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02-15

December 2002

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Income Segregation in Multi-Community Models*

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Dezember 2002, revised August 2003

Abstract

This paper presents necessary conditions for segregation of the population in multi-community models with housing markets and heterogeneous households. The conditions for the sorting of the population according to income classes or other dimensions of heterogeneity are established without explicitly describing the household utility function and budget constraint. They therefore apply to a broad class of models, including models with income taxation and property taxation. The segregation conditions in the existing literature are surveyed using a common framework and a series of new and less specific models are proposed. The analysis suggests that in models with income taxation, segregation can often only be established under very specific assumptions on the household's preferences. Furthermore, segregation cannot be ensured with progressive or regressive tax schemes.

Key Words: Income Segregation, Fiscal Federalism, Income Taxation, Local Public Goods
JEL-classification: H71, H73, R13

*The term 'stratification' has been replaced by 'segregation' in the course of revision.
‡I thank Klaus Neusser for helpful comments and his mathematical expertise.

1 Introduction

Over the past two decades, there has been ongoing research on multi-community models of urban agglomerations with heterogeneous agents. Multi-community models try to explain persistent differences in local tax levels and in the provision of local public goods in metropolitan systems of communities. A main finding of the literature is that these differences are accompanied by segregation of the population into groups with similar incomes and tastes. This paper establishes the general conditions that induce such a self-sorting process of the population.

The study of multi-community models originates in Tiebout's (1956) seminal work. While Tiebout was seeking to determine the optimal number of jurisdictions, *positive theories* of community systems take the political landscape, i.e. the number of jurisdictions and their physical size, as given. This strand of literature builds on Westhoff (1977), who analyzed a general equilibrium model with heterogeneous agents and a fixed number of competing jurisdictions. The individual communities provide a local public good which is financed by a proportional local income tax. The residents of a community agree on the tax rate and on the amount of public goods in a majority vote that respects the community's budget balance. Households choose the community that offers them the best combination of public goods provision and tax rate.

The consideration of local housing markets by Ellickson (1971) and Rose-Ackerman (1979) was a major step towards more realistic models of jurisdictional systems. This extension was accompanied by a shift away from the study of models with income taxation to models with *property taxation*. The shift was coherent with the U.S. institutional reality, but also a way to circumvent the technical problems associated with the housing market in income tax models. Property tax models have subsequently been investigated by Epple, Filimon and Romer (1984, 1993) and Epple and Romer (1991). This strand of literature has been comprehensively reviewed by Ross and Yinger (1999). The consideration of further dimensions of household heterogeneity by Epple and Platt (1998) allowed an empirical examination as undertaken by Epple and Sieg (1999) and Epple, Romer and Sieg (2001). There are only very few studies of models with local *income taxation* and housing markets, rare examples are Goodspeed (1989) and Hansen and Kessler (2001a). Besides posing a theoretically interesting problem, local income taxation exists in metropolitan areas in Switzerland.

The *segregation hypothesis* is the central proposition in multi-community models with heterogeneous households. Endogenous segregation means that different people choose different communities when the communities differ in tax rates, housing prices and public goods provision. The population is then segregated as groups of similar household attributes tend to live at the same places. While the Tiebout model focuses on preference heterogeneity, Ellickson and Westhoff turned the attention to income as the main dimension of difference.

Westhoff (1977) established income segregation by assuming that the *relative preference* for the public good varies with income. Westhoff's relative preference assumption is equivalent to the Spence-Mirrlees, also called single crossing condition, of incentive theory and information economics. The mathematical analogy is maintained in property tax models with a housing market. However, the same strategy does not generally apply in income tax models with housing markets. In Westhoff's original model, households make their residence choice by comparing a two-dimensional set of community characteristics, namely the public good and the income tax rate. This two-dimensionality is maintained in the property tax model, where households are only concerned with the after-tax housing price and not with the tax rate per se. Unfortunately, this reduction of dimensionality is only possible in very simplistic models with income taxation. Households have to make a choice which takes account of the three dimensions of community characteristics, namely the tax rates, the housing prices and the public goods provision. This paper investigates the segregation conditions in this general setting.[1]

The first part of the present paper establishes a set of segregation conditions formulated for a broad class of models, covering property tax models as well as income tax models. The property tax models in the existing literature and the few proposed income tax models can be treated as special cases of this general setting. This framework allows to study a variety of new and less restrictive income tax models. The second part of the paper presents a series of models and shows how these models satisfy the conditions described in the previous section. While in some models the proposed conditions are very naturally satisfied, one has to make specific assumptions of household preferences in others. The main

[1]Note that the natural generalization of the single crossing condition to multi-dimensional problems used in information economics cannot be adapted for the segregation condition. Guesnerie and Laffont(1984) describe how the single Spence-Mirrlees condition is adapted to a multi-dimensional decision space in mechanism design problems.

finding is that income segregation in income tax models can only be established under very specific assumptions of household preferences. A further result is that income segregation cannot be ensured with nonlinear, i.e. progressive or regressive, tax schemes.

2 The Model

The model economy is divided into J distinct communities. The area is populated by a continuum of heterogeneous households which differ in income $y \in [\underline{y}, \overline{y}]$, $\underline{y} > 0$, $\overline{y} < \infty$. Income is distributed according to the density function $f(y) > 0$. There are three goods in the economy: private consumption b, housing h and a local publicly provided good g. The latter may be a pure public good, a publicly provided consumption good or a pure transfer. It is local in the sense that it is only consumed by the residents of a community.

A household can move costlessly and chooses the community that maximizes its utility as place of residence. Each community j can individually set the amount of the local public good g_j and the local tax rate $t_j \in [0, 1]$. This decision is made in a majority vote by the residents who respect the budget balance in the community. At this point it is not specified whether the tax is based on property or income. Each community has a fixed amount of land L_j from which housing stock is produced. Households may be renters or house owners. The price for housing p_j in community j is determined in a competitive housing market. The private good is considered as the numeraire. A community j is fully characterised by the triple (t_j, p_j, g_j). The set of all possible community characteristics is given by $\Gamma = [0, 1] \times \mathbb{R}^{++} \times \mathbb{R}^{+}$.

2.1 Indirect utility

In the general framework, a household is described by its indirect utility function. Section 3 shows how the indirect utility function is derived from different sets of utility functions and household budget constraints. The indirect utility function is a function on $\Gamma \times \mathbb{R}^{+}$ such that:

$$(t, p, g, y) \rightarrow V(t, p, g, y) .$$

The indirect utility function is assumed to be twice continuously differentiable in all its arguments everywhere on its domain.

The following three assumptions on the form of the indirect utility function are necessary conditions for segregation of the population in equilibrium.

Assumption 1 *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$

$$V_t(t, p, g, y) := \left.\frac{\partial V}{\partial t}\right|_{t,p,g,y} < 0\,,$$

$$V_p(t, p, g, y) := \left.\frac{\partial V}{\partial p}\right|_{t,p,g,y} < 0\,,$$

$$V_g(t, p, g, y) := \left.\frac{\partial V}{\partial g}\right|_{t,p,g,y} > 0\,.$$

Assumption 1 is the standard assumption about the influence of prices, taxes and public goods on the household's well-being. Property 1 follows directly from applying Assumption 1 to the total differential of the indirect utility function.

Property 1 (Relative preferences)
For all $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$

$$M_{g,t}(t, p, g, y) := \left.\frac{dg}{dt}\right|_{dV=0,dp=0} = -\frac{V_t}{V_g} > 0\,,$$

$$M_{g,p}(t, p, g, y) := \left.\frac{dg}{dp}\right|_{dV=0,dt=0} = -\frac{V_p}{V_g} > 0\,,$$

$$M_{t,p}(t, p, g, y) := \left.\frac{dt}{dp}\right|_{dV=0,dg=0} = -\frac{V_p}{V_t} < 0\,.$$

Property 1 states that a household can be compensated for a tax increase either by more public good provision or by lower housing prices. It also states that a household is indifferent to higher housing prices if it is compensated by more public good provision. $M_{\cdot,\cdot}$ is called the marginal rate of substitution between two community characteristics. The marginal rates are well defined due to the strict inequalities in Assumption 1.

Assumption 2 (Constant sign of relative preferences)
At least one of the following three alternatives (a), (b) and (c) holds:
(a) *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$ *either*

$$(CS^+_{g,t}) \quad \frac{\partial M_{g,t}}{\partial y} > 0 \quad or \quad (CS^-_{g,t}) \quad \frac{\partial M_{g,t}}{\partial y} < 0\,.$$

(b) *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$ *either*

$$(CS^+_{g,p}) \quad \frac{\partial M_{g,p}}{\partial y} > 0 \quad or \quad (CS^-_{g,p}) \quad \frac{\partial M_{g,p}}{\partial y} < 0\,.$$

(c) *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$ *either*

$$(CS^+_{t,p}) \quad \frac{\partial M_{t,p}}{\partial y} > 0 \quad or \quad (CS^-_{t,p}) \quad \frac{\partial M_{t,p}}{\partial y} < 0\,.$$

Note that $M_{t,p} = -M_{g,p}/M_{g,t}$ by definition and hence

$$\frac{\partial M_{t,p}}{\partial y} = -\frac{\partial M_{g,p}/\partial y \cdot M_{g,t} - M_{g,p} \cdot \partial M_{g,t}/\partial y}{M^2_{g,t}}\,.$$

Assumption 2 states that the household's relative preferences for community characteristics change systematically with income. Alternative $CS^+_{g,t}$ implies that a rich household has to be compensated for a tax increase by strictly more public goods than a poor household. Alternative $CS^+_{g,p}$ supposes that the compensation for higher housing prices by public goods strictly increases with income. Alternative $CS^+_{t,p}$ means that the tax cut compensating for higher housing prices is strictly more substantial for a rich household than for a poor household. The alternatives $CS^-_{\cdot,\cdot}$ are interpreted analogously. Assumption 2 requires specific assumptions on the form of the utility function and/or the budget constraint. In Section 3 several examples in which Assumption 2 is naturally satisfied are presented.

Assumption 3 (Proportional shift of relative preferences)
One of the following two alternatives (a) and (b) holds:
(a) *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$ *either*

$$\frac{\partial M_{g,t}}{\partial y} = 0 \quad or \quad \frac{\partial M_{g,p}}{\partial y} = 0 \quad or \quad \frac{\partial M_{t,p}}{\partial y} = 0\,.$$

(b) *For all* $(t, p, g, y) \in \Gamma \times \mathbb{R}^+$ *both*

$$\frac{\partial M_{g,p}}{\partial y}\Big/\frac{\partial M_{g,t}}{\partial y} \quad \text{and} \quad \frac{\partial M_{t,p}}{\partial y}\Big/\frac{\partial M_{t,g}}{\partial y}$$

with $M_{t,g} = 1/M_{g,t}$ *are independent of* y.

Assumption 3a is a very strong assumption. Assumption 3b is a weaker but very technical. Both alternatives are difficult to interpret intuitively and seem difficult to justify empirically. However, as will become clear in the next section, Assumption 3 is an indispensable condition for the occurrence of segregation. Assumption 2 and Assumption 3 are a three-dimensional generalization of the single-crossing condition.

2.2 Location Choice

The indirect utility function V yields the utility of a household with income y in a community j with income tax t_j, housing prices p_j and public good provision g_j. V implicitly describes the indifference surfaces in the (t, p, g) space.

A household with income y chooses the community which maximizes the household's utility. Hence, given a set of community characteristics, (t_j, p_j, g_j) for $j = 1, ..., J$, a household prefers community j over community i if and only if

$$V(t_j, p_j, g_j, y) \geq V(t_i, p_i, g_i, y) \quad \textit{for all } i\,.$$

The graphical representation of the indirect utility function is extensively used in the following proofs and therefore introduced in detail. Figure 1 displays the indirect utility function graphically. The left picture in Figure 1 shows an indifference surface in the 3-dimensional (t, p, g) space. Point 1 represents a community with income tax t_1, housing price p_1 and public good provision g_1. The indifference surface $\bar{V}_{1,y} := \{(t, p, g) : V(t, p, g, y) = V(t_1, p_1, g_1, y)\}$ covers all community characteristics that yield the same utility as community 1 for a household with income y. Community 2 with (t_2, p_2, g_2) is an example of such a community. All triples above the indifference curves are preferred to community 1. As stated in Property 1, the indifference is increasing in p and in t.

The right picture in Figure 1 is an illustration of the same indifference surface in the 2-dimensional (t, g) policy space for a given level of housing prices p. Suppose for the moment that housing prices are fixed at p_1. The solid curve covers the set $\bar{V}_{1,y,p_1} := \{(t, g) : V(t, p_1, g, y) = V(t_1, p_1, g_1, y)\}$ and corresponds to the solid subset of $\bar{V}_{1,y}$ in the left picture. The solid curve represents all community characteristics that yield the same utility as community 1 given p_1. Consider now a different housing price level p_2. All community characteristics considered to be indifferent to community 1 given the new price p_2 are on the

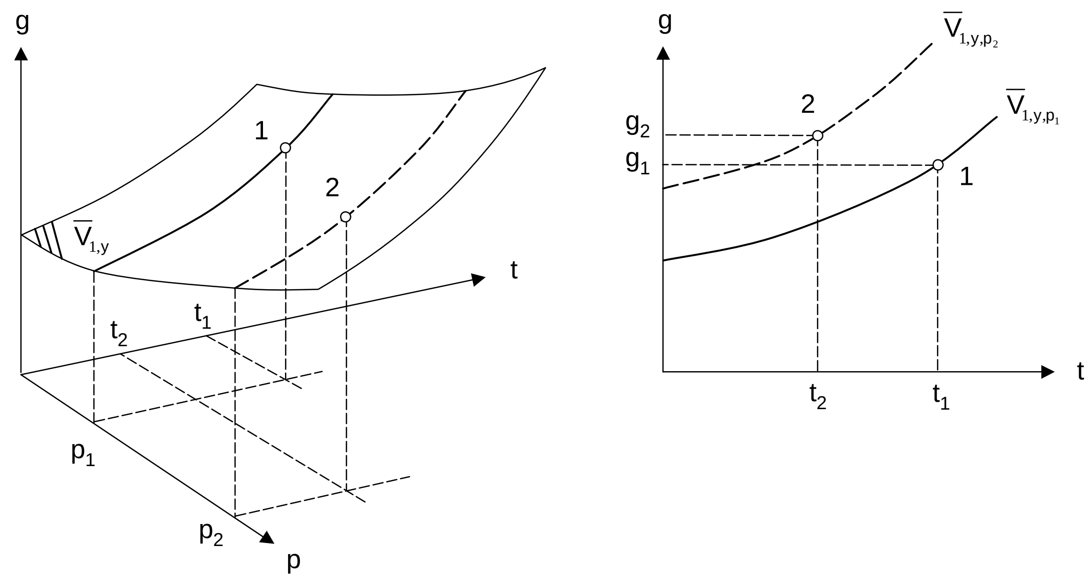

Figure 1: Indifference surface in the policy space.

dashed curve. Community 2 with (t_2, p_2, g_2) is an example of such a community. The dashed curve lies above the bold line when Property 1 holds. Note that the indifference surface depends on the household's income y.

In the following propositions, the allocation of the households across distinct communities induced by the conditions in Assumptions 2 and 3 is discussed. A first observation is that all households are indifferent between all communities when the communities have identical community characteristics, i.e. $(t_j, p_j) = (t_i, p_i)$ for all j, i. In this case the households settle such that all communities show the same income distribution. This situation is a possible equilibrium in all the models presented in Section 3. In addition, it is always possible to think of equilibria in which subsets of communities have identical characteristics, i.e. $(t_j, p_j) = (t_i, p_i)$ for some j, i. However, these equilibria may not be stable.[2] The focus of this paper is on the empirically interesting case of equilibria where all communities exhibit distinct characteristics.

[2]The notion of 'stability' in an intrinsically static model is rather peculiar. Nevertheless equilibria in static multi-community models are often judged by their 'dynamic' behavior. In this ad-hoc interpretation, an equilibrium is called 'stable' when the change of community characteristics induced by the migration of 'few' households gives these households an incentive to move back.

Proposition 1 (Boundary indifference)
When Assumption 1 holds and a household with income y' prefers to live in community j and another household with income $y'' > y'$ prefers to live in community i, then there exists a household with income $\hat{y}$, $y' \leq \hat{y} \leq y''$, which is indifferent between the two communities: $V(t_j, p_j, g_j, \hat{y}) = V(t_i, p_i, g_i, \hat{y})$.

Proof: Let $V_j(y) := V(t_j, p_j, g_j, y)$ be a household's utility in j and $V_i(y) := V(t_i, p_i, g_i, y)$ in i. The household with income y' prefers community j to i, hence $V_j(y') - V_i(y') \geq 0$. The opposite is true for a household with income y'': $V_j(y'') - V_i(y'') \leq 0$. $V_j(y) - V_i(y)$ is continuous in y since V is continuous in y. The intermediate value theorem implies that there is at least one $\hat{y}$ between y' and y'' s.t. $V_j(\hat{y}) - V_i(\hat{y}) = 0$. The existence of $\hat{y}$ follows from $f(y) > 0$. □

Proposition 1 states that for any pair of communities there is a 'border' household which is indifferent between the two.

Definition 1 (Perfect income segregation)
An allocation of households is called perfectly segregated by incomes if the J sets $I_j = \{y$: household with income y prefers community $j\}$ satisfy

- *I_j is an interval for all j,*
- *$I_j \neq \emptyset$,*
- *$I_j \cap I_i = \emptyset$ for all $j \neq i$,*
- *$I_1 \cup ... \cup I_J = [\underline{y}, \overline{y}]$.*

Definition 1 means that any community is populated by a single and distinct income class.

Proposition 2 (Perfect income segregation)
When Assumptions 1, 2 and 3 hold and all J communities exhibit distinct characteristics, $(t_j, p_j, g_j) \neq (t_i, p_i, g_i)$ for all $j \neq i$, then the allocation of households is perfectly segregated by incomes.

Proof: The proof proceeds in two steps. Firstly, income segregation is shown for a pair of two communities. Secondly, the result is extended to more than two communities.

(1) The proof refers to Figure 2. Consider two communities 1 and 2 and assume $CS^+_{g,t}$. The figure shows the indifference surface in the (g, t) space for three

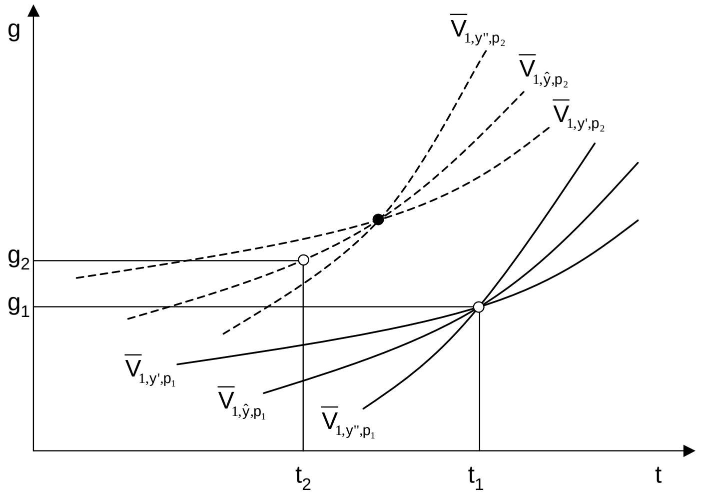

Figure 2: Indifference curves in the (t, g) policy space.

different income levels $y' < \hat{y} < y''$ and two levels of housing prices $p_1 < p_2$. The solid lines represent all (t, g) pairs indifferent to community 1, characterized by (t_1, p_1, g_1), given p_1. The indifference curves are increasing in t (Property 1) and become steeper as income rises (assumption $CS^+_{g,t}$). The dashed lines represent all (t, g) pairs that are considered to be indifferent to community 1 given p_2. They are shifted to the left of the solid curves (Property 1) and intersect at the same point (Assumption 3 and proof in Appendix A). Imagine now a community 2, characterized by (t_2, p_2, g_2), which is considered as good as community 1 by household $\hat{y}$. If the (t_2, g_2) lies to the left of the intersection, then all richer households $y > \hat{y}$, e.g. y'', prefer community 2 to community 1 and all poorer households $y < y$, e.g. y', prefer community 1. If (t_2, g_2) is on the right side of the intersection the preference order is inverted. No segregation occurs in the unlikely case that (t_2, g_2) is exactly the intersection. The analogous argument holds for the other alternatives of Assumption 2.

(2) Suppose that the household allocation is not segregated: y' as well as y'' prefer community i, but y''' strictly prefers community j. Then it follows from Proposition 1 that there is a $\hat{y}$, $y' \leq \hat{y} < y'''$. (1) implies that $y'' > y''' > \hat{y}$ strictly prefers j to i, which is a contradiction. □

Figure 3 shows a situation in which Assumption 3 does not hold. One can verify that the sketched indifference curves satisfy Assumption 2 ($CS^+_{g,t}$ and also $CS^+_{g,p}$ and $CS^+_{t,p}$). However, there is a richer household $y'' > \hat{y}$ that strictly prefers

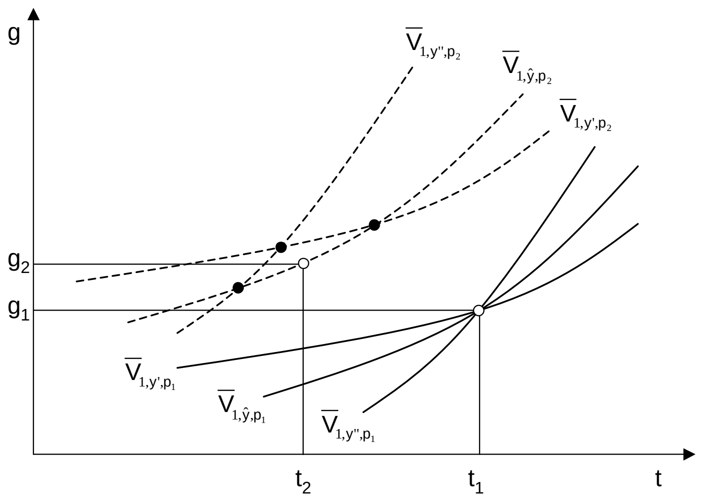

Figure 3: Indifference curves when Assumption 3 is not satisfied.

C_i and also a poorer household $y'' > \hat{y}$ that strictly prefers C_i. This contradicts Proposition 2 and shows that Assumption 3 is a necessary condition.

Note that very little can be said about the order of p, g and t across communities given the assumptions made so far. As Figure 2 shows, it is possible that rich people prefer the community with lower taxes or the one with higher taxes, that they live in the community with more or less public goods and that the rich communities have higher or lower housing prices. The properties of an equilibrium - if it exists - depends on the details of a fully specified model.

2.3 Taste Heterogeneity

In the previous sections, the sorting of the population with respect to income was discussed. There may be other sources of heterogeneity such as different tastes for housing or for the publicly provided good. Abstracting from income heterogeneity, one can use the above derived conditions and propositions for taste heterogeneity by simply replacing the variable y by a taste parameter, e.g. α, which satisfies all the assumptions respectively.

Combinations of income and taste heterogeneity result in more realistic models. Such models do not predict perfect segregation into income classes, but an allocation where the average household income still differs across communities. Models with both income and taste variation have been studied by Epple and

Romer (1998), Epple and Sieg (1999), Epple, Romer and Sieg (2001), Kessler and Lülfesmann (1999) and Schmidheiny (2002b). These models form the basis for empirical investigation.

3 A Survey of Models

This section presents a series of models and discusses the specific assumptions under which these models lead to income segregation. The models are categorized along three dimensions: the nature of the local tax, the character of the publicly provided good and the form of the housing demand. The model properties are analyzed in in the three-dimensional (t, p, g) space of the general framework. Some models could also be examined in a two-dimensional characteristics space with the proofs becoming more elegant. However, it is the aim of this paper to show the specifics of the whole model setup that generates segregation.

Each model is illustrated by an example. The household utility in these examples takes the form of a mixed CES/Stone-Geary function:

$$U(h, b, g) = [\gamma g^{\rho} + (1 - \gamma) w(h, b)^{\rho}]^{1/\rho}$$

with

$$w(h, b) = (h - \beta_h)^{\alpha} (b - \beta_b)^{1-\alpha} .$$

Appendix B explains the characteristics of this utility function and derives the properties of the different models. There are two important features of the chosen utility function. Firstly, it allows the public good to be a perfect substitute $(\rho = 1, \sigma \to \infty)$, a perfect complement $(\rho \to -\infty, \sigma = 1)$ or of any intermediate degree of substitutability, which is measured by the elasticity of substitution $\sigma = 1/(1 - \rho)$. Secondly, the budget share of housing can change with income and therefore the income elasticity can differ from unity.

Table 1 summarizes the parameters of the specified models and shows their properties and predictions.

3.1 Property Tax, Transfer and Elastic Housing Demand

Epple and Romer (1991) propose a model in which the local jurisdictions pay a lump sum transfer g to their inhabitants.[3] The transfer is financed by a proportional property tax t on the consumption of housing. The individual households choose their place of residence and their optimal consumption bundle of a composite private good b and housing h. The household problem is

$$\max_{h,b} U(h,b) \quad s.t. \quad y+g \geq p(1+t)h+b,$$

where p is the pre-tax price of housing. The rate of substitution between the community variables p, g and t can now be derived under standard assumptions[4] by applying the envelope theorem to the total differential of the indirect utility:

$$M_{g,t} = ph^* \,, \quad M_{g,p} = (1+t)h^* \,, \quad M_{t,p} = -\frac{1+t}{p} \,,$$

where $h^* = h(t,p,y)$ is the demand for housing. Assumptions $CS^+_{g,t}$ and $CS^+_{g,p}$ are satisfied as long as housing demand is strictly increasing in income. Assumption 3 holds as $M_{t,p}$ is independent of y.[5]

From the point of view of a household a monetary transfer is equivalent to a perfectly substitutable public good. Hence one can study transfers in the specified example by assuming that $\sigma \to \infty$. The corresponding properties are reported as case [1] in Table 1. The example shows that the transfer g in the poor community, i.e. the community populated by households from lower income classes, is higher than in the rich community. The opposite can be stated for the after-tax price of housing $p(1+t)$, yet there is no resulting order for the tax rate t itself.

Other than in the original proof the general framework allows to study the effect of a nonlinear tax scheme. Consider that the property tax rate $t \cdot r$ consists

[3] Epple and Romer (1998) extend this model and allow for heterogeneous tastes. The utility function depends on a household specific parameter α, which describes the taste for housing:

$$\max_{h,b} U(h,b,\alpha) \quad s.t. \quad y+g \geq p(1+t)h+b\,.$$

Conditional income segregation arises for any given taste. Conditional taste segregation for a given income level is only established by explicitly making Assumption 2 in terms of α. Assumption 3 in terms of α is satisfied by construction.

[4] The standard assumptions are the following: the utility function is increasing, continuous and twice continuously differentiable in all its arguments.

[5] From a household's viewpoint, a community is fully characterized by the transfer and the after-tax housing price $p(1+t)$. Segregation could therefore also be analyzed in the two-dimensional $(g, p(1+t))$ space, which simplifies the proof of segregation.

of a tax shifter t, set by the community, and an exogenous tax rate structure $r(h)$ which can depend on the consumed amount of housing $h = h^*(y)$ and consequently on income. In this case, Assumption 3a is not satisfied any longer as $M_{t,p} = -[r^{-1}(h^*) + t]p^{-1}$ depends on y. Thus, income segregation cannot be established under a regressive or a progressive tax rate.

3.2 Property Tax, Public Good and Elastic Housing Demand

In the Epple, Filimon and Romer (1984, 1993) model, communities use the revenue from a proportional property tax to finance a local public good.[6] Differently from above, the public good enters the utility function and not the budget constraint:

$$\max_{h,b} U(h,b,g) \quad s.t. \quad y \geq p(1+t)h + b\,.$$

The marginal rates of substitution between community characteristics become

$$M_{g,t} = p\,h^* \frac{U_b(h^*,b^*,g)}{U_g(h^*,b^*,g)}\,, \quad M_{g,p} = (1+t)h^* \frac{U_b(h^*,b^*,g)}{U_g(h^*,b^*,g)}\,, \quad M_{t,p} = -\frac{1+t}{p}\,.$$

Epple et al. make an explicit restriction on the household's preferences by assuming that $\partial[h^*\, U_b(h^*,b^*,g)/U_g(h^*,b^*,g)]/\partial y < 0$. This inequality guarantees that Assumptions $CS^-_{g,t}$ and $CS^-_{g,t}$ are satisfied. Assumption 3a is fulfilled by construction as $M_{t,p}$ is independent of income.[7]

The properties of different specifications of the example utility function are presented as cases [2] to [5] in Table 1. The example reveals that assumption made by Epple et al. is satisfied when the public good is either a complement (case [2]) or a substitute (case [4]). The order of community characteristics changes with the nature of the public good: Rich households will choose the community with high public good provision when they cannot easily substitute the public goods.

[6]Epple and Sieg (1999) extend this model and allow for heterogeneous tastes. The utility function depends on a household specific parameter α, which describes the taste for housing:

$$\max_{h,b} U(h,b,g,\alpha) \quad s.t. \quad y \geq p(1+t)h + b\,.$$

Conditional income segregation arises for any given taste. Conditional taste segregation for a given income level only occurs by explicitly making Assumption 2 in terms of α. Assumption 3 in terms of α is satisfied by construction.

[7]Again, this model could be studied in the $(g, p(1+t))$ space.

As in the previous section, income segregation cannot be established under a nonlinear tax scheme.

3.3 Income Tax, Transfer and Inelastic Housing Demand

Hansen and Kessler (2001a) present a model in which a pure monetary transfer is financed by a proportional local income tax. Hansen and Kessler assume that every household consumes one unit of housing independently of its income. This is a clearly unrealistic feature, but it allows a very elegant analysis.[8] The household problem is

$$\max_{b} U(h,b) \quad s.t. \quad y(1-t)+g \geq p+b\,.$$

The marginal rates of substitution between community characteristics are

$$M_{g,t}=y\,,\quad M_{g,p}=1\,,\quad M_{t,p}=-1/y\,.$$

Assumptions $CS^+_{g,t}$ and $CS^+_{g,p}$ are generically satisfied in this setup. Assumption 3a is fulfilled since $M_{g,p}$ is independent of y. Note, however, that the segregation conditions are only satisfied due to the extremely specific nature of the housing demand together with perfect substitutability of the public good.

Table 1 summarizes this model in case [6] and shows that poor households prefer the community with high transfer and high taxes.

3.4 Income Tax, Transfer and Elastic Housing Demand

The following model is a generalization of the model in the previous section. Housing demand is allowed to depend on income. The publicly provided good is still considered a pure transfer.[9] The household problem is

$$\max_{h,b} U(h,b) \quad s.t. \quad y(1-t)+g \geq ph+b\,.$$

The marginal rates of substitution between community characteristics are

$$M_{g,t}=y\,,\quad M_{g,p}=h^*\,,\quad M_{t,p}=-h^*/y\,.$$

[8]The housing price p in a community can formally be considered a reduction of the transfer g. This model can, therefore, easily be studied in the two-dimensional $(t, g-p)$ space.

[9]The same model is discussed in an unpublished study by Calabrese (1990). Unlike in the rest of the literature, Calabrese establishes the segregation conditions using restrictions from the public choice mechanism within the community. This proceeding may offer alternative ways to establish segregation conditions and deserves more attention.

Other than in the above models, this model cannot be analyzed in a reduced two-dimensional characteristics space. One can immediately see that assumption $CS^+_{g,t}$ is always satisfied and assumption $CS^+_{g,p}$ is met when housing demand is increasing in income. $CS_{t,p}$ is positive (negative) if the housing elasticity is smaller (bigger) than 1 since $\partial M_{t,p}/\partial y = h^*/y^2[1 - \partial h^*/\partial y \cdot y/h^*]$.

Assumption 3a is only fulfilled if housing demand h^* is a linear function of disposable income $y(1-t)$ through the origin, i.e. the preferences are homothetic and the income elasticity of housing is $\varepsilon_{h,y} = 1$ for all y. In this case $M_{t,p}$ is independent of y as h^*/y is a constant. Assumption 3b is satisfied if and only if housing demand is linear in income but not necessarily through the origin. Given this linearity $(\partial M_{g,p}/\partial y)/(\partial M_{g,t}/\partial y) = \partial h^*/\partial y$ and $(\partial M_{t,p}/\partial y)/(\partial M_{t,g}/\partial y) = y\partial h^*/\partial y - h^*$ are independent of y. Note that the assumption of linear housing demand is a necessary condition for segregation of the population in this model and not just a convenient simplification.

The example in Table 1 exemplifies the above reasoning in cases [7],[8] and [9]. The segregation conditions are always satisfied as the Stone-Geary subutility $w(h, b)$ generically leads to a linear housing demand function. If the income elasticity of housing is 1 (case [7]), then segregation is only driven by the nature of the public good. Hence, poor households prefer communities with higher transfers in this case.

Non-linear tax schemes are a main feature of a realistic income tax model. Consider that the income tax rate $t \cdot r$ consists of a tax shifter t, set by the community, and an exogenous tax rate structure $r(y)$ which depends on income. In this case, Assumptions 3a and 3b cannot be satisfied. Income segregation can therefore not be ensured under the prevalent progressive income tax schemes.

3.5 Income Tax, Public Good and Elastic Housing Demand

The attention is now turned to the most general model. Local jurisdictions provide a local public good financed by a proportional tax on income.[10] The household problem is the following:

$$\max_{h,b} U(h,b,g) \quad s.t. \quad y(1-t) \geq ph + b\,.$$

The marginal rates of substitution between community characteristics are

$$M_{g,t} = y\,\frac{U_b(h^*,b^*,g)}{U_g(h^*,b^*,g)}\,, \quad M_{g,p} = h^*\,\frac{U_b(h^*,b^*,g)}{U_g(h^*,b^*,g)}\,, \quad M_{t,p} = -\frac{h^*}{y}\,.$$

There are several sets of assumptions that generate segregation in this model. The first assumption is to set the income elasticity of housing $\varepsilon_{h,y}$ to unity for all y. In this case, housing demand is a constant fraction of disposable income. As in Section 3.4 $M_{t,p}$ is independent of y and Assumption 3a is fulfilled. Assumption 2a and 2b are satisfied when $\partial[h^*\,U_b(h^*,b^*,g)/U_g(h^*,b^*,g)]/\partial y$ is either > 0 or < 0. With this set of assumptions the segregation of the population is fully driven by the nature of the public good. This situation and its qualitative implications for the equilibrium values are resumed in cases [10] to [12] in Table 1.

Another important source for segregation is the income elasticity of housing. Assume that the elasticity of substitution between g and $w(h,b)$ is exactly one. Table 1 shows the model properties if the income elasticity of housing is below unity in case[13] and if it is above unity in case [14]. Assumptions $CS^-_{g,t}$, $CS^-_{g,p}$, and $CS^+_{t,p}$ (case[13]), respectively $CS^-_{t,p}$ (case[14]) as well as Assumption 3b are satisfied without further assumptions.

If the public good is either a substitute or a complement (cases [15] and [16]), the segregation conditions are only satisfied if the income elasticity of housing is one. Although the utility function in case [15] satisfies $CS^-_{g,t}$, $CS^-_{g,p}$, and $CS^+_{t,p}$ it

[10]This model has already been analyzed by Goodspeed(1989). The graphical proof of segration he provides in Goodspeed(1986) leads to Assumption 2 in this paper. However, he fails to observe the importance of my Assumption 3. The Stone-Geary specification in the numerical simulation

$$\max_{h,b} U(h,b,g) = (h-\beta_h)^{\alpha}(b-\beta_b)^{1-\alpha}(g-\beta_g)^{\gamma} \quad s.t. \quad y(1-t) \geq ph + b$$

satisfies Assumption 3b by chance and thereby prevents the detection of the missing assumption.

does fulfill neither Assumption 3a nor 3b. Therefore, given the specified utility function, segregation can be established if it is driven by either only the nature of the public good or by only the income elasticity of housing.

Note that segregation of the population cannot be established under a progressive or regressive income tax scheme as can easily be verified by looking at the conditions for Assumption 3 in Appendix B.

4 Conclusions

This paper presents a general framework in which the segregation of the population induced by local tax setting in multi-community models with heterogeneous agents and housing markets can be analyzed. A set of conditions leading to segregation of the households is derived. While these conditions are naturally satisfied in property tax models, very restrictive assumptions on the household's preferences are needed in models with local income taxes.

The results in this paper can be used to build more realistic multi-community models with local income taxation. Note that this paper does not investigate the entire general equilibrium model. Multi-community models have to be closed by specifying both the housing supply function and the public choice decision within communities. The latter is usually modelled as a majority vote. Existence of the equilibrium in the complete general equilibrium model is not generally guaranteed. Hansen and Kessler (2001b) show that segregation of the population is in many cases incompatible with the majority voting equilibrium within countries in models with local income taxation. Schmidheiny (2002) establishes equilibria in a model with local income taxes, a partly substitutable public good and elastic housing demand as outlined in Section 3.5.

The model class analyzed in this paper is designed to explain decentralized public choice in metropolitan areas. For this purpose it is justifiable to consider the households' residence choice independent from the location of its members' jobs. However, the disregard of the dependence of residence choice from the availability of suitable jobs - and vice-versa - limits the usefulness of these models for examining fiscal decentralization on the level of federal states or countries. The consideration of the location choice of firms would be a major step towards a better understanding of fiscal decentralization on a national scale.

Table 1: Overview of model properties.

	linear property tax					linear income tax										
Section	3.1	3.2				3.3	3.4			3.5						
Case	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]
Restrictions on model parameters																
$\sigma_{w,g}$	∞	≥ 1	$= 1$	≤ 1	> 1	∞	∞	∞	∞	> 1	$= 1$	< 1	$= 1$	$= 1$	< 1	$\neq 1$
β_h	n.r.	≤ 0	$= 0$	≥ 0	> 0	> 0	$= 0$	> 0	> 0	$= 0$	$= 0$	$= 0$	> 0	> 0	> 0	$\neq 0$
β_b	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.	$= 0$	> 0	> 0	$= 0$	$= 0$	$= 0$	> 0	> 0	> 0	$\neq 0$
ε_{h,y_d}	n.r.	n.r.	n.r.	n.r.	n.r.	$= 0$	$= 1$	< 1	> 1	$= 1$	$= 1$	$= 1$	< 1	> 1	< 1	$\neq 1$
Segregation conditions																
$\partial M_{g,t}/\partial y$	> 0	> 0	$= 0$	< 0	d.i.	> 0	> 0	> 0	> 0	> 0	$= 0$	< 0	< 0	< 0	< 0	d.i.
$\partial M_{g,p}/\partial y$	> 0	> 0	$= 0$	< 0	d.i.	$= 0$	> 0	> 0	> 0	> 0	$= 0$	< 0	< 0	< 0	< 0	d.i.
$\partial M_{t,p}/\partial y$	$= 0$	$= 0$	$= 0$	$= 0$	$= 0$	> 0	$= 0$	> 0	< 0	$= 0$	$= 0$	$= 0$	> 0	< 0	> 0	d.i.
Assumption 2	✓	✓	×	✓	×	✓	✓	✓	✓	✓	×	✓	✓	✓	✓	×
Assumption 3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×
Ordering of community characteristics in equilibrium																
g^{poor} vs. g^{rich}	$>$	$>$	$=$	$<$	-	$>$	$>$	$\gtreqless$	$\gtreqless$	$>$	$=$	$<$	$\gtreqless$	$\gtreqless$	-	-
t^{poor} vs. t^{rich}	$\gtreqless$	$\gtreqless$	$=$	$\gtreqless$	-	$>$	$\gtreqless$	$\gtreqless$	$\gtreqless$	$\gtreqless$	$=$	$\gtreqless$	$\gtreqless$	$\gtreqless$	-	-
p^{poor} vs. p^{rich}	$\gtreqless$	$\gtreqless$	$=$	$\gtreqless$	-	$\gtreqless$	$\gtreqless$	$\gtreqless$	$\gtreqless$	$\gtreqless$	$=$	$\gtreqless$	$\gtreqless$	$\gtreqless$	-	-
$p^{poor}_{a.t.}$ vs. $p^{rich}_{a.t.}$	$>$	$>$	$=$	$<$	-											

Notes: "n.r." indicates that no restrictions on the model parameter are made. "d.i." denotes that the sign depends on income, "✓" that an assumption is satisfied and "×" that an assumption is not satisfied. "$\gtreqless$" means that the community characteristics differ but that their order is not determined by the restrictions made, "-" means that income segregation cannot be established. $g, t, p^{poor/rich}$ is the equilibrium public good provision, tax rate and housing price in the community with poorer/richer households. $p_{a.t.}$ stands for the net off property tax price of housing.

Appendix A

This appendix shows the role of Assumption 3 in the proof of Proposition 2.

Consider three households with given income levels $y' < \hat{y} < y''$. The utility these households achieve in some community j with characteristics (t_j, p_j, g_j) is denoted by $V_j(y')$, $V_j(\hat{y})$ and $V_j(y'')$. The indifference surfaces associated with their achieved utilities in community 1 are implicitly defined as

$$\begin{aligned} f(t,p,g;y') &= V(t,p,g,y') - V_j(y') = 0\,, \\ f(t,p,g;\hat{y}) &= V(t,p,g,\hat{y}) - V_j(\hat{y}) = 0\,, \\ f(t,p,g;y'') &= V(t,p,g,y'') - V_j(y'') = 0\,. \end{aligned}$$

Note that $F : \Gamma \rightarrow \mathbb{R}^3$ defined by

$$F(t,p,g) = \begin{pmatrix} f(t,p,g;y') \\ f(t,p,g;\hat{y}) \\ f(t,p,g;y'') \end{pmatrix}$$

reaches $(0,0,0)'$ for $(t,p,g) = (t_j, p_j, g_j)$. Also note that F twice continuously differentiable in all its arguments since this was assumed for the indirect utility function. Given these definitions Assumption 3 can be stated in an alternative formulation.

Assumption 3 (Proportional shift, alternative formulation)

The Jacobian of F

$$DF(t,p,g) = \begin{bmatrix} V_t(t,p,g,y') & V_p(t,p,g,y') & V_g(t,p,g,y') \\ V_t(t,p,g,\hat{y}) & V_p(t,p,g,\hat{y}) & V_g(t,p,g,\hat{y}) \\ V_t(t,p,g,y'') & V_p(t,p,g,y'') & V_g(t,p,g,y'') \end{bmatrix}$$

is of rank 2 for any $(t,p,g) \in \Gamma$ and any triple of income levels $y' < \hat{y} < y'' \in \mathbb{R}^+$.

Assumption 3 states that - in a neighborhood around (t_j, p_j, g_j) and independent of the income levels - all three indifference surfaces *intersect in a common one-dimensional curve.* Note that they would intersect in single point in the case of full rank. As Assumption 3 holds for any (t,p,g) and any income levels the above argument extends to the whole interior of Γ.

The link of the above assumption to the formulation in Section 2.1 is discussed in the following. Note that the Jacobian cannot have rank 1 as the indifference surfaces do not coincide due to Assumption 2. Hence $Det(DF) = 0$ is a sufficient and necessary condition for Assumption 3 to hold.

Firstly, $Det(DF) = 0$ if two columns are proportional, i.e. either $V_t(t,p,g,y)/V_p(t,p,g,y)$ or $V_t(t,p,g,y)/V_g(t,p,g,y)$ or $V_g(t,p,g,y)/V_p(t,p,g,y)$ is independent of y. This is equivalent to the formulation of Assumption 3a in Section 2.1.

Secondly, $Det(DF) = 0$ if one column can be expressed as a linear combination of the other two. For this to hold, the equation system in $\lambda_1, \lambda_2 \in \mathbb{R}$

$$\begin{array}{rcccc} V_p(t,p,g,y') & = & \lambda_1 V_g(t,p,g,y') & + & \lambda_2 V_t(t,p,g,y') \\ V_p(t,p,g,\hat{y}) & = & \lambda_1 V_g(t,p,g,\hat{y}) & + & \lambda_2 V_t(t,p,g,\hat{y}) \\ V_p(t,p,g,y'') & = & \lambda_1 V_g(t,p,g,y'') & + & \lambda_2 V_t(t,p,g,y'') \end{array}$$

must have a unique solution. Solving the first two rows yields:

$$\lambda_1 = \frac{V_p(\hat{y})/V_t(\hat{y}) - V_p(y')/V_t(y')}{V_g(\hat{y})/V_t(\hat{y}) - V_g(y')/V_t(y')} = \frac{M_{t,p}(\hat{y}) - M_{t,p}(y')}{M_{t,g}(\hat{y}) - M_{t,g}(y')} \cong \frac{\partial M_{t,p}/\partial y}{\partial M_{t,g}/\partial y},$$

$$\lambda_2 = \frac{V_p(\hat{y})/V_g(\hat{y}) - V_p(y')/V_g(y')}{V_t(\hat{y})/V_g(\hat{y}) - V_t(y')/V_g(y')} = \frac{M_{g,p}(\hat{y}) - M_{g,p}(y')}{M_{g,t}(\hat{y}) - M_{g,t}(y')} \cong \frac{\partial M_{g,p}/\partial y}{\partial M_{g,t}/\partial y},$$

λ_1 and λ_2 will also solve the third row if they are independent of the income levels $\hat{y}$ and y'. This is equivalent to the formulation of Assumption 3b in Section 2.1.

Appendix B

This Appendix shows the calculations for the specified mixed CES/Stone-Geary model in Section 3. Case numbers [.] refer to Table 1.

The utility function of a household is given by

$$U(h, b, g) = [\gamma g^\rho + (1 - \gamma) w(h, b)^\rho]^{1/\rho}$$

and

$$w(h, b) = (h - \beta_h)^\alpha (b - \beta_b)^{1-\alpha} ,$$

where $\alpha, \gamma \in [0, 1]$ and $\rho \in [-\infty, 1]$. $w(h, b)$ is the subutility from housing and the private good g. This specification assumes that the preferences are weakly separable between the public good and the other two goods (h, b). The subutility of h and b is of the Stone-Geary form (see for a discussion e.g. Deaton and Muellbauer 1980). Although the parameters β_h and β_b can be interpreted as subsistence quantities they need not be positive. For $\beta_h = \beta_b = 0$ the Stone-Geary utility function reduces to a Cobb-Douglas function.

w can be interpreted as quantity index of non-public goods. The preference for the public good and the non-public goods takes the CES form. $\sigma = 1/(1 - \rho)$ is the elasticity of substitution between public and non-public goods. g and w are called substitutes when $\sigma > 1$ and complements for $\sigma < 1$. The CES function contains the Cobb-Douglas function ($\rho = 0, \sigma = 1$), perfect complementarity ($\rho = -\infty, \sigma = 0$) and perfect substitutability ($\rho = 1, \sigma = \infty$) as special cases.

Property Tax

In the case of a linear property tax, the housing demand is

$$h^*(t, p, y) = \frac{\alpha[y - p(1 + t)\beta_h - \beta_b]}{p(1 + t)} + \beta_h = \frac{\alpha(y - y_s)}{p(1 + t)} + \beta_h ,$$

where $y_s = p(1 + t)\beta_h - \beta_b$ are the minimal expenditures to reach the subsistence level. Note that the housing demand is independent of g because of the weak separability.

The marginal rates of substitution between the community characteristics are

$$M_{g,t} = \frac{[\alpha^\alpha (1 - \alpha)^{(1-\alpha)} p^{-\alpha} (1 + t)^{-\alpha} (y - y_s)]^\rho g^{1-\rho} (1 - \gamma) p h^*}{\gamma (y - y_s)} ,$$

$$M_{g,p} = \frac{[\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}(1+t)^{-\alpha}(y-y_s)]^\rho g^{1-\rho}(1-\gamma)(1+t)h^*}{\gamma(y-y_s)},$$

$$M_{t,p} = -\frac{1+t}{p},$$

where $h^* = h^*(t,p,y)$. Note that Property 1 is satisfied as long as the income is above subsistence level.

The segregation conditions of Assumption 2 are:

$$\frac{\partial M_{g,t}}{\partial y} = \frac{[\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}(1+t)^{-\alpha}(y-y_s)]^\rho g^{1-\rho}(1-\gamma)p(\rho h^* - \beta_h)}{\gamma(y-y_s)^2},$$

$$\frac{\partial M_{g,p}}{\partial y} = \frac{[\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}(1+t)^{-\alpha}(y-y_s)]^\rho g^{1-\rho}(1-\gamma)(1+t)(\rho h^* - \beta_h)}{\gamma(y-y_s)^2},$$

$$\frac{\partial M_{t,p}}{\partial y} = 0.$$

The following relationships hold if all households can afford the subsistence level, i.e $y > y_s$ and $h^* \geq \beta_h$:

$$\frac{\partial M_{g,t}}{\partial y}, \frac{\partial M_{g,p}}{\partial y} \begin{cases} > 0 & \text{if} \quad \rho h^* > \beta_h \quad \text{cases [1], [2]} \\ = 0 & \text{if} \quad \rho h^* = \beta_h \quad \text{case [3]} \\ < 0 & \text{if} \quad \rho h^* < \beta_h \quad \text{case [4]} \end{cases}.$$

Note that Assumption 3a is naturally satisfied in the property tax model since

$$\frac{\partial M_{t,p}}{\partial y} = 0.$$

Income Tax

In the case of a linear income tax, the housing demand is

$$h^*(t,p,y) = \frac{\alpha[y(1-t) - p\beta_h - \beta_b]}{p} + \beta_h = \frac{\alpha[y(1-t) - y_s]}{p} + \beta_h,$$

where $y_s = p\beta_h - \beta_b$ are the minimal expenditures to reach the subsistence level. Note that the housing demand is independent of g because of the assumed weak separability.

The income elasticity of housing is

$$\varepsilon(t,p,y) = \frac{\partial h^*}{\partial y}\frac{y}{h^*} = \frac{\alpha y(1+t)}{\alpha[y(1+t) - p\beta_h - \beta_b] + p\beta_h}.$$

Assuming that all households can afford more than the minimum required quantities, e.g. $y(1-t) > y_s$ for all y, it follows

$$\varepsilon \lesseqqgtr 1 \quad \text{iif} \quad (1-\alpha)p\beta_h - \alpha\beta_b \gtreqqless 0\,.$$

The marginal rates of substitution between the community characteristics are

$$M_{g,t} = \frac{\{\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}[y(1-t)-y_s]\}^\rho g^{1-\rho}(1-\gamma)y}{\gamma[y(1-t)-y_s]}\,,$$

$$M_{g,p} = \frac{\{\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}[y(1-t)-y_s]\}^\rho g^{1-\rho}(1-\gamma)h^*}{\gamma[y(1-t)-y_s]}\,,$$

$$M_{t,p} = -\frac{h^*}{y}$$

and satisfy Property 1 as long as income is above subsistence level.

The segregation conditions of Assumption 2 are:

$$\frac{\partial M_{g,t}}{\partial y} = \frac{\{\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}[y(1-t)-y_s]\}^\rho g^{1-\rho}(1-\gamma)[\rho y(1-t)-y_s]}{\gamma[y(1-t)-y_s]^2}\,,$$

$$\frac{\partial M_{g,p}}{\partial y} = \frac{\{\alpha^\alpha(1-\alpha)^{(1-\alpha)}p^{-\alpha}[y(1-t)-y_s]\}^\rho g^{1-\rho}(1-\gamma)(1-t)(\rho h^*-\beta_h)}{\gamma[y(1-t)-y_s]^2}\,,$$

$$\frac{\partial M_{t,p}}{\partial y} = \frac{(1-\alpha)p\beta_h - \alpha\beta_b}{py^2}\,.$$

The following relationships hold if all households can afford the subsistence level, i.e $y(1-t) > y_s$ and $h^* \geq \beta_h$:

$$\frac{\partial M_{g,t}}{\partial y}\begin{cases} >0 & \text{if} \quad \rho y(1-t) > p\beta_h + \beta_b \quad \text{cases [6]-[10]} \\ =0 & \text{if} \quad \rho y(1-t) = p\beta_h + \beta_b \quad \text{case [11]} \\ <0 & \text{if} \quad \rho y(1-t) < p\beta_h + \beta_b \quad \text{cases [12]-[15]} \end{cases}\,,$$

$$\frac{\partial M_{g,p}}{\partial y}\begin{cases} >0 & \text{if} \quad \rho h^* > \beta_h \quad \text{cases [7]-[10]} \\ =0 & \text{if} \quad \rho h^* = \beta_h \quad \text{cases [6],[11]} \\ <0 & \text{if} \quad \rho h^* < \beta_h \quad \text{cases [12]-[15]} \end{cases}\,,$$

$$\frac{\partial M_{t,p}}{\partial y}\begin{cases} >0 & \text{if} \quad \varepsilon < 1 \quad \text{cases [6],[8],[13],[15]} \\ =0 & \text{if} \quad \varepsilon = 1 \quad \text{cases [7],[10]-[12]} \\ <0 & \text{if} \quad \varepsilon > 1 \quad \text{cases [9],[14]} \end{cases}\,.$$

Case [6] is directly derived from section 3.3.

Neither Assumption 3a nor 3b is generally satisfied since

$$\frac{\partial M_{g,p}}{\partial y} \Big/ \frac{\partial M_{g,t}}{\partial y} = \frac{(1-t)(\rho h^* - \beta_h)}{\rho y(1-t) - y_s}$$

and

$$\frac{\partial M_{t,p}}{\partial y} \Big/ \frac{\partial M_{t,g}}{\partial y} = \frac{\{\alpha^\alpha (1-\alpha)^{(1-\alpha)} p^{-\alpha} [y(1-t) - y_s]\}^\rho g^{1-\rho} (1-\gamma)[(1-\alpha)p\beta_h - \alpha\beta_b]}{\gamma p[\rho y(1-t) - y_s]}$$

depend on income.

Assumption 3a is satisfied if $\varepsilon = 1$ (cases [7], [10], [11], [12]) since

$$\frac{\partial M_{t,p}}{\partial y} = 0\,,$$

and if $\rho = 1$ and $\alpha = 0$ (thus $\sigma \to \infty$ and $\varepsilon = 0$, case [6]) since

$$\frac{\partial M_{g,p}}{\partial y} = 0\,.$$

Assumption 3b is satisfied if $\rho = 0$ ($\sigma = 1$, cases [13], [14]) since

$$\frac{\partial M_{g,p}}{\partial y} \Big/ \frac{\partial M_{g,t}}{\partial y} = \frac{(1-t)\beta_h}{p\beta_h + \beta_b}\,,$$

$$\frac{\partial M_{t,p}}{\partial y} \Big/ \frac{\partial M_{t,g}}{\partial y} = \frac{[(1-\alpha)p\beta_h - \alpha\beta_b](1-\gamma)g}{p\gamma(p\beta_h + \beta_b)}$$

and if $\rho = 1$ ($\sigma \to \infty$, cases [8], [9]) since

$$\frac{\partial M_{g,p}}{\partial y} \Big/ \frac{\partial M_{g,t}}{\partial y} = \frac{(1-t)\alpha}{p}\,,$$

$$\frac{\partial M_{t,p}}{\partial y} \Big/ \frac{\partial M_{t,g}}{\partial y} = -\frac{[(1-\alpha)p\beta_h - \alpha\beta_b]\alpha^\alpha (1-\alpha)^{(1-\alpha)}}{p^{(1+\alpha)}}\,.$$

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