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On Information and Competition in Private Value Auctions

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Abstract

This paper studies the relationship between the auctioneer's provision of information and the level of competition in private value auctions. We use a general notion of informativeness which allows us to compare the efficient with the (privately) optimal amount of information provided by the auctioneer. We show that it is not optimal for the auctioneer to provide the efficient level of information. We also look at the effect of competition as parameterized by the number of participants in the auction. We find that both the optimal and the efficient level of information increase with the number of participants in the auction, and both converge when the number of bidders goes to infinity.

Keywords: Auctions, Competition, Private Values, Optimal and Efficient Provision of Information.

JEL classification numbers: D44, D82, D83.

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1 Introduction

This paper studies the auctioneer's incentives to provide information to bidders in private value settings and, more importantly, the relationship between the provision of information and the level of competition in the market measured by the number of bidders participating in the auction. We show that it is not optimal for the auctioneer to provide the efficient level of information, that both the optimal and the efficient level of information precision increase with the number of participants in the auction, and both converge when the number of bidders goes to infinity.

There are many situations in which the auctioneer can to some extent affect bidders' information. Take Internet auctions for example. In most of these auctions sellers have most of the information about the goods on sale and they decide how much information to reveal to bidders by posting electronic images, providing text descriptions, etc. Similarly, governments soliciting bids to execute a public project or a company selling a subsidiary have a lot of information on the goods at auction and control how and how much will reach bidders.

The question of whether an auctioneer with information that is useful to bidders should keep it hidden or disclose it has a very powerful answer, the so called "linkage principle" derived by Milgrom and Weber (1982) in an affiliated values environment. According to the linkage principle the expected-revenue-maximizing policy for the auctioneer is to commit to fully and publicly announce all information he has. Thus, the level of information provided to the market is the efficient one and is independent of competition.

As we will demonstrate below, this result does not hold in private value auctions. The reason is that the effect of disclosing information to bidders prior to the auction is fundamentally different in common value versus private value settings. In common value environments, bidders' preferences are aligned and bidders react symmetrically to the information revealed by the auctioneer and the revelation of information does not generate bidder rents. In private value environments bidders have heterogenous preferences, and their valuations depend on the match between their preferences and the characteristics of the object. This means that any information revealed by the auctioneer will be perceived differently by different bidders, raising the valuation of some bidders while reducing that of others. This asymmetric reaction to information may result in bidders' informational rents[1].

We show that, when facing the decision of how much information to reveal in a private value setting, the auctioneer faces two opposing forces: more information improves the efficiency of the match while it also increases informational rents. As the auctioneer balances improved efficiency (which raises revenues) with increased informational rents (which reduce them), the auctioneer will reveal an inefficient amount of information.

Further, we are interested in the relationship between the level of information in the market and competition. We prove that total surplus and the auctioneer's expected revenue are supermodular in the number of bidders and the level of information precision. This implies that the efficient and the auctioneer's optimal amount of information provision are increasing in the level of competition measured in terms of the number of bidders in the auction. In our setting, total surplus depends on the match between the object and the winning bidder's preferences, and the cost of information. If you add an extra bidder, this increases the opportunity of a better match and the marginal value of information, since this bidder reaps some efficiency gains from more

[1]As mentioned earlier, when talking about information we have in mind that the auctioneer provides information on the features of the object for sale. Other types of information could have different implications. For example, if the auctioneer were to reveal his valuation of the object in a private value setting (and his valuation was independent of the bidders' valuations) then this information would have no effect.

information without reducing the gains of existing bidders. Hence information and competition are complements when maximizing total surplus.

The effect on expected revenue is related. Auctioneer expected revenue is total surplus minus bidders' informational rents. Increasing competition naturally reduces bidder rents and thereby reduces the cost of providing information for the auctioneer. The compounded effect of competition on total surplus and informational rents is to increase the incentives of the auctioneer to provide more information. As the number of participants goes to infinity informational rents disappear, consequently total surplus and expected revenues converge and so do the optimal and the efficient levels of information.

Our results are demonstrated in the context of a standard private value auction. Prior to the auction we allow the auctioneer to provide bidders with information in the form of private signals correlated with their private valuations. The auctioneer controls the informativeness of the signals and chooses it so as to maximize his expected revenue from the auction. We use a general notion of informativeness which we refer to as *signal accuracy*. Once the auctioneer chooses the accuracy of the signals it becomes common knowledge, but the actual realizations of the private signals are known only by bidders. Each bidder uses this information (the level of accuracy and the realization of his private signal) to update his expected valuation of the object and then the auction takes place. We study both the socially efficient and the optimal (revenue maximizing) choice of information, and conclude by analyzing what happens as the number of bidders goes to infinity.

Note that we are modeling the match between the information revealed by the auctioneer and private preferences implicitly, via private signals which are correlated with agent's true and unknown valuations. An alternative approach is to model this match explicitly, i.e. have the auctioneer provide a public signal related to the characteristics of the object. Then, agents who have private information about their preferences interpret the signal and revise their expected valuations. Ganuza (2004) follows the direct approach in a Hotelling setting where an auctioneer sells a good to bidders who are located on a circle according to a uniform distribution. In this alternative symmetric setting, he finds the same tradeoff between efficiency and informational rents.

Another paper that is related to ours is Bergemann and Pesendorfer (2003). Their objective is to study the design of the optimal auction and the optimal information structure in private value settings. They do this by allowing the auctioneer to provide information to bidders asymmetrically. We take a different approach centered on the case where the auctioneer is constrained to provide information symmetrically. Such a constraint arises naturally in many real problems: in Internet and other auctions it can be very hard to identify active bidders until they actually make a bid, which considerably complicates the process of providing information in a personalized manner; also, in government-related auctions legal restrictions often require the auctioneer to publicly release information and explicitly forbid asymmetric information provision in order to avoid favoritism or corruption; furthermore, in some settings the information given to one bidder could be shared with other bidders or it could leak in some way, undermining the desired effects of information discrimination.

Moreover, the methodology of Bergemann and Pesendorfer (2003) is not well suited to the types of questions we pose here. We are interested in the interaction between information and competition. In their work, the information the auctioneer gives to bidders takes the form of partitions which are difficult to rank in terms of informational content or precision as the number of bidders changes. Our approach is closer to the work of Bergemann and Välimäki (2002), Persico (2000), and Athey and Levin (2001).

As in those papers we work with information structures that can be ordered in terms of their accuracy.

In private value auctions, Eső and Szentes (2003) assume that the auctioneer can fully commit to providing any given level of information precision prior to charging bidders for it. Thus, the auctioneer can extract all the informational rents ex-ante and the optimal amount of information released will be the efficient one. We study the case where the precision of information is known only when that information is made public (and then it is too late for the auctioneer to try to get bidders to pay him for it).

There are also a number of papers that focus on the incentives of bidders to acquire information rather than those of the auctioneer to reveal it: Tan (1992) compares sealed bid first and second price auction formats under different information acquisition technologies in private value settings; Stegeman (1996) shows that in private value auctions the sealed bid second price auction induces efficient information acquisition; Matthews (1984) and Persico (2001) study bidder information acquisition in the pure common value auction and affiliated values respectively. Compte and Jehiel(2004) compare bidder incentives to acquire information and their effect on revenues in static and dynamic auctions with private values and asymmetrically informed bidders.

A very interesting related empirical paper is that by Kavajecz and Keim (2004). It documents how some institutional investors use auctions to buy and sell a large number of shares as one package. It is related to this paper because in this setting brokers' (bidders') valuation of the package has a substantial private value component (it strongly depends on how the package of shares matches the demand from his other clients and which varies substantially from broker to broker). These auctions are called blind auctions because the auctioneer consciously provides relatively little detail on the shares in the package at auction.

The remainder of the paper is organized as follows. In Section 2 we introduce the model and describe how bidders' valuations depend on the information the auctioneer releases concerning the object. Section 3 studies the auctioneer's information release, characterizes the efficient solution and the auctioneer's optimal strategy. Section 4, concludes by discussing the scope and implications of the model. All proofs are relegated to a technical appendix.

2 The Model

An auctioneer wants to sell an object he values at zero to one of n (ex-ante) identical risk-neutral bidders (indexed by $i = 1, \ldots, n$). Bidders' valuations of the object are private and unknown. Bidder i's realized valuation after the auction is described by a random variable, V_i. We assume all agents are ex-ante identical so that for all $i = 1, \ldots, n$, V_i is independently distributed on $\mathcal{V} = [0, 1]$ according to a common distribution H with mean μ, where for all $v \in [0, 1]$, H describes the cumulative distribution of V, $H(v) = \Pr(V_i \leq v)$.

The utility obtained by bidder i from winning the auction is quasilinear. If the realized valuation is v_i and he makes a monetary payment of t_i, the utility obtained is given by

$$u_i(v_i, t_i) = v_i - t_i.$$

All bidders start with identical priors, described by H, and no other information on the object[2]. Hence, their expected valuations of the object will be the same and

[2]The model could start with each bidder having a private estimate of the value of the good (in addition to the common prior). This would add a great deal of technical complexity that would complicate the demonstration of our results. Nevertheless, as long as these estimates do not alter the ex-ante symmetry from the point of view of the auctioneer, we do not see any reason why our results should change in any qualitatively significant way.

equal to μ.

The auctioneer can reveal information on the object prior to the auction. The production of information is costly. By paying an amount $\delta \in [0, \infty)$ the auctioneer will generate information in the form of private signals $(X_i)_{i=1}^n$. Bidder i receives the private signal X_i and no other. The signals are independent and identically distributed random variables. We assume that these signals are drawn from the space of signals, $\mathcal{X} \subseteq \mathbf{R}$, and for each $i = 1, \ldots, n$, each X_i is informative only about bidder i's own true and unknown valuation of the object, v_i.

When the auctioneer decides how much to invest in providing information, δ, he determines the informational content of the private signals, $(X_i)_{i=1}^n$. Formally, by choosing δ, the auctioneer determines the information structure, where an information structure is a joint distribution, $\mathbf{F}_\delta$ over signals, $(X_i)_{i=1}^n$ and valuations $(V_i)_{i=1}^n$ indexed by δ. As the signals are independent, there exists a distribution $F_\delta(v, x) = \Pr(V \leq v, X \leq x)$, such that

$$\mathbf{F}_\delta \left(V_1 \leq v_1, \ldots, V_n \leq v_n, X_1 \leq x_1, \ldots, X_n \leq x_n \right) = \prod_{i=1}^{n} F_\delta(v_i, x_i)$$

We leave out the i subscripts on signals and valuations whenever they are clear from the context. With minor abuse of notation let $F_\delta(x)$ and $F_\delta(v)$ denote the marginal distributions of X and V respectively, and $F_\delta(x|v)$ and $F_\delta(v|x)$ the conditional distributions, where $F_\delta(x|v) = \Pr(X \leq x | V = v)$. As priors have to be consistent with the joint distribution, $F_\delta(v, x)$, then $F_\delta(v) = H(v)$. It will be convenient to assume that $F_\delta(x)$ is strictly increasing. From now on, we apply the following convention: the terms 'increasing' and 'greater than' mean 'non-decreasing' and 'no less than', and when it is important to distinguish between them we use mathematical notation which is unambiguous.

We will not yet formalize exactly how a higher δ generates better information, we discuss that in the next section, section 2.1. At this stage, it suffices to realize that the information structures are indexed by δ, that this δ serves to rank the information structures, and finally, that δ, the overall level of signal accuracy, is public information to all bidders. We will use the terms 'more information', 'better information', 'more precise information' and 'more accurate information' interchangeably.

After the auctioneer has released the information, the awarding process takes place. To participate in this process, each bidder combines his knowledge of δ and the realization of the private signal, x_i, to update his expected valuation of the object, also referred to as the interim valuation and denoted $w_i(x_i, \delta)$, using Bayes' rule. The auctioneer sells the object using a second-price sealed-bid auction.[3] For simplicity we abstract from reserve prices and assume that the object is always sold. Summarizing, the model is structured as follows:

1. Bidders start with common priors over their unknown valuations for the object.
2. Prior to the auction, the auctioneer, knowing the number of bidders, n, decides how much to spend on information, δ (the more he spends the more precise will the information be). This decision becomes public information.
3. Given δ, each bidder receives a private signal x_i over his valuation.
4. According to δ and the private signals, $(x_i)_{i=1}^n$, bidders update their valuations of the object.

[3]We have chosen the second-price sealed-bid auction for the sake of simplicity. As will be clear in the following, at the awarding stage bidders are symmetric in expected terms, risk neutral and their expected valuations of the object are independently distributed. Then, applying the revenue equivalence theorem, any auction mechanism in which the object is always awarded to the buyer with the highest valuation and where any bidder with the lowest valuation obtains zero surplus, yields the same expected revenue to the auctioneer. Thus, all "standard" auctions (second-price sealed-bid, first-price sealed-bid, oral ascending (English) or oral descending (Dutch)) and many non-standard auctions such as an "all-pay" auction would yield the same expected revenue to the auctioneer, bidders would make the same expected payments as a function of their valuations and, as a consequence, the same results would be obtained.

5. The second-price sealed-bid auction takes place.

We will now give more details on how a higher δ generates more informative signals and also look at how interim valuations, $w_i(X_i, \delta)$, are distributed. Then the analysis will proceed in the usual way: we characterize the Perfect Bayesian Equilibrium starting from the auction and move backwards. We focus on the auctioneer's decision about how much information to provide to the bidders. First, we characterize the efficient solution of this problem and then the auctioneer's optimal information release. Finally, we compare both solutions.

2.1 Information Structures

The auctioneer chooses how much to spend on information, δ, and determines the information structure faced by bidders, $\mathbf{F}_\delta$, which bidders use to evaluate the informational content of the signals they receive, X_i. We assume that for each δ, $F_\delta(x)$ has support on a subinterval of $\mathcal{X}$ and is strictly increasing on that subinterval.

Given any $\delta > 0$, the signals will be informative in the sense that given two signals x' and x, such that $x > x'$, receiving the larger signal, x, is good news in the sense of Milgrom (1981); i.e. the posterior distribution of true valuations conditional on x, $F_\delta(v|x)$, dominates the posterior distribution of true valuations conditional on x', $F_\delta(v|x')$, in terms of First Order Stochastic Dominance (FOSD) denoted $F_\delta(\cdot|x) \geq_{st} F_\delta(\cdot|x')$, (that is $F_\delta(v|x) \leq F_\delta(v|x')$ for all $v \in [0, 1]$). This implies that for any increasing function of the realized valuation, $\psi(v)$, $E[\psi(v)|x] \geq E[\psi(v)|x']$.

We want to formalize how a higher δ leads to better information. For this we will transform the realized signal X_i into a new random variable Π_i which has exactly the same informational content as X_i. This new Π_i is obtained using the probability integral transformation: $\Pi_i = F_\delta(X_i)$ so that $F_\delta^{-1}(\Pi_i) = X_i$, where F_δ^{-1} is the right-continuous inverse of the marginal distribution $F_\delta(x)$. We use Π_i instead of X_i because it is informationally equivalent and yet has a marginal distribution with a very useful property: the marginal distribution of Π_i is the uniform on $[0, 1]$ and hence independent of δ.

We can now define exactly how a higher δ leads to higher informational content of the signal X_i. Let $\delta > \delta'$ and define $\Pi = F_\delta(X) = F_{\delta'}(X)$. Then F_δ is more informative than $F_{\delta'}$ in the following sense:

Definition 1 *Let $\mathbf{F}_\delta$ and $\mathbf{F}_{\delta'}$ be two information structures with $\delta > \delta'$ and associated posterior distribution functions $F_\delta(v|x)$ and, $F_{\delta'}(v|x)$. The information structure F_δ is more accurate than $F_{\delta'}$ iff $\forall \pi, \pi' \in [0, 1]$, $\pi > \pi'$, $\forall v \in V$,*

$$F_\delta\left(v|F_\delta^{-1}(\pi')\right) - F_\delta\left(v|F_\delta^{-1}(\pi)\right) \geq F_{\delta'}\left(v|F_{\delta'}^{-1}(\pi')\right) - F_{\delta'}\left(v|F_{\delta'}^{-1}(\pi)\right)$$

Thus, a higher δ leads to conditional distributions over true valuations that are more sensitive to signal realizations.

This intuitive notion of accuracy is related to notions of informativeness proposed in the previous literature. In particular, it can be shown that our notion of accuracy is equivalent to the notion of stochastic supermodularity[4]. In order to put our notion in context we refer the interested reader to Athey and Levin (2001) where several different notions of informativeness of information structures are discussed.

2.2 Endogenous Bidder Valuations

In this section we want to study the effects of different information structures on the distribution of interim bidder valuations. Recall bidders start with only a prior over their valuation of the object, H. Then, the auctioneer spends δ and each bidder receives a private signal x_i. For fixed δ and signal x_i, the updated distribution of

[4]See remark 1 in the appendix and Topkis (1998) for additional details over stochastic supermodularity.

valuations for bidder i is $F_\delta(v|x_i)$, so that their expected valuations change from μ to $w_i(x_i, \delta)$. The expected value can be computed via a standard Riemann-Stieltjes integral

$$w(x, \delta) = \int_0^1 v \; \mathrm{d}F_\delta(v|x).$$

We have seen that for given δ, a higher signal implies better news and hence for all $\delta > 0$, $w(x, \delta)$ is an increasing function of the realization of the signal, x.

How about if we change δ? Recall we are using the transformed signal $\Pi = F_\delta(X)$ to make signals comparable across information structures. We will need the interim valuation (the updated expected valuation) of a bidder who receives the transformed signal Π: let $W(\pi, \delta) = w(F_\delta^{-1}(\pi), \delta)$. Notice that, as we have assumed that F_δ is strictly increasing, then $F_\delta^{-1}(\cdot)$ is also strictly increasing, π is a monotone increasing transformation of x and for all δ, $W(\pi, \delta)$ is increasing in π.

Then, a more accurate signal implies greater differences between expected valuations:

Lemma 1 *Let F_δ and $F_{\delta'}$ be two information structures such that $\delta > \delta'$ (F_δ is more accurate than $F_{\delta'}$) then for all $\pi > \pi'$*

$$W(\pi, \delta) - W(\pi', \delta) \geq W(\pi, \delta') - W(\pi', \delta'),$$

i.e. $W(\pi, \delta) - W(\pi', \delta)$ is increasing in δ.

We provide a direct proof of this in the Appendix. Also, this result follows directly from the equivalence between our notion of accuracy and stochastic supermodularity we have mentioned above.

Before we conclude this section we want to be very specific about what Lemma 1 means in terms of the distribution of interim valuations. Lemma 1 basically states that, a higher δ implies a more 'spread out' distribution of interim valuations. In the statistics literature there are several ways to formalize the notion of what it means for a distribution to be 'spread out'.

One notion is the following: a random variable X with cumulative distribution function F is said to be more *disperse* than another random variable Y with cumulative distribution function G, and denoted $X \geq_{disp} Y$ if for all $q, p \in [0,1]$, $q > p$

$$F^{-1}(q) - F^{-1}(p) \geq G^{-1}(q) - G^{-1}(p).$$

Using this definition, we can read Lemma 1 as follows, if F_δ is more accurate than $F_{\delta'}$ then the distribution of interim valuations generated by F_δ is more disperse than the distribution of interim valuations generated by $F_{\delta'}$.

Economists are more familiar with a different and yet related notion of 'spread out' distributions: a random variable X with cumulative distribution function F, finite mean and support on $A \subseteq \mathbf{R}$ is dominated in terms of second order stochastic dominance (SOSD) by a random variable Y with cumulative distribution function G, finite mean and support on $B \subseteq \mathbf{R}$, denoted $X \leq_{SSD} Y$, if the expected value of X is the same as that of Y and for all $z \in \mathbf{R}$

$$\int_{-\infty}^{z} F(x)\mathrm{d}x \geq \int_{-\infty}^{z} G(x)\mathrm{d}x.$$

In our setup, we can order the distribution of interim expected valuations according to second order stochastic dominance:

Corollary 1 *Let F_δ and $F_{\delta'}$ be two information structures such that $\delta > \delta'$ (F_δ is more accurate than $F_{\delta'}$), then $W(\Pi, \delta)$ is dominated by $W(\Pi', \delta')$ in the sense of SOSD*

Summarizing, the auctioneer can control how spread out is the distribution of expected valuations, in the sense that the difference between the expected valuation of a bidder with a signal in the q-th percentile and a bidder with a signal in the p-th percentile is greater if the signals come from a more accurate information system. But why should more information lead to a more spread out distribution of valuations? Intuitively, as bidders react asymmetrically to information, some increasing and some reducing their expected valuations, more information is more likely to lead to greater differences in updated bidders' valuations.

More formally, if the signals are more accurate, bidders give more weight to the realization of the signal in the calculation of their updated expected valuations. The increased importance of the realization of the signal comes with a reduction in the importance of the (common) prior. Thus the updated expected valuation is more sensitive to differences in realizations of the signals and the effect of receiving one signal rather than another implies a bigger effect, a bigger difference, in the interim expected valuations. Then, more information naturally leads to a distribution of expected valuations that is more 'spread out'.

3 Information Release

In this section, we study the accuracy of information the auctioneer provides. We want to contrast the optimal and the efficient level of accuracy and we start by characterizing the efficient level of accuracy: how much accuracy should the auctioneer provide?

3.1 The Efficient Level of Accuracy

The efficient level of accuracy is that which maximizes the total surplus at the time of the information release. In our setup, total surplus is defined as the sum of the auctioneer's revenue and the interim utility of the bidder with the highest expected valuation at the time of the auction.

Recall that the awarding mechanism is a second price auction. Given that for every information structure expected valuations are increasing in the signal, the highest bid will come from the bidder with the highest expected valuation and this bidder will be the one with the highest realization of the signal. We denote the highest realization of the signal by x_1. In terms of the transformed signal, Π, the winner will be the one who receives $\pi_1 \equiv F_\delta(x_1)$. If we take expectations prior to the information release, then the expected valuation of the winning bidder will depend on n and δ: $V_1(n,\delta) = E[W(\Pi_1,\delta)]$. The notation makes explicit the number of bidders as we shall be studying the effect of changing n. Let $U_{1:n}(p)$ be the cumulative distribution function of the first order statistic of n independent uniform random variables on $[0,1]$. As the transformed signals Π_1 are independent and uniformly distributed on $[0,1]$,

$$V_1(n,\delta) = \int_0^1 W(p,\delta)\mathrm{d}U_{1:n}(p)$$

The next result characterizes the relationship between the expected valuation of the winning bidder and the amount of information provided by the auctioneer.

Proposition 1 *The expected valuation of the winning bidder $V_1(n,\delta)$ is increasing in the accuracy of the information, δ.*

Proposition 1 rests on the fact that the winning bidder will be the bidder with the highest realization of the signal. The expected highest realization is greater than the mean, which is also the prior expectation. As more accurate signals lead to putting greater weight of the realization of the signal relative to the prior, then the greater the accuracy the greater the expected valuation of the winner, $V_1(n,\delta)$. As we said in the introduction, our model could be interpreted as a situation in which the auctioneer provides information about the features of the object and bidders have private information over their preferences. Under this interpretation Proposition 1 can be interpreted as: the larger the information provided to bidders, the better the matching between the features of the object and the preferences of the winning bidder.

Thus, the trade-off faced when deciding the efficient amount to spend on information, δ^E, is between increasing the expected valuation of the winning bidder and increasing the cost of providing that information to the market:

$$\delta^E \in \operatorname*{argmax}_{\delta} V_1(n, \delta) - \delta \tag{1}$$

The next proposition states the relationship between the efficient level of accuracy and the level of competition.

Proposition 2 *The total surplus is supermodular on δ and n.*

Proposition 2 states that the difference in terms of expected surplus between two levels of signal accuracy is larger the larger the number of bidders. Mechanically, more bidders imply that the highest realization of the signal will be higher. Thus, more competition increases the return from giving more weight to the highest realization of the signal and hence the incentive to increase signal accuracy. From an economic point of view, the larger the number of bidders, the larger the value of information. The intuition is that the larger the pool of bidder preferences, the larger the incentives to improve the matching by increasing the accuracy of information on the object.

Corollary 2 *The efficient amount of information, δ^E, is increasing in the number of bidders, $n \geq 1$.*

As the number of bidders in the auction increases the marginal effect of adding information is greater so that with more bidders it is efficient to spend more on information.

3.2 The Auctioneer's Optimal Information Release

Having characterized the efficient amount of information, let us now turn to the auctioneer's problem: how much information to release if one wants to maximize expected revenue from the auction.

We first characterize the bidder's optimal strategy given a fixed amount of expenditure on information, δ, and the realization of his private signal x_i. The nature of the second-price auction ensures that it is optimal for the bidder to bid his expected valuation: let $b_i(x_i, \delta)$ be the bid made by agent i on knowing δ and receiving signal x_i, then $b_i(x_i, \delta) = w(x_i, \delta) = W(F_\delta^{-1}(x_i), \delta)$. The winner of the auction will be the one with the highest signal, x_1. As bidders bid their valuations and we are in a second-price auction, the payment to the auctioneer will be equal to the expected valuation of the bidder with the second-highest signal, x_2. Using the transformed variables, the highest signal is π_1. Denote the second highest signal as $\pi_2 \equiv F_\delta^{-1}(x_2)$. As the transformed signals are independent and uniformly distributed on $[0, 1]$, $U_{2:n}(p)$, the cumulative distribution of the second order statistic in a sample of n iid uniform random variables on $[0, 1]$, is the cumulative distribution function of Π_2. Then:

$$V_2(n, \delta) = \int_0^1 W(p, \delta) \mathrm{d}U_{2:n}(p)$$

Where $V_2(n, \delta)$ is the expected price in the auction and the expected valuation of the bidder with the second highest signal realization. The next proposition states the relationship between expected price and the amount of information.

Proposition 3 *The expected price (the valuation of the second highest bidder), $V_2(n, \delta)$, is increasing in the amount of information, δ, if the number of bidders is larger than 3. The value of information to the auctioneer is in fact negative for $n = 2$.*

The intuition behind Proposition 3 is the same as behind Proposition 1 replacing the bidder with the highest realization of the signal with the one with the second highest realization. But, if there are only two bidders then in expected terms, the signal of the loser in the auction (and hence his expected valuation and the price) will be below the prior. Thus, improving information has a negative value as it lowers the expected price (and revenue).

The next proposition addresses the question of how the rents generated by higher information are distributed between the auctioneer and the winner in the auction. The expected informational rents of the winning bidder is the difference between his valuation and the valuation of the second closest bidder (which is the price paid for the object)

$$R_w(n,\delta) = V_1(n,\delta) - V_2(n,\delta)$$

Proposition 4 *The expected informational rents of the winning bidder are increasing in δ.*

Proposition 4 is linked with the fact that more information implies a more 'spread out' distribution of interim valuations. Hence Proposition 4 shows us the drawback of providing information to the market: it increases bidders' rents.

To establish the auctioneer's optimal strategy, we have to characterize the level of information, δ^A, that maximizes the difference between the expected price and the cost of providing more information:

$$\delta^A \in \operatorname*{argmax}_{\delta} V_2(n,\delta) - \delta \tag{2}$$

By comparing (1) and (2) it is easy to see that the structure of the auctioneer's problem is identical to that of total surplus maximization with $V_1(n,\delta)$ substituted by $V_2(n,\delta)$, so that the intuition behind the results presented in the following Proposition and its Corollary is the same as those for Proposition 2 and Corollary 2.

Proposition 5 *The auctioneer's expected profits are supermodular on δ and n.*

Hence the larger the number of bidders, the larger the incentives of the auctioneer to provide information.

Corollary 3 *The optimal amount of information, δ^A, is increasing in the number of bidders, n.*

Finally, the following proposition presents the main result of the paper.

Proposition 6 *The auctioneer provides less information to bidders than would be efficient, $\delta^A \leq \delta^E$. The difference between the efficient information release and the equilibrium information release converges to* 0 *as the number of bidders goes to infinity.*

To better understand the result one can rewrite the auctioneer's problem as:

$$\delta^A \in \underset{\delta}{\operatorname{argmax}}\, V_1(n,\delta) - \delta - R_w(n,\delta)$$

This formulation clarifies the trade-off faced by the auctioneer when providing information to the market. On the one hand, when the auctioneer provides more information, the efficiency of the auction process goes up ($V_1(n,\delta)$ is increasing in δ – Lemma 1). On the other hand, the increase in information also raises the informational rents of the winning bidder ($R_w(\delta)$ is increasing in δ – Proposition 4). The optimal balance of these two opposing effects leads the auctioneer to provide less information than would be efficient. In other words, the auctioneer will restrict the information released to the market in order to make the potential bidders more homogeneous, with the underlying goal of intensifying competition and increasing his expected revenue. As the number of bidders increases, the informational rents are reduced and the trade-off is weakened. In the limit, as the number of bidders goes to infinity, the informational rents disappear and with it the difference between efficient and optimal information release.

4 Conclusions

We were interested in what happens when the auctioneer can release information to bidders and how the amount of information released relates to the level of competition in private value auctions. We have set up a standard auction model and used a general notion of informativeness to study this question. We have shown that the optimal level of information released by the auctioneer is not the same as the efficient level, which contrasts with the "linkage principle". This is because in private value settings, there are two factors that determine the optimal provision of information: (i) improved information increases the efficiency of the auction; (ii) improved information also increases the informational rents of the winning bidder. These two effects represent opposing forces for the auctioneer: improved efficiency raises revenues while increased informational rents reduce revenues. Hence, the optimal amount of information released is below the efficient one.

Our second main result relates competition and the information provided to the market. We show that there is a complementarity between competition and information when maximizing total welfare and also, when maximizing auctioneer revenues. Then, both the efficient and the optimal level increase with the number of bidders. We conclude by showing that as the number of bidders goes to infinity, the difference between the efficient and optimal solutions vanishes.

The complementary between information and competition opens up a number of interesting avenues for future research. For example, this could be an ingredient in explaining the prevalence of incomplete contracting in many real life situations. In procurement, one can consider the possibility of reducing the degree of specificity in a contract in order to homogenize the market and inject an additional degree of competition into the procurement process.

Another possible extension is to consider endogenizing the degree of competition in the auction by adding an initial stage whereby firms decide whether to enter into the auction or not. The complementarity between the level of information provision and competition should intuitively lead to multiplicity of equilibria in the entry game. There will be equilibria with low competition and a low level of information and others with lots of competition and a high level of information.

A Appendix

A.1 Preliminary Result and Notation

Using the notation developed in the text, we will make repeated use of the following very well-known result, which we state as a lemma: if $X \geq_{st} Y$ then for all increasing functions ψ, $E[\psi(X)] \geq E[\psi(Y)]$.

Lemma 2 *Let X and Y be real-valued random variables with cumulative distribution functions F and G respectively, such that $F(z) \leq G(z)$ for all $z \in \mathbf{R}$. For all bounded real-valued increasing functions $\psi : \mathbf{R} \to \mathbf{R}$,*

$$\int_{\mathbf{R}} \psi(z)\mathrm{d}F(z) \geq \int_{\mathbf{R}} \psi(z)\mathrm{d}G(z)$$

We also use the following notation: $U_{i:j}(x)$ is the cumulative distribution function (cdf) of a random variable Y such that $U_{i:j}(x) = \Pr(Y \leq x)$. This random variable is the ith order statistic from a sample of j independently and identically uniform distributed random variables over $[0,1]$, where $U_{1:j}$ refers to the cdf of the maximum of the sample, $U_{2:j}$ to the cdf of the second highest realization in the sample and so on until $U_{j:j}$ which is the cdf of the minimum realization in the sample. We will also make use of the functional form of $U_{1:n}$ which is, for $\pi \in [0,1]$, $U_{1:n}(\pi) = \pi^n$.

A.2 Proofs

Proof of lemma 1: For $\pi > \pi'$, and using the properties of Riemann-Stieltjes integrals

$$\begin{aligned} W(\pi,\delta) - W(\pi',\delta) &= \int_V v \ \mathrm{d}F_\delta(v|F_\delta^{-1}(\pi)) - \int_V v \ \mathrm{d}F_\delta(v|F_\delta^{-1}(\pi')) \\ &= \int_V v \ \mathrm{d}\left[F_\delta(v|F_\delta^{-1}(\pi)) - F_\delta(v|F_\delta^{-1}(\pi'))\right] \geq 0 \end{aligned}$$

The last inequality arises from the assumption that $\pi' > \pi$ implies good news. Integrating by parts

$$W(\pi, \delta) - W(\pi', \delta) = -\int_V (F_\delta(v|F_\delta^{-1}(\pi)) - F_\delta(v|F_\delta^{-1}(\pi')))\mathrm{d}v \tag{3}$$

Similarly

$$W(\pi, \delta') - W(\pi', \delta') = -\int_V (F_{\delta'}(v|F_{\delta'}^{-1}(\pi)) - F_{\delta'}(v|F_{\delta'}^{-1}(\pi')))\mathrm{d}v \tag{4}$$

As $I(\delta)$ is more accurate than $I(\delta')$ and $\pi > \pi'$, then

$$F_\delta\left(v|F_\delta^{-1}(\pi')\right) - F_\delta\left(v|F_\delta^{-1}(\pi)\right) \geq F_{\delta'}\left(v|F_{\delta'}^{-1}(\pi')\right) - F_{\delta'}\left(v|F_{\delta'}^{-1}(\pi)\right) \tag{5}$$

Integrating over v on both sides of condition (5) and combining the outcome with equations (3) and (4) we get

$$W(\pi, \delta) - W(\pi', \delta) \geq W(\pi, \delta') - W(\pi', \delta')$$

∎

Corollary 4 *$W(\pi, \delta) - W(\pi, \delta')$ is increasing in π for $\delta > \delta'$.*

Proof of Corollary 4: This follows immediately from Lemma 1: for $\pi > \pi'$

$$\begin{aligned} W(\pi, \delta) - W(\pi', \delta) &\geq W(\pi, \delta') - W(\pi', \delta') \\ \Rightarrow W(\pi, \delta) - W(\pi, \delta') &\geq W(\pi', \delta) - W(\pi', \delta') \end{aligned}$$

∎

Proof of Corollary 1: Applying the law of iterated expectations: $\mu = E[E[v|X]] = E[W(\Pi, \delta)]$ and $\mu = E[E[v|X']] = E[W(\Pi', \delta')]$. If X and Y, two random variables, have the same mean and X is more disperse than Y, i.e. $X \geq_{disp} Y$, then $X \leq_{SSD} Y$ (Shaked and Shantikumar(1994), Theorem 2.B.10).

∎

PROOF OF PROPOSITION 1: We want to show that if $\delta > \delta'$ then $V_1(n,\delta) \geq V_1(n,\delta')$. This is equivalent to showing

$$\int_0^1 (W(p,\delta) - W(p,\delta'))\ \mathrm{d}U_{1:n}(p) \geq 0$$

By the law of iterated expectations, the expected valuation of the distribution of expected valuations, $E[W(\pi,\delta)]$, must not depend on the information structure. Let $U_{1:1}(\pi) = \pi$ denote the cumulative distribution function of the uniform. We can now write

$$\int_0^1 (W(\pi,\delta) - W(\pi,\delta'))\mathrm{d}\pi = \int_0^1 (W(\pi,\delta) - W(\pi,\delta'))\mathrm{d}U_{1:1}(\pi) = 0$$

By Corollary 4, the function $\psi(\pi) \equiv (W(\pi,\delta) - W(\pi,\delta'))$ is increasing in π. As $U_{1:n}(p) = p^n \leq U_{1:1}(p)$ for all $n \geq 1$ and $p \in [0,1]$ and $\psi(\pi)$ is increasing, we can apply Lemma 2 and the result follows.

∎

PROOF OF PROPOSITION 2: It suffices to show that $V_1(n+1,\delta) - V_1(n,\delta) \geq V_1(n+1,\delta') - V_1(n,\delta')$.

This is equivalent to showing

$$\begin{aligned} V_1(n+1,\delta) - V_1(n+1,\delta') &\geq V_1(n,\delta) - V_1(n,\delta') \\ \Leftrightarrow \int_0^1 (W(\pi_1,\delta) - W(\pi_1,\delta'))\mathrm{d}U_{1:n+1}(\pi_1) &\geq \int_0^1 (W(\pi_1,\delta) - W(\pi_1,\delta'))\mathrm{d}U_{1:n}(\pi_1) \end{aligned}$$

From Corollary 4, the function $\psi(\pi) \equiv (W(\pi,\delta) - W(\pi,\delta'))$ is increasing in π. As $U_{1:n+1}(p) = p^{n+1} \leq U_{1:n}(p) = p^n$ for all $p \in [0,1]$, we can apply Lemma 2.

∎

PROOF OF COROLLARY 2: Immediate from the results of Milgrom and Shanon (1994) and Proposition 2. ∎

Proof of Proposition 3: This follows by the same logic as the proof of Proposition 1 and the well-known result that $U_{2:n}(p) \leq U_{1:1}(p)$ for all $p \in [0,1]$ and $n \geq 3$.

∎

Proof of Proposition 4:

We want to show that for $\delta > \delta'$, $R_w(n,\delta) \geq R_w(n,\delta')$, i.e.

$$V_1(n,\delta) - V_2(n,\delta) \geq V_1(n,\delta') - V_2(n,\delta')$$

This is equivalent to

$$V_1(n,\delta) - V_1(n,\delta') \geq V_2(n,\delta) - V_2(n,\delta')$$

i.e.,

$$\int_0^1 (W(\pi_1,\delta) - W(\pi_1,\delta'))\mathrm{d}U_{1:n}(\pi_1) \geq \int_0^1 (W(\pi_2,\delta) - W(\pi_2,\delta'))\mathrm{d}U_{2:n}(\pi_2)$$

Again, using Corollary 4 and the stochastic dominace of the first order statistic over the second, $U_{1:n}(p) \leq U_{2:n}(p)$ for all $p \in [0,1]$. Applying Lemma 2 concludes the proof.

∎

Proof of Proposition 5: This follows by the same logic as the proof of Proposition 2 and the property that increasing the sample by one increases the second order statistic (in the stochastic sense), i.e. $U_{2:n+1}(p) \leq U_{2:n}(p)$ for all $p \in [0,1]$.

∎

Proof of Corollary 3: Immediate from the results of Milgrom and Shannon (1994) and Proposition 5.

∎

Proof of Proposition 6: The auctioneer's problem is

$$\delta^A \in \underset{\delta^A}{\operatorname{argmax}}\{V_2(n,\delta) - \delta\}$$

This problem is equivalent to

$$\delta^A \in \operatorname*{argmax}_{\delta^A}\{V_1(n,\delta) - \delta - R_w(n,\delta)\}$$

where $R_w(\delta)$ is as defined in the text.

Compare the formulation of the auctioneer's problem to the formulation of the social welfare maximization problem (equation 1). As δ^A solves the auctioneer's optimization problem

$$E[W(\Pi_1,\delta^A) - \delta^A - R_w(\delta^A)] \geq E[W(\Pi_1,\ \delta^E) - \delta^E - R_w(\delta^E)]$$

$R_w(\delta)$ is increasing (Proposition 4) so that if $\delta^A > \delta^E$, then this last equation would imply

$$E[W(\Pi_1,\delta^A) - \delta^A] \geq E[W(\Pi_1,\ \delta^E) - \delta^E]$$

but this contradicts the fact that δ^E maximizes social surplus, so that $\delta^A \leq \delta^E$.

To establish the second part of the Proposition, consider the informational rents, $R_w(n,\delta)$.

$$\begin{aligned}
R_w(n,\delta) &= V_1(n,\delta) - V_2(n,\delta) \\
&= \int_0^1 W(\pi,\delta)\mathrm{d}U_{1:n}(\pi) - \int_0^1 W(\pi,\delta)\mathrm{d}U_{2:n}(\pi) \\
&= \int_0^1 W(\pi,\delta)\mathrm{d}(U_{1:n}(\pi) - U_{2:n}(\pi))
\end{aligned}$$

We know $U_{1:n}(\pi) = \pi^n$ and $U_{2:n}(\pi) = n\pi^{n-1} - (n-1)\pi^n$.

$$\begin{aligned}
U_{1:n}(\pi) - U_{2:n}(\pi) &= n(\pi^n - \pi^{n-1}) \\
\Rightarrow \lim_{n\to\infty} U_{1:n}(\pi) - U_{2:n}(\pi) &= 0
\end{aligned}$$

As $W(\pi,\delta)$ is bounded and monotone in π, and $(U_{1:n}(\pi) - U_{2:n}(\pi))$ converges to zero then $R_w(n,\delta)$ also converges to zero

$$\lim_{n\to\infty} R_w(n,\delta) = \lim_{n\to\infty}\int_0^1 W(\pi,\delta)\mathrm{d}(n(\pi^n - \pi^{n-1})) = 0$$

Hence, the objective function of the auctioneer approaches total surplus as n goes to infinity.

$\blacksquare$

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