

UNIVERSITY OF CALIFORNIA  
Los Angeles

## **Essays in Law and Economics**

A dissertation submitted in partial satisfaction  
of the requirements for the degree  
Doctor of Philosophy in Economics

by

**Richard Scheelings**

2005

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2005

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*Aan mijn gezin, en ook, natuurlijk, aan haar.*

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ABSTRACT OF THE DISSERTATION

## Essays in Law and Economics

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The dissertation comprises three independent but thematically-related applications of game and contract theory to small business financing. Chapter 1 (Spousal Guarantees) explains the determinants of small business debt financing and collateral choice within an incomplete financial contracting framework. Personal assets generally involve shared ownership and hence intra-familial bargaining before they can be used as a security. On their face their use should be dominated by the use of business assets. Data show that about 10% of loans are nonetheless secured with a personal asset. It is shown that the determinants of small business collateralized asset choice depend on both firm characteristics and owner/family characteristics. The likelihood of the use of a personal asset as collateral is decreasing both in firm default risk and in firm size. Chapter 2 (Guarantees versus Collateral in Small Business Debt Financing) develops a model of committed debt choice within an optimal incomplete contracting environment. Owners of small businesses are invariably required by banks personally to commit to any loans made to the business. Commitments are either collateral

or guarantees. Both types of commitment get around the legal protection of limited liability, but collateralized loans are not subject to state-based homestead exemptions. The likelihood of the use of a committed loan is increasing in firm default-risk and decreasing in firm size, characteristics consistent with available empirical evidence. The model also explains a non-monotonicity at the 80-90% level in the variation of interest rates to differences in homestead exemption levels. Finally, emerging empirical evidence shows that the change in divorce laws in the 1970s improved the welfare of women within marriage. Chapter 3 (Marital Investments and Changing Divorce Laws) uses a finite two stage game in which team-members first vote (non-cooperatively) to invest and then vote to remain in the relationship. The model trades-off the benefits of making marital investments against the dis-amenity of remaining in a relationship with the wrong person. The unique separating equilibrium is shown to be superior under no-fault divorce compared to fault divorce under both types of marital property regimes found in the States.

# CHAPTER 1

## Spousal Guarantees

### 1.1 Introduction

This paper explains the determinants of collateralized asset-choice in small business debt financing. Data show (the NSSBF) that the majority of small business debt is bank-sourced; it is overwhelmingly backed by collateral; and the asset used as collateral is overwhelmingly sourced from the business.<sup>1</sup> Nonetheless, the data also show that about 10 percent of collateralized loans are backed instead by a personal asset such as a family home.<sup>2</sup> What is interesting about such a choice of asset as collateral is that it is an asset that usually involves shared ownership, so that the consent of the co-owner is often required. Concrete examples of people likely to be asked to act as third-party guarantees include wives guaranteeing the loans of husbands (and vice-versa), parents the loans of children, grandparents the loans of grandchildren.<sup>3</sup> On its face therefore using a co-owned asset is a

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<sup>1</sup>The importance of commitments to making loan funds available to small businesses has been shown in [Avery et al. \(1998\)](#) using 1987 and 1993 data on small business financing: debts without the backing of either guarantees and/or collateral never comprise more than 15% of all small business loans.

<sup>2</sup>See subsection [1.4.1](#) of this paper for the evidence.

<sup>3</sup>During the 1990s common law courts in the Anglosphere were asked increasingly to re-examine the legality of such guarantees, owing to concern about the possibility that such guarantees might be signed under ‘moral suasion’ or emotional and physical ‘coercion’, as well as to concern about the disproportionate impact of the potential loss of a family home on guarantors

higher transaction cost loan contract and so an interesting question is: why isn't its use dominated by the use of a business asset? Why do we see it used in the data at all?

The paper utilizes an 'incomplete financial contracting' model to fully characterize the determinants of collateralized asset choice in small business debt finance. I show that the choice of collateralized asset depends on two sets of characteristics: one set dealing with firm attributes (and in particular the two important ones of firm default risk and firm growth potential), and the other set of characteristics dealing with family attributes (such as relationship closeness and intrafamilial bargaining power). With this framework I show the following fundamental result: the likelihood of a personal asset being used to back a small business debt is decreasing both in firm growth potential and in firm default risk. We know empirically that compared to the universe of all small business loans the subset of collateralized loans are on average higher risk. That logic is not exacerbated with respect to the use of personal assets – they are used as backing only for firms that are solid (low risk of default) but not spectacular (steady growth).

This result is driven by the fundamental trade-off arising out of the optimal collateralized loan contract: the ex post cost of liquidating – after default – the chosen asset backing the loan versus the ex ante efficiency of enabling otherwise credit rationed firms with viable investment projects to be funded. When the risk of firm default is high, the contract design problem focuses more on the possible ex post waste of asset-liquidation. Asset liquidation is (ex post) inefficient because, in the case of the business asset, it is more productive to leave it in the hands of the owner, who has inside knowledge and specialist expertise; and in the case of a

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such as non-working wives and grandparents.

family home, it provides a flow of services to its occupants which is not represented by its fire-sale market price. Since an assumption of the model is that it is more inefficient to liquidate a family home rather than business assets, it follows that for high risk firms it is better to use business assets to secure the loan. For low risk firms the contractual design problem switches to the ex ante concern of freeing funds for viable projects, and the problem of providing the business owner with the right incentives not to renegotiate the loan when business is good. In that case the greater threat-value of an asset with higher ex post waste of liquidation becomes beneficial from the ex ante perspective, so that the personal asset is in general preferred. However, this preference depends also on the attributes of the ex post renegotiation which follows default for good firms – each asset brings with it a different renegotiation dynamic. In particular, with respect to the use of a personal asset, since strangers do not guarantee each other’s debts, it is intrinsic to such loan contracts that they involve some sort of relationship between the co-signer and the beneficiary of that guarantee (the business owner). The loan contract can ‘free ride’ on the pre-existing ‘relational contract’ between them, and so it is possible that, depending on the characteristics of the relationship (its ‘closeness’ and the business owner’s intra-familial bargaining power), that even for low risk firms the business asset might still be preferred after all.

**The economic significance of small businesses** Small businesses account for half of private-sector output, employ more than half of private-sector workers, and provide about three-fourths of net new jobs each year.<sup>4</sup> Their susceptibility to fluctuations in bank lending practices is an important transmission

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<sup>4</sup>See U.S. Small Business Administration, Office of Advocacy, 1998 State of Small Business, chapter 2 (<http://www.sba.gov/advo/statsa/>).



mechanism of monetary policy, and it is known that they are much more affected than larger firms by business cycle-related fluctuations.<sup>5</sup> On-the-ground circumstances have been changing rapidly in the small business sector, with the rise in the 90s of private, outside equity and debt markets, and in particular, the rise in certain industry sectors of venture capital financing and more developed IPO markets enlivening scholarly interest in the interface between the private and public corporate spheres and how firms transition between the two. The historically impressive economic boom of the 90s was driven by the entrepreneurial dynamics of small business corporate form: some of today's small businesses are the giants of tomorrow. On the other hand, the turn-over rate of small businesses is high.

**Relationship to literature** The paper contributes to that recent literature which analyzes financial decisions from the 'incomplete contracting' perspective inaugurated by [Grossman and Hart \(1986\)](#) and applied to the financial contracting setting by [Aghion and Bolton \(1992\)](#), [Hart and Moore \(1994\)](#) and [Hart and Moore \(1998\)](#). For a summary of the standard framework and related literature see chapter 5 of [Hart \(1995\)](#). The model is closest to that of [Bolton and Scharfstein \(1990\)](#) (though that paper is concerned with predation in industrial organization theory). Papers (like this one) extending this literature to include more than one investor are [Bolton and Scharfstein \(1996\)](#), [Dewatripont and Maskin \(1995\)](#), [Dewatripont and Tirole \(1994\)](#) and [Berglöf and von Thadden \(1994\)](#). The first two involve multiple investors with the same asset claim while the second two explore the effects of having different investors hold different asset claims. A related, earlier theoretical literature modelling debt contracts as arising from the asymmetric information that exists between the lender and lende

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<sup>5</sup>See [Gertler and Gilchrist \(1994\)](#).

and which therefore necessitates costly monitoring by the former includes (for the one period case) [Townsend \(1979\)](#) and [Gale and Hellwig \(1985\)](#), as well as (for the multiperiod case) [Gale and Hellwig \(1989\)](#). A paper similar to this one in that it analyzes the role of collateral in renegotiation is [Bester \(1994\)](#), though it does so in an asymmetric information framework rather than the incomplete contracting framework utilized here. We are only aware of one empirical paper on the use of collateral and secured guarantees in small business finance, namely, [Avery et al. \(1998\)](#). There do not appear to be any papers (either theoretical or empirical) dealing with third party guarantees *per se*, though there has been some discussion of this in the legal literature (see generally [Fehlberg \(1997\)](#)). There is no theoretical literature in economics modelling marriages and like emotion-dependent relationships as a ‘relational contract’, though [Scott and Scott \(1998\)](#) is a description in such terms within the legal literature.

**Outline of paper** Section [1.2](#) outlines the model while section [1.3](#) solves for the optimal contract and discusses the policy difficulties associated with third party guarantees. Section [1.4](#) explores the determinants of the pattern of collateralized small business loan finance while section [1.5](#) examines the case where the startup firm is predominantly constituted by entrepreneurial human capital, so that there might not be sufficient business assets to use as collateral. Section [1.6](#) concludes with directions for future research.

## 1.2 Model

### 1.2.1 The ex ante contract

**The agents** At date 0 a bank (denoted  $B$ ), an entrepreneur (denoted  $E$ ) and a guarantor (denoted  $G$ ) convene to sign a guaranteed loan contract to enable the entrepreneur to invest in a long-term, potentially profitable project.<sup>6</sup> The entrepreneur must borrow funds because he is wealth-constrained; in particular, we assume that he has zero (liquid) wealth ex ante, and we assume the same for the guarantor. All agents are risk neutral and discount factors are normalized to zero.

**The project** The project lasts two periods. There is no intertemporal interest rate. The project provides non-negative returns of  $\tilde{R}_1$  (a random variable with support  $\{0, x\}$ ) at date 1 and  $R_2 = r$  at date 2. In the first instance these returns accrue to the entrepreneur.<sup>7</sup> The commonly known distribution on  $\tilde{R}_1$  is given by:<sup>8</sup>

$$\tilde{R}_1 = \begin{cases} 0 & \text{with probability } 1 - \theta, \\ x & \text{with probability } \theta. \end{cases}$$

The project's initial cost is  $K > 0$ . The project is ex ante viable or productive since  $\theta x + r > K$ , where we also assume that  $K < x$ . Thus, we have biased

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<sup>6</sup>Hereafter the entrepreneur is referred to generically as 'he' and the guarantor as 'she'.

<sup>7</sup>These returns are specific to the entrepreneur - that is, neither the bank nor the guarantor can obtain these returns from the project without the entrepreneur. However, we do not model the process by which the entrepreneur generates these returns, assuming instead that they are exogenously given.

<sup>8</sup>Note that there is no loss of generality in confining attention to a two-state date 1 return, since even if  $R_1$  is an interval (in  $\mathbb{R}^+$ ) it would be easy to show that it would never be optimal for the entrepreneur to make a partial payment, so that default in that more general case would be defined as not paying anything at date 1.

the spousal guarantee problem in favor of financing indubitably worthwhile investments. The amounts  $R_1$  and  $r$  are uncontractable between entrepreneur and bank: that is, ex ante describable but ex post unenforceable (or ‘observable’ but not ‘verifiable’ in the language of [Grossman and Hart \(1986\)](#)).

**The loan** The entrepreneur borrows  $K$  from the bank at date 0 for a promise to repay  $P$  at date 1. The promised repayment amount  $P$  is also uncontractable. One way to think about this in concrete terms is to imagine the existence of a ‘savings account’ belonging to the entrepreneur into which the return is deposited when it accrues. Any amount in this ‘savings account’ is untouchable by the bank, even in the event that the entrepreneur defaults on the repayment of  $P$ . This is the ‘diversion’ or ‘stealing’ assumption of [Hart and Moore \(1998\)](#), a possibly extreme but nonetheless useful assumption designed to capture the more realistic phenomenon of managerial discretion in the use and disbursement of corporate funds. At least within the context of family businesses a reason for such untouchability lies in the ability of entrepreneurs potentially to divert business profits into family gifts and trusts.

Because the date 1 return is describable, the date 0 contract can stipulate the date 1 payment  $P$  to be conditional on  $\tilde{R}_1$ . Thus the contract can stipulate that at date 1 the entrepreneur should repay  $P_0$  when  $R_1 = 0$  and  $P_x$  when  $R_1 = x$ . However, because any contract terms conditioned on  $\tilde{R}_1$  are ex post unenforceable, we need to denote the *actual* payment made by the entrepreneur at date 1 by  $\hat{P}$ . Without loss we restrict this date 1 action set to be the same as the date 0 contractually specified repayment schedule:  $\hat{P} \in \{P_0, P_x\}$ . Actual date 1 repayment is contractible.

**Security asset** Because of the noncontractibility of the return stream, the bank requires security for the loaned funds  $K$ . There exist two types of asset which might act as security. A security on the assets (either or both) is contractible.

The first asset (asset  $A$ ) is a business asset that will be bought with the borrowed funds. It lasts one period. This asset is essential to the production process: in combination with the entrepreneur's skill it produces the return stream over the two periods. If the asset is liquidated at date 1, then the entrepreneur is unable to earn the date 2 return  $r$ . The date 1 liquidation value to the bank is  $L^A = \alpha r$ , where  $\alpha \in [0, 1)$ .

The second asset (asset  $H$ ) is a shared non-liquid relationship asset which is completely independent of the business. It has a deterministic market value of  $z$ , which can be interpreted as the value of (say) a family home to its occupants. The date 1 liquidation value to the bank is  $L^H = \lambda z$ , where  $\lambda \in [0, 1)$ . This modelling assumption captures the fact that the relationship asset is worth more when maintained as a relationship asset than when in the possession of the bank. Specifically, it captures the fact that a relationship asset like a family home provides a value to its occupants not encapsulated in liquidated sale price alone. Note that when both assets are used as security, the date 1 liquidation value of the assets  $AH$  to the bank is  $L^{AH} = \alpha r + \lambda z$ . These liquidation values constitute ex post exogenously determined inefficiencies which play an important role in the ex post renegotiation to be described below.

A security can also be placed over the combined assets, which we denote  $AH$ .

**Relationship closeness** Although the interpretation given of the parameter  $\lambda$  is that it represents the inefficiency of having the relationship asset liquidated by the bank rather than remaining in the hands of the owners, an alternative, related, interpretation is that  $\lambda$  represents the ‘closeness’ of the relationship. In this interpretation the magnitude of  $\lambda$  is inversely related to relationship closeness: the lower is  $\lambda$ , the more close the relationship between  $E$  and  $G$ ; the higher is  $\lambda$ , the less close the relationship.

**Relationship asset share** At date 2 (when the model ends) the relationship asset is sold and consumed by the entrepreneur and/or guarantor according to their exogenously determined share of the asset. Let  $S^E \in [0, 1]$  denote the entrepreneur’s date 2 share of the relationship asset (or the date 2 sale proceeds thereof). Hence the guarantor’s share is  $(1 - S^E)$ . When  $S^E = 1$  we have a pure personal guarantee, and when  $S^E = 0$  we have a pure third party guarantee. The most common case of using the matrimonial home as collateral will (depending on the family law property regime in place) fall between these two extremes, though the most usual family law default rule is  $S^E = 1/2$ .

**Entrepreneur’s promised payment to guarantor** The guarantor must be compensated for the risk of permitting (her share of) the relationship asset to act as security for the loan. We denote by  $y_0$  and  $y_x$  the amounts the entrepreneur promises to pay the guarantor at date 2 (conditional on the entrepreneur’s date 1 actual repayment  $\hat{P}$ ) in return for her permitting the relationship asset to be utilized as security. Note that the  $y$ ’s need not be interpreted as an explicit payment arising out of the guarantee contract but can be interpreted more expansively as the promise of a ‘standard of living’ arising out of the relationship. The  $y$ ’s are

enforceable since they are conditioned on actual date 1 repayment by the entrepreneur.<sup>9</sup> Such enforcement can be interpreted as divorce law in the case of spousal guarantees. We consider the interpretation of this repayment further in subsection 1.2.2 when we outline the ex post renegotiation regime.

**Contractual provision for default** We will assume that the assets are discrete so that they cannot be partially liquidated. In the case of a family home at least this assumption is realistic. The most general type of default provision then specifies that when the entrepreneur makes a date 1 payment  $\hat{P}$ , the bank has the right to liquidate the secured asset(s) with probability  $\beta(\hat{P}) \leq 1$ .<sup>10</sup> The date 0 contract will therefore specify that when the entrepreneur makes the payment  $P_x$  the bank has the right to liquidate the secured asset with probability  $\beta_x$ , and when the entrepreneur makes the repayment  $P_0$  the bank has the right to liquidate the secured asset with probability  $\beta_0$ . The  $\beta$ 's are enforceable since they are conditioned on actual date 1 repayment by the entrepreneur.

**Payoffs** Payoffs for the agents are described in the next section when the optimal guarantee is solved. They are linear in income/payments and (for the entrepreneur and guarantor) linear in asset share and (for the bank) linear in expected foreclosure value.

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<sup>9</sup>Nothing in the model precludes the payments to the guarantor being conditioned also on the foreclosure probabilities, which would then give four possible repayments rather than the two assumed here. However, since the results of the model do not depend upon this point, the simpler modelling choice has been adopted.

<sup>10</sup>A model of this type was first used in [Bolton and Scharfstein \(1990\)](#).

**Contractibility** It is useful to summarize the contractibility assumptions in the model. The return stream and agent payoffs are not contractible, while the asset(s) and the entrepreneur’s *actual* date 1 payments are contractible. Anything conditional on a non-contractible variable is non-contractible. These contractibility assumptions rest on the idea that it is easier to divert cash flow than physical, non-liquid assets, a distinction emphasized in [Hart and Moore \(1998\)](#).

**The date 0 contract** We assume that the entrepreneur has all the ex ante bargaining power and chooses the date 0 contract  $\Gamma = \{P_0, P_x, \beta_0, \beta_x, y_0, y_x\}$ . The first two terms (conditioned on  $\tilde{R}_1$ ) are not enforceable but the remaining terms (conditioned on  $\hat{P}$ ) are enforceable. Since the first two terms are not enforceable, at date 1 the entrepreneur might choose to ‘deviate’ from the loan repayment amounts specified in  $\Gamma$ .

The timeline is as follows. First nature moves determining whether the date 1 return is either  $x$  or  $0$ . Then the entrepreneur decides whether to pay  $P_x$  or  $P_0$ . Depending on this repayment amount, the bank acquires the right to foreclose on the secured asset(s) with probability  $\beta_x$  or  $\beta_0$ . However, liquidation of the secured asset(s) is not automatic because such liquidation is ex post inefficient. The agents would prefer to renegotiate the ex ante contractual terms  $\beta_x$  and  $\beta_0$ , setting them to zero and dividing amongst themselves the ex post surplus thereby saved. If renegotiation succeeds then the secured asset(s) is not liquidated and if it fails then it is liquidated by the bank. The specifics of renegotiation is outlined in subsection [1.2.2](#). This timeline is depicted in [figure 1.1](#) below.

**First-best** If a comprehensive contract could be signed, then given the assumptions on the productivity of the project the entrepreneur would have no dif-



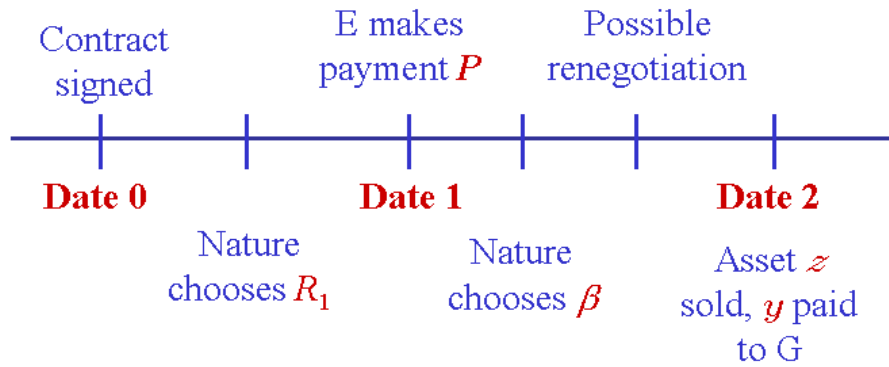


Figure 1.1: Timeline of Model

difficulty getting a bank to finance the project, and the first-best would be achieved. Note therefore that securing the loan would not be necessary and, if nonetheless undertaken, liquidation would never be part of a first-best outcome. However, the inability to contract on the return stream means that, without a mechanism to enforce date 1 repayment, no bank will lend to the entrepreneur in spite of the overall viability of the project. The usual mechanism in the literature is a security over the project asset  $A$  that is bought with the borrowed funds. Since the entrepreneur values continuance of the project, the possibility of liquidation of  $A$  at date 1 gives the bank leverage over the entrepreneur ensuring that the latter pays the loan out of the date 1 return stream. Securing the relationship asset rather than the project asset switches the leverage problem from the bank/entrepreneur

relationship to the entrepreneur/guarantor relationship. This is further explained in the following subsection.

### 1.2.2 The ex post renegotiation

**Renegotiation mechanism** Renegotiation takes the form of the entrepreneur bribing the bank not to exercise its right to liquidate the foreclosed asset(s). Since the entrepreneur has zero ex ante (liquid) wealth, such a bribe is only possible in the case  $R_1 = x$ . If the payment of  $P_0$  is called a *default*, then default can either be *strategic* or *necessary* according to whether it occurred when  $R_1$  equalled  $x$  or 0 respectively. Consequently there can be no renegotiation after a necessary default while after a strategic default renegotiation is possible since the funds potentially exist to ‘buy back’ the seized asset(s).

The ex post surplus over which the agents renegotiate depends on which asset(s) is used as security. The three different models examined in this paper are denoted  $A$ ,  $H$  or  $AH$ , depending on whether the security is over the project asset, the relationship asset, or both. To consolidate notation in the paper, define the indicator functions  $\kappa_r^i$  and  $\kappa_z^i$  for each value of  $i \in \{A, H, AH\}$ . We have

$$\kappa_r^i = \begin{cases} 1 & \text{when } i = AH, A \\ 0 & \text{when } i = H \end{cases}$$

and

$$\kappa_z^i = \begin{cases} 1 & \text{when } i = AH, H \\ 0 & \text{when } i = A \end{cases}$$

Denote by  $\Pi^i$  the social surplus salvaged by the parties when the liquidation of asset  $i$  is forestalled via renegotiation. These different amounts can then be expressed as  $\Pi^i = \kappa_r^i r(1 - \alpha) + \kappa_z^i z(1 - \lambda)$ . Note that they depend on the

exogenously given liquidation values mentioned in the previous subsection, so that the ex post surplus is also exogenous. Those liquidation values can also be succinctly summarized using indicator functions as  $L^i = \kappa_r^i \alpha r + \kappa_z^i \lambda z$ .

It is known empirically (see Dennis et al (1988)) that banks rarely forgive principal in the event of default, and so we assume that the bank is exactly compensated for the loss of liquidation value which it gives up. The entrepreneur and guarantor then engage in two-way bargaining over the surplus that remains. The agents are exogenously endowed with ex post bargaining power  $\tau_E$  for the entrepreneur and  $\tau_G$  for the guarantor. These bargaining parameters sum to one. The details of the generalized Nash bargaining are presented in Appendix A and the results summarized by  $g_E^i = \tau_E[\kappa_r^i r(1 - 2\alpha) + \kappa_z^i z(1 - 2\lambda)]$ .

It is worth pointing out that since the relationship asset  $H$  is assumed contractible and since it becomes liquid at date 2, the model leaves open the ability of the entrepreneur to propose at date 1 (say in exchange for forbearance on the part of the bank in foreclosing on the project asset  $A$ ) a share of his date 2 relationship asset proceeds. In some family law/property law jurisdictions such a contract would not be allowed, but this is not true for all. Allowing such an additional mechanism of ex ante commitment would introduce a ‘constant recontracting’ style of security over  $H$ .<sup>11</sup> For convenience we rule this out, just as we rule it out for the side payments between entrepreneur and guarantor (which would not be permitted anyway under most family law regimes).

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<sup>11</sup>See [Hart and Moore \(1998\)](#) or [Bulow and Rogoff \(1989\)](#) (the latter examining default and renegotiation in the context of international lending and sovereign debt) for examples of models which do not rule out constant recontracting.

**Leverage** In a one-period model the results are clear: the guarantor does not sign and the bank does not lend. In the two-period model, and in the absence of an outside asset, the bank only lends because it has leverage over the entrepreneur at date 1 (the time at which repayment to the bank is due). That leverage consists in the ability of the bank to withdraw from the entrepreneur's use the inside asset, so depriving the entrepreneur of the chance to earn the date 2 income  $r$ . When the outside rather than the inside asset is used as security, this leverage then switches to that between guarantor and entrepreneur, and now lies in the contractual payments  $y$  from the entrepreneur to the guarantor as well as in the extent to which the entrepreneur values the relationship asset as determined by exogenous ownership share.

### 1.3 The optimal collateralized loan contract

In this section we set out the optimization program that the entrepreneur solves at date 0 and use it to characterize the optimal contract for each of the three possible cases of secured asset ( $A, H, AH$ ). Without loss, we focus on the renegotiation-proof contract, where the possibility of future renegotiation is anticipated by the parties at date 0. Renegotiation-proofness manifests itself in the optimization program in the form of an added constraint.

**Set-up** At date 0 the entrepreneur solves the following linear program (call it  $(\star^{(i)})$ ), choosing over  $\Gamma^i = \{P_0, P_x, \beta_0, \beta_x, y_0, y_x\}$  to maximize his expected

payoff<sup>12</sup>

$$\begin{aligned} & \theta[x - P_x - y_x + (1 - \beta_x \kappa_r^i)r + (1 - \beta_x \kappa_z^i)S^E z + \beta_x g_E^i] \\ & + (1 - \theta)[-P_0 - y_0 + (1 - \beta_0 \kappa_r^i)r + (1 - \beta_0 \kappa_z^i)S^E z] \end{aligned} \quad (1.1)$$

subject to the individual rationality constraint of the bank

$$\theta[P_x + \beta_x L^i] + (1 - \theta)[P_0 + \beta_0 L^i] \geq K \quad (1.2)$$

as well as to the individual rationality constraint of the guarantor

$$\begin{aligned} & \theta[y_x + (1 - \beta_x \kappa_z^i)(1 - S^E)z + \beta_x g_G^i] \\ & + (1 - \theta)[y_0 + (1 - \beta_0 \kappa_z^i)(1 - S^E)z] \geq (1 - S^E)z \end{aligned} \quad (1.3)$$

and, in order to ensure that the entrepreneur does not strategically default when  $R_1 = x$ , subject also to the following ‘renegotiation constraint’

$$\begin{aligned} & x - P_x - y_x + (1 - \beta_x \kappa_r^i)r + (1 - \beta_x \kappa_z^i)S^E z + \beta_x g_E^i \\ & \geq x - P_0 - y_0 + (1 - \beta_0 \kappa_r^i)r + (1 - \beta_0 \kappa_z^i)S^E z + \beta_0 g_E^i \end{aligned} \quad (1.4)$$

and subject to the following ‘limited liability’ constraints for the entrepreneur and guarantor owing to the assumption of ex ante zero liquid wealth

$$P_0 \leq 0 \text{ and } P_x \leq x \quad (1.5)$$

$$0 \leq y_0 \leq r \text{ and } 0 \leq y_x \leq x + r - P_x \quad (1.6)$$

Note that these two constraints incorporate the ‘no constant recontracting’ assumptions made in the previous sections. Finally we have the feasibility constraints on the foreclosure probabilities

$$0 \leq \beta_0, \beta_x \leq 1 \quad (1.7)$$

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<sup>12</sup>Note that each of these contractual terms should also have an  $i$  superscript, but to avoid notational clutter we omit them.

The program depicts three models nested in one, depending on which asset is used as security ( $A, H, AH$ ). The payoffs of each of the three agents are written assuming that the contractual terms of  $\Gamma^i$  are honored. Thus, equation (1.1) shows the entrepreneur's payoff for both the case where  $R_1 = x$  and he pays the bank  $P_x$  (and the guarantor  $y_x$ ) and for the case where  $R_1 = 0$  and he pays the bank  $P_0$  (and the guarantor  $y_0$ ). When  $P_x$  is paid then with probability  $1 - \beta_x$  the entrepreneur keeps the secured asset(s) while with probability  $\beta_x$  he needs to buy it back and then split the surplus with the guarantor, giving him  $g_E^i$ . The payoffs for the bank and guarantor are derived in analogous way. Regarding equation (1.4), the LHS is taken from the LHS of (1.1) while the RHS has the same form except that now the entrepreneur has paid  $P_0$  so the other contractual terms ( $\beta$  and  $y$ ) conform to that payment. The second renegotiation constraint, ensuring that the entrepreneur pays  $P_x$  instead of  $P_0$  when  $R_1 = 0$ , is not needed since the entrepreneur is wealth constrained. For the same reason we need not include renegotiation payoffs for either the entrepreneur or the guarantor for the case when  $R_1 = 0$  since they are automatically zero.

**Characterizing the optimal contract** The following proposition will help to solve for the optimal contract.

**Proposition 1 (Contract Characterization).** *In the optimal contract*

- (i)  $P_0 = 0$ ,
- (ii)  $y_0 = y_0(1 - \kappa_r^i) \geq 0$ ,
- (iii)  $\beta_x = 0$ ,
- (iv) *both the bank's and guarantor's individual rationality constraints bind,*

(v) *the renegotiation constraint binds.*

**Proof** See Appendix B. ■

The proof of (i) follows immediately from assumptions on the contracting technology made in subsection 1.2.1.

Part (ii) shows that the corner solution for the payment to the guarantor is model-dependent. When only asset  $H$  is used, sale of  $H$  after necessary default still leaves the entrepreneur with asset  $A$  and the date 2 return. To ensure that the entrepreneur takes seriously the loss of the relationship asset when considering strategic default, the contract makes him hand the entirety of this future return to the guarantor. Indeed without such a stipulation, the guarantor would not participate in the contract. Note that in this part of the proof we have used the assumption that the date 2 return, while non-contractible between the entrepreneur and bank, *is* contractible between the entrepreneur and guarantor. This assumption can be justified as a modelling short-hand for the fact that third-party guarantees are signed within the context of a larger (long-term, relational) contract obtaining between the entrepreneur and guarantor.<sup>13</sup> It is known from the theory of relational contracting that a long-term relationship can transform non-contractible variables into de facto contractible ones. An alternative justification is that the institution of family law, in particular, laws governing the dissolution of marriages (which vary across states) provides the de facto commitment technology ensuring repayment by the entrepreneur to the guarantor.

Part (iii) is proved by showing that a strictly positive  $\beta_x$  cannot be optimal, since in that case decreasing  $\beta_x$ , without changing the payoffs of the bank and guarantor, strictly increases the entrepreneur's payoff. The intuition for the result

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<sup>13</sup>On marriage as a relational contract see [Scott and Scott \(1998\)](#).

is that the contract needs to provide the entrepreneur with incentives not to strategically default. Foreclosing on the secured assets when  $R_1 = x$  gives the entrepreneur precisely the opposite incentives from that point of view.

Part (iv) is proved by utilizing the fact that the entrepreneur, who has all the ex ante bargaining power, maximizes his payoff by paying both the bank and guarantor as little as possible.

The proof of part (v) is by contradiction - the relaxed program is solved and the result shown to contradict the ignored renegotiation constraint. Essential to the proof is the assumption that the  $y$ 's cannot be paid out of the guarantor's (or entrepreneur's) share of the relationship asset. The intuition is that the renegotiation constraint must bind in order to provide the incentive for the entrepreneur to repay the debt in the good income state. If there is no incentive for the entrepreneur to repay in the good income state, then the guarantor will not sign the contract.

**Contractual Inefficiency** Since  $\beta_0 = 0$  in the relaxed program leads to a contradiction, we have the following corollary.

**Corollary 1 (Second Best Efficiency).** *the optimal  $\beta_0$  is bounded away from zero.*

Even though asset foreclosure is ex post inefficient when  $R_1 = 0$ , nonetheless it will occur. This inefficiency arises from the twin effects of limited liability and contractual incompleteness. Recall that the first best involves  $\beta_0 = 0$ . The first best can never be achieved since there is always some asset liquidation in equilibrium. A positive  $\beta_0$  is needed to provide some disincentive to strategic default.



The standard debt contract ( $\beta_0 = 1$ ) provides the correct incentives, but involves too much punishment. From the proofs in appendix B it can be seen that total expected welfare in this model is

$$W^i = \theta x - K + r + S^E z - EL^i \quad (1.8)$$

where the first three terms are the net present value of the project in the absence of liquidation, and the final term, representing the efficiency loss due to contractual incompleteness, is defined in proposition 2. While  $\beta_0 = 1$  is a feasible solution, it is dominated for most parameter values by the optimal solution to be presented in proposition 2.

**Remark 1 (Standard Debt Contract).** *In general the standard debt contract is not optimal:  $\beta_0$  is less than one.*

It is the fact that liquidation occurs on the equilibrium path that makes a standard debt contract sub-optimal: it is socially wasteful to punish the entrepreneur more than is necessary to prevent strategic default.<sup>14</sup> This result provides some basis for judicial concern about the nature of third party guarantee contracts, which are standard debt contracts. We consider this question further below.

**Solving the linear program** Proposition 1 enables us to simplify ( $\star^i$ ) and consequently to find this optimal level of contractual inefficiency.<sup>15</sup>

**Proposition 2 (Contractual Inefficiency).**

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<sup>14</sup>This is similar to the argument in, say, Green and Porter (1984), or, more generally, in any repeated game model with noisy observation of (stage game) outcomes or probabilistic moves which make all paths in the game reachable with positive probability.

<sup>15</sup>Here we are re-inserting the superscript on the contract variable  $\beta$ .

(i) In the optimal contract, the efficiency loss due to contractual incompleteness is

$$EL^i \equiv (1 - \theta)\beta_0^i[\kappa_r^i r + \kappa_z^i z - L^i] \quad (1.9)$$

(ii) while the optimal foreclosure probability is

$$\beta_0^i = \frac{K - (1 - \kappa_r^i)y_0}{\theta[\kappa_r^i r + \kappa_z^i S^E z - g_E^i] + (1 - \theta)[L^i - \kappa_z^i(1 - S^E)z]} \quad (1.10)$$

(which will be a solution to  $(\star^i)$  provided that  $\beta_0^i$  is not greater than one).

**Proof** See Appendix B. ■

Equation (1.9) shows that the efficiency loss due to contractual incompleteness is the expected loss of surplus (the term in square brackets) when the date 1 return is zero (with probability  $1 - \theta$ ) and the bank gets the right to foreclose on the secured asset(s) (with probability  $\beta_0^i$ ). Equation (1.10) depicts the determinants of the ex ante (contractually) chosen probability of ex post foreclosure. (Recall from subsection 1.2.2 the equations for  $L^i$  and  $g_E^i$ .)

**Costly outsiders** To obtain some intuition for this result, note that equation (1.10) is just the renegotiation constraint after equations (1.2) and (1.3) have been substituted into it. It can be seen from the denominator of (1.10) that the effect of increasing the entrepreneur's outside asset share ( $S^E$ ) is independent of  $\theta$ : the greater his share, the lower is  $EL$ . Since  $S^E$  also enters  $W^i$  in equation (1.8) directly (and not just through  $EL$ ) this is not conclusive, but we have the following easily proved proposition.

**Proposition 3 (Costliness of TPGs).**  $W^i$  is monotone increasing in  $S^E$ .

As stated in the introduction, third party guarantees (defined as low  $S^E$  (and in the pure case by  $S^E = 0$ )), are not a low-cost financing option. This fact will

become prominent when determining the pattern of collateralized finance in the next section.

**The basic tradeoff** It can also be seen from the denominator of equation (1.10) that when  $\theta$  is high (low risk) concern revolves more about the entrepreneur's bargaining power than the liquidity value of the asset(s) - to prevent strategic default,  $\beta_0$  must be raised the more the entrepreneur is likely to receive a greater share of the ex post surplus after strategically defaulting. Thus 'weak' entrepreneurs can have lower finance costs. When  $\theta$  is low (high risk), concern shifts to the value of the asset(s) in liquidation. The greater the ex post inefficiency, the lower is  $EL$ . We summarize these statements in the following easily proved proposition.

**Proposition 4 (Efficiency Tradeoff).**

- (i)  $EL$  is monotone decreasing in  $\tau^E$ .
- (ii)  $EL$  is monotone increasing in  $\alpha$  and  $\lambda$ .

The model is characterized by the fact that the greater the ex post *inefficiency* the greater the ex ante *efficiency*. Stated another way, there exists a trade-off between the ex post cost of bankruptcy (wasteful asset liquidation) and the ex ante efficiency in ensuring that viable projects are undertaken. The paradox appears to be that, the closer the relationship, the more beneficial the personal collateral from a commercial perspective, but the greater the concern from a non-commercial perspective. The beneficial commitment effect of a relationship appears to be recognized by lenders. Thus, as the author of a survey of bank branch-level lending managers in the UK concluded, "private commitments enhanced public enforceability":

Lenders acknowledged the problems inherent in taking security from a person in an intimate relationship with the debtor, but they also emphasized the importance to them in commercial terms of the surety's emotional investments in both the relationship with the debtor and the home (where relevant).<sup>16</sup>

**Judicial concern** What makes the trade-off especially interesting in the context of spousal guarantees is that necessary default in the case of such guarantees involves important relationship asset loss, which can affect some classes of guarantor severely. This is especially so for those classes of guarantor recognized by common law courts as being especially in need of protection (such as stay-at-home wives or grandparents) because of an asymmetry in outside earning potential vis-a-viz the entrepreneur. With perhaps an excessive regard for the ex post regret obviously felt in those instances when loans or loved ones turn sour, third party guarantees have been dubbed in some legal scholarship a form of 'sexually transmitted debt'.<sup>17</sup>

During the nineties courts in the Anglo-American world grappled with the policy trade-offs involved in permitting the enforceability of third-party guarantees.<sup>18</sup> As an example, the leading House of Lords case (*Barclays Bank Plc v O'Brien* [1994] 1 AC 180.) involved a wife suing to prevent a bank foreclosing on the matrimonial home. She had co-signed a guarantee as backing for business interests in which her husband was involved (and which did not *directly* involve

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<sup>16</sup>See [Fehlberg \(1997\)](#) at page 204.

<sup>17</sup>See for example [Fehlberg \(1997\)](#).

<sup>18</sup>See the surveys of cases and jurisdiction by (for example) [Fehlberg \(1995\)](#) and [Trebilcock and Ballantyne-Elliot \(1998\)](#).

her). In their decision the law lords were aware that any desire for paternalistic circumvention of the usual legal and economic norms of freedom to contract should be balanced against the concern that ‘the wealth currently tied up in the matrimonial home does not become economically sterile’.<sup>19</sup> A ban on such guarantees would freeze forever all assets held in domestic use while unfettered freedom exposes a subset of guarantors to intolerable risk of primary asset loss. Proposition 2 informs us that the optimal guarantee contract trades off these concerns.

From the point of view of a court deciding between ex ante versus ex post efficiency, it is not obvious normatively which way the balance should be tipped. As a general rule it is true that the combination of proposition 4 and remark 1 suggests some validation for judicial suspicion of standard form debt guarantees with automatic foreclosure. By seeking to diminish the incidence of such guarantees, the courts are, in effect, attempting to decrease  $\beta_0$ , and there are efficiency-enhancing reasons for them to do so.

Aware of legal concern about ‘coercion’, banking associations in the United Kingdom and United States have drawn up conventions which branch managers must take into account when presenting third-party guarantees for signing.<sup>20</sup> Such conventions include the requirement on lenders to provide basic information about the nature and possible consequences of signing a guarantee (like a ‘health warning’) and also to urge guarantees to seek independent (that is, independent of the

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<sup>19</sup>The contractual legal doctrines protecting disadvantaged persons in common law countries fall under the rubric of equity. For a summary of equitable doctrines in contract see Hanbury and Martin: Modern Equity (2002).

<sup>20</sup>See for example the UK [BBA \(1994\)](#).

guarantee's representatives) legal advice before signing.<sup>21</sup>

## 1.4 The pattern of collateralized loans

In this section we explore the determinants of the pattern of collateralized small business finance. The first subsection outlines the empirical evidence of a pattern while subsection 1.4.2 presents the basic result. Subsection 1.4.3 gives a brief numerical example.

### 1.4.1 Motivation

Data on small business financing has traditionally been sparse. But beginning in 1987, the Federal Reserve Board, in association with the Office of Small Business Administration, has conducted five-yearly surveys of small businesses (defined as 500 employees or less) seeking especially information on owner characteristics and funding sources. Called the National Survey of Small Business Finances (NSSBF), these sequential cross-sectional samples (1987, 1993, 1998 and 2003) now comprise the main source of information on small business financial structure and during the 1990s have given rise to an empirical literature examining its findings.<sup>22</sup>

Figure C.1 in Appendix C depicts a table taken from Petersen and Rajan (1994). The table shows loan profile by firm size (Panel A) and firm age (Panel B). The data is taken from the 1987 version of the NSSBF. Note in both panels of the table column three, which states the percentage (with respect to the whole survey

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<sup>21</sup>These are the same type of measures (although perhaps strengthened) adopted by the House of Lords in its *Barclays Bank* decision.

<sup>22</sup>The data and supporting documents can be found at:  
<http://www.federalreserve.gov/pubs/oss/oss3/nssbftoc.htm>.

sample of over 3000 firms) of firms in each category with debt. Conditional on a firm having debt, the remaining columns show the percentage sources of loans. It can be seen from the bank column (column six) that the majority of loan finance is bank sourced (between 50 and 65 percent). Indeed, distinguishing between inside and outside sources of loans, the proportion of outside debt sourced from banks is overwhelming. The table supports the statement made in the introduction that the majority of small businesses have debts, and the majority of that debt is in bank loans.

The incidence of committed debt by small businesses is shown in Figure C.2 of Appendix C. It depicts a table taken from Avery et al. (1998). The data in the table is again based on the NSSBF, but this time both the 1987 and 1993 versions of the survey. The table shows, conditional on a firm having a loan, how much of those loans are backed by owner commitments. Committed debt is defined as loans backed either by guarantees or pledged assets. The table shows the four possible categories of commitment, as well as (when assets are pledged) a breakdown of which asset(s) was used to back the pledge (the business asset, personal asset, or both). The table supports the statement made in the introduction that the overwhelming majority of bank debt is collateralized.

In order to consider a conditioned sample of collateralized loans we ignore the first and last rows in the table depicted in Figure C.2. The percentage breakdown of collateralized loans (the remaining rows in the table of Figure C.2) according to asset(s) used is presented in Table 1.1 for both years (using the dollar value columns). It can be seen that the overwhelming majority of collateral is pledged using business assets, thus supporting the statement made in the introduction. While there is some variation over time (with the business cycle), and while that variation is itself interesting, nonetheless, the preponderance of business asset

	Percent Secured	
Asset	1987	1993
Business	93.2	86.1
Personal	3.4	10.2
Both	3.4	3.7

Table 1.1: **Percentage Incidence of Asset Use in Collateral**

collateral is a temporally stable stylized fact. From a business cycle perspective, note that in 1987 (a time of economic growth and capital gains), the proportion of personal assets pledged as collateral was much smaller than in 1993 (a time of recession - or immediate post-recession - and non-increasing house values). On the other hand, during a recession, the *relative* value of business assets to personal assets may have declined even more.

#### 1.4.2 Comparing Asset $A$ and Asset $H$

Using the results of proposition 2 and the parametric forms for  $g_E^i$  and  $L^i$  given in subsection 1.2.2 we can write the specific parametric forms for  $EL^A$  and  $EL^H$  as:

$$EL^A \equiv \frac{(1 - \theta)K[1 - \alpha]}{\theta[1 - \tau_E(1 - 2\alpha)] + (1 - \theta)\alpha}$$

$$EL^H \equiv \frac{(1 - \theta)\{K - r\}[1 - \lambda]}{\theta[S^E - \tau_E(1 - 2\lambda)] + (1 - \theta)[\lambda - (1 - S^E)]}$$

Define  $\Delta EL_H^A \equiv EL^A - EL^H$  and  $\tilde{\theta}_{A|H}$  as the cutoff theta at which the entrepreneur is indifferent between using asset  $A$  or asset  $H$ . The entrepreneur is



indifferent between using asset  $A$  or asset  $H$  when  $\Delta EL_H^A = 0$ , or:

$$\Delta EL_H^A \equiv \frac{K[1 - \alpha]}{\theta[1 - \tau_E(1 - 2\alpha)] + (1 - \theta)\alpha}(1 - \theta) - \frac{\{K - r\}[1 - \lambda]}{\theta[S^E - \tau_E(1 - 2\lambda)] + (1 - \theta)[\lambda - (1 - S^E)]}(1 - \theta) = 0$$

This implies that

$$K[1 - \alpha]\{\theta[S^E - \tau_E(1 - 2\lambda)] + (1 - \theta)[\lambda - (1 - S^E)]\} - \{K - r\}[1 - \lambda]\{\theta[1 - \tau_E(1 - 2\alpha)] + (1 - \theta)\alpha\} = 0$$

which can be written as (gathering  $\theta$  terms)

$$\theta(K[1 - \alpha]\{1 - \lambda - \tau_E(1 - 2\lambda)\} - \{K - r\}[1 - \lambda]\{1 - \alpha - \tau_E(1 - 2\alpha)\}) + K[1 - \alpha][\lambda - (1 - S^E)] - \{K - r\}[1 - \lambda]\alpha = 0 \quad (1.11)$$

which gives us finally

$$\theta = \frac{\{K - r\}[1 - \lambda]\alpha - K[1 - \alpha][\lambda - (1 - S^E)]}{K[1 - \alpha]\{1 - \lambda - \tau_E(1 - 2\lambda)\} - \{K - r\}[1 - \lambda]\{1 - \alpha - \tau_E(1 - 2\alpha)\}} \equiv \tilde{\theta}_{A|H}$$

**Risk profile** We now find the curve of indifference for an entrepreneur deciding between pledging asset  $A$  or  $H$ .<sup>23</sup> The goal is to focus attention on firm characteristics and to map these curves in risk-growth (or risk-sales) space and explore how changes in firm start-up size and relationship variables shift the pattern of collateral.<sup>24</sup> This is done with the following two propositions. We start

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<sup>23</sup>While we do not prove it here, a simple extension shows that it is never optimal to pledge both assets simultaneously.

<sup>24</sup>Recall that the firm variables are:  $\theta, K$  and  $r$ ; and that the relationship variables are:  $\lambda, \tau_E$  and  $S^E$ .

with the following proposition concerning which type of firm risk profile supports either asset choice.

**Proposition 5 (Risk Profile: Comparing Assets  $A$  and  $H$ ).** *Assume that  $K < r$ . Then equation (1.11) is monotone increasing in  $\theta$ , which implies that when  $\theta < \tilde{\theta}_{A|H}$  the entrepreneur prefers securing the project asset ( $A$ ) and when  $\theta > \tilde{\theta}_{A|H}$  the entrepreneur prefers securing the relationship asset ( $H$ ).*

**Proof** See Appendix B. ■

Recall that  $\theta$  is the probability of a high date 1 return (probability that  $\tilde{R}_1$  realizes  $x$ ). High risk firms prefer using inside assets because, with default more likely, the optimal contract places greater concern on possible liquidation than default deterrence, and when asset  $H$  is secure the entrepreneur stands to lose both house and  $r$  (recall that he must ‘bribe’ the guarantor with the full date 2 return in order to use asset  $H$  as security), while when only asset  $A$  is secured he at least gets to keep his share of the house (though he still loses  $r$ ). Conditional on default having occurred, from the entrepreneur’s perspective securing the house is payoff dominated by securing the business asset. This result is independent of the size of the entrepreneur’s asset share and of the relative inefficiencies of inside versus outside asset loss ( $\alpha$  versus  $\lambda$ ).

**Slope of  $\tilde{\theta}_{A|H}$  and comparative statics** The slope of the indifference curve depends on the relative magnitudes of the ex post inefficiencies of the two assets. The following proposition is easily proved.

**Proposition 6.** *Assume the same assumption as in proposition 5 above and also assume that  $\frac{1}{2} > \alpha > \lambda > 0$ . Then*

(i)  $\tilde{\theta}_{A|H}$  is monotone increasing in  $r$ , and

(ii)  $\tilde{\theta}_{A|H}$  is monotone decreasing in  $S^E$  and monotone increasing in  $\lambda$  and  $\tau_E$ .

**Proof** See Appendix B. ■

Part (i) of the proposition states that the entrepreneur's indifference curve is upward-sloping in  $\theta - r$  space. That the indifference curve is positively sloped might seem somewhat counter-intuitive (since low default risk and high growth prospects could be regarded as complements), but the result is driven by the fact that the use of a relationship asset de facto makes contractible - between entrepreneur and guarantor - the otherwise non-contractible second period return. To see this, start at a point on the indifference curve and move horizontally to the right. The entrepreneur is in the region where he prefers to use the business asset. This is because, by moving to the right, we have increased the future return of the firm and thereby increased the 'cost' to the entrepreneur of using the relationship asset as security rather than the business asset (recall that the entrepreneur 'bribes' the guarantor by offering her all of the second period return). To convince the entrepreneur to use the relationship asset instead of the business asset given the new future growth profile of the firm, the entrepreneur must be reassured that the firm is less risky. This is because, by using the relationship asset, the entrepreneur runs the risk after necessary default of losing everything: both the second period return (promised to the guarantor) and his share of the house. Therefore, he will want added reassurance that necessary default is less likely. That is, we must move vertically upwards.

Part (ii) of the proposition states how the indifference curve is effected by changes in family characteristics. An increase in the entrepreneur's share of the relationship asset increases the use of the outside asset as security. This is yet another statement that outside party involvement raises the costs of using a per-

sonal asset. Greater ex post inefficiency of liquidating the relationship asset also leads to its increased use (its benefits as a pre-commitment device is enhanced). And finally, increasing the entrepreneur's ex post intra-familial bargaining power decreases the use of the relationship asset, since it increases his incentives to strategically default when that asset is used and so makes it relatively less attractive as a pre-commitment technology compared to the business asset.

The basic lesson is that using the personal asset exposes the entrepreneur to greater ex post loss, which makes it an unattractive security option for high risk firms (where the contractual design problem focuses more on the possibility of ex post loss) but on the other hand does make it a more attractive security option for low risk firms (where the contractual design problem focuses more on the ex ante need to provide the entrepreneur with disincentives to strategically default). This greater exposure to ex post loss for the entrepreneur of using the relationship asset lies in the fact that the entrepreneur is forced to promise to hand over all his future return from the business to the guarantor to convince her to co-sign the security, and that this promise in turn is binding vis-a-vis entrepreneur and guarantor (as it could not be vis-a-vis the bank) because of the pre-existing relational contracting dynamic (not explicitly modelled) between them. The need for the entrepreneur to hand over all the second period return if he wants to use the relationship asset instead of the business asset is an artifact of the binary return space and the finiteness of the modelling environment - a model that allowed convexities in returns and payments, or which explicitly dynamised the relationship between entrepreneur and guarantor (so that the second period return from the business would be shared between them), would produce a less drastic-seeming outcome, although the basic intuition and trade-offs of the model would remain the same.

### 1.4.3 A brief numerical example

Figure C.3 in Appendix C shows in  $\theta-r$  space the  $\tilde{\theta}_{A|H}$  curve using the parameter choices shown in Table 1.2. The figure depicts a loan of  $K = 50,000$  when the value of the relationship asset (a home) is  $z = 100,000$ . Note that, because we confine ourselves to firms for which  $K < r$ , the vertical axis begins at  $r = 50,000$ . The values in the table are chosen for expository purposes only, and they are consistent with the assumptions in the propositions. That the chosen value of  $K$  is a reasonable ‘ballpark’ choice for expository purposes can be seen from the table depicted in Figure C.4 in Appendix C. That table is taken from [Hurst and Lusardi \(2004\)](#) and depicts average start up amounts across industry using the 1987 NSSBF. It can be seen from that table that median startup costs range from about \$9,500 for the construction industry to about \$55,000 for the retail trade, with some firms in some industries needing as much as \$200,000.

The slope of  $\tilde{\theta}_{A|H}$  is positive as required by proposition 6. The regions of asset choice, derived from proposition 5, are also shown in the diagram, namely, that firms lying to the right of the indifference curve use the business asset, while firms lying to the left use the relationship asset. It can be seen that likelihood of personal asset use is decreasing in both  $r$  and  $\theta$ .

In terms of comparative statics on  $K$ , it is easily shown that increasing  $K$  both shifts the curve to the right and makes it flatter. Thus the model predicts that, in industries with greater startup costs, we would expect to see a greater proportion of firms using personal assets, concentrated at the low default risk end of the firm distribution (alternatively, concentrated among relatively older firms).

Comparative statics on the relationship parameters (part (ii) of proposition 6) can be represented by shifts in and out of the  $\tilde{\theta}_{A|H}$  curve (though the slope

Parameter	Value
$\theta, K, r$	
$z$	100,000
$\alpha$	0.2
$\lambda$	0.1
$\tau^E$	0.6
$S^E$	0.5

Table 1.2: **List of parameter values used in numerical example**

of the curve also changes for some of them). As an example, an increase in the bargaining power of guarantors over time would be represented by an outward shift of the  $\tilde{\theta}_{A|H}$  curve, leading to the prediction that over the last few decades, with the rise of feminism and the increasing outside earning capacity of formerly stay-at-home spouses, we should expect to see a greater use of the family home as collateral for outside businesses.

## 1.5 Human Capital and Startups

The previous analysis assumed the entrepreneur had both types of asset at his disposal, and merely needed to decide which one to use. But many of the high-growth firms of the 1990s were characterized by low physical and high human capital. In such a firm, there are few business assets to secure and of course the entrepreneur's human capital is incapable of acting as a commitment technology. In that case, the only option may be to secure the outside asset. This case of the pure use of a personal asset as collateral leads to the following proposition.

**Proposition 7 (Risk Profile for Pure Personal Asset Case).**

Define  $\theta_X(\lambda, \tau_E, S^E) \equiv \frac{1 - S^E - \lambda}{1 - \lambda - \tau_E(1 - 2\lambda)}$ ,  $\theta^* \equiv \frac{K - r}{x}$  and  $\Delta \equiv \theta_X - \theta^*$

(i)  $K > r$  ( $K < r$ ) implies  $\theta > \theta_X$  ( $\theta < \theta_X$ ),

(ii) Define  $\Lambda \subset [0, 1]$  as the range of projects whose risk profile  $\theta$  is such that the projects are credit rationed.

**For the case of  $K > r$ :** If  $\Delta > 0$  then  $\Lambda = [\theta^*, \theta_X]$  (if  $\Delta \leq 0$  then  $\Lambda$  is empty)

**For the case of  $K < r$ :**  $\Lambda = [\bar{\theta}, 1]$

**Inside Asset Only:** When only asset  $A$  is available for use as a security,  $\Lambda$  is always empty.

(iii) For each of the arguments of  $\theta_X$  we have that  $\theta_X$  is decreasing in  $S^E$  and  $\lambda$ , and decreasing (increasing) in  $\tau_E$  when  $\lambda < \frac{1}{2}$  ( $\lambda > \frac{1}{2}$ ).

**Proof** See Appendix B. ■

$\theta_X$  is the denominator of  $\beta_0$  set to zero, while  $\theta^*$  is a rearrangement of the ex ante condition on project viability, namely  $\theta x + r \geq K$ . Note that the first is a function only of relationship variables while the second is a function only of firm variables. The proof of parts (i) and (ii) rely on the fact that  $\beta_0$  must be positive, as well as on a comparison of  $\theta_X$  and  $\theta^*$ . The proof of part (iii) involves finding the signs of the respective derivatives. The results depend sensitively on whether  $r$  is greater than or less than  $K$ . The case of  $K < r$  is likely to represent the sort of IT sector startups which featured prominently in the media during the 1990s, while examples of low  $r$  projects might be loan refinancing or extensions to extant lines of credit. Figure C.5 in Appendix C shows the proposition in  $\theta - r$

space. Note that the third claim of part (ii) of the proposition states that for firms which can use an inside asset (and only use an inside asset), there is no equivalent ‘credit rationed’ region as there is for firms confined solely to the use of an outside asset - yet another instance of the greater costliness of recourse to outsider-involvement funding.

Within the four quadrants of risk-growth space depicted in Figure C.5, firms located in the upper right hand quadrant are the least likely to have problems accessing financial backing (from any source). Although such firms are (seemingly anomalously) ‘credit rationed’ in this model, we know from empirical studies that the rise of venture capital markets and angel financing has been in precisely with respect to this quadrant (cream skimming). A better interpretation therefore is that the model does not expect such firms to need or use bank credit. Indeed, it is known from surveys of IT startups in the Silicon Valley in the 1990s that a major reason given by those who eschewed venture capital financing in favor of the stress and risk of mortgaging the house was that they wished to maintain control over the business, suggestive perhaps of the fact that for firms at least near the  $\theta_X$  boundary of the  $K < r$  half-space, personal collateral bank financing might be regarded as a substitute to VC financing.

For the polar opposite case, firms or projects in the lower left hand quadrant (high risk, low growth) are the least likely to be able to find funding (from any source) and such firms continue to be credit rationed even in the presence of a securitizable outside asset. The incidence of the benefits of personal guarantees falls to those firms in the upper left and lower right quadrants. Changes in the underlying parameters of the model will change the relative sizes of these two quadrants. In particular, the  $\theta^*$  line depends only on firm characteristics while the  $\theta_X$  depends only on family relationship characteristics.



One comparative static worth noting involves simultaneous changes in entrepreneurial bargaining power and relationship closeness. Recall that declining  $\lambda$  means increasing relationship closeness. It is not unreasonable to suppose that greater emotional bonds can make us more susceptible to moral suasion or emotional pressure to act contrary to our independently considered interests. It is easily verified that  $\lim_{\lambda \rightarrow 0^+, \tau_E \rightarrow 1^-} \theta_X \rightarrow 1$  (since  $\theta_X \in [0, 1]$ ). This result, combined with part (i) of proposition 7, informs us that for projects with low growth potential none will be financed (that is, regardless of risk profile) while for projects with high growth potential all will be. Whether coercion increases close to this limit is not obvious merely by inspection of part (iii) of proposition 7 since the effects of a decrease in  $\lambda$  and an increase in  $\tau_E$  work in opposite directions. However, taking the cross-partial we get

$$\frac{\partial^2 \theta_X}{\partial \tau_E \partial \lambda} = \frac{\partial^2 \theta_X}{\partial \lambda \partial \tau_E} = \frac{[1 - \lambda + \tau_E(1 - 2\lambda)] - 2S^E[\lambda + \tau_E - 2\tau_E\lambda]}{-[1 - \lambda - \tau_E(1 - 2\lambda)]^3}$$

and further taking the limit as  $\lambda \rightarrow 0^+$  and  $\tau_E \rightarrow 1^-$  we see that the effect of changing  $\lambda$  dominates ( $\lim_{\lambda \rightarrow 0^+, \tau_E \rightarrow 1^-} = -\infty$ ), so that increasing ‘closeness’ (decreasing  $\lambda$ ) means increasing  $\theta_X$ . Hence if we define ‘coercion’ as the *simultaneous* increase in  $\tau_E$  and decrease in  $\lambda$ , then we have the following corollary, which summarizes the above statements.

**Corollary 2.** *As ‘coercion’ increases, fewer projects of low growth potential will be funded and more projects of higher growth potential will be funded. In the limit (as  $\lambda \rightarrow 0^+$  and  $\tau_E \rightarrow 1^-$ ) only high growth projects will be funded.*

Close relationships between guarantor and guarantee should only be used to fund high-growth firms (startups). This result is mirrored by the comparative static on  $S^E$ . It is worth noting in this context that the fact situation underlying

the House of Lords decision of *Barclays Bank*, discussed in section 1.3, involved refinancing an existing loan facility rather than an investment in a project *de novo*.

## 1.6 Conclusion and directions for future research

This paper analyzes collateralized asset choice in an optimal incomplete financial contracting environment. The basic point is that personal assets are distinguished from business assets in that they often involve third parties and the relational dynamics that attend them. This outside-party aspect of personal collateral raises the cost of its use vis-a-viz business collateral and so we would expect, *ceteris paribus*, its use to be rarer and only by firms where the benefits of its use make its costs tolerable (firms with low default risk and stable growth prospects). The optimal collateralized loan contract trades off the benefits of deterring defaults against the desire of ensuring that unavoidable defaults are not too costly. The increased cost of using assets unavoidably associated with outside parties manifests itself in higher foreclosure probabilities because it unilaterally weakens the disincentive effect of using the outside asset. This does not mean that using the outside asset is never optimal. In fact, the model predicts that the pattern of asset use in collateralized loans is characterized by personal asset use for low-risk, low-growth firms. For those firms characterized by high human and low physical capital, so that there are likely insufficiently valuable inside assets to use as security, the model predicts that personal collateral (and closer relationships) are valuable for the funding of high risk startups such as was the case in the IT sector during the 1990s.

An obvious extension to this paper is to take the results and corollative

predictions of section 1.4 to small business data, especially to the NSSBF (briefly discussed in section 1.4.1), but also to the Survey of Consumer Finances (SCF) - like the NSSBF, another recurrent survey conducted by the Board of Governors of the Federal Reserve. The complete absence of data on pure third party guarantees necessitates recourse to the establishment of an original dataset, sourced predominantly by banks. Both of these research agendas are currently being undertaken by the author.

Further extensions to the model and the research agenda it embodies are foreseeable both theoretically and empirically. An obvious extension is to expand the shorthand assumption used in the paper regarding the contractibility of the date 2 return between the entrepreneur and guarantor by actually modelling the relational contract (repeated game) between them. While not salient in this paper, proposed future empirical work on the influence on small business secured credit of the heterogeneity of family law property regimes across the states of America would necessitate the more institutionally nuanced perspective an explicit, embedded modelling of the dynamic relationship between husbands and wives would provide.

An institution not considered in this paper is (personal) bankruptcy. This is because secured loans receive priority in any bankruptcy proceeding, and they also trump the 'homestead' exemptions which states otherwise afford personally bankrupt citizens. Consequently, the terms (collateralized) 'guarantee' and 'collateral' have been used interchangeably in this paper when in a different context the maintenance of the distinction for bankruptcy proceedings entailed in these two forms of committed loan finance would be crucial.

A final possible direction for future research involves very small businesses,

in which the distinction between family and business is almost completely non-existent. The model in this paper could accommodate an exploration of the added intricacies involved in home-office business environments via the introduction of complementarity between the business asset and the relationship asset. Such a modification would have implications especially for the issue of the influence of changes in the business cycle on the pattern of collateralized debt and the credit crunch which the smallest of small businesses are disproportionately subject to during downturns.

## CHAPTER 2

# Guarantees versus Collateral in Small Business Debt Financing

### 2.1 Introduction

The United States is unusual in having pro-debtor bankruptcy laws and, alone among the industrialized countries, it has a high and rapidly rising bankruptcy filing rate. The total number of bankruptcy filings has risen from under 300,000 per year in 1984 to 1.1 million in 1996 and 1.4 million in 1998.<sup>1</sup> The common knowledge in the finance and banking communities of this large and increasing number of personal and small business bankruptcy filings has led researchers to explore how bankruptcy affects consumers' and small business entrepreneurs' ex ante access to credit. When debtors in the United States file for personal bankruptcy, many types of debts are discharged, causing losses for creditors. Under the current law (soon to be changed) debtors who file under Chapter 7 of the U.S. Bankruptcy Code are absolved from the obligation to use future income to repay their debts and are only obliged to use current wealth to repay debt to the extent that that wealth exceeds predetermined, statutory exemption levels. Exemption levels in bankruptcy are set by the state in which the debtor lives

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<sup>1</sup>Bankruptcy filing data are easily obtained from the *Statistical Abstract of the United States*, various editions.

and they vary widely. Initial research has shown that these exogenous exemption levels affect the terms on which loans are made across states.<sup>2</sup>

This paper explains the determinants of committed debt in small business debt financing. Committed debt is a loan that has been either guaranteed and/or collateralized. Data show (the National Survey of Small Business Finances (hereafter ‘NSSBF’)) that the majority of small business debt is bank-sourced and that the majority of that debt is committed. In particular, the importance of commitments to making loan funds available to small businesses has been shown in [Avery et al. \(1998\)](#) using 1987 and 1993 NSSBF survey data on small business financing: debts without the backing of either guarantees and/or collateral never comprise more than 15% of all small business loans.<sup>3</sup> Committed debt serves the purpose of ensuring that a business owner’s personal assets are available to the creditor in the event of bankruptcy even for those small businesses set up as a corporation and so putatively having the protection of the corporate veil. But while both types of committed debt (guarantees or collateral) serve the purpose of removing limited liability for creditors, they are of differing status ex post in the event of bankruptcy - in particular, collateralized debt is not subject to state-based homestead exemptions whereas guaranteed debt is. In spite of this clear advantage, a downside of collateralizing loans is that they are a higher transaction cost form of loan (assets need to be valued and so on) than merely getting an owner to sign a guarantee in a bank branch.

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<sup>2</sup>Examples of articles which have shown such an affect empirically are [Berkowitz and White \(2004\)](#), [Lin and White \(2001\)](#) and [Berkowitz and Hynes \(1999\)](#).

<sup>3</sup>The NSSBF dataset is discussed in more detail in subsections [2.3.1](#) and [2.4.1](#).

**Description of model** The paper utilizes an ‘incomplete financial contracting’ model in the tradition of [Aghion and Bolton \(1992\)](#) to fully characterize the determinants of committed loan in small business debt finance. In the model a wealth constrained entrepreneur seeks funds from a bank, but cannot pre-committ to repay the bank when the firm begins showing profit (return streams are non-contractible). Thus, the entrepreneur and bank must find another way to pre-committ, without which the entrepreneur will be credit-rationed. That other way involves agreeing ex ante over the distribution of ‘control rights’ over the business assets, or, in the framework of bankruptcy, over when the creditor has the right to seize and sell business (or personal) assets. Since those assets are required by the entrepreneur in order to earn continuing business income, the possibility of such a change in ownership or control of those assets acts as leverage between the lender and lendee ensuring that the latter repays the loan. Such leverage is required because of the inability of the parties enforceably to contract on the business’s return stream, opening up the possibility of future strategic default and contract renegotiation. Without some mechanism of pre-commitment, the entrepreneur would be credit-constrained regardless of the viability of his project. There are two assets available to a creditor in bankruptcy in this model: a business asset and a personal asset. Uncommitted loans are modelled using only a business asset, while committed loans are modelled using both. The personal asset can also be secured.

**Description of results** The paper contains two types of result. First I show that the choice of committed loans depends on firm attributes such as default risk and size (sales), and in particular I show the following fundamental result: the likelihood of a loan being committed is increasing in firm default risk and decreing

in firm size. This result is consistent with the empirically established fact that, compared to the universe of all small business loans, the subset of collateralized loans are on average higher risk. The worst firms offer to commit loans made to them, without which commitment they would likely be credit rationed.

The second result offers an explanation for a *curiosum* in the data on small business financing first noted in Berkowitz and White (2004). In that article it was found that the interest rates on loans facing small businesses did not rise monotonically in homestead exemption level. The data used was the 1993 NSSBF. The authors noted a similar (and related) non-monotonicity in the variation with homestead exemption of the likelihood of a firm being credit-rationed and loan size. The authors note that they have no explanation for this surprising empirical finding. In section 2.4 we show that non-monotonicities of the type found by Berkowitz and White (2004) in the NSSBF data arise automatically from the proposed model, and that the model itself is a relatively simple formulation of the homestead exemption/personal guarantee environment. The non-monotonicity is due to the fact that, the higher the homestead exemption level, the more likely that average house prices fall under that level and so banks and business owners switch to (the higher transaction cost) collateralized form of loans rather than just guaranteeing loans.

**Outline of paper** This paper proceeds as follows. Section 2.2 describes and solves the model. Section 2.3 compares the three different types of loan contract (considered in this paper) in firm-characteristic space. Section 2.4 examines how loan interest rates (proxied by firm default risk) vary with variations in the homestead exemption level. Section 2.5 concludes.



## 2.2 Model

**Set-up** There are two agents, an entrepreneur seeking a loan and a bank considering giving one. At date 0 the entrepreneur solves the following linear program (call it  $(\star^{(i)})$ ), choosing over the contract  $\Gamma^i = \{P_0, P_x, \beta_0, \beta_x\}$  to maximize his expected payoff:<sup>4</sup>

$$\begin{aligned} & \theta[x - P_x + (1 - \beta_x)r + (1 - \beta_x\kappa_z^i)z + \beta_x g_E^i] \\ & + (1 - \theta)\{-P_0 + (1 - \beta_0)r + (1 - \beta_0\kappa_z^i)z + \kappa_z^i\beta_0[\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)]\} \end{aligned} \quad (2.1)$$

subject to the individual rationality constraint of the bank

$$\theta[P_x + \beta_x g_B^i] + (1 - \theta)[P_0 + \beta_0 L^i] \geq K \quad (2.2)$$

and, in order to ensure that the entrepreneur does not strategically default when  $R_1 = x$ , subject also to the following ‘renegotiation constraint’

$$\begin{aligned} & x - P_x + (1 - \beta_x)r + (1 - \beta_x\kappa_z^i)z + \beta_x g_E^i \\ & \geq x - P_0 + (1 - \beta_0)r + (1 - \beta_0\kappa_z^i)z + \beta_0 g_E^i \end{aligned} \quad (2.3)$$

and subject to the following ‘limited liability’ constraint for the entrepreneur owing to the assumption of ex ante zero liquid wealth

$$P_0 \leq 0 \text{ and } P_x \leq x$$

and finally subject to feasibility constraints on the foreclosure probabilities

$$0 \leq \beta_0, \beta_x \leq 1$$

---

<sup>4</sup>Note that each of these contractual terms should also have an  $i$  superscript, but to avoid notational clutter we omit them.

and where  $\eta \in [0, 1]$  and also  $\phi$  is an indicator function with

$$\phi = \begin{cases} 1 & \text{when unsecured} \\ 0 & \text{when secured} \end{cases}$$

This model contains two in one, depending on whether  $i = A$  or  $AH$ , where the letters stand for firm asset, or both firm asset and private asset (like a home) combined. The difference between these models depends on the different values of the indicator function  $\kappa_z^i$  for each value of  $i$ . In particular we have

$$\kappa_z^i = \begin{cases} 1 & \text{when } i = AH \\ 0 & \text{when } i = A \end{cases}$$

**Interpretation** A firm protected by the corporate veil would be represented by the model  $i = A$ . That is, in bankruptcy the bank only has access to the firm assets and not the entrepreneur's home. Requiring an entrepreneur to guarantee his firm's debts is represented by the model  $i = AH$ . That is, in bankruptcy the bank has access to both firm assets and the entrepreneur's home. It is clear that for most small businesses, regardless of corporate form, the correct model is  $i = AH$ . Thus we have:

When a firm is a corporation, limited liability implies that the owner is not legally responsible for the firm's debts. However, lenders to small corporations often require that the owner guarantee the loan and may also require that the owner give the lender a second mortgage on her house. This wipes out the owner's limited liability for purposes of the particular loan and makes small corporate firms in corporate/noncorporate hybrids.<sup>5</sup>

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<sup>5</sup>See [Berkowitz and White \(2004\)](#) at page 81.

Even when the business loan is personally guaranteed, a distinction exists between secured and unsecured loans. Secured loans trump homestead exemption laws whereas unsecured loans do not. For this reason, in bankruptcy, secured loans are invariably completely repaid whereas for unsecured loans this is often not the case.<sup>6</sup> The model when the loan or guarantee is secured has  $\phi = 0$ . The homestead exemption is then irrelevant to the extent of the security, represented by  $\eta$ .

**Timing and explanation of model** The timing of the two-period model is set out in figure 2.1.

The intuition of the model is as follows. A financially constrained entrepreneur seeks funds from an investor in order to exploit an investment opportunity with upfront cost  $K$ . The funds are used to buy a project asset which in turn generates a return stream. The return is binary stochastic in the first period ( $\tilde{R}_1$ ) and determinate in the second period (with value  $r$ ). The first period binary return gives  $x > 0$  with probability  $\theta$  and zero with probability  $1 - \theta$ . The project is assumed ex ante viable so that  $\theta x + r \geq K$ . The model involves the assumption that, at the time the loan contract is written, the parties to the contract are not able enforceably to condition on these future first period returns, so that the contract instead must specify who gets control in the first period of the project asset in the event the entrepreneur defaults in that period. Because the ex ante agreed loan repayment ( $P_j$ , where  $j = 0, x$ ) cannot be enforceably conditioned on the first period return stream (meaning that the contractually specified repayment amount can be renegotiated), default can occur strategically

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<sup>6</sup>The general legal principle (there is some fligree) is that secured transactions trump the state and federal homestead exemptions (see generally Baird (2000)).

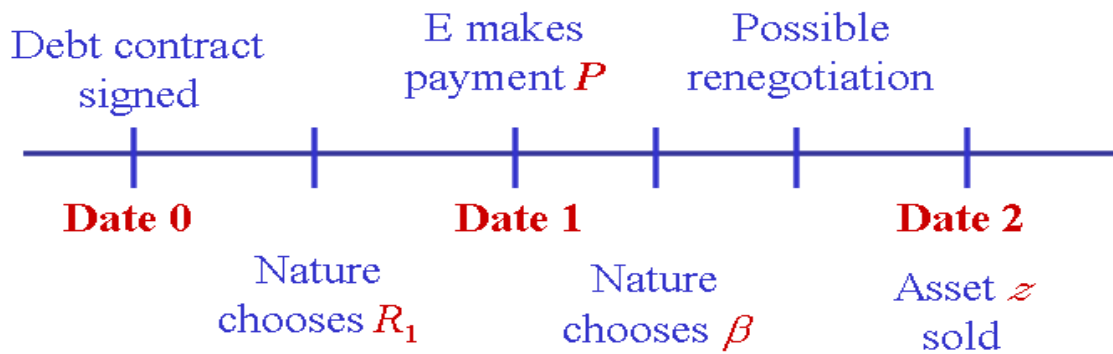


Figure 2.1: Timeline of Model

(that is, when  $\tilde{R}_1 = x$ ) and not just because the return was zero. To minimize the incentives for such strategic default the investor must liquidate part of the project asset in the event of non-strategic default, even though such liquidation is ex post inefficient. The reason such liquidation has the right incentive effects is because the second period return  $r$  (which accrues only to the entrepreneur) depends on the entrepreneur controlling the project asset. It is easily seen that in a one period model the entrepreneur would always default. The loan funds would therefore never be forwarded by the bank in that case (in spite of the project being ex ante viable). Thus it is the possibility of the bank's being able to deprive the entrepreneur of his second period return which gives the asset control decision

an important ‘leverage’ effect between entrepreneur and bank, enabling otherwise credit rationed firms to receive loans. In this paper there are two assets, a project asset and an outside asset completely independent of the project and its returns. Both assets (separately or in combination) can act as security for the loan.

**Ex post renegotiation** The contract  $\{P_0, P_x, \beta_0, \beta_x\}$  involves the entrepreneur agreeing to pay  $P_0$  when the first period return is zero and  $P_x$  when the first period return is  $x$ . In the event of default, the contract specifies that the bank will have the right to liquidate the project and/or the private asset (depending on the model) with probability  $\beta_0$  if the entrepreneur in fact paid  $P_0$  and with probability  $\beta_x$  if the entrepreneur in fact paid  $P_x$ . Sale of the asset(s) is however not automatic. This is because the asset(s) is/are worth more in the hands of the entrepreneur than what the bank can get for them on the market. Thus there exists an ex post surplus to be renegotiated over. If the bank sells the project asset  $A$  then it receives only a fraction of what the entrepreneur would have earned with it, that is, the bank gets  $\alpha r$ , where  $\alpha \in (0, 1)$ . And if the bank sells the private asset  $H$  then it receives less than what the asset, say a family home, is worth to its owner, that is, the bank gets  $\lambda z$ , where  $\lambda \in (0, 1)$  and  $z$  is the value of the private asset when kept in the entrepreneur’s hands.

The variable  $\gamma \in [0, \infty)$  represents the level of the homestead exemption: even when the bank liquidates the relationship asset  $H$ , the amount  $\gamma$  remains with the entrepreneur by law (provided the loan is unsecured). This is true provided that the value of the house is greater than the exogenously set homestead level; otherwise, the entrepreneur receives the full amount of the house  $z$ . The ‘liquidation value’ ( $L^i$ ) can then be defined as

$$L^i = [\alpha r + \kappa_z^i [\phi \max(0, \lambda \Delta) + (1 - \phi) z \lambda \eta]]$$

where  $\Delta \equiv z - \gamma$ .

Renegotiation - which does not occur in equilibrium, because it is fully anticipated - is in the form of generalized Nash bargaining, where each agent is exogenously endowed with bargaining strength  $\tau_i$  with  $i = E, B$  and where  $\sum \tau_i = 1$ . The amount which each agent receives after renegotiation is  $g_E^i$  or  $g_B^i$  respectively, which is simply the proportional share - determined by bargaining power - of the ex post surplus salvaged by the renegotiation. For the entrepreneur this amount is

$$g_E^i = \tau_E \{r(1 - \alpha) + \kappa_z^i [\phi(z - \lambda\Delta)1_{\Delta>0} + z(1 - \phi)(1 - \lambda\eta)]\}$$

It is assumed that the entrepreneur is not permitted to bargain away his home-stead exemption-protected private asset residual in any ex post renegotiations, an assumption in accordance with existing bankruptcy law.

**Solving the model** This is a model within the ‘incomplete financial contracting’ approach of [Aghion and Bolton \(1992\)](#), [Hart and Moore \(1994\)](#) and [Hart and Moore \(1998\)](#).<sup>7</sup> For a summary of the standard framework and related literature see chapter 5 of [Hart \(1995\)](#). The details of the solution of this contractual problem can be found in [Scheelings \(2004\)](#) (which is just chapter 1 above) and will not be repeated here. Obviously the first best could be achieved if comprehensive contracts could be written ex ante. But the twin effects of contractual incompleteness and a wealth constrained entrepreneur mean that some inefficiency is unavoidable. The following proposition states just that.

**Proposition 8 (Contractual Inefficiency).**

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<sup>7</sup>It is closely based on [Bolton and Scharfstein \(1996\)](#).

(i) In the optimal contract, the efficiency loss due to contractual incompleteness is

$$EL^i \equiv (1 - \theta)\beta_0\{r + \kappa_z^i z - \kappa_z^i[\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)] - L^i\} \quad (2.4)$$

(ii) while the optimal foreclosure probability is

$$\beta_0^i \equiv \frac{K}{\theta[r + \kappa_z^i z - g_E^i] + (1 - \theta)L^i} \quad (2.5)$$

(which will be a solution to  $(\star^i)$  provided the RHS is not greater than one).

**Proof** It is easily shown that  $\beta_x = P_0 = 0$  and that the first two constraints bind. Rewrite the program with these changes and then substitute the bank's IR constraint into both the objective function and the entrepreneur's IC constraint. This gives two equations in the one remaining choice variable, namely  $\beta_0$ . The new objective function becomes

$$\theta x - K + (r + z) - (1 - \theta)\beta_0\{r + \kappa_z^i z - \kappa_z^i[\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)] - L^i\}$$

so that  $EL^i$  in equation (2.4) is simply *defined* as the last term of this formula for social welfare in this model, while the new IC constraint for the entrepreneur is simply equation (2.5). ■

Equation (2.4) shows that the efficiency loss due to contractual incompleteness is the expected loss of surplus (the term in the braces) when the date 1 return is zero (with probability  $1 - \theta$ ) and the bank gets the right to foreclose on the secured asset(s) (with probability  $\beta_0^i$ ). Equation (2.5) depicts the determinants of the ex ante (contractually) chosen probability of ex post foreclosure. Because  $\beta_0^i$  is embedded in equation (2.4) we can see that the ex ante efficiency loss depends on the ex post inefficiency both directly (the term in square brackets) and indirectly (through  $\beta_0^i$ ). In fact, these effects work in opposite directions.

## 2.3 Comparison of debt contracts

### 2.3.1 Motivation

Data on small business financing has traditionally been sparse. But beginning in 1987, the Federal Reserve Board, in association with the Office of Small Business Administration, has conducted five-yearly surveys of small businesses (defined as 500 employees or less) seeking especially information on owner characteristics and funding sources. Called the National Survey of Small Business Finances (NSSBF), these sequential cross-sectional samples (1987, 1993, 1998 and 2003) now comprise the main source of information on small business financial structure and during the 1990s have given rise to an empirical literature examining its findings.<sup>8</sup>

The incidence of committed debt by small businesses was shown in Figure C.2 in appendix C. Recall that it depicts a table taken from Avery et al. (1998). The data in the table is based on the 1987 and 1993 versions of the NSSBF survey. The table shows, conditional on a firm having a loan, how much of those loans are backed by owner commitments. Committed debt is defined as loans backed either by guarantees or pledged assets. The table shows the four possible categories of commitment, as well as (when assets are pledged) a breakdown of which asset(s) was used to back the pledge (the business asset, personal asset, or both). It will be noted from the table that the overwhelming majority of bank debt is collateralized.

Note that the model in this paper does not capture every aspect of this table. In particular, there is no scope for modelling a security on the business asset(s)

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<sup>8</sup>The data and supporting documents can be found at:  
<http://www.federalreserve.gov/pubs/oss/oss3/nssbftoc.htm>.



(the most common type of security) using the model of section 2.2, nor is the whole of the category ‘Unguaranteed and secured by:’ captured by the model of section 2.2.

### 2.3.2 Analysis

In the NSSBF, both corporate and noncorporate firms are sampled. When firms are set up as a corporation then, technically, the liability of the firm’s debt lies with the firm and not personally with it’s owners or equity holders: lenders are legally not able to go after the personal assets of the business’ owners even if the assets in the firm are insufficient to recover outstanding loans. Such a situation is represented by the model with  $i = A$ . That is why the majority of small business owners are required (or volunteer) to commit personal assets to any loans taken out by the firm (represented by the model with  $i = AH$ ). This section outlines the conditions under which an entrepreneur is indifferent between committing and not committing the debt. Because committed debt can either be guaranteed or secured, there are in fact two indifference curves.

#### 2.3.2.1 Setup

From equations (2.4) and (2.5) we can write the parametric forms for  $EL^{AH}$  and  $EL^A$  as:

$$EL^{AH} = \frac{(1 - \theta)K\{r + z - [\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)] - L^{AH}\}}{\theta[r + z - g_E^{AH}] + (1 - \theta)L^{AH}}$$

and

$$EL^A \equiv \frac{(1 - \theta)K\{r - L^A\}}{\theta[r - g_E^A] + (1 - \theta)L^A}$$

An entrepreneur is indifferent between a committed and uncommitted transaction when  $EL^{AH} - EL^A = 0$ . In parametric form, this gives

$$\frac{(1 - \theta)K\{r + z - [\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)] - L^{AH}\}}{\theta[r + z - g_E^{AH}] + (1 - \theta)L^{AH}} - \frac{(1 - \theta)K\{r - L^A\}}{\theta[r - g_E^A] + (1 - \theta)L^A} = 0$$

which implies that

$$(1 - \theta)K\{r + z - [\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)] - L^{AH}\}\{\theta[r - g_E^A] + (1 - \theta)L^A\} \\ - \{(1 - \theta)K\{r - L^A\}\}\{\theta[r + z - g_E^{AH}] + (1 - \theta)L^{AH}\} = 0$$

Rearranging in terms of  $\theta$  gives us finally

$$\theta\{K(r - L^A)[r + z - g_E^{AH} - L^{AH}] + KX[r - g_E^A - L^A]\} - KXL^A + K(r - L^A)L^{AH} = 0 \quad (2.6)$$

or

$$\theta = \frac{XL^A - (r - L^A)L^{AH}}{(r - L^A)[r + z - g_E^{AH} - L^{AH}] + X[r - g_E^A - L^A]}$$

or

$$\theta = \frac{X\alpha - (1 - \alpha)L^{AH}}{(1 - \alpha)[r + z - g_E^{AH} - L^{AH}] + X[(1 - \alpha)(1 - \tau_E)]} \equiv \bar{\theta}_{AH|A} \quad (2.7)$$

where the substitutions  $g_E^A$  and  $L^A$  have been made and where for notational convenience we have set  $X \equiv r + z - [\phi \min(z, \gamma) + (1 - \phi)z(1 - \eta)] - L^{AH}$ . Note that the indifference curve is independent of the size of the loan,  $K$ . Equation (2.7) is really two equations in one, depending on whether a security is taken over the house or not. Explicitly we have

$$\bar{\theta}_{AH|A}^s = \frac{z\eta(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + z(1 + \eta)]} \quad (2.8)$$

and

$$\bar{\theta}_{AH|A}^u = \frac{(z - \gamma)(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + 2z - \gamma]} \quad (2.9)$$

where, in equation (2.9), we have only considered the case where  $\Delta = (z - \gamma) > 0$ , and the superscripts  $s$  and  $u$  represent the cases of when the house is secured or unsecured. The case where  $\Delta = (z - \gamma) < 0$  need not be considered because in that case,  $\bar{\theta}_{AH|A}^u = 0$  and we would never see a committed loan (because, if a security on the home is not an option, and the homestead exemption is higher than the private asset, the commitment is non-binding.)

Finally, for some of the results that follow, we require the following simple assumption.

**Assumption 1.**  $(\alpha - \lambda) > 0$  and  $z > 1$

We maintain this assumption throughout the rest of the paper. It implies that the loss of the house is more costly to the entrepreneur than the loss of the business assets.

### 2.3.2.2 Comparing uncommitted and committed debts in $\theta - r$ space

For the analysis of this subsection the relevant equations are (2.8) and (2.9). We wish to examine how the decisions to commit a loan, and whether to secure the commitment, depend on firm characteristics like default risk ( $\theta$ ) and firm size (or sales) ( $r$ ). We have the following proposition:

**Proposition 9.**

- (i) For both  $\bar{\theta}_{AH|A}^s$  and  $\bar{\theta}_{AH|A}^u$ , equation (2.6) is monotone increasing in  $\theta$ , which implies that when  $\theta < \bar{\theta}_{AH|A}^s$  ( $\bar{\theta}_{AH|A}^u$ ) the entrepreneur prefers to commit the loan, and when  $\theta > \bar{\theta}_{AH|A}^s$  ( $\bar{\theta}_{AH|A}^u$ ) the entrepreneur prefers not to commit the loan.

(ii) Both  $\bar{\theta}_{AH|A}^s$  and  $\bar{\theta}_{AH|A}^u$  are monotone decreasing in  $r$  in the relevant range.

### Proof

(i) This is obvious from the fact that the denominators of both curves are positive.

(ii) For  $\bar{\theta}_{AH|A}^s$  we have

$$\frac{\partial \bar{\theta}_{AH|A}^s}{\partial r} = \frac{-2z\eta(1-\alpha)^2(1-\tau_E)(\alpha-\lambda)}{\{(1-\alpha)(1-\tau_E)[2r(1-\alpha) + z(1+\eta)]\}^2}$$

which is negative by assumption 1. Similarly for  $\bar{\theta}_{AH|A}^u$ . ■

Note from part (i) of the proposition that the basic firm characteristics underlying the decision whether to commit the loan are independent of whether the commitment is secured or not, in particular, it is seen that in both cases high risk firms are more likely to have committed loans, a fact confirmed in empirical and survey studies (see for example Mann (1997b) and Mann (1997a)).<sup>9</sup> Both curves asymptote along the  $r$  axis in  $\theta - r$  space.

**Proposition 10.** Define  $\hat{\gamma} \equiv z(1-\eta)$ .

(i) When  $\gamma \leq \hat{\gamma}$  we have  $\bar{\theta}_{AH|A}^s > \bar{\theta}_{AH|A}^u$  for all  $r$  (the two curves intersect outside the relevant range), and

(ii) when  $\gamma > \hat{\gamma}$  the two curves intersect within the relevant range at the point  $(\dot{\theta}, \dot{r})$  where

$$\dot{\theta} = \frac{z\eta(\alpha-\lambda)[z(1-\eta) + \gamma]}{(1-\alpha)(1-\tau_E)[z\gamma(2+\eta) + z^2\eta(1-\eta)]} > 0$$

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<sup>9</sup>Recall that low  $\theta$  represents high default risk.

and

$$\dot{r} = \frac{-z[z(1-\eta) - \gamma]}{2(1-\alpha)[z(1-\eta) + \gamma]} > 0$$

such that for  $r < \dot{r}$  we have  $\bar{\theta}_{AH|A}^s < \bar{\theta}_{AH|A}^u$  and for  $r > \dot{r}$  we have  $\bar{\theta}_{AH|A}^s > \bar{\theta}_{AH|A}^u$ .

**Proof** Equating  $\bar{\theta}_{AH|A}^s$  and  $\bar{\theta}_{AH|A}^u$  gives

$$\frac{z\eta(\alpha - \lambda)}{(1-\alpha)(1-\tau_E)[2r(1-\alpha) + z(1+\eta)]} = \frac{(z-\gamma)(\alpha - \lambda)}{(1-\alpha)(1-\tau_E)[2r(1-\alpha) + 2z - \gamma]} \quad (2.10)$$

which after manipulation gives finally

$$r = \frac{-z[z(1-\eta) - \gamma]}{2(1-\alpha)[z(1-\eta) + \gamma]} \equiv \dot{r} \quad (2.11)$$

And substituting this back into  $\bar{\theta}_{AH|A}^s$  gives (after manipulation)

$$\theta = \frac{z\eta(\alpha - \lambda)[z(1-\eta) + \gamma]}{(1-\alpha)(1-\tau_E)[z\gamma(2+\eta) + z^2\eta(1-\eta)]} \equiv \dot{\theta}$$

It is easily seen by examination of the denominator of equation (2.11) that  $\dot{r} \leq 0$  whenever  $\gamma \leq \hat{\gamma}$ , so that *one* curve must lie always above the other in the relevant range. Substituting  $\gamma = 0$  into equation (2.10) then gives

$$\frac{\eta}{2r(1-\alpha) + z(1+\eta)} - \frac{1}{2r(1-\alpha) + 2z} > 0$$

because, given assumption 1,  $1-\eta < z[1-\eta]$ . When  $\gamma > \hat{\gamma}$  the intersection is in the relevant range ( $\dot{r} > 0$ ) and then, substituting  $\dot{r} + \epsilon$  (for small  $\epsilon$ , where  $\epsilon > 0$ ) into equation (2.10) gives

$$\frac{z\eta(\alpha - \lambda)}{(1-\alpha)(1-\tau_E)[2(\dot{r} + \epsilon)(1-\alpha) + z(1+\eta)]} - \frac{(z-\gamma)(\alpha - \lambda)}{(1-\alpha)(1-\tau_E)[2(\dot{r} + \epsilon)(1-\alpha) + 2z - \gamma]}$$

or

$$\frac{(\alpha - \lambda)}{(1-\alpha)(1-\tau_E)} \left( \frac{z\eta}{[2\dot{r}(1-\alpha) + z(1+\eta) + 2\epsilon(1-\alpha)]} - \frac{(z-\gamma)}{[2\dot{r}(1-\alpha) + 2z - \gamma + 2\epsilon(1-\alpha)]} \right)$$

and taking the derivative with respect to  $\epsilon$  gives

$$= -2 \frac{(\alpha - \lambda)}{(1 - \tau_E)} \left( \frac{z\eta}{[2\dot{r}(1 - \alpha) + z(1 + \eta) + 2\epsilon(1 - \alpha)]} - \frac{(z - \gamma)}{[2\dot{r}(1 - \alpha) + 2z - \gamma + 2\epsilon(1 - \alpha)]} \right. \\ \left. - \frac{z\eta + (z - \gamma)}{\{[2\dot{r}(1 - \alpha) + z(1 + \eta) + 2\epsilon(1 - \alpha)][2\dot{r}(1 - \alpha) + 2z - \gamma + 2\epsilon(1 - \alpha)]\}} \right)$$

Now as  $\epsilon \rightarrow 0$  this derivative becomes

$$= -2 \frac{(\alpha - \lambda)}{(1 - \tau_E)} \left( \frac{\{z\eta[2\dot{r}(1 - \alpha) + 2z - \gamma] - (z - \gamma)[2\dot{r}(1 - \alpha) + z(1 + \eta)]\} - z\eta + (z - \gamma)}{\{[2\dot{r}(1 - \alpha) + z(1 + \eta)][2\dot{r}(1 - \alpha) + 2z - \gamma]\}} \right)$$

or

$$= -2 \frac{(\alpha - \lambda)}{(1 - \tau_E)} \left( \frac{z\eta}{2\dot{r}(1 - \alpha) + z(1 + \eta)} - \frac{(z - \gamma)}{2\dot{r}(1 - \alpha) + 2z - \gamma} \right. \\ \left. - \frac{z\eta - (z - \gamma)}{[2\dot{r}(1 - \alpha) + z(1 + \eta)][2\dot{r}(1 - \alpha) + 2z - \gamma]} \right)$$

or (because at the point  $r = \dot{r}$  the first two terms are equal)

$$= 2 \frac{(\alpha - \lambda)}{(1 - \tau_E)} \left( \frac{z\eta - (z - \gamma)}{[2\dot{r}(1 - \alpha) + z(1 + \eta)][2\dot{r}(1 - \alpha) + 2z - \gamma]} \right)$$

which is positive within the relevant range (ie, when  $z(1 - \eta) \leq \gamma \leq z$ ). So to the right of the intersection point  $(\dot{\theta}, \dot{r})$  we have shown  $\bar{\theta}_{AH|A}^s > \bar{\theta}_{AH|A}^u$ . And analogously to the left of the intersection point. ■

The two parts of the proposition are most intuitively discussed in the context of figure 2.2a and figure 2.2b. Note first that both curves have negative slope, so that low risk and large sales are complements as we would expect. Also that above each curve we are in the region where no commitment is given, whereas below each curve we are in the region where a commitment is given (where the type of the commitment depends on the curve). From figure 2.2 it can be seen that the two curves do not intersect, so that they divide  $\theta - r$  space into three regions. In region A the entrepreneur prefers not to commit the loan, in region B he can

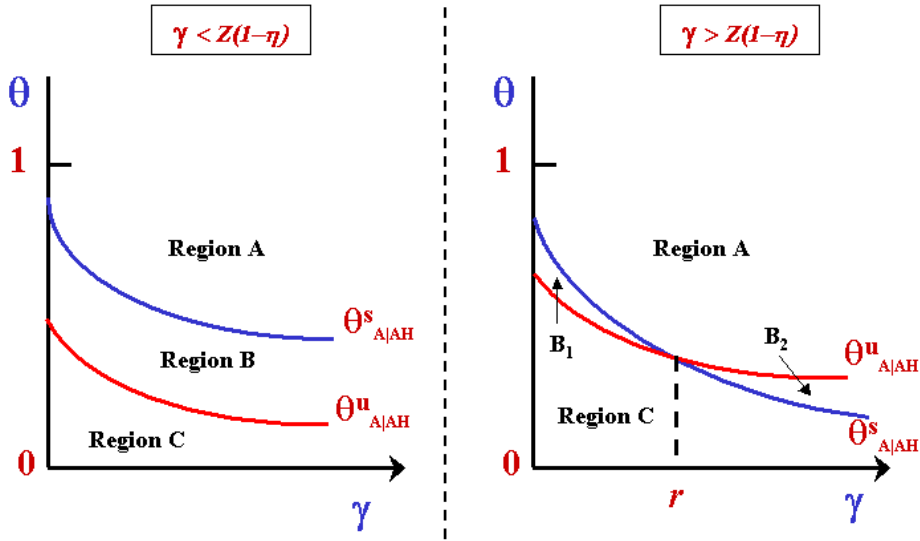


Figure 2.2: The indifference curves in  $\theta - r$  space for secured and unsecured commitments depending on whether a)  $\gamma \leq z(1 - \eta)$  or else b)  $\gamma > z(1 - \eta)$ .

choose between not committing the loan and committing it with a security (but not a guarantee), while in region C he always chooses to commit the loan, and can choose between guarantees or security. Alternatively, whenever  $\hat{\gamma} > z(1 - \eta)$  then the two curves intersect and  $\theta - r$  space is divided into four regions rather than three. With respect to large firms (right of  $\hat{r}$ ), then (depending on the riskiness of the firm) they choose (in increasing order of risk) between no commitments, an unsecured commitment or no commitment (region  $B_2$ ), and finally between unsecured and secured commitments. With respect to smaller firms (left of  $\hat{r}$ ), they choose between no commitments, secured commitments or no commitments (region  $B_1$ ), and finally between secured or unsecured commitments.

The next proposition deals with the comparative statics of the indifference curves in  $\theta - r$  space.

**Proposition 11.**

- (i)  $sign \left[ \frac{\partial \bar{\theta}_{AH|A}^s}{\partial z} \right] = -sign \left[ \frac{\partial \bar{\theta}_{AH|A}^u}{\partial z} \right] > 0$
- (ii)  $sign \left[ \frac{\partial \bar{\theta}_{AH|A}^s}{\partial \eta} \right] > 0$
- (iii)  $sign \left[ \frac{\partial \bar{\theta}_{AH|A}^s}{\partial \tau_E} \right] = sign \left[ \frac{\partial \bar{\theta}_{AH|A}^u}{\partial \tau_E} \right] < 0$
- (iv)  $sign \left[ \frac{\partial \bar{\theta}_{AH|A}^s}{\partial \lambda} \right] = sign \left[ \frac{\partial \bar{\theta}_{AH|A}^u}{\partial \lambda} \right] < 0$

**Proof** Each of the proofs rely on assumption 1 and the fact that, for the secured indifference curve,  $\gamma \leq z$ . We only prove the first part explicitly: the other parts are proved via a similar appeal to basic calculus.

- (i) The sign of the derivative is the same as the sign of the numerator, which is

$$sign \left[ \frac{\partial \bar{\theta}_{AH|A}^s}{\partial z} \right] = sign[\eta(\alpha - \lambda)(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha)]] > 0$$

and similarly for the second derivative

$$sign \left[ \frac{\partial \bar{\theta}_{AH|A}^u}{\partial z} \right] = sign\{-(\alpha - \lambda)(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + \gamma]\} < 0$$

The other parts of the proposition are proved in a like manner. ■

From part (i) of the proposition we can see that rising house prices will shift the  $\bar{\theta}_{AH|A}^s$  curve up and the  $\bar{\theta}_{AH|A}^u$  curve down, so that we would expect to see a rise in the proportion of loans backed by a security. From part (ii) we can see



that a rise in the amount of the asset that is secured shifts the  $\bar{\theta}_{AH|A}^s$  curve up, whereas increases in either the entrepreneur's bargaining power or the inefficiency of liquidating the personal asset shifts both curves down, so that in either case we would expect to see fewer committed loans.

Finally, comparing the case of  $\gamma > 0$  with the case of  $\gamma = 0$  we have:

**Proposition 12.** *Focusing solely on  $\bar{\theta}_{AH|A}^u$ , denote the case of  $\gamma = 0$  by  $\bar{\theta}_{\gamma=0}^u$ . Then we have  $\bar{\theta}_{AH|A}^u - \bar{\theta}_{\gamma=0}^u < 0$  for all  $r$  in the relevant range (ie, for  $\gamma \leq z$ ).*

**Proof** The sign of the difference between the two indifference curves is as follows:

$$\begin{aligned}\bar{\theta}_{AH|A}^u - \bar{\theta}_{\gamma=0}^u &= \frac{(z - \gamma)(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + 2z - \gamma]} - \frac{z(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + 2z]} \\ &= \frac{(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)} \left( \frac{(z - \gamma)}{[2r(1 - \alpha) + 2z - \gamma]} - \frac{z}{[2r(1 - \alpha) + 2z]} \right) \\ &= \frac{(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)} \left( \frac{(z - \gamma)[2r(1 - \alpha) + 2z] - z[2r(1 - \alpha) + 2z - \gamma]}{[2r(1 - \alpha) + 2z - \gamma][2r(1 - \alpha) + 2z]} \right)\end{aligned}$$

and now set  $r = 0$  to get

$$\bar{\theta}_{AH|A}^u - \bar{\theta}_{\gamma=0}^u = \frac{(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)} \left( \frac{-\gamma(2z - 1)}{2z(2z - \gamma)} \right) < 0$$

a result that is true for all  $r$  within the relevant range because the intersection point of the two curves is at  $r = \frac{-z}{2(1 - \alpha)} < 0$ . ■

The next section deals in greater detail with the comparative statics of the indifference curves with respect to  $\gamma$ . For now we need only note that rises in the homestead exemption shift the  $\bar{\theta}_{AH|A}^u$  curve down until, when  $\gamma = z$  we have that  $\bar{\theta}_{AH|A}^u = 0$ , and guarantees would never be used, only secured loans.

## 2.4 Changes in homestead exemption

### 2.4.1 Motivation

**The dataset** The 1993 NSSBF was a survey of small business owners/managers conducted under the auspices of the Board of Governors of the Federal Reserve System and the Office of Small Business Administration.<sup>10</sup> This survey covers a representative sample of US nonfinancial, nonfarm, for-profit businesses that have fewer than 500 employees, and which were in operation as of December 1993.<sup>11</sup> To ensure adequate representation across categories, the sample was stratified by census region, urban and rural location and by employment size.<sup>12</sup> There are approximately 1750 noncorporate firms and 2800 corporate firms in the sample, out of a population of small business firms across the nation of about 7.5 million. In [Berkowitz and White \(2004\)](#) this data is used, combined with their own data on the homestead exemption level across states, to test how certain small business loan variables (such as ‘credit rationing’, interest rate and loan size) varied with differences in homestead exemption levels.

‘Homestead exemption’ refers to that part of a debtor’s assets which are immune from creditors’ demands according to personal bankruptcy law. Depending on the exemption level in any given state, the family home (or part thereof) is exempt from forced sale during bankruptcy proceedings, legislation introduced

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<sup>10</sup>Three such surveys (independently sampled five years apart) have been jointly conducted by the Federal Reserve Board (FRB) and the Office of Small Business Administration (SBA): 1987, 1993 and 1998. Now called the Survey of Small Business Finances (SSBA), details of all three can be found at <http://www.federalreserve.gov/pubs/oss/oss3/nssbftoc.htm>.

<sup>11</sup>The actual surveying was conducted during 1994 and early 1995.

<sup>12</sup>See [Cole and Wolken \(1995\)](#) for a description of the dataset and the reasons underpinning the survey decisions.

for (family) welfare reasons. Different states have different levels of homestead exemption, and these levels vary widely, ranging from no exemption in one state (Maryland) to unlimited exemption in seven states (Arkansas, Florida, Iowa, Kansas, Minnesota, Oklahoma and Texas).<sup>13</sup> The median exemption level for real estate is \$15,000. All states also have personal property exemptions, ranging from \$1,200 (Montana) to \$60,000 (Texas), with a median amount of \$7,000.

**The positive change in interest rates to increases in homestead exemption** The authors used as dependent variable the following three variables: credit rationed, interest rate and loan size. The last two variables were direct questions in the survey. The first variable was inferable from the survey data because small business owners were asked whether, in the three years preceding the time of the survey, they had asked for credit, and whether, in the most recent of these credit requests, they had received the loan. Owners had also been asked whether they had *not* tried to seek a loan (even though they wanted one) during the three years prior to the survey because they were discouraged about the likelihood of receiving one. ‘Credit rationed’ was then specified in the data as the dummy variable ‘discouraged/denied’. The main independent variables of interest were the homestead and personal exemptions by state, entered as their dollar amounts and also as the square of their dollar amounts.<sup>14</sup> The authors summarize their findings about the relationship of these three variables to variations in

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<sup>13</sup>The existence of an unlimited exemption in Florida is a reason why O J Simpson, in anticipation of the civil law suit against him (which he eventually lost) by the family of victim Nicole Simpson, transferred residence from California to that state. Note that [Berkowitz and White \(2004\)](#) define ‘unlimited’ as the total amount of all the non-unlimited states’ exemption amounts (\$160,000), a separate dummy variable being included for those states which have unlimited exemptions.

<sup>14</sup>Many other independent variables were included in the regression, derived from previous papers using the same data set and showing them to be significant explanators.

homestead exemption as follows:

We find that small businesses are more likely to be denied credit if they are located in states with high rather than low homestead exemptions and that, if they receive loans, the loans are smaller and the interest rates are higher.<sup>15</sup>

These findings on the direction of the relationship of these three variables to variations in homestead exemptions (positive in the first and third cases, negative in the second case) accorded with the simple supply and demand credit market model expounded in their paper. More specifically, and focusing only on the ‘interest rate’ variable, the probability of a firm being credit rationed conditional on level of homestead exemption was calculated for both non-corporate and corporate firms in the dataset.<sup>16</sup> For non-corporate (corporate) firms located in a state with both homestead and personal property exemption levels at the 25th percentile, the *predicted* (if the homestead exemption level *only* rises, holding constant the personal property exemption) probability increase in moving to the 50th percentile is 0.60 (0.27), and the increase if the homestead exemption rises to the 75th percentile is 1.33 (0.56), and finally rises again from the 75th to the ‘unlimited’ percentile by another 0.22.<sup>17</sup>

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<sup>15</sup>See [Berkowitz and White \(2004\)](#) at page 70.

<sup>16</sup>For this out-of-sample predictive exercise [Berkowitz and White \(2004\)](#) focused solely on firms which were family-owned, non-minority owned, and which had average values for the other right hand side variables of their regression equation.

<sup>17</sup>These ‘predicted probabilities’ are calculated by initially substituting the numerical amounts of the 25th percentile for the homestead and personal property exemptions into the regression equation already containing previously estimated coefficients, calculating the value of the dependent variable thereby obtained, and then (holding the numerical value of the personal property exemption fixed) repeating the procedure with different numerical values for the homestead exemption.

**Non-monotonicity** However, for both corporate and non-corporate firms, this increase in predicted probability was not monotonic. In particular, for both types of firm the change in predicted probability fell when the homestead exemption level changed from the 75th percentile to somewhere in the 80th-90th percentile range, before rises again for the ‘unlimited’ case. Furthermore, this was true for the other two financial variables considered in their paper, namely ‘credit rationing’ and ‘loan size’, suggesting not only that the non-monotonicity is robust, but that there is something more fundamental at play here, since there is clearly a connection among these different variables and the underlying non-monotonicities.

This was something the authors of [Berkowitz and White \(2004\)](#) were not expecting and could not account for with their model. For the case of credit rationing, they found that:

[T]he probability of credit rationing is not monotonically increasing in the homestead exemption level. For noncorporates, for example, the probability drops to 0.154 at the 90th percentile and then rises to 0.161 if the homestead exemption is unlimited. We do not have a good explanation for why the probability of credit rationing displays this non-monotonic region when the homestead exemption level is not unlimited.<sup>18</sup>

While for the case of interest rates they state:

For both firm types, however [ie, non-corporate and corporate], the increase in interest rates is nonmonotonic when exemptions are around

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<sup>18</sup>See [Berkowitz and White \(2004\)](#) at page 79.

their 80th-90th percentiles. This result is surprising but consistent with the credit-rationing results.<sup>19</sup>

## 2.4.2 Analysis

For the model in this paper, loan size and credit rationing cannot be analyzed. Therefore, we focus only on the variation in interest rates (given variation in homestead exemption level), where firm default risk (in this paper, the inverse of  $\theta$ ) is used to proxy for the interest rate on the loan (since there is no explicit interest rate in the model).

Once again the relevant equations are (2.8) and (2.9). We graph each indifference curve in  $\theta - \gamma$  space, finding slopes and turning points where necessary.

The case of  $\bar{\theta}_{AH|A}^s$  is easy to analyst. Since the security over the house is taken, the existence of the homestead exemption doesn't matter. Thus we trivially have:

$$\bar{\theta}_{AH|A}^s = \frac{z\eta(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + z(1 + \eta)]}$$

and

$$\frac{\partial \bar{\theta}_{AH|A}^s}{\partial \gamma} = 0$$

and so it can be seen that in  $\theta - \gamma$  space is a horizontal line. For the case of  $\bar{\theta}_{AH|A}^u$  on the other hand we have the following proposition.

**Proposition 13.** *For the equation of  $\bar{\theta}_{AH|A}^u$  in  $\theta - \gamma$  space we have:*

$$(i) \quad \bar{\theta}_{\gamma=0}^u = \frac{z(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + 2z]} > 0$$

$$(ii) \quad \frac{\partial \bar{\theta}_{AH|A}^u}{\partial \gamma} = - \left\{ \frac{(z - \gamma)(\alpha - \lambda)(1 - \alpha)(1 - \tau_E) + (z - \gamma)(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + 2z - \gamma]}{\{(1 - \alpha)(1 - \tau_E)[2r(1 - \alpha) + 2z - \gamma]\}^2} \right\}$$

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<sup>19</sup>See [Berkowitz and White \(2004\)](#) at page 81.

- (iii) The two turning points of the graph are  $\gamma = z$  and  $\gamma = \gamma^* = \alpha - \lambda + 2r(1 - \alpha) + 2z$
- (iv)  $\bar{\theta}_{\gamma=\gamma^*}^u = \frac{z + [\alpha - \lambda] + 2r(1 - \alpha)}{(1 - \alpha)(1 - \tau_E)} > 1$  and  $\lim_{\gamma \rightarrow \infty} \bar{\theta}_{AH|A}^u = \frac{(\alpha - \lambda)}{(1 - \alpha)(1 - \tau_E)} > 0$
- (v)  $\theta_{\gamma=0} < \lim_{\gamma \rightarrow \infty} \bar{\theta}_{AH|A}^u < \theta_{\gamma=\gamma^*}$
- (vi) The graph of  $\bar{\theta}_{AH|A}^u$  touches the  $\gamma$  axis at  $z$
- (vii)  $\left. \frac{\partial \bar{\theta}_{AH|A}^u}{\partial \gamma} \right|_{\gamma=0} < 0$  and  $\lim_{\gamma \rightarrow \infty} \left[ \frac{\partial \bar{\theta}_{AH|A}^u}{\partial \gamma} \right] = -(1 - \alpha)(1 - \tau_E) < 0$
- (viii) The sign of  $\frac{\partial \bar{\theta}_{AH|A}^u}{\partial \gamma}$  is negative in the range  $[0, z] \cup [\gamma^*, \infty)$  and positive in the range  $(z, \gamma^*)$

**Proof** The various parts of the proposition are proved via basic algebra and calculus. The details are omitted. ■

The easiest way to understand the relationship of firm default risk to home-stead exemption is via figure 2.3. Recall that interest rates are the inverse of firm default risk, so that the graph of interest rate with respect to  $\gamma$  requires figure 2.3 to be flipped over. From which it follows that the variation is indeed positive for most of the range, with an intermediate negative range. Thus the required non-monotonicity arises automatically from the model of section 2.2.

## 2.5 Conclusion

Using an incomplete financial contracting environment in the tradition of [Aghion and Bolton \(1992\)](#) we have been able to outline the characteristics determining whether a business loan is committed or not, and, if committed, whether it will

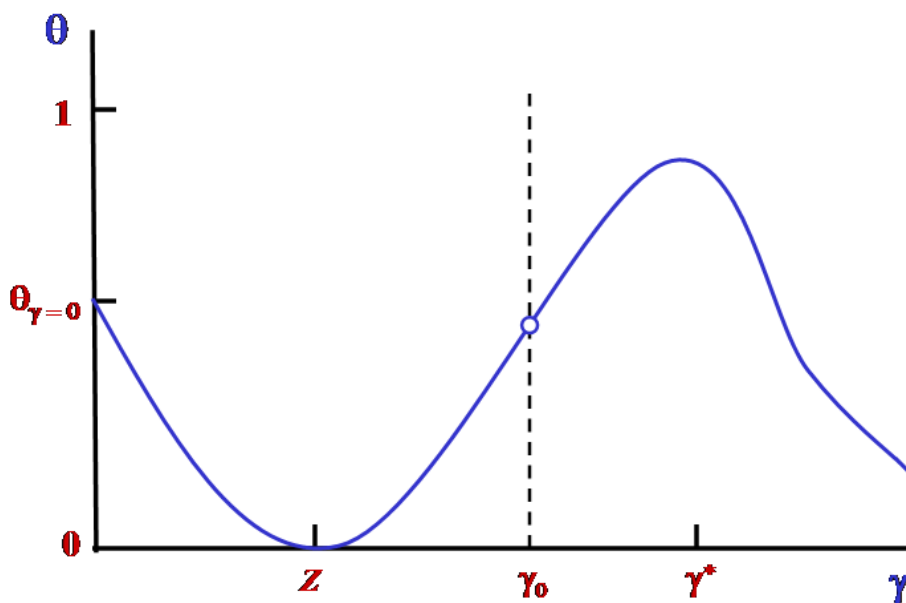


Figure 2.3: Non-monotonic relationship between  $\theta$  and  $\gamma$  for the indifference curve  $\bar{\theta}_{AH|A}^u$ .

be secured or not. It is shown that high-risk, small firms are more likely to be required to commit a loan, and that the higher the homestead exemption level, the greater the class of firms that are required to commit to their loans, to the point that when the exemption level becomes very high, unsecured commitments are unlikely to be used.

We have also been able to show that a straightforward model in the ‘incomplete financial contracting’ tradition can help explain an anomalous non-monotonicity in small business financing data first noted in [Berkowitz and White \(2004\)](#). In that paper it was shown that three small business finance variables



(‘credit rationed’, interest rate and loan size) appeared to vary non-monotonically when tested against the 1993 NSSBF survey data of small business finances, with increases in state homestead exemptions to creditor claims under the Chapter 7 personal bankruptcy provisions of the US Bankruptcy Code.

An obvious empirical extension is to test the results of section 2.3 against NSSBF data, while theoretical extensions include introducing multiple creditors with the same or differing claims (ie, priority differences), and introducing explicitly the transactional costliness of committed debt vis-a-viz uncommitted debt, and, within the class of committed loans, the increased costliness of collateralized vis-a-viz merely guaranteed loan, with a view to determining how the choice of loan contract changes with changing homestead exemptions.

## CHAPTER 3

# Marital Investments and Changing Divorce Laws

### 3.1 Introduction

In 1969 California was the first American state to introduce no-fault divorce. Whereas previously an unhappy spouse required the consent of the other spouse for marital dissolution, after the “no-fault revolution” divorce became unilateral. Over the next decade California’s legislative reform was replicated across other American states.<sup>1</sup> The change in divorce law was followed by a short-run rise in the divorce rate across the country.<sup>2</sup> In the 1990s some southern states have re-instituted a form of fault divorce and have given couples the option of choosing which regime they would like to be regulated under. These changes in divorce law gradually effected behavior and social norms. The effect of all these changes on the welfare of spouses and ex spouses, as well as on children, is an on-going area of research. Even if the divorce rate had not changed, a switch to unilateral divorce involves a switch in intra-marital bargaining power from the spouse seeking to maintain the relationship and the spouse wanting out. This redistribution of

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<sup>1</sup>See [Herbert \(1988\)](#) for the history.

<sup>2</sup>See [Wolfers \(2005\)](#) for the empirics.

power within marriages can raise intra-marital welfare by encouraging investment in marital goods and discouraging the production of marital ‘bads’. As Stevenson and Wolfers state in an article empirically exploring the effect of unilateral divorce on marital ‘bads’ like domestic violence and intimate homicide, “[I]n a society in which people can leave abusive partners, spouses may be less likely to be abusive,” (see [Stevenson and Wolfers \(2003\)](#) at page 2). In seeking to examine this question, attention must also be paid to the fact of different marital property law regimes across the states of America.

**Description of model** This paper explores the tension in marriage between exploiting investment opportunities versus the disamenity of remaining in the marriage with a spouse of the ‘wrong’ type. The model consists of two voting games played sequentially. In the first the agents must decide whether to make an investment and in the second whether to continue with the team (they are already in) or whether to vote for dissolution. In both voting games, the voting rule can be either majority or unanimous, giving a total of four possible ‘regimes’ for the game as a whole, representing the different combinations of marital property and divorce laws that exist in the United States cross-sectionally and that existed in the United States through time (see section [3.2](#) for a description of institutional variance across the states). The investment voting game is a coordination game with conflict - that is, each agent would prefer that the other agent made the investment, though each also prefers that the investment goes ahead rather than no. The decision to continue in the team or not is made by each agent on the basis of his or her guess about the type of the other agent. In this paper intra-team trust is modelled as uncertainty about the type of the person one is playing the game with. In particular, each agent can be either ‘normal’ or else a

‘disloyal’ type. A disloyal type is pre-programmed to play a certain way (as in the reputational game theory literature). Normal types of agents only prefer to stay in the team with another normal type and prefer to leave the team if with a disloyal type. It is in this way that the decision to invest or not in the firm voting game has a signalling aspect regarding one’s trustworthiness for the team that is independent of the value of the investment project for whose exploitation the team was established.

**Description of results** The paper focuses solely on symmetric equilibria and shows that whenever there exists a symmetric equilibrium then it must be separating. This is the equilibrium in which ‘loyal’ spouses invest and stay in the relationship while ‘disloyal’ spouses do not invest and the marriage ends in divorce. Under both types of marital property regimes, a no-fault divorce regime is superior to a fault divorce regime in the sense that it expands the range of parameters under which such an equilibrium exists. It is shown that, for a given level of spousal ‘disloyalty’ (or divorce rate) within a society, intra-marital welfare is higher under the communal (unilateral) than under the common law (joint) marital property regimes. It is also shown that for societies with high levels of spousal ‘disloyalty’ or divorce rates (such as modern, secular Western countries), a unilateral marital property regime is preferable as a matter of policy to a joint marital property regime, and that for societies with low levels of spousal ‘disloyalty’ or divorce rates (such as in developing and/or religious societies), this policy preference is reversed.

**Related literature** Within the literature on ‘family economics’ intrafamilial bargaining is modelled mostly as a cooperative game (with either ‘unitary’ or

conflicting preferences - ‘consensus’ or ‘non-consensus’ - among the family members: see the survey in [Pollak and Lundberg \(1996\)](#)). A brief summary of the potentialities within that field regarding those few exceptional papers which utilize a non-cooperative bargaining environment can be found in [Bergstrom \(1997\)](#). Within that small literature, the assumption of asymmetric information among family members is virtually non-existent, as is any modelling of pre-play communication. An article which briefly touches on this issue is [Peters \(1986\)](#) who states (at page 442, footnote 15):

The multi-period aspect of the marriage relationship may . . . reduce incentives for strategic bargaining. *The mistrust and ill-feelings engendered by strategic bargaining can be detrimental to a relationship based on trust and intimacy.* [my emphasis]

There is a literature on ‘relational contracting’ (that is, self-enforcing repeated game environments) within small teams (including marriages), which explores the interrelationship between formal and informal enforcement mechanisms when ease of relationship exit varies. See [Lindsey et al. \(2001\)](#) and [Sobel \(2002\)](#) for examples.

Within the mechanism design literature [Cramton et al. \(1987\)](#) analyzes the conditions for (ex post) efficient partnership or marital dissolution where the relationship asset or assets need to be divided, and where the division is obtained via an auction mechanism closer to an all-pay auction than to the more traditional auction formats found in practice. More elaborate mechanisms, when dealing with a more general partnership dissolution environment than found in [Cramton et al. \(1987\)](#) (such as, for example, non-independent valuations) are provided by

McAfee (1992). A theory of joint asset ownership (such as in common law marital property regimes) is provided by Cai (2003).

The model consists of two sequential voting games. The literature on sequential (or repeated) voting is sparse. Strategic voting such as is contained in this paper began with Austen-Smith and Banks (1996) and was extended by Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1998). That literature deals mostly with political elections and jury voting (some experimental support for strategic voting in a jury context can be found in Guarnaschelli et al. (2000)). The introduction of communication to the (strategic) voting environment is recent (see for example Gerardi and Yariv (2005)).

Finally, see Silbough (1998) and Scott and Scott (1998) for some legal academic perspectives of marriage, family law and divorce.

**Outline of paper** This paper proceeds as follows. Section 3.2 describes the relevant institutional context while section 3.3 outlines the model. Section 3.4 solves the model in pure strategies and section 3.5 outlines the main result for communal property regimes. Section 3.6 discusses analogous results for common law property regimes while section 3.7 concludes.

## 3.2 The heterogeneity of family law regimes among the states of America

**Introduction** The ability of marriages to exploit investment opportunities depends on implicit team rules regarding who has the right to decide to undertake the investment, and how the returns to that investment are shared within the

team. These implicit rules (the intra-familial power balance) within the team are in turn shaped and influenced by the explicit default rules provided by prevailing family law regimes.

Marital property rights in the US depend on which legal tradition obtains in the different states. Legal traditions divide into two: the great families of the common and civil law. Most English speaking countries are part of the common law tradition stemming from England, and the same is true of the majority of states in the US. However, owing to differing historical trajectories, nine states are part of the civil law tradition stemming from continental Europe. New York is an example of a common law state and California (because of the Spanish/Mexican heritage) an example of a civil law one.<sup>3</sup>

Both legal traditions had their own rules governing marriages. Consequently, currently across the states of the United States there exists cross-sectional heterogeneity in marital property law regimes.

**Marital property regimes** Wealth within a marriage may be held either jointly or separately. In this paper we deal only with jointly held wealth. In states with community property this jointly held wealth is called (if certain conditions are satisfied in its acquisition) community property.<sup>4</sup> Legislative intervention from the 1960s onwards in all community property states made both husband and wife manager of the jointly-held community property (where previously just the husband was deemed manager). While the details of each state's legislation differ, generally speaking this management could be exercised concurrently or

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<sup>3</sup>The nine community property states are Arizona, California, Idaho, Louisiana, Nevada, New Mexico, Texas, Washington and Wisconsin.

<sup>4</sup>On community property rules see generally [Reppy and Samuel \(2004\)](#).

separately. In particular this means that a characteristic of community property is that debt incurred by one managing spouse (if the purpose of the debt is for the community) is potentially recoverable by creditors on any part of the community property the debtor is entitled to manage, regardless of whether the other spouse knew about the debts. However, with respect to community *real* property most states have enacted additional legislation ensuring that both spouses are needed for the purpose of *entering a mortgage* - thus the consent of the other spouse is required prior to the encumbering of a community property such as a family home by one or the other spouse.<sup>5</sup> In this paper we deal only with real property.

In common law regimes there exist three main types of joint-ownership title: joint tenancy, tenancy in common, and tenancy by the entirety. The form of title which appears on its face to be closest to the civil law concept of community property is the 'tenancy by the entirety'. Unlike the other two types of title, but similar to community property, it is a form of title which can only exist between married couples, and only while the couple remains married. It involves the fiction that husband and wife are one - hence each has a non-severable interest over the entire property. Nonetheless, whether the rule governing consent by the other spouse to incumber real property is the same or the opposite of that mentioned above for community (real) property depends on which common law state we are dealing with. It should be noted that not all common law states have the title of tenancy by the entireties, and of those that do, they can be categorized into two broad classes, namely those that *do* require consent (such as Pennsylvania), thus making them akin to community property, and those that do not (such as New

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<sup>5</sup>Torts, for example, can still be unilaterally incurred. On the legislative rules dealing with encumbering real community property see [Oldham \(1993\)](#).



York), thus making them akin to a joint or common tenancy.<sup>6</sup>

In both types of family law regime (civil or common law) wealth not held jointly is held separately. Separately-held wealth obviously does not require the consent of the other, non-owning spouse before it is capable of being encumbered. On the other hand if the loan requires collateral of an amount greater than the amount of separate wealth owned by the spouse seeking the loan, then of course the consent of the other spouse, as owner of the other separate wealth upon which the encumbrance is additionally sought, must be obtained. This would then be a third-party guarantee.<sup>7</sup> A similar logic applies to the tenancy in common.

Once again, in this paper we focus only on jointly-held (real) assets. The governing example is the family home.

From the above brief discussion it can be seen that, and for the purposes of summarizing, the only time consent is required in order to use a *jointly-owned* (real) asset (since separate assets always require consent of the other owner to use his asset) within marriage for the purpose of supporting an investment is when the state is governed either by community property law or else is governed by common law marital rules, and the tenancy is by the entirety (assuming that type of title exists in the chosen state), and that the state is one in which the rules for such a title developed in a way that consent was deemed required. States which require consent for their titles by the entirety are: Delaware, District of Columbia, Florida, Indiana, Maryland, Missouri, Pennsylvania, Rhode Island,

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<sup>6</sup>On the different types of tenancy by the entirety across states see [Phipps \(1951\)](#), mentioning four categories, though in the intervening time since that article was written the categories have collapsed to (broadly speaking) two.

<sup>7</sup>See more generally [Harris et al. \(1996\)](#). Rules of thumb for determining when a spouse needs to co-sign or co-guarantee a loan are given in chapter 4 of [Atkinson \(2005\)](#), available at (as of 05/15/2005) <http://www.abanet.org/publiced/practical/books/family/home.html>.

Vermont, Virginia and Wyoming.<sup>8</sup>

All these differences in titles and legal traditions are schematized in figure 3.1, where of course ‘C’ stands for ‘consent’ and ‘NC’ for ‘no consent’. The columns represent ‘civil law’ and ‘common law’ and the rows separate and joint property titles. Since the focus of this paper is on joint asset ownership, the ‘C’ in the row marked ‘separate’ represents the case when the other property is needed to back a loan, that is, the case of third-party guarantees. Similar reasoning explains the ‘C’ in the box for the common law joint title ‘tenancy in common’.

**Divorce regimes** With respect to the law on divorce, by the 1980s most states had implemented some form of no-fault divorce regime. Prior to that fault had to be shown in order to leave a marriage. Some cross-state heterogeneity has been introduced in the last twenty years through the agency of state-based legislative intervention which attempted to give couples a choice of divorce regime, though the take-up within those states which have provided such a possibility of choice of stricter divorce rules has been low (see [Silbough \(1998\)](#) for a description of developments).<sup>9</sup>

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<sup>8</sup>States which do not are: Alaska, Arkansas, New Jersey, New York, and Oregon. As an aside, the requirement of consent is also the rule in Great Britain as well as those of her former colonies (such as Australia, Canada and New Zealand) which prefer to hew closely to legal developments in Britain (see [Fehlberg \(1997\)](#)).

<sup>9</sup>Recently the state of Louisiana has legislated to offer couples contemplating marriage a menu of marital dissolution regimes, ranging from no-fault to fault to the impossibility of divorce for any reason.

	<b>Civil</b>	<b>Common</b>		
<b>Separate</b>	C	C		
<b>Joint (real)</b>	C	C		<b>Common</b>
		NC		<b>Joint</b>
		NC	C	<b>Entireties</b>
		<i>New York</i>	<i>Delaware</i>	

Figure 3.1: Schematization of consent rules under different legal traditions and types of ownership title.

### 3.3 The model

#### 3.3.1 Timing

At date 0 there are 2 agents who convene with the intention of establishing a productive team relationship (marriage, partnership, joint venture and so on). The process by which they come together (say via a matching model in the case of marriage) is exogenous to the model. For convenience there is no intertemporal interest rate and no discount factors for the agents. Before date 1 the agents privately learn their types and then at date 1 they make a (publicly observable) non-cooperative investment decision and then at date 2 they make a

non-cooperative relationship decision (namely, whether to leave the relationship or not). Whether the investment in fact goes ahead, and whether the relationship in fact ends, depends on the type of legal/institutional regime prevailing (to be described later). At the end of the game the return on the investment (if undertaken) is realized and the agents' types are also revealed and payoffs for the game determined. The timeline for the model is depicted in Figure 3.2.

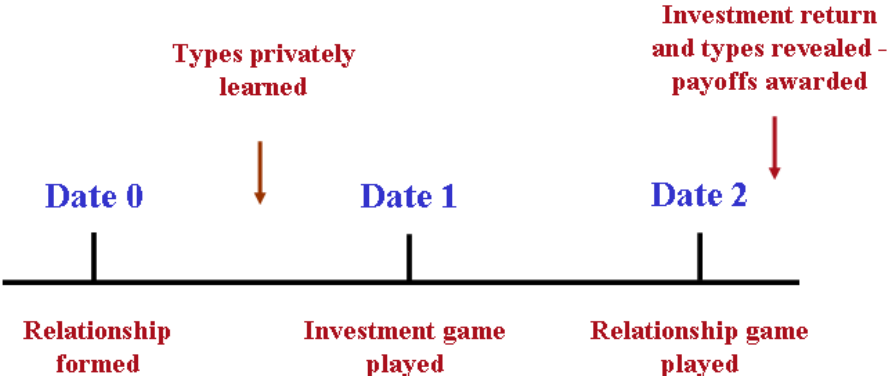


Figure 3.2: Timeline of game

**3.3.2 Investment versus maintaining a relationship**

The purpose of the paper is to explore the tension between exploiting an investment opportunity and maintaining a relationship under the two types of divorce

law regimes. These two aspects of the model are outlined below. Regimes are considered in the next subsection.

**The investment decision** At date 1 an investment opportunity presents itself. The state of the investment is described by a random variable  $\tilde{R}$  with support  $R = \{0, 1\}$  and with  $\gamma$  the probability that 1 is realized. A realization of zero indicates a bad outcome on the investment and a realization of one indicates a good outcome on the investment. The specific payoffs arising from the investment are detailed below.

The decision (at date 1) to invest or not is made simultaneously by both agents. Investment is a binary decision for both agents, with  $a_j^{t=1} \in A_j^{t=1} = \{i, ni\}$  (for  $j = 1$  and  $2$ ), where  $i$  means an agent decides to invest and  $ni$  means an agent decides not to invest. After the investment game, whether the project in fact goes ahead depends on the property sub-regime in place. These sub-regimes are outlined in subsection 3.3.3 below.

**The relationship decision** At date 2, the agents in the model (simultaneously) decide if they wish to remain in the relationship. This is a binary decision for both agents, with  $a_j^{t=2} \in A_j^{t=2} = \{c, l\}$  (for  $j = 1$  and  $2$ ), where  $c$  represents the decision to continue with the relationship and  $l$  represents the decision to leave. Whether the relationship ends or not depends on the exit sub-regime in place. The exit sub-regime is outlined in subsection 3.3.3 below.

### 3.3.3 Institutional regimes

Relationships, and investments made within them, occur within pre-existing institutional regimes. For the purpose of this paper, these in turn can be decomposed into two different types of sub-regimes: one governing the sharing rules regarding the joint relationship wealth (which we call the property sub-regime) and the other governing the rules of dissolution of the relationship (which we call the relationship sub-regime). Each subregime is a voting game. We consider these two games and the sub-regimes that apply to them in turn.

#### 3.3.3.1 Property subregime

At date 1 the simultaneous  $2 \times 2$  investment game is played with possible outcomes  $a^{t=1} \in A^{t=1} = \{A_j^{t=1}\}^2$ . The investment goes ahead if either one of the two agents voted for it (or both voted for it), that is, the investment regime is unilateral. The set of outcomes of the date 1 subgame is denoted by  $\Delta = \{I, NI\}$  (where  $I$  represents ‘invest’ and  $NI$  represents ‘not invest’). We have

$$\Delta = \begin{cases} I & \text{if } a^{t=1} = \{(i, i), (i, ni), (ni, i)\} \\ NI & \text{if } a^{t=1} = \{(ni, ni)\} \end{cases}$$

Consider the example of marriage. At date 0 there exists a relationship asset (say the family home) which (depending on the marital property law regime in place) may either require consent for its use as collateral to back the investment project or not. Clearly, given the heterogeneity across states described in section 3.2 above, the model constitutes a stylization of certain broad features discernable amongst that institutional heterogeneity. In particular, it can be seen from the explanation of marital property regimes given in section 3.2 that the unilateral sub-regime which will be used throughout most of this paper is descriptive of

common law joint tenancy and some states' tenancy in the entireties. States in the civil law tradition, as well as the eleven common law states mentioned towards the end of section 3.2, namely, those where the tenancy by the entireties both exists and also where that type of title requires the explicit consent of the other spouse, are dealt with by the joint voting sub-regime of section 3.6 of the paper.

### 3.3.3.2 Relationship subregime

At date 2 another simultaneous  $2 \times 2$  game is played, this time with outcomes  $a^{t=2} \in A^{t=2} = \{A_j^{t=2}\}^2$ . Whether the relationship continues or not depends on whether the right to exit the relationship is unilateral, or else requires the consent of both agents. Again, using marriage as an example, divorce laws (fault or no-fault divorce) determine the ease of exit from a marriage. Define the set of ultimate outcomes of the date 2 relationship game by  $\bar{\Delta} = \{C, E\}$  (where  $C$  represents 'continuing' and  $E$  represents 'ended'). Let the class of threshold voting rules be parameterized by  $\omega = 1, 2$ . Under voting rule  $\omega$ , the first alternative is chosen if and only if at least  $\omega$  agents vote in favor of it. Given a voting rule  $\omega$  and a profile of votes  $a^{t=2}$ , we let  $\psi_\omega(a^{t=2})$  denote the group's decision. Formally,  $\psi_\omega : A^{t=2} \rightarrow \bar{\Delta}$  is defined as follows:

$$\psi_\omega(a^{t=2}) = \begin{cases} C & \text{if } |\{j : a_j^{t=2} = l\}| < \omega, \\ E & \text{if } |\{j : a_j^{t=2} = l\}| \geq \omega. \end{cases}$$

It is straightforward to see that there are 4 types of game governing the relationship as a whole. We define a *regime* (which governs the whole game) as  $\Gamma(\omega)$ . We label the four possible types of regime as follows.

		Exit sub-regime	
		<i>Unilateral (No-fault)</i>	<i>Joint (Fault)</i>
Property sub-regime	<i>Joint</i>	Regime 1	Regime 3
	<i>Unilateral</i>	Regime 2	Regime 4

Table 1: Classification of four regimes in the two-person case

### 3.3.4 The agents

All agents are risk neutral.

#### 3.3.4.1 Types

We wish to model the possibility that deciding to invest or not invest involves a consideration additional to that of whether the investment is likely to be a good one or not, namely, that it can also be construed as a signal of an agent’s commitment to the relationship. Hence we assume that each agent is (independently) one of two possible types, with  $t_j \in T_j = \{N, D\}$  (for  $j = 1$  and  $2$ ), where  $N$  denotes a *normal* type and  $D$  denotes a *disloyal* type. The probability of a draw by nature resulting in agent  $j$  being type  $t_j$  is given by  $p_{t_j}$ . Each agent’s type is private knowledge throughout the relationship, and is only publicly revealed at the end of the game. The agents learn what type they are before the date 1 investment decision is to be made.

Type  $D$  is a “reputational” type (in the sense of [Fudenberg and Levine \(1989\)](#)). That is, it is always in type  $D$ ’s interest to play a specified way independent of the history of the game. The relationship *per se* is not important to the disloyal type. By assumption then a disloyal type always chooses not to invest in the date 1 decision and always chooses to leave in the date 2 decision.



It is therefore only the normal type who chooses which decision to make in each period on the basis of the tension between the income side and the trust side of the relationship. The specific payoffs of the normal agent are outlined below.

### 3.3.4.2 Payoffs

Each regime  $\Gamma(\omega)$  defines the following Bayesian game  $G_{\Gamma(\omega)}$ . Nature selects a profile of types according to the probability distribution  $p$ , then players learn their types, after which they vote simultaneously at date 1 and again at date 2. If the profiles of types and actions are  $t$  and  $(a^{t=1}, a^{t=2})$ , respectively, then (given regime in place) player  $j$  obtains  $u_j^{\Gamma(\omega)}(\Delta, \bar{\Delta}, t)$ .

Since the disloyal-type agents are pre-programmed to play a specified way, we need only specify the utility function of the normal types. Each normal type agent's utility possesses a wealth component and a relationship component. These components are assumed separable. Specifically, each normal type agent's preferences are represented by the utility function  $u_j^{\Gamma(\omega)} = w_j(\Delta) + \chi_j(\bar{\Delta}, t)$ . We consider these two components in turn.

**Wealth component** Wealth is denoted by the  $w_j$  term in each agent's utility function, so that (as required) agents are risk-neutral in wealth. If neither agent votes in favor of investing then both agent receive zero. It costs  $c > 0$  (where in addition  $c < r$ ) for the investment to be undertaken. If it is undertaken and the outcome is 0 (which can happen with probability  $1 - \gamma$ ) then both agents receive zero, and if it is undertaken and the outcome is 1 (which can happen with probability  $1 - \gamma$ ) then both agents receive  $r > 0$  (it is a public good within the team). In addition, whoever made the investment subtracts the cost from

the amount she received. If they both voted in favor of investing then the cost is shared equally between them. Clearly therefore either agent would prefer the other to make the investment, but both would rather have investment than non-investment as the aggregate outcome.

The payoffs from the  $t = 1$  investment game can be summarized as follows. For the case when  $\tilde{R} = 1$  (which occurs with probability  $\gamma$ ) we have that  $(w_1(\Delta), w_2(\Delta))$  equals

		Agent Two	
		$i$	$ni$
Agent	$i$	$(r - \frac{1}{2}c, r - \frac{1}{2}c)$	$(r - c, r)$
One	$ni$	$(r, r - c)$	$(0, 0)$

and for the case where  $\tilde{R} = 0$  (which occurs with probability  $\gamma$ ) we have that  $(w_1(\Delta), w_2(\Delta))$  equals

		Agent Two	
		$i$	$ni$
Agent	$i$	$(-\frac{1}{2}c, -\frac{1}{2}c)$	$(-c, 0)$
One	$ni$	$(0, -c)$	$(0, 0)$

or, in expectation

		Agent Two	
		$i$	$ni$
Agent	$i$	$(\gamma r - \frac{1}{2}c, \gamma r - \frac{1}{2}c)$	$(\gamma r - c, \gamma r)$
One	$ni$	$(\gamma r, \gamma r - c)$	$(0, 0)$

It is easily verified (given the assumption - without which investment would never be undertaken - that  $\gamma r - c > 0$ ) that this stage game has two pure strategy equilibria  $\{(i, ni), (ni, i)\}$  and one mixed strategy equilibrium.

**Relationship component** The relationship component of a normal-type agent's utility depends on two things: whether the agent is still in a relationship or not ( $\bar{\Delta}$ ) and the types of the other agents in the group ( $T_{-j}$ ). Thus the relationship component of the normal type's utility takes the form  $\chi_j(T_j, T_{-j}, \bar{\Delta})$ . Of course, if  $T_j = D$ , then that type of agent's payoffs are so configured as to ensure that the predetermined (behavioral) strategy is played regardless of the state of the relationship or the type of the other agent. Consequently, we need only consider the case where  $T_j = N$ .

A normal-type agent shares a relationship with two possible types of other agent: normal and disloyal. Although agents do not know the exact types of the other agents with whom they share the relationship (at least until the end of the game), their ultimate satisfaction from being in the relationship nonetheless depends on the other agent's type. Specifically, normal types prefer not to be in a relationship with disloyal types and prefer to be in a relationship with normal types. If we assume that agent  $j$  is a normal type, then the relationship component of agent  $j$ 's utility function (conditional on being in a relationship or not) takes the following form:

$$\chi_j(N, T_{-j}|C) = \begin{cases} 0 & \text{if } T_{-j} = N \\ -\delta & \text{if } T_{-j} = D \end{cases}$$

when agent  $j$  remains in a relationship at the end of the game, and similarly:

$$\chi_j(N, T_{-j}|E) = \begin{cases} -\delta & \text{if } T_{-j} = N \\ 0 & \text{if } T_{-j} = D \end{cases}$$

if agent  $j$  is no longer in a relationship at the end of the game. Similarly for the other agent in the team. Note that  $\delta \in (0, \infty)$ .

### 3.3.5 Equilibrium

The solution concept used in this paper is Perfect Bayesian Equilibrium.<sup>10</sup> As usual, in order to define this concept we need to define the strategies, best responses and beliefs of the agents.

A pure (behavioral) strategy for an agent  $j$  is a pair of mappings  $b_j = (b_j^{t=1}, b_j^{t=2})$ . The first mapping gives agent  $j$ 's date 1 decision conditional on his or her type,  $b_j^{t=1} : T_j \mapsto A_j^{t=1}$ . The second mapping gives agent  $j$ 's date 2 decision to remain in the relationship or not conditional on the (publicly observable) outcome of the date 1 investment game, thus  $b_j^{t=2} : A^{t=1} \mapsto A_j^{t=2}$ . A profile of such pure strategies for the game is  $b = (b_1, b_2)$ .

Mixed (behavioral) strategies are defined in an analogous way. Thus  $\Delta b_j^{t=1} : T_j \mapsto [1, 0]^2$  represents agent  $j$ 's probability choice over  $A_j^{t=1}$  (where  $\Delta b_j^{t=1}(a_j^{t=1})$  will be used to denote the probability that agent  $j$  plays  $a_j^{t=1} \in A_j^{t=1}$  in the date 1 investment game), and  $\Delta b_j^{t=2} : A^{t=1} \mapsto [1, 0]^2$  represents agent  $j$ 's probability choice over  $A_j^{t=2}$  (where  $\Delta b_j^{t=2}(a_j^{t=2})$  will be used to denote the probability agent  $j$  plays  $a_j^{t=2} \in A_j^{t=2}$  in the date 2 relationship game). The mixed (behavioral)

---

<sup>10</sup>For the above game, the set of Perfect Bayesian Nash Equilibria coincides with the set of sequential equilibria of [Kreps and Wilson \(1982\)](#) (see [Fudenberg and Tirole \(1991\)](#) for the required conditions).

strategy for agent  $j$  is thus the pair of mappings  $\Delta b_j = (\Delta b_j^{t=1}, \Delta b_j^{t=2})$  and a profile of such mixed strategies for the game is  $\Delta b = (\Delta b_1, \Delta b_2)$ . Such a profile induces a probability measure over the set of histories of the game.

A mixed profile for agent  $j$ , namely  $\Delta b_j = (\Delta b_j^{t=1}, \Delta b_j^{t=2})$ , is a best response to mixed strategy of the other agent  $\Delta b_{-j}$  if it maximizes agent  $j$ 's expected utility  $\Delta b_j = \arg \max_{\Delta \bar{b}_j} u_j(\bar{\Delta b}_j, \Delta b_{-j})$ . A strategy profile  $\Delta b$  is a *Bayesian Nash equilibrium* (BNE) if  $\Delta b_j$  is a best response to  $\Delta b_{-j}$  for  $j = 1, 2$ . If it is further required that out-of-equilibrium threats be credible, such as is required in Perfect Bayesian Nash Equilibria, then the agents in the relationship are required to make their decision whether to leave the relationship or not in the date 2 relationship game optimally with respect to some beliefs that do not contradict the common knowledge structure of the game. Let agent  $j$ 's beliefs about the type of the other agent  $-j$  when the game has reached date 2 be defined by the mapping  $\mu^j : T_{-j} \rightarrow [0, 1]^2$ .

**Definition 1 (Perfect Bayesian Nash Equilibrium).** *A strategy profile  $\Delta b$  is a Perfect Bayesian Nash equilibrium (PBNE) if:*

- (i)  $(\Delta b_1, \Delta b_2)$  are mutual best responses; and
- (ii) each agent  $j$ 's belief ( $\mu^j$ ) at date 2 about the type of the other agent is determined via a Bayesian updating of  $j$ 's prior belief on the other agent's type, where the updating is taken with respect to the actions chosen by the other agent at date 1.

In the rest of the paper we will confine ourselves to finding symmetric equilibria in pure (and mixed) strategies. Given the symmetry of the model, a solution which focuses on symmetric equilibria is intuitively reasonable. Of course, the

paper has nothing to say about the many asymmetric equilibria which also exist in this model. We should also add that we do not consider equilibria in weakly dominated strategies.

### 3.4 Solving the model

Symmetric pure strategy equilibria (if they exist) can be either separating and pooling. We consider each category in turn.

#### 3.4.1 Separating equilibrium

In this case loyal types play  $i$  at  $t = 1$  and disloyal types (obviously) play  $ni$  at  $t = 1$ . Note that on the equilibrium path each agent believes that the other agent is loyal with probability one if she sees  $i$  played by the other agent at  $t = 1$  and otherwise loyal with probability zero if she sees  $ni$  played by the other agent at  $t = 1$ . Denote agent one's belief that agent two is loyal by  $\mu^1$ . Then agent one's equilibrium-path beliefs are:

$$\mu^1 = \begin{cases} 1 & \text{if } a_2^{t=1} = i \\ 0 & \text{if } a_2^{t=1} = ni \end{cases}$$

Given these beliefs, at  $t = 2$ , a normal type of agent one will choose  $c$  when  $\mu^1 = 1$  and  $l$  when  $\mu^1 = 0$ .

In a unilateral relationship subregime the team continues only when both agents are loyal and ends when either or both agents are disloyal. The expected

payoff to agent one (of a normal type) is:<sup>11</sup>

$$p_N[\gamma r - \frac{1}{2}c] + (1 - p_N)[\gamma r - c]$$

In a joint relationship subregime the team also continues when it is comprised of only loyal types and ends whenever one or both agents is a disloyal type. Once again agent one's payoff in equilibrium is:

$$p_N[\gamma r - \frac{1}{2}c] + (1 - p_N)[\gamma r - c]$$

Note that, in a separating equilibrium, she is indifferent between the two regimes.

We now check to see that agent one doesn't seek to deviate at  $t = 1$ . Assume that agent one deviates at  $t = 1$  and plays  $ni$  instead. The investment still occurs, but now agent one has free-ridden on agent two's cost-bearing (if agent two is normal). At  $t = 2$  agent two believes that agent one is a disloyal type with probability one and chooses to leave the relationship. In a unilateral relationship subregime the relationship ends with certainty (both the normal and disloyal types leave), leading to the following expected payoff for agent one:

$$p_N[\gamma r - \delta]$$

and so agent one will only choose to deviate if:

$$p_N[\gamma r - \frac{1}{2}c] + (1 - p_N)[\gamma r - c] \geq p_N[\gamma r - \delta]$$

or

$$p_N[\gamma r - \frac{1}{2}c - \delta] \leq [\gamma r - c]$$

---

<sup>11</sup>Recall that the possible outcomes of the first stage game are:

	Player Two	
Player One	$ni, ni$	$i, ni$
	$ni, i$	$i, i$

. When describing the payoffs for a normal agent one, one must include the fact that she could be playing against either a normal or a disloyal agent two.

and so we have

$$p_N^{nf} \leq \frac{(\gamma r - c)}{(\gamma r - \frac{1}{2}c) - \delta} \quad (3.1)$$

provided that  $\delta < \delta^*$  where the superscript ‘nf’ indicates ‘no-fault’ and where we have defined  $\delta^*$  to be the value of the relationship disamenity such that  $\delta^* \equiv \gamma r - \frac{1}{2}c$ . (Note that when  $\delta > \delta^*$  then  $p_N^{nf}$  is negative and the inequality is reversed.)

In a joint relationship subregime, agent one (who believes agent two is normal with probability one) still chooses to stay in the relationship. Agent one’s payoff then is

$$p_N[\gamma r]$$

and so agent one will only choose to deviate if:

$$p_N[\gamma r - \frac{1}{2}c] + (1 - p_N)[\gamma r - c] \geq p_N[\gamma r]$$

or

$$p_N^f \leq \frac{\gamma r - c}{\gamma r - \frac{1}{2}c} \quad (3.2)$$

where the superscript ‘f’ indicates ‘fault’. We have the following proposition.

**Proposition 14 (Separating Equilibrium).**

- (i) *A symmetric pure strategy separating equilibrium exists under both regimes only if inequality 3.2 holds.*
- (ii) *It is characterized by a normal type playing  $i$  at  $t = 1$  and  $c$  at  $t = 2$ , with updated  $t = 2$  beliefs regarding the loyalty of the other agent as  $\mu_j(i) = 1$  and  $\mu_j(ni) = 0$  (for  $j = 1, 2$ ).*
- (iii) *In both regimes, in a separating equilibrium the first best efficiency is achieved.*

**Proof** Proved in the text. ■



### 3.4.2 Pooling equilibrium

In this case loyal types play  $ni$  at  $t = 1$  and disloyal types (obviously) play  $ni$  at  $t = 1$ . Note that on the equilibrium path there is no investment undertaken at  $t = 1$ . Also, on the equilibrium path each agent believes that the other agent is loyal with prior probability  $p_N$  if she sees  $ni$  played by the other agent at  $t = 1$ . That is, in equilibrium, she is not able to use the  $t = 1$  actions of the other agent to distinguish between loyal and disloyal types to any greater extent than she could before the investment subgame was played. Once again if we denote agent one's belief that agent two is loyal by  $\mu^1$  then agent one's equilibrium-path beliefs are  $\mu^1 = p_N$ . Agent one's expected utility from playing  $c$  in the  $t = 2$  subgame of the unilateral subregime is

$$EU_1(c) = p_N[p_N\chi_1(N, N|C) + (1 - p_N)\chi_1(N, N|E)] + (1 - p_N)[\chi_1(N, D|E)]$$

or

$$EU_1(c) = -\delta p_N(1 - p_N)$$

and her expected utility from playing  $l$  in the second subgame is

$$EU_1(l) = p_N[p_N\chi_1(N, N|E) + (1 - p_N)\chi_1(N, N|E)] + (1 - p_N)[\chi_1(N, D|E)]$$

or

$$EU_1(l) = -\delta p_N$$

Now

$$EU_1(c) - EU_1(l) = -\delta p_N(1 - p_N) + \delta p_N = \delta(p_N)^2 > 0$$

Hence, given her updated beliefs, agent one will play  $c$  in the relationship subgame for any positive  $p_N$  in the unilateral relationship subregime.

In addition, agent one's expected utility from playing  $c$  in the  $t = 2$  subgame of the joint subregime is

$$EU_1(c) = p_N[p_N\chi_1(N, N|C) + (1 - p_N)\chi_1(N, N|C)] + (1 - p_N)[\chi_1(N, D|C)]$$

or

$$EU_1(c) = -\delta(1 - p_N)$$

and her expected utility from playing  $l$  in the second subgame is

$$EU_1(l) = p_N[p_N\chi_1(N, N|C) + (1 - p_N)\chi_1(N, N|E)] + (1 - p_N)[\chi_1(N, D|E)]$$

or

$$EU_1(l) = -\delta p_N(1 - p_N)$$

Now

$$EU_1(c) - EU_1(l) = -\delta(1 - p_N) + \delta p_N(1 - p_N) = -\delta(p_N - 1)^2 < 0$$

Hence, given her updated beliefs, agent one will play  $l$  in the relationship subgame for any  $p_N$  in the joint relationship subregime.

On the equilibrium path therefore a normal type of agent one will receive in expectation

$$-\delta p_N(1 - p_N)$$

in a no-fault divorce regime and she will receive

$$-\delta p_N(1 - p_N)$$

in a fault divorce regime. Note that, for a pooling equilibrium, she is indifferent between the two regimes.

Given these beliefs and these  $t = 2$  actions, we now check to ensure that playing  $ni$  at  $t = 1$  is a best response. Assume that agent one deviates at  $t = 1$  and plays  $i$  instead. The investment now occurs. At  $t = 2$  agent two believes that agent one is a loyal type with probability one and so a normal agent two chooses to stay in the relationship regardless of subregime. Agent one's beliefs about agent two are still the same as previously (namely, the prior  $p_N$ ), and so in the joint relationship subregime agent one will leave and she receives in expectation

$$\gamma r - c + p_N \chi_1(N, N|C) + (1 - p_N) \chi_1(N, D|E)$$

or

$$\gamma r - c$$

while in a unilateral relationship subregime agent one will stay and so she receives in expectation

$$\gamma r - c + p_N \chi_1(N, N|C) + (1 - p_N) \chi_1(N, D|E)$$

or

$$\gamma r - c$$

Under both regimes therefore investing strictly dominates not investing at  $t = 2$ , and so we have the following proposition.

**Proposition 15 (Uniqueness).** *If a symmetric equilibrium exists, it can only be separating.*

**Proof** The sufficient conditions for the existence of a pure strategy separating equilibrium for both regimes was shown in proposition 14 above. To show uniqueness we need to show that there is no pooled and no (non-degenerate)

mixed strategy symmetric equilibria. The non-existence of a pooled pure strategy equilibrium was shown in the text above. This is true for all parameter values. The non-existence of a non-degenerate mixed strategy symmetric equilibrium follows from the fact that, in the second stage game, playing  $c$  weakly dominates playing  $l$  for a normal type agent, so that the final stage game has a unique equilibrium in pure strategies, regardless of type of relationship regime, and once equilibria in weakly dominated strategies are ruled out (as we do in this paper). This statement is proved in Appendix B. Given that both agents of the normal type will play  $c$  in the second stage game, in the first subgame playing  $i$  as a pure strategy weakly dominates any mix on  $i$  for a normal type and so the only mixed strategy is the degenerate separating strategy found in subsection 3.4.1. ■

## 3.5 Main result

### 3.5.1 The superiority of no-fault divorce

In the  $t = 1$  stage game played in isolation, both parties investing is not an equilibrium. A long term relationship can lead to cooperative investing and cost sharing. These are the separating equilibria shown in subsection 3.4.1. The question of the paper is, which type of divorce regime is better from the point of view of encouraging such cooperative, marital investments?

**Proposition 16 (The Superiority of No-Fault Divorce).** *A no-fault divorce regime expands the range of efficient separating equilibria.*

**Proof** We show that  $p_N^{nf} - p_N^f \geq 0$  for all  $\delta < \delta^*$ .

$$\begin{aligned} p_N^{nf} - p_N^f &= \frac{(\gamma r - c)}{(\gamma r - \frac{1}{2}c) - \delta} - \frac{(\gamma r - c)}{(\gamma r - \frac{1}{2}c)} \\ &= \frac{(\gamma r - c)\delta}{[(\gamma r - \frac{1}{2}c) - \delta](\gamma r - \frac{1}{2}c)} \end{aligned}$$

which is strictly positive whenever  $\delta < \delta^*$  and zero when  $\delta = 0$ . Since the range of separating equilibria are found whenever  $p_N$  is less than  $p_N^{nf}$  or  $p_N^f$  (equations 3.1 and 3.2), we have proved the proposition for  $\delta < \delta^*$ . For the case where  $\delta > \delta^*$ , the inequality in equation 3.1 reverses and the magnitude of the RHS becomes negative. This is true for the whole half-space. ■

Diagram 3.3 plots these respective (separating) equilibrium conditions in  $p_N - \delta$  space. The region in vertical (green) lines is that part of  $p_N - \delta$  space where a separating equilibrium exists for no-fault divorce rules but not for fault divorce rules. The region in diagonal (blue) lines is that part of  $p_N - \delta$  space where a separating equilibrium exists for both regimes, and the blank region is that part of  $p_N - \delta$  space where neither dissolution regime possesses a separating equilibrium (and we know from proposition 15 that no other type of symmetric equilibrium exists in that subspace). It can be seen therefore that switching from a fault to a no-fault divorce regime expands the space of efficient marital investment possibilities, as stated in proposition 16.

### 3.5.2 Trading off investment against relationship dis-amenity

From 3.1 we have the equation of the  $p_N^{nf}$  line

$$p_N^{nf} = \frac{(\gamma r - c)}{(\gamma r - \frac{1}{2}c) - \delta}$$

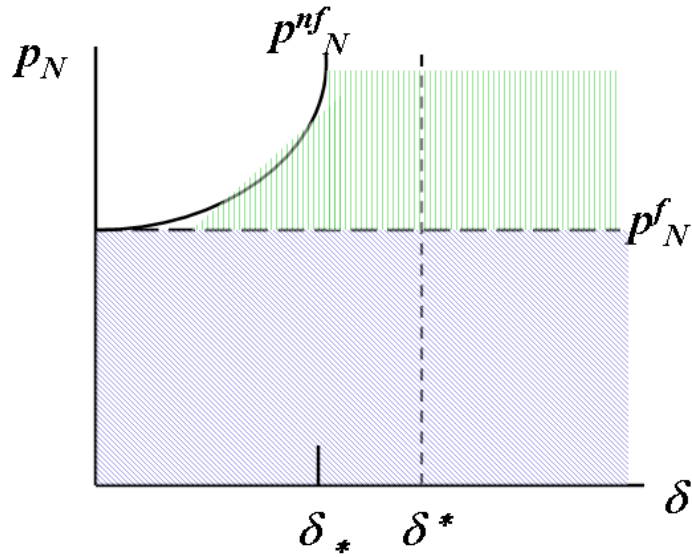


Figure 3.3: Proof of Proposition 16

Or rearranging gives

$$r = \frac{c(1 - \frac{1}{2}p_N^{nf}) - \delta p_N^{nf}}{(1 - p_N^{nf})\gamma} = \frac{c(1 - \frac{1}{2}p_N^{nf})}{(1 - p_N^{nf})\gamma} - \frac{p_N^{nf}}{(1 - p_N^{nf})\gamma}\delta$$

which is a downward-sloping straight line in  $r - \delta$  space. In order to maintain indifference between deviating from the equilibrium strategy or not, rises in the dis-amenity of being in the relationship with the wrong type ( $\delta$ ) must be matched by a lowering of the investment return ( $r$ ). Taking the derivative with respect to  $\delta$  gives us the trade-off between investment and relationship continuance in equilibrium

$$\frac{\partial r}{\partial \delta} = -\frac{p_N^{nf}}{(1 - p_N^{nf})\gamma}$$

A lowering of the riskiness of the investment (rising  $\gamma$ ) leads to a flatter tradeoff, while a rise in the likelihood of being with a loyal type (rising  $p_N$ ) makes the tradeoff steeper. Note that there is no tradeoff in the fault divorce regime.

## 3.6 Joint property subregimes

### 3.6.1 Separating versus pooling

Recall that the set of outcomes of the date 1 subgame is denoted by  $\Delta = \{I, NI\}$  (where  $I$  represents ‘invest’ and  $NI$  represents ‘not invest’). In a joint property subregime we have

$$\Delta = \begin{cases} I & \text{if } a^{t=1} = \{(i, i)\} \\ NI & \text{if } a^{t=1} = \{(i, ni), (ni, i), (ni, ni)\} \end{cases}$$

#### 3.6.1.1 Separating

When testing for  $t = 1$  deviations in the separating equilibrium, after a deviation the investment no longer occurs (unlike in the case of the unilateral property subregime). Assume agent one deviates at  $t = 1$ . Then agent two (of a normal type) believes that agent one is disloyal with probability one and chooses to leave the relationship at  $t = 2$ . Agent one still believes that agent two is loyal and so chooses to remain in the relationship at  $t = 2$ . In that case her payoff is  $-p_N\delta$  in the unilateral relationship subregime and zero in the joint relationship subregime. Both of these deviation payoffs need to be compared to

$$p_N[\gamma r - \frac{1}{2}c] - (1 - p_N)c$$

(which reduces to  $p_N(\gamma r + \frac{1}{2}c) - c$ ), the equilibrium payoff (the same across relationship regimes). The sign of  $p_N(\gamma r + \frac{1}{2}c) - c$  is ambiguous, depending on which

part of parameter space the model is located in. There are three possibilities.

**Proposition 17 (Existence).**

- (i)  $p_N(\gamma r + \frac{1}{2}c) - c > 0$  : *In this parameter range a separating equilibrium exists for both types of divorce regimes.*
- (ii)  $0 > p_N(\gamma r + \frac{1}{2}c) - c > p_N\delta$  : *In this parameter range there is a separating equilibrium in the no-fault divorce regime but not in the fault divorce regime.*
- (iii)  $-p_N\delta > p_N(\gamma r + \frac{1}{2}c) - c$  : *In this parameter range neither divorce regime possesses a separating equilibrium.*

**Proof** The proof follows from comparing deviation and equilibrium payoffs under each type of divorce regime. ■

### 3.6.1.2 Pooling

In the pooling equilibrium, deviating at  $t = 1$  by making an investment does not lead to investment occurring as it did in subsection 3.4.2. If agent one deviates, agent two now knows that she is loyal with probability one and so agent two (of a normal type) always chooses to stay in the  $t = 2$  relationship subgame. Agent one's beliefs about agent two remain as previously, so that she always leaves in the joint relationship regime and always stays in the unilateral relationship regime. Consequently, deviating gives her zero under both relationship subregimes whereas in equilibrium she gets  $-\delta p_N(1 - p_N)$  (a negative number) under both relationship subregimes. Hence deviating strictly dominates playing the equilibrium pooling strategy under a joint property subregime, regardless of divorce laws. The non-existence of a pooling equilibrium from proposition 15 still holds.



**Proposition 18 (Uniqueness).** *If a symmetric equilibrium exists, it is separating.*

**Proof** The above paragraph showed that a pooling equilibrium does not exist for any parameter values. To show that a mixed symmetric equilibrium also does not exist, we need to show that there is no non-degenerate mixed strategy equilibrium. The proof is similar to that given in proposition 15 and once again relies on ruling out equilibria in weakly dominated strategies. (See also Appendix B.) ■

### 3.6.2 Comparison of property regimes

We are now ready to compare all four types of regimes. Conditional on type of property subregime, in (separating) equilibrium, a normal agent obtains the same payoffs in either divorce subregime. Hence we need only compare the two different types of property subregime (under a separating equilibrium). We have

$$\begin{aligned} EU_{unilateral} - EU_{joint} &= p_N[\gamma r - \frac{1}{2}c] + (1 - p_N)[\gamma r - c] - \{p_N(\gamma r + \frac{1}{2}c) - c\} \\ &= \gamma r(1 - p_N) > 0 \end{aligned}$$

Hence a unilateral property subregime pareto dominates (in equilibrium) a joint property subregime. The intuition for this is obvious, since under a joint property subregime, a normal type playing the equilibrium with a disloyal type bears all the cost on her own without the benefits of reward for effort.

The equilibrium condition in the no-fault case is (confining attention to the case where a separating equilibrium exists for both types of divorce regime)

$$p_N^{nf}(\gamma r + \frac{1}{2}c) - c - \{-p_N^{nf}\delta\} \geq 0$$

or

$$p_N^{nf} \geq \frac{c}{(\gamma r + \frac{1}{2}c) + \delta} \quad (3.3)$$

Compare this with equation 3.1 from subsection 3.4.1 for the unilateral property subregime

$$p_N^{nf} \leq \frac{(\gamma r - c)}{(\gamma r - \frac{1}{2}c) - \delta}$$

Notice in particular that the inequality is reversed and that the slope with respect to  $\delta$  is negative rather than positive.

The equilibrium condition in the fault case is

$$p_N^f(\gamma r + \frac{1}{2}c) - c - 0 \geq 0$$

or

$$p_N^f \geq \frac{c}{\gamma r + \frac{1}{2}c} \quad (3.4)$$

Once again the  $p_N^f$  curve does not depend on  $\delta$ . Comparing equations 3.3 and 3.4 we obtain

$$\begin{aligned} p_N^{nf} - p_N^f &= \frac{c}{(\gamma r + \frac{1}{2}c) + \delta} - \frac{c}{\gamma r + \frac{1}{2}c} \\ &= \frac{-c\delta}{[(\gamma r + \frac{1}{2}c) + \delta](\gamma r + \frac{1}{2}c)} < 0 \end{aligned}$$

So that in the case of a joint property subregime, as in the case under the unilateral property subregime (though the reasoning is different), a no-fault divorce regime is better than a fault regime - better in the sense that the efficient separating equilibrium is possible for a larger range of parameters. Figure 3.4 is the analogous diagram for the joint property case to figure 3.3. Once again, in the figure the vertical (green) lines represent that part of parameter space in which a separating equilibrium exists under the no fault divorce regime but not the fault divorce regime, while the diagonal (blue) lines represent that part of parameter

space were a separating equilibrium exists for both types of divorce regime. The blank region is where there does not exist a separating equilibrium for either type of divorce regime. From a comparison of diagrams 3.3 and 3.4 it can be seen that a unilateral property subregime is better for low  $p_N$  (a society with more disloyal types: modern industrial societies, secular societies), and a joint property subregime is better for higher  $p_N$  (a society with fewer disloyal types: traditional societies, religious societies).

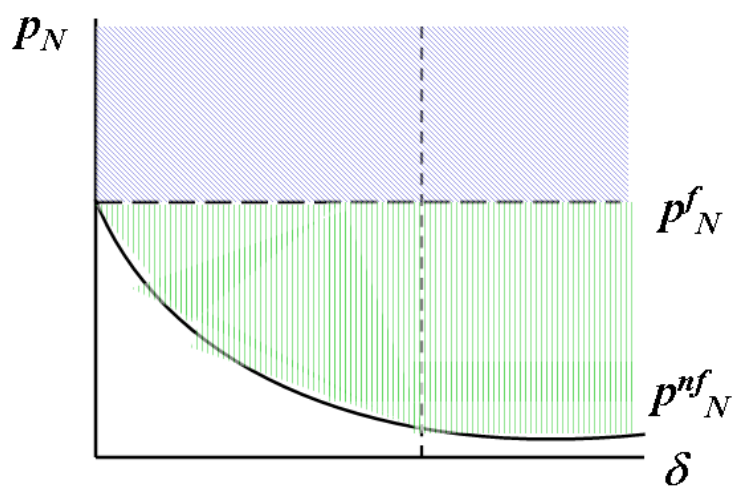


Figure 3.4: Separating equilibrium under both divorce regimes in the joint property case

### 3.7 Conclusion

This paper explores the tradeoff in marriages between investing in marital investment opportunities versus the dis-amenity of remaining in the marriage with a disliked spouse. The paper uses a finite sequential voting model in which spouses first vote to invest and then vote to remain in the marriage. The paper focuses solely on symmetric equilibria and shows that whenever there exists a symmetric equilibrium it must be separating. This is the equilibrium in which ‘loyal’ spouses invest and stay in the relationship while ‘disloyal’ spouses do not invest and the marriage ends in divorce. Under both types of marital property regimes (unilateral or joint), a no-fault divorce regime is superior to a fault divorce regime in the sense that it expands the range of parameters under which such an equilibrium exists. It is shown that, for a given level of disloyalty (or divorce rate) within a society, intra-marital welfare is higher under the unilateral than under the joint marital property regimes. It is also shown that for societies with high levels of spousal ‘disloyalty’ (or divorce rates), such as modern, secular Western countries, a unilateral marital property regime is preferable as a matter of policy to a joint marital property regime, and that for societies with low levels of spousal ‘disloyalty’ (or divorce rates), such as traditional, religious societies, this policy preference is reversed and the joint marital property regime is superior. Obviously any policy implications to be drawn from the model and conclusions of this paper would need to be made in the awareness of the stylized nature of the distinction drawn between joint and unilateral property regimes, and how that stylization compares when placed against the subtleties involved in the greater cross-state and cross-title heterogeneity described in section 3.2 above.

## APPENDIX A

### The generalized Nash bargaining of chapter 1

Let  $g_E^i$  denote the share of ex post surplus obtained by the entrepreneur during renegotiation (where  $i = A, H$  or  $AH$ ). Correspondingly, let  $g_B^i$  denote the share of the ex post surplus obtained by the bank and  $g_G^i$  the share of the ex post surplus obtained by the guarantor. Obviously  $g_E^i + g_B^i + g_G^i = \Pi^i$ . We always assume that the bank is exactly compensated for giving up its right to liquidate asset  $i$  so that  $g_B^i \equiv L^i$ . Let the exogenous bargaining powers of the entrepreneur and guarantor be  $\tau_E$  and  $\tau_G$  respectively, where  $\tau_E + \tau_G = 1$ . The generalized Nash bargaining problem for two agents takes the following form

$$\max_{g_E^i, g_G^i} \phi \equiv (g_E^i)^{\tau_E} (g_G^i)^{1-\tau_E}$$

subject to

$$g_E^i + g_G^i = \Pi^i - L^i$$

The first order condition is

$$-\frac{\phi(1-\tau_E)}{\Pi^i - L^i - g_E^i} + \frac{\phi\tau_E}{g_E^i} = 0$$

which after manipulation gives

$$g_E^i = \tau_E(\Pi^i - L^i)$$

Thus the entrepreneur's share of the ex post renegotiation surplus increases as his power within the relationship increases.

# APPENDIX B

## Mathematical proofs

### B.1 Proofs for chapter 1

#### Proof of Proposition 1 (Contract Characterization)

(i) From  $P_0 \leq 0$  in (1.5) we have either  $P_0 = 0$  or  $P_0 < 0$ . Assume the latter.

This means that the bank (it cannot be the guarantor, who also has zero (liquid) wealth) pays the entrepreneur something when  $R_1 = 0$ . But then it would be more socially efficient (since foreclosing on either the project or relationship asset is always inefficient) to increase  $P_0$  and so reduce  $\beta_0$ . Hence  $P_0 = 0$ .

(ii) For the cases of  $i = AH$  and  $A$ : From (1.6) we know that  $y_0 \geq 0$ . The amount  $y_0$  is paid at date 2 when the entrepreneur pays  $P_0$  at date 1. Under the assumption that the contractual terms are carried out as intended ex ante, such a payment occurs only when  $R_1 = 0$ . Under that scenario the assets are foreclosed with probability  $\beta_0$  and not foreclosed with probability  $1 - \beta_0$ . In the former case  $y_0 = 0$  since there is no income from either date 1 or date 2 with which to make the payment. In the latter case date 2 income ( $r$ ) does accrue to the entrepreneur so that a positive payment is not infeasible, but the assumption on the contracting technology made in subsection 1.2.1

(namely, that the parties to the contract are constrained to stipulate the same amount,  $y_0$ , in both cases) means that the agents choose the lesser amount when designing the contract at date 0. Hence  $y_0 = 0$ . Note that this last part of the proof only applies to the cases where the business asset is secured. When it is not secured (as in the case of  $i = H$ ) then even when the (relationship) asset is foreclosed, the business asset still exists to provide a date 2 return of  $r$ . The indicator function  $\kappa_r^i$  can be used to encapsulate all three models in one term, as shown in part (ii) of the proposition.

(iii) Suppose to the contrary that  $\beta_x$  is strictly positive at an optimum. Now reduce  $\beta_x$  by some infinitesimal amount, say  $\epsilon$ , without thereby changing the bank's and guarantor's payoffs (this can be effected if we simultaneously ensure that  $P_x$  is increased by  $\epsilon L^i$  in the bank's payoff and  $y_x$  is decreased by  $\epsilon[\kappa_z^i(1 - S^E)z - g_G^i]$  in the guarantor's payoff). With these changes, the entrepreneur's payoff changes by  $-\theta\epsilon[L^i - \kappa_z^i(1 - S^E)z + g_G^i - (\kappa_r^i r + \kappa_z^i S^E z) + g_E^i]$  and the LHS of the renegotiation constraint changes by  $-\epsilon[L^i - \kappa_z^i(1 - S^E)z + g_G^i - (\kappa_r^i r + \kappa_z^i S^E z) + g_E^i]$ . Hence, the assumption which began this proof will be contradicted provided that  $[L^i - \kappa_z^i(1 - S^E)z + g_G^i - (\kappa_r^i r + \kappa_z^i S^E z) + g_E^i] < 0$ . Note that  $g_G^i + g_E^i = \Pi^i - L^i$ . Incorporating this and rearranging gives  $\Pi^i - (\kappa_r^i r + \kappa_z^i z)$  which can easily be verified as negative for each case of  $i$ . These changes therefore strictly increase the entrepreneur's payoff while slackening the renegotiation constraint. Hence we have shown the contradiction in the assumption that a strictly positive  $\beta_x$  can be an optimum.

(iv) The bank's individual rationality constraint binds at an optimum since, if it did not, it would be possible to decrease  $P_x$  and consequently raise the

entrepreneur's payoff. Such a change would not effect the guarantor's payoff and would slacken the renegotiation constraint.

The guarantor's individual rationality constraint binds at an optimum since if it did not, it would be possible to decrease  $y_x$  and consequently raise the entrepreneur's payoff. Such a change would not effect the bank's payoff and would slacken the renegotiation constraint.

- (v) Suppose to the contrary that (1.4) is slack. We solve for the optimal contract assuming this and show that the solution to this relaxed program violates the renegotiation constraint. Using the results of parts (i)-(iv) of proposition 1 the optimization problem ( $\star^{(i)}$ ) can be reformulated as choosing over  $[P_x, \beta_0, y_x, y_0]$  to maximize

$$\begin{aligned} & \theta[x - P_x + r - y_x + S^E z] \\ & + (1 - \theta)[-y_0(1 - \kappa_r^i) + (1 - \beta_0 \kappa_r^i)r + (1 - \beta_0 \kappa_z^i)S^E z] \end{aligned} \quad (\text{B.1})$$

subject to:

$$\theta P_x + (1 - \theta)\beta_0 L^i - K = 0 \quad (\text{B.2})$$

$$\theta[y_x + (1 - S^E)z] + (1 - \theta)[y_0(1 - \kappa_r^i) + (1 - \beta_0 \kappa_z^i)(1 - S^E)z] - (1 - S^E)z = 0 \quad (\text{B.3})$$

$$\begin{aligned} & x - P_x + r - y_x + S^E z \\ & \geq x - y_0(1 - \kappa_r^i) + (1 - \beta_0 \kappa_r^i)r + (1 - \beta_0 \kappa_z^i)S^E z + \beta_0 g_E^i \end{aligned} \quad (\text{B.4})$$

$$P_x \leq x \quad (\text{B.5})$$



$$0 \leq y_0 \leq r \text{ and } 0 \leq y_x \leq x + r - P_x \quad (\text{B.6})$$

$$0 \leq \beta_0 \leq 1 \quad (\text{B.7})$$

Ignoring the renegotiation constraint (B.4), we can substitute (B.3) and (B.2) into (B.1) to obtain (after some manipulation and collecting the  $\beta_0$  terms) a reformulated objective function in terms of  $\beta_0$

$$\theta x - K + [r + S^E z] - (1 - \theta)\beta_0[\kappa_r^i r + \kappa_z^i z - L^i]$$

This objective function is linear in  $\beta_0$  so we have a corner solution. The feasibility constraint (B.7) on  $\beta_0$  means that the entrepreneur's payoff is maximized when  $\beta_0 = 0$  because  $[\kappa_r^i r + \kappa_z^i z - L^i]$  is positive for each of the three cases of  $i$  (as is easily be verified). Consequently, from (B.2) and (B.3), when  $\beta_0 = 0$  we have that  $P_x = \frac{K}{\theta}$  (which does not violate (B.5)) and that  $y_x = -\frac{(1 - \theta)}{\theta}y_0(1 - \kappa_r^i)$ . Returning now to the renegotiation constraint (B.4), it can be rewritten as

$$P_x + y_x < \beta_0[\kappa_r^i r + \kappa_z^i S^E z - g_E^i]$$

which, after substituting in the solutions  $\beta_0 = 0$ ,  $P_x = \frac{K}{\theta}$  and  $y_x = -\frac{(1 - \theta)}{\theta}y_0(1 - \kappa_r^i)$ , gives

$$\frac{K}{\theta} - \frac{(1 - \theta)}{\theta}y_0(1 - \kappa_r^i) < 0 \quad (\text{B.8})$$

For the two cases of  $i = A$  and  $AH$ ,  $\kappa_r^i = 1$  and this provides the required contradiction. For the case of  $i = H$  we have that  $\kappa_r^i = 0$  so that (B.8) becomes

$$\frac{K}{\theta} - \frac{(1 - \theta)}{\theta}y_0 < 0$$

which implies that  $K < (1 - \theta)y_0$  which gives the required contradiction. ■

**Proof of Proposition 2 (Contractual Inefficiency)** Using the results of proposition 1 the optimization problem ( $\star^{(i)}$ ) can be reformulated (as in the proof of part (v) of proposition 1) as choosing over  $[P_x, \beta_0, y_x, y_0]$  to maximize

$$\begin{aligned} & \theta[x - P_x + r - y_x + S^E z] \\ & + (1 - \theta)[-y_0(1 - \kappa_r^i) + (1 - \beta_0 \kappa_r^i)r + (1 - \beta_0 \kappa_z^i)S^E z] \end{aligned} \quad (\text{B.9})$$

subject to:

$$\theta P_x + (1 - \theta)\beta_0 L^i - K = 0 \quad (\text{B.10})$$

$$x - P_x + r - y_x + S^E z = x - y_0(1 - \kappa_r^i) + (1 - \beta_0 \kappa_r^i)r + (1 - \beta_0 \kappa_z^i)S^E z + \beta_0 g_E^i \quad (\text{B.11})$$

$$P_x \leq x \quad (\text{B.12})$$

$$0 \leq y_0 \leq r \text{ and } 0 \leq y_x \leq x + r - P_x \quad (\text{B.13})$$

$$0 \leq \beta_0 \leq 1 \quad (\text{B.14})$$

We prove each part of the proof in turn.

(i) Now both (B.10) and (B.11) can be rewritten as linear functions of  $\beta_0$  as follows

$$P_x = \frac{1}{\theta}K - \frac{(1 - \theta)}{\theta}\beta_0 L^i \quad (\text{B.15})$$

$$y_x = \frac{1}{\theta} \{ (1 - S^E)z(1 - \theta) - (1 - \theta)[y_0(1 - \kappa_r^i) + (1 - \beta_0\kappa_z^i)(1 - S^E)z] \} \quad (\text{B.16})$$

Substituting (B.15) and (B.16) into (B.9) gives (after manipulation) the entrepreneur's expected payoff from the contract

$$\theta x - K + (r + S^E z) - (1 - \theta)\beta_0[\kappa_r^i r + \kappa_z^i z - L^i] \quad (\text{B.17})$$

where the first three terms are the net present value of the project in the first best case of no liquidation, while the last term is the expected efficiency loss from the incompleteness of the contract, labeled  $EL^i$  in the proposition. This completes the proof of part (i) of the proposition.

- (ii) The renegotiation constraint (B.12) can also be rewritten as a linear function of  $\beta_0$  to give

$$P_x = y_0(1 - \kappa_r^i) - y_x + \beta_0[\kappa_r^i r + \kappa_z^i S^E z - g_E^i] \quad (\text{B.18})$$

Substituting (B.15) and (B.16) into (B.18) gives (after manipulation) the following reformulated renegotiation constraint

$$\beta_0 = \frac{K - y_0(1 - \kappa_r^i)}{\theta[\kappa_r^i r + \kappa_z^i S^E z - g_E^i] + (1 - \theta)[L^i - \kappa_z^i(1 - S^E)z]} \quad (\text{B.19})$$

The new linear program is to choose  $\beta_0$  and  $y_0$  to maximize (B.17) subject to (B.19) and (B.14). There are two cases. In the first case, when  $i = AH$  or  $A$ ,  $\kappa_r^i = 1$  and so the program is equivalent to choosing the minimum  $\beta_0$  compatible with (B.17) and (B.14). In the second case when  $i = H$ ,  $\kappa_r^i = 0$  and so any positive  $y_0$  is a possible solution. The  $y_0$  which minimizes  $\beta_0$  is  $y_0 = r$  (the maximum possible  $y_0$  in its range). In either case (B.19) is the solution provided that it falls between zero and one, which proves part (ii) of the proposition. ■

**Proof of Proposition 5 (Risk Profile: Comparing Assets  $A$  and  $H$ )**

Taking the derivative of (1.11) with respect to  $\theta$  we get

$$K[1 - \alpha]\{1 - \lambda - \tau_E(1 - 2\lambda)\} - \{K - r\}[1 - \lambda]\{1 - \alpha - \tau_E(1 - 2\alpha)\} > 0$$

so that when  $K < r$  we have that  $\Delta EL_H^A$  is increasing in  $\theta$  for all values of  $\theta$ . It follows that the entrepreneur secures the project asset when  $\theta < \tilde{\theta}_{A|H}$  and secures the relationship asset when  $\theta > \tilde{\theta}_{A|H}$ . ■

**Proof of Proposition 6** We prove only part (i). Part (ii) is proved via basic calculus. The sign of the derivative of  $\tilde{\theta}_{A|H}$  (with respect to  $r$ ) depends on the sign of the numerator, or more specifically depends on the sign of

$$K(1 - \alpha)(S^E - \tau_E(1 - 2\lambda)) - (K - r)(1 - \lambda)(1 - 2\alpha)(1 - \tau_E)$$

which is positive owing to the assumptions in the proposition. ■

**Proof of Proposition 7 (Risk Profile for Pure Personal Asset Case)**

- (i)  $\theta_X$  is obtained via manipulation of the denominator of (1.10) after substituting in the relevant parameters. This modified version of equation (1.10) is then set equal to zero and the  $\theta$  terms gathered on the LHS. Since (1.10) must be positive, the sign of the denominator of (1.10) will depend on the sign of the numerator (which case we are dealing with -  $K$  greater than or less than  $r$ ). This gives the two cases mentioned in the proposition.
- (ii) The case of  $K > r$  involves comparison of  $\theta^*$  and  $\theta_X$ . Since it is not rational for the entrepreneur to invest whenever  $\theta < \theta^*$ , feasible investments are only prevented when  $\theta^* < \theta_X$ . For the case of  $K < r$  we have that  $\theta^*$  can't be

negative (which it will be when  $K < r$ ) and so must instead be at its lower limit, namely  $\theta^* = 0$ . Thus all projects are ex ante viable and so from part (i) of the proposition,  $[\theta_X, 1]$  must be the non-empty set of unfundable risk profiles. For the final statement we need the equivalent of (1.10) for the case of asset  $A$ , namely

$$\beta_0^A \equiv \frac{K}{\theta r[1 - \tau_E(1 - 2\alpha)] + (1 - \theta)\alpha r}$$

and it is easily verifiable that both the numerator and the denominator are always positive.

(iii) We examine each case in turn.

Case  $S^E$ : Taking the derivative we have that

$$\frac{\partial \theta_X}{\partial S^E} = \frac{-1}{1 - \lambda - \tau_E(1 - 2\lambda)}$$

which is everywhere negative by the fact that the denominator of  $\theta_X$  is positive. Hence  $\theta_X$  is decreasing in  $S^E$ .

Case  $\lambda$ : The derivative gives (after manipulation of the numerator)

$$\frac{\partial \theta_X}{\partial \lambda} = \frac{\tau_E(2S^E - 1) - S_E}{[1 - \lambda - \tau_E(1 - 2\lambda)]^2}$$

The denominator is obviously positive so that the sign depends only on the numerator which is always negative and so  $\theta_X$  is decreasing in  $\lambda$ .

Case  $\tau_E$ : The derivative gives

$$\frac{\partial \theta_X}{\partial \tau_E} = \frac{(1 - S^E - \lambda)(1 - 2\lambda)}{[1 - \lambda - \tau_E(1 - 2\lambda)]^2}$$

Since by fact or assumption the other terms are positive, the sign of the derivative depends on  $(1 - 2\lambda)$ . This gives  $\lambda = \frac{1}{2}$  as the cut-off and accordingly gives the step function of the proposition. ■

## B.2 Proof that the relationship stage game of chapter 3 has a unique (stage game) equilibrium when equilibria in weakly dominated strategies are ruled out

**Proof** To show that the final stage game has a unique equilibrium if we are willing to rule out equilibria in weakly dominated strategies, recall that at the beginning of the  $t = 2$  subgame the space of  $t = 1$  histories (that is,  $A^{t=1}$ ) can be classified into the following four cases:

	Player Two	
Player One	$ni, ni$	$i, ni$
	$ni, i$	$i, i$

Label the cells in the above table as

	Player Two	
Player One	$A$	$B$
	$C$	$D$

Now let  $\mu_m^1 \equiv \Pr(\text{loyal}_2 | m = a^{t=1})$  be agent one's belief at  $t = 2$  given that the outcome of the  $t = 1$  investment game is  $m$ . For  $m = \{(ni, i), (i, i)\}$  we have that  $\mu_m^1 = 1$ , since a disloyal type does not mix and so  $i$  cannot be the outcome of a randomized strategy for that type. In those cases agent one (of a normal type) will strictly prefer play  $c$  in the second stage game. Hence we need only define agent one's beliefs for the two cases of  $m = \{(ni, ni), (i, ni)\}$ :

	<b>Agent Two</b>	
	$\Pr(\text{loyal}_2 m = \{(ni, ni), (i, ni)\})$	$\Pr(\text{disloyal}_2 m = \{(ni, ni), (i, ni)\})$
<b>Agent One</b>	$\mu_m^1 \equiv \frac{p_N(1-p^2)}{p_N(1-p^2)+(1-p_N)1}$	$1 - \mu_m^1$

where  $p^j$  is the probability that agent  $j$  invests in the  $t = 1$  investment subgame. In a mixed strategy equilibrium we have the following

$$\overline{EU}_1(c) = \overline{EU}_1(l)$$

where the upper bar indicates that the expected utilities are with respect to the second subgame payoffs only.

(i) Unilateral relationship regime:

We have four cases to consider. The  $D$  case is the simplest. Both agents believe that the other is a loyal type with probability one, and hence they both (when of the normal type) play  $c$ , and otherwise  $l$ . So the relationship continues iff they are both normal and ceases iff they are both disloyal. The cases of  $B$  and  $C$  are symmetric, so we need only consider one. We consider case  $B$ . In this case  $\mu_B^2 = 1$  and  $\mu_B^1 = \frac{p_N(1-p^2)}{p_N(1-p^2)+(1-p_N)1}$ . Player two, given his beliefs, plays  $c$  (if he is a normal type). Player one is indifferent between playing  $c$  or  $l$  if

$$\mu_B^1 \chi_1(N, N|C) + (1 - \mu_B^1) \chi_1(N, D|E) = \mu_B^1 \chi_1(N, N|E) + (1 - \mu_B^1) \chi_1(N, D|E)$$

or

$$0 = -\mu_B^1 \delta$$

which is a contradiction and hence agent one always prefers to play  $c$ . Once again the relationship continues iff both agents are loyal and ends iff both agents are disloyal. We are left with the  $A$  case, in which both agents have non-degenerate

beliefs of  $\mu_A^j$  for  $j = 1, 2$ , and in which a mixed equilibrium can be supported. Hence in three out of the four classifications of the history a normal agent strictly prefers to play  $c$ , while in the remaining one he is at best indifferent.

(ii) Joint relationship regime:

The existence of the joint relationship subregime instead of the unilateral relationship subregime makes unilateral deviations a matter of indifference for the agent deviating rather than, as before, being strictly dominated. The additional equilibria thereby created are ruled out since they are all in weakly dominated strategies. ■



# APPENDIX C

## Figures for chapter 1

Panel A: Sources of Borrowing: By Size										
Book Value of Assets (\$1,000)	Assets Percentile	Percentage of Firms with Debt	Debt (\$1,000)		Fraction Borrowed from Each Source					
			Mean	Median	Bank	Nonbank Financial Institution	Owner	Family	Other Firms	Fraction of Institutional Debt from Close Lenders
Less than 15	0-10	0.34	9	6	0.51	0.10	0.13	0.14	0.04	0.45
15-46	10-25	0.55	17	12	0.56	0.12	0.11	0.09	0.03	0.52
46-130	25-50	0.71	36	28	0.58	0.11	0.09	0.11	0.03	0.55
130-488	50-75	0.82	107	80	0.55	0.11	0.11	0.09	0.04	0.54
488-2,293	75-90	0.91	438	300	0.60	0.12	0.10	0.05	0.03	0.61
Over 2,293	90-100	0.91	2933	1585	0.62	0.14	0.07	0.03	0.04	0.62
<i>F</i> -statistic					2.10	0.40	1.59	6.07	0.21	2.74
<i>p</i> -value					0.06	0.85	0.16	0.00	0.96	0.02

Panel B: Sources of Borrowing: By Age										
Less than 2	0-10	0.79	648	40	0.49	0.09	0.10	0.17	0.05	0.52
2-5	10-25	0.77	395	61	0.54	0.12	0.14	0.11	0.03	0.60
5-10	25-50	0.77	334	53	0.58	0.11	0.10	0.09	0.04	0.57
10-19	50-75	0.74	410	54	0.63	0.12	0.08	0.05	0.03	0.57
19-30	75-90	0.71	695	96	0.60	0.14	0.10	0.04	0.04	0.55
Over 30	90-100	0.59	912	128	0.52	0.15	0.10	0.06	0.04	0.50
<i>F</i> -statistic					5.72	1.19	2.24	13.10	0.97	1.24
<i>p</i> -value					0.00	0.31	0.05	0.00	0.44	0.29

Figure C.1: Source Petersen and Rajan (1994). 1987 NSSBF.

Table 2  
 Percent distributions of sample loans and loan dollars by collateral and guarantees, by year,  
 NSSBF <sup>a</sup>

	1987		1993	
	Loans	Dollars	Loans	Dollars
Unguaranteed and unsecured	15.2	10.8	9.6	4.2
Unguaranteed and secured by:				
Business collateral only	56.9	50.0	44.7	37.0
Personal collateral only	1.8	0.9	3.7	1.8
Business and personal collateral	2.1	2.4	1.1	1.6
Total unguaranteed and secured	60.8	53.3	49.5	40.4
Personal Guarantee and Secured by:				
Business collateral only	17.4	28.4	23.8	37.9
Personal collateral only	0.9	0.4	4.4	5.4
Business and personal collateral	0.6	2.0	1.8	4.6
Total personal guarantee and secured	18.9	30.7	30.0	47.8
Personal guarantee and unsecured	5.0	5.2	10.9	7.4
Total loans	100.0	100.0	100.0	100.0

<sup>a</sup> Excludes leases, trade credit, credit card debt, and loans from owners.

Figure C.2: Source [Avery et al. \(1998\)](#). 1987 and 1993 NSSBF. 1993 dollars.

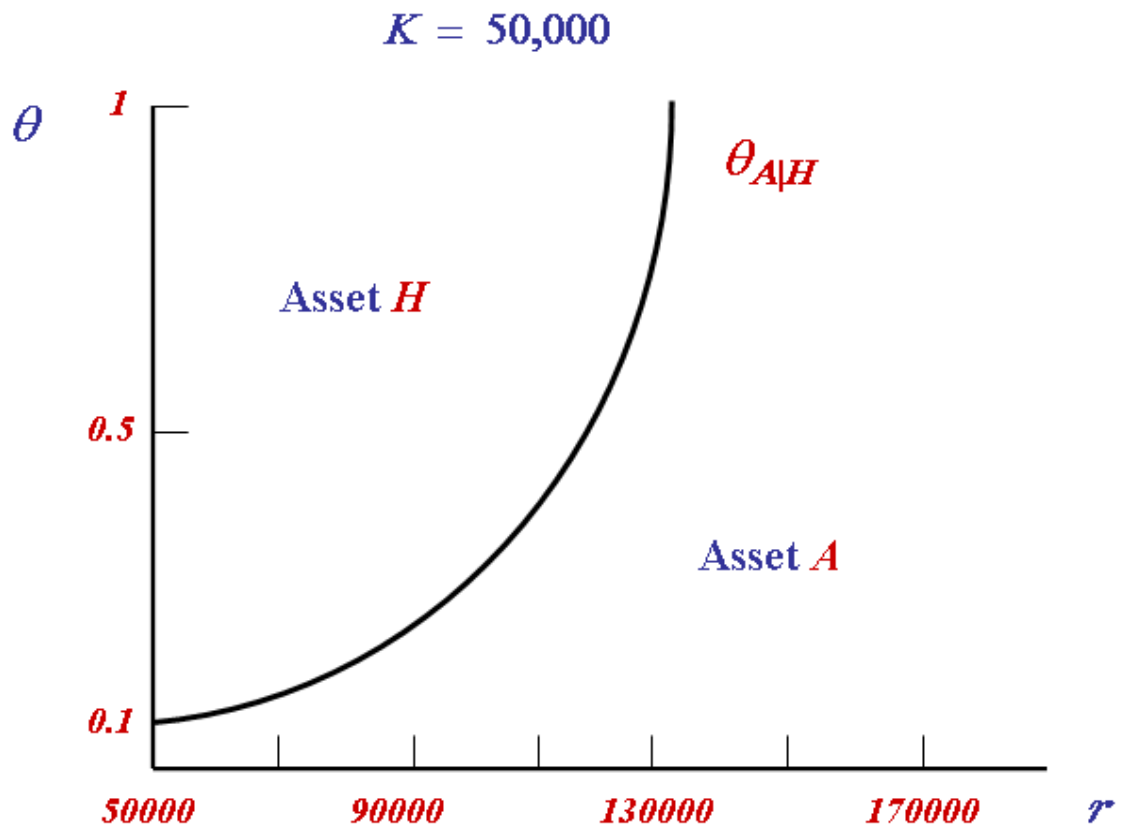


Figure C.3: The pattern of collateralized asset choice in risk-growth space, for  $K = \$50,000$ .

TABLE A1  
STARTING CAPITAL FOR NEW BUSINESS OWNERS BETWEEN 1980 AND 1987

INDUSTRY	STARTING CAPITAL VALUE			FIRMS IN INDUSTRY CATEGORY (%)
	First Quartile	Median	Third Quartile	
Low-starting capital industries:				
Construction	\$2,860	\$9,500	\$30,100	10.9
Services	\$3,450	\$19,400	\$62,719	30.3
High-starting capital industries:				
Mining	\$1,730	\$37,800	\$394,375	1.2
Transportation, communication, and public utilities	\$15,120	\$47,300	\$143,300	3.0
Finance, insurance, and real estate	\$7,900	\$36,500	\$173,260	4.8
Manufacturing	\$16,165	\$47,300	\$151,200	7.9
Wholesale trade	\$11,010	\$41,400	\$145,860	8.5
Retail trade	\$21,880	\$55,200	\$118,150	33.3

NOTE.—Data are taken from the 1987 National Survey of Small Business Finances. All values are reported in 1996 dollars. The sample includes 1,099 small businesses.

Figure C.4: Source: [Hurst and Lusardi \(2004\)](#) Table A1, 1987 NSSBF, 1996 dollars

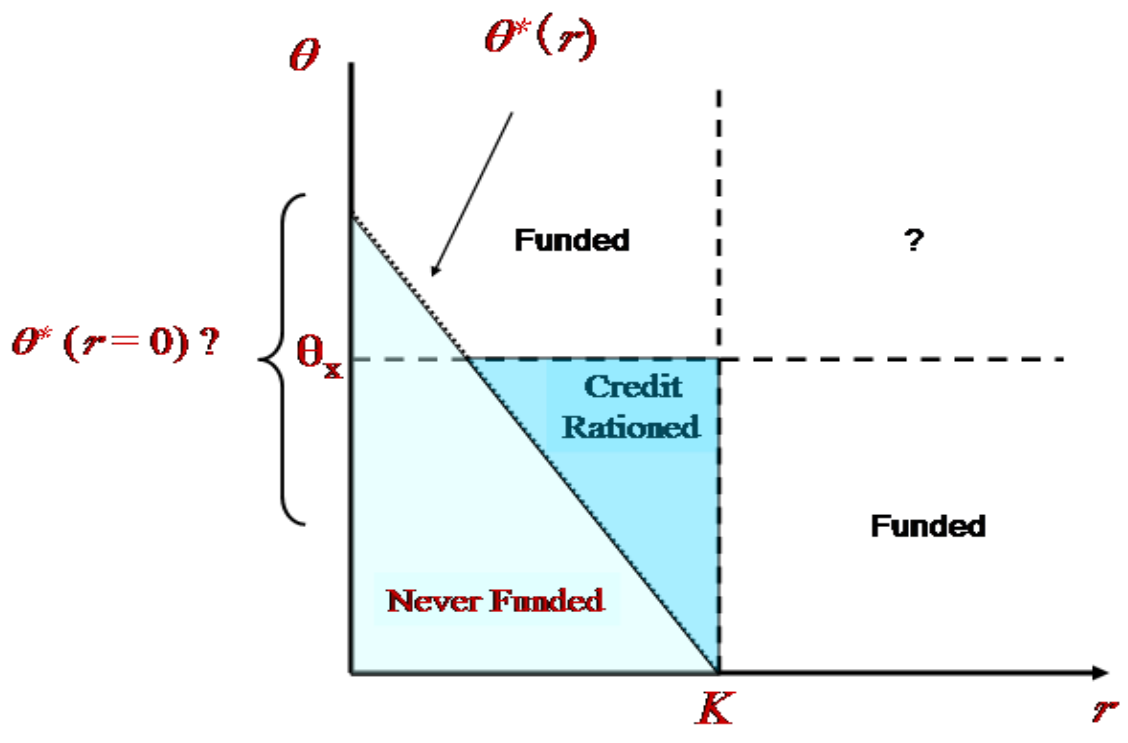


Figure C.5: Diagram depicting proposition 7 in section 1.5.

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