Asset Price Dynamics When Traders Care About Reputation*

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Abstract

What are the equilibrium features of a dynamic financial market where traders care about their reputation for ability? We modify a standard sequential trading model to include traders with career concerns. We show that this market cannot be informationally efficient: there is no equilibrium in which prices converge to the true value, even after an infinite sequence of trades. We characterize the most informative equilibrium of this game and show that an increase in the strength of the traders' reputational concerns has a negative effect on the extent of information that can be aggregated in equilibrium but a positive effect on market liquidity. The robustness of our results is probed from a variety of angles.

Keywords: Financial Equilibrium; Career Concerns; Information Cascades

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1 Introduction

The substantial increase in the institutional ownership of corporate equity around the world in recent decades has underscored the importance of studying the effects of institutional trade on asset prices.[1] Institutions, and their employees, may be guided by incentives not fully captured by standard models in finance. For example, consider the case of US mutual funds which make up a significant proportion of institutional investors in US equity markets. An important body of empirical work highlights the fact that mutual funds (e.g. Chevalier and Ellison [8]) and their employees (Chevalier and Ellison [9]) both face *career concerns*: they are interested in enhancing their *reputation* with their respective principals and sometimes indulge in perverse actions (e.g. excessive risk taking) in order to achieve this. Given the importance of institutions in equity markets, it is plausible to expect that such behavior may affect equilibrium quantities in these markets. What are the equilibrium features of a market in which a large proportion of traders care about their reputation?

While a growing body of literature examines the effects of agency conflicts on asset pricing, the explicit modeling of reputation in financial markets is in its infancy.[2] Dasgupta and Prat [11] present a two-period micro-founded model of career concerns in financial markets to examine the effect of reputation in enhancing trading volume. However, that analysis is done for a static market: each asset is traded only once.

In this paper, in contrast, we study a multi-period sequential trade market in which some traders care about their reputations. We show that the dynamic properties of this market are very different from those of standard markets.

1.1 Summary of Results

We present the most parsimonious model that captures the essence of our arguments. Much of our model is standard. We present a T-period sequential trade market for a single (Arrow) asset where all transactions occur via uninformed *market makers* who are risk neutral and competitive (following Glosten and Milgrom [16] and Kyle [17]) and quote bid and ask prices to reflect the informational content of order flow. In addition there is a large group of liquidity-driven *noise traders* who trade for exogenous reasons that are unrelated to the liquidation

[1] On the New York Stock Exchange the percentage of outstanding corporate equity held by institutional investors increased from 7.2% in 1950 to 49.8% in 2002 (NYSE Factbook 2003). For OECD countries as a whole, institutional ownership now accounts for around 30% of corporate equity (Nielsen [20]). Allen [1] presents persuasive arguments for the importance of financial institutions to asset pricing.

[2] For example, Allen and Gorton [2] and Dow and Gorton [15] examine the asset pricing implications of non-reputational agency conflicts. Reputational concerns are implicit in the contractual forms assumed in the general equilibrium models of Cuoco and Kaniel [10] and Vayanos [29].

value of the asset.

Our only innovation is that we introduce a large group of reputationally-concerned traders (whom we call *fund managers*), who trade on behalf of other (inactive) investors. These traders receive a payoff that depends both on the direct profits they produce and on the reputation that they earn with their principals.[3] Their reputation is determined endogenously by Bayesian investors, in a way that will be described shortly.

The fund managers can be of two types (smart or dumb) and receive informative signals about the asset liquidation value, where the precision depends on their (unknown) type. In each trading round either a randomly selected fund manager or a noise trader interact with the market maker. The asset payoff is realized at time T and all payments are made.

At time T, every fund manager is evaluated on the basis of all available information, with the exception of the agent's private signal. This implies that each investor can observe the liquidation of the asset and the portfolio choice of his own agent.[4] This assumption is plausible for relatively sophisticated investors, such as corporate pension plans, investment banks, insurance companies, and hedge fund clients. It may instead be an unrealistic requirement for retail mutual fund investors, who typically have limited knowledge of their fund's portfolio composition.[5]

We present the following results.

1. We begin with an impossibility result. We show that, in this market of career-concerned traders, *prices never converge to true liquidation value, even after an infinite sequence of trades.* If fund managers trade according to their private signal, the price evolves to incorporate such private information. Over time, the price should converge to the true liquidation value. However, as the uncertainty over the liquidation value is resolved, two things happen. First, the fund managers have less opportunity to make trading profits because the price is close to the liquidation value. The expected profit for a fund manager who trades according to his signal is always positive, but it tends to zero as the price becomes more precise. Second, taking a "contrarian" position (e.g. selling when the price has been going up) starts to carry an endogenous reputational cost: with high probability, the trade will turn out to be incorrect and the fund manager will "look dumb" in the eyes of (rational) principals. Because of the combination of these two effects, when the price becomes sufficiently precise fund managers begin to behave

[3] The principals may be line managers at mutual fund companies with oversight over the particular fund manager's activities, or, directly, the investors who have placed their funds with the company.

[4] For all our core results, it is irrelevant whether the investor observes the portfolio choice of other fund manager besides his.

[5] See Prat [23] for a discussion of the role of portfolio disclosure in delegated portfolio management with career concerns.

in a conformist way: their trade stops reflecting their private information. From then on, there is no information aggregation whatsoever and the price stays constant.

2. We then investigate how much information can be aggregated by equilibrium prices despite the presence of career concerns. We do this by characterizing the most informative trading strategies that can be sustained in equilibrium following each possible transaction price. We show that, as long as the price leaves sufficient uncertainty about liquidation values, sincere trade can be supported in equilibrium. Thus, each manager's signal can be fully revealed via his trade. However, as uncertainty is resolved, equilibrium trade becomes partially or completely uninformative.

   We show a number of monotonicity results, which relate the strength of career concerns with the extent of information revelation. For every price level, the amount of information revealed in equilibrium is decreasing in the importance of career concerns. We also consider the maximal and minimal ranges of equilibrium prices that can support completely sincere and fully conformist trading respectively, and characterize how such price ranges evolve with the importance of career concerns.

3. We consider the impact of career concerns on other core financial market variables: market liquidity and price volatility. We show that increasing career concerns increases liquidity and decreases volatility. Thus, increased institutional presence in a market decreases price informativeness, but has potentially beneficial impacts via liquidity and volatility. Our analysis provides theoretical underpinnings for a number of recent papers on herding in financial markets, which will be discussed in more detail in the Conclusion.

4. Finally, we examine a number of natural extensions of the model. The baseline model is presented with a binary asset liquidation value. We show that our impossibility result extends to richer payoff spaces. Further, in our baseline model we assumed that fund managers were unaware of their type. We extend the model to demonstrate that as long as self-knowledge is not too accurate, our main conclusion remains valid.

1.2 Related Literature

This paper brings together two influential strands of the literature. The first strand concerns the theory of dynamic financial markets with asymmetrically informed traders (Glosten and Milgrom [16] and Kyle [17]). The second strand focusses on the analysis of career concerns in sequential investment decision-making (Scharfstein and Stein [24]). Models in the first strand consider a full-fledged financial market with endogenously determined prices but do not allow traders to have career concerns. Models in the second strand do the exact opposite:

they analyze the role of reputational concerns in a partial equilibrium setting, where prices are exogenously fixed.

In the first strand, Glosten and Milgrom [16] have shown that in dynamic financial markets the price must tend to the true liquidation value in the long term. More recently, Avery and Zemsky [4] have shown that statistical information cascades à la Banerjee [5] and Bikhchandani, Hirshleifer, and Welch [6] are impossible in such a market.[6] After every investment decision, the price adjusts to reflect the expected value of the asset based on information revealed by past trades. Thus, traders with private information stand to make a profit by trading according to their signals. But by doing so, they release additional private information into the public domain. In the long run, the market achieves informational efficiency.

In the second strand, Scharfstein and Stein [24] have shown that managers who care about their reputation for ability may choose to ignore relevant private information and instead mimic past investment decisions of other managers.[7] This is because a manager who possesses "contrarian" information (for instance he observes a negative signal for an asset that has experienced price growth) jeopardizes his reputation if he decides to trade according to his signal. Scharfstein and Stein's analysis is carried out in partial equilibrium: prices play no informational role in such an analysis. For a general analysis of this class of partial equilibrium models see Ottaviani and Sorensen [21].[8]

Our results provide a clean theoretical link between the two types of economies represented in these two strands of the literature. On the one hand, in "Glosten-Milgrom type" economies, prices play an informational role and agents are motivated purely by trading profits. In such economies, agents always utilize their information and prices always converge to true liquidation value in the long run. On the other hand, in "Scharfstein-Stein type" economies prices are *assumed* to play no informational role and agents care only about ex post reputation

[6] A word of caution is in order here. There is almost universal agreement in the literature on the meaning of a *cascade*, which is the definition we have used above (an equilibrium event in which information gets trapped, and agents' actions no longer reveal any of their valuable private information). However, there is little agreement on the definition of the term *herds* (for example, substantively different definitions are used by Avery and Zemsky [4], Smith and Sorensen [27], and Chari and Kehoe [7]). In the interest of clarity, throughout this paper we shall restrict attention to cascades *only*.

Under additional assumptions, Avery and Zemsky [4] show that a form of herd behaviour may occur in the presence of prices. However, in all versions of their model cascades are absent and prices always converge to true liquidation value (Avery and Zemsky Proposition 2). Recently, Park and Sabourian [22] have explored generalizations of the necessary conditions for herds in Avery and Zemsky's model. As in Avery and Zemsky, however, cascades cannot arise in their model.

[7] Other more recent papers in this strand include, for example, Avery and Chevalier [3] and Trueman [28].

[8] The link between this paper and Ottaviani and Sorensen [21] is discussed in more detail at the end of Section 3.

for ability. In such economies, agents engage in conformist behavior in order to enhance their reputation. Our central observation is that if traders care even slightly about reputation in a Glosten-Milgrom type economy, then prices can play only a *limited* informational role. In order to converge to true value, prices must get close enough to true value. But when this happens, profits become unimportant, and reputational concerns become predominant. Then, the Glosten-Milgrom economy metamorphoses into a Scharfstein-Stein economy. But in the latter, conformism arises, and thus prices cannot incorporate further information.

In addition, by studying career concerns in financial equilibrium, we are able to study the effects of micro-founded reputation-driven conformism on financial market quantities (prices, informational efficiency, trade patterns, liquidity, and volatility), which leads to relevant predictions on observable market variables, as discussed above.

Other authors (Lee [18] and Chari and Kehoe [7]) have argued that information cascades can occur when prices are endogenous. However, their arguments hinge on a market breakdown: trade stops altogether.[9] Instead, in our model cascades occur in a functioning financial market with trade.

The rest of the paper is organized as follows. In the next section we present the model. Section 3 demonstrates the impossibility of full information aggregation. Section 4 characterizes the relationship between the importance of career concerns and the extent of equilibrium information aggregation. Extensions are examined in section 5. Section 6 concludes.

2 The Model

The economy lasts T discrete periods: $1, 2, ...T$. Trade can occur in periods $1, 2...T-1$. The market trades an Arrow security, which has equiprobable liquidation value $v = 0$ or 1, which is revealed at time T.

In practice, the asset could be a bond with maturity date T with a serious possibility of default. It could also be the common stock of a company which is expected to make an announcement of great importance (earnings, merger, etc.) at time T: all traders know that the announcement will occur but they may have different information on the content of the announcement.

There are a large number of fund managers and noise traders. At each period $t \in \{1, 2, ..., T-1\}$ either a fund manager or a noise trader enters the market with probabilities $1-\delta$ and $\delta \in (0, 1)$ respectively. The traders interact with a market maker and can

[9] In Lee's [18] model the existence of a transaction cost to trading can prevent traders with relatively inaccurate signals from trading, thus trapping private information in an illiquid market. In Chari and Kehoe [7], traders have the option of exiting the market (by making an outside investment) and may in equilibrium find it optimal to exit before further information arrives, thus, again, trapping private information.

issue market orders (a_t) to buy ($a_t = 1$) one unit or sell ($a_t = 0$) one unit of the asset. The market maker posts ask (p_t^a) and bid (p_t^b) prices at which he will sell or buy one unit of the asset respectively. As is standard in the literature (Glosten and Milgrom [16], Kyle [17]), we assume that the market maker is risk-neutral and competitive, and thus the quoted prices will be equal to the expected value of v conditional on the order history.

Denote the history of observed orders at the beginning of period t (not including the order at time t) by h_t. Let $p_t = E(v|h_t)$, $p_t^a = E(v|h_t,\text{buy})$, $p_t^b = E(v|h_t,\text{sell})$. Note that at any time t, p_t plays a dual role: on the one hand it is the *most recent transaction price*; on the other, it represents the *public belief* about v at the beginning of period t. We shall, therefore, refer to p_t below interchangeably as the "price" or the "public belief", depending on the context.

The fund manager can be of two types: $\theta \in \{b, g\}$ with $\Pr(\theta = g) = \gamma \in (0, 1)$. The type is independent of v. If at time t a fund manager appears, he receives a signal $s_t \in \{0, 1\}$ with distribution

$$\Pr(s_t = v|v, \theta) = \sigma_\theta,$$

where

$$\frac{1}{2} \leq \sigma_b < \sigma_g \leq 1.$$

Fund managers do not know their type. Noise traders buy or sell a unit with equal probability independent of v.

The profit obtained by the trader at time t is defined by:

$$\pi_t\left(a_t, p_t^a, p_t^b, v\right) = \begin{cases} v - p_t^a & \text{if } a_t = 1 \\ p_t^b - v & \text{if } a_t = 0 \end{cases}$$

If a fund manager traded at time t, his actions are observed at time T. Principals (e.g., line managers in the fund management firm) form a posterior belief about the manager's type based upon all observables, namely the whole history of trades and prices (h_T) and the realized liquidation value (v). We call this the manager's reputation and define it to be:

$$r_t(h_T, v) = \Pr(\theta_t = g|h_T, v).$$

The fund manager at time t receives utility

$$u_t = \beta\pi_t + (1-\beta)r_t,$$

where $1 - \beta \in (0, 1)$ measures the importance of career concerns.[10] A game Γ is defined by the values of five parameters $(\beta, \sigma_b, \sigma_g, \gamma, \delta)$.

[10] Our qualitative results hold for a much larger class of payoff functions: $u_t = \beta\chi(\pi_t) + (1-\beta)R(r_t)$ where χ and R are increasing and piecewise continuous functions. Such an extension increases algebra without adding to intuition. See Dasgupta and Prat [12] for details.

Let $\alpha_{s_t}^t(h_t)$ be the probability that the manager plays $a_t = 1$ given history h_t and signal realization s_t. A perfect Bayesian equilibrium of the game is a collection $\{\alpha_{s_t}^t(h_t)\}_{t=1}^{T-1}$ for every possible history h_t and signal realization s_t, satisfying the standard definition.

Finally, in a given PBE of the game, at a given time t, and for a given history h_t, we denote by $\Delta\pi_{s_t}^t$ the expected excess profit for the manager who has observed signal s_t from buying rather than selling. Similarly, denote the expected excess reputation by $\Delta r_{s_t}^t$ and the expected excess overall utility by $\Delta u_{s_t}^t$. Also, denote the private expectation of the manager about v after observing history h_t and his signal s_t by $v_{s_t}^t$. In our subsequent discussion, we will often hold time and history constant, and denote these simply by $\Delta\pi_s, \Delta r_s, \Delta u_s$ and v_s respectively.

Our model departs from Glosten and Milgrom's [16] only in that our informed traders – the fund managers – care about reputation as well as profit. If we set $\beta = 1$, our model becomes a special case of Glosten-Milgrom, and all their results apply as stated.

3 The Impossibility of Full Revelation

If there are no career concerns ($\beta = 1$), the unique equilibrium of this game is a *sincere equilibrium* (Glosten and Milgrom [16], Avery and Zemsky [4]): for any t, h_t, $\alpha_1^t(h_t) = 1$ and $\alpha_0^t(h_t) = 0$, that is, every trader who receives an opportunity to trade does so according to his information ($a_t = s_t$). This means that prices impound information and $p_t \to v$ as $t \to \infty$.

The interesting case is then when $\beta < 1$. We can now state our main result:

Proposition 1 *For any game Γ with $\beta < 1$, there exists $\underline{p} \in \left(0, \frac{1}{2}\right)$ such that in any equilibrium of the game, at all times, $p_t \in \left(\underline{p}, 1 - \underline{p}\right)$.*

For any $\underline{p} \in \left(0, \frac{1}{3}\right)$, there exists an open set of games Γ such that in any equilibrium of those games, at all times, $p_t \in \left(\underline{p}, 1 - \underline{p}\right)$.

For an equilibrium to be informative, the actions of traders who receive signal $s = 1$ must differ at least probabilistically from the actions of traders who receive signal $s = 0$, i.e., α_1 must be different from α_0. For a given history, and a corresponding set of public beliefs, informative equilibrium strategies must, therefore, satisfy either $\alpha_1 > \alpha_0$ or $\alpha_1 < \alpha_0$. Our proof shows that, when prices are sufficiently extreme, neither of these is possible. The proof of the result (as well as those of all subsequent results) is in the appendix. Here, we provide some intuition for why the result is true.

There are three crucial (endogenous) properties of our financial market that drive our results. The first property is that profit motives always encourage traders to trade sincerely. Private information is valuable, and in the presence of noise, prices reflect only part of this information. It always enhances the profits of traders to follow their private information. The second property is that when transaction prices, and therefore public beliefs, indicate that some liquidation value (say, $v = x$) of the asset is sufficiently more likely than the other, the reputational incentives of a career-concerned fund manager encourage him to act in a manner that will make the principal believe that the manager received the signal that is more likely to arise when $v = x$. This enhances the manager's reputation, because types are differentiated by their relative information precision. Finally, the third property is that when prices become sufficiently extreme, and thus sufficiently precise, trading profits become small because the beliefs of informed and uninformed traders converge.

We shall now argue that a combination of two or more of these ingredients rule out equilibria where, when prices are high or low enough, it is possible to have either $\alpha_1 > \alpha_0$ or $\alpha_1 < \alpha_0$.

First, consider the case in which $\alpha_1 > \alpha_0$. It is easy to see that the combination of the second and third properties rule out informative equilibria of this type for high enough or low enough prices. In an equilibrium with $\alpha_1 > \alpha_0$, when the principal sees a manager buy, he attaches high probability to the manager having received signal 1. This enhances the reputation of the manager, if, ex post, the liquidation value turns out to be 1. If instead the liquidation value turns out to be 0, the manager's reputation suffers. Consider a manager who has received signal 1 and suppose that transaction prices p get very small (we loosely write "$p \to 0$"). The third property implies trading profit becomes small ($\Delta\pi_1 \to 0$) and has a small impact on trading decisions. However, if $p \to 0$ in an informative equilibrium, it becomes very likely that $v = 0$. Thus, the second property implies that the manager's reputational incentives will encourage him to take the action that will make the principal believe that he has received signal 0. Thus, from a reputational perspective, this manager must prefer to sell instead of buy ($\Delta r_1 < 0$). As profit motivations diminish and reputational motivations become one-sided, eventually the latter dominates the former ($\Delta u_1 < 0$) and the manager ignores his private information: $\alpha_1 = 0$. Thus, for sufficiently extreme p, it cannot be the case that $\alpha_1 > \alpha_0$.

Consider next the case in which $\alpha_1 < \alpha_0$. The combination of the first two properties rules out this type of equilibrium when transaction prices are sufficiently extreme. In an equilibrium with $\alpha_1 < \alpha_0$, when the principal sees a manager buy, he attaches high probability to the manager having received signal 0. This enhances the reputation of the manager if, ex post, the liquidation value turns out to be 0. Consider a manager who has received signal 1. The first property implies that profit motivations drive this manager to buy ($\Delta\pi_1 > 0$). However, since $\alpha_1 < \alpha_0$, it must be the case that $\alpha_1 < 1$. This implies that $\Delta u_1 \leq 0$, which can only arise if the reputational value of buying is strictly lower than the reputational value of selling ($\Delta r_1 < 0$). Now suppose that $p \to 0$. In an informative equilibrium, this means that it is very likely that $v = 0$. The second property now implies that the manager can gain reputationally by signalling that he received $s = 0$, which he can do only by *buying*! Thus, it must be reputationally advantageous for him to buy rather than sell ($\Delta r_1 > 0$) for low enough public beliefs, contradicting our conclusion above. Thus, for sufficiently extreme p, it cannot be the case that $\alpha_1 < \alpha_0$.

Thus, the only possible equilibrium actions for sufficiently extreme prices involves $\alpha_1 = \alpha_0$. But since these actions are uninformative, such trades do not move the price further.

The price bounds identified in Proposition 1 are independent of history and time, and therefore of the length of the game T. This is because, while equilibrium strategies can in general be time and history dependent, we have shown that if prices ever attain our bounds, the continuation equilibrium is unique, independent of history and time, and dictates complete conformism.[11]

The second part of the proposition establishes that the non-revelation region is non-trivial. For any $\underline{p} < \frac{1}{3}$, there exist a positive measure of games (the space of games is the space on which the parameters $(\beta, \sigma_b, \sigma_g, \gamma, \delta)$ are defined) in which transaction prices can never be lower than $\underline{p}$ or higher than $1 - \underline{p}$. The $\underline{p} < \frac{1}{3}$ bound comes from the worst-case scenario for information revelation, namely when career concerns are very important ($\beta \to 0$), smart managers are very smart ($\sigma_g \to 1$), dumb managers are very dumb ($\sigma_b \to \frac{1}{2}$), and most managers are dumb ($\gamma \to 0$).

Our result bears a connection to Ottaviani and Sorensen [21], who provide a general analysis of reputational cheap talk and show that full information transmission is generically impossible. Of particular interest to the present paper is their Proposition 9, where they consider a sequence of experts providing reports on a common state of the world and they show that informational herding must occur. Our result is different because: (a) we show the necessity of informational cascades (while Ottaviani and Sorensen prove that herding must occur but they cannot exclude that the true value is revealed in the limit); (b) our experts have a profit motive as well as a reputational motive; and (c) most importantly, our model is embedded in a financial market.

4 How Much Information Can Be Aggregated?

We have just shown that the presence of reputational concerns preclude the existence of equilibria in which prices perfectly aggregate information in the long-run. We now turn to

[11] Needless to say, while are results are formally valid for all T, they are more *interesting* for large T. For sufficiently small T the range of possible transaction prices in the game *without* career concerns ($\beta = 1$) may lie within the bounds identified in Proposition 1.

a natural complementary question: How much information can be revealed in equilibrium, despite the presence of career concerns?

We address this question in two interrelated ways. First, we consider the most informative form of trading: sincere trading. We show that there exists a region of prices where, despite the presence of career concerns, sincere trading can be supported in equilibrium. These results are contained in Section 4.1.

Following this, we turn to the broader question of the relationship between the importance of career concerns and the extent of information aggregation. For each possible price, we define the *most informative* equilibrium strategies and provide a complete characterization of how such strategies vary depending on the importance of career concerns. These results are contained in Section 4.2.

While we have demonstrated our main impossibility result across the full spectrum of potential perfect Bayesian equilibria, for our comparative statics results we focus only on "non-perverse" equilibria with $\alpha_1^t(h_t) \geq \alpha_0^t(h_t)$ for all t and h_t. These are the only reasonable equilibria in a financial context. Other "perverse" equilibria feature strictly negative bid-ask spreads along the equilibrium path, which are very unrealistic in financial markets.[12] As we shall see shortly, a non-perverse equilibrium always exists.

4.1 Sincere Trading

In the absence of career concerns (when $\beta = 1$), sincere trading, i.e., trading which completely reveals individual signals, is the unique equilibrium outcome of our game (Glosten and Milgrom [16], Avery and Zemsky [4]). Our main result implies that in the presence of career concerns ($\beta < 1$) sincere trading cannot be sustained at all possible prices. We now consider whether, despite the presence of career concerns, sincere trading can be sustained for *some* prices. We show that if the price is sufficiently close to $\frac{1}{2}$, there always exists a sincere equilibrium. Let $\sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$.

Proposition 2 *If $p_t \in (1-\sigma, \sigma)$, it is an equilibrium for all fund managers to trade sincerely.*

The intuition for this result is straightforward. If $p_t \in (1-\sigma, \sigma)$, we have that $v_0^t < \frac{1}{2} < v_1^t$. The manager thinks that the high state is more likely if and only if he has a positive signal.

[12] A negative bid-ask spread would create an instantaneous risk-free arbitrage opportunity.This opportunity cannot be exploited in a Glosten-Milgrom setup like ours because there are no agents who can buy and sell at the same time. One could conceivably rule out perverse equilibria by adding uninformed short-lived arbitrageurs to the model. However, this would substantially complicate the model without generating additional insights on information aggregation. In addition, such a modification would take us further away from the well-known baseline model of sequential trade in the absence of career concerns, against which we currently benchmark our results.

It is easy to see that this implies that, if investors expect sincere play, a manager with a positive signal should indeed buy and one with a negative signal should indeed sell.

4.2 How Career Concerns Affect Information Aggregation

How does the ability of a market to aggregate information depend on the incentive structure of its traders? In this section we show that the amount of information that is revealed in equilibrium is decreasing in the strength of career concerns.

We begin our analysis by defining the concept of the *most informative equilibrium*. For any price p_t, the most informative equilibrium is the equilibrium at time t where the difference $\alpha_1 - \alpha_0$ is greatest. The following result is useful in characterizing most informative equilibria:

Proposition 3 *Suppose* $p_t \leq \frac{1}{2}$ $\left(p_t \geq \frac{1}{2}\right)$*. Then:*
(i) There exists an equilibrium in which $\alpha_1 \geq \alpha_0 = 0$ *(*$1 = \alpha_1 \geq \alpha_0$*).*
(ii) If in equilibrium $\alpha_1 > \alpha_0$*, then* $\alpha_0 = 0$ $(\alpha_1 = 1)$*.*

This result, together with the symmetry of the game, implies that the most informative equilibrium is determined by one variable $\bar{\alpha} \in [0, 1]$. If $p_t \leq \frac{1}{2}$, the most informative equilibrium is $(\alpha_0 = 0, \alpha_1 = \bar{\alpha})$. If $p_t > \frac{1}{2}$, the most informative equilibrium is $(\alpha_0 = 1 - \bar{\alpha}, \alpha_1 = 1)$. Informativeness is increasing in $\bar{\alpha}$, with the two extreme values, zero and one, denoting respectively a pooling equilibrium and a separating one.

Fix the parameters $(\gamma, \delta, \sigma_g, \sigma_b)$. For the remainder of this section restrict attention to $p_t \leq \frac{1}{2}$. All statements for $p_t > \frac{1}{2}$ are analogous.

We expect the most informative equilibrium to depend on the price and on the strength of career concerns. Let us make this dependence explicit by using the notation $\bar{\alpha}(\beta, p_t)$. In addition, we slightly abuse notation by including β and p_t explicitly as arguments of Δu_1.

For $p_t \leq \frac{1}{2}$ it is easy to see that:

$$\bar{\alpha}(\beta, p_t) = \begin{cases} 1 & \text{if } \Delta u_1(\alpha_1 = 1, \alpha_0 = 0, p_t, \beta) \geq 0 \\ 0 & \text{if } \Delta u_1(\alpha_1 = \alpha, \alpha_0 = 0, p_t, \beta) < 0, \ \forall \alpha > 0 \\ \max\{\alpha | \Delta u_1(\alpha_1 = \alpha, \alpha_0 = 0, p_t, \beta) = 0\} & \text{otherwise} \end{cases}$$

We can characterize how the most informative equilibrium varies as a function of β.

Proposition 4 *For all* $\beta'' > \beta'$*:*
(i) $\bar{\alpha}\left(\beta'', p_t\right) \geq \bar{\alpha}\left(\beta', p_t\right)$;
(ii) if $\bar{\alpha}\left(\beta', p_t\right) \in (0, 1)$*,* $\bar{\alpha}\left(\beta'', p_t\right) > \bar{\alpha}\left(\beta', p_t\right)$;

For a given vector of parameters, the most informative equilibrium for price p_t can be non-informative, partially informative, or fully informative. A decrease in the strength of career concerns (an increase in β) will weakly improve the informativeness of the most informative equilibrium. If the equilibrium is partially informative, it will strictly improve it.

To obtain intuition for this result, consider a given $p_t \leq \frac{1}{2}$ and let $\beta = \beta'$. Since profit motivations always drive manager towards sincere trading (i.e., $\Delta\pi_1 > 0$), if a manager with $s_t = 1$ is exactly indifferent between buying and selling (i.e., $\Delta u_1 = 0$) at a given p_t, under most-informative equilibrium strategy $\bar{\alpha}\left(\beta', p_t\right) < 1$, it must be the case that buying is reputationally costly for this manager (i.e., $\Delta r_1 < 0$). Now, increasing β (say, to β''), thus skewing incentives away from reputation, must make the manager strictly prefer to buy instead of sell under the proposed equilibrium strategies. Thus, $\bar{\alpha}\left(\beta', p_t\right)$ can no longer be the most informative equilibrium strategy at p_t with $\beta = \beta''$. There are two possibilities: either $\Delta u_1 = 0$ for a strictly larger (interior) equilibrium strategy $\bar{\alpha}\left(\beta'', p_t\right)$, or $\Delta u_1 > 0$ for all $\alpha_1 > \bar{\alpha}\left(\beta', p_t\right)$, in which case sincere trading is an equilibrium and thus $\bar{\alpha}\left(\beta'', p_t\right) = 1$.

We can also derive monotone comparative statics on the relevant boundaries of the equilibrium regions. Again, we restrict attention to $p \leq \frac{1}{2}$,(statements for $p \geq \frac{1}{2}$ are analogous) and define:

$$
\begin{aligned}
p_{\min}(\beta) &= \sup\{p_t : \bar{\alpha}(\beta, p_t) = 0\} \\
p_{\max}(\beta) &= \inf\{p_t : \bar{\alpha}(\beta, p_t) = 1\}
\end{aligned}
$$

The first bound, $p_{\min}$, is the highest price at which a non-informative equilibrium can be sustained. The second, $p_{\max}$, is the lowest price with a sincere equilibrium.[13] We can now state:

Proposition 5 *(i) If $\beta'' > \beta'$, then $p_{\min}\left(\beta''\right) \leq p_{\min}\left(\beta'\right)$ and $p_{\max}\left(\beta''\right) \leq p_{\max}\left(\beta'\right)$;*
(ii) $\lim_{\beta\to 1} p_{\max}(\beta) = \lim_{\beta\to 1} p_{\min}(\beta) = 0$;
(iii) $\lim_{\beta\to 0} p_{\max}(\beta) = \lim_{\beta\to 0} p_{\min}(\beta) = 1 - \sigma$.

The intuition of this result builds directly on Proposition 4, which showed that at any given p_t increasing β cannot decrease the amount of information revealed in the most informative equilibrium. Thus, increasing β can neither decrease the size of the sincere pricing region, $(p_{\max}(\beta), \frac{1}{2}]$, nor increase the size of the conformist region, $[0, p_{\min}(\beta))$.

As career concerns vanish (point ii), play becomes sincere for all prices, which confirms that Glosten and Milgrom [16] can be seen as a limit case of the present set-up. Point (iii) states that, as career concerns become more important, play is sincere only if $p_t > 1 - \sigma$, which shows that the bound identified in Proposition 2 is tight.

[13] The other two conceivable bounds are uninteresting. The lowest price with a non-informative equilibrium is always zero and the highest price (given that $p \leq \frac{1}{2}$) with a sincere equilibrium is always $\frac{1}{2}$.

4.3 Career Concerns and Financial Market Variables

One can study how the importance of career concerns affect other standard financial market variables. In this section, we consider widely used measured of liquidity, volatility, and trade predictability.

The bid-ask spread is the difference between the ask price and the bid price $(p_t^a - p_t^b)$, and it is a commonly used measure of market illiquidity. Price volatility can be defined as variance of the price of the asset at time $t+1$ given the price at t: $Var\left[p_{t+1}|p_t\right]$. Finally, trade-predictability is the ability to predict the sign of the trade at time t based on public information. We measure it by $\frac{1}{Var(a_t|p_t)}$.

We show that each of these variables is monotonically related to the importance of career concerns:

Proposition 6 *For any given p, in the most informative equilibrium, the bid-ask spread and price volatility are non-decreasing, and trade predictability is non-increasing, in β.*

Increasing β weakens career concerns. Thus, stronger career concerns make markets more liquid and less volatile, and makes trades more predictable. In sequential trade models with risk-neutral and competitive market makers, the bid-ask spread, and thus illiquidity, arises out of adverse selection. The more the informed traders (fund managers) utilize their private information in their trades, therefore, the higher the bid-ask spread, and thus the greater the amount of information revealed in equilibrium. Proposition 4 shows that the higher is β (i.e., the less important are career concerns) the more informative the trades of fund managers. Thus, increasing β increases the bid-ask spread. For the same informational reason, career concerns also make market prices and trades more predictable. As less information is revealed, the price is more stable.

The results of sections 4.2 and 4.3 provide theoretical underpinning to a number of existing empirical findings, and also suggest new avenues for empirical work. We discuss the connection to the empirical literature in the conclusion.

5 Extensions

5.1 A More General Set-Up

The baseline model was presented for a simple binary structure where liquidation values could take only two possible values. We extend it here to a generic discrete set of possible values $V \in \Re$. Denote the maximum and minimum possible values of $v \in V$ by $v_{\max}$ and $v_{\min}$. The ex ante distribution of v is determined by any arbitrary probability mass function.

Each fund manager of type $\theta \in \{b, g\}$ (with $\Pr(\theta = g) = \gamma$ as before) receives a signal distributed according to $\Pr(s = 1|v, \theta) = \sigma_{v,\theta}$, with the following properties:

A1 Full support: $\sigma_{v,\theta} \in (0, 1)$ for all θ and v.

A2 Monotone Likelihood Ratio Property (MLRP): For every pair of liquidation values $v'' > v'$,

$$\frac{\sigma_{v'',g}}{\sigma_{v'',b}} > \frac{\sigma_{v',g}}{\sigma_{v',b}}.$$

A3 Informativeness: $\sigma_{v,\theta}$ increasing in v for all θ, and $\sigma_{v_{\max},g} > \sigma_{v_{\max},b}$ and $\sigma_{v_{\min},g} < \sigma_{v_{\min},b}$.

The first assumption (A1) is crucial. It implies that the signal is never fully informative: for all s and v: $\Pr(v|s) < 1$. If a manager knows he has the truth, he would follow his signal even if all his predecessors had traded in the opposite direction.

First, note that if there are no career concerns ($\beta = 1$) there exists a fully informative equilibrium (see Avery and Zemsky [4]). When, instead, career concerns are present, we shall see that full information revelation is impossible.

Proposition 7 *For $\beta < 1$, there exists no equilibrium for which $\lim_{t\to\infty} p_t = v$ for more than one liquidation value v.*

The intuition parallels the case with binary liquidation values, and we therefore provide only a concise summary here. Assumptions A2 and A3 guarantee that for all but possibly one liquidation value, either $\sigma_{v,g} > \sigma_{v,b}$ or $\sigma_{v,g} < \sigma_{v,b}$. For each such v we show that there cannot exist informative equilibria when p is close enough to v. Consider the possibility that there is an informative equilibrium with $\alpha_1 > \alpha_0$. As $p \to v$, profits become unimportant, and the manager finds it desirable to indicate via her action that she has received a particular reputation-enhancing signal. If, for example, v is such that $\sigma_{v,g} < \sigma_{v,b}$ it is better for the manager to sell, which indicates that she was likely to have received signal 0. But since this is true even for a manager with $s = 1$, we must have $\alpha_1 = 0$, and thus the equilibrium cannot be informative. Alternatively, consider the possibility that there is an informative equilibrium with $\alpha_1 < \alpha_0$. Then, as we have argued earlier in the main model, the manager with $s = 1$ must always prefer to sell from a reputational perspective: $\Delta r_1 < 0$. Suppose again that $p \to v$ where $\sigma_{v,g} < \sigma_{v,b}$. Then, the manager must find it reputationally beneficial to indicate that she has received $s = 0$, but can only do this (since $\alpha_1 < \alpha_0$) by buying, which contradicts the fact that $\Delta r_1 < 0$. Thus for p close enough to v there cannot be informative equilibria with $\alpha_1 < \alpha_0$.

5.2 Self-Knowledge

The baseline analysis was carried out under the assumption that the manager did not know his type. What happens if the fund managers have some self-knowledge? In this section we examine the extent to which our results extend to such more complex settings. In doing so we illustrate the nature of interaction between conformism and self-knowledge, and show that the central economic message of our baseline model is robust to the presence of substantial self-knowledge.

We now let each fund manager receive two signals: the now familiar s_t, and a new signal z_t, with $\Pr(z_t = \theta|\theta) = \rho \in \left(\frac{1}{2}, 1\right)$. The rest of the model is exactly as in the main analysis.

We assume that

$$\rho < \frac{\sigma_g}{\sigma_g + \sigma_b}. \tag{1}$$

Note that the right-hand side of the inequality varies between $\frac{1}{2}$ (when $\sigma_g = \sigma_b$) to $\frac{2}{3}$ (when $\sigma_g = 1$ and $\sigma_b = \frac{1}{2}$). This assumption can be given a very natural economic interpretation. It is straightforward to show that condition (1) is equivalent to assuming that $\Pr(\theta = g|s = v) > \Pr(\theta = g|z = g)$. Thus, this condition guarantees that the manager's reputation is more sensitive to the precision of his signal about the asset payoff than to his signal about his own type.

Denote by α_{sz} the probability with which a manager who has received liquidation value signal s and self knowledge signal z chooses to buy. It is easy to see that, in the absence of career concerns ($\beta = 1$) – and provided that the proportion of noise traders is sufficiently high – the game has a unique equilibrium: $\alpha_{1g} = \alpha_{1b} = 1$ and $\alpha_{0b} = \alpha_{0g} = 0$. This equilibrium has the property that trades fully reveal the liquidation value signal s. We demonstrate that, for sufficiently extreme prices, no such equilibrium can exist when $\beta < 1$. We also rule out the possibility of the existence of an *informationally equivalent* "perverse" equilibrium with $\alpha_{1g} = \alpha_{1b} = 0$ and $\alpha_{0b} = \alpha_{0g} = 1$. We show that:

Proposition 8 *If condition (1) is satisfied, there exists a threshold $\bar{p} \in (0,1)$, such that if $p < \bar{p}$ or $p > 1 - \bar{p}$, there is neither an equilibrium with $\alpha_{1g} = \alpha_{1b} = 1$ and $\alpha_{0b} = \alpha_{0g} = 0$ nor an equilibrium with $\alpha_{1g} = \alpha_{1b} = 0$ and $\alpha_{0b} = \alpha_{0g} = 1$.*[14]

The intuition behind this result clarifies the interaction between conformism and self-knowledge. A manager can signal the quality of his type either by making the (ex post)

[14] The proof in the appendix shows that, for sufficiently extreme prices, not only is there no equilibrium with $\alpha_{1g} = \alpha_{1b} = 1$ and $\alpha_{0b} = \alpha_{0g} = 0$ but there is actually no equilibrium with $\alpha_{1g} \geq \alpha_{1b} \geq \alpha_{0b} \geq \alpha_{0g}$ where at least one inequality is strict. However, no such generalization is available for the second part of the proposition.

correct trade or by taking an action that reveals that he received a positive signal about his own type. For a given history, these two actions may not be identical. As long as the parameters of the model are such that the manager's reputation is helped more by revealing that he received the ex post correct signal about asset payoffs rather than by revealing that he received a good signal about his type, our baseline results go through. A manager with a good type-signal and a "contrarian" value-signal does not then find it worthwhile to try to reveal his type-signal at the risk of taking the incorrect contrarian action. Thus, just as in the baseline case, it becomes optimal for him to behave in a conformist manner.

Propositions 8 implies that allowing for career concerns "slows down" the rate of information aggregation via prices compared to the case with no career concerns: it is no longer possible that the trades of fund managers will reveal their signals in each period once prices are sufficiently extreme. It is worth noting that this result is weaker than the impossibility result in Proposition 1. While there we were able to prove the necessity of a full informational cascade, here we can only exclude a certain class of informative equilibria.[15]

5.3 Other Extensions

It is possible to extend our model in a variety of other directions. As we have noted earlier, our qualitative results go through for a much richer class of payoff functions:

$$u_t = \beta\chi(\pi_t) + (1-\beta)R\left(r_t\right)$$

where χ and R are increasing and piecewise continuous functions. The extension to such payoff functions increases algebraic complexity without adding to the intuition behind our results.

In addition, instead of having managers derive utility from their absolute reputation, we could allow them to care about reputation *relative* to their peers. For example, we could redefine manager t's reputational payoff by $R_t(r_1, ..., r_T)$, where r_i represents the realized reputation of manager i, with $\frac{\partial R_t}{\partial r_t} > 0$ and $\frac{\partial R_t}{\partial r_\tau} < 0$ for $\tau \neq t$. Even in this more complex case, it is possible to show that sincere trading cannot be sustained as an equilibrium. To see why, imagine that we are in an equilibrium with sincere trade, and consider the incentives of the last manager. Since this manager's actions cannot affect the principal's beliefs about his peers, he is just like a manager in our baseline model. He will conform for sufficiently extreme prices.

It is also possible to consider introducing informed non-career concerned (individual) traders into our model. Informed individuals devoid of career concerns would trade sincerely,

[15] There remains the possibility that, in the self-knowledge case, full-information revelation is achieved in the long term through less informative equilibria. While we cannot exclude that possibility, we were unable to construct examples in which this occurred.

and thus, in the presence of such traders, prices would eventually converge to true value. However, the basic intuition of the main result remains unchanged: once prices were close enough to true value, career concerned institutional traders would begin to ignore their own information. Thus convergence to true value would take place much more slowly than in the case without fund managers, and the extent of slowdown in convergence would depend on the proportion of career concerned traders in the market. In addition, conformist trading by institutional traders would still occur in the presence of informed individual traders, in keeping with empirical findings (e.g. Sias [25]).

Finally, it is possible to micro-found the utility function assumed in this paper. This, and the other extensions alluded to in this subsection, are discussed in greater detail in Dasgupta and Prat [12].

6 Conclusion

The central message of this paper is that we should expect the presence of traders with reputational concerns to affect the dynamic properties of financial markets. In particular, we have shown that stronger career concerns necessarily lead to more conformist behavior among traders, less precise information aggregation through prices, and better market liquidity. This paper creates a link between two sets of variables: the incentive structure faced by traders and the dynamics of asset markets. As both sets of variables are potentially measurable, our comparative statics results lead to clear-cut testable predictions.

In particular, our analysis provides theoretical underpinnings for a number of empirical findings. First, in all equilibria of our game, institutional investors exhibit conformist trading at some prices. Such conformism introduces high serial correlation in institutional trade. This prediction provides a theoretical rationale for the results of Sias [25]. Sias examines the quarterly SEC 13-F reports of US institutional money managers from 1983 to 1997 and finds a strong positive relationship between the fraction of institutions buying individual stocks over adjacent quarters, consistent with money managers herding behind each other's trades.[16] In addition, our results indicate the extent of institutional conformism (e.g., measured by the informativeness of their trading strategies or by the range of prices over which they herd) is linked to the incentive structure of their traders. This prediction finds indirect support

[16] This finding is complemented by Dasgupta, Prat, and Verardo [13] who examine SEC 13-F reports from 1983 to 2003 and find (amongst other things) that a vast majority of institutional traders exhibit conformist trading patterns. They excessively buy (sell) stocks that have been persistently bought (sold) by their peers consecutively over a period of 5 or more quarters. See also Dennis and Strickland [14]. Sias [26] provides a recent survey and reconciliation of the growing literature on momentum trading and herding by institutional traders.

in the work of Massa and Patgiri [19]. Massa and Patgiri study data on US mutual funds for the period 1994-2003, and quantify the extent of profit-based incentives in the contracts of fund managers. They find that those managers who receive higher profit-based incentives (i.e., have higher β in our setting) exhibit less conformism. Finally, some of our predictions point to potentially interesting new empirical exercises. For example, it would be interesting to examine whether there is a relationship between the incentives of money managers and the liquidity and volatility of the stocks they trade. Our stylized model predicts that, ceteris paribus, career concerned fund managers will increase liquidity and reduce volatility of the assets they trade.

Our model is stylized. We believe that it is important to build richer and more realistic models of dynamic financial markets with career concerned traders. The increasing importance of professional money managers in financial markets make such extensions topical. Our results establish a benchmark against which such future findings can be understood.

7 Appendix

Proof of Proposition 1:

We first characterize some crucial properties of beliefs in our model:

Lemma 1 *Let* $v_{s_t}^t = \Pr(v = 1|h_t, s_t)$. *Then,*

(a) $v_{s_t}^t$ *is strictly increasing and continuous in* p_t;

(b) $v_{s_t}^t = 1\,(0)$ *if* $p_t = 1\,(0)$;

(c) $v_1^t > v_0^t$ *if* $p_t \in (0, 1)$

Proof. By Bayes' rule,

$$v_{s_t}^t = \frac{p_t \Pr(s_t|v=1)}{p_t \Pr(s_t|v=1) + (1-p_t)\Pr(s_t|v=0)}$$

Now parts (a) and (b) follow immediately. To see part (c) note that

$$v_1^t = \frac{p_t}{p_t + (1-p_t)\frac{\Pr(s_t=1|v=0)}{\Pr(s_t=1|v=1)}} = \frac{p_t}{p_t + (1-p_t)\frac{1-\sigma}{\sigma}}$$

where $\sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$. Similarly

$$v_0^t = \frac{p_t}{p_t + (1-p_t)\frac{\sigma}{1-\sigma}}$$

Since $\sigma_g > \sigma_b > \frac{1}{2}$, $\sigma > \frac{1}{2}$. Thus $\frac{1-\sigma}{\sigma} < 1 < \frac{\sigma}{1-\sigma}$. This then implies $v_1^t > v_0^t$ which completes the proof of the lemma. ■

Given that the action set of every manager is binary, it is easy to see that the game has at least one PBE. Focus on time t. Suppose that given history h_t (with price p_t), equilibrium play dictates strategy $\alpha_0^t(h_t)$ and $\alpha_1^t(h_t)$.

The strategy of the proof is as follows. There are three cases: $\alpha_0^t(h_t) < \alpha_1^t(h_t)$, $\alpha_0^t(h_t) > \alpha_1^t(h_t)$, and $\alpha_0^t(h_t) = \alpha_1^t(h_t)$. We shall identify a lower bound and an upper bound to price such that the first two cases are impossible if the price is above the upper bound or below the lower bound. As it will become apparent, those two bounds are independent of time and history. The third case denotes uninformative play on the part of the manager at time t. Note that, if at a certain time t the price goes above the upper bound or below the lower bound, uninformative play guarantees that the price will not change in the next round; hence, play will be uninformative from then on.

In the remainder of the proof, we hold history and time constant. For simplicity, therefore, we drop the history and time arguments (e.g. $\alpha_0^t(h_t)$ become α_0).

For any (α_1, α_0) we can compute the following quantities. The bid and ask prices as follows:

$$
\begin{aligned}
p_a &= \frac{\delta\frac{1}{2} + (1-\delta)\left[\alpha_1\sigma + \alpha_0(1-\sigma)\right]}{\delta\frac{1}{2} + (1-\delta)\left[\alpha_1\Sigma_t + \alpha_0(1-\Sigma_t)\right]} p \\
p_b &= \frac{\delta\frac{1}{2} + (1-\delta)\left[(1-\alpha_1)\sigma + (1-\alpha_0)(1-\sigma)\right]}{\delta\frac{1}{2} + (1-\delta)\left[(1-\alpha_1)\Sigma_t + (1-\alpha_0)(1-\Sigma_t)\right]} p,
\end{aligned}
$$

where $\sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$, and $\Sigma = p\sigma + (1-p)(1-\sigma)$.

The manager's equilibrium strategy fully determines investors' beliefs (the beliefs do not depend on history or price directly – they only depend on history and price through α_1 and α_0):[17]

$$
\begin{aligned}
r(a=1, v=1) &= \frac{\alpha_1\sigma_g + \alpha_0(1-\sigma_g)}{\alpha_1\sigma + \alpha_0(1-\sigma)}\gamma \\
r(a=1, v=0) &= \frac{\alpha_1(1-\sigma_g) + \alpha_0\sigma_g}{\alpha_1(1-\sigma) + \alpha_0\sigma}\gamma \\
r(a=0, v=1) &= \frac{(1-\alpha_1)\sigma_g + (1-\alpha_0)(1-\sigma_g)}{(1-\alpha_1)\sigma + (1-\alpha_0)(1-\sigma)}\gamma \\
r(a=0, v=0) &= \frac{(1-\alpha_1)(1-\sigma_g) + (1-\alpha_0)\sigma_g}{(1-\alpha_1)(1-\sigma) + (1-\alpha_0)\sigma}\gamma
\end{aligned}
$$

Suppose the manager observes signal $s = 1$. The difference in his expected payoff if he plays $a = 1$ instead of $a = 0$ can be denoted with

$$\Delta u_1 = \beta\Delta\pi_1 + (1-\beta)\Delta r_1,$$

[17] This key property of beliefs in our game is due to the assumption that investors observe the liquidation value v and that the managers' signals are mutually independent given v.

where the profit component is

$$\Delta\pi_1 = (v_1 - p_a) - (p_b - v_1)$$

and the reputational component is

$$\Delta r_1 = v_1\left(r\left(a=1, v=1\right) - r\left(a=0, v=1\right)\right) + \left(1 - v_1\right)\left(r\left(a=1, v=0\right) - r\left(a=0, v=0\right)\right)$$

Case with $\alpha_0 < \alpha_1$ **:**

Suppose first that $\alpha_0 < \alpha_1$. We first show that in all equilibria either the manager with the high signal or the manager with the low signal play a pure strategy.

Lemma 2 *There are no mixed strategy equilibria in which* $0 < \alpha_0 < \alpha_1 < 1$ *for any* t.

Proof. Consider a putative equilibrium in which $1 > \alpha_0 > 0$, i.e. the agent at time t who receives signal zero is exactly indifferent between buying and selling. We will show that in this equilibrium, it must be the case that the agent who receives signal 1 at time t must strictly prefer to buy rather than sell. Consider the expected profit difference between buying and selling: $\Delta\pi_s$. This can be written as

$$v_s\left(\left(1 - p_a\right) - \left(p_b - 1\right)\right) + \left(1 - v_s\right)\left(\left(0 - p_a\right) - \left(p_b - 0\right)\right)$$

Since $(1 - p_a) - (p_b - 1) > 0 > (0 - p_a) - (p_b - 0)$, and by Lemma 1 $v_1 > v_0$, it is clear that $\Delta\pi_1 > \Delta\pi_0$.

Now consider the expected reputational payoff difference between buying and selling: Δr_{s_t}. This can be expressed as:

$$v_s[r\left(a=1, v=1\right) - r(a=0, v=1)] + \left(1 - v_s\right)\left[r\left(a=1, v=0\right) - r\left(a=0, v=0\right)\right]$$

Notice that $r\left(a=1, v=1\right) > r(a=0, v=1)$. To see why note that

$$\frac{\alpha_1\sigma_g + \alpha_0\left(1-\sigma_g\right)}{\alpha_1\sigma + \alpha_0\left(1-\sigma\right)} \leq \frac{\left(1-\alpha_1\right)\sigma_g + \left(1-\alpha_0\right)\left(1-\sigma_g\right)}{\left(1-\alpha_1\right)\sigma + \left(1-\alpha_0\right)\left(1-\sigma\right)} \Rightarrow \left(\sigma_g - \sigma\right)\left(\alpha_1 - \alpha_0\right) < 0$$

which is a contradiction since $\sigma_g - \sigma > 0$ and $\alpha_1 - \alpha_0 > 0$. A similar argument establishes that $r\left(a=1, v=0\right) < r\left(a=0, v=0\right)$. Thus,

$$r\left(a=1, v=1\right) - r(a=0, v=1) > 0 > r\left(a=1, v=0\right) - r\left(a=0, v=0\right).$$

Given Lemma 1, we know that $v_1 > v_0$, and thus $\Delta r_1 > \Delta r_0$.

Putting these together, we have $\Delta u_1 > \Delta u_0 = 0$. Thus, if $0 < \alpha_0 < 1$, then $\alpha_1 = 1$. An identical argument establishes that if $0 < \alpha_1 < 1$, then $\alpha_0 = 0$. ∎

At a given price p, consider $\Delta\pi_s$, the profit incentives of an agent who has received s to buy vs sell:

$$2v_s(p) - p_a(p) - p_b(p)$$

Since $p_a \geq 0$ and $p_b \geq 0$, and $v_0(p) < v_1(p) = \frac{\sigma}{\Sigma}p$, it is immediate that $\Delta\pi_s$ is bounded above by:

$$2\frac{\sigma}{\Sigma}p$$

At the same price p consider Δr_s the reputational incentives of this agent to buy vs sell:

$$v_s\left(\frac{\alpha_1\sigma_g + \alpha_0(1-\sigma_g)}{\alpha_1\sigma + \alpha_0(1-\sigma)} - \frac{(1-\alpha_1)\sigma_g + (1-\alpha_0)(1-\sigma_g)}{(1-\alpha_1)\sigma + (1-\alpha_0)(1-\sigma)}\right)\gamma +$$
$$(1-v_s)\left(\frac{\alpha_1(1-\sigma_g) + \alpha_0\sigma_g}{\alpha_1(1-\sigma) + \alpha_0\sigma} - \frac{(1-\alpha_1)(1-\sigma_g) + (1-\alpha_0)\sigma_g}{(1-\alpha_1)(1-\sigma) + (1-\alpha_0)\sigma}\right)\gamma$$

Lemma 2 allows us to restrict attention to cases where either $\alpha_1 = 1 > \alpha_0 \geq 0$ or $1 \geq \alpha_1 > \alpha_0 = 0$. It is then not difficult to see that Δr_s is bounded above by

$$v_s\left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + (1-v_s)\left(1 - \frac{\sigma_g}{\sigma}\right)\gamma,$$

which, in turn, is bounded above by

$$\frac{\sigma}{\Sigma}p\left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + (1-\frac{\sigma}{\Sigma}p)\left(1 - \frac{\sigma_g}{\sigma}\right)\gamma.$$

Thus, an upper bound on the expected utility difference enjoyed by this agent from buying vs selling at p is

$$\beta\left[2\frac{\sigma}{\Sigma}p\right] + (1-\beta)\left[\frac{\sigma}{\Sigma}p\left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + (1-\frac{\sigma}{\Sigma}p)\left(1-\frac{\sigma_g}{\sigma}\right)\gamma\right]$$

This is linear and increasing in $\frac{\sigma}{\Sigma}p = v_1(p)$, which in turn, is increasing in p. It crosses 0 exactly once at $p = \widehat{p}_1$ which is defined by:

$$v_1(\widehat{p}_1) = \frac{(1-\beta)\left(\frac{\sigma_g}{\sigma} - 1\right)\gamma}{2\beta + (1-\beta)\left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} + \frac{\sigma_g}{\sigma} - 1\right)\gamma} > 0$$

Since $v_1(p) > 0$ if and only if $p > 0$, we know that $\widehat{p}_1 > 0$. Thus we have proved that if $p < \widehat{p}_1$ managers will sell regardless of their signals. A symmetric proof establishes that for $p > 1 - \widehat{p}_1$ managers will buy regardless of their signals. Thus, for $p < \widehat{p}_1$ and $p > 1 - \widehat{p}_1$ it cannot be the case that $\alpha_1 > \alpha_0$.

Case with $\alpha_0 > \alpha_1$:

We now move on to the case where $\alpha_0 > \alpha_1$. As before, we define

$$\Delta u_1 = \beta\Delta\pi_1 + (1-\beta)\Delta r_1$$

If $\alpha_0 > \alpha_1$, a manager who observes $s = 1$ plays $a = 0$ with positive probability. It must be that $\Delta u_1 \leq 0$. As $\Delta \pi_1 > 0$, a necessary condition for the existence of such an equilibrium is that $\Delta r_1 < 0$. We shall show that this condition cannot hold if p is sufficiently low.

As before,

$$\Delta r_1 = v_1 \left(r(a = 1, v = 1) - r(a = 0, v = 1)\right) + (1 - v_1) \left(r(a = 1, v = 0) - r(a = 0, v = 0)\right)$$

The necessary condition can thus be re-written as

$$\frac{v_1}{1 - v_1} > -\frac{r(a = 1, v = 0) - r(a = 0, v = 0)}{r(a = 1, v = 1) - r(a = 0, v = 1)}$$

Let

$$\begin{array}{ll} a = \alpha_1 \sigma_g + \alpha_0 (1 - \sigma_g) & A = \alpha_1 \sigma + \alpha_0 (1 - \sigma) \\ b = \alpha_1 (1 - \sigma_g) + \alpha_0 \sigma_g & B = \alpha_1 (1 - \sigma) + \alpha_0 \sigma \end{array}$$

Note that

$$\begin{aligned} \frac{1}{\gamma} \left(r (a = 1, v = 1) - r (a = 0, v = 1)\right) &= \frac{a}{A} - \frac{1 - a}{1 - A} = \frac{a - A}{A (1 - A)} = \frac{(\alpha_1 - \alpha_0) (\sigma_g - \sigma)}{A (1 - A)} \\ \frac{1}{\gamma} \left(r (a = 1, v = 0) - r (a = 0, v = 0)\right) &= \frac{b}{B} - \frac{1 - b}{1 - B} = \frac{b - B}{B (1 - B)} = \frac{- (\alpha_1 - \alpha_0) (\sigma_g - \sigma)}{B (1 - B)} \end{aligned}$$

The necessary condition becomes

$$\frac{v_1}{1 - v_1} > \frac{A (1 - A)}{B (1 - B)} \equiv K \tag{2}$$

We are interested in the lower bound $\inf_{\alpha_1 < \alpha_0} K$. If it is strictly greater than zero, then for p low enough the necessary condition (2) cannot be satisfied.

Lemma 3 $\inf_{\alpha_1 < \alpha_0} K = \frac{1 - \sigma}{\sigma}$.

Proof. First, assume that the infimum is reached at an interior point: $0 < \alpha_1 < \alpha_0 < 1$. As K is twice differentiable, such point satisfies the two first-order conditions

$$\begin{aligned} \frac{\partial}{\partial \alpha_0} \frac{A (1 - A)}{B (1 - B)} &= 0 \\ \frac{\partial}{\partial \alpha_1} \frac{A (1 - A)}{B (1 - B)} &= 0 \end{aligned}$$

These can be expressed as

$$\begin{aligned} \sigma (1 - 2A) B (1 - B) &= (1 - \sigma) (1 - 2B) A (1 - A) \\ (1 - \sigma) (1 - 2A) B (1 - B) &= \sigma (1 - 2B) A (1 - A) \end{aligned}$$

Since $\frac{\sigma}{1-\sigma} > \frac{1-\sigma}{\sigma}$ and $A, B \in (0,1)$, the only way these two hold together is if $A = B = \frac{1}{2}$ which is impossible since $\alpha_1 \neq \alpha_0$.

Consider instead the corner solution: $0 = \alpha_1 < \alpha_0 \leq 1$. Now $A = \alpha_0 (1-\sigma)$ and $B = \alpha_0 \sigma$. Thus,

$$K = \frac{\alpha_0 (1-\sigma)(1-\alpha_0(1-\sigma))}{\alpha_0 \sigma (1-\alpha_0 \sigma)} = \frac{1-\sigma}{\sigma} \frac{1-\alpha_0(1-\sigma)}{1-\alpha_0 \sigma}$$

Since $\sigma > \frac{1}{2}$ this is clearly increasing in α_0. Thus, the infimum can be obtained by taking

$$\lim_{\alpha_0 \to 0} K = \lim_{\alpha_0 \to 0} \frac{1-\sigma}{\sigma} \frac{1-\alpha_0(1-\sigma)}{1-\alpha_0 \sigma} = \frac{1-\sigma}{\sigma}$$

The other potential corner solution is obtained by: $0 \leq \alpha_1 < \alpha_0 = 1$. Now $A = \alpha_1 \sigma + 1 - \sigma$ and $B = \alpha_1 (1-\sigma) + \sigma$. Thus,

$$K = \frac{\sigma}{1-\sigma} \frac{(\alpha_1 \sigma + 1 - \sigma)}{(\alpha_1 (1-\sigma) + \sigma)}$$

which is minimized for $\alpha_1 \to 0$, with value 1. Hence, the infimum is $\frac{1-\sigma}{\sigma}$. ■

Thus, there exists $\widehat{p}_2 > 0$ such that for $p < \widehat{p}_2$ the necessary condition for $\alpha_0 > \alpha_1$ fails. A corresponding upper bound of $1 - \widehat{p}_2$ follows from a symmetric proof. Thus, we have shown that for $p < \widehat{p}_2$ and $p > 1 - \widehat{p}_2$ it is not possible to have $\alpha_0 > \alpha_1$.

Price bounds:

We have now shown that there exists $\widehat{p}_1 > 0$ and $\widehat{p}_2 > 0$ such that for $p \notin [\widehat{p}_1, 1-\widehat{p}_1]$ it is not possible to have $\alpha_1 > \alpha_0$ and for $p \notin [\widehat{p}_2, 1-\widehat{p}_2]$ it is not possible to have $\alpha_1 < \alpha_0$. Now define $\widehat{p} = \min(\widehat{p}_1, \widehat{p}_2)$. Thus for $p \notin [\widehat{p}, 1-\widehat{p}]$ it is not possible to have $\alpha_1 \neq \alpha_0$.

In order to compute the lowest possible transaction price that can potentially be reached, we compute the bid-price at $\widehat{p}$ under the assumption that play is sincere and that there are no noise traders. This is given by

$$\underline{p} \equiv \Pr[v = 1 | \widehat{p}, s = 0] = \frac{(1-\sigma)\widehat{p}}{\sigma(1-\widehat{p}) + (1-\sigma)\widehat{p}}$$

A price below $\underline{p}$ can never be reached, because it would imply an informative trade following a transaction price (public belief) of $p = \widehat{p}$ or lower. An upper bound on prices of $1 - \underline{p}$ follows by symmetry. Note that for $\delta > 0$ (i.e., with noise traders) transaction prices of $\underline{p}$ and $1 - \underline{p}$ are never actually reached. Thus, for all t, $p_t \in (\underline{p}, 1-\underline{p})$. This completes the proof of the first part of the proposition.

Second part of the proposition:

For the case where $\alpha_1 > \alpha_0$ note that

$$\lim_{\beta \to 0, \sigma_g \to 1, \sigma_b \to \frac{1}{2}, \gamma \to 0} v_1(\widehat{p}_1) = \frac{1}{3}.$$

It is easy to see that in this situation, since $\sigma \to \frac{1}{2}$, $\lim \underline{p} = \frac{1}{3}$ as well. By continuity (of Δu over all the parameters of the game as well as p), one sees that for any $\underline{p} < \frac{1}{3}$ there is a set of parameter values with positive measure such that for prices below $\underline{p}$ there are no non-perverse equilibria in all games with parameters in that set.

For the case where $\alpha_1 < \alpha_0$, we find the maximal value of $\frac{1-\sigma}{\sigma}$, which is attained under the same limiting values used for the case with $\alpha_1 > \alpha_0$. Note that $\frac{v_1}{1-v_1} > \frac{1-\sigma}{\sigma}$ if and only if $v_1 > 1 - \sigma$. $\lim_{\sigma_g \to 1, \sigma_b \to \frac{1}{2}, \gamma \to 0} \sigma = \min \sigma = \frac{1}{2}$. In this limit, there is no equilibrium with $\alpha_1 < \alpha_0$ if $v_1 \leq \frac{1}{2}$. Given that $\sigma \to \frac{1}{2}$, $v_1 \to p$. This shows that $\lim_{\sigma_g \to 1, \sigma_b \to \frac{1}{2}, \gamma \to 0} \widehat{p}_2 = \frac{1}{2}$. This would yield a boundary $\underline{p} = \frac{1}{2}$.

Comparing the perverse and the non-perverse case, we see that the lower boundary is $\underline{p} = \frac{1}{3}$. This completes the proof of the second part of the proposition.

Proof of Proposition 2:
To prove this proposition, we need to show that for $p_t \in (1-\sigma, \sigma)$ $\alpha_0^t = 0$ and $\alpha_1^t = 1$ is an equilibrium. For the fund manager with $s_t = 1$, the expected profit difference from buying instead of selling is

$$2v_1^t - p_t^a - p_t^b$$

It is easy to see that this is always strictly positive, since $v_1^t > p_t^a > p_t^b$. If $\alpha_0^t = 0$ and $\alpha_1^t = 1$, then the reputational benefit of buying instead of selling is

$$\begin{aligned} & v_1^t \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma + (1 - v_1^t) \left(\frac{1-\sigma_g}{1-\sigma} - \frac{\sigma_g}{\sigma} \right) \gamma \\ = \; & (2v_1^t - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma \end{aligned}$$

which is positive if and only if $v_1^t \geq \frac{1}{2}$. Since $v_1^t = \frac{\sigma p_t}{\sigma p_t + (1-\sigma)(1-p_t)}$, $v_1^t \geq \frac{1}{2}$ if and only if $p_t \geq 1 - \sigma$. Thus, for $p_t \geq 1 - \sigma$ the manager with $s_t = 1$ would prefer to buy from both profit and reputational perspectives.

Now consider the case of a manager with $s_t = 0$. This manager profit difference from buying instead of selling is

$$2v_0^t - p_t^a - p_t^b$$

which is strictly negative since $p_t^a > p_t^b > v_0^t$. If $\alpha_0^t = 0$ and $\alpha_1^t = 1$, then the reputational benefit of buying instead of selling is

$$\begin{aligned} & v_0^t \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma + (1 - v_0^t) \left(\frac{1-\sigma_g}{1-\sigma} - \frac{\sigma_g}{\sigma} \right) \gamma \\ = \; & (2v_0^t - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma \end{aligned}$$

which is negative if and only if $v_0^t \leq \frac{1}{2}$. Since $v_0^t = \frac{(1-\sigma)p_t}{\sigma(1-p_t)+(1-\sigma)p_t}$, $v_0^t \leq \frac{1}{2}$ if and only if $p_t \leq \sigma$. Since $\sigma > \frac{1}{2}$, $1-\sigma < \sigma$, and thus for a strictly positive measure region $(1-\sigma, \sigma)$ sincere play is an equilibrium.

Proof of Proposition 3:

We provide the proof for the case where $p_t \leq \frac{1}{2}$. The other case is symmetric.

To prove (i), assume that $\alpha_0 = 0$. It is easy to check that, for all values of $\alpha_1 > 0$,

$$\Delta\pi_0(\alpha_1, \alpha_0 = 0) \leq 0 \text{ and } \Delta r_0(\alpha_1, \alpha_0 = 0) \leq 0.$$

The first is immediate. For the second, first note that it is easy to check that $\Delta r_0(\alpha_1, \alpha_0 = 0)$ is increasing in p. At $p = \frac{1}{2}$, $v_0(p) = 1 - \sigma$, and thus for $\alpha > 0$:

$$\begin{aligned}\Delta r_0(\alpha_1 = \alpha, \alpha_0 = 0) &= (1-\sigma)\left(\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma}\right)\gamma + \sigma\left(\frac{1-\sigma_g}{1-\sigma} - \frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma}\right)\gamma \\ &= -\frac{(\sigma_g - \sigma)\left[1 - \alpha(1 - \sigma(1-\sigma))\right](2\sigma - 1)\gamma}{\sigma(1-\alpha\sigma)(1-\alpha+\alpha\sigma)(1-\sigma)} < 0\end{aligned}$$

So, if $\alpha_1 > 0$, we know that $\Delta u_0(\alpha_1, \alpha_0 = 0) \leq 0$ and $\alpha_0 = 0$ is a best response. Consider two cases: (a) $\lim_{\alpha_1 \to 0^+} \Delta u_1(\alpha_1, \alpha_0 = 0) \geq 0$ and (b) $\lim_{\alpha_1 \to 0^+} \Delta u_1(\alpha_1, \alpha_0 = 0) < 0$.

In case (a), by continuity of Δu_1, either $\Delta u_1(\alpha_1 = 1, \alpha_0 = 0) \geq 0$ or there exists an interior α_1 such that $\Delta u_1 = 0$. Either subcase corresponds to an equilibrium.

In case (b), there exists a pooling equilibrium where $\alpha_1 = \alpha_0 = 0$. To see this, it is sufficient to set the beliefs generated by the out-of-equilibrium action $a = 1$ as if this action comes from someone with $s = 1$. In this case, one can easily see that

$$\Delta u_1(\alpha_1 = 0, \alpha_0 = 0) = \lim_{\alpha_1 \to 0^+} \Delta u_1(\alpha_1, \alpha_0 = 0),$$

which, as we are in case (b), must be negative.

For part (ii), consider an equilibrium with $\alpha_1 > \alpha_0$, and consider the manager who has received signal $s = 0$. It is clear at this point that $\Delta\pi_0 < 0$ for all p. We now show that for $p \leq \frac{1}{2}$, $\Delta r_0 \leq 0$. It is easy to check that Δr_0 is increasing in p. Thus, it is sufficient to show that $\Delta r_0\left(\frac{1}{2}\right) \leq 0$.

Suppose for contradiction that $\alpha_0 = \alpha > 0$. Lemma 2 and the assumption that $\alpha_1 > \alpha_0$ imply that $\alpha_0 = 1$. Let $p = \frac{1}{2}$. Thus, $v_0(p) = 1 - \sigma$. Using the values of r computed earlier, we now write:

$$\begin{aligned}\Delta r_0\left(\frac{1}{2}\right) &= (1-\sigma)\left(\frac{\sigma_g + \alpha(1-\sigma_g)}{\sigma + \alpha(1-\sigma)} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + \sigma\left(\frac{1-\sigma_g+\alpha\sigma_g}{1-\sigma+\alpha\sigma} - \frac{\sigma_g}{\sigma}\right)\gamma \\ &= -\frac{(\sigma_g - \sigma)(2\sigma-1)(1-\alpha)}{[\sigma + \alpha(1-\sigma)][1-\sigma+\alpha\sigma]}\gamma < 0,\end{aligned}$$

which generates a contradiction.

Proof of Proposition 4:
Since we are considering $p \leq \frac{1}{2}$ where by Proposition 3 we know that $\alpha_0 = 0$, we suppress α_0 in the proof. We thus write $\Delta u(\alpha_1, p, \beta)$ for $\Delta u(\alpha_1, \alpha_0 = 0, p, \beta)$, and similarly for $\Delta\pi$ and Δr. It is easy to check that the function $\Delta u(\alpha, p, \beta)$ is continuous in α (for $\alpha > 0$).

Suppose first that $\bar{\alpha}\left(\beta', p_t\right) = 1$. This means that

$$\Delta u\left(1, p, \beta'\right) = \beta' \Delta\pi(1, p) + \left(1 - \beta'\right) \Delta r(1, p) \geq 0.$$

We know that $\Delta\pi > 0$ for all values. If $\Delta r\left(1, p, \beta'\right) \geq 0$, then it is immediate that

$$\beta'' \Delta\pi(1, p) + \left(1 - \beta''\right) \Delta r(1, p) \geq 0.$$

If instead $\Delta r\left(1, p, \beta'\right) < 0$, we see that

$$\beta'' \Delta\pi(1, p) + \left(1 - \beta''\right) \Delta r(1, p) > \beta' \Delta\pi(1, p) + \left(1 - \beta'\right) \Delta r(1, p) \geq 0.$$

In both cases, $\Delta u\left(1, p, \beta''\right) \geq 0$ and $\bar{\alpha}\left(\beta'', p_t\right) = 1$.

Next, assume that $\bar{\alpha}\left(\beta', p_t\right) \in (0, 1)$. It must then be that

$$\Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta'\right) = \beta' \Delta\pi\left(\bar{\alpha}\left(\beta', p_t\right), p\right) + \left(1 - \beta'\right) \Delta r\left(\bar{\alpha}\left(\beta', p_t\right), p\right) = 0.$$

As $\Delta\pi > 0$, this implies that $\Delta r\left(\bar{\alpha}\left(\beta', p_t\right), p\right) < 0$. Hence,

$$\Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta''\right) > \Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta'\right) = 0$$

As $\Delta u(\alpha, p, \beta)$ is continuous in α, at least one of the following statements must be true: (i) There exists $\alpha'' \in \left(\bar{\alpha}\left(\beta', p_t\right), 1\right)$ such that $\Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta''\right) = 0$ (in which case there exists an informative equilibrium with $\alpha = \alpha''$); or (ii) $\Delta u\left(1, p, \beta''\right) \geq 0$ (in which case there exists a separating equilibrium). Either way, $\bar{\alpha}\left(\beta'', p_t\right) > \bar{\alpha}\left(\beta', p_t\right)$.

Proof of Proposition 5:
Start with (i). If $\widehat{p} \in \{p_t : \bar{\alpha}(\beta, p_t) = 0\}$ then by definition $\Delta u_1(0, \widehat{p}, \beta) \leq 0$ and $\Delta u_1(\alpha, \widehat{p}, \beta) < 0$ for all $\alpha \in (0, 1]$. Hence, $p_{\min}(\beta)$ must satisfy the following two conditions: $\Delta u_1(0, p_{\min}(\beta), \beta) \leq 0$ and $\Delta u_1(\alpha, p_{\min}(\beta), \beta) \leq 0$ for all $\alpha \in (0, 1]$. Thus, $\Delta u_1(\alpha, p_{\min}(\beta), \beta) \leq 0$ for all $\alpha \in [0, 1]$.

Let $\beta = \beta''$. $\Delta u_1(\alpha, p_{\min}\left(\beta''\right), \beta'') \leq 0$ for all $\alpha \in [0, 1]$. Consider $\beta' < \beta''$. Since $\Delta\pi_1 > 0$ and $\Delta r_1 < 0$, $\Delta u_1(\alpha, p_{\min}\left(\beta''\right), \beta) < 0$ for all $\alpha \in [0, 1]$. But this implies that $p_{\min}\left(\beta''\right) \in \{p_t : \bar{\alpha}(\beta, p_t) = 0\}$. Thus, $p_{\min}\left(\beta''\right) \leq p_{\min}\left(\beta'\right)$.

If $\widehat{p} \in \{p_t : \bar{\alpha}(\beta, p_t) = 1\}$ then by definition $\Delta u_1(1, \widehat{p}, \beta) \geq 0$. Hence, $p_{\max}(\beta)$ must satisfy $\Delta u_1(1, p_{\max}(\beta), \beta) = 0$. Let $\beta = \beta'$. Since Δu_1 is continuous in p and we know from Proposition 1 that $p_{\max}(\beta) > 0$, it must be the case that $\Delta u_1(1, p_{\max}(\beta'), \beta') = 0$, which implies that $\Delta \pi_1(1, p_{\max}(\beta')) > 0$ and $\Delta r_1(1, p_{\max}(\beta')) < 0$. Consider $\beta'' > \beta'$. It is now clear that $\Delta u_1(1, p_{\max}(\beta'), \beta'') > 0$, which means that $p_{\max}(\beta') \in \{p_t : \bar{\alpha}(\beta'', p_t) = 1\}$. Thus, $p_{\max}(\beta') \geq p_{\max}(\beta'')$.

For (ii), simply note that for all α and $p > 0$

$$\lim_{\beta \to 1} \Delta u_1(\alpha, p, \beta) = \Delta \pi_1(\alpha, p) > 0$$

Hence, for all $p > 0$,

$$\lim_{\beta \to 1} \bar{\alpha}(\beta, p) = 1,$$

which shows that $\lim_{\beta \to 1} p_{\max}(\beta) = \lim_{\beta \to 1} p_{\min}(\beta) = 0$.

For (iii), first we show that $\lim_{\beta \to 0} p_{\max}(\beta) = 1 - \sigma$. Note that

$$\begin{aligned} \lim_{\beta \to 1} \Delta u_1(1, p, \beta) &= \Delta r_1(1, p) \\ &= v^1(p)\left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + (1 - v^1(p))\left(\frac{1-\sigma_g}{1-\sigma} - \frac{\sigma_g}{\sigma}\right)\gamma \end{aligned}$$

where $v^1(p) = \frac{\sigma p}{\sigma p + (1-\sigma)(1-p)}$. It is easy to see that $\Delta r_1(1, p) \geq 0$ if and only if $p \geq 1 - \sigma$.

Now consider $\lim_{\beta \to 0} p_{\min}(\beta)$. For any α, $\lim_{\beta \to 1} \Delta u_1(\alpha, p, \beta) = \Delta r_1(\alpha, p)$, where

$$\Delta r_1(\alpha, p) = v^1(p)\left(\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma}\right)\gamma + (1 - v^1(p))\left(\frac{1-\sigma_g}{1-\sigma} - \frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma}\right)\gamma$$

We wish to find p such that $\Delta r_1(\alpha, p) < 0$ for all $\alpha > 0$ and $\Delta r_1(0, p) \leq 0$. We observe that $\Delta r_1(\alpha, p)$ is strictly increasing in p. Thus, for any α there exists a $p(\alpha)$ such that $\Delta r_1(\alpha, p) < 0$ if $p < p(\alpha)$. We compute $p(\alpha)$ for all α and minimize with respect to α. This gives $\lim_{\beta \to 0} p_{\min}(\beta)$.

For any α $p(\alpha)$ is defined by $\Delta r_1(\alpha, p(\alpha)) = 0$ which is equivalent to:

$$v_1(p(\alpha)) = \frac{1}{\frac{\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma}}{\frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma} - \frac{1-\sigma_g}{1-\sigma}} + 1}$$

Since $v_1(p)$ is increasing in p, we minimize $v_1(p(\alpha))$ with respect to α, which is equivalent to solving the following problem:

$$\max_{\alpha} \frac{\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma}}{\frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma} - \frac{1-\sigma_g}{1-\sigma}}$$

Upon some simplification, this can be shown to be equivalent to:

$$\max_{\alpha} \frac{1-\sigma}{\sigma} \frac{1-\alpha(1-\sigma)}{1-\alpha\sigma}$$

The maximand is monotone increasing in α since $\sigma > \frac{1}{2}$, and thus

$$\min_{\alpha} v_1(p(\alpha)) = v_1(p(1)) = \frac{1}{2} \Rightarrow \min_{\alpha} p(\alpha) = p(1) = 1-\sigma$$

Thus, $\lim_{\beta\to 0} p_{\min}(\beta) = 1-\sigma$.

Proof of Proposition 6:

Bid-ask spread. Focus on $p \in \left[0, \frac{1}{2}\right]$. Recall that in the most informative equilibrium a manager with $s_t = 0$ sells and a manager with $s_t = 1$ buys with probability $\bar{\alpha}(\beta, p)$. The ask price and the bid price are

$$\begin{aligned} p_t^a &= \frac{\delta\frac{1}{2} + (1-\delta)\bar{\alpha}\sigma}{\delta\frac{1}{2} + (1-\delta)\bar{\alpha}\Sigma_t} p_t \\ p_t^b &= \frac{\delta\frac{1}{2} + (1-\delta)\left[(1-\bar{\alpha})\sigma + (1-\sigma)\right]}{\delta\frac{1}{2} + (1-\delta)\left[(1-\bar{\alpha})\Sigma_t + (1-\Sigma_t)\right]} p_t, \end{aligned}$$

It is easy to check that the former is increasing in $\bar{\alpha}$, which, by Proposition 4, is non-decreasing in β.

Price-volatility. Given p_t, the random variable p_{t+1}takes two values: p_t^a with probability $\Lambda = \delta\frac{1}{2} + (1-\delta)\bar{\alpha}\Sigma_t$ and p_t^b with probability $1-\Lambda = \delta\frac{1}{2} + (1-\delta)\left[(1-\bar{\alpha})\Sigma_t + (1-\Sigma_t)\right]$. The variance is

$$\begin{aligned} Var\left[p_{t+1}|p_t\right] &= \Lambda\left(p_t^a - p_t\right)^2 + (1-\Lambda)\left(p_t^b - p_t\right)^2 \\ &= \frac{\left(\bar{\alpha}(1-\delta)(\sigma-\Sigma_t)\right)^2}{\Lambda} p_t^2 + \frac{\left(-\bar{\alpha}(1-\delta)(\sigma-\Sigma_t)\right)^2}{1-\Lambda} p_t^2 \\ &= p_t^2 (1-\delta)^2 (\sigma-\Sigma_t)^2 \frac{\bar{\alpha}^2}{\Lambda(1-\Lambda)} \end{aligned}$$

It is then easy to check that $Var\left[p_{t+1}|p_t\right]$ is increasing $\bar{\alpha}$, and hence non-decreasing in β.

Trade-predictability. This is immediate because

$$Var\left(a_t|p_t\right) = \Lambda(1-\Lambda)$$

and (for $p < \frac{1}{2}$) Λ is increasing in $\bar{\alpha}$. Thus, trade predictability, $\frac{1}{Var(a_t|p_t)}$, is decreasing in $\bar{\alpha}$, and thus non-increasing in β.

Proof of Proposition 7:

Note that there is at most one v for which $\sigma_{v,g} = \sigma_{v,b}$. Denote this by v_{equal}, so that for $v < v_{equal}$, $\sigma_{v,g} < \sigma_{v,b}$ and for $v > v_{equal}$, $\sigma_{v,g} > \sigma_{v,b}$. Consider any arbitrary (true) liquidation value $v^* < v_{equal}$ (the case for $v^* > v_{equal}$ is symmetric and is omitted). Suppose for contradiction that the equilibrium is such that $\lim_{t\to\infty} p_t = v^*$. Namely, for every $\Pr[v^*|h_t]$, there must be an informative equilibrium with either $\alpha_1 > \alpha_0$ or $\alpha_0 > \alpha_1$.

Case with $\alpha_1 > \alpha_0$

First note that, reusing the notation of the baseline model, $\Delta\pi_1 > 0 > \Delta\pi_0$. In addition, for $v^* < v_{equal}$ and $\alpha_1 > \alpha_0$ it is easy to show that $r(v^*,1) - r(v^*,0) < 0$. Thus, since $\Delta\pi_0 < 0$, $r(v^*,1) - r(v^*,0) < 0$, and $r(v,1) - r(v,0)$ is bounded for all v, there exists an $\epsilon > 0$ such that for $\Pr(v = v^*|p_t) > 1 - \epsilon$, $\Delta u_0 < 0$ and thus $\alpha_0 = 0$.[18] Now for histories implying that $\Pr(v = v^*|p_t) > 1 - \epsilon$, we can set $\alpha_0 = 0$, and write:

$$r(v^*,1) - r(v^*,0) = \left(\frac{\sigma_{v^*,g}}{\sigma_{v^*}} - \frac{\sigma_{v^*,g}(1-\alpha_1) + 1 - \sigma_{v^*,g}}{\sigma_{v^*}(1-\alpha_1) + 1 - \sigma_{v^*}}\right)\gamma$$

where $\sigma_{v^*} = \gamma\sigma_{v^*,g} + (1-\gamma)\sigma_{v^*,b} > \sigma_{v^*,g}$. It follows that we can find a strictly negative upper bound for $r(v^*,1) - r(v^*,0)$ since

$$\frac{\sigma_{v^*,g}}{\sigma_{v^*}} - \frac{\sigma_{v^*,g}(1-\alpha_1) + 1 - \sigma_{v^*,g}}{\sigma_{v^*}(1-\alpha_1) + 1 - \sigma_{v^*}} \leq \frac{\sigma_{v^*,g}}{\sigma_{v^*}} - 1 < 0$$

Now, consider the agent with $s = 1$. Given the boundedness of $\Delta\pi_1$ and $r(v,1) - r(v,0)$ for all v, we can write:

$$\lim_{\Pr(v=v^*|p_t)\to 1} \Delta u_1 = (1-\beta)\left(r(v^*,1) - r(v^*,0)\right) \leq (1-\beta)\left(\frac{\sigma_{v^*,g}}{\sigma_{v^*}} - 1\right)\gamma < 0$$

Thus, $\alpha_1 = 0$, and for $\Pr(v = v^*|p_t)$ high enough, the equilibrium cannot be informative.

Case with $\alpha_1 < \alpha_0$

In order to have an informative equilibrium, it must be the case that $\Delta u_1 \leq 0$ and since $\Delta\pi_1 > 0$ it must be the case that $\Delta r_1 < 0$. That is

$$\sum_v \Pr[v|h_t, s=1]\left(r(v,1) - r(v,0)\right) < 0.$$

Thus,

$$\Pr[v = v^*|h_t, s=1]\left(r(v^*,1) - r(v^*,0)\right) < \sum_{v\neq v^*} \Pr[v|h_t, s=1]\left(r(v,0) - r(v,1)\right)$$

It is clear that for any $\alpha_1 < \alpha_0$, $r(v,1) - r(v,0) > 0$ if and only if $\sigma_{v,g} < \sigma_{v,b}$. In particular, for any $\alpha_1 < \alpha_0$ the maximum value of $r(v,0) - r(v,1)$ is attained at $v = v_{\max}$. This is

[18] Note that, as in the baseline model, we can rule out totally mixed equilibria with $\alpha_1 > \alpha_0$.

because, for any given $\alpha_1 < \alpha_0$, $r(v,0)$ is increasing in v and $r(v,1)$ is decreasing in v. To see that (we omit the $r(v,0)$ case), note that

$$r(v,1) = \frac{\sigma_{v,g}\alpha_1 + (1-\sigma_{v,g})\alpha_0}{\sigma_v\alpha_1 + (1-\sigma_v)\alpha_0},$$

where, as before, $\sigma_v = \gamma\sigma_{v,g} + (1-\gamma)\sigma_{v,b}$. By A2, the ratio $\frac{\sigma_{v,g}}{\sigma_v}$ is increasing in v and the ratio $\frac{1-\sigma_{v,g}}{1-\sigma_v}$ is decreaing in v. As $\alpha_0 > \alpha_1$, this implies that the whole ratio $r(v,1)$ is decreasing in v.

Thus, the maximum value that can be taken by the right hand side of the above inequality is $\sum_{v\neq v^*} \Pr[v|h_t, s=1](r(v_{\max},0) - r(v_{\max},1))$. Hence, a necessary condition for $\Delta r_1 < 0$ at $v = v^*$ is

$$\Pr[v=v^*|h_t,s=1](r(v^*,1)-r(v^*,0)) < (1-\Pr[v=v^*|h_t,s=1])(r(v_{\max},0)-r(v_{\max},1))$$

Which can be rewritten as follows:

$$\frac{r(v^*,1)-r(v^*,0)}{r(v_{\max},0)-r(v_{\max},1)} < \frac{1-\Pr[v=v^*|h_t,s=1]}{\Pr[v=v^*|h_t,s=1]}$$

Define

$$E = -\frac{r(v^*,1)-r(v^*,0)}{r(v_{\max},1)-r(v_{\max},0)}$$

We shall show that $\inf_{\alpha_1<\alpha_0} E > 0$, which means that for histories implying $\Pr[v=v^*|h_t,s=1]$ high enough, the necessary condition must fail. The proof is a convoluted version of the relevant subcase of the proof of the main result. Define

$$b = \sigma_{v^*,g}\alpha_1 + (1-\sigma_{v^*,g})\alpha_0 \qquad B = \sigma_{v^*}\alpha_1 + (1-\sigma_{v^*})\alpha_0$$
$$a = \sigma_{v_{\max},g}\alpha_1 + (1-\sigma_{v_{\max},g})\alpha_0 \quad A = \sigma_{v_{\max}}\alpha_1 + (1-\sigma_{v_{\max}})\alpha_0$$

where $\sigma_{v^*} = \gamma\sigma_{v^*,g} + (1-\gamma)\sigma_{v^*,b}$, and similarly for $\sigma_{v_{\max}}$. Now it is easy to show that

$$E = \frac{\sigma_{v^*} - \sigma_{v^*,g}}{\sigma_{v_{\max},g} - \sigma_{v_{\max}}}\frac{A(1-A)}{B(1-B)}$$

Note that $\frac{\sigma_{v^*}-\sigma_{v^*,g}}{\sigma_{v_{\max},g}-\sigma_{v_{\max}}} > 0$ and independent of α_0, α_1. Thus, finding the infimum reduces to finding

$$\inf_{\alpha_1<\alpha_0}\frac{A(1-A)}{B(1-B)}$$

As before, interior solutions are ruled out by the facts that $\sigma_{v_{\max}} > \sigma_{v^*}$ and $A \neq B$. The remaining possibilities are that $\alpha_1 = 0$ and $\alpha_0 > 0$, in which case the infimum can be shown to be $\frac{1-\sigma_{v_{\max}}}{1-\sigma_{v^*}} > 0$ and $\alpha_1 < 1$ and $\alpha_0 = 1$, in which case the infimum can be shown to be $\frac{1-\sigma_{v_{\max}}}{1-\sigma_{v^*}}\frac{\sigma_{v_{\max}}}{\sigma_{v^*}} > 0$.

Proof of Proposition 8:

We first show that for sufficiently extreme prices, there cannot be any equilibrium in which $\alpha_{1g} \geq \alpha_{1b} \geq \alpha_{0b} \geq \alpha_{0g}$ with at least one strict inequality. This implies the first part of the proposition.

First note that because $\sigma_g > \sigma_b \geq \frac{1}{2}$, it is easy to check that $\frac{\sigma_g}{\sigma_g+\sigma_b} < \frac{1-\sigma_b}{2-\sigma_g-\sigma_b}$. Thus, condition (1) also guarantees that $\rho < \frac{1-\sigma_b}{2-\sigma_g-\sigma_b}$ which can be shown to be equivalent to $\Pr(\theta = g|s \neq v) < \Pr(\theta = g|z = b)$.

There are now four types of managers, determined by the values of s and z. Accordingly, any equilibrium, given p, is fully described by the probabilities that the four types play $a = 1$, which we denote as α_{0g}, α_{1g}, α_{0b}, and α_{1b} (where, for instance, $\alpha_{0b} = \Pr(a = 1|s = 0, z = b)$).

First, note that$\Delta\pi_{sz} = 2v_{sz} - p_a - p_b$, where $v_{sg} = \Pr[v = 1|s, z]$. This implies a strict ordering: $\Delta\pi_{1g} > \Delta\pi_{1b} > \Delta\pi_{0b} > \Delta\pi_{0g}$.

Next, note that the reputational benefit is

$$\Delta r_{sz} = v_{sz}\left(r\left(a = 1, v = 1\right) - r\left(a = 0, v = 1\right)\right) + \left(1 - v_{sz}\right)\left(r\left(a = 1, v = 0\right) - r\left(a = 0, v = 0\right)\right)$$

As the equilibrium is non-perverse and informative, $\alpha_{0z} \leq \alpha_{1z'}$ for $z \in \{b, g\}$ and $z' \in \{b, g\}$ with at least one strict inequality. We show that, under condition 1, this implies:

$$r\left(a = 1, v = 1\right) > r\left(a = 0, v = 1\right) \text{ and } r\left(a = 1, v = 0\right) < r\left(a = 0, v = 0\right)$$

We show that $r(a = 1, v = 1) > \gamma$. This automatically implies that $r(a = 0, v = 1) < \gamma$.

$$r\left(a = 1, v = 1\right) = \frac{1}{1 + \frac{1-\gamma}{\gamma}\frac{\sigma_b(1-\rho)\alpha_{1g}+\sigma_b\rho\alpha_{1b}+(1-\sigma_b)(1-\rho)\alpha_{0g}+(1-\sigma_b)\rho\alpha_{0b}}{\sigma_g\rho\alpha_{1g}+\sigma_g(1-\rho)\alpha_{1b}+(1-\sigma_g)\rho\alpha_{0g}+(1-\sigma_g)(1-\rho)\alpha_{0b}}}$$

Suppose that $r(a = 1, v = 1) \leq \gamma$. This can happen if and only if

$$\begin{aligned} & \sigma_b(1-\rho)\alpha_{1g} + \sigma_b\rho\alpha_{1b} + (1-\sigma_b)(1-\rho)\alpha_{0g} + (1-\sigma_b)\rho\alpha_{0b} \\ \geq \; & \sigma_g\rho\alpha_{1g} + \sigma_g(1-\rho)\alpha_{1b} + (1-\sigma_g)\rho\alpha_{0g} + (1-\sigma_g)(1-\rho)\alpha_{0b} \end{aligned}$$

which can be rearranged as follows:

$$\left(\alpha_{1b} - \alpha_{0b}\right)\left[\rho(\sigma_g + \sigma_b) - \sigma_g\right] \geq \left(\alpha_{1g} - \alpha_{0g}\right)\left[\rho\sigma_g - (1-\rho)\sigma_b\right] - \left(\alpha_{0b} - \alpha_{0g}\right)\left(2\rho - 1\right)$$

Condition 1, implies that $\rho(\sigma_g + \sigma_b) - \sigma_g < 0$ and $\rho\sigma_g - (1-\rho)\sigma_b > 2\rho - 1$. Thus the LHS is negative. The first term on the RHS is positive. The second is positive or negative depending on the sign of $\alpha_{0b} - \alpha_{0g}$. If this is negative, then the RHS is positive. If, on the other hand, $\alpha_{0b} - \alpha_{0g} > 0$, then note further that $\alpha_{1g} - \alpha_{0g} \geq \alpha_{0b} - \alpha_{0g}$ and $\rho\sigma_g - (1-\rho)\sigma_b > 2\rho - 1$. Thus the RHS is still positive. Thus, we have a contradiction, unless both the sides of the inequality are exactly zero, which happens only when $\alpha_{1b} = \alpha_{0b} = \alpha_{0g} = \alpha_{1g}$ which makes the equilibrium uninformative. This shows that $r(a = 1, v = 1) > \gamma > r(a = 0, v = 1)$. The case for $r(a = 1, v = 0) < r(a = 0, v = 0)$ is symmetric.

Given this, we now have a strict ordering on reputational payoff differences as well:$\Delta r_{1g} > \Delta r_{1b} > \Delta r_{0b} > \Delta r_{0g}$, in turn implying a strict ordering on the total payoff differential: $\Delta u_{1g} > \Delta u_{1b} > \Delta u_{0b} > \Delta u_{0g}$.

If there exists an informative equilibrium it must be that$\Delta u_{1g} \geq 0$ and $\Delta u_{0g} \leq 0$.We now show that if p is small enough, the condition $\Delta u_{g1} \geq 0$ is violated. For this, first note that, since $\Delta \pi_{0b} < 0$, for small enough p, it must be the case that $0 > \Delta u_{0b} > \Delta u_{0g}$, implying that $\alpha_{0b} = \alpha_{0g} = 0$, which then implies that:

$$\begin{aligned}\max_{\alpha_{1g},\alpha_{1b}} r(a = 1, v = 0) &= r(a = 1, v = 0|\alpha_{1g} = 1, \alpha_{1b} = 0) = \frac{(1-\sigma_g)\rho\gamma}{(1-\sigma_g)\rho\gamma + (1-\sigma_b)(1-\rho)(1-\gamma)} \\ \min_{\alpha_{1g},\alpha_{1b}} r(a = 0, v = 0) &= r(a = 0, v = 0|\alpha_{1g} = \alpha_{1b} = 0) = \gamma\end{aligned}$$

Hence,

$$r(a = 1, v = 0) - r(a = 0, v = 0) \leq \frac{(1-\sigma_g)\rho\gamma}{(1-\sigma_g)\rho\gamma + (1-\sigma_b)(1-\rho)(1-\gamma)} - \gamma$$

which is negative if

$$(1-\sigma_g)\rho < (1-\sigma_b)(1-\rho),$$

which is guaranteed under condition (1). Therefore,

$$\lim_{p\to 0} \Delta u_{g1} = (1-\beta)\left[r(a = 1, v = 0) - r(a = 0, v = 0)\right] \leq \frac{(1-\sigma_g)\rho\gamma}{(1-\sigma_g)\rho\gamma + (1-\sigma_b)(1-\rho)(1-\gamma)} - \gamma < 0$$

This completes the proof of the first part of the result.

Second part of the proposition: Since $\alpha_{0g} = 1$, $\Delta u_{0g} \geq 0$. Since $\Delta \pi_{0g} < 0$, it must be the case that $\Delta r_{0g} > 0$. We shall show that for sufficiently extreme p, this condition is violated, because $r(a = 1, v = 1) < \gamma < r(a = 0, v = 1)$ in this equilibrium. To see, assume to the contrary that $r(a = 1, v = 1) \geq \gamma$. Utilizing the expression derived in the proof of Proposition 8, we know that for any α_{sz}, this is equivalent to:

$$(\alpha_{1b} - \alpha_{0b})\left[\rho(\sigma_g + \sigma_b) - \sigma_g\right] \leq (\alpha_{1g} - \alpha_{0g})\left[\rho\sigma_g - (1-\rho)\sigma_b\right] - (\alpha_{0b} - \alpha_{0g})(2\rho - 1)$$

Substituting in equilibrium values gives: $\rho(\sigma_g + \sigma_b) - \sigma_g \geq \rho\sigma_g - (1-\rho)\sigma_b$, which is impossible since $\rho(\sigma_g + \sigma_b) - \sigma_g < 0$ and $\rho\sigma_g - (1-\rho)\sigma_b > 0$. For $p \to 1$, $\Delta r_{0g} \to r(a = 1, v = 1) - r(a = 0, v = 1) < 0$ which violates $\Delta r_{0g} > 0$.

References

[1] Franklin Allen. Do Financial Institutions Matter? *Journal of Finance* 56 (4): 1165-75, 2001.

[2] Franklin Allen and Gary Gorton. Churning Bubbles. *Review of Economic Studies* 60(4): 813-36. 1993.

[3] Christopher Avery and Judith Chevalier. Herding Over the Career. *Economics Letters*, 63: 327-333. 1999.

[4] Christopher Avery and Peter Zemsky. Multidimensional Uncertainty and Herd Behavior in Financial Markets. *American Economic Review*, 88(4): 724-748.

[5] Abhijit Banerjee. A Simple Model of Herd Behavior. *Quarterly Journal of Economics*, 107: 797-817. 1992.

[6] Sushil Bikhchandani, David Hirshleifer and Ivo Welch. A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades. *Journal of Political Economy*, 100: 992-1026. 1992.

[7] Varadarajan Chari and Patrick Kehoe. Financial Crises as Herds: Overturning the critiques. *Journal of Economic Theory*, forthcoming. 2003.

[8] Judith Chevalier and Glenn Ellison. Risk Taking by Mutual Funds as a Response to Incentives. *Journal of Political Economy* 105 (6): 1167-1200, 1997.

[9] Judith Chevalier and Glenn Ellison. Career concerns of mutual fund managers. *Quarterly Journal of Economics* 114(2), 389-432, 1999.

[10] Domenico Cuoco and Ron Kaniel. Equilibrium Prices in the presence of Delegated Portfolio Management. Working Paper, University of Pennsylvania, 2001.

[11] Amil Dasgupta and Andrea Prat. Financial Equilibrium with Career Concerns. *Theoretical Economics*, 1(1), 67-93, March 2006.

[12] Amil Dasgupta and Andrea Prat. Asset Price Dynamics When Traders Care About Reputation. CEPR Discussion Paper No. 5372, December 2005.

[13] Amil Dasgupta, Andrea Prat, and Michela Verardo. The Price of Conformism. Working paper, London School of Economics. 2005.

[14] Patrick Dennis and Deon Strickland. Who Blinks in Volatile Markets, Individuals or Institutions? *Journal of Finance* 57(5): 1923-1950. 2002.

[15] James Dow and Gary Gorton. Noise trading, delegated portfolio management, and economic welfare. *Journal of Political Economy* 105(5): 1024–1050, 1997.

[16] Lawrence R. Glosten and Paul R. Milgrom. Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders. *Journal of Financial Economics* 14(1): 71-100, 1985.

[17] Albert S. Kyle. Continuous Auctions and Insider Trading. *Econometrica* 53(6): 1315-35, 1985.

[18] In Ho Lee. Market Crashes and Informational Avalanches. *Review of Economic Studies*, 65: 741-759.

[19] Massimo Massa and Rajdeep Patgiri. Compensation and Managerial Herding: Evidence from the Mutual Fund Industry. Working paper, INSEAD. 2005.

[20] Kasper Nielsen. Institutional Investors and the Market for Corporate Equity. Working paper, University of Copenhagen, September 2003.

[21] Marco Ottaviani and Peter Sorensen. Professional Advice. Journal of Economic Theory, 126: 120-142. 2006.

[22] Andreas Park and Hamid Sabourian. Herding in Models of Sequential Trade and Monotonic Signals. Mimeo, University of Toronto. November 2004.

[23] Andrea Prat. The Wrong Kind of Transparency. *American Economic Review* 95: 862-877. 2005.

[24] David Scharfstein and Jeremy Stein. Herd behavior and investment. *American Economic Review* 80: 465–479, 1990.

[25] Richard Sias. Institutional Herding. Forthcoming, *The Review of Financial Studies*. October 2002.

[26] Richard Sias. Reconciliable Differences: Momentum Trading by Institutions. Working paper, Washington State University, 2003.

[27] Lones Smith and Peter Sorensen. Pathological Outcomes of Observational Learning. *Econometrica* 68: 371-398. 2000.

[28] Brett Trueman. Analyst Forecasts and Herding Behavior. *Review of Financial Studies* 7: 97-124.

[29] Dimitri Vayanos. Flight to Quality, Flight to Liquidity, and the Pricing of Risk. Working paper, MIT, 2003.