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Streams of events and performance of queuing systems: The basic anatomy of arrival/departure processes, when the focus is set on autocorrelation

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STREAMS OF EVENTS AND PERFORMANCE OF QUEUING SYSTEMS: THE BASIC ANATOMY OF ARRIVAL/DEPARTURE PROCESSES, WHEN THE FOCUS IS SET ON AUTOCORRELATION

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Abstract

Judging from the vast number of articles in the field of queuing simulation, that assumes i.i.d. in one or more of the stochastic processes used to model the situation at hand, often without much validation, it seems that sequence independence must be a very basic property of many real life situation or at least a very sound approximation.

However, on the other hand, most actual decision making is based upon information taken from the past - where else! In fact the only real alternative that comes into my mind is to let a pair of dices fully and completely rule behaviour, but I wonder if such a decision setup is that widespread in consequent use anywhere. So, how come that sequence independence is so relatively popular in describing real system processes?

I can only think of three possible explanations to this dilemma - (1) either sequence dependence is present, but is mostly not of a very significant nature or (2) aggregate system behaviour is in general very different from just the summing-up (even for finite sets of micro-behavioural patterns) and/or (3) it is simply a wrong assumption that in many cases is chosen by mere convention or plain convenience.

It is evident that before choosing some arrival processes for some simulation study a thorough preliminary analysis has to be undertaken in order to uncover the basic time series nature of the interacting processes. Flexible methods for generating streams of autocorrelated variates of any desired distributional type, such as the ARTA method or some autocorrelation extended descriptive sampling method, can then easily be applied. The results from the Livny, Melamed and Tsiolis (1993) study as well as the results from this work both indicates that system performance measures as for instance average waiting time or average time in system are significantly influenced by the taken i.i.d. versus the autocorrelations assumptions. Plus/minus 35% in performance, but most

likely a worsening, is easily observed, when comparing even moderate (probably more realistic) autocorrelation assumptions with the traditionally and commonly used i.i.d. assumptions.

Keywords: Autocorrelation, queuing systems, TES method, ARTA method, Descriptive/Selective sampling, Simulation, Job/flow-shop, performance, control.

INTRODUCTION

Judging from the vast number of articles in the field of queuing simulation, that assumes i.i.d. in one or more of the stochastic processes used to model the situation at hand, often without much validation, it seems that sequence independence must be a very basic property of many real life situation or at least a very sound approximation.

However, on the other hand, most actual decision making is based upon information taken from the past - where else! In fact the only real alternative that comes into my mind is to let a pair of dices fully and completely rule behaviour, but I wonder if such a decision setup is that widespread in consequent use anywhere. So, how come that sequence independence is so relatively popular in describing real system processes?

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To put this discussion further into perspective it can be noted, that methods for introducing autocorrelation into a simulation study are in fact numerous. The ARTA-method suggested by Cario and Nelson (1998), where ARTA denotes ”Auto-Regressive-To-Anything”, and the TES-method developed by Jagerman and Melamed (1992,a & b), where TES denotes ”Transform Expand Sample”, are nice representatives of the various methods for generating uniform variates, that incorporates autocorrelation. Even finite sets of variates methods, as for instance ”selective” sampling originally developed by Brenner (1963), or ”descriptive” sampling as originally described by Saliby (1989), may at least in principle also be used to create autocorrelation patterns of almost any imaginable nature, if only it is possible to devise a relevant ”scrambling” procedure with a resulting desired sequence dependence.

There are, in my opinion, several good reasons for the need to look closer into the phenomenon of process autocorrelation in general and in relation to queuing and job/flow shop systems in particular. Autocorrelation is in my view, and I find it well supported by practical experience, a much more predominant phenomenon in socio-economic systems, than is the case of independence. And results presented by Livny, Melamed and Tsiolis (1993) corroborates this view that it is immensely important to take proper account of any potential autocorrelation present, by showing that autocorrelation mostly has a profound negative effect on the functioning of a simple queuing system.

One could also think of another situation where we quite deliberately chooses to introduce autocorrelation into some event stream, namely whenever the ”Shortest Processing Time” rule, in short the SPT-rule, is applied for prioritizing jobs in a queue. This is in contrast a situation where autocorrelation is systematically introduced and utilized in order to explicitly improve the overall system performance.

So autocorrelation has seemingly both the potential to introduce positive as well as negative effects into the systems functioning and so it obviously has to be of interest to know more about the basic anatomy of event streams with respect to autocorrelation and also how some given autocorrelation phenomenon can be expected to propagate its way through for example a job/flow-shop system.

Let’s therefore approach the discussion of queuing systems and the relevance of autocorrelation by first clarifying, that there can of course be good reason sometimes to assume independence in event processes, typically arrival processes. From the Palm-Khintchine Theorem (1969) we know that under fairly mild conditions on the individual (n’th) arrival source (s_n), that the ”superposition process” that results from the mixing of all the (s_n) processes with $n \to \infty$ is known to converge towards a Poisson process. This is popularly speaking sort of a ”Central Limit Theorem” for arrival processes, that kicks in and govern the nature of mixtures of an infinity of independent sub-processes.

However, when the number of processes that is mixing is limited, or as it often will be the case in reality, consists of only a few processes say 1 or 2, sequence independence of event inter-arrival times of the mix-process is not guaranteed by any means. In fact, if the individual process components in the mix process show sign of autocorrelation, then certainly also does the mix-process to some extent.

The present work will take its offset in the results presented by Livny, Melamed and Tsiolis (1993) and start asking the question as to what extent these findings simply can be ascribed to the level of the 1st order autocorrelation or if they are dependent on the characteristics of the whole TES-setup? It will be tested by recomputing some of the Livny, Melamed and Tsiolis results with an alternative, more flexible method for generating autocorrelated variates with comparable 1st order autocorrelations, but otherwise significantly different ACF/PACF profile, where by the way ACF denotes the ”AutoCorrelation-Function” and PACF denotes the ”Partial AutoCorrelation-Function”. Livny, Melamed and Tsiolis (1993) themselves also investigates one alternative method for generating autocorrelated uniform variates, called the Minification method, however, though the Minification method is different from the TES method with respect to the ACF/PACF profiles, it is just as inflexible as the TES method, with respect to the shaping of the full autocorrelation profile. The more flexible ”TES-deviating” setups, that will be considered in this work will be discussed in two variants, one that obeys a certain distributional shape asymptotic (ARTA) and one that obeys a certain distributional shape relative to a given finite sample exact distributional form (Extended Descriptive Sampling with Autocorrelation). The last setup effectively separates, at least in principle, the autocorrelation effect onto the performance in its most pure form.

Having, hopefully, by now established whether just a first order approach to autocorrelation in event streams essentially is of importance or a more full approach is called for, this work will continue by taking an analytical offset in the autocorrelation characteristics of some output stream of events from a simple M/M/1 queuing system operated by some specific queuing discipline as for instance the

SPT-rule. The question that comes naturally in this situation is whether TES, ARTA etc. are methods that can be fitted well enough to approximate/simulate the input to succeeding systems, that otherwise typically would take the output from a few simple M/M/1-like queuing systems, operated, of course, by some given queuing discipline, as input?

Autocorrelation in Event Streams

Whether an event process is autocorrelated or not can seldom be judged simply by looking at its graphical appearance. Let for example x be i.i.d. exponential distributed and let $X_n = X_{n-1} + x_n$ be the n'th x-event time of occurrence ($X_0 = 0$). Let further y be not independent but still i.d. exponential distributed and let $Y_n = Y_{n-1} + y_n$ be the n'th y-event time of occurrence ($Y_0 = 0$). Plotting the event streams for both these processes will look like the following, where each "tic" denoting an event

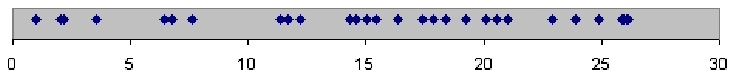

Figure 1: Occurrence of "x"-events

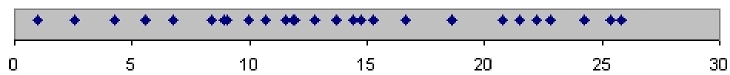

Figure 2: Occurrence of "y"-events

Clearly, the sample is too small to make any judgement at all, however, if on the other hand the sample was larger the eye would not be able to see anything, but a mess of "tics".

So, there is called for a more elaborate approach in order for example to uncover a possible first order autocorrelation effect. A plot of x against its lagged value might give some insight.

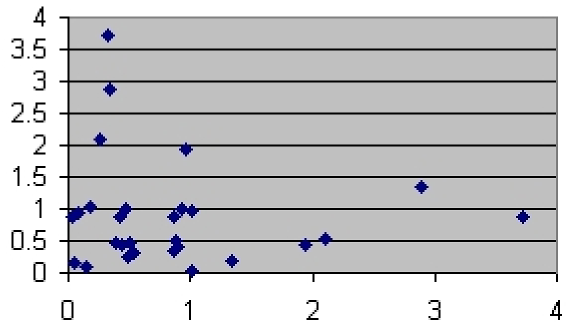

Figure 3: x against lag(x)

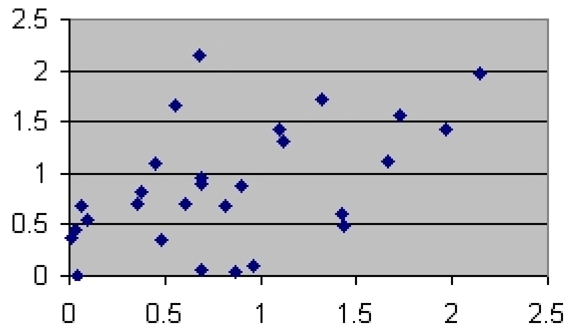

Figure 4: y against lag(y)

However, despite the pictures does not give any conclusive information, due to a still too small a sample size, they non the less appear to be of a distinctive different nature. Further performing a plot of the autocorrelation as well as the partial autocorrelation functions for this very scarce sample size does, as should be expected, not add significantly to the findings above, remembering that 5% of the observations are to be expected to be significant by mere chance.

Finally, please observe that in both plots (Figure 5 & 6), as will also be the case whenever this type of ACF/PACF graphical representation is applied elsewhere in this paper, the ”Series 1”-label denotes the ACF profile, the ”Series 2”-label denotes the PACF profile and the ”Series 3 & 4” denotes the 95% confidence limits for both the ACF and the PACF profiles.

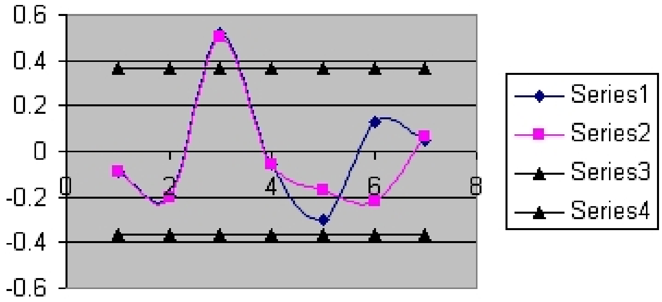

Figure 5: ACF and PACF for x

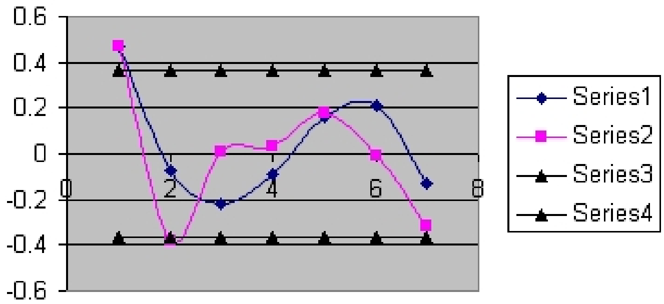

Figure 6: ACF and PACF for y

As a contrast to the above (lack of) findings, if we repeat the plotting of the autocorrelation as well as the partial autocorrelation functions based now on 20000 observations, we get a whole lot of a very different story told. The ACF and PACF plots in figure 7 & 8 tells us that the x-variate is clearly i.i.d. whereas the y-variate is most likely an AR(2)-process with $\rho_1 \approx 0.4$ and $\rho_2 \approx -0.2$. By the way, if necessary, the parameters of the underlying AR(2)-process can easily be computed from the ρ values by exercising the socalled Yule-Walker equations (see for a full description).

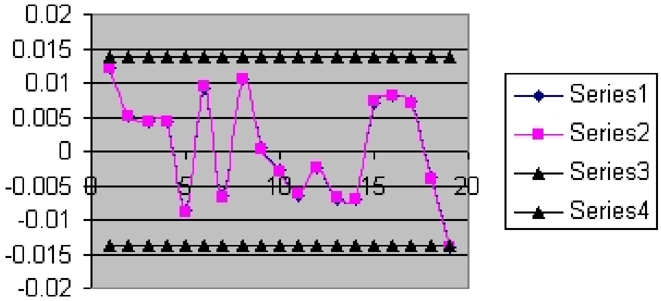

Figure 7: ACF and PACF for x

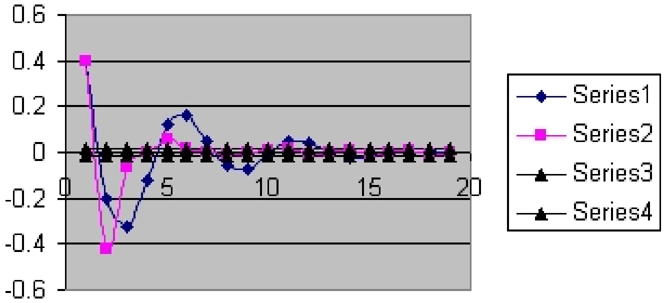

Figure 8: ACF and PACF for y

So, let us now turn our attention towards the output processes emanating from queuing systems, which typically is the input to succeeding stages in a production flow. Let us begin by taking a closer look in terms of autocorrelations at the stream of events flowing from a simple M/M/1 queuing system governed by different queue priority rules such as for instance SPT and LPT, where LPT denotes ”Largest Processing Time”. The different setups will be considered for different levels of traffic intensity - say 0.25, 0.75 and 0.95.

The following six M/M/1 queuing system setups, which are ”x-rayed” below (figure 9 through 14) for their autocorrelation properties, are all simulated for a pass-through of 20000 units, customers or items of goods if you like.

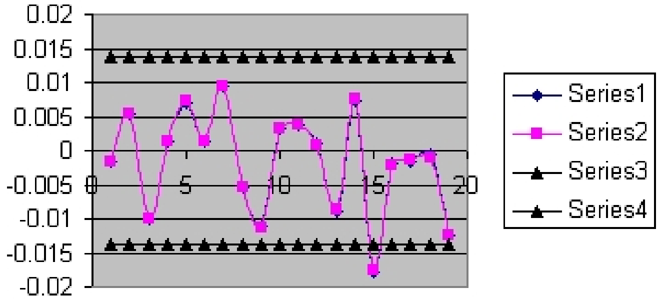

Figure 9: ACF and PACF for M/M/1/SPT output - traffic intensity=0.25

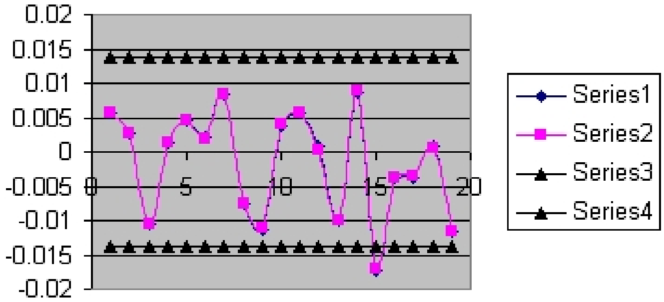

Figure 10: ACF and PACF for M/M/1/LPT output - traffic intensity=0.25

As should be expected the filtering effect of a given queue priority setup only kicks in for relative high levels of utilisation. For traffic intensities of 0.75 and 0.95 we see a pronounced autocorrelation generating effect by both the SPT and LPT rule. Negative autocorrelations for the SPT-rule, and positive ones for the LPT-rule. It can also be noticed that in the SPT case the first order autocorrelation coefficient is absolutely and almost solely dominant, whereas in the LPT case a much more slowly decreasing full set of autocorrelation coefficients is the case.

The simulation results reported by Livny, Melamed and Tsiolis (1993) in their paper with the title ”The Impact of Autocorrelation on Queuing Systems” mainly tells us that it is autocorrelation in the arrival process that hurts the most and definitely more than autocorrelation in the service process, at least for M/M/1/FIFO systems. Both possitive as well as negative autocorrelation patterns has

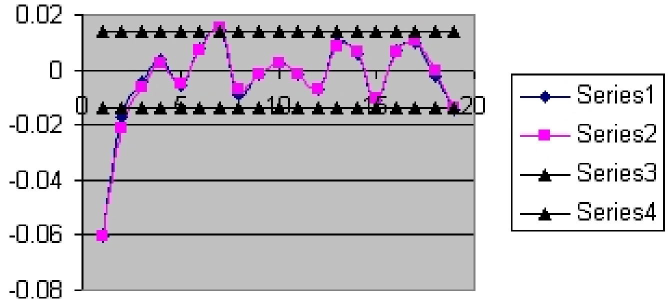

Figure 11: ACF and PACF for M/M/1/SPT output - traffic intensity=0.75

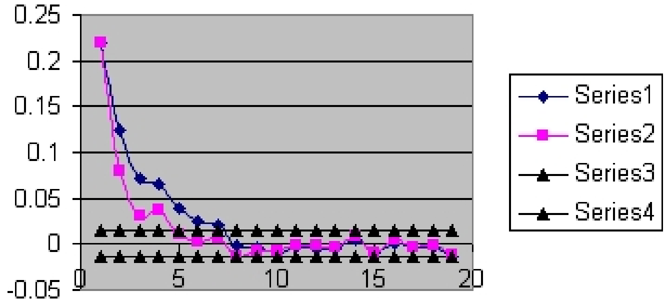

Figure 12: ACF and PACF for M/M/1/LPT output - traffic intensity=0.75

according to their findings a strong deteriorating effect on typical performance measures, such as for example the average waiting times, whenever the system utilization level is moderate to high.

From the autocorrelation plots (Figure 11 & 13) for the M/M/1/SPT queuing system it seems clear that the first order autocorrelation plays the dominant role which implies that it is important to be able to model autocorrelation in arrival processes by the magnitude of this first order autocorrelation relationship, but is this sufficient? Or is there a reasonable need for the ability to be able to model higher order autocorrelation properties? The TES method, the one that is used by Livny, Melamed and Tsiolis (1993), for generating uniformly distributed variates is computational extremely effective in a simulation context, but allows only for the specification of the first order autocorrelation coefficient

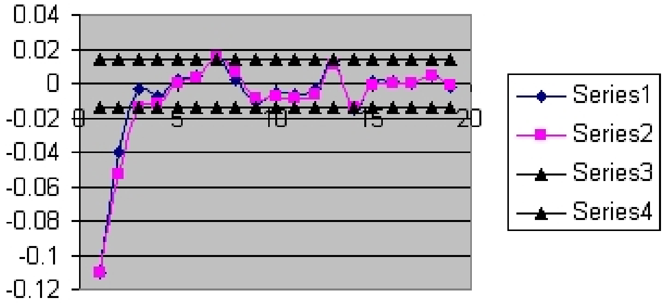

Figure 13: ACF and PACF for M/M/1/SPT output - traffic intensity=0.95

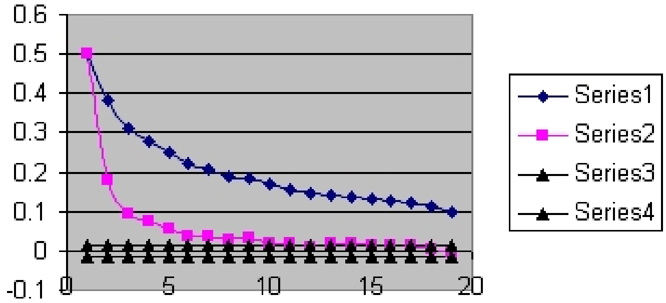

Figure 14: ACF and PACF for M/M/1/LPT output - traffic intensity=0.95

by which the whole ACF/PACF structure is determined. It would of course, primarily due to the computational aspect, be nice if TES-type autocorrelation processes could be viewed as sound and generally applicable approximations, a kind of first order approximation, to any type of autocorrelation present in a given analysis. This is unfortunately not the case! We consequently have to look broader. The ARTA method for generating autocorrelation properties in event processes is much more flexible in that it allows in principle the specification of the full spectrum of autocorrelation coefficients, however confined to the class of covariance-stationary autoregressive time series models, where the Yule-Walker equations constitutes a solid link between the autocorrelation coefficients and the specification of the data generating AR-process. The ARTA method is in principle as computational efficient and easy to use as the TES method.

The following section will deal with and dig a little bit deeper into the differences between the TES- and the ARTA methods as judged by their impact on typical performance measures relating to the M/M/1/FIFO queuing system.

TES versus ARTA

The TES method is a very efficient method for generating autocorrelated uniform variates given by the following transformation scheme that can be built into any good uniform random number generator.

$$\begin{aligned} U_0^+ &= Z_0 \\ U_i^+ &= \left\langle U_{i-1}^+ + L + (R-L)\cdot Z_i \right\rangle \\ U_i^- &= U_i^+ \quad \text{if } i \text{ is even} \\ U_i^- &= 1 - U_i^+ \quad \text{if } i \text{ is odd} \\ \text{for } i &= 1,\ldots,N \end{aligned} \tag{1}$$

where the Z_i are i.i.d. uniform random variates and $-0.5 < L < R \leq 0.5$ are the pair of parameters that parameterise the TES method. The notation $\langle x \rangle$ denotes modulo-1 arithmetic. The two variates U_i^+ and U_i^- are respectively the positive and negative autocorrelated TES variates. Some example of TES-ACF/PACF paths are illustrated in the figures 15 & 16.

The ARTA method (AutoRegressive-To-Anything) is a method based on the standard covariance stationary autoregressive time series setup

$$Y_i = const + \alpha_1 \cdot Y_{i-1} + \ldots + \alpha_n \cdot Y_{i-n} + \epsilon_i \tag{2}$$

where ϵ_i are normally distributed. Due to the Yule-Walker equations a well defined relation between the autocorrelation coefficients and the process parameters exists for this type of time series processes, which makes them very well suited for generating variates of any type. The Y variate is normally distributed and has consequently a known distribution function that takes Y into the uniform domain, and from there it is standard to transform to any other type of distribution. To make sure that the desired autocorrelations prevail in the final distribution ARTA makes the necessary corrections to the "Yule-Walker" given AR parameters. Some example of ARTA-ACF/PACF paths are illustrated in the figures 17 through 20.

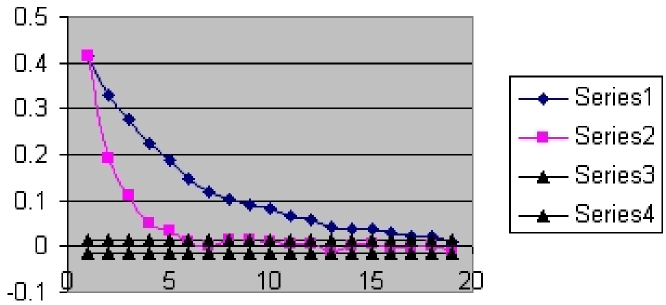

Figure 15: ACF and PACF for TES($\rho_1 = 0.4$) generated arrival stream - Exponential distributed with mean=1

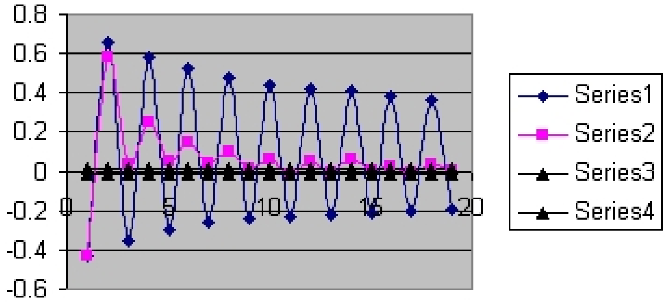

Figure 16: ACF and PACF for TES($\rho_1 = -0.4$) generated arrival stream - Exponential distributed with mean=1

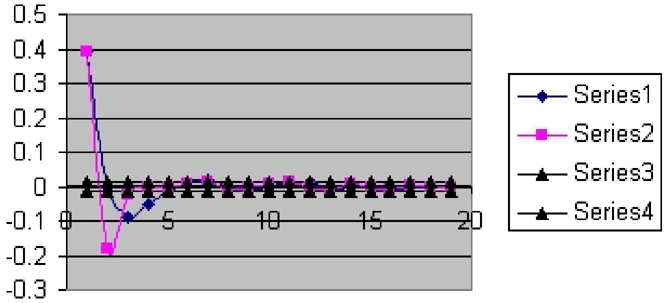

Figure 17: ACF and PACF for ARTA(a) generated arrival stream - Exponential distributed with mean=1, (see table 1 for further specifications of the process-parameters (the head-column).)

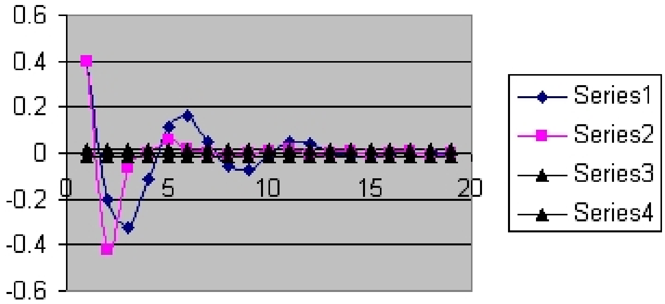

Figure 18: ACF and PACF for ARTA(b) generated arrival stream - Exponential distributed with mean=1, (see table 1 for further specifications of the process-parameters (the head-column).)

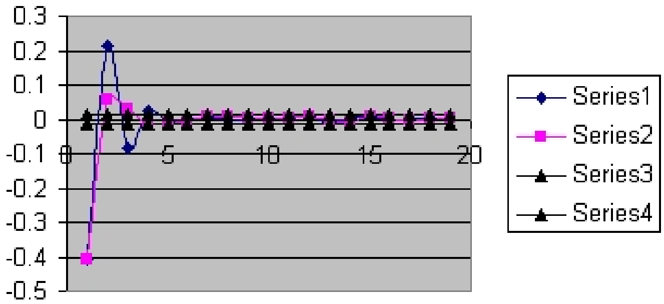

Figure 19: ACF and PACF for ARTA(c) generated arrival stream - Exponential distributed with mean=1, (see table 2 for further specifications of the process-parameters (the head-column).)

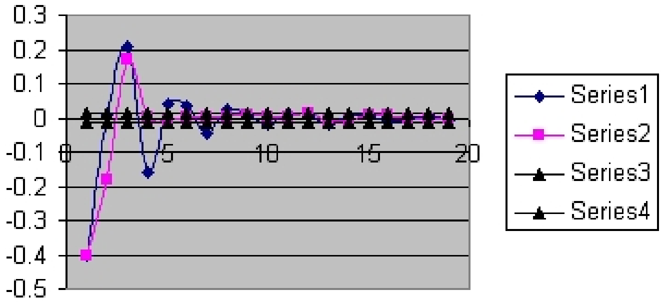

Figure 20: ACF and PACF for ARTA(d) generated arrival stream - Exponential distributed with mean=1, (see table 2 for further specifications of the process-parameters (the head-column).)

The following M/M/1/FIFO system experiments will be conducted with only autocorrelation present in the arrival stream and will be based on the same patterns as described in the figures 15 through 20. The service process will be standard i.i.d. exponential distributed.

The findings reported in table 1 & 2 (below) by and large tells us that positive first order autocorrelations tends to deteriorate performance compared to the i.i.d. base case, whereas negative first order autocorrelations in fact has the potential to improve performance compared to the i.i.d.

base case, except for such extreme ACF-patterns, which are implied by the TES-method, that on the contrary causes a very dramatic deterioration in performance. This is strongly in accordance with the results in the Livny, Melamed and Tsiolis (1993) paper.

Traffic Intencity	0.25	0.75	0.95
I.I.D.	0.33 (0.33//0.33)	2.98 (2.92//3.05)	18.80 (17.18//20.42)
TES, $\rho_1 = 0.4$	0.46 (0.45//0.46)	8.45 (8.25//8.65)	60.24 (50.29//70.19)
ARTA(a), $\rho_1 = 0.4$, $\rho_2 = -0.00001$	0.36 (0.36//0.36)	3.60 (3.54//3.66)	22.08 (19.90//24.26)
ARTA(b), $\rho_1 = 0.4$, $\rho_2 = -0.2$	0.35 (0.35//0.35)	2.99 (2.95//3.04)	17.55 (15.90//19.19)

Table 1: Estimated M/M/1/FIFO Average System Time / Flow = 100000 units / Transient Period = 10000 / Replications = 10

It is also worth noting that the observed performance improvement caused in the cases ARTA(c) and ARTA(d) very well can be and probably has to be attributed to significant higher order (> 2) autocorrelation effects, that is in other words, the full ACF/PACF profile! The applied ARTA specifications are in this study only specified up to the second order and as can be seen in the ARTA(d) case (see table 2), though $\rho_2 \approx 0$, figure 20 tells us that $\rho_3 \approx 0.2$ and $\rho_4 \approx -0.15$! It clearly would be interesting to get a more complete and systematic understanding of the full ACF/PACF patterns and

Traffic Intencity	0.25	0.75	0.95
I.I.D.	0.33 (0.33//0.33)	2.98 (2.92//3.05)	18.80 (17.18//20.42)
TES, $\rho_1 = -0.4$	0.31 (0.31//0.31)	5.72 (5.38//6.07)	141.32 (104.47//178.17)
ARTA(c), $\rho_1 = -0.4$, $\rho_2 = 0.2$	0.31 (0.31//0.31)	2.35 (2.32//2.39)	13.79 (12.59//14.98)
ARTA(d), $\rho_1 = -0.4$, $\rho_2 = 0.00001$	0.31 (0.31//0.31)	2.25 (2.22//2.29)	13.10 (12.05//14.16)

Table 2: Estimated M/M/1/FIFO Average System Time / Flow = 100000 units / Transient Period = 10000 / Replications = 10

their systematic relation and influence on system performance in general.

Descriptive sampling with Autocorrelation

Descriptive or selective sampling is mostly known as a procedure for scrambling N numbers z_i given by $\frac{1}{2}\left(\frac{2 \cdot i-1}{N}\right)$ where $i = 1, \ldots, N$, independently by picking numbers in a random fashion based on some well behaved congruent mechanism, which then results in a finite set of independent (truly)uniformly distributed variates u_i where $i = 1, \ldots, N$.

Descriptive sampling is a method that is not accepted by all simulation analysts as being a sound procedure - but I think it offers a method of last resort, in cases where the ARTA method is not able to suggest any generating method at all, given some observed set of data. The autocorrelated (extended) descriptive sampling method can be made working on virtual any set of observed data, no strings attached, which I think is quite a nice property.

A scrambling procedure, whereby some desired autocorrelation property can be incorporated into exactly N numbers u_i is, however, not a stright foreward matter. Non the less, one scheme for scrambling a finite set of figures z_i given by $\frac{1}{2}\left(\frac{2 \cdot i-1}{N}\right)$ where $i = 1, \ldots, N$ could be as follows:

- Generate or observe N successive values and perform a rescale down to the 0-1-interval, resulting in $y_1, y_2, \ldots, y_N$.
- Now consider a nonlinear mathematical programming problem in $N \times N$ binary variables $\gamma_{i,j}$ given as follows

$$\begin{aligned}
\text{MIN Z} &= \sum_{i=1}^{N} \Phi_i^2 \\
\text{Subject to} & \\
\Phi_i &= y_i - \sum_{j=1}^{N} \gamma_{i,j} \cdot z_j \quad \text{for } \; i = 1, \ldots, N \\
\sum_{j=1}^{N} \gamma_{i,j} &= 1 \qquad \text{for } \; i = 1, \ldots, N \\
\sum_{i=1}^{N} \gamma_{i,j} &= 1 \qquad \text{for } \; j = 1, \ldots, N \\
\\
\text{all } \; \gamma_{i,j} & \quad \text{are} \quad \text{binary}
\end{aligned} \tag{3}$$

In essence the solution to this optimization problem will produce the best-fit permutation of the z data in relation to the generated y sample. Solution values $\gamma^*_{i,j} = 1$ denotes the placement of z_j at sequence index i, that is $u_i = \sum_{j=1}^{N} \gamma^*_{i,j} \cdot z_j$ where $i = 1, \ldots, N$.

Unfortunately, the above formulated optimization problem is quite cumbersome to solve exactly in practice. However, an approximate solution method, based on a repeatedly pair-wise interchange of z values in order to step-wise minimize $\sum_{i=1}^{N} \Phi_i^2$, has ”proven” in my experience to ”copy” enough of the autocorrelation characteristics from the y sample, in a manageable number of iterations, to be a satisfactory procedure. But still this partial interchange method is not a computational very efficient method and the computational effort increases by N^2. The generation of 500 variates is still manageable, but beyond this number it becomes rapidly quite impractically.

The results from using this method is showing the same general tendency as the results from the TES and the ARTA method, when these specific characteristics are ”copied”, even on very small data sets, but as the computational efficiency is very low, I will not continue to present any more detailed analysis in this study based on this extended descriptive sampling method.

Concluding Remarks

In the Livny, Melamed and Tsiolis (1993) paper the conclusion is amongst others that TES should be utilized whenever the analyst is in need for a conservative benchmark on the systems performance. This makes sense because by comparing the results in this work especially figure 12 & 14 with figure 15 it can be seen that TES operated with a significant positive first order autocorrelation effect is much the same as if the arrival generating process was another queue-sub-system ruled by the not very efficient ”units with the largest processing time come first” or in short the LPT rule.

Now taking a summarising look at the autocorrelation patterns generated by a queue-sub-system ruled by the SPT-rule it seems that much resemblance can be found when comparing the figures 11 & 13 with the figures 19 & 20. The performance results for these instants are also quite reassuring, given common experience, and in addition they give a slightly deeper understanding of the reasons for the popularity of the SPT-rule in terms of autocorrelation patterns.

First order negative autocorrelation is obviously beneficial, however, from the figures 16 & 20 and their corresponding table-results it is quite clear that full autocorrelation pattern is of vital importance for the overall systems performance outcome.

It is evident that before choosing some arrival processes for some simulation study a thorough preliminary analysis in order to uncover the basic time series nature of the interacting processes must be undertaken. Having done so, flexible methods for generating streams of autocorrelated variates of any desired distributional type such as the ARTA method or some autocorrelation extended descriptive sampling method can more or less easily be applied, and as the results from the Livny, Melamed and Tsiolis (1993) study as well as the results from this work indicates, the system performance measures are heavily influenced by the i.i.d. versus the autocorrelations assumptions done. Plus/minus 35% in performance is easily observed even when comparing moderate and probably more realistic autocorrelation assumptions with the traditionally and commonly used i.i.d. assumptions.

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