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Luxury Prices: An Expository Note*

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Abstract

We study a model in which "useless" luxury goods are a mechanism for redistribution.

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1 Introduction

Economists generally associate the redistribution of resources to the apparatus of taxes and transfer payments. Such redistributions are done by the power of the authorities. However, resources are redistributed by other means as well. People give away income in a variety of ways, deliberate and unintentional. In this paper, agents transfer consumption goods in return for a good which lacks material qualities and affects their preferences because it has "value".

An example of a real life commodity without intrinsic value is diamonds. The standard consumption value of a diamond is low, especially as artificial diamonds look the same as real ones to the naked eye. Many luxury goods such as art pieces have similar characteristics. Owning a house in a "rich" area could have extra benefits in consumption and yet, one important motivation is simply to live in an expensive neighborhood. By holding such a good an agent refrains from acquiring resources that he could consume and thus, frees resources for other agents.

Our thinking on this issue originates from a naive question: Why are wealthy people striving to be wealthier? It is quite absurd to think that rich people wish to increase the consumption of standard goods as assumed in the classical consumer model. Indeed, by acquiring an expensive diamond or an original piece of art, a consumer demonstrates his avoidance of consumption.

We construct and analyze an elementary competitive equilibrium model in which the value of "useless" jewels enter the preferences of consumers. We show that if jewels are scarce, they can trade at a positive price despite having no intrinsic utility. Under fairly natural assumptions, a positive price equilibrium redistributes consumption from the rich to the poor. Thus, the model demonstrates the common wisdom that the existence of luxury goods can have a positive effect on income distribution.

This paper is mainly expository. The idea of consumption goods lacking intrinsic value and of prices entering directly the utility functions is not original. Veblen (1899) pointed out that some goods are consumed by some people for reasons not connected with their intrinsic values. Veblen referred to consumption intended to impress and to signal wealth as conspicuous consumption.

Arrow and Hahn (1971) (Chapter 6) (see also Kalman (1968)) discussed a general equilibrium model in which agents care not only about the consumption but also about the price vector. Under standard assumptions, they proved existence of competitive equilibrium. In our model, the existence of equilibria is not an issue as there always exists an equilibrium where jewels have zero price. Our interest is in the existence of equilibria in which the price of jewel is not zero.

Ng (1987) treats "diamonds" explicitly as goods that "are valued not because their intrinsic consumption effects but because they are costly." He considers a consumer whose utility function is of the type $U(p^1x^1/p^n, x^2, x^3, ...x^n)$, where the first good is "diamonds" and the last good is the "numeraire". The maximization of this function given the standard budget constraint $\sum p^i x^i = w$ yields demands for which a change in p^1 does not change the demand of other commodities and does not affect the utility. Ng uses this fact to point out that imposing taxes on diamonds is an efficient way of raising revenue as it does not have real effects on the economy.

Bagwell and Bernheim (1996) study a model in which the utility of a consumer is a function of his consumption and an action taken by a "representative agent" (the society) who observes whether the consumer does or does not hold a jewel. The representative agent does not observe the initial wealth of the consumer and his action is a response to the information he infers from conspicuous consumption. Thus, a consumer faces a trade-off between consuming and showing that he is "wealthy" by buying a jewel.

Other models consider equilibria with goods that do not have intrinsic value. See, for example, Frank (1988). In this paper, we focus on a tradeable good whose possession generates utility endogenously only as a function of the price. We treat "jewels" as goods which satisfy the psychological need of owning a precious commodity and not as signals of status aimed at obtaining more standard consumption goods. In particular, we show that there can be equilibrium prices inducing different levels of utility yet revealing the same information about the consumers who possess the good. Although many conspicuous commodities do serve as signals of status, the same cannot be said of many goods with no intrinsic value such as old stamps or coins.

2 A Simple Model

Consider an economy with one divisible consumption good and one indivisible good called a "jewel". For simplicity, we will assume that a consumer cannot own more than one jewel. Each individual enters the market with a pair (c, d), where $c \in \Re_+$ is the amount of consumption that he owns and $d \in \{0, 1\}$ indicates whether he owns or does not own a jewel. Let $\mu_n(c)$ denote the mass of consumers who initially do not own jewels and whose initial endowment of consumption is below c, and $\mu_o(c)$ the mass of the initial owners of jewels whose initial endowment of consumption is below c. Denote by $\mu_n(\infty)$ the mass of consumers who initially do not own jewels and by $\mu_o(\infty)$ the mass of consumers who initially do. The measures $\mu_n(c)$ and $\mu_o(c)$ are defined on the interval $[0, \infty)$. We assume that there exists points c_i, $i = n, o$, such that $\mu_i(c) = \mu_i(\infty)$ for any $c \geq c_i$ and $\mu_i(c)$ is strictly increasing on the interval $(0, c_i)$.

Denote by p the price of a jewel in terms of consumption. The decision that each consumer faces is binary, that is, each consumer decides whether to hold the jewel or not; if he holds the jewel initially, he decides whether to sell it for p units of consumption; if he does not holds the jewel initially, he decides whether to buy it for p units of consumption.

The pair (c, p) denotes consumption of c and ownership of a jewel priced p. We assume that consumers maximize a preference relation $\succsim$, identical for all consumers, over the space of pairs (c, p). The pair $(c, 0)$ denotes consumption of c and either ownership of a jewel priced 0 or not owning a jewel. We assume that consumers are indifferent between not having a jewel and having a jewel priced 0. The value of a jewel is in its price.

We make several assumptions about the preference relation $\succsim$:

P1: The preferences are monotonic, strictly convex, and continuous.

P2: For any $x > 0$, $(x, 0) \succ (0, x)$ (consumers prefer to consume x units to a jewel priced x).

P3: For $c' > c$ and $p > 0$, if $(c, p) \succsim (c + p, 0)$ then $(c', p) \succ (c' + p, 0)$ (if a consumer wishes to have a jewel, a richer consumer also wishes to have it).

Any preference relation represented by a utility function $u(c, p)$ which satisfies (i) strong monotonicity, (ii) quasi-concavity, (iii) continuity, (iv) $\frac{\partial^2 u}{\partial p \partial c} - \frac{\partial^2 u}{\partial c^2} > 0$ and (v) $u(x, 0) > u(0, x)$, satisfies assumptions P1-P3. To see that such a utility function satisfies P3, note that

$$u(c,p)-u(c+p,0)=\int_0^p[\frac{\partial u}{\partial p}(c+p-t,t)-\frac{\partial u}{\partial c}(c+p-t,t)]dt$$

and, by (iv), the difference between the partial derivatives in the integral is increasing in c.

A candidate for a *competitive equilibrium* is a price p and two functions $B:[0,\infty)\to[0,1]$ and $S:[0,\infty)\to[0,1]$. The function B describes the behavior of a consumer with initial bundle $(c,0)$: The proportion of agents with initial bundle $(c,0)$ who buy a jewel is $B(c)$. The function S describes the behavior of a consumer with initial bundle $(c,1)$: The proportion of agents with initial bundle $(c,1)$ who sell the jewel is $S(c)$. Thus, whenever there is a positive mass of consumers with initial bundle (c,i), we implicitly treat the mass as a continuum of consumers.

An equilibrium is a triple (p,B,S) satisfying

(i) For any c, the behavior described by the functions B and S is optimal given the price p (namely, $B(c)=1$ if $(c-p,p)\succ(c,0)$, $B(c)=0$ if $(c,0)\succ(c-p,p)$, etc.).

(ii) Equality of demand and supply of jewels: $\int B(c)d\mu_n(c)=\int S(c)d\mu_o(c)$

2.1 Buying a jewel

Under assumptions P1-P3, there is a cut-off c^* such that:

(i) consumers who do not have a jewel and are endowed with consumption below c^* do not buy the jewel regardless of the price, and

(ii) consumers who do not have a jewel and are endowed with consumption above c^* buy the jewel if its price does not exceed a reserve price.

Claim 1 *Assume P1-P3. There exists an increasing, continuous function $q:[0,\infty)\to[0,\infty]$ such that a consumer endowed with c and no jewel demands the jewel if $p<q(c)$ and does not demand it if $p>q(c)$. The function q is strictly increasing at any point $c>c^*=\max\{c|q(c)=0\}$.*

Proof: Consider the set D of all pairs (c,p) for which a consumer who has consumption level c and no jewel is not worse off buying the jewel for the price p. By the continuity the set D is closed. By P2, if $(c,p) \in D$ and $c > 0$ then $p < c$. By convexity, if $(c,p) \in D$ then, for any $p' < p$, $(c,p') \in D$. By P3, if $(c,p) \in D$ then, for any $c' > c$, $(c',p) \in D$. By definition, $(c,0) \in D$ for any c. Define $q(c) = \max\{p : (c,p) \in D\}$. The function q is continuous and strictly increasing for any c for which $q(c) > 0$. A potential buyer with initial consumption level c will demand the jewel if $p < q(c)$ and will not if $p > q(c)$. QED

The above claim give us a characterization of the demand for jewels. Given a composite function $f(g(x))$, let $f^+(g(x))$ and $f^-(g(x))$ denote the right limit and the left limit at x. For a positive price, the demand is the correspondence

$$\Delta(p) = [\mu_n(\infty) - \mu_n^+(q^{-1}(p)), \mu_n(\infty) - \mu_n^-(q^{-1}(p))].$$

2.2 Selling a jewel

In the next result, we characterize the supply of jewels. Let $c^s(p)$ be the largest $c \in [0, c_o]$ such that $(c+p, 0) \succsim (c,p)$. Note that, by P2, $c^s(p) > 0$ for any p and that, by P1 and P3, $(c,p) \succ (c+p,0)$ if and only if $c > c^s(p)$. For a positive price, the supply is the correspondence:

$$\Sigma(p) = [\mu_n^-(c^s(p)), \mu_n^+(c^s(p))].$$

Since a change in the price p implies also a change in the wealth level of a potential seller, we need additional assumptions for a tidy characterization.

P4: For $c > 0$ and $p' > p > 0$, if $(c,p) \succsim (c+p,0)$ then $(c,p') \succ (c+p',0)$

P4 implies that if a consumer who owns a jewel prefers to keep it, he will still prefer to keep it if the price increases. Any preference relation represented by a utility function $v(c)\,(1+p)$, where $v' > 0$, $v'' < 0$, and $v(0) = 0$ satisfies assumptions P1-P4.

Claim 2 *Assume P1-P4. There exists a decreasing, continuous function* $r : [0, \infty) \to [0, \infty]$ *and* $c^l, c^h \in [0, \infty]$ *such that: (i) if* $c \leq c^l$*, an owner sells the jewel for any positive price; (ii) if* $c^l < c < c^h$*, an owner sells the jewel if* $p < r(c)$ *and does not sell it if* $p > r(c)$*; (iii) if* $c > c^h$*, an owner does not sell it the jewel for any positive price. The function* $r(c)$ *is strictly decreasing at any point* c *for which* $r(c)$ *is finite and positive.*

Proof: Consider the set Z of all pairs (c, p) for which a consumer who holds initially consumption level c and a jewel is not worse off by selling the jewel at price p. By continuity, the set Z is closed. By P2, for any p we have $(0, p) \in Z$.

By P4, if $(c, p) \notin Z$ then $(c, p') \notin Z$ for any $p' > p$. By P3, if $(c, p) \notin Z$ then $(c', p) \notin Z$ for any $c' > c$. Define $r(c) = \max\{p : (c, p) \in Z\}$ ($r(c)$ can be infinite). The function r is continuous and strictly decreasing when it has a finite positive value. Thus, there exist c^l and c^h such that: (i) if $c \leq c^l$, an owner sells the jewel regardless of the price, (ii) if $c^l < c < c^h$, an owner of the jewel sells the jewel if $p < r(c)$ and does not sell it if $p > r(c)$ and (iii) if $c > c^h$, an owner does not sell the jewel regardless of the price. QED

Note that, under assumption P4, the supply of jewels is downward sloping. If P4 is replaced by the following assumption, the supply of jewels is upward sloping.

P4$'$: For $c > 0$ and $p' > p > 0$, if $(c+p, 0) \succsim (c, p)$ then $(c+p', 0) \succ (c, p')$

P4$'$ implies that a consumer who sells a jewel will continue selling it if the price increases. Any preference relation represented by a utility function $v(p)c$, where $v' > 0$, $v'' < 0$, and $v(0) > 0$ satisfies assumptions P1-P3, and P4$'$.

Claim 3 *Assume P1-P3 and P4$'$. There exists an increasing, continuous function* $r : [0, \infty) \to [0, \infty]$ *and* $c^l, c^h \in (0, \infty]$ *such that: (i) if* $c \leq c^l$*, an owner sells the jewel for any positive price; (ii) if* $c^l < c < c^h$*, an owner sells the jewel if* $p > r(c)$ *and does not sell it if* $p < r(c)$*; (iii) if* $c > c^h$*, an owner does not sell it for any positive price. The function* $r(c)$ *is strictly increasing at any point* c *for which* $r(c)$ *is finite and positive.*

Proof: The proof is analogous to the proof of Claim 2 and is omitted. QED

2.3 Equilibria

A "no-value equilibrium", in which the price of a jewel is null and the allocation of jewels is arbitrary, always exists. In the next proposition, we provide conditions for the existence of equilibria in which jewels are traded and have positive value. Recall that $\mu_i(c_i) = \mu_i(\infty)$, $i = n, o$.

Proposition 4 *Assume P1-P3. Suppose that $\mu_n(c_n) - \mu_n(c^*) > \mu_o(c_o)$. Then, there exists an equilibrium with a non-zero price and trade.*

Proof: For any small $\epsilon > 0$, when the price is positive and sufficiently small all points in $\Delta(p)$ are at least as large as $\mu_n(c_n) - \mu_n(c^*) - \epsilon$ by Claim 1. Since $\mu_n(c_n) - \mu_n(c^*) > \mu_o(c_o)$, the demand for jewels exceeds the supply of jewels when the price is positive and sufficiently small.

If $p > q(c_n)$, demand is zero but supply is positive by P2 and the assumption that μ_o is strictly increasing on its support $[0, c_o]$. Since the mass functions μ_o and μ_n are strictly increasing on the support and the functions q and r are continuous, demand and supply are upper hemi-continuous, convex valued correspondences. Hence, an equilibrium price with trade exists. QED

Proposition 4 does not rely on assumption P4. Under P4, equilibria with unbounded prices and no trade can exist when every jewel owner wishes to keep the jewel for some positive price. Such equilibria are not unrealistic. As Baumol (1986) pointed out, trade in art pieces is very infrequent.

Proposition 5 *Assume P1-P4. Suppose that all owners have an initial consumption endowment bounded below by a number $\tilde{c} > c^l$. Then, there exists $\bar{p}$ such that any $p^* > \bar{p}$ is an equilibrium price for which no jewels are traded.*

Proof: Take any $p^* > \bar{p} = \max\{q(c_n), r(\tilde{c})\}$. By the Claims 1 and 2, no consumer wants to buy a jewel and no consumer wants to sell a jewel. QED

The following example illustrates the equilibrium under assumptions P1-P4.

Example: Consider the case that the consumers' preferences are represented by $u(c,p) = (1+p)\sqrt{c}$. This utility function satisfies P1-P4. On the demand side, we get $c^* = 1/2$ and $q(c) = \frac{1}{2}c + \frac{1}{2}\sqrt{4c+c^2} - 1$. On the supply side, we get $c^l = 0$, $c^h = 1/2$ and $r(c) = \dfrac{1}{c} - 2$.

Poor non-owners ($c < 1/2$) never buy. Rich non-owners ($c > 1/2$) will buy if $p < \frac{1}{2}c + \frac{1}{2}\sqrt{4c+c^2} - 1$. Rich jewel owners ($c \geq 1/2$) never sell. Poor jewel sell if the price is below $\dfrac{1}{c} - 2$.

As always, the market has a zero price equilibrium: $p = 0$ is a trivial equilibrium price for which nobody cares about owning a jewel. In any (non-degenerate) equilibrium, $\mu_o(1/(2+p)) = \mu_n(\infty) - \mu_n(\frac{1}{p+2}(p+1)^2)$. Suppose that α_n non owners of jewelry and α_o owners are uniformly distributed on $[0,1]$. Then in equilibrium $\alpha_o\left(\frac{1}{p+2}\right) = \alpha_n(1 - \frac{1}{p+2}(p+1)^2)$.

Let $\alpha_o/\alpha_n = k$. The equilibrium equation $p + 2 - (p+1)^2 - k = 0$ is solved by $p = \frac{1}{2}\sqrt{5-4k} - \frac{1}{2}$, which is positive for $k < 1$.

Consider now μ_n with support bounded by some c_n and μ_o such that $\mu_o(\tilde{c}) = 0$ for some $\tilde{c} > 0$. Then, any $p > \max\{\dfrac{1}{\tilde{c}} - 2, \frac{1}{2}c_n + \frac{1}{2}\sqrt{4c_n + c_n^2} - 1\}$ is an equilibrium price with no trade.

Assumption P3 implies that in equilibrium, if consumer with a consumption level c buys a jewel for a price p, no initial owner with consumption of $c - p$ or higher will sell the jewel. In equilibrium, the highest c of a seller is smaller by at least p than the lowest c of a buyer. Thus, by any measure of inequality, the equilibrium trade improves the equality of the distribution of consumption in the population. Also note that a trade equilibrium always Pareto dominates a no-trade equilibrium.

Comment: Competitive equilibria in standard models are Pareto efficient. In our model, the higher is the price in the no-trade equilibrium of Proposition 5, the "better" it is for the consumers who hold the jewel. For any such equilibrium, there is another one which Pareto dominates it. In particular, after trade at price p^*, all jewel owners have consumption above $r^{-1}(p^*)$ and thus, by Proposition 5, there are new equilibria with "high" prices and no trade which make all owners better off and leave the non-owners indifferent.

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