Strategic Patent Breadth and Entry Deterrence with Drastic Product Innovations

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1. Introduction

Innovating firms choose to patent their innovations when patenting allows the appropriation of more rents than do other forms of intellectual property protection (e.g., trade secrecy). The degree of appropriability of innovation rents enabled by a patent is mainly defined by two elements – patent length and patent breadth (Merges and Nelson 1990, Klemperer 1990). While the innovator cannot affect patent length since it is standardized and predetermined by law (i.e., 20 years for most patents) he plays a crucial role in the determination of the breadth of protection granted to the patent. The innovator's claims in the patent application specify the breadth of protection sought for the innovation and constitute the basis on which the Patent Office decides on the breadth of protection granted to the patent, if any, and the courts rule on patent validity and infringement issues (Merges and Nelson 1990, Miller and Davis 1990, Cornish 1989).

The purpose of this paper is to theoretically examine the innovator's optimal patent breadth strategy; the patent breadth choice that maximizes the innovator's ability to appropriate innovation rents. The analysis of the innovator's patenting behavior in the existing economic literature has primarily focused on the innovator's decision to patent the innovation or to keep it a secret (Horstmann et al. 1985, Waterson 1990). Lerner (1995) empirically examined some other aspects of the innovator's patenting behavior, namely, the decision to patent in certain patent subclasses given competitors' patent subclass choices and legal costs. There is no formal framework of analysis of the innovators' patent breadth choice once the decision to patent has been made, however. Instead, it has been traditionally assumed that the innovator has an incentive to claim 'as much as possible' (Lenz 1988).

Our paper explicitly models the innovator's patent breadth decision and examines the optimal patent breadth strategy that the innovator should employ when faced with entry by products

of superior quality and the possibility that the breadth of the patent will be legally challenged. Patent breadth is defined in terms of the area in a vertically differentiated product space that the patent protects. The theoretical model developed considers the efficiency of patent breadth as an entry deterrent. As a consequence, the model also explicitly examines the assumption that the innovator has an incentive to claim the broadest scope of patent protection possible.

Analytical results show that in most cases the optimal patent breadth strategy for the innovator is to claim a patent breadth which is less than the maximum possible. The analysis also shows that that it is possible under some conditions for an innovator to use patent breadth to deter entry – when this is possible, the optimal patent strategy is to always deter entry. These conditions occur under certain combinations of the entrant's R&D effectiveness and trial cost values (i.e., low R&D effectiveness – which results in high R&D costs – and high trial costs). When these specific conditions do not hold, the optimal strategy for the innovator is to allow a new competitor to enter the market. When allowing entry, the innovator chooses patent breadth so that the benefits of increased product differentiation that result from greater patent breadth are traded off with the increased likelihood of patent challenge and invalidation that comes with greater patent breadth. One of the conclusions of the paper is that the innovator will choose the maximum patent breadth when patent infringement is never an optimal strategy for the entrant. The innovator may also choose maximum patent breadth when entry deterrence is not possible and it is optimal for the innovator to induce patent infringement. This occurs under a very specific set of conditions (i.e., a combination of very low R&D effectiveness values and low monopoly profits).

The rest of the paper is organized as follows. Section two gives a background discussion of the relationship between patent breadth and innovation rents and outlines inefficiencies related to the patent granting process. Section three describes the theoretical development of the strategic patent breadth model; it describes the market conditions, defines patent breadth and models the choice of patent breadth as a sequential game of complete information. Section four provides the analytical solution of the model. Finally, section five concludes the paper.

2. Background

The innovator's patent breadth choice is a strategic decision. Patent breadth defines the technological territory claimed and protected by the patent. It plays an important role in the determination of the degree of competition in the market and the effective patent life, which in turn determine the true reward to the innovator. On the one hand, the greater is the breadth of patent protection, the harder it is for potential competitors to enter into the patentee's market with noninfringing innovations and thus, the longer the patentee can maintain the limited monopoly that the patent grants (Gallini 1992). At the same time, however, a patent that is too broad increases the likelihood of both infringement and patent validity challenges by competitors and/or third parties (Merges and Nelson 1990). Consequently, broad patent protection may reduce the effective patent life, and thus the innovation rents that can be captured with the patent, as patents may be revoked during infringement trials and patent validity challenges (Barton 2000). This concern is especially critical in light of the increase in patent litigation during the last decades, particularly in the field of biotechnology, and the increase in the number of patents that are invalidated after being challenged (Barton 2000, Lanjouw and Schankerman 2001, Harhoff and Reitzig 2000). Thus, a broad patent protection may impede the innovator's ability to safeguard and/or defend the technological territory protected by his patent.

The assumption that the innovator has an incentive to follow a 'claim as much as possible' strategy is mainly based on the premise of an efficiently operating Patent Office that will prune back or reject broad and/or erroneous claims during the patent granting process. If the Patent Office could grant an 'optimal' patent then the innovator would be better off claiming broad patent

protection as the patent breadth granted cannot be greater than the patent breadth claimed. Evidence shows, however, that the United States Patent and Trademark Office (USPTO) often grants broad patents that cannot survive a validity attack and patents that appear to overlap leading to disputes that have to be resolved through costly litigation or settlement (Voss 1999, Barton 2000, Lenz 1988, Lerner 1994). Barton (2000) claims that, due to the increase in patent applications over the last decade and resources limitations in the USPTO, patent examiners spend on average only twenty five to thirty hours examining a patent application, time that is not enough to conduct effective searches and evaluate patent claims.

The inefficiencies present in the patent granting process suggest that the innovator cannot always rely on the Patent Office for help in refining his patent claims. This is especially true for pioneering/drastic innovations. According to the Patent Office's policy, drastic innovations are usually granted broader protection (EPO 2000, USPTO 1999).¹ Merges and Nelson (1990) observe that claims to drastic innovations are often allowed to cover areas beyond the area examined and disclosed by the innovator while the narrowing of the claims of drastic innovations is usually left to the courts.²

Existing patent breadth studies have mainly focused on the determination of a socially optimal patent policy and have thus assumed an efficient patent granting process (Gilbert and Shapiro 1990, Klemperer 1990, Gallini 1992, Green and Scotchmer 1995, Chang 1995, Matutes et. al 1996, O' Donughue 1998). In these studies, a regulator (e.g., Patent Office) determines a socially optimal patent breadth; a patent breadth that rewards the innovator/patentee 'sufficiently' at the least social costs.

¹ According to the European Patent Office (EPO) (2000) 'an invention that opens up a whole new field is entitled to more generality in the claims than an invention that is concerned with advances in a known field of technology'.

 $^{^{2}}$ This is due to the fact that the more drastic is the innovation, the harder it is for an examiner to find support in the prior art to object to broad claims demonstrating that embodiments of the claimed invention would be impossible to make without undue experimentation. Thus, when drastic innovations are concerned, the burden falls on the examiner who must disprove enablement (Merges and Nelson 1990).

This paper follows a different approach. We seek to determine a privately rather than a socially optimal breadth of patent protection. In our analysis the innovator determines the breadth of patent protection claimed that maximizes his ability to appropriate innovation rents given the inefficiencies present in the patent granting process. Our analysis focuses on drastic product innovations; innovations that generate new demand or meet demand not previously met. The focus is on drastic innovations because, the Patent Office's role in refining the innovator's patent claims is limited and the innovation rents that are at stake are substantial increasing the probability of a patent challenge (Cornish 1989, Lanjouw and Schankerman 2001). Consequently, the innovator, in our model, does not rely on the Patent Office to structure his claims. He is aware of both the inefficiencies in the determination of patent breadth in the Patent Office and that his effort to safeguard his technological territory does not usually conclude with the granting of the patent.

The section that follows describes the theoretical development of the strategic patent breadth model.

3. The Strategic Patent Breadth Model

3.1 Model Assumptions

The model is based on a number of assumptions. The optimal patent breadth strategy is determined in a sequential game of complete information. The agents in the game are an incumbent/patentee who, having invented a patentable drastic product innovation and having decided to seek patent protection, decides on the patent breadth claimed and a potential entrant who decides on whether to enter the patentee's market and, if entry occurs, where to locate in a vertically differentiated product space. Both the incumbent and the entrant are risk neutral and maximize profits. It is assumed that the regulator (e.g., Patent Office) always grants the patent as claimed; thus, the regulator is not explicitly modeled. The assumption that the Patent Office plays no role in refining the patent claims is a realistic assumption for drastic innovations.

The patentee's investment decision that led to the development of a new product is not examined – this decision is treated as exogenous to the game. In addition, it is assumed that the patentee and the entrant each produce at most one product and that the entrant does not patent her product since further entry is not anticipated (see footnote 4 below). The production process for the entrant is assumed to be deterministic, so that once the entrant chooses a location she can produce the chosen product with certainty. It is also assumed there is no time lag between making and realizing a decision.

The patentee and the entrant, if she enters, operate in a vertically differentiated product market that can support at most two products. Consumers differ according to some attribute λ , uniformly distributed with unit density $f(\lambda) = 1$ in the interval $\lambda \in [0,1]$, each buying one unit of either the patentee's or the entrant's product but not both. The patentee is assumed to have developed a product that provides consumers with utility $U_p = V + \lambda q_p - p_p$, where V is a base level of utility, q_p is the quality of the patentee's product p_p is the price of the product produced by the patentee. The entrant's product has quality $q_e > q_p$, $q_e \in (0,1]$, that provides consumers with utility $U_e = V + \lambda q_e - p_e$, where p_e is the price of the entrant's product. Without affecting the qualitative nature of the model, the quality of the patentee's product q_p is set equal to zero (i.e., $q_p = 0$). As a result, the entrant's quality q_e is interpreted as the difference in quality between her product and that of the patentee, or more generally as the distance the entrant has located away from the patentee.³

³ With $q_p \neq 0$, equation (1) becomes $\lambda^* = \frac{(p_e - p_p)}{q_e - q_p}$. Since the quality difference, $q_e - q_p$, in the denominator is the relevant parameter of interest in the subsequent analysis, the assumption that $q_p = 0$ can be made to ease the notation

Product i (i = p, e) is consumed as long as $U_i \ge 0$ and $U_i > U_j$. It is assumed that V is large enough so that $V \ge p_i \ \forall i = p, e$ and the market is always served by at least one product. The consumer who is indifferent between the two products has a λ denoted by λ^* , where λ^* is determined as follows: $V \ge p_i \ \forall i = p, e$

(1)
$$U_p = U_e \Longrightarrow \lambda^* = \frac{p_e - p_p}{q_e}$$

Since each consumer consumes one unit of the product of her choice, the demand for the products produced by the patentee and the entrant are given by $y_p = \lambda^*$ and $y_e = 1 - \lambda^*$, respectively.

The patentee has already incurred the development costs associated with the product quality that he has patented. Thus, the R&D costs for the patentee are sunk. For the entrant, however, market entry can only occur if she develops a higher quality product. To do so, she incurs R&D costs $F_e(q_e)$, where $F_e = \beta \frac{q_e^2}{2}$ and $\beta \ge \frac{4}{9}$. The restriction on the parameter β ensures that the quality chosen by the entrant, q_e , is bounded between zero and one. Note that with this formulation, $F'_e(q_e) > 0$ and $F''_e(q_e) > 0$, thus, it is increasingly costly for the entrant to locate away from the patentee in the one-dimensional product space (i.e., to produce the better quality product). In addition, since q_e represents the quality difference between the patentee's and the entrant's product the filing of a patent by the patentee provides the entrant with knowledge of how to produce the patentee's product (i.e., $F_e(q_p) = 0$ – the assumption of perfect information disclosure by the patente entret is made). The R&D costs are assumed sunk once they have been incurred and neither the patentee nor the entrant find it optimal to relocate once they have chosen their respective qualities. Once the R&D costs are incurred, production of the products by both the patentee and the entrant occur at

without affecting the qualitative nature of the model.

zero marginal cost.4

The patent breadth claimed and granted to the patentee's product is denoted by b and it defines the area in the one-dimensional product space that the patent protects, thus, $b \in (0,1]$. Patent breadth values close to zero indicate protection of the patented innovation only against duplication. It is assumed that when the entrant locates at a distance $q_e < b$ away from q_p a trial always takes place, either because the patentee files an infringement lawsuit or because the entrant directly challenges the validity of the patent. It is further assumed that the filing of an infringement lawsuit is always met with a counterclaim by the accused infringer that the patent is invalid.⁵ The costs incurred during the infringement trial/validity attack by the patentee and the entrant are denoted by C_p and C_E , respectively. These costs are assumed to be independent of the breadth of protection and of the entrant's location. The trial costs will only be incurred if $q_e < b$ and they are assumed to be sunk – once made they cannot be recovered by either party.⁶

The patent system being modeled is assumed to be that of the fencepost type, in which patent claims define an exact border of protection. Under the fencepost system, infringement will always be found when an entrant locates within the patentee's claims, unless the entrant proves that the patent is invalid (Cornish 1989).⁷ In the fencepost system the probability that infringement is found does not depend on how close the entrant has located to the patentee. The implication of

⁴ Note that the market conditions outlined above imply that the Finiteness Property introduced by Shaked and Sutton (1982) holds; products are vertically differentiated, the burden of quality improvements falls on fixed rather than on variable costs and the unit variable costs increase in quality slower than the willingness to pay for quality $-\forall \lambda > 0$. Thus, this market will be concentrated irrespective of its size and the level of fixed costs. Moreover, given the assumption that consumer preferences are such that the market can support at most two products this market is a natural duopoly.

⁵ This is a standard defence adopted by accused infringers (Cornish 1989, Merges and Nelson 1990).

⁶ With this assumption we exclude the possibility of the court awarding lawyers' fees to either party.

⁷ In contrast, a signpost patent system implies that claims provide an indication of protection and the claims are interpreted using the doctrines of equivalents and reverse equivalents. Under a signpost system the closer the entrant locates to the patentee the easier it is to prove infringement using the doctrine of equivalents. In addition, infringement may be found even when the entrant locates outside the patentee's claims using the doctrine of reverse equivalents.

assuming a fencepost patent system is that the probability that infringement will be found (given that the entrant has located at $q_e < b$ distance away from q_p) is equal to the probability that the validity of the patent will be upheld. Thus, the fencepost patent system implies that the events that the patent is found to be infringed and that the patent is found to be invalid can be treated as mutually exclusive and exhaustive.⁸

Patent validity is directly linked to patent breadth. In general, the broader is the patent protection, the harder it is to show novelty, nonobviousness and enablement (Miller and Davis 1990). Thus, the broader is patent protection the harder it is to establish validity. In addition, evidence from the literature shows that courts tend to uphold narrow patents and invalidate broad ones (Waterson 1990, Cornish 1989, Merges and Nelson 1990). To capture this element, the probability that the patent will be found to be valid or equivalently that infringement will be found, denoted by $\mu(b)$, is assumed to be inversely related to patent breadth, $\mu'(b) < 0$.

3.2 The Game

The strategic patent breadth game consists of three stages. In the first stage of the game, the patentee applies for a patent, claiming a patent breadth, b. In the second stage of the game, a potential entrant observes the patentee's product and the breadth of protection granted to it and chooses whether or not to enter the market. If the entrant does not enter she earns zero profits while the patentee operates as a monopolist in the third stage of the game and earns monopoly profits Π_p^M . If the entrant enters, she does so by choosing the quality q_e of her product relative to that of the patentee. This decision determines whether the entrant infringes the patent or not.

If the entrant chooses a quality greater than or equal to the patent breadth claimed by the

⁸ Note that, our analysis and results are not affected by whether only certain claims are invalidated during the infringement/validity trial or the entire patent; that is, when patent breadth is narrowed rather than the entire patent revoked. This occurs because further entry is not anticipated in our model (see footnote 4).

patentee (i.e., $q_e \ge b$), then no infringement occurs, and she and the patentee compete in prices in the third stage of the game and earn duopoly profits Π_e^{NI} and Π_p^{NI} , respectively. If the entrant locates inside the patent breadth claimed by the patentee (i.e., $q_e < b$), the patent is infringed and a trial occurs in which the validity of the patent is examined. With probability $\mu(b)$, the patent is found to be valid (i.e., infringement is found), the entrant is not allowed to market her product and the patentee operates as a monopolist in the third stage of the game. With probability $1 - \mu(b)$, the patent is found to be invalid, and the entrant and the patentee compete in prices. The payoffs for the patentee and the entrant when the entrant chooses $q_e < b$ are $E(\Pi_e^I)$ and $E(\Pi_p^I)$, respectively. Figure 1 illustrates the extensive form of the game outlined above.



Figure 1. The Game in Extensive Form

The solution to this game is found by backward induction. The third stage of the game in which the patentee and the entrant – when applicable – compete in prices is examined first, followed by the

second stage in which the entrant makes her entry decision, and then the first stage in which the patentee makes his decision regarding patent breadth.

4. Analytical Solution of the Game

4.1 Stage 3 – The Pricing Decisions

In the third stage of the game, two cases must be considered – the case where the entrant has entered and the case where the entrant has not entered. Considering the last case first, in the absence of entry by the entrant, the patentee will charge $p_p = V$ and earn monopoly profits $\prod_p^M = V - F_p$.

If entry occurs, the problem facing duopolist i is to choose price p_i to maximize profit

$$\pi_i = p_i y_i - F_i$$
 $(i = p, e)$, where $y_p = \frac{p_e - p_p}{q_e}$ and $y_e = \frac{q_e + p_p - p_e}{q_e}$. Recall that the R&D costs,

 F_p and F_e for the patentee and the entrant, respectively, are assumed to be sunk at this stage in the game. The Nash equilibrium in prices, as well as the resulting outputs and profits, are given by:

(2) Patentee:
$$p_p^* = \frac{q_e}{3}, y_p^* = \frac{1}{3}, \pi_p^* = \frac{q_e}{9}$$

(3) Entrant:
$$p_e^* = \frac{2q_e}{3}, y_e^* = \frac{2}{3}, \pi_e^* = \frac{4q_e}{9}$$

Since the entrant has the higher quality product, she charges the higher price. Profits are increasing in the distance q_e between the patentee's and the entrant's location. The greater is the difference in quality between the two products, the less intense is competition at the final stage of the game and the greater are the profits for both the incumbent and the entrant.⁹

⁹ This is a well-established result in the product differentiation literature in simultaneous games. When competitors first simultaneously choose their locations in the product space and then compete in prices they choose maximum differentiation to relax competition in the pricing stage that would curtail their profits (Lane 1980, Motta 1993, Shaked and Sutton 1982).

4.1 Stage 2 – The Location Decision

As outlined above, the entrant must choose one of three options – Not Enter, Enter and Not Infringe the Patent, or Enter and Infringe the Patent. For any given patent breadth, b, the entrant will choose the option that generates the greatest profit.

The outcome of the Not Enter option is straightforward – the entrant earns zero profits. The outcomes of the other two options depend on a number of factors, including patent breadth, R&D costs and trial costs. The benefits and costs associated with the Enter and Not Infringe option are examined below, followed by an examination of the benefits and costs associated with the Enter and Infringe option. Once the net benefits of each option are formulated, the most desirable option for the entrant is determined for any given patent breadth.

• Entry with No Infringement $(q_e \ge b)$

For the entrant to enter without infringing the patent, the entrant must choose a quality location that is greater than or equal to the patent breadth – i.e., $q_e \ge b$. Let q_e^* be the optimal quality the entrant would choose when the patent breadth is not binding, where q_e^* solves the following problem:

(4)
$$\max_{q_e} \Pi_e = \pi_e - F_e = \frac{4q_e}{9} - \beta \frac{q_e^2}{2}$$

Optimization of equation (4) yields the optimal quality q_e^* :

(5)
$$q_e^* = \frac{4}{9\beta}$$

Equation (5) indicates that the less costly it is to produce the better quality product (i.e., the smaller is β), the further away from the incumbent the entrant locates.

As long as $q_e^* \ge b$, the patent breadth does not affect the location chosen by the entrant, since the entrant can choose her optimal quality without fear of infringement. Thus, patent breadth will only be binding if $q_e^* < b$. Since an increase in quality beyond q_e^* results in a reduction in profits, the entrant's profit is decreasing in q_e for all $q_e > q_e^*$. As a result, the entrant, when faced with a binding patent breadth, will always choose a quality equal to the patent breadth chosen by the patentee (i.e., $q_e = b$).

Thus, a profit-maximizing entrant that wishes to not infringe the patent will choose her entry location q_e^{NI} as follows:

(6)
$$q_e^{NI} = \begin{cases} \frac{4}{9\beta} & \text{if } b < \frac{4}{9\beta} \\ b & \text{if } b \ge \frac{4}{9\beta} \end{cases}$$

while the profits earned by the entrant are:

(7)
$$\Pi_e^{NI} = \begin{cases} \frac{8}{81\beta} & \text{if } b < \frac{4}{9\beta} \\ \frac{4}{9}b - \frac{\beta}{2}b^2 & \text{if } b \ge \frac{4}{9\beta} \end{cases}$$

• Entry with Infringement $(q_e < b)$

If the entrant enters and infringes the patent filed by the patentee, a trial takes place. If the patent is found to be valid during trial, the entrant cannot enter and the patentee has a monopoly position in the market. If the patent is found to be invalid, the entrant is allowed to market her product and the patentee and the entrant operate as duopolists. The probability that the patent is found to be valid is given by $\mu(b)$, with $\mu(b)$ having the functional form $\mu(b) = 1 - \alpha b$.¹⁰ Thus, $1 - \mu(b) = \alpha b$ is the probability that the patent will be found to be invalid. For an given patent breadth, the greater is the

¹⁰ Patent breadth is not the only factor affecting the validity of the patent. A patent may also be invalidated because of unallowable amendments during patent examination and because the innovation is not regarded an invention under the patent law (Cornish 1989). By assuming that the innovator has generated a patentable innovation we have excluded the latter case. To keep the analysis simple we assume that the probability of patent invalidation due to unallowable amendments is negligible.

validity parameter α , the greater is the probability that the patent will be found invalid. With this background, the quality chosen by the entrant is determined by solving:

(8)
$$\max_{q_e} E\left(\Pi_e^I\right) = (1-\mu) \cdot \pi_e - F_e - C_e = \alpha b \frac{4q_e}{9} - \beta \frac{q_e^2}{2} - C_e$$

The optimal quality chosen is given by:

(9)
$$q_e^I = \frac{4\alpha b}{9\beta}$$

Equation (9) shows that when the entrant infringes the patent she finds it optimal to locate at a distance proportional to the breadth of the patent. Because there is uncertainty with respect to whether the entrant will be able to continue in the market, she 'underlocates'; to reduce the R&D costs, which are incurred with certainty, the entrant locates closer to the patentee than she would have done had infringement not been a possibility.

The expected profits for the entrant are given by equation (10):

(10)
$$E\left(\Pi_e^I\right) = \frac{8\alpha^2 b^2}{81\beta} - C_e.$$

When patent breadth is negligible (i.e., *b* approaches zero), the expected profits from infringement approach $-C_e$, since the probability of the patent being found valid approaches one. As patent breadth increases, expected profits from infringement also increase, a reflection of the rising probability that the patent will be found invalid.

The Entry/Infringement Decision

The decision made by the entrant whether to enter, and if entry occurs, whether to infringe the patent, depends on patent breadth, b, and three variables that are treated as exogenous in this study – the R&D cost parameter β , the trial costs C_e and the validity parameter α . As shown above, when patent breadth is such that $b \le q_e^*$ the entrant always finds it profitable to enter the market

locating at her most preferred location (q_e^*) without triggering the trial outcome. For patent breadth values such that $b > q_e^*$, however, the entrant may be deterred from entering the market or, if entry cannot be deterred, she may always finds it profitable to enter without infringing the patent or she may enter and be induced to either infringe or not infringe the patent. These cases where $b > q_e^*$ are examined below.

Case I – Entry Deterrence

The entrant can be deterred from entering the market when there exists a, \hat{b} , where \hat{b} ensures that the following conditions are satisfied: $\Pi_e^{NI}(\hat{b}) \leq 0$; $E(\Pi_e^I(\hat{b})) \leq 0$ and $\hat{b} \in (q_e^*, 1]$. In fact, there might be a range of patent breadths that deter entry. Define \hat{b}_I as the patent breadth that makes the entrant indifferent between entering the market and infringing the patent on the one hand and not entering the market on the other hand. Then \hat{b}_I must ensure that the following conditions are met: $E(\Pi_e^I(\hat{b}_I)) = 0$ and $\hat{b}_I \in (q_e^*, 1]$.¹¹ Also, define \hat{b}_{NI} as the patent breadth that makes the entrant indifferent between entering the market without infringing the patent on the one hand and not entering the market on the other hand. Then \hat{b}_{NI} must satisfy the following conditions:

 $\Pi_e^{NI}(\hat{b}_{NI}) = 0 \text{ and } \hat{b}_{NI} \in (q_e^*, 1]. \text{ It is straightforward to show that } \hat{b}_I = \sqrt{\frac{81\beta C_e}{8\alpha^2}} \text{ and } \hat{b}_{NI} = \frac{8}{9\beta};$

since $\hat{b}_{NI} \in (q_e^*, 1]$, \hat{b}_{NI} exists only for β values such that $\beta \ge \frac{8}{9}$. Given the above, any $b \in (q_e^*, 1]$ such that $b \le \hat{b}_I$ makes entry under infringement unprofitable for the entrant while any $b \in (q_e^*, 1]$ such that $b \ge \hat{b}_{NI}$ makes entry under no infringement unprofitable for the entrant. Thus, the entrant

¹¹ The assumption is made that when the entrant is indifferent she will not enter.

will not find it profitable to enter the market if a $\hat{b} \in (q_e^*, 1]$ such that $\hat{b}_{NI} \leq \hat{b} \leq \hat{b}_I$ exists. Case I is illustrated in Figure 2, panels (i) and (ii).

Case II – Entry and No Infringement

The entrant will always enter and not infringe when the entrant's trial costs and R&D effectiveness are such that, $\forall b \in (q_e^*, 1], \ \Pi_e^{NI} > E(\Pi_e^I)$ and $\Pi_e^{NI} > 0$. Case II is illustrated in Figure 2, panel (iii). *Case III – Entry and Inducement of Infringement/Non Infringement*

Let \tilde{b} be the patent breadth that makes the entrant indifferent between infringing and not infringing the patent, while still generating positive profits for the entrant – i.e., $\tilde{b} \in (q_e^*, 1]$ and solves $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I(\tilde{b})) > 0$. The entrant will enter and not infringe when $b \in (0, \tilde{b}]$, while the entrant will enter and infringe when $b \in (\tilde{b}, 1]$ (the assumption is made that when the entrant is indifferent she will choose to not infringe the patent). The expression for \tilde{b} is derived in the Appendix.

The patent breadth \tilde{b} is a function of the R&D effectiveness parameter, β , the validity parameter, α , and the trial costs C_e . The relationship between \tilde{b} and the above parameters is such that, the greater are the costs of producing the higher quality product, the greater is the validity parameter and the smaller are the trial costs, the smaller is the breadth of the patent that makes the entrant indifferent between infringing and not infringing the patent, $\frac{\partial \tilde{b}}{\partial \beta} \leq 0$, $\frac{\partial \tilde{b}}{\partial \alpha} < 0$ and

$$\frac{\partial \tilde{b}}{\partial C_e} > 0 \forall \beta \ge \frac{4}{9}, \ \alpha \in (0,1] \text{ and } C_e \ge 0 \text{ (for a proof see the Appendix). The above results occur$$

because, the more costly it is to produce the better quality product, the closer the entrant is forced to locate to the patentee and the smaller is the breadth of patent protection that makes it unprofitable for the entrant to not infringe the patent. In addition, the greater is the value of the validity parameter, the greater is the effect that patent breadth has on the probability that the validity of the patent will be upheld and the smaller is the patent breadth that makes it profitable for the entrant to infringe the patent. Finally, the greater are the trial costs, the less appealing is infringement to the entrant. The entrant in this case will infringe only if the breadth is so large that her cost structure does not allow her to locate outside the patentee's patent claims. Case III is illustrated in Figure 2, panel (iv).



Figure 2. The Entrant's Profits under Infringement and No Infringement When Entry Can be Deterred – Panels (i) and (ii) – and When Entry Cannot be Deterred – Panels (iii) and (iv)

Figure 3 illustrates the combinations of β and C_e values, for a given α value, ($\alpha = 0.5$) that give rise to each of the three cases. Entry deterrence (Case I), where there exists a patent breadth \hat{b} such that $\hat{b}_{NI} \leq \hat{b} \leq \hat{b}_I$, is represented by the dotted area on Figure 3 and occurs for relatively high trial costs, C_e , and high β (low R&D effectiveness) values. Entry with no infringement (Case II), where there is no patent breadth \hat{b} that can deter entry and no patent breadth \tilde{b} that can induce non infringement, is represented by the horizontally hatched area in Figure 3 and occurs for relatively high trial costs C_e , and low β (high R&D effectiveness) values. Finally, entry and inducement of infringement/no infringement (Case III), where there exists a patent breadth \tilde{b} such that $\hat{b}_I < \tilde{b} < \hat{b}_{NI}$, occurs for low trial cost values, C_e .



Figure 3. Combinations of β and C_e values for a given α value (α =0.5) that generate Cases I, II and III

As demonstrated in Figure 4, the validity parameter α affects the precise combination of β and C_e values that gives rise to a particular case. Specifically, the larger is α , the smaller is the parameter area in which the entrant will enter and not infringe the patent (the area to the left of locus $\tilde{b} = 1$ and for $\beta \in [\frac{4}{9}, \frac{8}{9})$), and the smaller is the parameter area that can deter entry (the area to the right of locus $\hat{b}_I = \hat{b}_{NI}$ and for $\beta \ge \frac{8}{9}$). These results follow directly from the impact that α has on the probability that the validity of the patent will be upheld during trial, μ . As α becomes larger, the greater is the probability that the patent will be found invalid, for any given patent breadth, b. As a consequence, entry is harder to deter and when entry does occur, the entrant is less likely to not infringe the patent.



Figure 4. Combinations of β and C_e values that give rise to Cases I, II and II, for $\alpha=1, \alpha=0.75$ and $\alpha=0.25$

The relationship between the existence of a patent breadth $\hat{b} \in (q_e^*, 1]$ that can deter entry and a patent breadth $\tilde{b} \in (q_e^*, 1]$ the makes the entrant indifferent between infringing and not infringing the patent is formally described in the propositions that follow.

Proposition 1. If a $\tilde{b} \in (q_e^*, 1]$ and a $\hat{b} \in (q_e^*, 1]$ do not exist it is never optimal for the entrant to infringe the patent.

Proof:

At the entrant's most preferred location q_e^* non infringement is always more profitable than

infringement for the entrant. That is, for $b = q_e^* = \frac{4}{9\beta}$, $E(\Pi_e^I) - \Pi_e^{NI} = \frac{128\alpha^2}{6561\beta^3} - C_E - \frac{8}{81\beta} < 0 \forall$

 $\beta \ge \frac{4}{9} \land \alpha \in (0,1] \land C_e \ge 0$. In addition, at q_e^* , $\Pi_e^{NI} = \frac{1}{9\beta} > 0 \forall \beta \ge \frac{4}{9}$. The above conditions imply

that if a $\tilde{b} \in (q_e^*, 1]$ does not exist (i.e., there is no patent breadth that makes $E(\Pi_e^I) = \Pi_e^{NI} > 0$), then $E(\Pi_e^I) - \Pi_e^{NI} < 0 \forall b \in (0,1]$ which implies that $\Pi_e^{NI} > E(\Pi_e^I) \forall b \in (0,1]$. Since there is no $\hat{b} \in (q_e^*, 1]$ either there is no \hat{b}_{NI} such that $\Pi_e^{NI} = 0$ which implies that $\Pi_e^{NI} > 0 \forall b \in (0,1]$. This result is depicted in Figure 2 in panel (iii) and in Figure 3 as the horizontally hatched area. \Box

Proposition 2. If $a \ \tilde{b} \in (q_e^*, 1]$ does not exist, the only patent breadth $\hat{b} \in (q_e^*, 1]$ that can deter entry is the patent breadth that satisfies the non-entry condition under no infringement, i.e., \hat{b}_{NI} .

Proof:

From Proposition 1 it is known that for $b = q_e^*$, $E(\Pi_e^I) - \Pi_e^{NI} < 0$. If \tilde{b} that makes $E(\Pi_e^I) = \Pi_e^{NI} > 0$ does not exist then $\forall b \in (0,1]$ $E(\Pi_e^I) - \Pi_e^{NI} < 0 \Rightarrow \Pi_E^{NI} > E(\Pi_E^I)$. If there is a patent breadth \hat{b}_{NI} that satisfies the non-entry condition under no infringement this implies that for

 $b = \hat{b}_{NI} \quad \prod_{E}^{NI} \le 0$. Given that $\prod_{E}^{NI} > E(\prod_{E}^{I})$, when $b = \hat{b}_{NI}$ the entry deterrence condition is also satisfied. Thus, any $b \in [\hat{b}_{NI}, 1]$ can deter entry. This case is depicted in Figure 2 in panel (ii). \Box

4.2 Stage 1 – The Patent Breadth Decision

In stage 1 of the game, the patentee chooses the patent breadth *b* that maximizes profit, given his knowledge of the entrant's behavior in the second stage of the game. Since the entrant's behavior depends on the values of C_e , α and β , the patent breadth chosen by the patentee also depends on these parameters. Specifically, three situations are possible, each one corresponding to one of the cases outlined above. These situations are presented in Figure 5 and are analyzed below.



Figure 5. The Patentee's Strategic Patent Breadth Decision

Scenario A – Choose Patent Breadth to Deter Entry

If there are values of β , α and C_e are such that entry can be deterred – i.e., if there exists a $\hat{b} \in (q_e^*, 1]$ – then the patentee should always choose to deter entry. By deterring entry, the patentee earns monopoly profits Π_p^M . Since these profits are higher than what can be earned under a duopoly, the patentee always finds it optimal to deter entry.

Scenario B – Choose Maximum Patent Breadth

When the values of β , α and C_e are such that the entrant will always enter and not infringe the patent, regardless of the patent breadth (i.e., case II), the patentee always chooses the maximum patent breadth. The reasoning is straightforward. With both firms operating in the market, the profits of the patentee are increasing in the quality chosen by the entrant – i.e., $\pi_p^* = \frac{q_e}{9}$ (see

equation (2)). As equation (5) indicates, the entrant will choose $q_e = b$ for $b \ge \frac{4}{9\beta}$. Thus, the

patentee can earn maximum profits by choosing the largest possible patent breadth, which in turn causes the entrant to chose the largest possible value of q_e .

Scenario C – Allow Entry and Induce Either Infringement or Non Infringement

If the values of β , α and C_e are such that the entrant will enter and either infringe or not infringe depending on patent breadth, the patentee must decide whether to induce infringement or not. Consider first the profits the patentee earns if he induces the entrant to not infringe. Recall from equation (2) that the patentee's profits equal $\pi_p^* = \frac{q_e}{9}$ when the entrant enters without infringing.

Recall also (see equation (6)) that the entrant will always choose $q_e^{Ni} = \frac{4}{9\beta}$ if $b < \frac{4}{9\beta}$, while the

entrant chooses $q_e^{NI} = b$ when $b \ge \frac{4}{9\beta}$. Thus, if the patentee induces non infringement, his profits

are given by:

(11)
$$\Pi_{p}^{NI} = \begin{cases} \frac{4}{81\beta} & \text{if } b < \frac{4}{9\beta} \\ \frac{b}{9} & \text{if } \frac{4}{9\beta} \le b < \hat{b} \end{cases}$$

Since the patentee's profits can always be increased by choosing $b \ge \frac{4}{9\beta}$, the patentee can earn maximum profits *and* not induce infringement by choosing $b^{NI} = \tilde{b}$. The patentee's profits are thus:

(12)
$$\Pi_p^{NI} = \frac{\widetilde{b}}{9}.$$

The profits earned from inducing non infringement have to be compared to the expected profits earned by inducing infringement. Recall that the entrant chooses $q_e^I = \frac{4\alpha b}{9\beta}$ when she enters and infringes the patent, and that the probability of the patent being found valid is $\mu = 1 - \alpha b$. The patentee's expected profits are given by: $E(\Pi_p^I) = \mu \Pi_p^M + (1 - \mu)\pi_p - C_p$. The problem facing the patentee is thus:

(13)
$$\max_{b} E(\Pi_{p}^{I}) = (1-ab)\Pi_{p}^{M} + \frac{4\alpha^{2}b^{2}}{81\beta} - C_{p}$$
$$s.t. \quad \widetilde{b} + e \le b \le 1 \quad where \quad e \to 0$$

The patent breadth b that solves equation (13) does not result in maximum profits for the

patentee, since the second-order conditions do not hold – i.e., $\frac{\partial^2 E(\Pi_p^I)}{\partial b^2} = \frac{8\alpha}{81\beta} > 0$. Thus, the

 $b^{I} = 1$. Note, however, that under this scenario the patent breadth chosen must violate the entry deterrence condition, that is, $\hat{b}_{I} < b^{I} < \hat{b}_{NI}$. When $b^{I} = 1$ the condition $\hat{b}_{I} < b^{I} < \hat{b}_{NI}$ holds only for values of β such that, $\frac{4}{9} \le \beta < \frac{8}{9}$. With $b^{I} = 1$, the patentee's expected profits are:

(14)
$$E(\Pi_{p}^{I})_{b^{I}=1} = (1-\alpha)\Pi_{p}^{M} + \frac{4\alpha^{2}}{81\beta} - C_{p}$$

while with $b^{I} = \tilde{b} + e$, the expected profits are:

(15)
$$\lim_{e \to 0} E(\Pi_p^I)_{b^I = \widetilde{b} + e} = (1 - \alpha \widetilde{b}) \Pi_p^M + \frac{4\alpha^2}{81\beta} \widetilde{b}^2 - C_p$$

Assuming the patentee induces infringement, the patentee chooses $b^{I} = 1$ when $E(\prod_{p}^{I})_{b^{I}=1} > E(\prod_{p}^{I})_{b^{I}=\tilde{b}}$ and $\frac{4}{9} \le \beta < \frac{8}{9}$. This condition is satisfied when $\frac{4\alpha}{81\beta}(1+\tilde{b}) > \prod_{p}^{M}$ and

 $\frac{4}{9} \le \beta < \frac{8}{9}$. Thus, the patentee is more likely to induce infringement by choosing the maximum patent breadth when α is large, β is small, \tilde{b} is large and Π_p^M is small.

The above results show that the smaller are the monopoly profits that the patentee makes when his patent is found valid at trial, the greater is the patentee's incentive to claim the maximum breadth of protection and risk having his patent revoked. This occurs because under infringement the entrant's location is proportional to the breadth of the patent (i.e., $q_e^I = \frac{4\alpha}{9\beta}b$) so the greater is patent breadth, the further away from the patentee the entrant locates and the greater are the profits at the last stage of the game for both players. Thus, in this case, the effect of the loss of monopoly profits due to the large patent breadth is smaller than the effect of the increased profits brought by the increased level of differentiation between the two products. The reverse is true for large values of the monopoly profits. Having determined the optimal patent breadth decision and the patentee's expected profits when he induces infringement and non infringement the next step to the analysis is to determine when the patentee will find it optimal to induce infringement or non infringement. Figure 6 depicts the possible outcomes of a comparison between the patentee's expected profits when he induces infringement and his profits when he induces non infringement when the optimal patent breadth under inducement of infringement is $b^{I} = \tilde{b} + e$ (panel (i)) and $b^{I} = 1$ (panel (ii)).



Figure 6. The Patentee's Expected Profits under Infringement and his Profits under Non Infringement under Scenario C

Even though a direct comparison of the patentee's profits when he induces infringement and when he induces non infringement is not possible without knowledge of the values of the parameters that affect the patent breadth decision, i.e., β , α , Π_p^M , C_e and C_p , we can observe the impact of some of the exogenous parameters on the incentive to induce infringement when $b^I = \tilde{b} + e$ and when $b^I = 1$. Let $Z_p^1 = E(\Pi_p^I)_{b^I=1} - \Pi_p^{NI}$ and $Z_p^2 = E(\Pi_p^I)_{b^I=\tilde{b}+e} - \Pi_p^{NI}$. Then it can be shown that, the greater are the monopoly profits, the greater is the patentee's incentive to induce infringement, with the increase greater for the case where $b^{I} = \tilde{b} + e$ (i.e., $\frac{\partial Z_{p}^{1}}{\partial \Pi_{p}^{M}} \ge 0$,

$$\frac{\partial Z_p^2}{\partial \Pi_p^M} \ge 0 \text{ and } \frac{\partial Z_p^1}{\partial \Pi_p^M} \le \frac{\partial Z_p^2}{\partial \Pi_p^M} - \text{ for a proof see the Appendix). This result occurs because the only}$$

chance the patentee has to realize monopoly profits is when his patent is infringed and its validity is upheld during the infringement trial. At the same time, as expected, the greater are the patentee's trial costs, the smaller are the benefits (greater are the losses) from inducing infringement (i.e.,

$$\frac{\partial Z_p^1}{\partial C_p} < 0, \ \frac{\partial Z_p^2}{\partial C_p} < 0 - \text{for a proof see the Appendix}.$$

4. Concluding Remarks

Existing studies have limited the analysis of the innovator's patenting behavior to the study of his decision to patent or not to patent his innovation. The innovator's patent breadth decision that affects, whether the patent will be granted, the breadth of protection granted and the viability of the patent after grant and thus determines the innovation rents that can be captured with the patent, have not been explicitly modeled in the literature. Instead, it has been traditionally assumed that the innovator will apply for the broadest protection possible.

In this paper a simple game theoretic model is used to describe the patenting behavior of an innovator who, having invented a patentable drastic product innovation and having decided to seek patent protection, determines the breadth of protection that maximizes the appropriability of the innovation rents enabled by the patent. To determine the optimal breadth of patent protection claimed, the patentee acts strategically, choosing the breadth of protection that induces the desired behavior by the entrant. The patentee is foresighted and anticipates that he may have to incur costs to enforce and/or defend his patent rights. The model suggests that the breadth of patent protection

that maximizes the innovators ability to appropriate innovation rents, depends on the entrant's R&D cost structure, the patentee's and the entrant's trial costs and the effect that patent breadth has on the probability that the validity of the patent will be upheld during an infringement/validity trial.

Contrary to what it is traditionally assumed, the results show that it is not always optimal for the patentee to claim the maximum patent breadth possible. In fact, only for certain values of the parameters that determine the patent breadth decision it is optimal for the patentee to claim the maximum breadth of patent protection. The patentee claims maximum patent protection when he cannot deter entry and the entrant's R&D effectiveness and trial costs are such that she always finds it optimal to not infringe the patent (i.e., when the entrant's R&D costs are very low). The maximum breadth of patent protection may also be claimed when the patentee cannot deter entry and he finds it optimal to induce infringement. This case occurs, however, only for relatively small monopoly profits and when the entrant's R&D costs are very low.

The results hold under the assumption of a fencepost patent system, which implies that the events that the patent is infringed and the patent is invalid can be treated as mutually exclusive and exhaustive. In addition, it has been assumed that the market can only support two products, and that the R&D process is deterministic. Relaxing the above assumptions is the focus of future research.

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APPENDIX

• Existence of patent breadth \widetilde{b} .

If a patent breadth \tilde{b} that makes the entrant indifferent between infringing and not infringing the patent, while still generating positive profits for the entrant, exists it should satisfy the conditions $\tilde{b} \in (q_{e}^{*}, 1]$ and

$$\Pi_e^{NI}(\widetilde{b}) = E(\Pi_e^I(\widetilde{b})) > 0. \text{ The solution of } \Pi_e^{NI}(\widetilde{b}) = E(\Pi_e^I(\widetilde{b})) \Longrightarrow (\frac{8\alpha^2}{81\beta} + \frac{\beta}{2})\widetilde{b}^2 - \frac{4}{9}\widetilde{b} - C_e = 0 \text{ in terms of } E(\Pi_e^I(\widetilde{b})) \Longrightarrow (\frac{8\alpha^2}{81\beta} + \frac{\beta}{2})\widetilde{b}^2 - \frac{4}{9}\widetilde{b} - C_e = 0$$

 \widetilde{b} yields the following two roots: $\widetilde{b}_{1,2} = \frac{9(4\beta \pm \sqrt{2}\sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})}{16\alpha^2 + 81\beta^2}$. The root

$$\widetilde{b}_1 = \frac{9(4\beta - \sqrt{2}\sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})}{16\alpha^2 + 81\beta^2} \le 0 \quad \forall \beta \ge \frac{4}{9}, \ \alpha \in (0,1] \land C_e \ge 0 \text{ and it is thus rejected}$$

since
$$q_e^* < \widetilde{b} \le 1$$
. The root $\widetilde{b}_2 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})}{16\alpha^2 + 81\beta^2} \ge 0 \quad \forall \beta \ge \frac{4}{9}, \ \alpha \in (0,1] \land C_e \ge 0$

and it is accepted as a possible solution. If $\tilde{b} = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})}{16\alpha^2 + 81\beta^2}$ exists it should

also satisfy the conditions $q_e^* < \tilde{b} \le 1$, $\Pi_e^{NI}(\tilde{b}) > 0$ and $E(\Pi_e^I(\tilde{b})) > 0$. It is easily verified that the condition $\tilde{b} - q_e^* > 0$ is satisfied $\forall \beta \ge \frac{4}{9}$, $\alpha \in (0,1] \land C_e \ge 0$. That is,

$$\widetilde{b} - q_e^* = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})}{16\alpha^2 + 81\beta^2} - \frac{4}{9\beta} > 0 \quad \forall \quad \beta \ge \frac{4}{9}, \ \alpha \in (0,1] \land \ C_e \ge 0. \text{ The condition}$$

 $\tilde{b} \leq 1$ is satisfied for certain combinations of β , α and C_e values. To determine the combinations of β , α and C_e values which satisfy the condition $\tilde{b} \leq 1$, the pairs of β , α and C_e values that satisfy the above constraint as an equality ($\tilde{b} = 1$) are determined first. The solution of $\tilde{b} - 1 = 0$ with respect to C_e yields:

$$C_e = \frac{16\alpha^2 - 72\beta + 81\beta^2}{162\beta}$$
. The combination of β and C_e values, for a given α value, for which $\tilde{b} - 1 = 0$

is represented by the locus $\tilde{b} = 1$ in Figure 3. The area to the right of the locus $\tilde{b} = 1$ represents all

combinations of β and C_e values, for a given α value, for which $\tilde{b} < 1$. If \tilde{b} exists it must also satisfy the conditions $\Pi_e^{NI}(\tilde{b}) > 0$ and $E(\Pi_e^I(\tilde{b})) > 0$. Thus, \tilde{b} must take values in the interval $\hat{b}_I < \tilde{b} < \hat{b}_{NI} - \tilde{b}$ must not satisfy the entry deterrence condition. To determine the combination of β , α and C_e values for which $\hat{b}_I < \tilde{b} < \hat{b}_{NI}$ the locus $\hat{b}_I = \hat{b}_{NI}$ must first be determined. The locus $\hat{b}_I = \hat{b}_{NI}$ depicted in Figure 3

refers to the pairs of β , α and C_e values for which $\frac{8}{9\beta} = \sqrt{\frac{81C_E\beta}{8\alpha^2}}$ holds true. Solution of the above

condition with respect to C_e yields: $C_e = \frac{512\alpha^2}{6561\beta^3}$. All combinations of β and C_e values, for a given α

value, below the locus $\hat{b}_I = \hat{b}_{NI}$ are such that $\hat{b}_I < \tilde{b} < \hat{b}_{NI}$.

Given the above, \tilde{b} exists for all combinations of β and C_e values, for a given α value, in the area below the locus $\tilde{b} = 1$ and below the locus $\hat{b}_I = \hat{b}_{NI}$ represented by the vertically hatched area in Figure 3. This case is also depicted in Figure 2, panel (iv).

• The effect of β , α and C_e on \tilde{b} .

$$\frac{\partial \tilde{b}}{\partial \beta} = \frac{(9(16\alpha^2 - 81\beta^2) \left(\sqrt{2}C_e(16\alpha^2 + 81\beta^2)^2 + 8(2\sqrt{2}\beta + \sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})\right)}{\sqrt{2}\sqrt{\beta}(16\alpha^2 + 81\beta^2)^2 \sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2}} \le 0$$

$$\forall \beta \geq \frac{4}{9}, \alpha \in (0,1] \text{ and } C_E \geq 0$$

$$\frac{\partial \widetilde{b}}{\partial \alpha} = -\frac{\left(144\alpha\sqrt{\beta}\left(\sqrt{2}C_e(16\alpha^2 + 81\beta^2) + 8(2\sqrt{2}\beta + \sqrt{\beta}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2})\right)\right)}{(16\alpha^2 + 81\beta^2)^2\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2}} < 0$$

$$\forall \beta \geq \frac{4}{9}, \alpha \in (0,1] \text{ and } C_E \geq 0.$$

$$\frac{\partial \widetilde{b}}{\partial C_E} = \frac{9\sqrt{\beta}}{\sqrt{2}\sqrt{16C_E\alpha^2 + 8\beta + 81C_E\beta^2}} > 0, \ \forall \beta \ge \frac{4}{9}, \alpha \in (0,1] \ and \ C_E \ge 0.$$

• The effect of Π_p^M and C_p on Z_p^1 and Z_p^2 .

$$Z_p^1 = (1 - \alpha)\Pi_p^M + \frac{4\alpha^2}{81\beta} - C_p - \frac{\widetilde{b}}{9}$$
$$\frac{\partial Z_p^1}{\partial \Pi_p^M} = 1 - \alpha \ge 0, \ \frac{\partial Z_p^1}{\partial C_p} = -1 < 0$$
$$Z_p^2 = (1 - \alpha \widetilde{b})\Pi_p^M + \frac{4\alpha^2 \widetilde{b}^2}{81\beta} - C_p - \frac{\widetilde{b}}{9}$$
$$\frac{\partial Z_p^2}{\partial \Pi_p^M} = 1 - \alpha \widetilde{b} \ge 0, \ \frac{\partial Z_p^2}{\partial C_p} = -1 < 0.$$