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Scheduling participants of Assessment Centres

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Abstract

Assessment Centres are used as a tool for psychologists and coaches to observe a number of dimensions in a person's behaviour and test his/her potential within a number of chosen focus areas. This is done in an intense course, with a number of different exercises which expose each participant's ability level in the chosen focus areas. The participants are observed by assessors with the purpose of gathering material for reaching a conclusion on each participant's personal profile.

We consider the particular case that arises at the company *Human Equity* (www.humanequity.dk), where Assessment Centres usually last two days and involve 3-6 psychologists or trained coaches as assessors.

An entire course is composed of a number of rounds, with each round having its individual duration. In each round, the participants are divided into a number of groups with prespecified pairing of group sizes and assessors. The scheduling problem amounts to determining the allocation of participants to groups in each round. We have developed a model and solution approach for this particular scheduling problem, which may be viewed as a rather extensive generalization of the Social Golfer Problem.

1 Introduction

This paper considers a rather extensive generalization of the *Social Golfer Problem* (see, e.g., [2]), which arises regularly as a planning problem at the company *Human Equity.*

Human Equity offers, among other things, courses of development in which they use *Assessment Centres* (a tool for psychologists and coaches, see [1]) for observing various dimensions in the individual participant's behaviour. Observations are made by *assessors* (psychologists and coaches) while participants do exercises which expose each participant's ability level in chosen focus areas.

An entire course contains a prespecified number of rounds. In each round, the entire set of participants must be partitioned into groups. The durations of the rounds, and the pairing of group sizes and assessors, are given in advance and may vary from one round to another.

The planning problem is to determine the allocation of participants to groups in each round.

2 Modelling

We introduce the following notation. Let P denote the number of participants (indexed $1, \ldots, P$), R denotes the number of rounds (indexed $1, \ldots, R$), and Q is the number of assessors (indexed $1, \ldots, Q$). Moreover, $D(r)$ is the duration of round r, and $G(r)$ is the number of groups (indexed $1, \ldots, G(r)$) in round r, for $r = 1, \ldots, R$.

In addition, $N(r, g)$ and $A(r, g)$ denote the size (cardinality) and assessor, respectively, of group g in round r, for $r = 1, \ldots, R$, $g = 1, \ldots, G(r)$. We assume that the sum of group sizes in each round equals P.

There are several possibilities for modelling of objectives in this practical case. We have chosen to measure solution quality in terms of deviations from certain prespecified target intervals. Each interval represents a desired total time that a pair of persons spend together during the entire course. For pairs of players, we have two target intervals; the interval of first priority is $[PP_1^-; PP_1^+]$, and the interval of second priority is $[PP_2^-; PP_2^+]$, where $PP_1^- \leq PP_2^-$ and $PP_1^+ \geq PP_2^+$. Similarly, for pairs of participant/assessor, we have two target intervals; the interval of first priority is $[PA_1^-; PA_1^+]$, and the interval of second priority is $[PA_2^-; PA_2^+]$, where $PA_1^- \leq PA_2^-$ and $PA_1^+ \geq PA_2^+$.

For a given schedule, we can then assess its quality based on absolute deviations from the target intervals. We consider the minimization of the total (weighted) deviations from the two first priority intervals to be the most important. However, in case of ties wrt. deviations from the first priority intervals, we attempt to minimize the total weighted deviations from the two second priority intervals. Hence, our model is a *lexicographic goal programming model.* It contains the following variables:

1. x_{prg} (for $p = 1, \dots, P$, $r = 1, \dots, R$, $g = 1, \dots, G$), where $x_{prg} = 1$ represents that participant p is allocated to group g in round r, and $x_{prg} = 0$ otherwise.
2. TPP_{ij} denotes the time spent together by participants i and j.
3. TPA_{ij} denotes the time spent together by participant i and assessor j.
4. DPP_{ij}^{α} denotes TPP_{ij}'s absolute deviation from target interval of priority α, for $\alpha = 1, 2$.
5. DPA_{ij}^{α} denotes TPA_{ij}'s absolute deviation from target interval of priority α, for $\alpha = 1, 2$.
6. Z_α denotes the total weighted absolute deviation from target interval of priority α, for $\alpha = 1, 2$.

We then obtain the following model:

$$\min Z_1 = w_p \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} DPP_{ij}^1 + w_a \sum_{i=1}^{P} \sum_{j=1}^{Q} DPA_{ij}^1 \tag{1}$$

$$\min Z_2 = w_p \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} DPP_{ij}^2 + w_a \sum_{i=1}^{P} \sum_{j=1}^{Q} DPA_{ij}^2 \tag{2}$$

$$\text{s.t.:} \qquad \sum_{g=1}^{G(r)} x_{prg} = 1, \text{ for } p = 1, \dots, P;\ r = 1, \dots, R \tag{3}$$

$$\sum_{p=1}^{P} x_{prg} = N(r, g), \text{ for } r = 1, \dots, R;\ g = 1, \dots, G(r) \tag{4}$$

$$TPP_{ij} = \sum_{r=1}^{R} \sum_{g=1}^{G} D(r) x_{irg} x_{jrg}, \text{ for } i, j = 1, \dots, P;\ i \neq j \tag{5}$$

$$TPA_{ij} = \sum_{r=1}^{R} \sum_{g|A(r,g)=j} D(r) x_{irg}, \text{ for } i = 1, \dots, P;\ j = 1, \dots, Q \tag{6}$$

$$DPP_{ij}^{\alpha} = \max\{PP_{\alpha}^{-} - TPP_{ij}, TPP_{ij} - PP_{\alpha}^{+}, 0\}, \text{ for } \alpha = 1, 2 \tag{7}$$

$$DPA_{ij}^{\alpha} = \max\{PA_{\alpha}^{-} - TPA_{ij}, TPA_{ij} - PA_{\alpha}^{+}, 0\}, \text{ for } \alpha = 1, 2 \tag{8}$$

As mentioned, our model is a lexicographic goal programming model, which means that for any two given solutions, say A and B, we prefer solution A over B if

$Z_1^A < Z_1^B$ or ($Z_1^A = Z_1^B$ and $Z_2^A < Z_2^B$). The two weights $w_p \geq 0$ and $w_a \geq 0$ are parameters which are specified in advance by the user.

Before considering solution approaches, we would like to emphasize that certain well-known problems arise as a consequence of particular settings of weights and target intervals.

The *Social Golfer Problem* (SGP) is obtained by settting $w_p > 0$, $w_a = 0$, $D(r) = 1$ for $r = 1, \ldots, R$, $PP_1^- = 0$ and $PP_1^+ = 1$. With these setttings, a solution with $Z_1 = 0$ is a schedule in which each pair of participants meet at most once, as desired in the Social Golfer Problem ([4]).

The *Debating Tournament Problem* (DTP) is a generalization of the SGP in which each pair of participants is required to meet at least β and at most γ times (see [3]). This is also a special case of our model and is obtained by settting $w_p > 0$, $w_a = 0$, $D(r) = 1$ for $r = 1, \ldots, R$, $PP_1^- = \beta$ and $PP_1^+ = \gamma$.

3 Algorithms

Given that our problem is a generalization of the SGP, we found it natural to consider possible generalizations of approaches to the SGP. We comment on these possibilities in the following. Before continuing, however, we wish to note that the SGP may be viewed both as a constraint satisfaction problem and as an optimization problem. In the optimization version, the objective is to maximize the number of rounds such that no pair of participants meet more than once.

The characteristics of the SGP imply that certain constructions may be useful for its exact or heuristic solution. In particular, *Mutually Orthogonal Latin Rectangles* (MOLR) may be used for constructing an optimal or heuristic solution to the optimization version of the SGP. For example, this is done in [4]. For certain combinations of the group size and the number of groups, it is known that an optimal solution can be constructed using MOLR. In our case, however, it may easily be the case that the number of groups in some round is smaller than the largest group in some other round, so the orthogonality restriction does not apply in our case.

Another approach for the SGP is *local search.* While being heuristic in nature and as such not guaranteeing optimal solutions, it is also much more flexible wrt. the types of problems that may be handled. One such heuristic is the tabu search heuristic presented in [2], in which a move is defined as a swap of two participants (players) between two groups in the same round.

We adopted the idea in [2] of using local search to make improvements of one round at a time. However, we also considered other neighbourhood structures than swapping only two partipants. More generally, the whole class of *cyclic transfer* *algorithms* [5] comes into consideration when choosing a neighbourhood structure.

We implemented a multi-start heuristic, which contains the following ingredients. The inital solution is obtained by generating, for each round r, a random sequence of the set of participants. The first $N(r, 1)$ participants in the random sequence are then assigned to the first group, the next $N(r, 2)$ participants are assigned to the second group, and so on.

Given the initial solution, we iteratively attempt to improve the current solution, For that purpose we considered two types of moves, a *2-swap* and a *3-swap*, respectively. Each type of move changes only one round at a time. A 2-swap simply swaps two participants between two groups (in the same round), whereas a 3-swap assigns each of 3 participants to another group; specifically, a 3-swap takes an ordered triplet (i, j, k) of participants, currently assigned to three different groups (g_i, g_j, g_k), and moves i to group g_j, j to group g_k, and k to group g_i. In our implementation, we continue to improve a round until no more improvements are found, before considering the next round. When no round can be improved, we have a candidate solution.

In our experiments we found that using the 3-swap does not lead to major improvements of the resulting solution. In fact, in most cases the final solution is not affected if 3-swaps are deactivated. Based on this observation we chose not to implement any neighbourhoods involving more than 3 participants.

Our implementation takes as input a parameter which specifies the number of candidate solutions (being equal to the number of initial solutions) that should be generated. The program then outputs the best among all candidate solutions. We have found that generating 100 candidate solutions gives a reasonable trade-off between running time and solution quality.

We programmed our heuristic as a console application using Microsoft Visual C++ 2005. Input is read from a single text file containing the user specified parameters. The program outputs another text file containing the schedule and the resulting measures of deviations from the target intervals.

4 Practical use

Compared to the previous manual planning process, there are many advantages of using the described planning programme.

The most important advantage is the increased efficiency. The process is very easy and intuitive which remarkably decreases the planning time of the Assessment Centres. A process that previously could take hours - depending on the number of participants and the varying duration of the different exercises - is now done in a matter of minutes. Using the software, it is very easy to experiment while optimizing the solution, by modifying the input parameters. This process was earlier very time-consuming and mistakes were easily made. Furthermore the given solutions have higher quality, meaning the participants are better allocated concerning the varying duration of the different exercises.

Another big advantage of the planning programme is that the planning no longer is as dependent on specific employees, meaning that the planning is not as dependent on the employees who are logical thinkers and have a large breadth of view or the employee who normally plans the courses. They can now be planned by any employee who has insight into the practical aspects of the Assessment Centres.

5 Conclusion

This paper has described a practical scheduling problem that arises in relation to the use of Assessment Centres. The scheduling problem, which may be viewed as a generalization of the Social Golfer Problem, is modelled as a lexicographic goal programming model. We chose to solve the problem using a multi-start heuristic which involves swaps of participants between groups in a round.

The practical use of the planning tool has several advantages relative to the earlier manual process. The planning tool reduces solution time, leads to higher quality solutions, and reduces the dependency of specific employees in the planning process.

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