Markov Equilibrium in Models of Dynamic Endogenous Political Institutions

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Abstract

This paper examines existence of Markov equilibria in a class of *dynamic political games* (or DPGs). DPGs are infinite horizon games in which political institutions are endogenously determined each period. Specifically, at each date t, a social choice rule determines both the current public policy and the social choice rule to be used in date $t+1$. These rules are instrumental choices in the sense that they do not affect payoffs or technology directly.

We show that any dynamic political game can be transformed into a stochastic game in which the political institutions are reinterpreted as "public players" in a noncooperative, stochastic game played against private citizens. The public players' preferences may be dynamically inconsistent due to the fact that naturally occurring changes in the economic state, such as evolution of the wealth distribution, alter the way a political institution aggregates preferences of the citizenry over time. The paper characterizes this transformation, and establishes existence of Markov equilibria in which the Markov strategies are smooth functions of the state. Applicability of the result is demonstrated in an example with endogenous voting rules.

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1 Introduction

This paper introduces and examines equilibrium existence in a class of infinite horizon games in which political institutions are determined endogenously. We refer to these games as *dynamic political games* (or DPGs). In a DPG, future political institutions are chosen by the existing political institution.

At each date t, private decisions of individual citizens are combined with a public policy decision such as a tax rate or a quantity of a public good. A *political rule*, summarized by a parameter θ_t, is a social choice correspondence defined on the preference profiles of these citizens. The political rule θ_t determines both the payoff-relevant decisions in the current period and a future political rule θ_{t+1} for next period. The potential rules for choosing decisions in period $t+1$ are therefore objects of choice in period t.

A key feature of this recursive process is that political rules are instrumental choices: the rules do not affect payoffs or technology directly. Decision makers in a DPG thus modify existing rules, not because the details of rules enter into their utility functions, but rather because these details alter the process of policy determination in future periods.

A *politically feasible* strategy is an endogenous law of motion for policies, private decisions, and political rules such that at each date and after any history, each citizen chooses his private decision, while the current policy and future political rule are selected from the prevailing social choice correspondence. Political feasibility is analogous to Subgame Perfection in the sense that the strategies must yield implementable social choices after any history.

The paper's main result establishes existence of politically feasible strategies. In particular, we establish existence of politically feasible *Markov* strategies, i.e., strategies that vary only with current state. Moreover, these Markov strategies are shown to be smooth functions of the state.

Though "equilibrium existence" often connotes a purely technical exercise, the result offers some broader insights to the problems of dynamically endogenous institutional change. First, the existence issue brings to light some internal weaknesses of many common political institutions in making intertemporal decisions. To elaborate, the basic idea of the result is that any dynamic political game can be transformed into a more familiar looking object. Political institutions, because they are represented here by social choice rules, may be rationalized by social welfare functions or *aggregators* without loss of generality. These aggregators resemble the dynamic payoffs of an individual decision maker. Taking this interpretation one step further, a game with n citizens and a political aggregation rule is transformed into a *stochastic game* in which all public decisions are determined by an additional "public player."

In the case where a political rule is a dictatorship by one of the citizens, this is easy to see.

The "public player" is simply the dictator. In general, however, the game must be transformed into a stochastic game with a dynamically *in*consistent player. This happens despite the fact that every citizen's preferences are (by assumption) perfectly dynamically consistent.

To be clear about what this means, the "dynamic inconsistency" in the present model subsumes the common notion of "time inconsistency" (although the two terms are often used interchangeably) but can also be something quite different from it. Both terms refer to a tension between a decision maker's incentives at different points in time. However, time consistency typically refers to the tension created by one's inability to make commitments and the credibility of one's choices given the reaction of *other* individuals in the game.[1] By contrast, "dynamic inconsistency" can also refer to a structural "defect" in an individual's own preferences. The latter can occur even in the absence of any choices of others.

Dynamic inconsistencies — especially the latter sort — arise quite naturally in political rules such as wealth-weighted voting. For example, consider a voting rule in which each individual is allocated a number of votes proportional to his current wealth. A variation of this rule was used in Prussia in the mid 19th century. There the electorate was divided into thirds, each third given equal weight in the voting. The wealthiest individuals who accounted for the first third of taxes paid accounted for 3.5% of the population. The next wealthiest group that accounted for the middle third accounted for 10-12% of the population. The remainder accounted for the remaining third (see Finer (1997)).

More prosaic examples of wealth-weighted voting exist in contemporary institutions. In fact, any polity in which money plays a role in determining policy (e.g., through campaign contributions, political advertising, direct bribes, etc.) can yield wealth-weighted voting outcomes.

To see why wealth-weighted voting rules are dynamically inconsistent, consider an economy in which self-interested individuals differ only by wealth, and a pivotal voter exists. Then under quite normally occurring changes in the wealth distribution, the identity of a pivotal voter will change over time. This would occur if, for instance, the variance in the wealth distribution increases as time passes. Hence, the identity of the pivotal voter tomorrow will be fundamentally different than the identity of the pivotal voter today. But since the current pivotal voter evaluates future decisions according to own identity, rather than that of the future pivotal voter, a standard inconsistency arises.[2]

So why does the transformation of rules into dynamically inconsistent players matter for a theory of endogenous institutions? Notice that the choice of θ_{t+1} by the "public player" in period t amounts to a strategic delegation problem. The date t public player delegates decision authority to $(t+1)$-player. The simple example suggests that the inconsistency must

[1] The large literature following Kydland and Prescott's (1977) influential work typically interprets time consistency in this sense.

[2] Dynamic inconsistencies can also arise, of course, in an unweighted median voter model, but it would require more controversial modeling assumptions about the wealth distribution.

influence that choice to a large degree. Namely, it creates a desire to foreclose the inevitable loss of power by the current pivotal voter. In other words, the *internal* inconsistency in the institution's structure, rather than *external* commitment problems, can be the critical determinant of endogenous change of political institutions.[3] This stands in contrast to much of the literature on dynamically endogenous institutions, for example Gradstein and Justman (1999), Acemoglu and Robinson (2000, 2001, 2005), Lagunoff (2001), Greif and Laitin (2004), Jack and Lagunoff (2006a,b), Cervellati, et. al. (2006), and Gradstein (2007). In most of the literature, the inability to commit to policies that could eliminate external threats create a rationale for change.

The idea that dynamic inconsistency is a structural attribute of a political institution is, of course, not new. There are numerous models of majority voting which exhibit tensions between present and future median voters. A sample includes Alesina and Tabellini (1990), Krusell and Ríos-Rull (1996), Amador (2003), Nataraj Slavov (2006), Messner and Polborn (2004), and Lagunoff (2006). The last two take it one step further and use this inconsistency to model some form of institutional *change*. Messner and Polborn model preference reversals of voters as they age. The model helps to explain why current pivotal voters may opt for more conservative supermajority rules in the future. The Lagunoff paper is a companion to this paper, using the present model to examine stability of political rules.

The present paper adds to this literature by demonstrating the breadth of this problem via the equilibrium existence logic. The result applies to all forms of rule-based inconsistency in which the aggregator preserves the time separability of individual preferences. Hence the most commonly modeled forms of dynamic inconsistency, including, for instance, hyperbolic preferences are included.

The result does, however, have limitations. It does not, for example, produce any new voting aggregation result for static games. In particular, we take as a starting point those environments and rules that admit solutions in static models. Since the public decision problem each period is inherently multi-dimensional, this basically means that either the rule is not a voting rule, or more reasonably, that payoffs are restricted so that voting admits a solution (e.g., majority voting under order-restricted preferences (Rothstein (1980)).

The existence result does, however, make progress in extending aggregation results to dynamic environments. The extension is nontrivial because of the "political fixed point problem" that arises in all dynamic models of political preference aggregation. The problem is as follows. Voting aggregation in period t depends on citizens' preference orderings over all contingent polices starting from date t. But these preference orderings depend on the anticipated effect of the current policy on the future (equilibrium) policy strategy in period $t+1$. The date $t+1$ policy strategy, however, comes from the political preference aggregation in date $t+1$ which, repeating by the same argument as before, depends on political preference aggregation in date

[3] See Lagunoff 2006 for a fuller development of this idea.

$t+2$, and so on. This means that a rule admits a solution in date t only if all admissible rules admit solutions in all future periods and in all future states.

For obvious reasons, this problem does not arise in static models. Hence, while the present existence theorem does not resolve the problem for all conceivable political institutions (since many of these institutions are problematic in the simpler static environments), it does apply to a large variety of rules and environments that are not covered under previous equilibrium existence theorems. One such example is laid out in Section 2.

The theorem itself adapts some elements of an elegant result of Horst (2005). Horst proves the existence of smooth Markov equilibria in a class of standard stochastic games. The critical assumption is a "Moderate Social Influence (MSI)." Roughly speaking, MSI bounds the effect of others' actions one one's own marginal payoff uniformly over all states and in all periods of the game. MSI establishes uniformly Lipschitzian bounds on the players' marginal best replies. In turn, this fact is used to establish continuity of each player's "Bellman's" operator. The present model faces an additional complication that does not arise in Horst or related other results. Namely, that in the transformation of each rule as a "player," this player's preferences are dynamically inconsistent. This, in turn, implies that the player's current value map is not a Bellman's operator. Despite this, the proof shows that as long as the inconsistency preserves time (though possibly not state) separability of each player's dynamic preferences, the uniform Lipschitzian bounds property still holds. Hence, the result can be extended to these preferences.

The paper is organized as follows. The general model and equilibrium concept are described in Section 2. A canonical example is developed which illustrates the applicability of the main result. The result itself is described in Section 3. Section 4 offers concluding remarks and discusses related literature in more detail. The proof of the main result is in the Appendix.

2 Dynamic Political Games

Informally, a dynamic political game is a standard stochastic game with the addition of an explicit social choice rule that maps preference profiles to collective decisions at each date t. One part of this decision is the rule to be used in date $t+1$. The setup is necessarily complicated by the fact that one critical primitive — the rule by which preferences are aggregated — is defined on endogenous dynamic preferences. Section 2.1 first specifies all the components of the model. Section 2.2 then summarizes these components in the definition of a dynamic political game, and proceeds with a definition of political feasibility.

From here on, all defined sets are assumed to Borel measurable subsets of Euclidian spaces, and all defined functions and correspondences are assumed measurable. In addition, all feasible choice spaces (e.g., the space of feasible policies) are assumed to be compact.

2.1 The General Model

Formally, time is discrete and indexed by $t = 0, 1, \ldots$. Let $I = \{1, \ldots, n\}$ denote a society of infinitely lived individuals. An individual is denoted by $i \in I$. At each date t, two types of decisions, private and public, are determined simultaneously. An individual's *private* decision is one that affects him exclusively. Individual i's private action e_{it} at date t is chosen from a feasible set E. Later, we will explicitly assume that e_{it} is actually *chosen* by individual i. For now, however, we leave the issue of "who chooses what" open. Let $e_t = (e_{1t}, \ldots, e_{nt})$ denote a vector of private decisions. A *policy* decision p_t is determined collectively by society as a whole from the feasible set P. As with private decisions, we defer the discussion of precisely what "determined collectively" means until later.

The date t stage payoff of each individual depends on the profile of private decisions, e_t, on the policy p_t, and on a state variable ω_t drawn from a set Ω. Let $u = (u_1, \ldots, u_n)$ be a profile of stage game payoff functions, where each u_i is continuous, nonnegative, and expressed by $u_i(\omega_t, e_t, p_t)$. Citizen i's dynamic payoff is:

$$E\left[\sum_{t=0}^{\infty} \delta^t (1-\delta)\ u_i(\omega_t,\ e_t,\ p_t\ )\Big|\ \omega_0\right] \tag{1}$$

In (1), δ is the common discount factor, ω_0 is the initial state, and the expectation is taken with respect to a stochastic transition technology that determines how current states, private actions and public policies pin down distributions over future states. Formally, let $q(B|\ \omega_t, e_t, p_t)$ denote the probability that ω_{t+1} belongs to the Borel measurable subset $B \subseteq \Omega$, given the current state ω_t, the private decision profile, e_t, and the policy p_t. The Markov kernel q is assumed to be weakly continuous on $\Omega \times E^n \times P$, and the expected value in (1) is assumed to be finite for any initial state and feasible sequence of decisions.

The process that determines all decision variables is summarized by a parameter θ_t. This θ_t identifies a social choice function that aggregates individual preferences. We refer to the parameter θ_t as a *political rule*. Let Θ denote the set of admissible political rules for this society. Ordinarily social choice rules are straightforward objects: mappings from preferences to outcomes. However, in this case the notion is complicated by the fact that the domain of a rule is the space of individuals' dynamic preference profiles. These preferences may depend endogenously on how others' condition on the realized history in future periods. To address this issue, we proceed cautiously as follows.

The composite state at date t is summarized by the pair (ω_t, θ_t), consisting of an "economic" state and a political rule. A history at date t is

$$h^t = ((\omega_0, \theta_0, e_0, p_0), \ldots, (\omega_{t-1}, \theta_{t-1}, e_{t-1}, p_{t-1}), (\omega_t, \theta_t))$$

which includes all past data up to that point in time, as well as the current composite state (ω_t, θ_t). The initial history is $h^0 = (\omega_0, \theta_0, \emptyset)$. Let H denote the set of all histories at all dates.

For each citizen i, a private strategy $\sigma_i : H \to E$ defines a history-contingent action $\sigma_i(h^t)$ describing i's private action after observing history h^t. A profile of private strategies is given by $\sigma = (\sigma_i)_{i\in I}$. Similarly, a *public strategy* for this society is a pair $\psi : H \to P$ and $\mu : H \to \Theta$ that define, respectively, the policy $p_t = \psi(h^t)$ and the subsequent period's political rule $\theta_{t+1} = \mu(h^t)$. To re-iterate, these strategies map from histories that include the current economic state and political rule. A public strategy determines both a current policy and the subsequent period's political rule.

Denote the set of private, policy and institutional strategies by Σ, Ψ and M, respectively. A *strategy profile* is a triple $(\sigma, \psi, \mu) \in \Sigma^n \times \Psi \times M$ which produces an action profile $(\sigma(h^t), \psi(h^t), \mu(h^t)) = (e_t, p_t, \theta_{t+1})$ after every history $h^t \in H$. These strategies, together with the Markov kernel q, determine a stochastic process over realized histories. Given any realized history $h^t \in H$, a citizen's preference function over profiles (σ, ψ, μ) in the continuation of the game is given by

$$V_i(h^t;\ \sigma, \psi, \mu) \ = \ E_{q,\mu}\left[\sum_{t=\tau}^{\infty} \delta^{\tau-t}(1-\delta)\ u_i(\omega_\tau,\ \sigma(h^\tau),\ \psi(h^\tau)\ )\ \Big|\ h^t\right] \tag{2}$$

In (2), the expectation $E_{q,\mu}$ is taken with respect to the distribution on future states $(\omega_\tau, \theta_\tau),\ \tau \geq t$ induced by q and μ. Notice that the institutional strategy μ enters the payoff V_i only indirectly through the private and public strategies, σ and ψ, since these may vary with the current and past θs. A more convenient way to express (2) is given by

$$V_i(h^t;\ \sigma, \psi, \mu) \ = \ (1-\delta)\ u_i(\omega_t,\ \sigma(h^t),\ \psi(h^t)\ ) \ + \ \delta\ E_q\left[V_i(h^{t+1};\ \sigma, \psi, \mu)\ \Big|\ h^t, \sigma(h^t), \psi(h^t), \mu(h^t)\right] \tag{3}$$

Equation (3) defines the recursive form of the dynamic utility function $V_i(h^t;\ \cdot)$ over strategies (σ, ψ, μ) following h^t.

The tools are now in place to define the political rules explicitly. A *class* of political rules is a pair (Θ, C) where Θ is an index set and C is a social choice map defined such that, for any history h^t, C is a cylinder set $C(\ V_1(h^t;\ \cdot), \ldots, V_n(h^t;\ \cdot)\ ;\ \omega_t, \theta_t\ ) \subset \Sigma^n \times \Psi \times M$ (in the sense that no restrictions are placed on decisions $(\sigma(h^\tau), \psi(h^\tau), \mu(h^\tau)),\ \tau < t$ prior to date t). Hence, by writing $(\sigma, \psi, \mu) \in C(\ V_1(h^t;\ \cdot), \ldots, V_n(h^t;\ \cdot)\ ;\ \omega_t, \theta_t\ )$, the meaning is that (σ, ψ, μ) is the social choice under preference profile $(V_1(h^t;\ \cdot), \ldots, V_n(h^t;\ \cdot)\ )$ following history h^t. Notice that it is not assumed at the outset that C is non-empty valued. This is precisely part of the equilibrium existence problem that we address later on.

Putting together all the components of the model, a *Dynamic Political Game (DPG)* is defined as the collection

$$G \equiv \langle\ u, q, \Omega, E, P,\ \Theta, C,\ \omega_0, \theta_0 \rangle$$

Finally, given a dynamic political game G, we seek an equilibrium concept that does not assume that a political rule at date t can commit to a fixed law of motion governing political rules and policies for the indefinite future. Arguably, a commitment of this type would be impossible in any real society. Our equilibrium definition below therefore builds in the lack of strategic commitment by requiring implementable decisions at date t to be consistent with implementable decisions at date $t+1$.

Definition A strategy profile (σ, ψ, μ) as *politically feasible* if it constitutes a social choice for this society at each date and after any history. In other words, a politically feasible (σ, ψ, μ) satisfies

$$(\sigma, \psi, \mu) \in C(\, V_1(h^t;\, \cdot), \ldots, V_n(h^t;\, \cdot)\, ;\, \omega_t, \theta_t\,) \;\; \forall\, h^t,\, \forall\, t = 0, 1, \ldots \tag{4}$$

Politically feasible strategies that vary only with the current (Markov) state (ω_t, θ_t) are referred to as *politically feasible Markov strategies.*

2.2 Some Comments on the General Model

At this point, a few remarks may help clarify the modeling choices. First, by summarizing the political process by (Θ, C), we adopt the "detail free" approach of social choice rather than the fully explicit approach of noncooperative game theory. One could, of course, make the case that the noncooperative approach is preferable. For instance, the political rule could define a noncooperative voting game whereby today's votes determine which voting game is to be used tomorrow. Consequently, endogenous institutions could arise as Nash equilibrium outcomes of a standard stochastic game. The issue becomes: which game? While there are agreed upon canonical social choice representations of voting, there are relatively few such games. For one thing, strategic models of politics are notoriously sensitive to minor details. For another, the dynamics of detailed political rules are hard to model, which may explain why the literature is sparse.[4] Ultimately, the trade-off is one of "explicitness" versus "representativeness" and tractability. The approach taken here favors the latter.

Second, "political rules" are modeled for now as objects that subsume both public *and* private decisions. This is largely just a notational convenience, since C can always encode any decision left up to the individual. A natural presumption is that private decisions are determined privately. We will in fact assume this in the next Section.[5]

Third, though C can vary in the state s_t it is assumed to have no *structural* dependence on past history. Indirect dependence can arise, however, through citizen preferences. This is not a purely technical assumption. Rather, it preserves the intuitive idea that there are critical differences between *de jure* and *de facto* political processes. The de jure rule, expressed by C, is the explicit constitutional prescription that formally dictates who can participate, how votes

[4]See Gomes and Jehiel (2005) for one promising "hybrid" approach that largely avoids this problem.

[5]There are cases where private actions are not determined privately. Theocracies, for example, prescribe and enforce detailed behavior for individuals. In some cases, these prescriptions leave little room for individual choice.

are counted, etc. It may use information about the current state, but not typically past history. The de facto process would additionally include historical precedents and norms, as expressed in the citizens' equilibrium strategies.[6]

We illustrate the meaning of a politically feasible strategy in the following example. In both the notation below and in the remainder of the paper, an omitted subscript refers to the vector-valued profile, i.e., $V(h^t;\ \cdot) = (V_1(h^t;\ \cdot), \ldots, V_n(h^t;\ \cdot))$.

Example 2.1. *Suppose $E = \emptyset$, i.e., there are no private decisions (and so the Example omits e from the notation). Now suppose there are two possible political rules which we categorize as "Democracy" and "Dictatorship by Player 1." In this case, $\Theta = \{\theta^M, \theta^D\}$ where, if $\theta = \theta^M$ then the resulting public decision must be Majoritarian, i.e., resulting in public outcomes that dominate any alternative in a pairwise majority vote. If $\theta = \theta^D$ then the strategies are imposed by Citizen $i = 1$. Then define politically feasible (public) decisions by:*

$$C(\ V(h^t;\ \cdot)\ ;\ \omega_t, \theta_t\ ) = \begin{cases} \textit{set of Majoritarian outcomes} & \textit{if}\ \ \theta_t = \theta^M \\ \textit{arg}\max_{\psi,\mu} V_1(h^t;\ \psi, \mu) & \textit{if}\ \ \theta_t = \theta^D \end{cases}$$

where (ψ, μ) is a Majoritarian outcome or "Condorcet Winner" if for all $(\hat{\psi}, \hat{\mu})$,

$$|\{i \in I :\ V_i(h^t;\ \psi, \mu) < V_i(h^t;\ \hat{\psi}, \hat{\mu})\}|\ \leq\ \frac{n}{2}.$$

Other examples of politically feasible rules are easy construct. For instance θ could identify the fraction of individuals required to pass a public decision (endogenous supermajority rules). Alternatively, θ might identify the subset of individuals who currently possess the right to vote (endogenous voting franchise) or the collection of goods to be determined by the public rather than private sector.

2.3 "Rules as Players": Public Aggregators and Private Decisions

Political feasibility is essentially a Subgame Perfection idea for social choice rules. Strategies must be politically feasible at each date and after any history. In this section, the model is further specialized in two ways. First, we adopt the literal meaning of "private actions" and assume that each citizen i chooses the action e_{it} in date t. Next, we identify the public part of C with a social welfare function, referred to as an *aggregator*. Informally, an aggregator is a social welfare function that aggregates the preferences of the citizens each period to produce a social ranking

[6]See also Acemoglu and Robinson (2006) who provide a more explicit model that examines the difference between the de facto and de jure institutions.

over public decisions p_t and θ_{t+1}, taking as given the strategies of private citizens. Formally, an aggregator is a function $F : \mathbb{R}^n_+ \times \Omega \times \Theta \to \mathbb{R}_+$ expressed as $F\left(V(h^t;\ \sigma, \psi, \mu),\ \ \omega_t, \theta_t\right)$.[7] The social choice problem is thereby transformed into a noncooperative game consisting of the n private citizens and the aggregator. A class of rules (C, Θ) is *jointly rationalized by the citizens and the aggregator* if for any politically feasible strategy (σ, ψ, μ),

$$\sigma_i(h^t) \in \arg\max_{e_i}\ (1-\delta)u_i(\omega_t, e_i, \sigma_{-i}(h^t), \psi(h^t)\ ) + \delta E_q[V_i(h^{t+1};\ \sigma, \psi, \mu) \mid h^t, e_i, \sigma_{-i}(h^t), \psi(h^t), \mu(h^t)] \tag{5}$$

and

$$(\psi(h^t), \mu(h^t)) \in \arg\max_{p_t, \theta_{t+1}} F\left((1-\delta)u(\omega_t, \sigma(h^t), p_t\ ) + \delta E_q[V(h^{t+1};\ \sigma, \psi, \mu) \mid h^t, \sigma(h^t), p_t, \theta_{t+1}],\ \omega_t, \theta_t\right) \tag{6}$$

The problem now has the look of a more standard formulation. Notice that public and private decisions are now chosen separately. Each period, the n private citizens each choose their own private actions, while a "public player," whose payoff function is a social welfare function F chooses the public decisions taking as given the private strategies of citizens. Consequently, a dynamic political game maps to a stochastic game with n private agents and $|\Omega| \times |\Theta|$ public ones.

2.4 Dynamic Inconsistency within a Single Rule

Unfortunately, the transformed game described above may not be not very tractable. There are $|\Omega| \times |\Theta|$ "public" players, but Ω is not generally finite or even bounded. A more tractable approach — the one adopted here — is to transform the dynamic political game into a stochastic game with a *single* public player. In this case the payoff function of the "public" agent will not generally be dynamically consistent.[8]

To understand the source of the inconsistency, we illustrate the problem under a *single* political rule. Let an economic state be given by $\omega_t = (G_t, y_t)$, where $y_t = (y_{1t}, \ldots, y_{nt})$ with y_{it} the wealth of Citizen i at date t, and G_t is the stock of a durable public good at t. Let stage game utility be of the form $u_i = \kappa(y_{it})u(G_t, e_{it}, p_t) + v(G_t, e_{it}, p_t)$ where κ is strictly increasing. Suppose that at $t = 0$ the initial wealth distribution is y_{i0} and $y_{10} \leq \cdots \leq y_{n0}$. We omit the description of the technology for G_t since it is not relevant to the example.

For purposes of illustration, suppose that there there is only one admissible political rule, i.e., $\Theta = \{\theta\}$. Let θ be defined as follows. Fixing a private strategy σ for the remainder of the

[7] Recall that $V(h^t;\ \sigma, \psi, \mu)$ in this expression is a vector-valued profile of preferences of all citizens.

[8] See Lagunoff (2006) for a precise definition of dynamic consistency. For our purposes, we use the term in the typical sense that the set of solutions to the "full commitment problem" is different than the set of sequentially rational solutions.

example, define θ such that politically feasible (public) strategies (ψ, μ) satisfy: for all h^t,

$$\sum_{j\in J} y_{jt} \;\geq\; \sum_{j\notin J} y_{jt}$$

where $J \equiv$

$$\Big\{j \in I :\; V_j(h^t;\; \sigma, \psi, \mu) \geq (1-\delta)u_j(\omega_t, \sigma(h^t), p_t\;) + \delta E_q[V(h^{t+1}; \sigma, \psi, \mu)|\; h^t, \sigma(h^t), p_t, \theta_{t+1}]\Big\} \tag{7}$$

indicating the set of individuals who weakly prefer the public action pair $(\psi(h^t), \mu(h^t))$ to the action pair (p_t, θ_{t+1}), given the continuation $V(h^{t+1}; \sigma, \psi, \mu)$. Of course, in this example $\theta_{t+1} = \theta_t = \theta$. According to this rule, public decisions are those that survive pairwise comparisons against any other public decision when each citizen is endowed with y_{it} votes to cast in the election. This is sometimes referred to as the *Wealth-as-Power rule.*[9]

Given a law of motion for wealth distribution, it is easy to see how the identity of the pivotal voter under this rule can change over time. To illustrate this starkly, suppose (1) the median wealth level, denoted by y_{M0} satisfies $y_{M0} = \frac{1}{2}\sum_{j=1}^{n} y_{j0}$. This implies that the initial pivotal decision maker under the Wealth-as-Power rule is precisely the median voter; (2) suppose that for $t \geq 1$, $y_{i\,t+1} = y_{it} + z_i$ with $z_i > 0$, $z_n > 1/2$, and $z_1 < \cdots < z_n$. In this example, the ordering of the wealth distribution does not change over time (hence, the identity of the median voter remains constant). However, wealthier citizens accumulate wealth faster than poorer ones. In fact, for t sufficiently large, $y_{nt} > \frac{1}{2}\sum_{j=1}^{n} y_{jt}$. In other words, the richest citizen eventually accumulates over half the total wealth.

It is clear that under the Wealth-as-Power rule, the identity of the pivotal voter changes from the median to the richest individual $i = n$. Hence, the rule gives rise to a natural dynamic inconsistency. Namely, the tension between today's pivotal voter and tomorrow's creates a natural bias not unlike that of a hyperbolic decision maker.

By our restriction to a single rule θ, we illustrate that dynamic inconsistency arises because of intertemporal conflicts arising from de facto changes of power, in this case, under a *single* rule, and not because of conflicts *between* rules. One could, of course, observe that in a richer example, i.e., one with more admissible political rules to choose from, the decisive voter at date 0 might opt for a different rule — one that preserves his *de facto* power rather than face the loss of power under the same (*de jure*) rule. This is precisely the subject of an aforementioned companion paper which examines the properties of the equilibrium and asks when and whether a rule is stable. For now, our interest in the potential for dynamic inconsistency. Depending on the richness of the set Θ, an example with some additional options can still display an inconsistency.[10]

[9] See, for instance, Jordan (2006).

[10] Clearly, in any model of dynamic inconsistency, a richer set of options or strategies that build in commitment can eliminate the inconsistency. One could then ask: why put any *a priori* restrictions on the set of admissible

2.5 A Parametric Example

Consider a second example which contains a specific parametric specification for the economy. Let $\Omega \subset \mathbb{R}^n_+$ be convex. The economic state is $\omega_t = (\omega_{1t}, \ldots, \omega_{nt})$ where ω_{it} is Citizen i's wealth at date t. Policy $p_t \in [0,1]$ taxes wealth to generate revenue $p_t \sum_{j=1}^n \omega_{jt}$ used to produce public capital. Each period, a citizen's future wealth is produced by augmenting existing wealth with public investment that takes as inputs labor and tax revenue, the latter using a flat tax p_t on wealth. Formally:

$$\omega_{i\,t+1} \;=\; \omega_{it}(1-\gamma) \;+\; \nu_i B_i \cdot (p_t \sum_j \omega_{jt})^\eta (\sum_j e_{jt})^{1-\eta}$$

where $e_{it} \in [e^{min}, e^{max}]$ is i's labor effort, γ is the depreciation rate, B_i is a citizen-specific productivity parameter, and ν_i is a random shock with compact support $[\nu^{min}, \nu^{max}]$. Stage game payoffs are $u_i = (A + \omega_{it}(1-p_t))^\beta - e_{it}^2$ where A is a positive constant, and $0 < \beta \leq 1$. Here, individuals differ only by wealth, and u_i is increasing in after-tax wealth $(1-p)\omega_{it}$ and decreasing in labor effort e_{it}.

Private sector actions are determined individually, while the public sector decisions are as follows. Let Θ be a compact subset of $\mathbb{R}^n_+$. Now for each θ_t a politically feasible (public) strategy (ψ, μ) is one that satisfies: for all h^t,

$$\sum_{j\in J} \theta_{jt}\omega_{jt} \;\geq\; \sum_{j\notin J} \theta_{jt}\omega_{jt}$$

where, one may recall, J is defined in (7) as the set of individuals who weakly prefer the public actions $(\psi(h^t), \mu(h^t))$ to (p_t, θ_{t+1}).

In other words, θ_{it} defines the weight placed on i's income in the aggregation of votes. If $\theta_{it} = \frac{1}{n}$ then the each individual is allocated ω_{it} votes in a majority vote — as in the previous example. Alternatively, if $\theta_{it} = (\frac{\omega_{it}}{\sum_j \omega_{jt}})^{-1}$ then all individuals are weighted equally, and the rule is a simple majority voting rule in that period. In between these extremes, citizens face a classic choice: how much should wealth matter in determining political power?[11]

Unfortunately, this straightforward parameterization does not admit a closed form solution. On the positive side, the joint assumption on preferences and technology satisfy the conditions

political rules? First, there may be technological reasons that certain political institutions are not available. Second, and more significantly, note that Θ is an index set, and "the set of all indices" is not a well defined object. One could then look at a proscribed set of "voting rules" but even then the boundaries are ad hoc. Without exogenous limits the problem can quickly degenerate into an infinite regress problem (see Vassilakis (1991) and Lagunoff (1992)). In our case, we take as given a limited set of admissible rules, and examine the consequences for equilibrium existence.

[11]This choice is sometimes phrased as whether or not individuals favor a "timocracy," (a term coined by Aristotle (350 BCE), describes a rule by which property is decisive).

for Grandmont's (1978) Intermediate Preference Assumption. In particular, notice that the wealth ordering on I is preserved across all states. Hence, these preferences satisfy the order restriction property of Rothstein (1990). Rothstein's Theorem can therefore be applied to show that at each date, public decisions are rationalized by at least one individual's recursive payoff function. To see this, order the individuals, $\theta_{1t}\omega_{1t} < \theta_{2t}\omega_{2t} < \cdots < \theta_{nt}\omega_{nt}$. Then let

$$i_t = \arg\max_i \{\theta_{it}\omega_{it} : \sum_{j\geq i} \theta_{jt}\omega_{jt} \geq \sum_{j<i} \theta_{jt}\omega_{jt}\}$$

Notice that individual i_t is the pivotal individual whose inclusion is decisive in comprising a winning coalition. Therefore, F is simply defined by

$$F\left(V(h^t;\ \psi,\mu),\ \omega_t,\theta_t\right) = V_{i_t}(h^t;\ \psi,\mu) \tag{8}$$

Significantly, despite the richness of this particular set of voting rules, the identity of individual i_t may nevertheless change over time. The reason is that the delegation of decision authority from one i_t to a different i_{t+1} may be a necessary commitment device in order to induce more favorable private effort from the rest of the citizenry.[12] Nevertheless, despite the dynamic inconsistency, it is later verified that this example satisfies conditions sufficient to imply the existence of a unique politically feasible Markov profile which is almost everywhere differentiable in the state.

3 The Main Result

3.1 An Outline of the Problem

In this Section we state an existence theorem for politically feasible strategies when the aggregator F satisfies a separability assumption specified below. The theorem establishes existence of Markov strategies that are, in fact, smooth functions of the economic state. Smoothness is highly desirable in many applications. In particular, in most dynamic models of policy, equilibria are characterized by their Euler equations. These Euler equations are more elaborate than those in single agent problems since they include intertemporal effects that other agents' future policy functions have on a decision maker's policy decision in the current period. Computing these effects require solutions to higher order differential equations of future policy functions.[13] Hence, smooth existence is required in order to validate the computational solutions.

[12] See Jack and Lagunoff (2006). That paper has an example similar to this one, except that there is no random shock, $A = 0$, $\beta = 1$, and there is accumulation in a public good rather than in private wealth. In that case a closed form for a Markov equilibrium exists and satisfies $i_{t+1} = Ki_t$ for some constant K with $K \neq 1$.

[13] See Basar and Olsder (1999) for a general formulation. See Klein, Krusell, and Rios-Rull (2002) for an Euler equation characterization of macro policy games, and Jack and Lagunoff (2004) for an Euler equation characterization for political games.

Existence of Markov equilibria in stochastic games has long been known to be problematic. Recent contributions include Amir (1996), Curtat (1996), and Novak (2007) who make use of convexity assumptions on transitions, and supermodularity restrictions on preferences and transitions. Our result makes use of an innovation from Horst (2005) who proves existence of Lipschitz-continuous (hence, almost everywhere smooth) Markov Perfect equilibria in dynamic games. His result makes use of a "moderate social influence" (MSI) assumption whereby the interactions between players are sufficiently weak. The MSI assumption apparently originates from a restriction on payoffs in a local interaction model of Horst and Scheinkman (2002). The idea, roughly, is that one's own actions have a relatively greater effect on one's own marginal dynamic payoff than those of all other individuals combined. MSI overcomes a common problem in dynamic games. Generally, continuity of each player's "Bellman's" operator fails because it conflicts with conditions required for compactness of the function space to which the operator applies. One can recover continuity of the Bellman operator if there are uniform, Lipschitzian bounds on the players' marginal best replies in each state in the dynamic game. The MSI assumption establishes existence of such bounds.

Our upcoming existence theorem differs from Horst's smooth existence theorem in two respects. First, the present result applies to dynamically inconsistent preferences. From the example in Section 2.4, it is clear that attempt to transform the dynamic political game into a standard stochastic game will be thwarted by the implied dynamic inconsistency of the "public decision maker." For instance, in our examples of Sections 2.4 and 2.5 the dynamic preferences of i_t over continuations starting from $t+1$ onward will not generally coincide with that of i_{t+1} who "takes over" at the beginning of $t+1$. For this reason, the problem does not translate into a standard stochastic game since the preferences of the current public decision maker are dynamically inconsistent. In this sense, the present result represents an important extension of known results on equilibrium existence, even before considering the additional extension to dynamic political games.

Second, the fixed point argument differs. Horst first applies a fixed point argument for bounded state spaces. These fixed points are smooth Markov equilibria of the game with bounded states. He then extends the result by taking a uniform limit of fixed points as the bound is sequentially relaxed. In the present environment, it is not clear how to proceed with this line of argument since the state space itself identifies the characteristics of a player-aggregator, independently of which state the system is currently in. The aggregator in state ω_t when the state space is Ω may be different from the aggregator in state ω_t when the state space is $\tilde{\Omega} \supset \Omega$. The problem does not arise in the present argument because we apply the fixed point argument directly to the full, unbounded state space. The trade off is that our theorem employs stronger boundedness assumptions throughout the state space (see (A2) below).

3.2 Separability

Our existence theorem requires one additional assumption on the aggregator. It happens that this assumption is implicit in virtually all dynamic models of welfare or social aggregation. An aggregator F is *separable* if for every pair (ω_t, θ_t),

$$\begin{aligned} & F\Big(\ (1-\delta)u(\omega_t, e_t, p_t\ )\ +\ \delta\ E_q[V(h^{t+1}) \mid h^t, e_t, p_t, \theta_{t+1}],\ \omega_t, \theta_t\Big) \\ = \ & (1-\delta)F\left(u(\omega_t, e_t, p_t\ ),\ \omega_t, \theta_t\right)\ +\ \delta\ E_q[F\Big(V(h^{t+1}),\ \omega_t, \theta_t\Big) \mid h^t, e_t, p_t, \theta_{t+1}] \end{aligned} \tag{9}$$

In other words, a separable F is time separable and linearly homogeneous in the discount weights $(1-\delta, \delta)$. To understand just how common this assumption is, observe that the standard formulation of social welfare is given by $F(V_1, \ldots, V_1,\ \omega_t, \theta_t) = \sum_i \beta_i V_i$ which is separable. The Wealth-as-Power rule introduced in Section 3 is also separable. As is the recently popular hyperbolic aggregator.[14] The last two are dynamically inconsistent while the first is not. The welfare function $\left(\sum_i V_i\right)^{1/2}$ is not separable (and is dynamically inconsistent). Generally, nonseparable aggregators are inconsistent, but inconsistent aggregators may or may not be separable. Separability therefore rules out certain forms of inconsistency such as those due to "income effects" in the future consumption arising from changes in current consumption - as in this square root example. But it preserves some natural inconsistencies due to the rule's direct dependence on the state, independently of its indirect dependence through citizens' preferences. In the Wealth as Power rule, the aggregator that decides on p_t and θ_{t+1} is forever identified by ω_t even in all future states $\omega_\tau, \tau > t$. This puts a type ω_t aggregator in conflict with future types ω_τ that later inhabit the same decision authority.

We call a Dynamic Political Game, G, *separable* if its class of political rules is jointly rationalized by the citizens and a separable aggregator F. In the remainder, we restrict attention to DPGs that are separable.

3.3 Transformation to an Extended Stochastic Game

In this Section, we temporarily abstract away public decisions p_t and θ_{t+1} and from the political aggregation problem. Consider an $(n+1)$-player game that contains the basic elements of a standard stochastic game. However, in this game, individuals have possibly dynamically inconsistent preferences of a particular form described below.

To distinguish this game notationally from the original dynamic political game, define an *extended stochastic game* by

$$\bar{G} = \langle\ (\bar{V}_{1t}, \ldots, \bar{V}_{n+1\ t}\ )_{t=0}^{\infty},\ \ \bar{E}_1, \ldots, \bar{E}_{n+1},\ \ \bar{q},\ \bar{\Omega},\ \bar{\omega}_0\ \rangle$$

[14] See, for example, Krusell, Kuruscu, and Smith (2002).

In $\bar{G}$, $\bar{e}_{it} \in \bar{E}$ is each player's action in period t, the state is $\bar{\omega}_t \in \bar{\Omega}$, and $\bar{q}(\cdot|\bar{\omega}_t, \bar{e}_t)$ denotes the Markov Probability distribution over future states $\bar{\omega}_{t+1}$. The dynamic payoff to individual i at date t is given by

$$\bar{V}_{it} \;=\; (1-\delta)v_i(\bar{\omega}_t, \bar{e}_t) \;+\; \delta\; E_{\bar{q}}\;[\sum_{\tau=t+1}^{\infty}(1-\delta)\delta^{\tau-t-1}\pi_i(\bar{\omega}_\tau, \bar{e}_\tau;\; \bar{\omega}_t)\Big|\; h_t] \tag{10}$$

In (10) the first payoff v_i can be interpreted as the stage payoff of the current decision maker in the current date. The subsequent payoffs, all defined by the function π_i, are the stage payoffs to this same decision maker in subsequent periods. The payoff in (10) clearly satisfies the separability assumption, but it allows for two avenues of possible inconsistency. One comes from the difference between v_i and π_i. The other comes from the dependence of future payoffs on the current state ω_t. Intertemporal trade-offs between present and future stage payoffs may be different than the trade-offs between any pair of future stage payoffs. Of course, in the special case where $v_i(\bar{\omega}_\tau, \bar{e}_\tau) = \pi_i(\bar{\omega}_\tau, \bar{e}_\tau;\; \omega_t),\; \forall\tau$ we have the standard (dynamically consistent) formulation of a stochastic game.

As is standard, a subgame perfect equilibrium in $\bar{G}$ is a profile $\bar{\sigma}$ of history contingent actions that are sequentially rational for each player after every history. A Markov Perfect equilibrium is a subgame perfect equilibrium in Markov strategies.

With one additional caveat, any separable n-person dynamic political game G may be transformed into a $(n+1)$-player extended stochastic game $\bar{G}$. The one caveat is that one must allow for mixed strategies on the institutional choice of θ_{t+1} in order to convexify it. Let $\Delta(\Theta)$ denote the set of probability distributions on Θ (recall that Θ is compact though possibly not convex). Let $\beta \in \Delta(\Theta)$ an element of this set. An institutional strategy μ^* is a mixed behavioral strategy expressed as $\mu^*(h^t) = \beta_t$. The definition of political feasibility can be extended to μ^* in the obvious way.

We transform a dynamic political game G to an extended stochastic game $\bar{G}$ as follows. Let $\bar{\Omega} = \Omega \times \Theta$. Let $\bar{E}_i = E$ for each $i = 1, \ldots, n$ and let $\bar{E}_{n+1} = P \times \Delta(\Theta)$. Hence, a state is $\bar{\omega}_t = (\omega_t, \theta_t)$ as before. The technology, expressed as a density,[15] is defined as:

$$d\bar{q}(\bar{\omega}_{t+1}|\bar{\omega}_t, \bar{e}_t) = dq(\omega_{t+1}|\omega_t, e_t, p_t)d\beta(\theta_{t+1})$$

Payoffs are as follows. For each private citizen $i = 1, \ldots, n$,

$$v_i(\bar{\omega}_\tau, \bar{e}_\tau) = \pi_i(\bar{\omega}_\tau, \bar{e}_\tau;\; \bar{\omega}_t) = u_i(\omega_t, e_t, p_t)$$

Finally, for the public player $i = n+1$ his payoffs are

$$v_{n+1}(\bar{\omega}_t, \bar{e}_t) = F(u(\omega_t, e_t, p_t);\; \omega_t, \theta_t)$$

[15]The notation for the density may be expressed by $dq(\cdot|\omega, e, p)$, i.e., the instantaneous change in distribution q.

in date t, and

$$\pi_{n+1}(\bar{\omega}_\tau, \bar{e}_\tau;\ \bar{\omega}_t) = F(u(\omega_\tau, e_\tau, p_\tau);\ \omega_t, \theta_t).$$

in any date $\tau > t$

Notice that under this transformation, only the public player (the aggregator) has dynamically inconsistent preferences. However, we clearly utilize the separability assumption by associating stage payoffs in the transformed game to the aggregator's stage payoff in the original DPG. With this transformation, we have the following result:

Theorem 1 *Let G be any separable dynamic political game with n citizens, and $\bar{G}$ the transformation of G to an extended stochastic game with $n+1$ players. Then G admits a politically feasible Markov strategy profile (σ, ψ, μ^*) whenever $\bar{G}$ admits a Markov Perfect equilibrium, $\bar{\sigma}$.*

Our approach will therefore be to establish conditions under which a Markov Perfect equilibrium exists in the transformed game. Specifically, $\bar{\sigma}$ will vary only with $\bar{\omega}_t$, the current state.

3.4 An Existence Theorem for Extended Stochastic Games

Assumptions are now given for $\bar{G}$ which collectively imply existence of a smooth Markov equilibrium. However, to make sense of these assumptions, the following definitions and notational conventions are necessary. First, let $H : \mathbb{R}^\ell \to \mathbb{R}^k$ be any C^∞ (smooth) function, and let $|| \cdot ||_r$ denote the sup norm on the r^{th} derivative $D^r H : \mathbb{R}^\ell \to \mathbb{R}^{k\ell^r}$ as defined by

$$||D^r H||_r \;=\; \sup_{x'} \;\sup_{q=1,\ldots,k} \;\sup_{j_1,\ldots,j_r} ||\frac{\partial^r H_q}{\partial x_{j_1} \cdots \partial x_{j_r}}(x')||.$$

In this notation, $|| \cdot ||_0$ is the standard sup norm on H. Next, endow the class of all such functions with the topology of C^∞-uniform convergence on compacta. Formally, $H^m \to H$ in this topology if, for any compact set $Y \subset \mathbb{R}^\ell$, $\{H^m\}$ converges to H C^∞-uniformly on Y (i.e., for each r and each rth partial derivative, $||D^r H^m - D^r H||_r \to 0$ on Y). The function H is C^∞-uniformly bounded if it is smooth and there is some some finite number $L > 0$ that uniformly bounds H and bounds all its higher order derivatives in sup norm.

Next, define a real valued function, $g : \mathbb{R}^\ell \to \mathbb{R}$ to be *α-concave* with $\alpha > 0$ if $g(x) + \frac{1}{2}\alpha||x||^2$ is concave.[16] Notice that $\alpha = 0$ corresponds to the standard definition of concavity. When $\alpha > 0$, then α-concavity is obviously a stronger curvature condition. α-concavity is used elsewhere in the literature to bound higher order derivatives via the Implicit Function Theorem. It is used here for a similar purpose.

[16] An equivalent definition is: g is α-concave if the matrix $D^2 g + \alpha I$, with I denoting the identity matrix, is negative semi-definite.

In what follows we fix an extended game $\bar{G}$. However, we drop the cumbersome "bar" notation and write simply e_t and ω_t to denote action profiles and states, respectively. In addition, we adopt the usual convention of using primes, e.g., ω' to denote subsequent period's variables, ω_{t+1}, with double primes, ω'' for ω_{t+2}, and so on.

(A1) Each E_i is a compact, convex subset of a Euclidian space, and Ω is a convex subset of a Euclidian space.

(A2) (Uniform Bounds) For each i, there are numbers $L_i, L_{-i}, K > 0$ such that the payoff functions v_i and π_i are smooth and C^∞-uniformly bounded by L_i. In particular, v_i, π_i are uniformly bounded above by some $K \leq L_i$ and $D_{e_{-i}}[Dv_i]$ and $D_{e_{-i}}[D\pi_i]$ are uniformly bounded by $L_{-i} < L_i$. Also, $q(\cdot|\,\omega, e)$ admits a norm continuous density with respect to Lebesgue measure, and there is an $M > 0$ such that for each ω' and each ω, the density $dq(\omega'|\,\omega, \cdot)$ as a function of e is assumed to be C^∞-uniformly bounded by M.

(A3) (Concavity) For each i, there is an $\alpha_i > 0$ such that for each ω, v_i is α_i-concave in e_i.

(A4) (Moderate Social Influence) There exists a $0 < \gamma < 1$ such that for each i, the bounds M, K, L_{-i}, and α_i jointly satisfy:

$$\frac{(1-\delta)L_{-i} + 2\delta KM}{\alpha_i} \leq \gamma(1-\delta).$$

Assumptions (A1), (A3), and a closely related version of (A4) are also assumed in Horst (2005). Assumption (A4) is the Moderate Social Influence (MSI) discussed earlier. Assumption (A2) is stronger than the boundedness assumptions of Horst, in that it places uniform bounds on all higher order derivatives, rather than just a uniform Lipschitzian bound on first differences. The stronger assumption is needed for dealing with the added problem of dynamic inconsistency mentioned earlier. On the upside, we obtain the stronger conclusion that the equilibrium is almost everywhere smooth (C^∞) rather than just Lipschitz. When applied to the transformed game, these assumptions imply that a political rule must be jointly rationalized an F and private preferences that satisfy (A1)-(A4). The class of weighted rules in Section 2.5 is one example that satisfies the assumptions (provided that individual preferences do so as well).

THEOREM A *Let $\bar{G}$ denote an extended stochastic game satisfying (A1)-(A4). Then $\bar{G}$ admits a Markov Perfect equilibrium $\bar{\sigma}$ that is almost everywhere smooth in the state ω_t.*

The proof of Theorem A is in the Appendix. Together, the rules-into-players transformation and Theorem A imply:

Theorem 2 *Let G be any separable dynamic political game such that the transformation of G into an extended stochastic game $\bar{G}$ satisfies (A1)-(A4). Then G admits a politically feasible Markov profile (σ, ψ, μ^*) that is almost everywhere smooth in the composite state (ω_t, θ_t).*

To get a sense of the kinds of environments that are covered by the theorem, observe that the Parametric Model in Section 2.5 satisfies the conditions of the Theorem in an open set of parameter values of A, β, γ, η, the discount factor δ, the initial wealth distribution ω_0, and productivity parameters (B_i). To see this, consider any smooth density on ν_i with compact support $[\mu^{min}, \nu^{max}]$ and mean $B_i(p \sum_j \omega_j)^\eta (\sum_j e_j)^{1-\eta}$. To verify (A1), notice first that all choice sets are compact and the state space is obviously convex. As for (A2), let $B^{max} = \max_i B_i$. Then if $\omega_{i0} \leq \left(\frac{\gamma}{\nu^{max} B^{max} n (e^{max})^{1-\eta}}\right)^{\beta/(\eta\beta-1)} \equiv \bar{\omega}$, then one can verify that u_i is uniformly bounded above. Also, norm continuity of the density dq follows from norm continuity in the mean, and both u_i and dq are C^∞-uniformly bounded if η is sufficiently small and A sufficiently large.[17] Regarding (A3), the payoff u_i is α_i-concave for $\alpha_i = \max\{-2, \beta(1-\beta)A^\beta (\frac{\bar{\omega}}{A})^2\}$. Finally, the MSI assumption (A4) holds if δ is not too large.

4 Summary Discussion

This paper proves an existence theorem for a class of dynamic political games (DPGs). In DPGs, political institutions are instrumental objects of choice each period. Decisions are determined by social choice rules that aggregate profiles of dynamic recursive preferences of individuals. The public sector decisions include parameters of future political rules. Hence, rules used in date $t+1$ are a part of the decision determined by rules in date t.

Our approach to modeling political institutions as social choice rules is reminiscent of static models of "self-selected rules" of Koray (2000) and Barbera and Jackson (2000), and models on infinite regress in choice of rules in Lagunoff (1992) and Vassilakis (1992). These all posit social orderings on the rules themselves based on the outcomes that these rules prescribe. The present model differs from these in that institutional choice occurs in real time. This makes possible an analysis of explicit dynamics of change. More importantly, however, the present paper shares the stance taken by these models that social choice is a useful starting point for a theory of endogenous institutions.

In particular, social choice admits convenient and canonical notions of rules while avoiding detail-sensitive specifications inherent in many strategic voting games. Many common dynamic voting games can, in fact, be mapped into a social choice setting. In the companion to the present paper, Lagunoff (2006, Section 4.4) has an example in which a noncooperative delegation game is mapped into the present model. In that example, the constructed aggregator F is dynamically inconsistent and so the present result can be used to resolve the issue of equilibrium existence. Other "transformable" games include some dynamic legislative bargaining games, notably Kalandrakis (2004), Battaglini and Coate (2007a,b), and Bowen and Zahran (2007).

[17] Note ω_i is bounded by $\bar{\omega}$ as defined above. In turn, rth-order derivatives of u_i then converge to 0 as $r \to \infty$ if $A > \omega_i$.

In these games the political rule θ is fixed, and at each date a randomly selected proposer from a group of legislators chooses a policy which is then ratified by the remaining legislators using a voting protocol. While the details of the transformation are omitted, one could anticipate how it works: ω identifies both a status quo allocation and the identity of the proposer whose preferences are given by $F(\cdot, \omega)$; meanwhile, the ratification decisions are a part of the private decisions of citizen-legislators. In other words, the proposer's problem is given by (6) in the present paper, while those of the ratifiers are given by (5).[18] Significantly, the resulting F in the Kalandrakis and Bowen and Zahran models will be dynamically inconsistent because different proposers have inherently different preferences toward the policy.[19]

The social choice approach is common in many dynamic macro models of political economy and has been for some time. Building on the time consistent policy literature, Krusell, Quadrini, and Ríos-Rull (1997) posited an exogenous "political aggregator function" (a social welfare function) to determine policy. Even in their case where the political institution is taken as exogenous, the "existence" problem is nontrivial. The problem is resolved in Krusell, et. al., and in most of the literature that followed, by assuming either that there are only two types of agents, or that the policy space is uni-dimensional and per period payoffs have a special form to generate single peaked recursive preferences.[20]

In the present model, single peaked preference assumptions are not sufficient since the public decisions are inherently multi-dimensional: both the current policy and the future political rule are determined each period. A few papers examine dynamic models of voting that specifically allow for multi-dimensional choice spaces (though maintaining an exogenous voting rule). These include Bernheim and Nataraj (2004), Kalandrakis (2004), and Banks and Duggan (2003). However useful, these models have limited applicability in the present context because different institutions often manifest different types of inconsistencies. To my knowledge, no one has examined the general existence problem when institutions are endogenous.

Surprisingly, not so much is known about dynamic aspects of endogenous institutional change. Papers that do address such aspects include Roberts (1998, 1999), Gradstein and Justman (1999), Acemoglu and Robinson (2000, 2001, 2005), Lagunoff (2001), Barbera, Maschler, and Shalev (2001), Messner and Polborn (2004), Jack and Lagunoff (2006a,b), Gradstein (2007), Powell (2004), Cervellati, et al. (2006), Greif and Laitin (2004), and Egorov and Sonin (2005). These all posit useful dynamic/repeated game models in which one specific type of reform

[18]An appropriate modification is required to account for the fact that the stage game in these models is sequential rather than simultaneous.

[19]In their models, the policy is purely redistributive, and so each legislator prefers a distribution that allocates the largest possible share of the pie for his/her district. Interestingly, the inconsistency does not arise in the Battaglini and Coate models. The reason is that in their models, the redistributive component in the legislators' payoff functions is additively separable from the part that directly interacts with the state variable. Hence all legislators have the same preferences toward the dynamic part of their payoffs. Consequently, the F that they construct is dynamically consistent.

[20]See, more recently, Persson and Tabellini (2001) and Hassler, et. al. (2003) for other references.

(e.g., the voting franchise, supermajority threshold, or the assassination of the current ruler) is examined.[21]

Historically, institutional changes have often been multi-faceted and varied. Institutional choice in the present paper is therefore not restricted. DPGs admit a broad array of institutional changes. In a given game, all types of reforms can occur including changes in the voting rule (majority vs supermajority rules), changes in voting rights (e.g. larger vs smaller voting franchise), and changes in the scope of the public sector (e.g., expansions vs contractions of regulatory authority).

Apart from this attribute, the main point of this exercise is to demonstrate how the problem of existence of politically feasible strategies brings out some important attributes of preference aggregation embodied by the political rule itself. Perhaps the most significant of these attributes is the inherent dynamic inconsistency in some rules. These inconsistencies becomes apparent when the game is transformed into a more familiar stochastic game. The main result utilizes this transformation to establish existence of smooth Markov equilibrium of the dynamic political game.

Clearly, the conditions of the theorem apply to some interesting environments, but not to others. This leaves open the question of just how far the analysis can go. For example, do similar transformations exist if the political rules are jointly rationalized by citizens and by aggregators that are not time separable? We leave this and other issues for future research.

5 Appendix

Proof of Theorem A We prove Theorem A in a series of Lemmata.

Fix an *extended stochastic game*

$$\bar{G} = \langle\ (\bar{V}_{1t}, \ldots, \bar{V}_{n+1\,t})_{t=0}^{\infty},\quad \bar{E}_1, \ldots, \bar{E}_{n+1},\quad \bar{q},\ \bar{\Omega},\ \bar{\omega}_0\ \rangle$$

satisfying the assumptions of Theorem A. In what follows we drop the "bar" notation in the description above and write simply e_i instead of $\bar{e}_i$, etc.

Let $\mathcal{X}$ denote the set of all uniformly bounded, Lipschitz continuous functions, $x : \Omega^2 \to [0, K]^{n+1}$ expressed by $x = (x_1, \ldots, x_{n+1})$. Each x_i has uniform Lipschitz bound given by $(1-\delta)L_i + \delta KM$. The value of x is expressed as $x(\omega', \omega)$. Standard results show that $\mathcal{X}$ is compact in the topology of uniform convergence on compacta (see, for example, Mas-Colell

[21] See the companion paper Lagunoff (2006) both for a more detailed discussion of the literature, and for a broader motivation, citing a number historical examples in which political institutions are modified.

(1985, Theorem K.2.2). For each such function $x \in \mathcal{X}$, define a one shot game by the payoffs,

$$H_i(\omega, e,\ x) \ = \ (1-\delta)v_i(\omega, e) \ + \ \delta \int x_i(\omega', \omega) dq(\omega' |\ \omega, e) \tag{11}$$

for each $i = 1, \ldots, n+1$. Then let $H = (H_i)_{i=1}^n$ be the vector valued function with components defined by (11).

Lemma 1 *For each state ω and each continuation value $x \in \mathcal{X}$, the one shot game defined by payoff profile, $H(\omega, \cdot,\ x)$, has a unique pure strategy Nash equilibrium profile,*

$$(\xi_1(\omega, x), \ldots, \xi_n(\omega, x)\ )$$

of private decisions. The profile ξ is smooth with uniformly bounded first derivatives in ω, and is uniformly bounded and continuous in x.

Proof of Lemma 1 Observe first that by Assumptions (A1)-(A3), H_i is a smooth and C^∞-uniformly bounded function of (ω, e) (in the relative topology), with uniform bound given by

$$(1-\delta)L_i + \delta KM \tag{12}$$

Clearly, this bound is independent of x since x is itself uniformly bounded by K. Consequently, H_i is uniform bounded on its entire domain.

Next, define $\alpha_i^* \equiv \alpha_i(1-\delta) \ - \ \delta KM$. Observe that $\alpha_i^* > 0$ by (A4). We show that for each state ω, H_i is α_i^*-concave in e_i. To show this, we must show that for each ω, $D^2_{e_i} H_i(\omega, e\ x) + \alpha_i^* I$ is negative semi definite. To this end, fix ω. Observe that by α-concavity on stage utility functions v_i and the uniform boundedness of x_i and dq (Assumption (A2)), we have for every pair of profiles, $\hat{e}$ and e,

$$\begin{aligned}
& e_i^T \cdot D^2_{e_i} H_i(\omega, \hat{e}\ x) \cdot e_i, \\
\leq \ \ & e_i^T \cdot \left[D^2_{e_i}(1-\delta)v_i(\omega, \hat{e}) + \delta K \int D^2_{e_i}\ dq(\omega' \Big|\ \omega, \hat{e}) \right] \cdot e_i \\
\leq \ \ & -\alpha_i(1-\delta)||e_i||^2 \ + \ \delta KM||e_i||^2 \\
= \ \ & -\alpha_i^* ||e_i||^2
\end{aligned}$$

Since, $\alpha_i^* > 0$, it follows that H_i is α_i^*-concave. Consequently, by compactness of E (Assumption (A1)), and by the smoothness and strict concavity of H_i in e_i, the best response

$$g_i(\omega, e_{-i}, x) \ \equiv \ \arg\max_{e_i \in E} H_i(\omega, e, x)$$

for each i is nonempty and single valued.

Consider the best response function, g_i. There are two possibilities: either $g_i(\omega, e_{-i}, x)$ defines a critical point, i.e.,

$$D_{e_i} H_i(\omega, g_i(\omega, e_{-i}, x), e_{-i}, x) = 0.$$

or $g_i(\omega, e_{-i}, x)$ is locally constant. In the latter case, g_i is obviously smooth with uniformly bounded derivatives. In the case of the former, the Implicit Function Theorem can be applied since H_i is strictly concave. It follows that g_i is a locally smooth function in a neighborhood of (ω, e_{-i}) (in the relative topology). In this neighborhood, the Implicit Function Theorem implies

$$Dg_i \; = \; -[D^2_{e_i} H_i]^{-1} \cdot [D_{\omega, e_{-i}} D_{e_i} H_i]$$

Given the C^∞-uniform bound on H_i given by (12), the α_i^*-concavity of H_i implies that there is a uniform bound on Dg_i given by $\frac{1}{\alpha_i^*}[(1-\delta)L_i + \delta K M]$. Finally, since the choice of (ω, e_{-i}) was arbitrary, every such point is a regular point and so g_i is once again smooth with uniformly bounded first derivative.

We now show that there is a unique Nash equilibrium, $\xi(\omega, x)$ of the game with payoffs, $H(\omega, \cdot, x)$. Fixing ω and x, consider the best response map

$$e \mapsto (g_1(\omega, e_{-1}, x), \ldots g_n(\omega, e_{-n}, x)\ ) \,.$$

By the arguments above, the conditions for Brouwer's Theorem are met and so this map has a fixed point ξ. Since all best responses are interior — as shown above — the fixed point must be an interior point in E^n. To verify that this fixed point is unique, it suffices to show that the best response difference map

$$e \mapsto e - (g_1(\omega, e_{-1}, x), \ldots g_n(\omega, e_{-n}, x)\ ) \tag{13}$$

has no critical points. It suffices then to show that the Jacobian of this map at differentiable points is nonsingular. In turn, the Jacobian is nonsingular if it has a dominant diagonal. The Jacobian has a dominant diagonal if

$$||D_{e_{-i}} g_i||_1 < 1, \ \forall i, \tag{14}$$

at points (ω, e_{-i}, x) of the best response map, g_i. To verify (14), consider the best response map, g_i. Then we have:

$$
\begin{aligned}
||D_{e_{-i}} g_i||_1 &= || - [D^2_{e_i} H_i]^{-1} \cdot [D_{e_{-i}} D_{e_i} H_i]||_2 \\
&\leq \frac{1}{\alpha^*_i} ||(1-\delta) D_{e_{-i}} D_{e_i} v_i + \delta D_{e_{-i}} D_{e_i} \int x_i(\omega', \omega) dq(\omega' \Big| \, \omega, e)||_2 \\
&\leq \frac{1}{\alpha^*_i}[(1-\delta)L_{-i} + \delta K M] \\
&= \frac{(1-\delta)L_{-i} + \delta K M}{(1-\delta)\alpha_i - \delta K M} \\
&< \frac{(1-\delta)L_{-i} + \delta K M}{(1-\delta)\alpha_i - \frac{\delta K M}{\gamma}} \\
&\leq \gamma \; < \; 1
\end{aligned}
\tag{15}
$$

The first equality is the Implicit Function Theorem.[22] The first *inequality* follows from the definition of α^*-concavity. The second follows from the Uniform Bounds Assumption (Assumption (A2)). The next (second) equality follows from the definition of α^*_i, and the last *in*equality follows from the MSI condition (Assumption(A4)).

Next, we show that the profile ξ is smooth with uniformly bounded first derivatives in ω with the bound uniform across all x as well. Using the nonsingularity of (13) and the Implicit Function Theorem (IFT), $D_\omega \xi$ is smooth and defined by

$$D_\omega \xi \; = \; [I - D_e g]^{-1}[D_\omega g]$$

Note that the inverse $[I - D_e g]^{-1}$ exists and is uniformly bounded over all ω and all x by the dominance diagonal condition, (15). Consequently, $D_\omega \xi$ exists everywhere and is uniformly bounded over all ω and x.

Finally, we now prove that ξ is Lipschitz continuous in x with uniform Lipschitz constant. This follows from a result of Montrucchio (1987, Theorem 3.1). In particular, their result implies that for each i, each ω, and any pair x, x',

$$||g_i(\omega, \cdot, x) - g_i(\omega, \cdot, x')||_0 < \gamma ||x - x'||_0 \tag{16}$$

where γ is the MSI bound in Assumption (A4). Using the difference map in (13) to define the fixed points, the Implicit Function Theorem again implies that (16) applies to the fixed point, ξ, as well to the best response map g. ∎

[22] if e_i is one dimensional, then the sup norm picks out one such term,

$$\frac{\partial g_i}{\partial e_j} \; = \; (\frac{\partial^2 H_i}{\partial e_i^2})^{-1} \frac{\partial^2 H_i}{\partial e_j \partial e_i}, \; \forall j \neq i$$

Using Lemma 1, let $\xi = (\xi_1, \ldots, \xi_{n+1})$ be the map that defines the unique Nash equilibrium

$$\xi(\omega, x) = (\xi_1(\omega, x), \ldots, \xi_{n+1}(\omega, x)\ )$$

for the one shot game with payoffs, $H_i(\omega, e, x),\ i = 1, \ldots, n+1$.

Now for any pair of states $(\omega, \tilde{\omega})$, define the payoff function H_i^* by

$$H_i^*(\omega, \tilde{\omega}, e, x) = (1-\delta)\pi_i(\omega, e;\ \tilde{\omega}\ )\ +\ \delta \int x_i(\omega', \tilde{\omega}) dq(\omega' |\ \omega, e\ ) \tag{17}$$

Payoff function H^* is payoff value where π_i replaces v_i in the payoff function.

Lemma 2 $(H_i^*(\cdot, \cdot, \xi(\cdot, x), x)\ )_{i=1}^{n+1}\ \in \mathcal{X}$. *That is, H_i^* is a smooth function of the pair $(\omega, \tilde{\omega})$ with a uniformly bounded first derivative. This bound is uniform across all x.*

Proof of Lemma 2 By definition,

$$H_i^*(\omega, \tilde{\omega}, \xi(\omega, x), x) = (1-\delta)\pi_i(\omega, \xi(\omega, x);\ \tilde{\omega}\ )\ +\ \delta \int x_i(\omega', \tilde{\omega}) dq(\omega' |\ \omega, \xi(\omega, x)\ )$$

The smoothness therefore follows from the smooth of π_i and x_i directly and from the smoothness of $\xi(\omega, x)$ in ω established in Lemma 1. The uniform boundedness of first derivatives in $(\omega, \tilde{\omega})$, follows from the C^∞-uniform boundedness of π_i, x_i and dq and the uniform boundedness of first derivatives of ξ established in Lemma 1. ∎

Now define the operator T on $\mathcal{X}$ by

$$(Tx)(\omega, \tilde{\omega}) = (H_1^*(\omega, \tilde{\omega}, \xi(\omega, x), x), \ldots, H_{n+1}^*(\omega, \tilde{\omega}, \xi(\omega, x), x)\ ) \tag{18}$$

By Lemma 2, the function (Tx) is smooth in $(\omega, \tilde{\omega})$ with uniformly bounded first derivative, this bound being uniform over all x. That is, Tx has a uniform Lipschitz bound. Consequently, $Tx \in \mathcal{X}$ for all $x \in \mathcal{X}$.

Lemma 3 *T is a continuous operator.*

Proof Lemma 3 Let $\{x^\ell\}$ be a sequence such that $x^\ell \in \mathcal{X}$ for all ℓ and $x^\ell \to x \in \mathcal{X}$ with the convergence uniform on each compact set $Y \subset \Omega$ as $\ell \to \infty$. By Lemma 2, we also know that by Lipschitz continuity of ξ in x, $||\xi(\cdot, x^\ell) - \xi(\cdot, x)|| \to 0$ uniformly in a pair $(\omega, \tilde{\omega})$. Consequently, by the smoothness properties of v_i and π_i for each i and of dq we can fix $\epsilon > 0$ and let $\bar{\ell}$ satisfy for all $\ell \geq \bar{\ell}$, all ω' and all i, $||\pi_i(\cdot, \xi(\cdot, x^\ell);\ \cdot\ ) - \pi_i(\cdot, \xi(\cdot, x);\ \cdot\ )||_0 < \epsilon$, and $|dq(\omega' \big| \omega, \xi(\omega, x)\ ) - dq(\omega' \big| \omega, \xi(\omega, x^\ell)\ )| < \epsilon$.

Now by Lemma 5.2 in Horst (2005), given $\epsilon > 0$, for all i, and for all $\ell \geq \bar{\ell}$,

$$|| \int x_i(\omega', \cdot) dq(\omega' \Big| \cdot, \xi(\cdot, x)\ ) \ - \ \int x_i^\ell(\omega', \cdot) dq(\omega' \Big| \cdot, \xi(\cdot, x^\ell)\ ) ||_0 \ < \ \epsilon$$

With these results we see that:

$$\begin{aligned}
& ||Tx_i^\ell(\cdot,\cdot) - Tx_i(\cdot,\cdot)||_0 \\
= \ & ||H_i^*(\cdot,\cdot,\xi(\cdot,x^\ell),x^\ell) - H_i^*(\cdot,\cdot,\xi(\cdot,x),x)||_0 \\
\leq \ & (1-\delta)||\pi_i(\cdot,\xi(\cdot,x^\ell);\ \cdot\ ) - \pi_i(\cdot,\xi(\cdot,x);\ \cdot\ )||_0 \\
& \quad + \ \delta|| \int x_i(\omega', \cdot) dq(\omega' \Big| \cdot, \xi(\cdot, x)\ ) \ - \ \int x_i^\ell(\omega', \cdot) dq(\omega' \Big| \cdot, \xi(\cdot, x^\ell)\ ) ||_0 \\
< \ & (1-\delta)\epsilon \ + \ \delta\epsilon \ = \ \epsilon
\end{aligned}$$

Hence T is continuous. ∎

Using Lemma 3, T maps continuously from the compact set $\mathcal{X}$ into $\mathcal{X}$. By Schauder's Fixed Point Theorem (see, for example, Aliprantis and Border (1999)), T has a fixed point, x^*:

$$x^* = Tx^*.$$

Therefore, the profile, $\bar{\sigma} = (\bar{\sigma}_1, \ldots, \bar{\sigma}_{n+1}\ )$ defined by $\bar{\sigma}_i(\omega) = \xi_i(\omega, x^*)$ is a Markov perfect equilibrium that is an almost everywhere smooth function of ω. ∎∎

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