Short wavelength far infrared laser polarimeter with silicon photoelastic modulators

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(Presented 13 May 2008; received 8 May 2008; accepted 22 June 2008; published online 31 October 2008)

A short wavelength far infrared laser whose wavelength $\lambda$ is about 50 $\mu$m is preferable for a polarimeter and an interferometer for high density operations in the Large Helical Device (LHD) and on future large fusion devices such as ITER. This is because the beam bending effect ($\sim \lambda^2$) in a plasma, which causes fringe jump errors, is small and the Faraday and the Cotton–Mouton effects are moderate. We have developed a polarimeter with highly resistive silicon photoelastic modulators (PEMs) for the CH$_3$OD laser ($\lambda = 57.2$ and 47.7 $\mu$m). We performed bench tests of the polarimeter with a dual PEM and demonstrated the feasibility for the polarimeter. Good linearity between actual and evaluated polarization angles is achieved with an angular resolution of 0.05° and a temporal resolution of 1 ms. The baseline drift of the polarization angle is about 0.1° for 1000 s. © 2008 American Institute of Physics. [DOI: 10.1063/1.2957936]

I. INTRODUCTION

Measurement of the safety factor $q$ (or the rotation transform $\iota$) profile is indispensable for operations of magnetically confined fusion plasmas. A polarimeter which is combined with an interferometer, which measures the Faraday rotation and the density profile, is one of the conventional tools to evaluate the $q$ or the current profiles. The Faraday rotation $\alpha$ in a plasma is given as follows:

$$\alpha(\text{deg}) = 1.50 \times 10^{-11} \lambda^2 (m)^2 \int n_e (m^{-3}) B(T) \cdot ds(m),$$

where $\lambda$ is the wavelength of the probe beam, $n_e$ is the electron density, $B$ is the magnetic field strength, and $s$ is the beam path length in the plasma.

So far, far infrared (FIR) lasers, whose wavelengths are 339 $\mu$m (HCN laser), 195 $\mu$m (DCN), 119 $\mu$m (CH$_3$OH), have been used for the light sources of polarimeters. These wavelengths, however, are too long to use for the recent high density operation of the Large Helical Device (LHD) and on future large fusion devices such as ITER. This is because the beam bending effect, which causes fringe jump errors of the interferometer, is proportional to the electron density and the wavelength squared. Furthermore, the coupling between the Faraday ($\sim \lambda^2$) and the Cotton–Mouton effects ($\sim \lambda^3$), which makes the evaluation of the magnetic field difficult, cannot be ignored when long wavelengths are used because these effects become so large. Hence, we have been developing a short wavelength FIR laser, CH$_3$OD laser (simultaneous oscillation at wavelengths of 57.2 and 47.7 $\mu$m), for the polarimeter.

Many recent polari-interferometers utilize the polarization rotation method, which was invented by Dodel and Kunz. The method has a high temporal resolution, which can measure magnetic fluctuations. Polarization detection using photoelastic modulators (PEMs) is one of the alternative methods to measure the Faraday rotation angle. The optical system with PEMs has good compatibility with an interferometer because the Faraday rotation can be measured by dividing a probe beam of a usual heterodyne interferometer after passing through a plasma and by inserting a couple of PEMs and a polarizer into the divided beam path. In addition, the angle and temporal resolutions obtained in Refs. 17 and 18 are expected to be adequate for equilibrium analyses.

Since there was no available PEM for a wavelength around 50 $\mu$m, we have developed the PEM with a photoelastic element made of highly resistive silicon. The performance was experimentally evaluated and measurement of the polarization angle with the single PEM was demonstrated. This paper describes performance of the dual PEM system, which is upgraded in order to improve temporal and angular resolutions.

II. OPTICAL SYSTEM

A. Principle of measurement

Figure 1 shows the optical setup of the polarimeter which consists of a couple of PEMs and a polarizer. For description of the state of polarization and its temporal evolution, a reduced Stokes vector $s$ and a plasma Mueller matrix $M$ are introduced here.

The Stokes vector $s_0 = (s_0^0, s_0^1, s_0^2)$ of the linearly polarized beam with the Faraday rotation $\alpha$ after passing through a plasma or a half-wave plate is described as $(\cos 2\alpha, \sin 2\alpha, 0)$. The Mueller matrix of the PEM $M_{\text{PEM}}(\rho, \gamma)$ with a retardation $\rho$ and its angle of optical axis $\gamma$ is given by the following expression:
$$M_{PEM}(\rho, \gamma) = \begin{pmatrix} G + H \cos 4\gamma & H \sin 4\gamma & -\sin \rho \sin 2\gamma \\ H \sin 4\gamma & G - H \cos 4\gamma & \rho \sin 2\gamma \\ \sin \rho \sin 2\gamma & -\sin \rho \cos 2\gamma & \cos \rho \end{pmatrix},$$

where $\rho = \rho_0 \sin \omega_m t$, $G = (1 + \cos \rho)/2$, and $H = (1 - \cos \rho)/2$. $\rho_0$ and $\omega_m$ are the maximum retardation and the drive frequency of the PEM with a piezoelectric transducer, respectively. Then, the Stokes vector $s^1$ after passing through PEMs, which are aligned with optical axis angles $\gamma$ of $\pi/4$ and 0 degrees, is

$$s^1 = M_{PEM}(\rho_2, 0)M_{PEM}(\rho_1, \pi/4)s^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \rho_2 & \sin \rho_2 \\ 0 & -\sin \rho_2 & \cos \rho_2 \end{pmatrix} \begin{pmatrix} \cos \rho_1 & 0 & -\sin \rho_1 \\ 0 & 1 & 0 \\ \sin \rho_1 & 0 & \cos \rho_1 \end{pmatrix} \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} \cos \rho_2 \cos 2\alpha \\ \cos \rho_1 \cos 2\alpha \\ -\sin \rho_2 \sin 2\alpha + \sin \rho_1 \cos 2\alpha \end{pmatrix}.$$

The detected signal $I$ after passing through the polarizer whose transmission axis angle is 22.5° is given by following expression:

$$2I/I_0 = 1 - \cos 2(\pi/8 + \delta)s^1 + \sin 2(\pi/8 + \delta)s^2. \quad (1)$$

$I_0$: constant intensity; $\delta$: setting error of the transmission angle of polarizer). Here, $\cos \rho_k$ and $\sin \rho_k$ in $s^1$ and $s^2$ are expanded by using the Bessel function $J_n$ as follows;

$$\cos \rho_k = \cos(\rho_0^k \sin \omega_m^k t) = J_0(\rho_0^k) + 2 \sum_{n=1}^{\infty} J_{2n}(\rho_0^k) \cos(2n\omega_m^k t),$$

$$\sin \rho_k = \sin(\rho_0^k \sin \omega_m^k t) = 2 \sum_{n=1}^{\infty} J_{2n-1}(\rho_0^k) \sin((2n-1)\omega_m^k t).$$

By extracting amplitudes of $2\omega_m^1$, $2\omega_m^2$ components from $I$ with lock-in amplifiers, we obtain

$$I(2\omega_m^1) = -I_0 \cos 2(\pi/8 + \delta)J_2(\rho_0^1) \cos 2\alpha,$$

$$I(2\omega_m^2) = I_0 \sin 2(\pi/8 + \delta)J_2(\rho_0^2) \sin 2\alpha.$$

Then, the evaluated polarization angle $\alpha_{eval}$ is as follows;

$$\alpha_{eval} = -\frac{1}{2} \tan^{-1} \left\{ \frac{I(2\omega_m^2)}{I(2\omega_m^1)} \right\} = -\frac{1}{2} \tan^{-1} \left\{ -\frac{J_2(\rho_0^1)}{J_2(\rho_0^2)} \tan(\pi/8 + \delta) \tan 2\alpha \right\}. \quad (2)$$

Ideally, $\rho_0^1$ and $\rho_0^2$ should be equal and $\delta$ should be zero. Figure 2 shows the relationships between the actual and the evaluated polarization angles with (a) differences between $\rho_0^1$ and $\rho_0^2$, which are determined by the applied voltage to each photoelastic element, and with (b) various setting errors of the polarizer $\delta$. For example, the difference in the retardations ($\rho_0$ of 1.30 and 1.40 radians) by about 7% causes nonlinearity between actual and evaluated angles and the difference in the angle is about 10% at an actual polarization angle of 22.5°. A setting error of 1° also causes similar nonlinearity and an error of about 1° in the polarization angle at an actual angle of 22.5°. The difference in the retardation can be compensated by fine adjustment of the angle of the transmission axis of the polarizer. By making the coefficient in front of $\tan 2\alpha$ unity, good linearity can be obtained as shown in Fig. 2(c). In this figure, a difference in the retardation by 7% is compensated by shifting the transmission axis by 1.78° from the ideal angle.

**B. Optical setup**

The maximum output power of laser light at wavelengths of 57.2 and 47.7 µm are 1.6 and 0.8 W, respectively. Although these two wavelengths oscillate simultaneously for a two-color interferometer, bench tests of the polarimeter were performed separately; one wavelength can be selected with a polarizer because the polarization angles are perpendicular to each other. The drive frequencies of the PEMs (HINDS Instruments) are 50 and 40 kHz. The retardations were set at about 1.4 rad for both PEMs although slight difference was expected between them. The photoelastic material is highly resistive silicon whose absorptions of the 57.2 and 47.7 µm laser light are the small. Both sides of the photoelastic element are anti-
reflection (AR) coated. Multiple reflections inside the photoelastic element also cause significant nonlinearity between the actual and the evaluated polarization angles. This is because each phase of the multiple reflections is modulated by a small change in the optical length in the element due to the piezodrive, and the resulting interference with the transmitted light causes a modulation of \( I_0 \) with frequencies of \( n \omega_{\text{m}} \) (\( n: \) integer) in Eq. (1). These additional modulation terms lead to an error in the evaluated polarization angle. The coating material is Parylene, a kind of plastic, with a thickness of 8.7 \( \mu \text{m} \) which is optimized for 57.2 \( \mu \text{m} \) light. Although SiO\(_2\) is often used as the material for AR coating in this terahertz region, the strong residual stress which is attributed to the difference in coefficients of heat expansion might lead to an unintentional photoelastic effect. The Faraday rotation in a plasma is simulated with a half-wave plate made from crystal quartz (Tokyo Denpa Co.). The detector is a liquid helium cooled gallium-doped germanium photoductor (QMC Instruments Ltd.). The amplitudes of the second harmonic components are obtained with two lock-in amplifiers (5610B, NF Corporation). Since silicon is not transparent for visible light, beam alignment is carried out with a cw neodymium-doped yttrium aluminum garnet (Nd:YAG) laser beam with a wavelength of 1.06 \( \mu \text{m} \) (IRCL-300-1064, CrystaLaser) superposed to the FIR laser beam. The Nd:YAG laser beam can transmit silicon and can be visualized easily with an infrared sensor card and an infrared viewer (IRV-1700, Newport).

### III. EXPERIMENTAL RESULTS

The Parylene coating could reduce the reflectivity from 0.30 to 0.03 for the single reflection of 57.2 \( \mu \text{m} \) light. The maximum transmissivities of the 57.2 and 47.7 \( \mu \text{m} \) light through the PEM are about 70% and 60% for each PEM, respectively. Remaining multireflection components can be suppressed by tilting the incident angle of the laser light slightly based on the etalon effect. The maximum retardations \( \rho_0 \) after Parylene coating are about 1.4 and 1.7 rad for 57.2 and 47.7 \( \mu \text{m} \) lights, respectively.

### A. Linearity

Figure 3 shows relationship between the actual polarization angle (twice the rotation angle of the half-wave plate) and the evaluated one with Eq. (2). After the fine adjustment of the transmission axis of the polarizer to make the coefficient multiplied by the \( \tan 2\alpha \) in Eq. (1) unity, good linearity between the actual and evaluated polarization angles is obtained as simulated in Sec. II A. Actually, this fine adjustment is done to make the amplitudes of \( 2\omega_{\text{m}}^{-1} \), \( 2\omega_{\text{m}}^{2} \) equal when an actual polarization angle is set at 22.5°.

### B. Long time stability

Since plasma operation in future fusion devices is expected to be steady state, long time stability is required; drifts of the zero level of the polarization angle should be suppressed. For example, in the case of ITER, the plasma duration of a hybrid operation is planned to be longer than 1000 s. Figure 4 shows the temporal behaviors of the amplitudes of the second harmonics and the evaluated polarization angle. The drift is typically about 0.1° for 1000 s even though there are slight variations in the amplitudes. Equilibrium analyses in Ref. 27 show that a \( q \)-profile recovery error of about 5% is caused by an angle error of 0.2° in

![FIG. 2. (Color online) Relationships between actual and evaluated polarization angles with differences in the retardation (a) and setting errors of the transmission angle of the polarizer (b). The nonlinearity due to a difference of 0.1 rad in the retardations can be compensated by fine adjustment of the transmission angle of the polarizer.](attachment://fig2.png)

![FIG. 3. (Color online) Relationship between actual and measured polarization angles.](attachment://fig3.png)
Hence, the measurement shown in Fig. 4 was carried out over only 200 s. It is possible that the polarization angle drifted by about 0.4° without operating the air conditioner and by inserting a polarizer in front of the half-wave plate to fix the polarization angle. In order to improve the long-time stability more, it will be effective to cover the whole optical system. The other reason for the drift is reappearance of the additional modulated components almost zero without the polarizer in front of the detector. However, they gradually and slightly increased during the experiment, which is speculated to be caused by the slight change in the optical path of the laser light or slight change in the optical constant of the photoelastic elements due to the temperature variations. Here, it is assumed that an amplitude of 0.1 mV due to the multiple reflections, which was sometimes observed during experiments, is added to only an amplitude of I(80 kHz) of 8.5 mV. Then, the resultant error in the polarization angle is 0.06°, which is comparable to the drift in Fig. 4. Hence improvement of the AR profile is caused by about 0.2° at 57.2 μm, a little improvement of the stability of the baseline is necessary. The reasons for the drift of the polarization angle are as follows. One is the effects of airflow. When there was airflow by an air conditioner, the measured polarization angle drifted by about 0.4° over only 200 s. It is possible that the polarization angle was affected by the airflow although the reason is not yet clear. Hence, the measurement shown in Fig. 4 was carried out without operating the air conditioner and by inserting a polarizer in front of the half-wave plate to fix the polarization angle. In order to improve the long-time stability more, it will be effective to cover the whole optical system. The other reason for the drift is reappearance of the additional modulation components due to the multiple reflections. At the beginning of the measurement, the incident angle of the laser light is adjusted to make the additional modulation components almost zero without the polarizer in front of the detector. However, they gradually and slightly increased during the experiment, which is speculated to be caused by the slight change in the optical path of the laser light or slight change in the optical constant of the photoelastic elements due to the temperature variations. Here, it is assumed that an amplitude of 0.1 mV due to the multiple reflections, which was sometimes observed during experiments, is added to only an amplitude of I(80 kHz) of 8.5 mV. Then, the resultant error in the polarization angle is 0.06°, which is comparable to the drift in Fig. 4. Hence improvement of the AR coating or inserting an aperture after the PEMs to cut the multiple reflections, which have walk-off from the transmitted light, will be effective to suppress these drifts.

C. Angular and temporal resolutions

In this paper, the standard deviation of 1 s long period of angle data sampled with a frequency of 100 kHz is defined as the angular resolution. The relationship between the angular resolution and the time constant of the lock-in amplifiers is shown in Fig. 5. The minimum time constant is determined by the lock-in amplifier. The present angular resolution is 0.05° (0.02°) with a time constant of 1 ms (10 ms). In ITER, the maximum Faraday rotation angle is about 14° for 57.2 μm light in the case of the density profile of 1 × 10^{20}(1−ρ^3) m^{-3}. Considering that a required temporal resolution is 10 ms for equilibrium analyses in ITER and the q-profile recovery error mentioned above, the present resolution is preferable to the equilibrium analyses. For measurements of magnetic field fluctuations, improvement of the temporal resolution is necessary. The replacement of lock-in amplifiers and the application of a digital lock-in technique are planned.

IV. SUMMARY

We developed a polarimeter with highly resistive silicon PEMs for 57.2 and 47.7 μm laser lights, which are adequate wavelengths for polarimeter interferometers in large and high-density fusion devices. Good linearity between actual and evaluated polarization angles is obtained. The drift of the baseline is about 0.1° for 1000 s, and it is expected to be caused by airflow or the increase in additional modulated components due to multiple reflections in the photoelastic elements. The present angular resolution is 0.05 (0.02)° with a time constant of 1 (10) ms.

ACKNOWLEDGEMENTS

This work is supported by a Grant-in-Aid for Scientific Research on Priority Areas “Advanced Diagnostics for Burning Plasmas” (16082208). The authors thank Dr. Theodore C. Oakberg and Dr. Linda Hirschky of HINDES Instruments for the fabrication of the PEMs and discussions. The authors thank Dr. Byron J. Peterson for his proofreading.