Partnerships: A Potential Solution to the Common-Property Problem but a Problem for Antitrust Authorities

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Abstract The common property problem, first analyzed in the context of overfishing (Gordon, 1954), is ubiquitous: independent tax authorities will overtax the same base (Berkowitz and Li, 2000), and independent researchers will exert excessive effort to make the same breakthrough (Wright, 1983). We propose a "Partnership Solution" to this common property problem. Each of n players maximizes his payoff by joining a partnership in the first stage and by choosing his effort at the second stage. Under the rules of a partnership, each member must pay his own cost of effort but receives an equal share of the partnership's revenue. The incentive to free ride created by such partnerships can be beneficial since it naturally offsets the incentive to exert excessive effort inherent in common property problems. In our two-stage game, this institutional arrangement can, under specified circumstances, induce socially optimal effort in a subgame-perfect equilibrium: no one has a unilateral incentive (1) to switch partnerships (or create a new partnership) in the first stage or (2) to deviate from socially optimal effort in the second stage. Not all consequences of partnerships are so benign. Cartel members can use partnerships to solve their "problem" of excessive output so as to achieve monopoly profits; infinitely-repeated interactions are unnecessary. Service professionals frequently organize themselves into such partnerships as do plywood producers and crews on fishing vessels. In Japan, crews of different fishing vessels sometimes form partnerships to share their revenues (Platteau and Seki, 2000), reportedly for the reasons we analyze.

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1 Introduction

If N individuals independently exploit a common property resource, aggregate effort will be excessive because of congestion externalities. If, on the other hand, everyone must share the fruits of his labor equally with the other $N-1$ individuals, aggregate effort will be insufficient because of free-riding. Each of these two extremes is a special case of the following arrangement: players partitioned into partnerships simultaneously choose effort levels, with each partnership's share of aggregate revenue equal to its share of aggregate effort and every member of each partnership required to share equally with his colleagues the gross revenue he brings in. In the first of the extremes above, there are N "solo" partnerships while in the second, there is 1 "grand" partnership to which all N individuals belong.

Since too little effort occurs when there is 1 partnership and too much effort occurs when there are N partnerships, one might expect that aggregate effort increases with the number of partnerships. We verify this conjecture analytically and Schott et. al. (2005) verify it experimentally. Socially optimal effort can, therefore, be induced (or approximated if there are integer problems) by dividing the N players exogenously into an intermediate number of partnerships in such a way that each agent's tendency to work too hard is exactly offset by his tendency to free ride. We refer to this as the "Partnership Solution."[1]

In reality, of course, the Partnership Solution is viable if and only if a person assigned to a given group has no incentive to switch to some other partnership (pre-existing or new). We refer to such partnerships as "stable." We investigate the stability of the Partnership

[1] While Kandel and Lazear (1992) emphasized that the free-riding inherent in partnerships is a problem to be overcome, Schott(2001) was the first to recognize that, in the context of common property, free-riding may be part of the solution. In particular, he introduced the idea of grouping users of a common-property resource into an optimal number of independent output-sharing partnerships. He did not investigate the stability of these partnerships nor did he show that partnerships can also solve the cartel's problem of excessive output.

Solution in a two-stage game where partnerships are formed at the first stage and effort is chosen simultaneously at the second stage. Whether a partnership is stable or not turns out to depend on the advantages of team production over solo production. This follows since the principal source of first-stage instability is going into business for oneself.

Since common-property problems are ubiquitous, our Partnership Solution has many potential applications. While environmental problems (overfishing, excessive hunting, excessive pumping of water or oil) come immediately to mind, common-property problems arise in other situations as well. When many researchers independently work to make the same discovery, there is excessive research effort due to the negative externalities that each researcher imposes on the others. If researchers were grouped into stable "research partnerships" and paid a share of revenues rather than a wage, the problem of excessive effort could be attenuated much as it is on fishing vessels where the entire crew shares the catch.[2] Similarly, when many tax authorities independently tax the same base, there is excessive taxation due to the negative externalities that each imposes on the others. If tax authorities grouped themselves into stable partnerships and had to share what they collected while bearing their own collection costs, the problem of excessive taxation could be resolved.[3]

Indeed our Partnership Solution applies to other collective action problems besides the common-property problem—some of which we might prefer to leave *unsolved.* Consider a homogeneous cartel which operates over a finite horizon under complete information. Under such circumstances, Cournot profits are predicted to occur in every period since any scheme to collect higher profits would unravel from the end. But if the firms in an industry were partitioned into stable partnerships (a common form of organization within some service

[2] Wright (1983), among others, has shown that competition to make a discovery results in excessive research activity due to a congestion externality "equivalent to that noted by H. Scott Gordon (1954) with respect to fishing." Wright (p. 694) credits Usher (1964) as the first to note the equivalence between these two problems but lists many contributors to the literature on inventive activity who have emphasized its common-property aspect.

[3] In their analysis of multiple tax authorities in Russia, Berkowitz and Li (2000) point out that the tax base is a common property resource and the excessive taxation is a "tragedy of the commons."

industries) enough free-riding could be induced within each partnership to elevate industry profits to the monopoly level without any need for complex, history-dependent strategies over an unbounded horizon. This would be one potential explanation for the existence of the partnership as an organizational form.

A more benign explanation for partnerships has recently been advanced by Levin and Tadelis (2004). They show that a firm's choice to organize as a partnership instead of a corporation can reassure consumers unable to observe the quality of a service prior to purchase that a firm's employees are of high quality. To isolate this effect, they assume that members of the applicant pool differ in their intrinsic quality; as a simplification, Levin and Tadelis assume that each individual works equally hard in any organizational environment regardless of the incentives he faces. While their elegant model clearly captures one reason why firms choose to organize as partnerships, it cannot explain why fishermen share their catch, why plywood employees (Craig and Pencavel, 1992) share their revenues, or why wait-staff share their tips.

The motives of the fishermen of Toyama Bay who for nearly half a century have formed groups which pool their revenues (net of some costs) has been investigated empirically in two fascinating articles by Platteau, Seki, and Carpenter (Platteau and Seki (2000) and Carpenter and Seki (2004)). Since 1992, the fishermen have divided into 5 partnerships: the crews of seven vessels have constituted one partnership, the crews of two vessels have constituted a second partnership, and the crews of each of the remaining three vessels have constituted the other three partnerships (each sharing the catch of its own vessel). As Plateau and Seki (2000) emphasize, these fishermen are relatively homogeneous: they come from the same region, use the same technology to catch the same prey (Japanese glass shrimp or "shiroebi") and market it through the same cooperative. To identify the benefits they derive from partnerships, Platteau and Seki interviewed the skippers of the 12 boats and, when feasible, used more objective measures to validate their responses. It turns out that partnerships are not formed for insurance purposes: "The most prominent result emerging from this exercise is certainly the fact that stabilization of incomes was not mentioned a single time by the 12 skippers interviewed." Instead, the main motive is to reduce congestion: "The desire to avoid the various costs of crowding while operating in attractive fishing spots appears as the main reason stated by Japanese fishermen for adopting pooling arrangements." The fishermen also mentioned that by sharing catch and reducing excessive effort, they can obtain higher prices: "Fishermen believe that by limiting effort they can cause fish prices to rise." Statistical analysis of price data confirmed this effect. These shiroebi fishermen are hardly unique. There were 147 such fishing groups in Japan that engaged in some form of pooling as of the census of 1988.

Our goal is to identify the circumstances when such partnership solutions would be (1) advantageous to participants and (2) stable. To do so, we assume that each worker chooses his effort level to maximize his payoff and hence responds to effort incentives. As a simplification, appropriate in the case of the Toyama Bay fishermen, we assume that workers are homogeneous. This pair of assumptions provides a useful complement to Levin-Tadelis's analysis, which assumes instead no response to effort incentives but heterogeneity.[4] Our analysis should be of interest both to regulators attempting to solve common-property problems and to anti-trust authorities trying to thwart collusion.

We proceed as follows. In Section 2, we introduce our notation, define the goal of socially optimal effort, and discuss the determinants of equilibrium effort in the second-stage of our game. In Section 3, we provide conditions sufficient for the Partnership Solution to be stable. Section 4 generalizes the analysis to account for situations where some costs are shared and where agents have market power; it also shows how partnerships can be used to improve everyone's payoff even when they cannot attain the first-best. Section 5 concludes

[4] Also see Farrell and Scotchmer (1988) for other examples of partnerships and an analysis of partnerships using cooperative game theory.

the paper.

2 Decentralization in a Two-Stage Partnership Game

To begin, we define the notation that will be used throughout this paper.

m_i = *number of members of group i*

x_{ik}= *effort level of agent k in group i*

Y_i^{-k} = *aggregate effort level of members of group i other than agent k*

X_{-i}= *aggregate effort of other groups*

X = *total effort level (sum of all agents' efforts)*

f(X) = *aggregate production function*

c = *constant marginal cost of effort*

n = *number of groups*

N = *total number of agents*

$A(\cdot) = \frac{f(X)}{X} =$ *average product*

$\bar{x}_{ik} = \frac{\left(x_i + Y_i^{-k}\right)}{m_i} =$ *mean effort level in group i*

Until Section 4, we make the assumption standard in the common-property literature that the price of output is a constant (normalized to unity). In addition, we assume that (1) $A(X)$ is bounded, strictly positive, strictly decreasing, and twice continuously differentiable; (2) $A(0) - c > 0$; and (3) the Novshek (1985) condition, $A'(X) + XA''(X) < 0$, holds for all $X \geq 0$.

Socially optimal effort ($X^* =$ argmax $X(A(X) - c)$) must satisfy the following first-order condition:

$$A(X^*) + X^* A'(X^*) - c = 0. \tag{1}$$

Since the Novshek condition holds, X^* is unique. This aggregate effort level is the goal we seek to achieve by decentralization through our Partnership Solution.

In the first stage of our two-stage game, agents choose a partnership to which to belong. Let $n \leq N$ denote the number of distinct groups specified by the agents and index these groups $i = 1, \ldots, n$. Then, in the second stage, agents simultaneously choose their effort after observing each agent's choice of group.[5] To verify that the partnership solution is subgame-perfect, we must show that it forms a Nash equilibrium in every subgame. We approach this through backwards induction, considering the problem of effort choice first.

2.1 Equilibrium Effort Choice in Second-Stage Subgames

Consider second-period subgames in which individuals grouped into partnerships simultaneously choose their effort levels.

An individual in group i would choose his own effort level (x_{ik}) taking as given the aggregate effort level of his colleagues in partnership i ($Y_i^{-k} = \sum_{l \neq k} x_{il}$) as well as the aggregate effort levels of the other partnerships (X_{-i}). Hence, he would maximize

$$\pi_{ik} = \underset{x_{ik}}{Max} \left\{ \frac{1}{m_i} \left[\frac{x_{ik} + Y_i^{-k}}{x_{ik} + Y_i^{-k} + X_{-i}} \right] \cdot f(x_{ik} + Y_i^{-k} + X_{-i}) - c x_{ik} \right\},$$

where m_i is the number of partners in his group.This is equivalent to maximizing:

$$m_i \pi_{ik} = \left(x_{ik} + Y_i^{-k} \right) \cdot A \left(x_{ik} + Y_i^{-k} + X_{-i} \right) - m_i c x_{ik}. \tag{2}$$

[5] The assumption that agents observe the composition of their partnership before exerting effort seems plausible; however, it is not innocuous. If effort choices had to be made without observing the partnership partition, then there would be no pure-strategy Nash Equilibria.

To find the best response of member k in partnership i, we differentiate the objective function (2) with respect to x_{ik} and substitute $X = x_{ik} + Y_i^{-k} + X_{-i}$ to arrive at the following N first-order conditions:

$$A(X) + \left(x_{ik} + Y_i^{-k}\right) \cdot A'(X) - cm_i = 0 \text{ for } i = 1, \ldots, n \text{ and } k = 1, \ldots, m_i. \qquad (3)$$

Each first-order condition in (3) clarifies why player i reduces his effort in a multiperson partnership compared to his effort operating solo, for unchanged effort of the other $N-1$ players. There are two effects, each of which leads him to reduce his effort: the "internalization effect" and the "diversion-of-benefits effect." First, since in a multiperson partnership, player i receives a *share* of the receipts generated by his partners, he would refrain from imposing as large a negative externality on them as he would if he operated solo. That is, the first factor in the second term is larger by Y_i^{-k} than it would be if he operated solo. This "internalization effect" would induce him to reduce his effort in a multiperson partnership even if $c = 0$ but the effect would disappear if under the rules of the partnership he received nothing from his partners. Second, since in a multiperson partnership, player i must *relinquish* a share of the benefits of his effort but must pay the full cost of generating them, he would reduce his effort. That is, the second factor in the last term is $m_i > 1$ times as large as it would be if he were operating solo. This "diversion-of-benefits effect" would persist even if his partners were not, like him, required to share their own benefits but would disappear if $c = 0$.

The $N = \sum_{i=1}^{n} m_i$ first-order conditions in (3) plus the two equations defining Y_i^{-k} and X in terms of the individual effort levels (x_{ik}) determine the N effort levels and these two aggregates.[6] Rewriting the first factor of the second term in terms of $\bar{x}_i$ gives us:

$$A(X) + m_i \bar{x}_i \cdot A'(X) - cm_i = 0, \quad \text{for } i = 1, \ldots, n. \tag{4}$$

These n equations plus the equation $X = \sum_{i=1}^{n} m_i \bar{x}_i$ uniquely determine the n mean effort levels $\{\bar{x}_i\}_{i=1}^{n}$ and X. We can solve (4) for $\bar{x}_i$, the mean effort level in group i:

$$\bar{x}_i = \left(\frac{1}{-A'(X)} \right) \left(\frac{A(X)}{m_i} - c \right). \tag{5}$$

2.2 Partnership Effects on Effort Choice

If partnerships of different sizes form at the first stage, then their mean effort levels will differ at the second stage. In particular,

Proposition 1 *In any equilibrium, strictly larger groups have strictly smaller mean effort levels.*

Proof: As (5) reflects, the strictly positive mean effort level at the i^{th} partnership can be represented as the product of two positive factors. The second factor will be smaller at a partnership with a larger number of members (m_i) while the first factor will be the same for all the partnerships. Hence, the larger the partnership the smaller the mean effort.■

Intuitively, the larger the group, the more free-riding occurs within it.

Next we verify that *aggregate* effort in the second-stage depends only on the *number* (n) of groups formed at the first stage and not on the distribution of agents among the different groups:

[6] In this model, *aggregate* effort within each partnership is uniquely determined but individual effort within each partnership is indeterminate. To understand why, consider any solution to the $N+2$ first-order conditions. If the effort within any partnership is reassigned internally without affecting the partnership's aggregate effort, then each of these $N+2$ equations still holds. Intuitively, such a reassignment does not affect anyone's marginal incentives to alter his effort unilaterally. An expansion in effort still has the same marginal cost (c) and, since it still has the same effect on the total effort of the group and the same effect on the aggregate effort of all groups, it has the same marginal benefit as before the reassignment.

Proposition 2 *Aggregate effort (X) in the second stage depends only on the number of groups formed in the first stage and not on the size of those groups.*

Proof: Adding together the n first-order conditions in (4), we obtain the following condition:[7]

$$nA(X) + XA'(X) - cN = 0. \tag{6}$$

Thus aggregate effort (X) induced in the Nash equilibria of second-stage subgames depends only on the number of groups formed at the first stage and not on the specific partition. ■

A monotonic relationship exists between the number of partnerships formed at the first stage and the aggregate effort expended at the second stage.

Proposition 3 *The larger the number of groups formed at the first stage, the larger the aggregate effort level at the second stage.*

Proof: Differentiating (6) implicitly, we obtain:

$$\frac{dX}{dn} = \frac{A(X)}{-[(n+1)A'(X) + XA''(X)]} > 0,$$

where the inequality follows from $A(X) > 0, A'(X) < 0$, and the Novshek condition.■

Since aggregate effort in our game is a continuous function of the number of groups formed at the first stage and since $n = 1$ induces too little aggregate effort and $n = N$ generates too much, some unique intermediate number of groups will (if we provisionally ignore integer constraints) induce the socially optimal level of effort at the second stage. We can find this number by plugging X^* into (6) and then solving for n^*.

Proposition 4 *If* $n^* = \frac{c(N-1)}{A(X^*)} + 1$ *groups form at the first stage, then the aggregate effort chosen in the Nash equilibrium of the second stage will be socially optimal.*

[7] Our proposition reinterprets the result in Bergstrom and Varian (1985) that, in an interior equilibrium of a Cournot oligopoly model with constant marginal costs, aggregate output depends only on the sum of the marginal costs.

Proof: Substitute $n^* = \frac{c(N-1)}{A(X^*)} + 1$ and X^* into (6). This gives us:

$$\left(\frac{c(N-1)}{A(X^*)} + 1\right) A(X^*) + X^* A'(X^*) - cN = 0.$$

Simplifying, we obtain:

$$A(X^*) + X^* A'(X^*) - c = 0$$

which is the same as (1), the condition defining X^*.■

Proposition 4 implies that whenever $c = 0$, the social optimum is achieved by putting everyone in a single partnership ($n = 1$). For suppose everyone is in a single partnership and exerting an N^{th} of the optimal aggregate effort. If any individual varied his effort in either direction, his costs would remain zero, the revenues that he contributes to the pool would change but the revenues his partners would contribute to the pool would change by an exactly offsetting amount (since there could be no first-order change in aggregate producer surplus). The individual would, therefore, have no incentive to deviate.

Contrast this with the case where $c > 0$. In that case, the social optimum cannot be supported by putting everyone in a single partnership since this same individual would now have a strict incentive to decrease his effort; for contracting effort would lower his costs without any first-order change in his gross revenues. As illustrated below, when $c > 0$ the social optimum is achieved with *more than one* partnership. For then, if an individual unilaterally increases his effort the revenues obtained by his own partnership must strictly increase (exactly offsetting the revenue *decrease* experienced by each of the other partnerships) by enough that his share of his partnership's gain exactly offsets his additional cost.

To illustrate, suppose that $N = 12$ players earn their livelihood working in an activity plagued by a congestion externality. Assume aggregate production (and hence aggregate revenue) is $f(X) = 19X - X^2$, where X represents aggregate effort. Suppose that the cost per unit of effort is $c = 3$. It is straightforward to see that the socially optimal effort level is $X^* = 8$. Since $N = 12$, then Proposition 4 implies that:

$$n^* = \frac{3(11)}{19 - 8} + 1 = 4.$$

That is, if the 12 players divide into 4 partnerships, then the resulting aggregate effort will be socially optimal. If n^* is not an integer, the Partnership Solution can only approximate the maximum social surplus.[8] Henceforth, we assume that n^* is an integer.

3 Equilibrium Partnership Choice in the First Stage

To implement the Partnership Solution, consider the following two-step procedure:

1. Step 1

   Partition the N players into n^* groups in such a way that no two groups differ in size by more than one member.[9]

2. Step 2

   Recommend that every player observe the number of groups which form at the first stage, use (6) to compute the aggregate effort expected in the second stage, and then set his own effort level equal to the mean effort of his group as given in (4).

No player would have an incentive to deviate unilaterally from the recommendation in Step 2 since he would anticipate that the others are making the recommended efforts then

[8]Suppose in the previous example that $N = 8$ instead. As before, $X^* = 8$ but now $n^* = \frac{32}{11} = 2.91$. There are two possible integer solutions, $n = 2$ or $n = 3$. If $n = 3$, then we find that $X = 8.25$ which yields a social product of 63.94, while if $n = 2$ then $X = 4.67$ and the social product is 52.89. So $n = 3$ is optimal given the integer constraint. Setting $n = 3$ allows society to obtain 99.9% of the maximal social product, while only 40% can be achieved under the common property solution (where $n = N$).

[9]This can always be done. We simply compute the number of members in the smallest group by taking the largest integer, $Q \leq \frac{N}{n^*}$. If $\frac{N}{n^*}$ is an integer, then all groups will have Q members. If $\frac{N}{n^*}$ is not an integer, there will be a remainder of $R < n^*$ people left over, each of whom can be assigned to a different group. There will then be $n^* - R$ groups, each with Q members, and R groups, each with $Q + 1$ members. For a concrete analogy to dealing playing cards sequentially, see footnote 13.

and these recommendations form a Nash equilibrium.

But does any agent have an incentive to deviate in the first stage from the partnership to which he is assigned given that he anticipates sharing the workload of that partnership equally at the second stage? If not, we will have established one way to implement the Partnership Solution. If so, we will have established that the Partnership Solution cannot be implemented. For, in that case, the Partnership Solution also can not be implemented with partners making asymmetric efforts.[10]

Deviations at the first stage fall into two categories: (1) an agent can abandon the colleagues in his prescribed group for the members of some other group or (2) he can abandon his prescribed group to go into business for himself. As the following proposition shows, the first type of deviation is never advantageous.

Proposition 5 *If groups differ in size by at most one member, then no one can strictly improve his payoff by joining another group.*

Proof: First note that, from Proposition 2, a deviation which maintains the number of groups formed at the first stage will not alter aggregate effort (X^*) exerted at the second stage. Second, assuming homogeneous effort within groups (as discussed above), note that the payoff to each player in group i (π_i) is:[11]

$$\pi_i = \bar{x}_i(A(X^*) - c). \tag{7}$$

This is strictly increasing in $\bar{x}_i$ since $A(X^*) - c > 0$. Each member's payoff is larger in groups with a larger mean level of effort. Proposition 1 tells us that a group with a smaller number of members will have a larger mean effort since its smaller size will discourage

[10]Consider a partition of the players into k partnerships. This uniquely determines the aggregate effort level of every partnership and hence the *gross revenue* of the members of each partnership. An individual in partnership i will receive t he same gross revenue no matter how aggregate efforts are distributed within any of the k partnerships. However, his *payoff* equals the common gross revenue of his partnership *less* his own effort cost. Hence, if effort were reallocated within his partnership so that he undertook more than his share, he would have a stronger incentive to deviate.

[11]To see this, begin with the objective function (2) and see that, if player k makes effort x_{ik} in a group with mean effort $\bar{x}_i$ when aggregate effort is X, then his payoff is: $\pi_{ik} = \bar{x}_i A(X) - cx_{ik}$.

free-riding. Hence, the only way to strictly increase one's payoff by defecting to another group is to switch to a group which, even *after* the defector is added, is strictly smaller than his original group. But there are no such opportunities to increase one's payoff if groups initially differ in size by at most one member.■

Consider the second type of deviation: an agent deviates to form a new, singleton, group. Whether this is profitable or not depends upon the disadvantage of solo production compared to team production. The literature on the theory of the firm identifies the disadvantages of organizing multi-agent firms. Such firms are rife with incentive problems to which single-agent firms are immune. But, since multi-agent firms abound, there must be a countervailing advantage to such arrangements—individuals working in teams must be able to produce more output per man-hour than those working alone.[12] Following the literature on team production, therefore, we assume that a team can produce more than an individual working by himself the same number of man-hours; in extreme cases, a team may be necessary in order to produce at all.

Suppose that to duplicate the efforts of 1 man-hour of team effort, a single individual must work $1/\beta$ hours, for $\beta \in (0, 1]$. Then, if we continue to express effort in man-hours of team effort, the marginal cost of effort for an individual working alone would be $\frac{1}{\beta}c$.

Partition the N players into n groups in such a way that no two groups differ in size by more than 1 member. For any n there is a unique partition that satisfies this restriction.[13] In the case where some partnerships are one member larger than others, these larger partnerships will generate more free-riding in the equilibrium of the second stage (Proposition 1). Anticipating lower payoffs in the second stage, every member of a larger partnership would have a stronger incentive to deviate to a solo partnership at the first stage. Let

[12] Alchian and Demsetz (1972) were the first to emphasize the importance of team production in the theory of the multi-person firm and their insights have now percolated down to undergraduate treatments of that theory. For an extensive discussion, consult the textbooks by Eaton et. al (Chapter 19, 2002) and Campbell (Chapter 2.5, 1995).

[13] One might visualize dealing out N agents sequentially (as if they were cards in a deck) to each of n partnerships until all N agents had been dealt out. At most, some partnerships would have one more agent than other partnerships.

$g(n, \beta)$ denote the gain a member of a larger partnership would achieve by setting up his own partnership. If $g(n, \beta) \leq 0$ then he has no incentive to deviate and *a fortiori* neither does any member of a smaller partnership; hence the partition under consideration is stable. If, however, $g(n, \beta) > 0$ then he has an incentive to deviate and the partition under consideration is unstable. By analyzing properties of the $g(\cdot, \cdot)$ function, we show below that for any n, including n^*, there is a unique $\beta(n) \in (0, 1]$ such that the Partnership Solution is stable for all $\beta \leq \beta(n)$.

3.1 Team Production is Essential ($\beta = 0$)

In many applications, "it takes two workers to perform a given task" (Holmstrom and Tirole, p. 67). That is, solo production is infeasible. For example, no matter how hard a person works he/she cannot catch a whale by himself; nor can he/she stay awake every day and night of his medical career to help patients with their medical emergencies. In other applications deviating to solo groups may be illegal since many partnership agreements contain 'non-compete' clauses which prevent an individual, when leaving a partnership, from competing in the same market as the group he is leaving.[14]

Whenever solo production is infeasible, $g(n, 0) < 0$ and we can conclude:

Proposition 6 *When solo production is infeasible, the Partnership Solution solves the common-property problem.*

Proof: As we have verified, no unilateral deviation to an existing partnership is strictly advantageous to any agent. Moreover, since $g(n, 0) < 0$, no deviation to a solo partnership is profitable for any n, including n^*. ■

[14] Various courts have upheld such clauses, including the Georgia Supreme Court in Rash v. Toccoa Clinic Med. Assoc., 253 Ga. 322, 320 S.E.2d 170 (1984).

3.2 Solo Production Is Feasible ($\beta \in (0, 1]$)

If solo partnerships are legal and feasible, we must investigate further. Social welfare can never be maximized as long as any solo partnership is involved. For, if there are any solo partnerships, then even if in equilibrium optimal effort (X^*) results, the cost of achieving it will strictly exceed cX^*, which a planner could achieve just by assembling a team of all N players and commanding that level of effort. So we assume that $n = 1, 2, \ldots, \lfloor N/2 \rfloor$ partnerships, where $\lfloor Z \rfloor$ denotes the greatest integer less than or equal to Z. For example, if $N = 15$, there are at most $\lfloor 15/2 \rfloor = 7$ partnerships: six with two members and one with three members.

Equation (6) implicitly defines the aggregate effort which would result from n partnerships, each of which has two or more members. Denote the aggregate effort implicitly defined by this equation as $X(n)$. If $X(\lfloor N/2 \rfloor) \geq X^*$, then the Partnership Solution can potentially achieve the first best by generating more free riding and thereby bringing effort down toward X^*.

Denote the payoff of a potential deviator, prior to his deviation, as π^C and his payoff after going solo as π^D. π^C is independent of β. A partner who deviates, therefore, gains $g(n, \beta) = \pi^D - \pi^C$. His gain from going solo, his effort, everyone else's effort, and aggregate effort, will depend on the parameter β. Define $\underline{\beta}$ such that for any $\beta > \underline{\beta}$, the deviator going solo would make strictly positive effort while for any smaller β he would make zero effort. When $\beta \in [0, \underline{\beta}]$, the deviator would receive a zero payoff ($\pi^D = 0$) following his deviation. Hence, $g(n, \beta) = g(n, 0) = -\pi^C < 0$ for any $\beta \in [0, \underline{\beta}]$. When $\beta \in (\underline{\beta}, 1]$ the consequences of one agent's going solo are described by the four variables π^D, X, X_{-1}, and $\bar{x}_1$ which are defined by equations (8)-(11) below, where for simplicity we assign the index "1" to the deviator's solo partnership (and therefore denote his effort as $\bar{x}_1$ and the aggregate effort of all others as X_{-1}):

$$\pi^D = \bar{x}_1(A(X) - \frac{c}{\beta}) \quad (8)$$

$$A(X) + \bar{x}_1 A'(X) - \frac{c}{\beta} = 0 \quad (9)$$

$$nA(X) + X_{-1}A'(X) - (N-1)c = 0 \quad (10)$$

$$\bar{x}_1 + X_{-1} = X. \quad (11)$$

Equation (10) is obtained by adding up the first-order conditions of the n original partnerships after effort levels have adjusted in response to the deviation.

Proposition 7 *$g(n, \beta)$ is a continuous function of β for any β in $(\underline{\beta}, 1)$.*

Proof: Since π^C is independent of β, it is sufficient to show that π^D is continuous in β. Use (11) to eliminate X from (8)-(10). Equation (10) does not involve β. Given the Novshek condition, $(n+1)A' + X_{-1}A'' \neq 0$; therefore, the implicit function theorem insures that, in a neighborhood of any solution $(\bar{x}_1, X_{-1}, X)$ induced by $\beta \in (\underline{\beta}, 1)$ we can write (10) as $X_{-1} = f(\bar{x}_1)$ where $f(\cdot)$ is a continuous function with derivative $f' = -\frac{nA'+X_{-1}A''}{(n+1)A'+X_{-1}A''} \in (-1, 0)$. Equation (9) does involve β. Replace X_{-1} in this equation by $f(\bar{x}_1)$. Given the Novshek condition and $A' < 0$, $(1+f')(A'+\bar{x}_1A'')+A' \neq 0$; therefore, the implicit function theorem insures that we can write (9) locally as $\bar{x}_1 = h(\beta)$ for some continuous function $h(\cdot)$ with derivative $h' = -\frac{c/\beta^2}{A'+(1+f')(A'+x_1A'')} > 0$. Substituting both of these continuous functions into (8), we obtain:

$$\pi^D(\beta) = h(\beta)\left[A\left(h(\beta) + f(h(\beta))\right) - c/\beta\right].$$

Since $A(\cdot)$ is continuous and since sums, products, and compositions of continuous functions are continuous, π^D is a continuous function of β in a neighborhood of any solution $(\bar{x}_1, X_{-1}, X)$ induced by $\beta \in (\underline{\beta}, 1)$. Given this conclusion, there can be no $\beta \in (\underline{\beta}, 1)$ where π^D is discontinuous. It follows that $g(n, \beta)$ is continuous for β in the open interval $(\underline{\beta}, 1)$. ■

Since $g(n, \beta) = \pi^D(\beta) - \pi^C$, we can differentiate to obtain the partial derivative, $g_\beta(n, \beta)$ anywhere in the open interval:

Proposition 8 *$g_\beta(n, \beta) > 0$ for any β in $(\underline{\beta}, 1)$.*

Proof: Since $\bar{x}_1 > 0$ for any β in $(\underline{\beta}, 1)$, $h(\beta) > 0$. Recall that $A' < 0$. Differentiating our expression for $g(n, \beta)$ and using (9) to simplify (an application of the envelope theorem) we conclude that:

$$g_\beta(n, \beta) = h'[A + hA' - c/\beta] + h[A'f'h' + c/\beta^2] = h[A'f'h' + c/\beta^2] > 0$$

for any β in $(\underline{\beta}, 1)$.■

We have shown that $g(n, \beta)$ is continuous and strictly increasing in β in the open interval $(\underline{\beta}, 1)$ and $g(n, \beta) = -\pi^C$ for $\beta \in [0, \underline{\beta}]$. The following lemma establishes that there is no discontinuity at the boundary $\beta = \underline{\beta}$.

Lemma 1 *The function $g(n, \beta)$ is continuous in β at the point $\underline{\beta}$.*

Proof: Since $g(n, \beta) = -\pi^C$ for $\beta \in [0, \underline{\beta}]$, it suffices to verify that $\lim_{\beta \downarrow \underline{\beta}} g(n, \beta) = \lim_{\beta \downarrow \underline{\beta}} (\pi^D - \pi^C) = -\pi^C$. But this follows from (8) since $\lim_{\beta \downarrow \underline{\beta}} \bar{x}_1 = 0$ and A is bounded. ■

We can therefore, conclude:

Proposition 9 *If the partition indexed by n is stable for some β, then it is stable for all smaller β.*

Proof: This follows from Proposition 8. ■

We now use the results above to prove the existence and uniqueness of a 'threshold' $\beta(n)$ which separates stable from unstable partitions.

Proposition 10 *For any $n \leq \lfloor N/2 \rfloor$, there exists a unique $\beta(n) \in (\underline{\beta}, 1]$ such that for any $\beta < \beta(n)$, the partition indexed by n can be supported as a subgame-perfect equilibrium, while for $\beta > \beta(n)$ the partition can never be supported.*

Proof: For any given n, suppose that at $\beta = 1$, $g \leq 0$. Then that partition can be supported as an subgame-perfect equilibrium for any $\beta \in (0, 1]$ and we can define $\beta(n) = 1$. Now suppose that at $\beta = 1$, $g > 0$. Then by continuity (Proposition 7), there will exist one or more roots, $\beta \in (0, 1)$, such that $g(n, \beta) = 0$. Denote any root as $\beta(n)$. Uniqueness of $\beta(n)$ then follows since g is strictly increasing (Proposition 8).■

This makes precise the intuitive notion that the socially optimal partnership partition is stable if team production is "sufficiently advantageous": β needs to be smaller than $\beta(n^*)$. Alternatively, for any given β, $\beta(n)$ also defines partnership partitions which are stable:$\{n : \beta(n) \leq \beta\}$.

Is the Partnership Solution *always* stable even when team production confers no advantage whatsoever($\beta = 1$)? A single counterexample suffices to eliminate this possibility. Recall the example introduced at the outset where $N = 12$ producers in an industry, each with constant marginal cost of $c = 3$, face an inverse demand curve of $P = 19 - X$ and attempt to achieve monopoly profits by dividing into $n = 4$ partnerships of equal size. It is easily verified that for any $\beta \leq .39$, full monopoly profits (\$64) can be achieved, but for $\beta > .39$ the configuration of four partnerships is unstable. There remains the possibility that for at least some example satisfying our assumptions, the partnership solution is stable even in the absence of advantages to team production. This seems unlikely since, in general, a partition with fewer than n^* partnerships can never be stable.[15]

4 Generalizations

Until now, we have assumed that no costs were *shared* within a partnership. We have also assumed that no individual or partnership has the power to change the price of output. We now relax both assumptions by reinterpreting our previous analysis. In addition, we show

[15]When $\beta = 1$, $n < n^*$ is never stable. Recall that the only partitions we need consider are those where partnerships differ by at most one member and where effort is shared equally among the partners. Pick a partnership and designate someone as a potential deviator. Before going solo, he would earn exactly the same payoff as everyone else in his partnership; after going solo, he would earn at least as much as his ex-partner(s) since he would eliminate free-riding and $\beta = 1$. If, for the sake of argument, he did not strictly benefit from going solo then (1) the payoff of his ex-partners would likewise not increase and (2) the payoff of everyone else must strictly decrease. But then the sum of the payoffs would strictly decrease which contradicts the fact that the aggregate profit function is increasing in the number of partnerships to the left of n^*. An analogous argument establishes the strict profitability (when $\beta = 1$) of a "marginal" deviation in the neighborhood of n^* partnerships. See our earlier working paper: Heintzelman, Salant, and Schott (2004).

how partnerships can be valuable as a way to increase payoffs even when the first-best is unattainable.

Suppose we partition N homogeneous agents into n payoff-sharing groups indexed by i, each playing a simultaneous-move game. Assume agent k in group i chooses x_{ik} to maximize $\frac{1}{m_i}\left[x_{ik}+Y_i^{-k}\right]\cdot G(x_{ik}+Y_i^{-k}+X_{-i})-cx_{ik}$. If we make the same assumptions about $G(X)$ that we made about $A(X)$ then we will get the corresponding results. So assume that (1) $G(X)$ is strictly positive, strictly decreasing, and twice continuously differentiable; (2) $G(0)-c>0$; and (3) the Novshek (1985) condition, $G'(X)+XG''(X)<0$, holds for all $X \geq 0$. These assumptions are sufficient to insure the existence of a pure-strategy Nash equilibrium in the simultaneous-move game. Because $G(\cdot)$ is downward-sloping, there is a negative externality: agent k is adversely affected by increases in X_{-i}. We have derived conditions sufficient for the aggregate payoff, $X(G(X)-c)$, to be maximized: provided $n^* \leq \lfloor N/2 \rfloor$ and $\beta < \beta(n^*)$, the optimum can be achieved by setting up n^* partnerships differing in size by at most one member.

Suppose $G(X)=A(X)-K$, where K denotes cost per unit effort for those costs shared within the partnership. Then the Partnership Solution maximizes producer surplus. Since price is constant, this maximizes social welfare as well.

Next suppose $G(X)=P(f(X))A(X)-K$, where $P(\cdot)$ is the industry price when aggregate output $f(X)$ is put on the market. This generalization fits the case of the fishermen of Toyama Bay, who share some but not all costs and who use their partnerships not merely to curb congestion but to raise price. Again, the Partnership Solution maximizes producer surplus.

Finally, suppose $G(X)=P(X)-K$, where X is now interpreted as *output* and K (respectively, c) as the cost per unit *output* rather than effort, which is shared (respectively, not shared) within the partnership. In this case, the Partnership Solution curbs excessive output and permits a cartel to maximize profits without any need for supergame strategies.

In these last two cases, producer surplus is maximized but social welfare could be increased by raising production. To see this, note that a marginal increase in production by anyone would have no first-order effect on producer surplus but would strictly increase consumer surplus.

In cases where $\beta > \beta(n^*)$, the advantages of team production are insufficient to achieve the first-best using the Partnership Solution. In such cases, a generalization of the Partnership Solution can nonetheless lead to a *second-best* equilibrium with a large increase in the aggregate payoff. To illustrate, recall the example where $N = 12, c = 3$, and $G(X) = 19-X$. In that case $n^* = 4$ and $\underline{\beta} = .39$. Suppose as in our earlier example that $\beta = .56 > .39$. Then dividing the agents into four partnerships of equal size is not feasible since each member would have an incentive to go solo. However, if the 12 agents are divided into six partnerships of equal size, then industry profit is \$54.12—not the first-best level of \$64 but approximately *triple* the result in the oligopoly (or common property) solution.

5 Conclusion

In this paper, we examined the viability of the Partnership Solution to the common property problem and showed that this proposal was also a potential solution to the problem of organizing a cartel to achieve monopoly profits. The Japanese fishermen who have formed partnerships to pool the revenues (and some costs) from their various vessels report that their goal is to reduce congestion and raise price. These are, in fact, the consequences to be expected from partnerships.

We showed that the Partnership Solution suffers from a single weakness: the temptation to flee one's free-riding partners and go solo. Going solo is sometimes infeasible for technological or legal reasons. Even when it is feasible, however, going solo ceases to be as attractive when there are substantial benefits from team production or substantial fixed costs of setting up a private practice (office rent, support staff to handle billing and third party reimbursement, etc.). In such circumstances, the Partnership Solution can sometimes be used to maximize the aggregate payoff.

Throughout, we assumed that a partnership had to admit every applicant. It might have been more realistic to assume that members of an existing partnership could deny admission to anyone if opposition to him within the partnership was "sufficiently widespread." This change in assumption would in fact have *increased* the scope of the Partnership Solution. For, every solution we identified as stable would continue to be stable since no one in such solutions has any incentive to join an existing partnership even when assured of admission. But partitions we identified as unstable under our old assumption would become stable under this new assumption. To illustrate, suppose going solo was infeasible and we set up n^* non-solo partnerships some of which differed by two or more members. Such an arrangement could not achieve the first-best under our old assumption because every member of the largest partnership would deviate unilaterally to a smaller partnership with less free-riding. But this same arrangement *would* achieve the first-best under the new assumption since admitting him would be blocked unanimously by existing members who anticipated that expanding the number of partners would stimulate free-riding and would lower each of their payoffs. In assuming that no applicant could be rejected by existing members, therefore, we *understated* the usefulness of partnerships in solving the common-property and cartel problems.

Our conclusions contain both good and bad news. The good news is that partnerships can eliminate all or much of the deadweight loss associated with the common property problem. The bad news is that they can also eliminate all or much of the loss in monopoly profits experienced by cartels. Anti-trust authorities would be well-advised to take our analysis into account when investigating professions where firms are organized as partnerships.

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