UNIVERSITÄT
BERN

Faculty of Economics and
Social Sciences

Department of Economics

Does a Platform Owning Monopolist Want Competition?

Andras Niedermayer

05-17

December 2005

DISCUSSION PAPERS

Schanzeneckstrasse 1
Postfach 8573
CH-3001 Bern, Switzerland
http://www.vwi.unibe.ch

Does a Platform Owning Monopolist Want Competition?

Andras Niedermayer*

December 8, 2005

Abstract

We consider a software vendor selling both a monopoly platform (e.g. operating system) and an application that runs on this platform. He may face competition by an entrant in the applications market. Consumers are heterogeneous in their preferences for both the platform and the applications. They first buy the platform and then the applications. Their utility over the horizontally differentiated applications is known only after they bought the platform. In equilibrium the platform seller can be better off with a competitor in the applications market for three reasons. First, the platform vendor makes more profits with his platform. Second, the competitor's entry serves as a credible commitment to lower prices for applications. Third, higher ex ante expectations of product diversity lead to a higher demand for his application. Competition may be profit enhancing even if the first two effects are absent, i.e. the product diversity effect can be sufficient. The model also gives an answer to the much debated question why Microsoft prices MS Office significantly higher than its operating system.

Keywords: Two-sided markets, platforms, entry, complementary goods, price commitment, product diversity, Microsoft
JEL-Classification: D41, D43, L13, L86

---

*Economics Department, University of Bern, Schanzeneckstrasse 1, CH-3001 Bern, Switzerland. Email: niedermayer@vwi.unibe.ch. I thank Philipp Ackermann, Alain Egli, Winand Emons, Roland Hodler, Simon Loertscher, Lucy White and participants of seminars at the University of Bern for very helpful comments. Any errors are mine.

1 Introduction

Platforms play an important role in many markets. A platform gives two sides (e.g. sellers and buyers) the possibility to interact (e.g. trade) with each other. The platform owner can get part of the generated surplus.

In software markets platforms seem to play a crucial role: it would be too costly to develop a new application for every possible combination of hardware, versions of operating systems, file formats, etc.[1] A software platform provides a common interface between different applications and different configurations of users' systems. Hence, it enables application developers on one side and end users on the other side to interact with each other. We will use the term "software platform" with a very broad meaning: it can mean an operating system (such as Windows or Linux[2]), a file format (e.g. Adobe's PDF, Microsoft Word documents, OpenOffice documents), virtual machines (e.g. Sun's Java Platform, Microsoft's .NET Platform), database access interfaces (e.g. the Structured Query Language) or game consoles (e.g. Sony's Playstation 2 and Microsoft's XBox). We will consider applications running on this platform, i.e. pieces of software that are only usable in conjunction with the platform. Examples are the spreadsheet calculation programs MS Excel and Lotus 1-2-3 for MS Windows and the file creation software Adobe Acrobat Standard[3] and PDF Writer for the PDF file format. Two interesting observations arise when considering these examples. First, the platform owner often also owns one or more (but not all) of the applications running on his platform. Second, the platform owner makes most of his profits with the applications – the platform may even be a "loss

[1] For a survey of the economic role of software platforms in computer-based industries see Evans, Hagiu & Schmalensee (2004) [5].

[2] Evans, Hagiu & Schmalensee (2004) [5] note that an operating system actually connects three sides: application developers, end users and *hardware suppliers*. We are going to abstract away from the hardware in our model.

[3] The free Acrobat Reader can only display PDF files, the Standard and Professional versions can also create files.

leader" (as in the case of the XBox). The two-sided markets literature has provided many interesting insights about the second observation, however, the first observation (especially that the platform owner owns *part* of the applications market) hasn't been treated extensively. This paper looks in more detail at the specific effects arising in markets with the aforementioned ownership structure.

Related Literature. Our model is related to the recent strain of literature on two-sided markets. Caillaud & Jullien (2003) [2], Rochet & Tirole (2003) [10] and Armstrong (2002) [1] consider platform owners as intermediaries who help matching a continuum of sellers and buyers. The focus in this literature is usually on the platform. Nocke, Peitz & Stahl (2004) [8] look at the impact of ownership structures on platform size and product variety. They consider the cases where either all sellers (application vendors in our terminology) or none of them own the platform. Hagiu (2004) [7] considers the effects of commitment to a platform price.

The question considered in this paper has similarities to the questions investigated in the network externalities literature. Parker & Van Alstyne (2000) [9] consider a platform owner who induces more competition in the applications market to get higher profits in the platform market. Economides (1996) [4] looks at a monopolist who is willing to induce competition as a means of committing to higher quantities. Our model also shows the effects described by Parker & Van Alstyne and Economides (with the difference that it has price and not quantity commitment), but introduces a third effect: the application diversity effect.

This paper discusses a different question, but bears resemblance to the second-sourcing literature which considers a monopolist (e.g. a patent holder) who is willing to allow competition in order to commit to lower future prices. Farrell & Gallini (1988) [6] look at a two-period game in which a monopolist is willing to accept Bertrand competition and zero profits in the second period in order to convince consumers to incur high setup costs and buy his product in the first period.

In our model we look at the specific setup of a platform owner who also owns an application running on his platform. An independent firm considers developing a further, horizontally differentiated application for the platform. Consumers are heterogeneous in both their preferences for the platform and the applications. They buy the platform at the first stage of the game, at the second stage they get to know their preferences about the applications.

As long as consumers do not know their preferences over applications, they form expectations over their utility derived from purchasing the applications. The higher the expectations, the more willing consumers are to buy the platform. An entrant in the applications market increases consumers' expectations and thus demand for the platform. At the same time he takes away market shares from the former monopolist in the applications market. This paper will argue that the positive effect of competition for the monopolist may offset the negative effect. We will also illustrate the different channels through which the positive effect of competition works.

This article is structured in the following way. Section 2 describes the setup of the basic model. Sections 3 and 4 treat the two cases where the potential entrant either stays out of the market or enters. Section 5 compares the monopolist's profits for the two cases. We will show that the platform monopolist may be better off with a competitor in the applications market. This has three reasons. First, the platform vendor makes more profits with his platform. Second, the competitor's entry serves as a credible commitment to lower prices for applications. Third, higher ex ante expectations of product diversity lead to a higher demand for his application. In order to show that the third effect can be sufficient, the first two effects are eliminated in Sections 6 and 7. Section 6 removes the first effect by assuming that the platform vendor cannot make profits with the platform (because he has to give it away for free). Section 7 eliminates the second effect as well with the assumption that the platform vendor gives away the platform for free *and* can credibly commit to low application prices without a competitor. In Section 7 we make some simplifying assumptions about the distribution of consumer preferences in order to keep the model tractable, however, it can be shown for numeric examples that the basic results carry over to a setup without the simplifications.

Section 8 discusses the monopolist's optimal pricing of the platform and the application for the example of Microsoft Windows and Office.

2 Model

Consider a software market with two firms, A and B. Firm A produces two goods: a platform and an application. B considers developing an application for A's platform (see Fig. 1). B is the only firm capable of producing its application, either because of its unique expertise in programming this piece of software or because of legal issues (e.g. copyright laws, patents or noncompetition clauses for its lead developers). B's application is usable with A's platform only. One can think of the platform as being an operating system (e.g. MS Windows) and the applications being software written for this operating system (e.g. MS Excel, Lotus 1-2-3). Another possibility is the platform being a file standard (e.g. PDF – Portable Document Format) and the applications being software for creating files complying with this standard (e.g. Adobe Acrobat Standard, PDF Writer).

Now let us consider the potential buyers of the platform and the applications. We assume a continuum of consumers with heterogeneous preferences over the platform $y \in [0, \infty)$ and over the applications $x \in [0, 1]$ (see Fig. 2). One can imagine y as the distance of a consumer from the platform: the

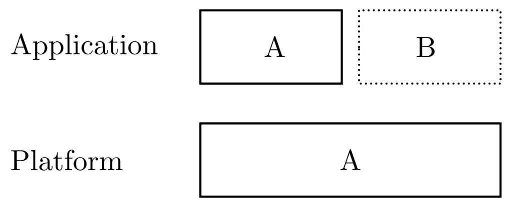


Figure 1: Products offered by A and B

further one is (i.e. the greater y), the less willing one is to buy the platform.[4] x is the location of the consumer in a Hotelling competition between applications A and B with fixed firm location where A is located at 0 and B at 1. This means that consumers with a small x are more willing to buy A and less willing to buy B than consumers with a large x. Consumers' utility is set to 0 for the case they do not buy the platform (and hence cannot buy any of the applications either), $v_0 = s - p - y$ if they buy the platform without any applications[5], $v_A = v_0 + s_A - p_A - tx$ if they buy the platform with application A and $v_B = v_0 + s_B - p_B - t(1 - x)$ if they buy it with application B. s is the intrinsic value of the platform, p is the price of the platform, s_A and s_B are the gross utilities (without "transportation costs") consumers derive from applications A and B respectively, p_A and p_B are the prices of the applications and t represents the "transportation costs" in the choice of the application.[6] We will assume that consumers learn their preferences over applications x only after having bought the platform.[7] Like

[4] Or one can consider y to be the outside option of a consumer as in Nocke, Peitz & Stahl (2004) [8].

[5] The possibility of buying the platform with neither application A nor application B can be justified by the idea that the platform is bundled with an application or that there is a further application C which is not competing with applications A and B.

[6] I.e. the higher the "transportation costs" the less willing consumers are to buy an application which is further away from their preferred type of application.

[7] This can be a learning by doing effect: only trying different applications can show which is suitable for one's own needs. Alternatively one can consider applications A and B as future releases of software, one doesn't know one's preferences about software which hasn't been released yet.

in the standard Hotelling setup, we assume that consumers cannot or do not want to buy both applications.

We further assume a constant density of consumers $\rho(x, y) = \alpha$ for $0 \leq x \leq 1$ and $y \geq 0$ and $\rho(x, y) = 0$ otherwise. (For the application pricing part we only need the assumption of uniformity over x, i.e. $\rho(x, y) = \rho(y)$ for $0 \leq x \leq 1$.[8])

To simplify the description of the model we will call all consumers with the same y a *consumer unit*. (An alternative interpretation of the model is that *one* consumer has a specific y and stochastic preferences over the applications determined by x. Then x is a random variable uniformly distributed between 0 and 1 and is only known to consumers in stage 2. According to this interpretation a consumer unit is equivalent to a consumer.)

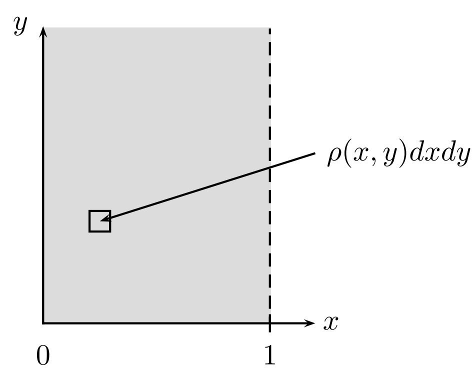


Figure 2: Distribution of consumers' preference parameters x (applications) and y (platform)

We will consider the following timing:

- Stage 0: A already has a platform and an application, B decides whether to enter,
- Stage 1: A sets price for platform, consumers buy platform,

[8]One could easily extend the platform pricing part with a stepwise uniform density, e.g. $\rho(y) = \alpha_1$ for $0 \leq y < y_1$ and $\rho(y) = \alpha_2$ for $y \geq y_1$.

- Stage 2: A and B set prices for applications, consumers learn their x and buy applications.

We will first consider the case where B decides not to enter (A thus having a monopoly both in the platform and the applications market) and set up and solve the model backwards.

In the second case we consider the situation where B enters and solve the model backwards again. If B's revenues from entering are higher than the fixed costs it incurs from developing the application, B will be willing to enter.

3 No Market Entry by Competitor B

We will first consider profits from application sales and consumer surplus per consumer unit at stage 2.[9] Afterwards, at stage 1, we will look at the platform choice of consumers and thus determine the number of consumer units. Assuming subgame perfection, at stage 2 players take the outcomes of stage 1 as given and do not have to fulfill any promises or threats.

3.1 Stage 2

Consider stage 2 of the case where B doesn't enter. In this case A is a monopolist in the applications market as well. Let us only consider consumers who have bought the platform. They have to decide whether they want to buy application A additionally to the platform or want to use the platform alone. Consumers not buying the application derive utility v_0 from the usage of the platform alone. Consumers buying application A have a utility of v_A. To simplify analysis we will only consider excess utility compared to v_0: excess utility for using the platform alone is 0, for using application A $v_A - v_0 = s_A - p_A - tx$.

[9] According to the alternative interpretation where one consumer has a specific y and a stochastic x, we calculate ex ante expected consumer surplus *per consumer*. Because the x of a consumer isn't known to the firms even at stage 2, they maximize expected profits per consumer.

Now let us consider a consumer unit whose members have bought the platform.[10] According to the assumption made previously consumers are uniformly distributed along the x-axis (i.e. the density of consumers at point (x, y) is $\rho(x, y) = \rho(y)$ for $0 \leq x \leq 1$ and $y \geq 0$), therefore we get a one-sided version of the standard fixed location Hotelling setup where a monopolist sells goods to consumers with heterogeneous preferences.

We will assume that A has an incentive to sell to all consumers (Fig. 3). For this, we need to assume that transportation costs are low enough (or that the gross utility derived from application A is high enough):

$$s_A \geq 2t. \tag{1}$$

Proposition 1. *If the gross utility derived from application A is high enough ($s_A \geq 2t$), the monopolist will sell to all consumers and will set the outermost consumer indifferent between buying and not buying.*

For a formalization and a proof of this proposition and for a treatment of the alternative case where the monopolist doesn't sell to consumers far away from him see Appendix A. The effect we intend to show is even stronger in the alternative case.

With full market coverage, the monopolist will set the outermost consumer indifferent between buying his application or using the platform without the application, i.e. for $\hat{x} = 1$

$$s_A - p_A - t\hat{x} = 0$$

where $\hat{x}$ is the location of the indifferent consumer (see Fig. 3)

Thus, under the assumption of full market coverage, we get the optimal price:

$$p_A^* = s_A - t. \tag{2}$$

[10] Note that x isn't known in the first period, therefore only perceived heterogeneity in y exists for consumers when deciding whether to buy the platform. In pure strategies either all consumers with a specific y buy the platform or none.

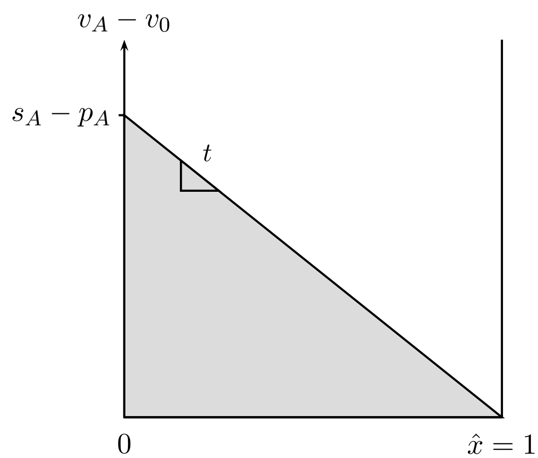


Figure 3: Monopolist A selling the application to all consumers who have bought the platform. The shaded area under the curve denotes the consumer surplus.

For equilibrium profits per consumer unit from sales of the application we get

$$\pi_A^* = p_A^* \hat{x} = s_A - t.$$

under the assumption of zero marginal costs.

For the sake of clarity, profits per consumer unit at stage 2 will be denoted with a lower case π, total profits at stage 1 will be denoted with a upper case Π.

The consumer surplus per consumer unit is the integral of consumers' utilities over x, as denoted in the shaded area in Fig. 3:

$$\mathrm{EU} = \int_0^{\hat{x}} (s_A - p_A^* - tx)dx.$$

Substituting p_A^* and $\hat{x}$ we get

$$\mathrm{EU} = \frac{t}{2}.$$

We denote consumer surplus with EU because it is the utility that consumers expect to derive from the purchase of the application when they form expectations at stage 1.

Having calculated the outcome of stage 2, we can proceed to stage 1, where consumers buy the platform.

3.2 Stage 1

At stage 1 consumers decide whether to buy the platform. As they do not know their preferences for the application (i.e. their x) they form expectations over x. Their expected utility for buying the platform is

$$s - p - y + \mathrm{EU}. \tag{3}$$

There is an indifferent consumer unit $\hat{y}$ for whom

$$s - p - \hat{y} + \mathrm{EU} = 0. \tag{4}$$

(see Fig. 4)

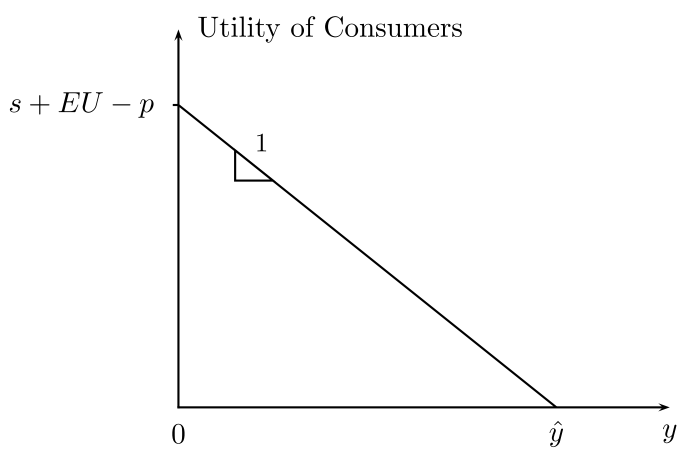


Figure 4: Platform Choice

One can get the number of consumer units (i.e. all consumers with the same y) who are willing to buy the platform by integrating the density function from 0 to $\hat{y}$:

$$N = \int_0^{\hat{y}} \int_0^1 \rho(x,y)dxdy = \alpha\hat{y} = \alpha(s + \mathrm{EU} - p). \tag{5}$$

Firm A makes profits from selling its platform (pN) and its application at stage 2 ($\pi_A^* N$). The overall profit of firm A is thus

$$\Pi = pN + \pi_A^* N \tag{6}$$

and the profit maximization problem

$$\Pi^* = \max_p N(p + \pi_A^*). \tag{7}$$

The profit maximizing price p^* for the platform is

$$p^* = \frac{1}{2}\left(s + \mathrm{EU} - \pi_A^*\right)$$

or, after substituting,

$$p^* = \frac{1}{2}\left(s + \frac{3}{2}t - s_A\right). \tag{8}$$

p^* is nonnegative if

$$s + \mathrm{EU} \geq \pi_A^*. \tag{9}$$

$$\Leftrightarrow$$

$$s \geq s_A - \frac{3}{2}t.$$

We assume that either s is sufficiently large so that condition (9) is satisfied or that firm A has the possibility to set a negative p^* (i.e. subsidize its platform).[11]

For the number of consumer units buying the platform we get

$$N^* = \frac{\alpha}{2}\left(s + \mathrm{EU} + \pi_A^*\right)$$

[11] E.g. by offering free support for the platform or by offering an application C additionally to the platform for free, where C is not substitutable with applications A and B.

or

$$N^* = \frac{\alpha}{2}(s + s_A - t).$$

Because both EU and π_A^* are positive N^* is strictly positive for all nonnegative values of s, therefore we don't have to make further assumptions to ensure that $N^* \geq 0$.

Equilibrium total profits of firm A are

$$\Pi^* = \frac{\alpha}{4}\left(s + \mathrm{EU} + \pi_A^*\right)^2 \tag{10}$$

or

$$\Pi^* = \frac{\alpha}{4}(s + s_A - t)^2. \tag{11}$$

3.3 Stage 0

We set B's profits to 0 for the case that it doesn't enter the market.

4 Market Entry by Competitor B

Now we can look at the case when B enters the market.

4.1 Stage 2

Consider stage 2 of the case where B enters. Again, let us only consider consumers who have bought the platform. They have to decide whether they want to buy application A or B additionally to the platform or do not want to buy any of the applications. Consumers not buying any of the applications derive utility v_0 from the usage of the platform alone. Consumers buying application A have a utility of v_A, those buying B a utility of v_B. Excess utility for using the platform alone is 0, for using application A $v_A - v_0 = s_A - p_A - tx$, for B $v_B - v_0 = s_B - p_B - t(1-x)$.

Now let us consider a consumer unit whose members have bought the platform. Because of the uniform distribution of consumers' preferences along the x-axis we get a standard fixed location Hotelling setup with firm A located at $x = 0$ and firm B at $x = 1$. The only difference to the standard model is that s_A isn't necessarily equal to s_B.

Here we will assume an equilibrium as depicted in Fig. 5. To exclude special cases we make some restrictions on the ranges of s_A, s_B and t:

$$s_A + s_B > 3t \tag{12}$$

$$-3t < s_A - s_B < 3t. \tag{13}$$

We assume that the whole market is covered (there are no consumers who do not buy any of the applications) and that the consumer who is indifferent between A and B has a strictly positive excess utility (Eq. (12)). We further assume that both firms can sell strictly positive quantities of their application (i.e. neither firm's application is so much better than the other's that it could capture the whole market, Eq. (13)). See Appendix B for a derivation of these restrictions and for a treatment of the cases where these assumptions aren't satisfied. As noted in subsection 3.1 comparing these alternative cases with the cases mentioned in subsection 3.1 (full and partial market coverage) gives us even stronger results.

Under the aforementioned conditions all consumers buy an application (see Fig. 5). The indifferent consumer $\tilde{x}$ derives the same excess utility from applications A and B:

$$s_A - p_A - t\tilde{x} = s_B - p_B - t(1 - \tilde{x}) \tag{14}$$

Consumers to the left of $\tilde{x}$ buy A, those to the right of $\tilde{x}$ buy B.

Demand per consumer unit for application A is

$$\tilde{x} = \frac{1}{2} + \frac{1}{2t}(s_A - s_B + p_B - p_A)$$

and for B

$$1 - \tilde{x} = \frac{1}{2} - \frac{1}{2t}(s_A - s_B + p_B - p_A).$$

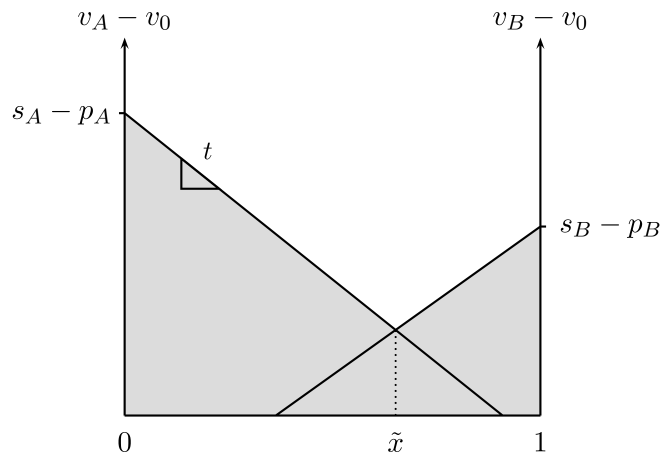


Figure 5: Application Pricing. The shaded area denotes consumer surplus.

Profits per consumer unit from the sales of the applications are

$$\pi_A = p_A \tilde{x} \tag{15}$$

$$\pi_B = p_B(1 - \tilde{x}). \tag{16}$$

We assume profit maximization in equilibrium:

$$p_A^* = \arg\max_{p_A} \pi_A(p_A, p_B^*) \tag{17}$$

$$p_B^* = \arg\max_{p_B} \pi_B(p_A^*, p_B). \tag{18}$$

From (14), (15), (16), (17) and (18) we get the Nash equilibrium

$$p_A^* = t + \frac{\Delta}{3} \tag{19}$$

$$p_B^* = t - \frac{\Delta}{3} \tag{20}$$

with $\Delta = s_A - s_B$. The indifferent consumer is at location

$$\tilde{x}^* = \frac{1}{2} + \frac{\Delta}{6t}$$

and equilibrium profits are[12]

$$\pi_A^* = \left(t + \frac{\Delta}{3}\right)\left(\frac{1}{2} + \frac{\Delta}{6t}\right) \tag{21}$$

$$\pi_B^* = \left(t - \frac{\Delta}{3}\right)\left(\frac{1}{2} - \frac{\Delta}{6t}\right). \tag{22}$$

The consumer surplus per consumer unit is the integral of consumers' utilities over x, as denoted in the shaded area in Fig. 5:

$$\text{EU} = \int_0^{\tilde{x}^*} (s_A - p_A^* - tx)dx + \int_{\tilde{x}^*}^1 (s_B - p_B^* - t(1-x))dx, \tag{23}$$

substituting p_A^*, p_B^* and $\tilde{x}^*$ we get

$$\text{EU} = \frac{\Delta^2}{36t} + \frac{s_A}{2} + \frac{s_B}{2} - \frac{5}{4}t. \tag{24}$$

Again, we can use stage 2 results for stage 1.

4.2 Stage 1

As in the case where B doesn't enter, consumers' valuation for the platform depends on the intrinsic value of the platform plus the expected value of the applications at stage 2. The only difference is that here consumers anticipate that they might buy application B instead of A at stage 2 and adjust their expectations accordingly. Their expected utility for buying the platform is

$$s - p - y + \text{EU}. \tag{25}$$

Consumers with $y \in [0, \tilde{y}]$ buy the platform where the location of the indifferent consumer is given by

$$\tilde{y} = s - p - \text{EU}.$$

The number of consumer units is

$$N = \int_0^{\tilde{y}} \int_0^1 \rho(x, y)dxdy = \alpha\tilde{y} = \alpha(s + \text{EU} - p). \tag{26}$$

[12] These results are consistent with the standard Hotelling model where $s_A = s_B$. In the standard Hotelling model equilibrium prices are $p_A^* = p_B^* = t$ and equilibrium profits are $\pi_A^* = \pi_B^* = t/2$. Substituting $\Delta = 0$ into (19), (20), (21) and (22) gives us the same results.

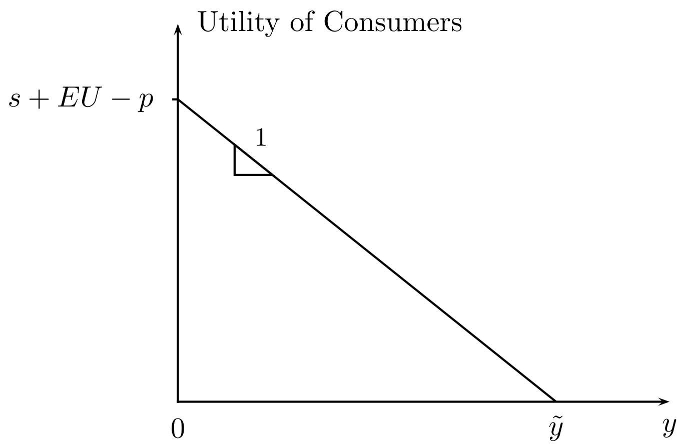


Figure 6: Platform Choice

Firm A's overall profits are still

$$\Pi = pN + \pi_A^* N$$

but with a different π_A^* this time.

By analogy to subsection 3.2 we get

$$\begin{aligned} p^* &= \frac{1}{2}\left(s + \mathrm{EU} - \pi_A^*\right) \\ \Pi^* &= \frac{\alpha}{4}\left(s + \mathrm{EU} + \pi_A^*\right)^2 \end{aligned} \tag{27}$$

for platform price and total profits.

Substituting the values of EU and π_A^* for the case where B enters the market, we get

$$p^* = \frac{1}{2}\left(s - \frac{\Delta^2}{36t} + \frac{1}{6}s_A + \frac{5}{6}s_B - \frac{7}{4}t\right) \tag{28}$$

and

$$\Pi^* = \frac{\alpha}{4}\left(s + \frac{\Delta^2}{12t} + \frac{5}{6}s_A + \frac{1}{6}s_B - \frac{3}{4}t\right)^2. \tag{29}$$

As in Section 3.2 we assume that A can either subsidize the platform or that the condition

$$s \geq \frac{\Delta^2}{36t} + \frac{1}{6}s_A + \frac{5}{6}s_B - \frac{7}{4}t \tag{30}$$

is satisfied and thus we do not have to care about the constraint $p^* \geq 0$.

Again, as in Section 3.2 N^* is positive for nonnegative values of s.

4.3 Stage 0

Before entering the market, B anticipates revenues per consumer unit π_B^* for stage 2 and the number of consumer units N^* buying the platform for stage 1. If B's total revenues $\pi_B^* N^*$ exceed its development costs f_B, B will enter the market.

Market entry condition for B:

$$\pi_B^* N^* - f_B \geq 0. \tag{31}$$

5 Comparison of Profits

Having calculated A's profits for both cases (B enters/B doesn't enter) we can look at the central question of this article: Does a Monopolist Want Competition?

We will denote A's profits in the competition case

$$\Pi^{*C} = \frac{\alpha}{4}\left[s + \frac{\Delta^2}{12t} + \frac{5}{6}s_A + \frac{1}{6}s_B - \frac{3}{4}t\right]^2 \tag{32}$$

from Eq. (29). A's profits in the case of being a monopolist are

$$\Pi^{*M} = \frac{\alpha}{4}[s + s_A - t]^2 \tag{33}$$

as calculated in Eq. (11).

The expressions in the brackets in (32) and (33) are nonnegative, therefore one can skip the $\alpha/4$ and the square and compare the expressions in the brackets directly[13]:

$$\Pi^{*C} \overset{?}{>} \Pi^{*M}$$

$$\Leftrightarrow$$

$$s + \frac{\Delta^2}{12t} + \frac{5}{6}s_A + \frac{1}{6}s_B - \frac{3}{4}t \overset{?}{>} s + s_A - \frac{t}{2}.$$

By regrouping and multiplying by $12t$ we get

$$\Delta^2 - 2t\Delta - 3t^2 \overset{?}{>} 0. \tag{34}$$

As the coefficient of Δ^2 is positive, the expression will be positive for very large and very negative values of Δ. To find out whether it can be negative in between, we have to find the roots of the polynomial in Δ. The roots are

$$\Delta_{1,2} = t \pm 2t.$$

The left hand side in Eq. (34) will be negative if Δ is between the roots $-t$ and $3t$ and positive otherwise.

Remember that we have assumed that in the competition case neither firm can dominate the applications market ($-3t < \Delta < 3t$). Restricting Δ to this relevant range we get

$$\Pi^{*C} < \Pi^{*M} \text{ for } -t < \Delta < 3t \text{ and}$$
$$\Pi^{*C} > \Pi^{*M} \text{ for } -3t < \Delta < -t.$$

Thus if B's product is better than A's ($s_A - s_B < -t$), but not good enough to take over the whole market ($s_A - s_B > -3t$) A is better off if B enters the market. Area I in Fig. 7 shows the combinations of s_A, s_B and t for which competition is desirable for the monopolist.

[13] This can be seen by looking at the intermediary steps for the calculation of total first stage profits (10) and (27): We assume that the platform has a nonnegative intrinsic value to consumers ($s \geq 0$). The consumer surplus per consumer unit EU and per consumer unit profits from selling application A π_A^{*2} are also both nonnegative. Thus their sum (the expression in the brackets) has to be nonnegative as well.

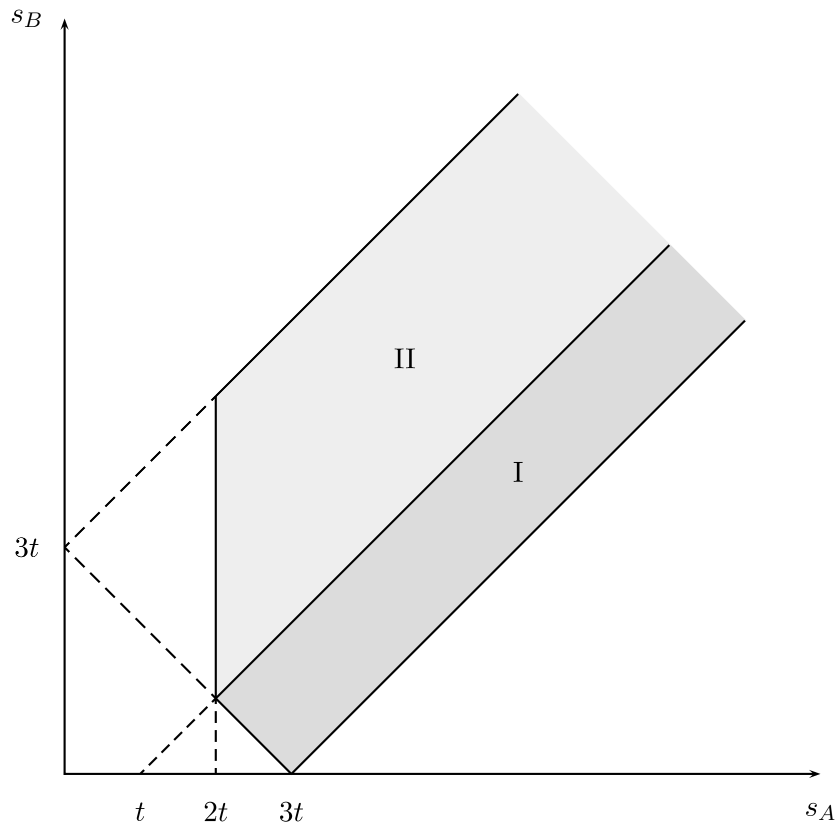


Figure 7: Areas I and II are permissible under the assumptions made ($s_A \geq 2t$, $s_A + s_B > 3t$ and $-3t < s_A - s_B < 3t$). In area I the platform monopolist has higher profits in the competition case. (s_A: quality of application A, s_B: quality of application B, t: "transportation costs")

6 Modification: Zero Price Platform

We have seen that under certain conditions firm A is better off if firm B enters the market. But one could argue that this doesn't mean that he is really happy about competition, he's just happy about competition in a market complementary to his platform. He still has a monopoly on the platform and can always make money there. In an extreme case when he cannot sell his application at all, we have the case of two complementary goods (the platform of A/the application of B). It has already been shown that a firm is willing to induce more competition in a complementary market.

So what's the difference in this paper? We can show that firm A can be better off after a market entry of B *even if it gives away its platform for free* and thus has to make its profits with its application only.

One can consider the zero price of the platform to be exogenously given (e.g. the platform is an open-source operating system or an open standard). Alternatively one can think of the case where the platform price p^* cannot be negative and the nonnegativity conditions (9) and (30) are not satisfied. In this case the corner solution $p^* = 0$ comes up.

In this alternative setup the results from stage 2 shown in the previous sections still hold.

However, stage 1 changes.

The price of the platform is $\bar{p} = 0$. There is no optimization problem for firms to be solved here.[14] Consumers form expectations about consumer surplus at stage 2 and decide whether to use the platform.

Note that even with zero prices not all consumers are willing to use the platform.[15]

We get for the marginal consumer $\tilde{y} = s + \mathrm{EU}$ and for the number of consumer units

$$N = \alpha(s + \mathrm{EU}).$$

Profits for firm A are thus

$$\Pi^* = \alpha\pi_A^*(s + \mathrm{EU}).$$

Now we can substitute the results from stage 2 for the different cases and compare total profits of firm 1.

[14]Or $\bar{p} = p^* = 0$ is the corner solution of the optimization problem.

[15]This may sound counterintuitive at first sight. However we often observe it in reality: e.g. not everyone uses the open-source operating system Linux, not everyone uses the free browser Mozilla Firefox. Many possible explanations have been named for this phenomenon: there are costs arising from the effort of installation, retraining for the usage of the new software, migration of legacy systems, paying external staff for the maintenance of the system, etc.

For the case where firm B enters and there is an inner equilibrium at stage 2. Substituting π_A^* and EU gives

$$\Pi^{*C} = \alpha \left(\frac{\Delta^2}{18t} + \frac{\Delta}{3} + \frac{t}{2} \right) \left(s + \frac{\Delta^2}{36t} + \frac{s_A}{2} + \frac{s_B}{2} - \frac{5}{4}t \right). \tag{35}$$

For the case that B doesn't enter and A covers the whole market at stage 2 we get

$$\Pi^{*M} = \alpha(s_A - t) \left(s + \frac{t}{2} \right). \tag{36}$$

$\Pi^{*C} - \Pi^{*M}$ is a polynomial of fourth degree in s_A and s_B. An analytical answer to the question when $\Pi^{*C} - \Pi^{*M}$ is positive would be intractable, however substituting different parameter values into the result shows that at least for some parameters it can be positive. E.g. for $s_B = 5$, $t = 1$ and $s = 0$ we have

- $\Pi^{*C} > \Pi^{*M}$ if $5.12 < s_A < 8$
- $\Pi^{*C} < \Pi^{*M}$ if $2 < s_A < 5.12$.

Note that because of the restriction $-3t < s_A - s_B < 3t$ made in Eq. (1) the parameter s_A can only have values in the range $(2, 8)$. Note further that α doesn't change the roots of the polynomial, it merely scales the profits.

Fig. 8 shows $\Pi^{*C} - \Pi^{*M}$ for different values of s_A.

This means that under some conditions A is better off if B enters even if A makes his profits with his applications only.

7 Modification: Zero Price Platform and Possibility of Price Commitment

We have shown that competition may be attractive for the monopolist even if he has to make profits in the applications market alone. Now there are only two effects of competition left: price commitment and product diversity. In order to separate the diversity effect we will exclude the price commitment effect of competition by assuming that the monopolist has a means to commit to a price for his application.

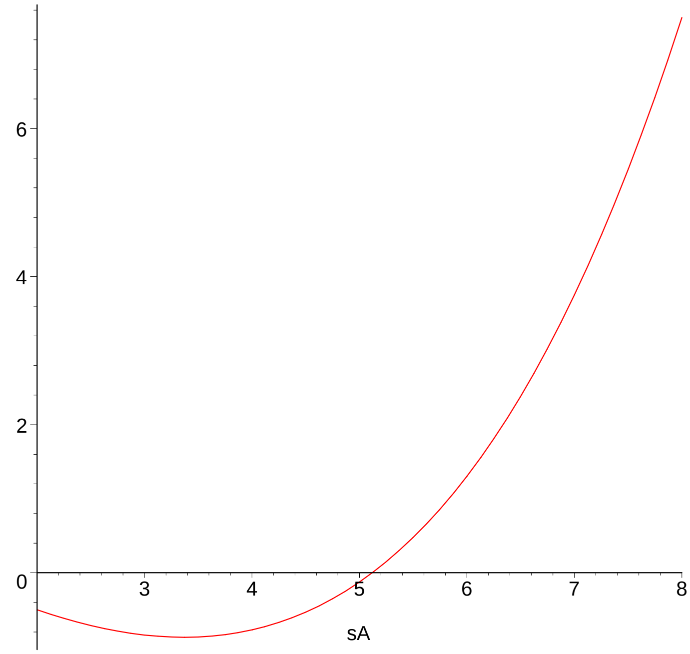


Figure 8: $\Pi^{*C} - \Pi^{*M}$ for different values of s_A with $s_B = 5$, $t = 1$, $s = 0$ and $\alpha = 1$. The allowed range for s_A is $(2, 8)$.

To keep the model tractable, we assume a different distribution of consumer preferences: consumers are homogeneous with respect to their preferences for the platform and all have the parameter value y_1 as depicted in Fig. 9. (We can show with numerical examples that our results also hold in a setting with the constant density of consumers we assumed in the previous sections.)

We can describe the density of consumers with the Dirac delta function $\delta(\cdot)$ used in physics:

$$\rho(x,y) = \begin{cases} \delta(y - y_1) & \text{for } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

The number of consumers between 0 and $\tilde{y}$ is thus

$$N = \int_0^{\tilde{y}} \int_0^1 \rho(x,y) dx dy = \begin{cases} 1 & \text{if } \tilde{y} \geq y_1, \\ 0 & \text{otherwise,} \end{cases}$$

i.e. either all consumers buy the platform or none. We will first look at the

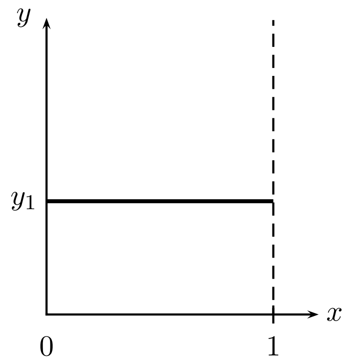


Figure 9: Consumers with homogeneous preferences $y = y_1$ over the platform

monopoly case in this setup and then at the competition case. We will show that it is possible that a monopolist cannot sell his platform even if he can commit to the application price at stage 1. Then we shown that in such a situation competition can be a remedy.

7.1 Monopoly

We assume again full market coverage, i.e. the monopolist sets the application price such that consumers with all values of x are willing to buy the application. However, contrary to the previous sections, the outermost consumer ($x = 1$) isn't necessarily set indifferent between buying and not buying (see Fig. 10), because the monopolist may be willing to commit to a lower p_A at stage 1 to convince consumers to buy the platform.

For stage 2 profits and expected consumer surplus we get

$$\pi_A = p_A$$
$$\mathrm{EU} = s_A - p_A - \frac{t}{2}.$$

The condition for full market coverage at stage 1 is

$$s_A - p_A \geq t. \tag{37}$$

At stage 1, consumers are willing to buy the platform if their y is not

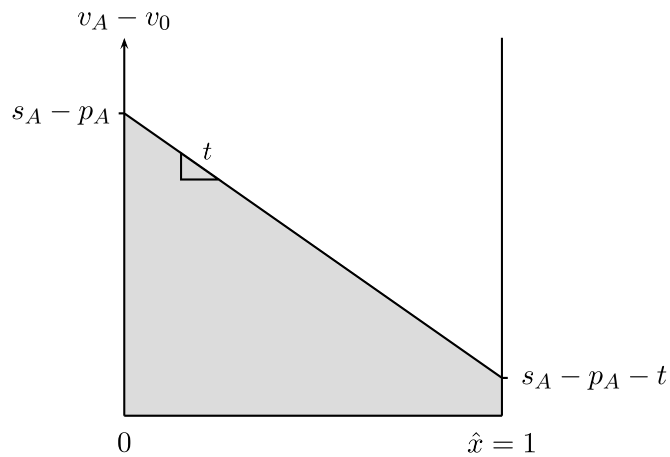


Figure 10: Full coverage with price commitment at stage 1. The shaded area below the curve denotes consumer surplus.

above

$$\tilde{y} = s + \mathrm{EU} = s + s_A - p_A - \frac{t}{2}.$$

Because all consumers have $y = y_1$, the monopolist has to commit to a price p_A at stage 1 such that

$$\tilde{y} \geq y_1 \tag{38}$$

to ensure that consumers are willing to buy his platform.

The profit maximization problem of the monopolist consists of setting p_A as high as possible such that conditions (37) and (38) are still satisfied. We take the case where condition (38) is stronger that condition (37) and the monopolist sets p_A such that (38) is just binding:

$$y_1 = s + s_A - p_A - \frac{t}{2}.$$

For the equilibrium application price we get

$$p_A^* = s + s_A - \frac{t}{2} - y_1$$

and for overall profits

$$\Pi^* = \pi_A^* N^* = p_A^* \times 1 \times 1 = s + s_A - \frac{t}{2} - y_1.$$

Now let us consider the case where

$$y_1 > s + s_A - \frac{t}{2}. \tag{39}$$

In this case the firm would have to set a negative price p_A for the application to convince consumers to buy its platform. Hence, in this case it is not possible for the monopolist to get positive profits.

7.2 Competition

If B enters the market, both firms commit to application prices at stage 1. They face the same problem as at stage 2 in the previous sections with the additional constraint that consumers should be willing to buy the platform:

$$\tilde{y} \geq y_1 \tag{40}$$

where $\tilde{y} = s + \text{EU}$ is the maximal distance at which consumers are still willing to buy the platform.

We consider the case where (40) is non-binding. In this case we can use the results obtained in the previous sections, the only difference is that prices are set at stage 1 and not at stage 2. In equilibrium stage 2 profits and expected consumer surplus are

$$\begin{aligned} \pi_A^* &= \left(t + \frac{\Delta}{3}\right)\left(\frac{1}{2} + \frac{\Delta}{6t}\right) \\ \pi_B^* &= \left(t - \frac{\Delta}{3}\right)\left(\frac{1}{2} - \frac{\Delta}{6t}\right) \\ \text{EU} &= \frac{\Delta^2}{36t} + \frac{s_A}{2} + \frac{s_B}{2} - \frac{5}{4}t \end{aligned} \tag{41}$$

as in subsection 4.1.

Firm A's profits are

$$\Pi^* = \pi_A^* N^* = \pi_A^* \times 1.$$

7.3 Comparison of Profits

In the case where the monopolist cannot achieve positive profits, but with competition profits are strictly positive, firm A is (trivially) better off with competition.

This case occurs for parameter values which satisfy both conditions (39) and (40). Proposition 2 states when both conditions can be satisfied simultaneously.

Proposition 2. *For $\Delta \in (-3t, (9-6\sqrt{3})t)$ conditions (39) and (40) can both be satisfied at once if neither firm dominates the market.*

Proof. Substituting (41) into (40) gives

$$y_1 \leq \frac{\Delta^2}{36t} + \frac{s_A}{2} + \frac{s_B}{2} - \frac{5}{4}t.$$

Combining this with (39) yields

$$s + s_A - \frac{t}{2} < y_1 \leq \frac{\Delta^2}{36t} + \frac{s_A}{2} + \frac{s_B}{2} - \frac{5}{4}t.$$

The range of y_1 which allows for both conditions to be satisfied is non-empty if

$$s + s_A - \frac{t}{2} < \frac{\Delta^2}{36t} + \frac{s_A}{2} + \frac{s_B}{2} - \frac{5}{4}t$$

$$\Leftrightarrow$$

$$\Delta^2 - 18t\Delta - 27t^2 > 0. \tag{42}$$

The roots of the left-hand side of (42) are

$$\Delta_{1,2} = (9 \pm 6\sqrt{3})t \approx \{-1.4t, 19.4t\}.$$

For values of Δ not between the roots Δ_1 and Δ_2 Eq. (42) is satisfied. Combining this with the assumption that neither firm dominates the market ($-3t < \Delta < 3t$, see Eq. (13)) we get

$$-3t < \Delta < (9-6\sqrt{3})t.$$

□

8 Applying the Results: Pricing of MS Windows vs. MS Office

An often asked question during the anti-trust case against Microsoft was why Microsoft Windows is much cheaper than Microsoft Office, even though Microsoft has a monopoly in the operating systems market.

As Economides & Viard (2000) [3] note there have been difficulties answering this question.

Our model gives a possible answer to this question.

We want to explain why the price of MS Windows is lower than the price of MS Office, i.e. why

$$p^* < p_A^* \tag{43}$$

in our model.

We will first consider the monopoly and then the competition case.

8.1 Monopoly

Substituting the results obtained in Section 3 (Eqs. (2) and (8)) into $p^* < p_A^*$ yields

$$s + \frac{3}{2}t - s_A < s_A - t$$
$$\Leftrightarrow$$
$$s + \frac{5}{2}t < 2s_A.$$

I.e. if the gross utility s_A derived from the application is sufficiently large compared to the intrinsic value s of the platform[16], it is optimal for the monopolist to charge more for the application than for the platform. Furthermore, lower “transportation costs” t mean that consumers are less heterogeneous with respect to their preferences over applications and it is thus easier for A to charge close consumers a higher price for the application without losing the consumers who are further away.

[16] e.g. because setup costs for the platform are higher than for the application

8.2 Competition

One can do the same comparison for the competition case. Substituting the results from Section 4 (Eqs. (19) and (28)) into $p^* < p_A^*$ gives

$$\frac{1}{2}\left(s - \frac{\Delta^2}{36t} + \frac{1}{6}s_A + \frac{5}{6}s_B - \frac{7}{4}t\right) < t + \frac{\Delta}{3},$$

regrouping yields

$$s < \frac{15}{4}t + \frac{\Delta^2}{36t} + \frac{s_A}{6} - \frac{s_B}{2}. \tag{44}$$

For the allowed ranges of s_A, s_B and t the right-hand side of (44) is increasing in s_A, decreasing in s_B and increasing in t.

Hence, we get the results that Microsoft is willing to price Windows higher than Office if 1. the intrinsic value s of Windows is sufficiently low, 2. the substitutability of Office and competing applications is sufficiently low (i.e. t is sufficiently large), 3. the gross utility derived from Office s_A is sufficiently high and 4. the gross utility derived from competing products s_B is sufficiently low.

9 Conclusions

If a potential application of an innovative competitor is better than its own application (but not too much better) a platform owning monopolist is better off if the competitor enters. He will lose market shares to the competitor, but the growth of the applications market will offset this effect and lead to higher overall profits. This may be an explanation why Microsoft encourages third party developers to develop software for Windows even if it competes with its own applications.[17]

We have furthermore shown that for certain parameter combinations the platform owner can be better off after an entry of a competitor in the ap-

[17] One could argue that Microsoft considers its applications a “loss-leader” and prefers making money with the operating system. However, this is inconsistent with the observation that the price of MS Office is much higher than the price of MS Windows and the market share of the Office suite and the operating system are approximately equal.

plications market even if he can only earn profits in the applications market itself (e.g. because the platform is an open file standard or an open source operating system). This is a possible explanation of Adobe's strategy to open its PDF file format. If users want to create PDF files, they have the choice between Adobe Acrobat Standard and a large number of commercial (e.g. PDF Writer) and free (e.g. PDF Creator) software. Adobe lost market shares in the PDF creation application market to competitors, but the market grew sufficiently to offset this effect. Our model can also explain why commercial firms like Oracle and IBM have invested significant resources in the open source operating system Linux instead of developing an own proprietary operating system.[18]

We have further shown for a simplified distribution of consumers' preferences (homogeneity in platform preferences y) that the product diversity effect of competition is sufficient to increase the profits of the platform vendor. We have shown this by introducing the assumptions of a zero price platform and of the possibility of price commitment, and thus eliminated the complementary goods effect and the price commitment effect of competition.

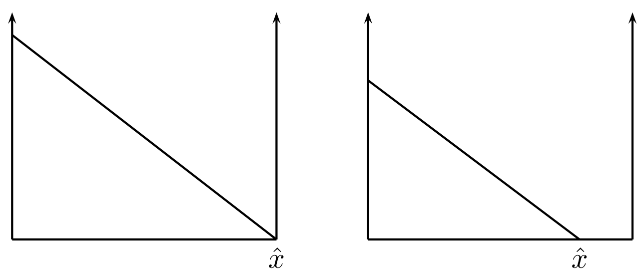
Finally, we have given a possible explanation for the observation that MS Office costs significantly more than MS Windows.

[18]IBM did of course take the effort to develop proprietary operating systems for Intel based PCs (IBM DOS and OS/2) but they haven't been successful with it.

Appendix

A Alternative Cases of Monopoly

If B doesn't enter, A is a monopolist at stage 2. Here two possibilities exist: if s_A is sufficiently large ($s_A \geq 2t$) A will serve all consumers (full market coverage, see Fig. 11(a)), otherwise ($s_A < 2t$) A will charge such a high price that some of the consumers will not buy the application (partial market coverage, Fig. 11(b)).



(a) Full Market Coverage (b) Partial Market Coverage

Figure 11: Cases of monopolistic pricing by A. The vertical axis denotes excess utility $v_A - v_0$ derived from the usage of application A.

We will derive the condition that separate the two cases.

Firm A's profits from application sales are

$$\pi_A = p_A \hat{x}$$

where $\hat{x}$ denotes the location of the consumer furthest away from A who is still willing to buy the application. If only part of the consumers buys the application $\hat{x}$ is the indifferent consumer with

$$s_A - p_A - t\hat{x} = 0$$

$$\Leftrightarrow$$

$$\hat{x} = \frac{s_A - p_A}{t}.$$

If all consumers are willing to buy the platform, i.e. even the consumer at location 1 has a non-negative utility from buying the platform

$$s_A - p_A - t \times 1 \geq 0$$

and $\hat{x}$ is equal to 1.

Formally:

$$\hat{x} = \begin{cases} \frac{1}{t}(s_A - p_A) & \text{if } \frac{1}{t}(s_A - p_A) < 1, \\ 1 & \text{otherwise.} \end{cases}$$

Proposition 3 derives the separating condition and show the equilibrium for the full coverage case.

Proposition 3. *(Formalization of Proposition 1) If $s_A \geq 2t$ it is optimal for the monopolist to set $p_A^* = s_A - t$. The consumer at $x = 1$ derives a utility 0 from buying the application.*

Proof. Substituting $p_A = p_A^*$ and $x = 1$ into excess utility

$$v_A - v_0 = s_A - p_A - tx$$

yields

$$v_A - v_0 = 0.$$

Therefore, for $p_A^* = s_A - t$ the consumer at $x = 1$ is just indifferent between buying and not buying. Demand is hence 1 and profits are

$$\pi_A^* = p_A^* = s_A - t.$$

It doesn't pay off to choose a lower price $p_A^l < p_A^*$ because demand cannot be larger than 1 and profits are hence

$$\pi_A^l = p_A^l < p_A^* = \pi_A^*.$$

It doesn't pay off either to choose a higher price $p_A^h > p_A^*$. For a higher price demand would be less than 1:

$$\hat{x} = \frac{s_A - p_A}{t}.$$

Profits would be

$$\pi_A^h = p_A^h \frac{s_A - p_A^h}{t}.$$

The derivative of the profit function is

$$\frac{\partial \pi_A^h}{\partial p_A^h} = \frac{s_A}{2} - p_A.$$

At $p_A^h = p_A^*$ (and hence at $\hat{x} = 1$) the derivative is

$$\left.\frac{\partial \pi_A^h}{\partial p_A^h}\right|_{p_A^h = p_A^*} = t - \frac{s_A}{2}.$$

If $s_A \geq 2t$ the derivative of the profit function is non-positive at $p_A^h = p_A^*$ and decreasing in p_A^h, therefore, $\pi_A^h \leq \pi_A^*$ and the firm isn't willing to increase its price. □

For the case where $s_A \leq 2t$ the monopolist sells only to a part of the consumers.

His profit maximization problem is

$$\pi_A^* = \max_{p_A} p_A \hat{x} = \max_{p_A} p_A \frac{s_A - p_A}{t}.$$

Solving the first order condition for p_A yields

$$p_A^* = \frac{s_A}{2}.$$

The location of the marginal consumer and profits are hence

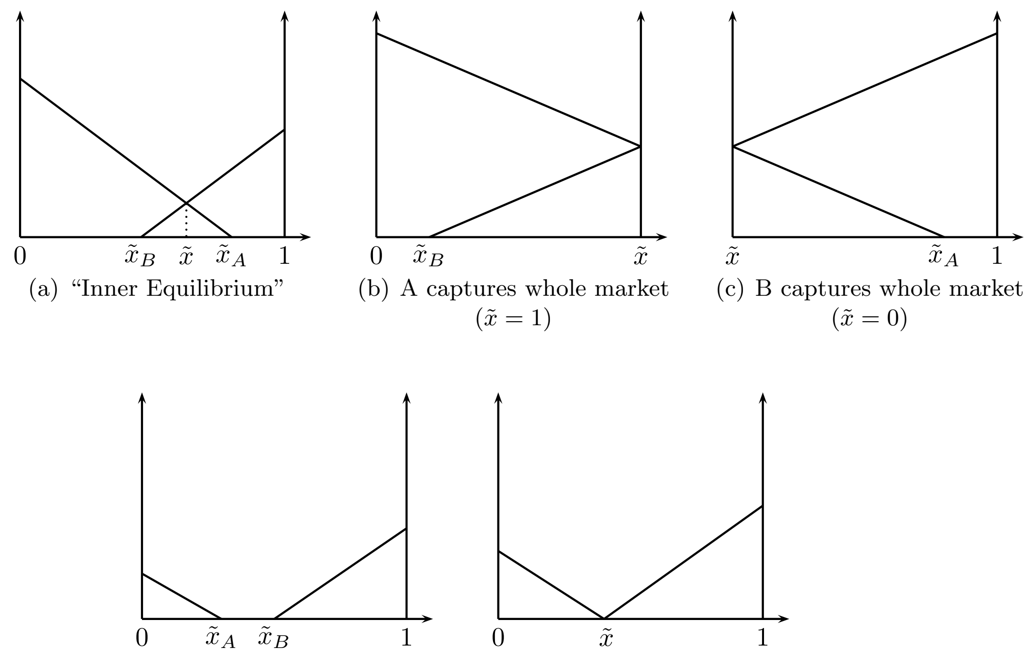
$$\hat{x}^* = \frac{s_A}{2t}$$
$$\pi_A^* = \frac{s_A^2}{4t}.$$

Consumer surplus is

$$\text{EU} = \int_0^{\hat{x}^*} (s_A - p_A - tx)dx = \frac{s_A^*}{8t}.$$

B Alternative Cases of Competition

Five cases can be distinguished in a fixed location Hotelling setup: 1. an "inner equilibrium" ($s_A + s_B > 3t$ and $-3t < s_A - s_B < 3t$, see Fig. 12(a)), 2. market domination by A ($s_A + s_B > 3t$ and $s_A - s_B \geq 3t$, Fig. 12(b)), 3. market domination by B ($s_A + s_B > 3t$ and $s_A - s_B \leq -3t$, Fig. 12(c)), 4. two local monopolies ($s_A + s_B \leq 2t$, Fig. 12(d)) and 5. a "limiting case" where prices are too low for a local monopoly, but too high for competition ($2t < s_A + s_B \leq 3t$, Fig. 12(e)).



(a) "Inner Equilibrium"

(b) A captures whole market ($\tilde{x} = 1$)

(c) B captures whole market ($\tilde{x} = 0$)

(d) Local Monopolies

(e) "Limiting Case" ($\tilde{x} = \tilde{x}_A = \tilde{x}_B$)

Figure 12: Different cases in a Hotelling setup. The vertical axis on the left denotes the excess utility $v_A - v_0$ derived from the usage of application A, the vertical axis on the right denotes the excess utility $v_B - v_0$ from B.

We will derive these conditions and the equilibria arising in the different cases.

We will denote the consumer indifferent between applications A and B with $\tilde{x}$, the consumer indifferent between buying application A and not buying any application with $\tilde{x}_A$ and the consumer indifferent between B and not buying with $\tilde{x}_B$. Formally:

$$s_A - p_A - t\tilde{x} = s_B - p_B - t(1 - \tilde{x})$$
$$s_A - p_A - t\tilde{x}_A = 0$$
$$s_B - p_B - t(1 - \tilde{x}_B) = 0.$$

Regrouping yields

$$\tilde{x} = \frac{1}{2} + \frac{1}{2t}(s_A - s_B + p_B - p_A) \tag{45}$$
$$\tilde{x}_A = \frac{1}{t}(s_A - p_A) \tag{46}$$
$$\tilde{x}_B = 1 - \frac{1}{t}(s_B - p_B). \tag{47}$$

We will call the demand for application A x_A and the demand for application B $(1 - x_B)$ where

$$x_A = \begin{cases} 0 & \text{if } \tilde{x} < 0, \\ 1 & \text{if } \tilde{x} > 1, \\ \tilde{x}_A & \text{if } \tilde{x}_A < \tilde{x}, \\ \tilde{x} & \text{otherwise,} \end{cases} \tag{48}$$

and

$$x_B = \begin{cases} 0 & \text{if } \tilde{x} < 0, \\ 1 & \text{if } \tilde{x} > 1, \\ \tilde{x}_B & \text{if } \tilde{x}_B > \tilde{x}, \\ \tilde{x} & \text{otherwise.} \end{cases} \tag{49}$$

The five cases can be formally defined as follows:

- "Inner Equilibrium": $\tilde{x}_B < \tilde{x}_A$ and $0 < \tilde{x} < 1$
- Domination by A: $\tilde{x} \geq 1$

- Domination by B: $\tilde{x} \leq 0$
- Local Monopolies: $\tilde{x}_A < \tilde{x}_B$
- "Limiting Case": $\tilde{x} = \tilde{x}_A = \tilde{x}_B$

The following propositions state the conditions for the cases and the resulting equilibria. As in the main section, we will use Δ as a shorthand for $s_A - s_B$.

Proposition 4. *If $s_A + s_B > 3t$ and $-3t < \Delta < 3t$ there is an "inner equilibrium" ($\tilde{x}_B < \tilde{x}_A$ and $0 < \tilde{x} < 1$) with equilibrium prices $p_A^* = t + \Delta/3$ and $p_B^* = t - \Delta/3$.*

Proof. Substituting p_A^* and p_B^* into $\tilde{x}_A$ and $\tilde{x}_B$ yields

$$\tilde{x}_A^* = \frac{2s_A + s_B}{3t} - 1, \quad \tilde{x}_B^* = -\frac{2s_B + s_A}{3t} + 2.$$

The condition $\tilde{x}_B < \tilde{x}_A$ becomes thus

$$-\frac{2s_B + s_A}{3t} + 2 < \frac{2s_A + s_B}{3t} - 1$$
$$\Leftrightarrow$$
$$3t < s_A + s_B$$

which is fulfilled by assumption.

Substituting p_A^* and p_B^* into $\tilde{x}$ we get

$$\tilde{x} = \frac{1}{2} + \frac{\Delta}{6t}.$$

The condition $0 < \tilde{x} < 1$ can be rewritten as

$$0 < \frac{1}{2} + \frac{\Delta}{6t} < 1$$
$$\Leftrightarrow$$
$$-3t < \Delta < 3t$$

which is again fulfilled by assumption.

Because both $\tilde{x}_B < \tilde{x}_A$ (and thus $\tilde{x}_B < \tilde{x} < \tilde{x}_A$) and $0 < \tilde{x} < 1$ hold we can write the demand functions specified in (48) and (49) as

$$x_A = \tilde{x} \quad \text{and} \quad 1 - x_B = 1 - \tilde{x}.$$

The Nash equilibrium is hence

$$\begin{aligned} p_A^* &= \arg\max_{p_A} p_A \tilde{x}(p_A, p_B^*) \\ p_B^* &= \arg\max_{p_B} p_B(1 - \tilde{x}(p_A^*, p_B)). \end{aligned}$$

Solving the first order conditions of the two maximization problems for p_A and p_B yields

$$\begin{aligned} p_A^* &= t + \frac{\Delta}{3} \\ p_B^* &= t - \frac{\Delta}{3}. \end{aligned}$$

□

Proposition 5. *If $s_A + s_B > 3t$ and $\Delta \geq 3t$ A will capture the whole market ($\tilde{x} \geq 1$) and equilibrium prices are $p_A^* = s_A - s_B - t$ and $p_B^* = 0$.*

Proof. Substituting p_A^* and p_B^* into $\tilde{x}$ yields

$$\tilde{x}^* = \frac{1}{2} + \frac{1}{2t}(s_A - s_B + p_B^* - p_A*)$$

$$\Leftrightarrow$$

$$\tilde{x}^* = 1.$$

B has no incentive to deviate from $p_B^* = 0$: with a negative price his profits would be non-positive, with a higher price his demand would remain zero.

A has no incentive to deviate either. With a lower price his demand would still be 1, therefore, his profits would decrease.

The reason why he wouldn't set a higher price is the following. At $p_A = p_A^* = s_A - s_B - t$ the derivative of the profit function is

$$\begin{aligned} \left.\frac{\partial \pi_A}{\partial p_A}\right|_{p_A = s_A - s_B - t} &= \left[\frac{1}{2} + \frac{s_A - s_B - 2p_A}{2t}\right]_{p_A = s_A - s_B - t} \\ &= \frac{3t - (s_A - s_B)}{2t}. \end{aligned}$$

The derivative is non-positive at p_A^* for $s_A - s_B \geq 3t$ and linearly decreasing in p_A. Therefore, A has no interest in increasing the price.

□

Proposition 6. *If $s_A + s_B > 3t$ and $\Delta \leq -3t$ B will capture the whole market ($\tilde{x} \leq 0$) and equilibrium prices are $p_A^* = 0$ and $p_B^* = s_B - s_A - t$.*

Proof. By analogy to Proposition 5. □

Proposition 7. *If $s_A + s_B < 2t$ there are local monopolies ($\tilde{x}_A < \tilde{x}_B$) and equilibrium prices are $p_A^* = s_A/2$ and $p_B^* = s_B/2$.*

Proof. Substituting p_A^* and p_B^* into $\tilde{x}_A$ and $\tilde{x}_B$ yields

$$\tilde{x}_A = \frac{s_A}{2t}, \quad \tilde{x}_B = 1 - \frac{s_B}{2t}.$$

Substituting this into $\tilde{x}_A < \tilde{x}_B$ gives

$$\frac{s_A}{2t} < 1 - \frac{s_B}{2t} \quad \Leftrightarrow \quad s_A + s_B < 2t,$$

which is fulfilled by assumption.

$\tilde{x}$ has to be between $\tilde{x}_A$ and $\tilde{x}_B$

$$\tilde{x}_A \leq \tilde{x} \leq \tilde{x}_B.$$

Therefore, we can write demand as

$$x_A = \tilde{x}_A \quad \text{and} \quad 1 - x_B = 1 - \tilde{x}_B.$$

The two local monopolists do not compete with each other, hence the two firms maximize profits independently

$$\pi_A^* = \max_{p_A} p_A \tilde{x}_A(p_A)$$
$$\pi_B^* = \max_{p_B} p_B(1 - \tilde{x}_B(p_B)).$$

Solving the first order conditions gives

$$p_A^* = \frac{s_A}{2}, \quad p_B^* = \frac{s_B}{2}.$$

□

When neither of the aforementioned cases occurs ($2t \leq s_A + s_B \leq 3t$), we have the "limiting case" with $\tilde{x} = \tilde{x}_A = \tilde{x}_B$.

References

[1] Mark Armstrong, *Competition in two-sided markets*, Industrial Organization 0505009, Economics Working Paper Archive EconWPA, May 2005, presentation at ESEM meeting 2002.

[2] Bernard Caillaud and Bruno Jullien, *Chicken & egg: Competition among intermediation service providers*, RAND Journal of Economics **34** (2003), no. 2, 309–28.

[3] Nicholas Economides and Brian Viard, *Pricing of complementary goods and network effects*, Industrial organization, Economics Working Paper Archive EconWPA, July 2004.

[4] Nicholas Economides, *Network externalities, complementarities, and invitations to enter*, European Journal of Political Economy **12** (1996), no. 2, 211–233.

[5] David S. Evans, Andrei Hagiu, and Richard Schmalensee, *A survey of the economic role of software platforms in computer-based industries*, Cesifo working paper series, CESifo GmbH, 2004.

[6] Joseph Farrell and Nancy T Gallini, *Second-sourcing as a commitment: Monopoly incentives to attract competition*, The Quarterly Journal of Economics **103** (1988), no. 4, 673–94.

[7] Andrei Hagiu, *Platforms, pricing, commitment and variety in two-sided markets*, Ph.D. thesis, Princeton University, 2004.

[8] Volker Nocke, Martin Peitz, and Konrad O. Stahl, *Platform ownership*, Cepr discussion papers, October 2004.

[9] Geoffrey G. Parker and Marshall W. Van Alstyne, *Information complements, substitutes, and strategic product design*, William Davidson Institute Working Papers Series 299, March 2000.

[10] Jean-Charles Rochet and Jean Tirole, *Platform competition in two-sided markets*, Journal of the European Economic Association **1** (2003), no. 4, 990–1029.