all w_i, $\widehat{w}_i$.

Consider the strategy of party i that consists in following $\sigma_i(\widehat{w}_i)$ during the first stage, and to exercise the outside option if no agreement is proposed by the end of this stage, or if the proposed agreement entails receiving a payment smaller than w_i. The expected payoff associated with that strategy when party i is of type w_i and parties j, $j \neq i$ follow $\sigma_j(w_j)$ is denoted $\bar{U}_i(\widehat{w}_i, w_i)$, and it satisfies:

$$\bar{U}_i(\widehat{w}_i; w_i) \equiv E_{\sigma_{-i}, w_{-i}}[\max(\widetilde{\mathbf{t}}_i^{1, \widehat{w}_i; w_i}, w_i)\phi^1(\widehat{w}_i, w_{-i}) + (1 - \phi^1(\widehat{w}_i, w_{-i}))w_i \mid w_i]$$

Because strategies are in equilibrium, the deviations above must be deterred, which implies that conditions $\bar{U}_i(w_i; w_i) \geq \bar{U}_i(\widehat{w}_i; w_i)$ hold for all w_i, $\widehat{w}_i$. Now observe that $U_i(w_i; w_i) = \bar{U}_i(w_i; w_i)$,[17] and that $\bar{U}_i(\widehat{w}_i; w_i) \geq U_i(\widehat{w}_i; w_i)$.[18] So the incentives constraints $U_i(w_i; w_i) \geq U_i(\widehat{w}_i; w_i)$ must hold for all w_i, $\widehat{w}_i$. It follows that the direct mechanism defined by the transfer rules $t_i(w)$ must be an efficient direct truthful mechanism with veto rights. But, we have seen that no such mechanism exists, thereby showing that no equilibrium of any non-binding bargaining protocol whatsoever can induce an efficient outcome.[19]

3.3 Other Applications

We now observe that our inefficiency result equally applies to other well known problems.

[17] This is because all transfer realizations $\widetilde{t}_i^1$ in the support of $\widetilde{\mathbf{t}}_i^{k, w_i, w_{-i}}$ are such that $\widetilde{t}_i^1 \geq w_i$.

[18] This is because for any (w_i, w_{-i}), $E \max\{\widetilde{\mathbf{t}}_i^{1, w_i, w_{-i}}, w_i\} \geq \max\{E\widetilde{\mathbf{t}}_i^{1, w_i, w_{-i}}, w_i\}$.

[19] The above considerations allow us to conclude that inefficiencies must occur in any equilibrium of any game whether static or dynamic in which parties can quit the bargaining table at any point in time. However, the set of feasible alternatives in a multi-stage framework should account for the date at which the agreement is reached. Thus, the second-best analysis of a dynamic framework may *a priori* differ from the second-best analysis of static frameworks depending on how the cost of delay is modelled.

The seller/buyer problem: Agent 1, the seller, owns an object which he considers selling to agent 2, the buyer. The seller's valuation for the object is given by v_S; the buyer's valuation for the object is given by v_B. The seller knows his valuation v_S but not that of the buyer v_B. Symmetrically, the buyer knows her valuation v_B, but not that of the seller v_S. Agents also know that (v_B, v_S) is drawn from a joint distribution with bounded and smooth density with support $(0, \overline{v})^2$.

For this application, our result (Theorem 1) establishes that if the seller and the buyer can receive no subsidy ex post (i.e. the sum of side-payments received by the two agents can never exceed 0) efficiency cannot be achieved whenever each agent must get at least his reservation utility in any event (that is, in any event the seller must get at least v_S and the buyer must get at least 0).

To see formally how to apply Theorem 1, call $p_i(\widehat{v})$ the payment to agent i, $i = S, B$ (it may be negative) in exchange for a trade between the seller and the buyer after the announcements $\widehat{v}_B$ and $\widehat{v}_S$ are made by the buyer and the seller, respectively. Ex post veto rights mean that in any event the seller must receive at least her valuation v_S in case of transaction (that is, $p_S(\widehat{v}) \geq v_S$ in case of trade) and that the buyer must get at least 0 in any event (that is, $v_B + p_B(\widehat{v}) \geq 0$ in case of trade). The ex post no subsidy constraint means that the sum of monetary transfers received by the seller and the buyer cannot exceed 0 ($p_S + p_B \leq 0$).

This trade problem can be cast into a bargaining problem with outside options, where the size of the pie V, outside options and transfers are defined as follows: $V = \overline{v}$, $w_S = v_S$, $w_B = \overline{v} - v_B$, $t_S(w) = p_S(v)$ and $t_B(w) = \bar{v} + p_B(v)$. It is readily verified that the inefficiency result in the seller/buyer problem is equivalent to the inefficiency result in this bargaining with outside option problem.[20]

The public good problem: A representative must decide whether or not to provide a public good. There are n agents $i = 1, ...n$. The cost of the public good is C. Agent i values the public good at $\theta_i \in (\underline{\theta}, \overline{\theta})$. Each agent i knows the value of θ_i, but not of θ_j, $j \neq i$. Everybody knows that $(\theta_1, ..., \theta_n)$ is distributed according to a joint distribution that has a bounded and smooth density with support $(\underline{\theta}, \overline{\theta})^n$, and we assume that $n\overline{\theta} - C \leq \overline{\theta} - \underline{\theta}$. That is, the maximum surplus from the public good

[20]Indeed, $w_S + w_B < V$ is equivalent to $v_S < v_B$; the no subsidy constraint $t_S(w) + t_B(w) \leq V$ is equivalent to $p_S(v) + p_B(v) \leq 0$; the ex post participation contraints $t_S(w) \geq w_S$ and $t_B(w) \geq w_B$ are respectively equivalent to $p_S(v) \geq v_S$ and $p_B(v) + v_B \geq 0$.

does not exceed the uncertainty about any agent's valuation for the public good.[21]

Efficiency would require to build the public project whenever $\sum_i \theta_i > C$. Besides, we assume that the community cannot receive ex post subsidies (that is, the sum of financial payments made by the agents must be at least equal to the cost C of the public good).[22]

Our analysis shows for this application that efficiency cannot be achieved whenever agents have the right to veto the public project (thereby enjoying a reservation utility of 0). As in the previous application, inefficiency is inevitable even if the distributions of willingness to pay are correlated and whatever the degree of correlation.

To see more formally the connection to Theorem 1, let $p_i(\widehat{\theta})$ denote the payment requested from agent i when the profile of announcements is $\widehat{\theta}$. The ex post veto right means that an agent i with type θ_i will refuse to make any payment greater than θ_i. The no subsidy constraint means that for any $\widehat{\theta}$ one should have $\sum_i p_i(\widehat{\theta}) \geq C$. Efficiency means that the public good should be implemented whenever $\sum_i \theta_i > C$.

No mechanism permits the implementation of the efficient decision rule whenever $(\theta_1, ...\theta_n)$ is distributed on $(\underline{\theta}, \overline{\theta})^n$ where we assume that $0 < n\overline{\theta} - C < \overline{\theta} - \underline{\theta}$ and the density is assumed to be smooth and bounded by a strictly positive number on its support.

This can be seen as a corollary of Theorem 1 where we define the bargaining problem $V = n\overline{\theta} - C$, with outside options $w_i = \overline{\theta} - \theta_i$.

The transfers in the bargaining problem $t_i(\widehat{w})$ should be identified with $\overline{\theta} - p_i(\widehat{\theta})$, and it is readily verified that the incentive constraints and veto right constraints in the bargaining problem are identical to the incentive constraints and veto right constraints in the public good problem, thereby establishing the inefficiency in the public good decision problem as a corollary of Theorem 1.

4 Ex post participation constraints versus ex post veto constraints

In this Section we explore how the ex post veto constraints as modelled above relate to the more familiar ex post participation constraints. We also discuss how the

[21] This reflects the idea that a single agent's lack of enthusiasm for the public project may undermine the desirability of making the public project.

[22] We also assume that building the public good requires the consent of every agent.

efficiency analysis is affected by the choice of one or the other form of constraints.

Ex post participation constraints assert that a party i with outside option w_i should get at least w_i in equilibrium whatever the realization of types w_j of other parties j, $j \neq i$. Thus, $t_i(w_i, w_{-i}) \geq w_i$, which is the same as (2) in Section 2. Note that these constraints alone give no guarantee that party i should get at least w_i off the equilibrium path, i.e. after announcing $\widehat{w}_i \neq w_i$. That is, with the usual approach, whenever w_i announces $\widehat{w}_i$ his payoff is:

$$U_i(\widehat{w}_i; w_i) = E_{w_{-i}} \left\{ t_i(\widehat{w}_i; w_{-i})\phi(\widehat{w}_i, w_{-i}) + w_i(1 - \phi(\widehat{w}_i, w_{-i})) \mid w_i \right\} \tag{8}$$

and not (3) as in Section 2. $U_i(\widehat{w}_i; w_i)$ takes this form because now announcing $\widehat{w}_i$ should be understood to mean that i follows fully the strategy of party i with type $\widehat{w}_i$ and not merely whenever this strategy gives no less than w_i, party i's true outside option. This distinction is relevant whenever for a non-zero measure of w_{-i} (given w_i), there is a positive probability of agreement at $(\widehat{w}_i, w_{-i})$, i.e. $\phi(\widehat{w}_i, w_{-i}) > 0$, and $t_i(\widehat{w}_i; w_{-i}) < w_i$.

Ex post participation constraints - we believe - can hardly be interpreted in terms of quitting rights, as it seems hard to justify that quitting rights could only be exerted on the equilibrium path.[23]

One alternative justification for ex post participation constraints is in terms of no regret. Party i should feel no regret, after seeing the outcome of the mechanism, for not having exerted his outside option before playing the game. Two objections though might be raised against this justification. First, the no regret idea, while appealing, cannot be derived from standard equilibrium constraints that individual strategies should satisfy: parties are comparing the outcome of the mechanism to an outside option that is no longer available. Besides, if one is willing to rule out the possibility that an agent feels regret about his decision to participate, why not also rule out the possibility that the agent feels regret about the strategy that he uses within the mechanism. The latter idea would lead to stronger notions of implementation such as dominant strategy or posterior implementation (see Green and Laffont (1987) for first introducing the idea of posterior implementation). So, in our view the no regret approach should lead to consider stronger forms of implementation (than the usual Nash Bayes implementation) together with ex post participation constraints.

[23] This is in contrast with the approach we have followed, in which quitting rights can be exerted *on and off* the equilibrium path.

In spite of these issues concerning the interpretation of ex post participation constraints, we now ask ourselves how our inefficiency result (Theorem 1) is affected by the choice of ex post participation constraints (i.e. where $U_i(\widehat{w}_i; w_i)$ should be expressed as (8)) or ex post veto constraints (i.e. where $U_i(\widehat{w}_i; w_i)$ should be expressed as (3)). We shall make the following observations.

(i) Under the condition of Theorem 1 (i.e. , with a smooth and bounded distribution of types $g(\cdot)$ that has full support on $[0, V]^n$) inefficiency also prevails under the ex post participation constraint scenario. This is despite the fact that the incentive constraints are less stringent under the ex post participation constraint scenario.

(ii) We shall illustrate below however that under alternative assumptions, the two types of constraints yield different conclusions. If $g(\cdot)$ does not have full support, or if the pie V depends decreasingly on the outside options (and if additional constraints prevail on the set of outside options a party with outside option w_i can pretend to be), efficiency is achievable with ex post participation constraints but not with ex post veto constraints.

Under the condition of Theorem 1, inefficiency must prevail with the ex post participation constraint approach. To see why, observe that our proof of Theorem 1 makes only use of the upward incentive constraints (i.e., party i with type w_i should not gain by pretending he is of type $\widehat{w}_i > w_i$, see subsection 3.1 and Appendix). The ex post participation constraint of party i with type $\widehat{w}_i$ implies that $t_i(\widehat{w}_i; w_{-i}) \geq \widehat{w}_i$ whenever $\phi(\widehat{w}_i, w_{-i}) > 0$. But, $\widehat{w}_i > w_i$ implies that $\max[t_i(\widehat{w}_i; w_{-i}), w_i] = t_i(\widehat{w}_i; w_{-i})$ and thus (3) and (8) take the same form whenever $\widehat{w}_i > w_i$.

When $g(\cdot)$ does not have full support, the situation is different because then the ex post participation constraint of say $\widehat{w}_i$ need not be satisfied if $(\widehat{w}_i; w_{-i})$ falls outside the support of $g(\cdot)$. This in turn opens the door to the possibility of punishment $(t_i(\widehat{w}_i; w_{-i}) < 0)$ when $(\widehat{w}_i; w_{-i})$ falls outside the support of $g(\cdot)$, which may facilitate the incentive constraints (thereby inducing an efficient outcome while preserving the ex post participation constraints).

For the sake of illustration, suppose that the support of $g(\cdot)$ coincides with the set $\Gamma = \{w \mid \sum_i w_i \leq V\}$ and that $g(\cdot)$ is bounded and smooth on its support. Consider any transfer scheme such that for all i, (i) $w_i \to t_i(w_i, w_{-i})$ is increasing in w_i on Γ, (ii) $t_i(w_i, w_{-i}) \geq w_i$ on Γ, and such that (iii) $t_i(w_i, w_{-i}) = -P$ for $(w_i, w_{-i}) \notin \Gamma$. It is readily verified that when P is set sufficiently large, such a transfer scheme allows to implement the efficient allocation rule in the ex post participation scenario.[24] By

[24]The monotonicity of t_i ensures that deviating downwards is not profitable and upwards devi-

contrast, with the veto right approach, inefficiency still holds in this case (see the discussion after Theorem 1).

To illustrate further the difference between ex post participation constraints and ex post veto constraints, we now consider the following situation. *The size of the pie is not constant* and it depends on the signals held by the various parties.[25] There are several applications one could think of with this feature. In a bargaining setup, the pie can be thought of as the output of a joint production, and this output may depend on characteristics of the parties. In a public good setup, the cost may depend on the characteristics of the agents for example because the final implementation of the public good will have to meet requirements that may be related to these specific characteristics.

Formally, we let $V(w_i, w_{-i})$ denote the size of the pie when party i's type is w_i. In the following example, we assume that $V(.,.)$ is decreasing.[26] We also assume that the entire surplus $V(w_i, w_{-i})$ must be distributed between the parties and that the agents cannot pretend that they have an outside option that is larger than their real outside option (they can only lie downwards).[27] The fact that V is decreasing (and that the entire surplus must be distributed) makes it attractive to pretend that one has a low outside option. This, in turn, explains why the ex post participation constraint approach and the ex post veto constraint approach are not equivalent, and, as it turns out, efficiency can be achieved with the former when it cannot be achieved with the latter.

A simple example: There are two parties $i = 1, 2$, (w_1, w_2) is uniformly distributed on $[0, 1]^2$, and $V(w_1, w_2) = \left| \begin{array}{c} 1 \text{ if } w_1 > w^* \text{ or } w_2 > w^* \\ V \text{ if } w_1 \leq w^* \text{ and } w_2 \leq w^* \end{array} \right.$ where $V > 1$ and $w^* < \frac{1}{2}$. Our assumption that the entire pie *must* be distributed between the

ations are deterred by the punishment P for P sufficiently large.

[25] In the next Section we suggest an extension of our model to the case where V may vary with the types of the agents. We show that our inefficiency result can be extended to such a framework when either V is increasing or V does not vary too much with the types of agents. In this Section we simply illustrate that when V is decreasing, the ex post participation constraints and the ex post veto constraints may lead to different predictions.

[26] One motivation for V being decreasing in the bargaining example may be that if one has a low outside options one is more willing to exert effort internally (within the team) to increase the size of the pie.

[27] A motivation for this may be that parties have to certify that they have at least the outside option they pretend to have. See Green and Laffont (1986) for a first approach to mechanism design with partially verifiable information.

two parties writes as:

$$t_1(w_1, w_2) + t_1(w_1, w_2) = V(w_1, w_2)$$

whenever $\phi(w_1, w_2) > 0$. We further consider direct truthful mechanisms in which party i with type w_i cannot pretend he is type $\widehat{w}_i > w_i$ (he can pretend he is type $\widehat{w}_i < w_i$).[28]

We shall make two claims:

Claim A: Suppose $V > 2 - w^*$. Then the first-best cannot be achieved with the ex post veto approach.

Claim B: Suppose that $V < 2 - w^* + \frac{(1-2w^*)^2}{2w^*}$. Then the first-best can be achieved with the ex post participation constraint approach.

These claims have the following corollary:

Corollary: Whenever $2 - w^* + \frac{(1-2w^*)^2}{2w^*} > V > 2 - w^*$ the first-best can be achieved with the ex post participation constraint approach, but not with the ex post veto constraint approach.

We start with the proof of claim A.

Proof of claim A: Suppose the first-best can be achieved. There must exist $x < w^*$ or $y < w^*$ such that $E_{w_2}[t_1(x, w_2) \mid w_2 \leq w^*] \geq \frac{V}{2}$ or $E_{w_1}[t_2(w_1, y) \mid w_1 \leq w^*] \geq \frac{V}{2}$ (as otherwise $E_{w_1,w_2}[t_1(w_1, w_2) + t_2(w_1, w_2) \mid w_1,w_2 \leq w^*] < V$ violating the premise that $t_1(w_1, w_2) + t_2(w_1, w_2) = V$ whenever $w_1,w_2 \leq w^*$).

Suppose this holds for x, set $z > w^*$, and consider party 1 of type $w_1 = z$. At best, party 1 with type w_1 gets $1 - w_2$ when $w_2 \leq 1 - z$ and w_1 otherwise. That is, by telling the truth, party 1 with type w_1 gets at most:

$$\int_0^{1-z} (1 - w_2)dw_2 + \int_{1-z}^{1} w_1 dw_2 = \left[\frac{-(1 - w_2)^2}{2}\right]_0^{1-z} + w_1 z = \frac{1 + z^2}{2} \tag{9}$$

in expectation.

By pretending he is type x, party 1 with type $w_1 = z$ gets at least

$$z(1 - w^*) + \frac{V}{2} w^* \tag{10}$$

Take now $z = 1 - w^*$.[29] (10) is larger than (9) whenever

$$V > 2 - w^*. \tag{11}$$

[28] The restriction to direct truthful mechanisms is legitimate because our structure satisfies the *Nested Range Condition* of Green and Laffont (1986).

[29] This is the value of z that maximizes the difference (10)-(9).

Thus, when (11) holds, the first-best cannot be achieved with the ex post veto constraint approach. **Q. E. D.**

We now turn to the proof of Claim B.

Proof of Claim B: Consider the efficient allocation rule, i.e. $\phi(w_1, w_2) = 1$ when $w_1 + w_2 \leq 1$ and $\phi(w_1, w_2) = 0$ when $w_1 + w_2 > 1$. Define party 1's transfer for $w = (w_1, w_2)$, $w_1 + w_2 \leq 1$ as:

$$t_1(w_1, w_2) = \left| \begin{array}{c} V/2 \text{ for } w_1 \leq w^* \text{ and } w_2 \leq w^* \\ w_1 \text{ for } w_1 \leq w^* \text{ and } w_2 > w^* \\ 1 - w_2 \text{ for } w_1 > w^* \text{ and } w_2 \leq w^* \\ w_1 + \frac{1-w_1-w_2}{2} \text{ for } w_1 > w^* \text{ and } w_2 > w^* \end{array} \right.$$

and symmetrically for party 2's transfer. Clearly, the ex post participation constraints are satisfied and $t_1(w_1, w_2) + t_2(w_1, w_2) = V(w_1, w_2)$ for all $w = (w_1, w_2)$, $w_1 + w_2 \leq 1$.

It remains to check the incentive constraints. For types $w_1 \leq w^*$, downward incentive constraints are immediately satisfied. For types $w_1 > w^*$, we just have to check that party 1 prefers announcing his true type $\widehat{w}_1 = w_1$ rather than $\widehat{w}_1 = w^*$ (because other announcements are dominated by either $\widehat{w}_1 = w_1$ or $\widehat{w}_1 = w^*$). This yields the condition:

$$\frac{V}{2}w^* + w^* \int_{w^*}^{1-w^*} dw_2 + w_1 \int_{1-w^*}^{1} dw_2 \leq \int_0^{1-w_1} t_1(w_1, w_2) dw_2 + w_1 \int_{1-w_1}^{1} dw_2$$

It is easy to check that this condition holds for all $w_1 > w^*$ if and only if it holds at the limit where w_1 tends to w^*, which yields the condition[30] $V < 2 - w^* + \frac{(1-2w^*)^2}{2w^*}$.
Q. E. D.

5 Extension and future directions

5.1 Pies of interdependent size

Following the example of Section 4, we now consider an extension of the model where we allow the size of the pie to be a function of the signals received by the

[30]Indeed, at the limit, the inequality becomes

$$\frac{V}{2}w^* + w^*(1 - 2w^*) \leq w^*(1 - \frac{w^*}{2}) + (1 - 2w^*)(\frac{1}{4} + \frac{w^*}{2})$$

which is equivalent to the desired condition.

parties, thereby resulting in some form of interdependence. Formally, the pie is now of size $V(w_i, w_{-i})$. We assume that $\frac{\partial V}{\partial w_i}(w) < a < 1$ and w is distributed according to a smooth and bounded density $g(\cdot)$ with support on $\times_{i=1}^{n}[0, \overline{w}_i]$ where $\overline{w}_i$ satisfies $\overline{w}_i > V(\overline{w}_i, 0)$.

Direct mechanisms with veto rights are defined as in Section 2. Direct truthful mechanisms are direct mechanisms with the additional requirement that in equilibrium parties report their true types to the third party and proposed agreements are ratified.[31] The incentive and ex post veto constraints take the same form as in Section 2. The main change compared to the private value case lies in the writing of the ex post no subsidy constraint, as now, one should have that

$$\sum_i t_i(w) \leq V(w) \tag{12}$$

whenever $\phi(w) > 0$. An agreement may be viewed as the decision to form a productive team that delivers a monetary output of $V(w)$. In this interpretation, $t_i(w)$ is the monetary compensation or wage received by i to participate in the joint production, and the third party is the residual claimant obtaining $V(w) - \sum_i t_i(w)$ for herself.

In the case where the pie has a constant size V, the condition (12) arises as before due to the quitting right of the third party: in no event this third party should make losses. With a pie of interdependent size, (12) arises as the requirement that the third party should *believe* that she makes no loss given the *equilibrium inference* that follows from the announcement. Since in equilibrium each party i reports his true type $\widehat{w}_i = w_i$, (12) follows.

It should be noted however that assuming that the third party gets to learn the announcement $\widehat{w}_i$ of each party i is not without loss of generality, as one could a priori imagine that the announcements are made to a fourth party who would not be the residual claimant and whose role would consist in transmitting information. From this broader perspective, what is being disclosed to the third party should itself be endogenized: it need not be the full reports of types, but only the transfers that ought to be made to each party i.[32]

When the third party is sufficiently risk averse, it is not difficult to see that the best case for efficiency is when the announced types are fully disclosed to the third party (this is because under infinite risk aversion hiding some aspects of $\widehat{w}$ can only

[31] The assumption that reports are directly sent to the third party has a bite in our context with pies of interdependent size. This will be discussed at length below.

[32] A similar observation appears in Forges (1999).

make the acceptance of the third party worse), and thus (12) appears as a necessary condition.

When the risk aversion of the third party is less extreme however, the treatment of the quitting rights of the third party is more complex and it should be the subject of further work.[33] In this more general case (and no matter what the risk attitude of the third party is), we note that one should have that for all w with $\phi(w) > 0$:

$$\sum_i t_i(w) \leq \overline{V} \tag{13}$$

where $\overline{V} = \sup_{w \in \times_{i=1}^{n} [0, \overline{w}_i]} V(w)$.

We refer to (12) as the uniform no subsidy constraints and to (13) as the weak no subsidy constraint. We have:

Theorem 2: *(i) Assume that V is non-decreasing in w_i for all i. Then inefficiencies must arise in equilibrium in any direct truthful mechanism with veto right (in which the uniform no subsidy constraint applies). (ii) Whenever V does not vary too much in its domain (i.e $\sup_{w,\, w'} [V(w) - V(w')]$ is sufficiently small), inefficiencies must arise under the weak no subsidy constraint scenario.*

5.2 Ex post no subsidy versus Ex ante no subsidy

We return to the case of pies with constant size. In our analysis, we have assumed that parties could receive no subsidy ex post. One may wonder what happens if we only require that the parties receive no subsidy ex ante. We wish to illustrate here that for some distributions over outside options, efficiency can be achieved while satisfying the ex post veto constraints, if only the ex ante no subsidy constraint is required.

To this end, we assume there are two parties $i = 1, 2$, and we consider a distribution over outside options defined as follows.[34] With probability $p > 0$, outside options are distributed according to a density g_0 with full support on $[0, V]^2$. With probability $1 - p$, outside options are distributed uniformly on $F = \{(w_1, V - w_1), w_1 \in [0, V]\}$. We construct below transfers that implement the efficient outcome.

[33] In the context of the simple example of Section 4, since the pie had to be entirely distributed, the third party had to know whether $w_i < w^*$ for $i = 1, 2$ or not, which was enough to make claim A.

[34] The example falls outside the class of distributions covered in Theorem 1. Yet, we conjecture that a slight modification would allow us to provide an example falling in this class.

Specifically, we set

$$t_i(w_1, w_2) = w_i \text{ when } w_1 + w_2 < V$$

and

$$t_i(w_1, w_2) = w_i + T(w_i) \text{ when } w_1 + w_2 = V$$

Intuitively, the idea is to subsidize agreement ex post by a substantial amount $T(w_i)$ whenever the announcement falls on the frontier. When party i overstates his outside option, and announces $\widehat{w}_i > w_i$, he obtains a transfer equal to $\widehat{w}_i$ instead of w_i with probability $p \Pr_{g_0}\{w_j < V - \widehat{w}_i \mid w_i\}$. However, with probability $(1-p)$, he loses the subsidy. So choosing the subsidy $T(w_i)$ so that

$$(1-p)T(w_i) = p \max_{\widehat{w}_i}(\widehat{w}_i - w_i) \Pr_{g_0}\{w_j < V - \widehat{w}_i \mid w_i\} \tag{14}$$

ensures that party i has incentives to report his outside option truthfully.

Having defined $T(w_i)$ for all w_i, it remains to check whether ex ante, these subsidies remain smaller than the expected surplus generated by the agreement. To do that, it is sufficient to check that conditional on each w_i, the expected subsidy $(1-p)T(w_i)$ is smaller than half the expected surplus, that is,

$$(1-p)T(w_i) \leq \frac{1}{2} p E_{g_0}(V - w_i - w_j \mid w_i). \tag{15}$$

It is easy to check that (14) and (15) are compatible for a class of distributions g_0. [35]

5.3 The Second-Best

We have seen in Section 2 that inefficiencies are inevitable even if the distribution of outside options exhibit correlation whenever parties can exert their veto right at any point in time and the pie has constant size V. An interesting next step would be to analyze the form of the second-best in such situations.

[35] For example, if $g_0(w_1, w_2)$ is proportional to $w_1 w_2$, one obtains

$$\begin{aligned}(1-p)T(w_i) &= \frac{p}{2} \max(\widehat{w}_i - w_i) \frac{(V - \widehat{w}_i)^2}{2} \\ &= \frac{p}{27}(V - w_i)^3\end{aligned}$$

and

$$\frac{1}{2} p E_{g_0}(V - w_i - w_j \mid w_i) = \frac{1}{2}\frac{p}{2}\frac{1}{6}(V - w_i)^3.$$

Since $\frac{1}{27} < \frac{1}{24}$, we get the desired inequality.

Abstracting from the additional constraints imposed by the veto rights of agents, it should be mentioned that in the case of correlated types, it is not possible to infer uniquely the expected transfers to be given to agents from the allocation rule. This makes the analysis of the second-best much harder in the correlated case.

Abstracting from correlation, a few researchers have tried to characterize when the second-best with interim participation constraints can be achieved with the more demanding ex post participation constraints. Attempts along these lines include Myerson and Satterthwaite's original work and Gresik (1991). Myerson and Satterthwaite observe in the buyer/seller problem that when valuations are uniformly distributed on some interval, the second-best can be implemented using the split-the difference mechanism (first studied by Chatterjee and Samuelson (1983)),[36] which satisfies the ex post participation constraints. Gresik (1991) extends this observation to all distributions that are unimodal. The second-best with ex post participation constraints is not known for more general distributions of types (still assumed to be independent across agents).

Combining the two difficulties (plus the additional observation that ex post participation constraints need not be equivalent to ex post veto constraints) makes it very hard to characterize the second-best, and more work is required for that task. In some special cases though, we may take advantage of existing results and the observation that the second-best is independent of the distribution of types w s.t. $\sum_i w_i > V$ (see the discussion following Theorem 1) to characterize the second-best.

For the sake of illustration, assume that there are two parties $i = 1, 2$ and that conditional on $w_1 + w_2 \leq V$ outside options are uniformly distributed on $\{w \mid w_1 + w_2 \leq V\}$.[37] Such a distribution allows for correlation as it makes no assumption on the specification of the distribution on $\{w \mid w_1 + w_2 > V\}$. In particular, we may well have $g(w_1, w_2) = 0$ whenever $w_1 + w_2 > V$ in which case it is common knowledge that an agreement is beneficial.

We claim that the second-best can be implemented through a direct mechanism with veto rights, which refer to as the *Nash bargaining protocol*,[38] characterized by

[36] In this mechanism both the seller and the buyer quote a price; if the seller's price is below the buyer's price, there is trade at a price that is equal to the average of the two quoted prices; there is no trade otherwise.

[37] A similar argument can be made for situations in which conditional on $w_1 + w_2 \leq V$, outside options (w_1, w_2) are unimodally distributed so that Gresik's analysis holds.

[38] This protocol is the analog of the split-the-difference mechanism in the buyer/seller problem.

the probabilities

$$\phi(\widehat{w}_1, \widehat{w}_2) = 1 \text{ if } \widehat{w}_1 + \widehat{w}_2 \leq V, \text{ and } \phi(\widehat{w}_1, \widehat{w}_2) = 0 \text{ otherwise},$$

and by transfers

$$\tau_i(\widehat{w}_1, \widehat{w}_2) = \widehat{w}_i + \frac{V - \widehat{w}_1 - \widehat{w}_2}{2} = \frac{V + \widehat{w}_i - \widehat{w}_j}{2}$$

That is, a proposal is made if and only if the announcements are compatible with the size of the pie V. Transfers are chosen so that each party i obtains, in addition to $\widehat{w}_i$, half the surplus $V - \widehat{w}_1 - \widehat{w}_2$. Exploiting Myerson-Satterthwaite's analysis, we get:

Claim C: Suppose that conditional on $w_1 + w_2 \leq V$, (w_1, w_2) is uniformly distributed on $\{(w_1, w_2) \mid w_1 + w_2 \leq V\}$. Then the second-best can be implemented through the *Nash bargaining protocol*, and it leads to an agreement if and only if $w_1 + w_2 \leq \frac{3V}{4}$.

References

[1] Akerlof, G. (1970): "The Market for Lemons: Quality, Uncertainty and the Market Mechanism," *Quarterly Journal of Economics* ***84***, 488-500.

[2] Chatterjee K. and W. Samuelson (1983): "Bargaining under Incomplete Information," *Operations Research* **31**, 835–851

[3] Crémer, J. and R. McLean (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica* **56**, 1247–1257.

[4] Forges, F. (1999): "Ex post Individually Rational Trading Mechanisms", in *Current trends in Economics,* Springer-Verlag, 157-175, Alkan,A. C. Aliprantis and N. Yannelis (eds).

[5] Green, J and J. J. Laffont (1986): "Partially Verifiable Information and Mechanism Design," *Review of Economic Studies* **53**, 447-456.

[6] Green, J and J. J. Laffont (1987): "Posterior Implementation in a Two-Person Decision Problem," *Econometrica* **55**, 69-94.

[7] Gresik, T. (1991): "Ex ante Efficient, ex post individually rational trade," *Journal of Economic Theory*, **53**, 131-145.

[8] Heifetz, A. and Z. Neeman (2006): "On the Generic (Im)possibility of Full Surplus Extraction in Mechanism Design" *Econometrica*, **74**, 213 - 233.

[9] Kosmopoulou, G. and S. Williams (1998),"The robustness of the independent private value model in Bayesian mechanism design", *Economic Theory*, **12**, 393-421.

[10] Laffont J. J. and D. Martimort (2000) "Mechanism Design with Collusion and Correlation," *Econometrica* **68**, 309-342.

[11] Mailath, G. and A. Postlewaite (1990), "Asymmetric Information Bargaining Problems with Many Agents", *Review of Economic Studies* **57**, 351-367

[12] Matsuo, T. (1989): "On Incentive Compatible, Individually Rational, and Ex Post Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, **49**, 189-194.

[13] Matthews, S. and Postlewaite, A. (1989), "Pre-play Communication in Two-Person Sealed-Bid Double Auctions", *Journal of Economic Theory*, **48**, 238–263.

[14] McAfee, R. P and P. Reny (1992): "Correlated Information and Mecanism Design." *Econometrica* **60**, 395-421.

[15] Myerson, R. and M. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," with M. Satterthwaite, *Journal of Economic Theory*, **29**, 265-281.

[16] Neeman, Z (2004): "The Relevance of Private Information in Mechanism Design, *Journal of Economic Theory*," **117**, 55-77.

[17] Robert J. (1991): "Continuity in Auction Design," *Journal of Economic Theory* **55**, 169-179.

6 Appendix

Proof of Theorems 1 and 2:

The differentiable case:

We start by showing Theorem 1 in the case where $t_i(w)$ is differentiable. We will later show how to relax this assumption and how to generalize the proof to cover Theorem 2.

We first derive a condition on transfers implied by incentive compatibility conditions. Party i should prefer reporting he is of type w_i rather than of type $\widehat{w}_i = w_i + \varepsilon$. When he reports $\widehat{w}_i$ (rather than w_i), he gains $t_i(\widehat{w}_i, w_{-i}) - t_i(w_i, w_{-i})$ whenever $\widehat{w}_i + w_{-i} \leq V$, and he loses no more than $t_i(w_i, w_{-i}) - w_i$ in events where $w_i + w_{-i} \leq V$ and $\widehat{w}_i + w_{-i} > V$. (In other events, there is no loss because he cannot expect more than his outside option payoff.) Incentive compatibility conditions thus require that

$$\int_{\widehat{w}_i + w_{-i} \leq V} (t_i(\widehat{w}_i, w_{-i}) - t_i(w_i, w_{-i}))g(w)dw_{-i} \leq \int_{\substack{\widehat{w}_i + w_{-i} > V \\ w_i + w_{-i} \leq V}} (t_i(w) - w_i)g(w)dw_{-i} \tag{16}$$

When $\widehat{w}_i + w_{-i} > V$, the surplus is at most equal to ε. Since $t_i(w) - w_i$ cannot exceed the surplus, the right hand side of (16) is comparable to ε^2. Dividing by ε on both sides and taking the limit of this comparison as ε goes to 0 yields (thanks to the differentiability assumption on t_i):

$$\int_{w_i + w_{-i} \leq V} \frac{\partial t_i}{\partial w_i}(w_i, w_{-i})g(w_i, w_{-i})dw_{-i} \leq 0. \tag{17}$$

Define the following function for every $w_i \in (0, V)$.

$$H_i(w_i) \equiv \int_{w_i + w_{-i} \leq V} (V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))g(w_i, w_{-i})dw_{-i} \tag{18}$$

We will prove that $H_i(w_i) = 0$ for all $w_i \in (0, V)$. Given that $V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}) \geq 0$ is non-negative (we know from (5) that $V - \sum_{j \neq i} w_j$ is the maximum transfer that party i can hope to get when each party j's outside option is given by w_j), we will deduce that for all (w_i, w_{-i}) such that $w_i + w_{-i} \leq V$:

$$t_i(w_i, w_{-i}) \equiv V - \sum_{j \neq i} w_j.$$

To establish that $H_i(\cdot) \equiv 0$, observe that[39]

$$\begin{aligned} \frac{dH_i(w_i)}{dw_i} &= -\int_{w_i + w_{-i} \leq V} \frac{\partial t_i}{\partial w_i}(w_i, w_{-i})g(w_i, w_{-i})dw_{-i} \\ &\quad + \int_{w_i + w_{-i} \leq V} (V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))\frac{\partial g}{\partial w_i}(w_i, w_{-i})dw_{-i} \end{aligned}$$

Using (17) we get:

$$\frac{dH_i(w_i)}{dw_i} \geq \int_{w_i + w_{-i} \leq V} (V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))\frac{\partial g}{\partial w_i}(w_i, w_{-i})dw_{-i} \tag{19}$$

[39] The term corresponding to the variation of the domain of integration does not appear because at the boundary the veto constraint together with the ex post no subsidy constraint imply that for w such that $\sum_j w_j = V$, $t_i(w_i, w_{-i}) = w_i$ and thus $V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}) = 0$.

But, note that

$$H_i(V) = 0,$$

since when $w_i = V$ the domain of w_{-i} such that $w_i + w_{-i} \leq V$ has measure 0.

Thus, when $\frac{\partial g}{\partial w_i}(w_i, w_{-i}) \geq 0$ for all $w_i \in (0, V)$, (19) allows us to conclude that $\frac{dH_i(w_i)}{dw_i} \geq 0$ for all $w_i \leq V$. Since $H_i(w_i)$ is non-negative everywhere (by the no ex post subsidy requirement) and since $H_i(V) = 0$, we conclude that $H_i(w_i) = 0$ everywhere, as desired.

In the general case where the variations of g may be arbitrary, observe that the fact that g has a strictly positive lower bound on its support and that g varies smoothly with w_i guarantee that there must exist a constant a (possibly negative) such that for all $(w_i, w_{-i}) \in \Gamma_V$:

$$\frac{\partial g}{\partial w_i}(w_i, w_{-i}) > ag(w_i, w_{-i}).$$

Given the non-negativeness of $V - \sum_{j \neq i} w_j - t_i(w_i, w_{-i})$, we infer from (19) that:

$$\frac{dH_i(w_i)}{dw_i} \geq aH_i(w_i).$$

Thus, $H_i(V) \geq \exp(a(V - w_i))H_i(w_i)$. Since $H_i(V) = 0$, and $H_i(w_i) \geq 0$, we conclude that $H_i(w_i) = 0$, as desired. **Q. E. D.**

The general case:

This case will cover both Theorem 1 and Theorem 2 (ii), where variations of the size of the pie are bounded by ε. Also we will no longer restrict our attention to differentiable transfer functions. We consider a direct truthful mechanism with veto rights that is efficient and that satisfies the ex post no subsidy constraint, and we establish an upper bound on

$$H_i(w_i) = \int_{w_i + w_{-i} \leq \bar{V} - \varepsilon} g(w_i, w_{-i})(\bar{V} - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))dw_{-i}.$$

We will prove that there exists a constant a independent of ε such that

$$H_i(w_i) \leq a\varepsilon. \tag{20}$$

For all (w_i, w_{-i}) such that $w_i + w_{-i} \leq V(w_i, w_{-i})$, the constraint $t_i(w_i, w_{-i}) \geq w_i$ must hold. Since $V(w_i, w_{-i}) \geq \bar{V} - \varepsilon$, inequality (20) in turn imply a lower bound on player i's expected utility. Let $S = \bar{V} - \sum_j w_j$ denote an upperbound on total surplus. We have:

$$\begin{aligned}
E(U_i - w_i) &\geq \int_{w_i+w_{-i}\leq V(w_i,w_{-i})} g(w_i,w_{-i})(t_i(w_i,w_{-i})-w_i)dw \\
&\geq \int_{w_i+w_{-i}\leq \bar{V}-\varepsilon} g(w_i,w_{-i})(t_i(w_i,w_{-i})-w_i)dw \\
&\geq \int_{w_i+w_{-i}\leq \bar{V}-\varepsilon} g(w_i,w_{-i})(\bar{V}-\sum_j w_j)dw - \int_{w_i+w_{-i}\leq \bar{V}-\varepsilon} H(w_i)dw_i \\
&\geq \int_{w_i+w_{-i}\leq \bar{V}} g(w_i,w_{-i})Sdw - \varepsilon \Pr(S\in[0,\varepsilon]) - a\varepsilon\bar{V}
\end{aligned}$$

Adding these inequalities for all players, and since

$$\sum_i E(U_i - w_i) \leq \int_{w_i+w_{-i}\leq \bar{V}} g(w_i,w_{-i})Sdw,$$

we obtain:

$$(N-1)\int_{w_i+w_{-i}\leq \bar{V}} g(w_i,w_{-i})Sdw \leq N\varepsilon(\Pr(S\in[0,\varepsilon]) + aV)$$

which is impossible for ε small.

We now turn to the critical part of the proof, which consists in showing that inequality (20) holds.

First observe that the ex post participation and the no subsidy constraints together imply that for all (w_i, w_{-i}) such that $w_i + w_{-i} \leq \bar{V} - \varepsilon$,

$$\bar{V} - \sum_{j\neq i} w_j \geq t_i(w_i,w_{-i}) \geq w_i, \tag{21}$$

which implies that $H_i(w_i) \geq 0$. We now use incentive compatibility constraints to derive an upper bound on $H_i(w_i)$. Incentive compatibility requires that for all $\widehat{w}_i > w_i$,

$$\begin{aligned}
\int_{w_i+w_{-i}\leq V(w_i,w_{-i})} g(w_i,w_{-i})t_i(w_i,w_{-i})dw_{-i} &\geq \int_{\widehat{w}_i+w_{-i}\leq \bar{V}-\varepsilon} g(w_i,w_{-i})t_i(\widehat{w}_i,w_{-i})dw_{-i} + \\
&\quad + w_i \int_{\substack{\widehat{w}_i+w_{-i}>\bar{V}-\varepsilon \\ w_i+w_{-i}\leq V(w_i,w_{-i})}} g(w_i,w_{-i})dw_{-i},
\end{aligned}$$

Since $\bar{V} - \sum_{j\neq i} w_j - t_i(w_i,w_{-i}) \geq 0$, we obtain:

$$\begin{aligned}
H_i(w_i) &\leq \int_{w_i+w_{-i}\leq V(w_i,w_{-i})} g(w_i,w_{-i})(\bar{V}-\sum_{j\neq i} w_j - t_i(w_i,w_{-i}))dw_{-i}. \\
&\leq \int_{\widehat{w}_i+w_{-i}\leq \bar{V}-\varepsilon} g(w_i,w_{-i})(\bar{V}-\sum_{j\neq i} w_j - t_i(\widehat{w}_i,w_j))dw_j \\
&\quad + \int_{\substack{\widehat{w}_i+w_{-i}>\bar{V}-\varepsilon \\ w_i+w_{-i}\leq \bar{V}}} g(w_i,w_{-i})(\bar{V}-\sum_j w_j)dw_{-i}
\end{aligned}$$

Let $\Delta = \widehat{w}_i - w_i$. The last term is bounded by $(\varepsilon + \Delta)\Pr\{0 \leq S \leq \varepsilon + \Delta\}$, hence it is below $b(\varepsilon+\Delta)^2$ for some constant b independent of ε and Δ. To bound the first term, Remember that there exist $m > 0$ and M such that $m \leq g(w_i, w_{-i})$ and $\mid \frac{\partial g}{\partial w_i} \mid \leq M$, $i = 1, 2$, hence we have:

$$\begin{aligned} g(w_i, w_j) &\leq g(\widehat{w}_i, w_j) + \mid g(w_i, w_{-i}) - g(\widehat{w}_i, w_{-i}) \mid \\ &\leq g(\widehat{w}_i, w_j)(1 + M\Delta/m). \end{aligned}$$

We thus obtain:

$$H_i(w_i) \leq H_i(\widehat{w}_i)(1 + \frac{M}{m}\Delta) + b(\varepsilon + \Delta)^2$$

Let $\rho = \max(M/m, b)$ and consider $w_i \leq v - \varepsilon$. We choose $\Delta = \frac{\bar{V} - \varepsilon - w_i}{N}$, where N is set so that $\varepsilon/2 \leq \Delta \leq \varepsilon$. We obtain the sequence of inequalities:

$$\begin{aligned} H_i(w_i) &\leq H_i(w_i + \Delta)(1 + \rho\Delta) + \rho(\varepsilon + \Delta)^2 \\ &\leq H_i(w_i + 2\Delta)(1 + \rho\Delta)^2 + \rho(\varepsilon + \Delta)^2(1 + (1 + \rho\Delta)) \\ \ldots &\leq H_i(w_i + n\Delta)(1 + \rho\Delta)^n + \rho(\varepsilon + \Delta)^2 \sum_{k=0}^{n-1}(1 + \rho\Delta)^k \\ \ldots &\leq \rho(\varepsilon + \Delta)^2 \sum_{k=0}^{N-1}(1 + \rho\Delta)^k \text{ (since } H_i(w_i + N\Delta) = H_i(\bar{V} - \varepsilon) = 0) \\ &\leq \rho\frac{(\varepsilon + \Delta)^2}{\Delta}\Delta N(1 + \rho\Delta)^N \end{aligned}$$

As N gets large, the term $\Delta N(1 + \rho\Delta)^N$ remains bounded (by $Ve^{\rho V}$). Since the inequalities hold for all N, $H_i(w_i)$ remains bounded above by $\rho\frac{(2\varepsilon)^2}{\varepsilon/2} = 8\rho\varepsilon$, which concludes the proof. **Q. E. D.**

The case where V is increasing (Theorem 2 (i)).

Let $\overline{\overline{w}}_i$ such that $\overline{\overline{w}}_i = V(\overline{\overline{w}}_i, 0)$. We define, for all $w_i \in [0, \overline{\overline{w}}_i]$,

$$H_i(w_i) = \int_{w_i + w_{-i} \leq V(w_i, w_{-i})} g(w_i, w_{-i})(V(w_i, w_{-i}) - \sum_{j \neq i} w_j - t_i(w_i, w_{-i}))dw_{-i}.$$

and show that we must have $H_i(.) = 0$. This will permit us to conclude as before. Consider $w_i < \overline{\overline{w}}_i$ and $\widehat{w}_i \in (w_i, \overline{\overline{w}}_i]$. Incentive compatibility requires that for all $\widehat{w}_i > w_i$,

$$\begin{aligned} \int_{w_i + w_{-i} \leq V(w_i, w_{-i})} g(w_i, w_{-i})t_i(w_i, w_{-i})dw_{-i} &\geq \int_{\widehat{w}_i + w_{-i} \leq V(\widehat{w}_i, w_{-i})} g(w_i, w_{-i})t_i(\widehat{w}_i, w_{-i})dw_{-i} + \\ &+ w_i \int_{\substack{\widehat{w}_i + w_{-i} > V(\widehat{w}_i, w_{-i}) \\ w_i + w_{-i} \leq V(w_i, w_{-i})}} g(w_i, w_{-i})dw_{-i}, \end{aligned}$$

Using this inequality, the definition of $H_i(.)$ and since V is increasing, we obtain:

$$\begin{aligned} H_i(w_i) \leq & \int_{\widehat{w}_i + w_{-i} \leq V(\widehat{w}_i, w_{-i})} g(w_i, w_{-i})(V(\widehat{w}_i, w_{-i}) - \sum_{j \neq i} w_j - t_i(\widehat{w}_i, w_j))dw_j \\ & + \int_{\substack{\widehat{w}_i + w_{-i} > V(\widehat{w}_i, w_{-i}) \\ w_i + w_{-i} \leq V(w_i, w_{-i})}} g(w_i, w_{-i})(V(w_i, w_{-i}) - \sum_j w_j)dw_{-i} \end{aligned}$$

Let $\Delta = \widehat{w}_i - w_i$. The last term is bounded by $b(\Delta)^2$ for some constant b independent Δ. We use the same technique as before to bound $g(w_i, w_j)$. The rest of the proof is identical:[40] we choose increments $\Delta = \frac{\overline{\overline{w}}_i - w_i}{N}$ and obtain, $H_i(w_i) \leq \rho\Delta[\Delta N(1 + \rho\Delta)^N]$, which tends to 0 as Δ tends to 0 (or N gets large), which concludes the proof. **Q. E. D.**

Before proving Claim C, we state the following claim which constructs an equilibrium of the Nash bargaining protocol. This claim can be seen as a corollary of Chatterjee and Samuelson (1983):

Claim D: Suppose that conditional on $w_1 + w_2 \leq V$, (w_1, w_2) is uniformly distributed on $\{(w_1, w_2) \mid w_1 + w_2 \leq V\}$. Then the announcement strategies defined for each party i with type w_i by the announcement $\widehat{w}_i = a(w_i)$ where

$$a(w_i) = \frac{1}{4}V + \frac{2}{3}w_i$$

are in equilibrium. There is agreement when $w_1 + w_2 \leq \frac{3V}{4}$. The outside option alternative is implemented when $w_1 + w_2 > \frac{3V}{4}$.

For completeness, we will provide a brief proof of this claim shortly. Also it will illustrate that Chatterjee and Samuelson's analysis does not depend on the specification of the distribution of over types for events where the best alternative is the status quo (here the outside option). Let us see first how Claim D can be used to show Claim C.

Proof of Claim C: Observe that $\left\{(w_1, w_2) \mid w_1 + w_2 \leq \frac{3V}{4}\right\}$ is the domain of agreement in the second-best problem in which outside options are uniformly distributed on $[0, V]^2$ and only the interim participation constraints together with the Nash-Bayes incentive constraints and the ex ante budget-balancedness are required (this follows from Myerson and Satterthwaite (1983), see their characterization on pages 276-277).

This allows us to conclude that the Nash bargaining protocol implements the second-best when parties have the right to veto the agreement at any point in time,

[40] The only difference is that now, ε should be set equal to 0.

and conditional on $w_1 + w_2 \leq V$, outside options (w_1, w_2) are uniformly distributed on $\{(w_1, w_2) \mid w_1 + w_2 \leq V\}$.

To see this, suppose that this is not the second-best, so that there is a mechanism that generates a strictly higher expected welfare when conditional on $w_1 + w_2 \leq V$, outside options (w_1, w_2) are uniformly distributed on $\{(w_1, w_2) \mid w_1 + w_2 \leq V\}$. We could then also improve upon the second-best of Myerson-Satterthwaite when each w_i is independently and uniformly distributed on $(0, V)$ by considering a mechanism stipulating the same transfers and allocations for $\widehat{w} = (\widehat{w}_1, \widehat{w}_2)$ with $\widehat{w}_1 + \widehat{w}_2 \leq V$ and no agreement with no transfer when $\widehat{w}_1 + \widehat{w}_2 > V$. This would obviously contradict Myerson and Satterhwaite's analysis.**Q. E. D.**

We now turn to the proof of Claim D.

Proof of Claim D: The expected gain of party 1 with type w_1 when announcing $\widehat{w}_1$ is

$$\begin{aligned} G(w_1, \widehat{w}_1) &= \int_{\frac{V - \widehat{w}_1 + a(w_2)}{2} > w_2} \max(w_1, \frac{V + \widehat{w}_1 - a(w_2)}{2}) \frac{dw_2}{V - w_1} \\ &+ w_1 \int_{\frac{V - \widehat{w}_1 + a(w_2)}{2} < w_2} \frac{dw_2}{V - w_1} \end{aligned}$$

We now check that it is optimal for party 1 to announce $\widehat{w}_1 = a(w_1)$. Given the form of $a(\cdot)$ it is readily verified that whenever the announcements are compatible, i.e. $a(w_1) + a(w_2) < V$, we have that $a(w_i) > w_i$ for $i = 1, 2$, hence the Nash bargaining share of each party i is above w_i. This allows us to simplify the expression of $G(w_1, \widehat{w}_1)$ when $\widehat{w}_1$ lies in a neighborhood of $a(w_1)$ into:

$$\begin{aligned} G(w_1, \widehat{w}_1) &= \int_{a(w_2) < V - \widehat{w}_1} \frac{V + \widehat{w}_1 - a(w_2)}{2} \frac{dw_2}{V - w_1} \\ &+ w_1 \int_{a(w_2) > V - \widehat{w}_1} \frac{dw_2}{V - w_1} \end{aligned}$$

Differentiating $G(w_1, \widehat{w}_1)$ with respect to $\widehat{w}_1$ yields:

$$\frac{\partial G(w_1, \widehat{w}_1)}{\partial \widehat{w}_1} = \frac{1}{V - w_1} [(1/2) b(V - \widehat{w}_1) - b'(V - \widehat{w}_1)(\widehat{w}_1 - w_1)]$$

where $b(w) = -\frac{3}{8} V + \frac{3}{2} w$ is the inverse of function $a(\cdot)$. Straightforward computations show that

$$\left. \frac{\partial G(w_1, \widehat{w}_1)}{\partial \widehat{w}_1} \right|_{\widehat{w}_1 = a(w_1)} = 0. \quad \mathbf{Q.\ E.\ D.}$$