The Revealed Preference Implications of Reference Dependent Preferences[†]

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Abstract

Köszegi and Rabin (2005) propose a theory of reference dependent utility in which the ultimate choice also serves as the reference point. They analyze dynamic choice problems with uncertainty and interpret their model as a description of the individuals psychological process. We focus on static and deterministic choice problems and identify the revealed preference implications of their model.

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1. Reference Dependent Utility

A reference dependent utility function U, associates a utility with each reference point $z \in X$ and each choice object $x \in X$. Köszegi and Rabin (2005) (henceforth Köszegi-Rabin) propose a novel theory to determine the reference point. Let X be a finite set[1] and let $U : X \times X \to I\!R$, where $U(x, z)$ is the utility of x given the reference z. A personal equilibrium (Köszegi (2004)) for a decision-maker facing the choice set A is any $x \in A$ such that

$$U(x, x) \geq U(y, x) \tag{2}$$

for all $y \in A$. Hence, Köszegi-Rabin define the reference point as the x that ultimately gets chosen. It follows that an alternative $x \in A$ is optimal (i.e., a possible choice) for a Köszegi-Rabin decision-maker if (and only if) condition (2) above is satisfied. Köszegi-Rabin assume that U has the form

$$U(x, y) = \sum_{k \in K} u_k(x) + \sum_{k \in K} \mu(u_k(x) - u_k(y)) \tag{3}$$

where μ is an increasing function with $\mu(0) = 0$ and K is some finite set indexing the relevant hedonic dimensions of consumption. Köszegi-Rabin also require that

$$U(x, y) \geq U(y, y) \text{ implies } U(x, x) > U(y, x) \tag{4}$$

for all $x, y \in X$.

In this note, we provide a revealed preference analysis of the Köszegi-Rabin model. Let Y be the set of all nonempty subsets of X. A function $c : Y \to Y$ is a choice function if $c(A) \subset A$ for all $A \in Y$. Given any state dependent utility function U, define $C(\cdot, U)$ as follows:

$$C(A, U) = \{x \in A \,|\, U(x, x) \geq U(y, x) \forall y \in A\}$$

A choice function c is *a general Köszegi-Rabin choice function* if there exists a reference dependent utility function U such that $c = C(\cdot, U)$. If the U also satisfies (3) and (4)

[1] An element $x \in X$ may be uncertain (i.e., may be a lottery).

then c is a *a special Köszegi-Rabin choice function.* For any binary relation $\succeq$, define the function $C_{\succeq}$ as follows:

$$C_{\succeq}(A) = \{x \in A \,|\, x \succeq z \forall z \in A\}$$

It is easy to construct examples such that $C_{\succeq}(A) = \emptyset$ unless certain assumptions are made on $\succeq$. We say that the choice function c is induced by the binary relation $\succeq$ if $c(A) = C_{\succeq}(A)$ for all $A \in Y$. It is well-known that $C_{\succeq}$ is a choice function whenever $\succeq$ is complete ($x \succeq y$ or $y \succeq x$ for all $x, y \in X$) and transitive ($x \succeq y$ and $y \succeq z$ implies $x \succeq z$ for all $x, y, z \in X$). However, transitivity is not necessary for $C_{\succeq}$ to be a choice function. The proposition below characterizes Köszegi-Rabin choice functions[2]:

Proposition: *The following three conditions are equivalent:*

(i) c is a general Köszegi-Rabin choice function

(ii) c is a choice function induced by some complete binary relation

(iii) c is a special Köszegi-Rabin choice function

Proof: See Appendix

Note that $c = C_{\succeq}$ is a choice function implies $\succeq$ is complete. Hence, we may omit the word complete in the above proposition. The equivalence of (i) and (ii) establishes that abandoning transitivity is the only revealed preference implication of the Köszegi-Rabin theory. The equivalence of (ii) and (iii) implies that the particular functional form (3) and condition (4) are without loss of generality.

The revealed preference analysis answers the following question: suppose the modeler could not determine the individual ingredients that go into the representation, how can he check whether or not the decision-maker behaves in a manner consistent with such a representation? Or to put it differently, how is the behavior of a Köszegi-Rabin decision maker different from a standard decision-maker? For the case of deterministic choice, the answer is that the Köszegi-Rabin decision-maker may fail transitivity.

[2] Kim and Richter (1986) provide a condition on a demand function that is equivalent to maximizing a (possibly) nontransitive binary relation over a standard neoclassical consumption set.

2. Proof

First, we will show that (i) implies (ii): Suppose c is a general Köszegi-Rabin choice function. Then, there exists a reference dependent utility function U such that $c = C(\cdot, U)$. Define $\succeq$ as follows: $x \succeq y$ if $U(x,x) \geq U(y,x)$. Then, for all $A \in Y$,

$$c(A) = C(A,U) = \{x \in A \,|\, U(x,x) \geq U(y,x)\} = \{x \in A \,|\, x \succeq y\} = C_{\succeq}(A)$$

as desired.

To prove that (ii) implies (iii), assume that $c = C_{\succeq}$ and let n be the cardinality of X. Recall that $\succeq$ is a complete, reflexive, binary relation. We write $x \succ y$ for $x \succeq y$ and $y \not\succeq x$. Let $K = X \times X$. For $k = (w,z) \in K$, we define the function $u_{(w,z)} : X \to \{-2, 0, 2, 3\}$ as follows:

$$u_{(w,z)}(x) = \begin{cases} 3 & \text{if } x = w = z \\ 2 & \text{if } x = w \text{ and } w \succ z \\ -2 & \text{if } x = z \text{ and } w \succ z \\ 0 & \text{otherwise.} \end{cases}$$

Define the function μ as follows:

$$\mu(t) = \begin{cases} 16nt & \text{if } t \in \{-4, -3, 4\} \\ t & \text{if } t \in \{-2, 0, 2, 3\} \end{cases}$$

Clearly, μ is strictly increasing and $\mu(0) = 0$. Let

$$U(x,y) = \sum_{k \in K} u_k(x) + \sum_{k \in K} \mu(u_k(x) - u_k(y))$$

To complete the proof, we will show that $C_{\succeq} = C(\cdot, U)$; that is $x \succeq y$ iff $U(x,x) \geq U(y,x)$ for all $x, y \in X$, and

$$U(x,y) \geq U(y,y) \text{ implies } U(x,x) > U(y,x) \tag{4}$$

Note that

$$2n \geq \sum_{k \neq (x,x)} u_k(x) \geq -2n \tag{$*$}$$

Let $K_{x,y} = K\backslash\{(y,y),(x,x),(x,y),(y,x)\}$ and note that for $k \in K_{x,y}$

$$2 \geq u_k(x) - u_k(y) \geq -2 \tag{**}$$

Equations (*) and (**) and the definition of μ imply that

$$4n \geq \sum_{K_{x,y}} (u_k(x) - u_k(y)) = \sum_{K_{x,y}} \mu(u_k(x) - u_k(y)) \geq -4n$$

Let $x \succeq y$. Note that $\mu(u_{(x,y)}(y) - u_{(x,y)}(x)) \leq 0$ and, since $x \succeq y$, we also have $\mu(u_{(y,x)}(y) - u_{(y,x)}(x)) \leq 0$. It follows that

$$\begin{aligned}
U(x,x) - U(y,x) &= \sum_{k \in K} (u_k(x) - u_k(y)) - \sum_{k \in K} \mu(u_k(y) - u_k(x)) \\
&\geq -4n - \sum_{K_{x,y}} \mu(u_k(x) - u_k(y)) - \\
&\quad - \mu(u_{(x,x)}(y) - u_{(x,x)}(x)) - \mu(u_{(y,y)}(y) - u_{(y,y)}(x)) \\
&\geq -8n + 48n - 3 > 0
\end{aligned}$$

Conversely, let $y \succ x$. Then, $\mu(u_{(x,y)}(y) - u_{(x,y)}(x)) = 0$ and $\mu(u_{(y,x)}(y) - u_{(y,x)}(x)) = 64n$. Therefore,

$$\begin{aligned}
U(x,x) - U(y,x) &= \sum_{k \in K} (u_k(x) - u_k(y)) - \sum_{k \in K} \mu(u_k(y) - u_k(x)) \\
&\leq 4n - \sum_{K_{x,y}} \mu(u_k(y) - u_k(x)) - \mu(u_{(x,x)}(y) - u_{(x,x)}(x)) \\
&\quad - \mu(u_{(y,y)}(y) - u_{(y,y)}(x)) - \mu(u_{(y,x)}(y) - u_{(y,x)}(x)) \\
&\leq 8n - 3 + 48n - 64n < 0
\end{aligned}$$

Finally, suppose $U(x,y) - U(y,y) \geq 0$. Then,

$$\begin{aligned}
U(x,x) - U(y,x) &\geq U(x,x) - U(y,x) - U(x,y) + U(y,y) \\
&= -\sum_{k \in K} [\mu(u_k(y) - u_k(x)) + \mu(u_k(x) - u_k(y))] \\
&= -2(\mu(-3) + \mu(3)) = 2(48n - 3) > 0
\end{aligned}$$

completing the proof that (ii) implies (iii). That (iii) implies (i) is immediate. $\square$

References

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