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Horizontally Differentiated Market Makers

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Horizontally Differentiated Market Makers

Simon Loertscher*

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Abstract

I present a model of competition between two market makers who are horizontally differentiated. I first show that absent a search market for buyers and sellers, there is a continuum of symmetric equilibria. These equilibria are payoff equivalent for market makers, but affect buyers' and sellers' welfare in opposite ways. Second, I analyze the model when buyers and sellers can also exchange the good in search markets. The model with search markets shares many features with existing models, yet allows competing intermediaries to net a profit in equilibrium. Interestingly, the model exhibits a complementarity between intermediaries' profits in the presence of search markets. Third, I show that every equilibrium in a game with market makers is also an equilibrium in an appropriately defined game with matchmakers.

Keywords: Market making, intermediation and search, horizontal differentiation, market microstructure.
JEL-Classification: C72, D41, D43, L13.

1 Introduction

I analyze imperfect competition between market makers who are horizontally differentiated. I study both the cases where they face competition from search markets and where they do not. To the best of my knowledge, this is the first paper to address horizontal differentiation between market makers in the presence of search markets. Spulber (1999, p.68-71) analyzes Bertrand competition between market making intermediaries that are horizontally differentiated. He shows that market makers net a profit in equilibrium if they are differentiated,

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but he does not analyze the case when intermediaries face competition from search markets. Moreover, his analysis is stripped down in that it takes as given the demand and supply functions facing the intermediaries, rather than deriving them explicitly from a model like Hotelling (1929)'s or Salop (1979)'s.

The following definition of a market maker is taken from the U.S. Securities and Exchange Commission (SEC):[1]

> A "market maker" is a firm that stands ready to buy and sell a particular stock on a regular and continuous basis at a publicly quoted price.[2]

This definition of market maker is narrow insofar as it is strictly confined to a dealer in stock exchanges. In a broader sense, market makers encompass all firms that buy and sell goods at *publicly observable* bid and ask prices in order to make profits.[3] According to this broader definition, intermediaries in job and housing markets and travel agents are considered as market makers, too.[4] Moreover, since retailers and supermarkets intermediate between producers and consumers, they act as market makers as well. Though I assume that intermediation is costless, it is straightforward to extend the model to account for costly production when the intermediaries use a one-to-one technology that transforms one unit of input into one unit of output. So, all firms that set public prices both on the input and on the output market can be subsumed as market makers. In particular, merchants typically are market makers.

For the case of homogenous goods, Bertrand competition between market makers has been analyzed by Stahl (1988), Gehrig (1993), Fingleton (1997), Spulber (1999) and Rust and Hall (2003).[5] Ju et al. (2004) and Loertscher

[1] http://www.sec.gov/answers/mktmaker.htm.

[2] The quote goes on: "Many OTC stocks have more than one market-maker. You'll most often hear about market makers in the context of the Nasdaq or other "over the counter" (OTC) markets. Market makers that stand ready to buy and sell stocks listed on an exchange, such as the New York Stock Exchange, are called "third market makers". Market-makers generally must be ready to buy and sell at least 100 shares of a stock they make a market in. As a result, a large order from an investor may have to be filled by a number of market-makers at potentially different prices."

[3] The requirement for publicity of prices is what distinguishes market makers from other intermediaries who are sometimes called middlemen and who trade at prices that are only privately communicated, e.g., after a buyer or seller has contacted the middleman by phone.

[4] Job and housing market intermediaries typically do not buy the goods. Thus, they are more like matchmakers (or platforms) who facilitate or enable matches between a seller and a buyer, whereas travel agents buy travels and therefore stand ready to sell; see section 6 for a discussion of the similarities and differences of market makers and matchmakers.

[5] Spulber (1996) develops a model in which price setting firms intermediate between buyers and sellers. However, in his model prices are not public information. Instead, a buyer or a seller learns a firm's price only if matched to that firm in the search market. Adhering to

(2005) analyze price competition between market makers when goods are homogeneous and market makers set capacity constraints prior to competing in prices (see also Kremer and Polkovnichenko, 2000).[6] Related, yet somewhat orthogonal to this literature is the paper by Neeman and Vulkan (2003) that studies the conditions under which centralized markets, whose microstructure is not modelled, drive out trade based on bilateral negotiations.

Another strand of literature to which my paper relates is the recent literature on two-sided markets and platform competition (see, e.g., Caillaud and Jullien, 2001, 2003; Rochet and Tirole, 2002, 2004). A platform like a matchmaker for men and women, is said to be two-sided because the agents on one side of the platform (say, men) care about the number and nature of agents on the other side (in this case, women), giving rise to coordination problems and multiple equilibria. Interestingly, the two-sided nature in this paper does not pertain to the intermediated markets, but to the search markets. This is so because I adhere to the definition of the SEC by assuming that market makers are committed ("stand ready") to buy at the bid prices they quote. Consequently, sellers are not concerned about the number of buyers attracted by a market maker, whereas those sellers who consider participating in a search market certainly care about the number of buyers participating in the search market. Nonetheless, I also show that the analysis extends easily to platforms insofar as every equilibrium in the game with market makers is also an equilibrium in an appropriately defined game with platforms or matchmakers.

The industrial organization literature offers two interpretations for horizontal differentiation, both of which are relevant for the present paper. According to the first, firms (or products) are geographically differentiated. For example, consider a job market intermediary. Clearly, workers in a given city will, all else equal, prefer the intermediary who has job offers that come from that city. Similarly, a firm that seeks to employ someone via an intermediary may prefer the intermediary whose workers come from the same region to those who commute a longer distance since these workers will be more flexible and eventually better

the terminology of Rust and Hall (2003), Spulber's model is therefore a model of middlemen rather than market makers.

[6] The paper by Ju et al. is primarily an empirical investigation of the effect of the collapse of a market maker (Enron) on price dispersion in a search market. My paper is purely theoretical and deals with the mixed strategies Ju et al. do no consider. Similarly, Kremer and Polkovnichenko (2000) analyze price competition between capacity constrained intermediaries who trade a homogenous good. In their setting, though, demand and supply intermediaries face are independent in the following way: The equilibrium price intermediaries get on the output market depends only on their sales capacity or stock, but not on the quantity they buy. Consequently, their ask and bid windows are basically independent operations.

motivated due to the shorter commute. According to the second interpretation, the differentiation is in a characteristics space. Consumers and producers can, e.g., choose between trading with a retailer specialized in organic food and another one specialized in genetically manipulated food. In all likelihood, some consumers and producers will prefer trading with the former, while others will prefer buying from or selling to the latter.

The remainder of the paper is structured as follows. In section 2, I develop the basic model. In section 3, I analyze the model when there are two market makers but no search markets. Search markets are introduced in section 4, where I first derive equilibrium when there are no market makers and then derive equilibrium when both market makers and search markets are active. In section 5, I analyze asymmetric market structures and show in particular that there is a complementarity between intermediaries' profits. Section 6 shows that every equilibrium in the game with a market makers is also an equilibrium in a corresponding game when the intermediaries are platforms or matchmakers. Section 7 concludes. The Appendix contains a somewhat cumbersome derivation, a longer proof and two generalizations. The first generalization extends the model to parameter ranges that are excluded from the main part of the paper. The second one analyzes the model when search markets are not fully efficient.

2 The Model

The following is a natural adaptation of the models of Hotelling (1929) and Salop (1979) to market making intermediaries. I assume that there is a continuum of buyers and a continuum of sellers, each with measure $\frac{1}{2}$. Buyers and sellers are uniformly distributed along the North and South semi-circles of a circle with circumference 1, respectively. Each buyer has a gross valuation for the good v, and he either buys one unit of the good or none. Each seller has production costs of zero, and he either produces one unit or none. Both sellers and buyers bear a constant cost $t > 0$ per unit of distance they have to travel with the good. I assume that v is so large in comparison to t that in any equilibrium all buyers consume, i.e., I assume

$$\frac{v}{t} > \frac{3}{2}.$$

This condition guarantees that competition between intermediaries will be so tough that equilibrium profits of the intermediaries are independent of v. It is the same as the condition that guarantees full market coverage in equilibrium in the standard Hotelling model.[7] Because of this assumption, no buyers and sellers will be inactive in equilibrium, which contrasts with the models of Gehrig (1993) and Rust and Hall (2003).

There are two market places, one located at the Westernmost point of the circle and the other one at the Easternmost, as illustrated in Figure 1. The former is labelled W and the latter E. As an accounting convenience, I let the locations of buyers and sellers increase from 0 to $\frac{1}{2}$ from West to East. That is, the buyer and seller with location 0 is situated in W, and the buyer and seller with location $\frac{1}{4}$ are at the North and South pole of the circle. Accordingly, the location of the buyer and seller in E is $\frac{1}{2}$. Each market place can host a market making intermediary or a search market or both. An intermediary in k with $k = E, W$ sets a pair of ask and bid prices (a^k, b^k) at which it is willing to sell and buy. An intermediary is obliged to buy any quantity supplied at the bid price it sets.[8] The quantity it sells is the minimum of the quantity buyers demand at its ask price and the quantity supplied.

Motivation and Discussion The assumption that search markets and market makers occupy the same locations maintains a property of models with homogenous market makers. Under this assumption, the buyers and sellers who have the most to gain from search market participation (i.e., the high valuation buyers and the low cost sellers) also have the most to gain from trading with intermediaries.[9] Modelling the economy in the present manner amounts to locating all buyers and sellers on separate Hotelling lines, and connecting these at two points. It is straightforward for two market makers (and two search markets), but imposes an odd asymmetry for three and more market makers. Alternatively, one could allocate buyers and sellers uniformly along the full circle and assume additionally that they can exchange the good only at "official" markets, i.e., at search or intermediated markets. If search markets were located at places different from intermediated markets, some buyers and sellers would be closer to search markets and others would be closer to intermediated markets. Allocating buyers and sellers along the full circle and requiring search

[7] The cases with $\frac{v}{t} < \frac{3}{2}$ are treated in Appendix A.

[8] On the other hand, an intermediary is not obliged to serve all buyers. If quantity demanded exceeds its stock, some buyers will be rationed.

[9] See Gehrig (1993), Fingleton (1997) and Rust and Hall (2003). This contrasts with Yavas (1994), where low valuation buyers and high cost sellers join the intermediary. In Yavas' model, an intermediary does not set ask and bid prices but charges a percentage of the surplus generated by a match, which enables the intermediary to price discriminate. Consequently, those buyers and sellers who expect to generate a high surplus gain less by joining the intermediary.

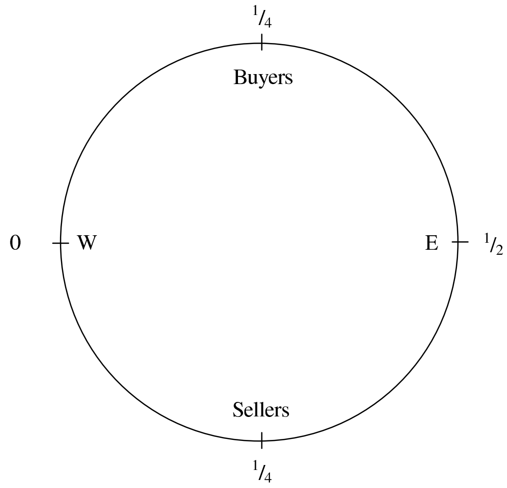

Figure 1: Buyers and sellers uniformly distributed along a circle.

and intermediated markets to be at the same locations (and to be located symmetrically along the circle) would allow to study the entry and exit decisions of market makers. Though this is certainly an interesting line of research, I leave it for future work.

The present specification also maintains the linearity of the demand and supply functions intermediaries face in equilibrium. It is most appropriate when the horizontal differentiation is geographical. For example, suppose that E and W are two cities, each of which has its own regional newspaper, and consider intermediaries in labor markets. A firm that seeks an employee can either place an ad in one of the newspapers, whereby it participates in the search market in this city. Alternatively, it can contact an intermediary in this city. Consequently, search markets and intermediated markets have the same locations.

I first describe the model without search markets, where each market place is host to one intermediary. The details of the organization of the search market will be outlined when the interactions of search and intermediated markets are analyzed.

3 Equilibrium without Search Markets

In this section, I first derive the equilibrium when there are no search markets and when buyers and sellers care equally about the degree of horizontal differentiation (i.e., have the same $t > 0$). Second, I look at the case where only agents on one side (sellers or buyers) care about horizontal differentiation.

3.1 Equilibrium Analysis

Let $\tilde{x}$ be the location of the buyer who at ask prices a^W and a^E is indifferent between buying from the intermediary in W and the intermediary in E.[10] Denote by $U_c^W(x)$ and $U_c^E(x)$ the net utility of a buyer (or consumer) located at x when buying from the intermediary in the West and East, respectively. Then, buyer $\tilde{x}$ is indifferent between the two intermediaries if and only if

$$\begin{aligned} U_c^W(\tilde{x}) = v - t\tilde{x} - a^W &= v - t\left(\frac{1}{2} - \tilde{x}\right) - a^E = U_c^E(\tilde{x}) \\ &\Leftrightarrow \\ \tilde{x}(a^W, a^E) &= \frac{1}{4} + \frac{a^E - a^W}{2t}, \end{aligned} \tag{1}$$

where $\tilde{x}(a^W, a^E)$ is the demand the intermediary in W faces. Analogously, the demand the intermediary in E faces consists of the remaining buyers, i.e., $\frac{1}{2} - \tilde{x}(a^W, a^E)$.

Similarly, let $U_p^W(y)$ and $U_p^E(y)$ be the utility of a seller (or producer) at location y when selling to the intermediary in W and E, respectively. Given bid prices b^W and b^E, the seller with location $\tilde{y}$ is indifferent between selling to the intermediary in W and the one in E when

$$\begin{aligned} U^W(\tilde{y}) = b^W - t\tilde{y} &= b^E - t\left(\frac{1}{2} - \tilde{y}\right) = U^E(\tilde{y}) \\ &\Leftrightarrow \\ \tilde{y}(b^W, b^E) &= \frac{1}{4} + \frac{b^W - b^E}{2t}. \end{aligned} \tag{2}$$

That is, $\tilde{y}(b^W, b^E)$ is the quantity supplied to the intermediary in W and $\frac{1}{2} - \tilde{y}(b^W, b^E)$ is the quantity supplied to the one in E.

I assume that intermediation is costless. Then, the profit of the intermediary in W is given by

$$\Pi^W(a^W, b^W; a^E, b^E) = a^W \min\left\{\tilde{x}(a^W, a^E), \tilde{y}(b^W, b^E)\right\} - b^W \tilde{y}(b^W, b^E), \tag{3}$$

because the quantity sold is the minimum of quantity demanded $\tilde{x}$ and quantity suppliers sell $\tilde{y}$. The objective of the intermediary is to maximize Π^W over a^W and b^W. Clearly, it cannot be in the interest of an intermediary to buy more

[10] I assume that in case a buyer is rationed by an intermediary he can join the other intermediary at no cost in excess of the cost he would have incurred had he joined the other intermediary at the outset. This amounts to assuming that the travel cost are incurred only if the good is actually bought.

than it sells. Therefore, $\tilde{x} = \tilde{y} \equiv q^W$ will hold, where q^W denotes the quantity traded by the intermediary in W.[11]

Replacing $\tilde{x}$ and $\tilde{y}$ by q^W in equations (1) and (2) and solving for a^W and b^W yields the inverse demand and supply functions the intermediary in W faces. These are functions of its quantity traded and the prices set by the intermediary in E. These functions are given, respectively, by

$$A^W(q^W, a^E) = a^E + \frac{t}{2} - 2tq^W \tag{4}$$

and

$$B^W(q^W, b^E) = b^E - \frac{t}{2} + 2tq^W. \tag{5}$$

The profit of the intermediary in W can now be written as a function of the quantity q^W it trades, i.e.,

$$\Pi^W(q^W, a^E, b^E) = \left(A^W(q^W, a^E) - B^W(q^W, b^E)\right) q^W = \left(a^E - b^E + t - 4tq^W\right) q^W. \tag{6}$$

Maximizing (3) over (a^W, b^W) subject to $\tilde{x} = \tilde{y}$ is thus equivalent to maximizing (6) over q^W, which yields as a first order condition

$$0 = a^E - b^E + t - 8tq^W. \tag{7}$$

Notice that q^W only depends on the spread $(a^E - b^E)$ set by the competitor in E.

Everything being symmetric, the optimal quantity traded by the intermediary in E, denoted as q^E, given prices a^W and b^W will be given as

$$0 = a^W - b^W + t - 8tq^E, \tag{8}$$

which in turn only depends on the spread $(a^W - b^W)$ set by the intermediary in W.

Equilibrium requires

$$a^k = A^k(q^k, a^{-k})$$

and

$$b^k = B^k(q^k, b^{-k})$$

[11] Assume the intermediary in W chooses prices such that $\tilde{x} > \tilde{y}$ (which implies $\frac{1}{2} - \tilde{x} < \frac{1}{2} - \tilde{y}$ so that buyers will be served with certainty in E), and denote by $\alpha \in [0,1]$ the probability that a buyer is served by the intermediary in W. Then, because of the assumption that a rationed buyer can still join the other intermediary buyer, x' is indifferent between going to the intermediary in W and in E if and only if $\alpha(v - tx' - a^W) + (1-\alpha)(v - t\left(\frac{1}{2} - x'\right) = v - t\left(\frac{1}{2} - x'\right)$. Solving for x' reveals that $x' = \tilde{x}$. Thus, the number of buyers joining the intermediary in W is not affected by rationing some buyers. On the other hand, $\tilde{y} > \tilde{x}$ is not profitable because buying excess stock is costly. Hence, it follows that inducing rationing is not profitable for an intermediary.

for $k = E, W$, where $-k$ means not k. Taken together, the equilibrium conditions for the intermediary in W read

$$\begin{aligned} 0 &= a^E - b^E + t - 8tq^W \quad (9) \\ a^W &= a^E + \frac{t}{2} - 2tq^W \quad (10) \\ b^W &= b^E - \frac{t}{2} + 2tq^W. \quad (11) \end{aligned}$$

Subtracting (11) from (10) yields

$$a^W - b^W = a^E - b^E + t - 4tq^W. \quad (12)$$

By symmetry, $a^W - b^W = a^E - b^E$ will hold in any equilibrium, so that

$$q^{W*} = \frac{1}{4}$$

follows from (12). Plugging $q^W = \frac{1}{4}$ into equation (9) reveals then that the equilibrium spread $z^* \equiv a^W - b^W = a^E - b^E$ is given as

$$z^* = t.$$

Notice that this spread is the same as the mark-up in the standard Bertrand model of product differentiation.

By completely symmetric reasoning,

$$q^{E*} = \frac{1}{4}$$

can be established.

Notice that though the equilibrium spread is determined, equilibrium prices are not. When the intermediary in W sets a high bid price b and a high ask price a, then it is also optimal for the intermediary in E to set high prices. On the other hand, if the opponent sets low prices, then it is best to set low prices as well. Therefore, there will be a multiplicity of equilibria, some of which are better for buyers, and others that are better for sellers.

Of course, there are some boundary conditions equilibrium prices have to satisfy, which are that buyers and sellers get nonnegative utility when buying from or selling to an intermediary. The seller who has to travel the largest distance is located $\frac{1}{4}$ away from the intermediary to whom he sells. Thus, he incurs a travel cost of $\frac{1}{4}t$. Consequently, his net utility is nonnegative if and only if

$$b - \frac{1}{4}t \geq 0.$$

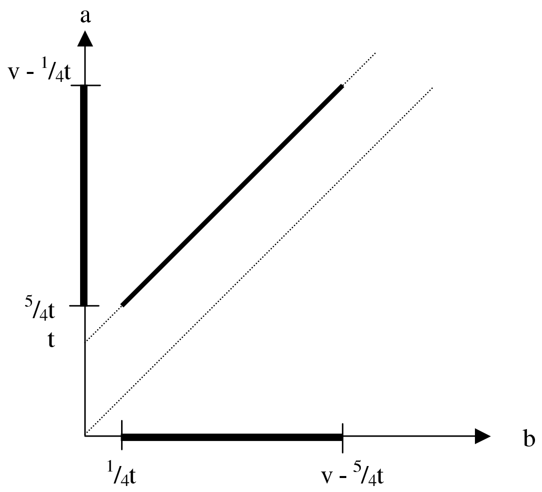

Figure 2: The Set of Equilibrium Ask and Bid Prices.

Since the equilibrium spread is t, a lower bound for equilibrium prices is $a = \frac{5}{4}t$ and $b = \frac{1}{4}t$. On the other hand, for the buyer with the largest travel cost to get a nonnegative utility, the equilibrium ask price a must be such that

$$v - \frac{1}{4}t - a \geq 0.$$

Thus, equilibrium ask prices must be weakly smaller than $v - \frac{1}{4}t$, so that the upper bounds for equilibrium prices is $a = v - \frac{1}{4}t$ and $b = v - \frac{5}{4}t$. Moreover, the highest ask and bid prices must be at least as large as the lowest ones, i.e., $v - \frac{1}{4}t \geq \frac{5}{4}t$ and $v - \frac{5}{4}t \geq \frac{1}{4}t$ must hold. Since

$$v > \frac{3}{2}t$$

is assumed, this requirement is met. In summary, therefore, all ask and bid prices

$$a^* \in \left[\frac{5}{4}t, v - \frac{1}{4}t\right] \quad \text{and} \quad b^* \in \left[\frac{1}{4}t, v - \frac{5}{4}t\right] \tag{13}$$

with $a^* = b^* + t$ are consistent with equilibrium. The set of equilibria is illustrated in Figure 2, and the results I have just derived are summarized in the following proposition.

Proposition 1 *There is a continuum of equilibria in the market maker game without search markets. In every equilibrium,*

- *both intermediaries set the same ask price (i.e., $a^W = a^E$) and the same bid price ($b^W = b^E$)*

- *buyers and sellers with $x \leq \frac{1}{4}$ and $y \leq \frac{1}{4}$ join the intermediary in W and the other ones join the intermediary in E*
- *and the spread is*
$$z^* \equiv a^* - b^* = t,$$

where a^* and b^* are given by (13). Each market maker nets a profit of

$$\Pi^* = \frac{1}{4}t.$$

The multiplicity of equilibria has perhaps some interesting implications. First, all equilibria are socially efficient, but they differ with respect to distributional aspects. Second, the multiplicity suggests that high price and high wage countries like, e.g., Switzerland and low price and low wage countries like, say, Spain may represent the play of different equilibria.

3.2 Differentiation on One Side Only

In some instances, it is plausible to argue that the horizontal differentiation pertains to only one side of the intermediation business. For example, geographical location may be more important for workers than for firms because the workers travel to the firm. So as to account for this possibility and keeping the worker-firm example in mind, I assume now that buyers (firms) are indifferent between sellers (workers), so that each buyer gets a net utility $v > 0$ from getting the good from a seller. Sellers are still uniformly distributed along a line of length $\frac{1}{2}$ and bear the unit cost of transportation $t > 0$. Market makers are still located at the East and West end of this line.

As before, I assume that in case a buyer does not get served by one intermediary he can try to get the good from the other intermediary at no additional cost. Note also that because of the homogeneity of buyers, the rationing rule does not matter. Consequently, the market where competition between the two intermediaries will matter is the input market. To see this, note that though an intermediary that sets a slightly lower ask price than its competitor would attract all demand, it cannot satisfy this demand unless it has also attracted all sellers. Since sellers only care about the price they get (net of transportation) and disregard the number of buyers attracted by an intermediary, the essential market is the input market. This gives immediately way to the following lemma.

Lemma 1 *In any equilibrium in which both intermediaries attract some sellers, $a^W = a^E = v$ holds.*

Proof: Setting ask prices greater than v would yield zero sales, which is clearly worse than selling something at a positive price. So, once I have shown that each intermediary can sell all it buys from sellers when setting $a = v$, no matter what ask price the other intermediary sets, the lemma will be proved. Note that aggregate demand at an ask price $a \leq v$ is $\frac{1}{2}$, which is weakly more than the aggregate quantity intermediaries can buy. Let $q < \frac{1}{2}$ (where $q < \frac{1}{2}$ must hold because the lemma only applies to equilibria where both intermediaries attract some sellers) be the number of sellers selling to intermediary $-k$ and assume $-k$ sets an ask price $a^{-k} \leq v$. Residual demand for intermediary k when setting $a^k = v$ is then $\frac{1}{2} - q$, which is weakly more than the number of sellers attracted by k. Thus, given that $-k$ sets an ask price weakly smaller than v, setting $a = v$ is the best response of k. Hence, the lemma is proved.■

Let $\tilde{y} = \frac{1}{4} + \frac{b^W - b^E}{2t}$ be the seller indifferent between intermediary E and W. Taking Lemma 1 into account, intermediary W's profit as a function of b^W and b^E is

$$\Pi^W(b^W, b^E) = (v - b^W)\tilde{y}(b^W, b^E) = (v - b^W)\left(\frac{1}{4} + \frac{b^W - b^E}{2t}\right).$$

Maximizing $\Pi^W(b^W, b^E)$ with respect to b^W yields the reaction function

$$b^{W*}(b^E) = \frac{v}{2} - \frac{1}{4}t + \frac{b^E}{2}.$$

In equilibrium, $b^{E*} = b^{W*} = b^*$ holds, whence

$$b^* = v - \frac{1}{2}t.$$

Because $b^* = v - \frac{1}{2}t$ is the unique point of intersection of $b^{W*}(b^E)$ with the 45-degree line, the equilibrium is unique. Notice that the equilibrium bid price is larger than the highest equilibrium bid price when both sides are heterogenous, which is a reflection of the fact that competition for the essential input good is most intense.

The implied equilibrium spread is $v - b^* = \frac{1}{2}t$, which is smaller than t, the equilibrium spread when both sellers and buyers care about differentiation. Consequently, equilibrium profit is

$$\Pi^* = \frac{1}{8}t,$$

which is half the profit when both sides are heterogenous. Note that for the seller at $\frac{1}{4}$ to get nonnegative utility, $b^* - \frac{1}{4}t = v - \frac{1}{2}t - \frac{1}{4}t > 0$ must hold, requiring

$$v > \frac{3}{4}t,$$

which holds if v/t is sufficiently large. Thus, I have shown:

Proposition 2 *In the unique equilibrium of the game where all sellers and no buyers care about product differentiation, both market makers set*

$$a^* = v \quad \textit{and} \quad b^* = v - \frac{t}{2}$$

and net a profit of

$$\Pi^* = \frac{t}{8}.$$

Homogenous Sellers, Heterogenous Buyers So as to complete the analysis of one-sided differentiation, I now consider the case where buyers but not sellers are differentiated. Specifically, I assume that each seller has a cost of zero and that buyers are distributed uniformly on $[0, \frac{1}{2}]$. In case both intermediaries set the same bid price $b \geq 0$, each gets $\frac{1}{4}$, while if their bid prices differ, the intermediary with the larger bid price gets the whole supply of $\frac{1}{2}$. I assume that $\frac{v}{t} > 1$, which makes sure that an intermediary who happens to buy all the supply wants to sell all that it has bought.[12] Note that the present model can be seen as a model of price competition with differentiated products and capacity constraints, where each intermediary's capacity constraint is $\frac{1}{4}$ if both are active (i.e., if both have attracted some sellers). Because an intermediary need, in general, not set a market clearing price, some buyers may be rationed. Therefore, so as to complete the model, a rationing rule must be specified. The two rationing schemes most frequently used in the literature are the so called proportional (or random) rationing rule and the efficient rationing rule. Under efficient rationing, if the intermediary in W attracts y sellers and x buyers with $y < x$, then the buyers with locations in $[0, y]$ get served by W and those in $(y, x]$ get rationed. Under proportional rationing, each buyer in $[0, x]$ is served with probability $\alpha \equiv \frac{y}{x}$ and rationed with probability $1 - \alpha$.

Equilibrium Ask Prices Independent of the rationing rule, both intermediaries set $a = v - \frac{1}{4}t$ in equilibrium if both have a capacity of $\frac{1}{4}$ (i.e., if both set the same bid price). The result is fairly trivial for efficient rationing because then no buyer who is further away from an intermediary than $\frac{1}{4}$ will ever get served by that intermediary. Consequently, an intermediary cannot gain anything by setting $a < v - \frac{1}{4}t$. On the other hand, prices $a > v - \frac{1}{4}t$ will not be optimal either. The reason for that is that demand is price elastic for all $q \leq \frac{1}{2}$. Thus, an intermediary would rather sell more than $\frac{1}{4}$ at a lower

[12] More precisely, it makes sure that for any admissible number of sellers an intermediary attracts, it will be in its interest to sell everything. A monopoly who sells quantity q earns a revenue of $R(q) = (v - tq)q$, provided $q \leq \frac{1}{2}$. Maximizing $R(q)$ via q yields $q^* = \frac{v}{2t}$, which is greater than $\frac{1}{2}$ for $\frac{v}{t} > 1$. Thus, under the assumption $\frac{v}{t} > 1$ the maximal revenue of an intermediary who has bought the whole supply of $\frac{1}{2}$ is $R^* = (v - \frac{1}{2}t)\frac{1}{2} = \frac{v}{2} - \frac{1}{4}t$.

price than selling less at a higher price. Therefore, the equilibrium ask price will be $a = v - \frac{1}{4}t$ for efficient rationing. With proportional rationing, the argument needed to establish that the unique optimal ask price for equal capacity is $a = v - \frac{1}{4}t$ is slightly more involved and is therefore relegated to Appendix B.

Equilibrium Bid Prices The analysis for the input market in this model is somewhat different from the previous models because bidding for inputs is now homogenous Bertrand competition. Quite clearly, in equilibrium neither intermediary must have an incentive to slightly overbid the competitor's bid price b. Since both intermediaries attract $\frac{1}{4}$ sellers when setting the same bid price, both will set ask prices equal to $v - \frac{1}{4}t$ when setting the same bid price $b \geq 0$. Each intermediary's expenditure will be $\frac{b}{4}$. On the other hand, when slightly overbidding the bid b, an intermediary attracts $\frac{1}{2}$ sellers. In this case, its expenditure is approximately $\frac{b}{2}$, while the revenue will be $\frac{v}{2} - \frac{1}{4}t$ (see the footnote above). Consequently, an equilibrium condition is

$$\left(v - \frac{1}{4}t - b\right)\frac{1}{4} \geq \frac{v}{2} - \frac{1}{4}t - \frac{b}{2}. \tag{14}$$

Solving (14) for b yields

$$b \geq v - \frac{3}{4}t.$$

Clearly, if both intermediaries set $b = v - \frac{3}{4}t$ and $a = v - \frac{1}{4}t$, this is an equilibrium, and each intermediary's equilibrium profit is

$$\Pi^* = \left(v - \frac{1}{4}t - \left(v - \frac{3}{4}t\right)\right)\frac{1}{4} = \frac{1}{8}t.$$

However, note that any larger bid prices will not affect revenue of an intermediary, whether it is the only seller or not. Consequently, any larger bid price will not be overbid either. Therefore, provided the bid price allows nonnegative profits, any larger bid price will be consistent with equilibrium. As ask prices will be $v - \frac{1}{4}t$ if both set the same input price, any bid price

$$b \leq v - \frac{1}{4}t \tag{15}$$

will therefore also be consistent with equilibrium. Thus, there is, again, a continuum of equilibria, where any bid price

$$b \in \left[v - \frac{3}{4}t, v - \frac{1}{4}t\right]$$

is associated with an equilibrium. Note, though, that in contrast to the model where differentiation is two-sided, intermediaries are not indifferent between these equilibria but unambiguously prefer the equilibria with lower bid prices to those with higher bids. Moreover, the zero-profit equilibrium with $b = v - \frac{1}{4}t$ is quite risky insofar as an intermediary who sets this price makes a loss if the other one deviates and sets a lower price. This risk is not small since as no intermediary makes a profit in this equilibrium, either one may just as well deviate and set a lower price, in which case it still makes zero profits. Note also that this risk exists for every $b > v - \frac{1}{2}t$, though the incentives to deviate are much smaller, given that each intermediary can make positive profits if both set $b \in [v - \frac{1}{2}t, v - \frac{1}{4}t)$.

Discussion Though profits are still positive when one side is homogenous and the other one is heterogenous, equilibrium spreads and profits are unambiguously smaller with homogenous sellers or buyers.[13] Nonetheless, the result that both intermediaries can make positive profits when only sellers or buyers are differentiated shows that the zero-profit outcome obtained by Stahl (1988) depends, among other things, quite critically on the homogenous goods assumption. If both sides or if only agents on one side are heterogenous, then intermediaries make positive profits in (almost every) equilibrium.

It is also noteworthy that the result does not hinge on the assumption that intermediaries do not sell forward contracts. To see this, assume that they do so and that the default penalty is severe enough to deter default. Buyers are homogenous, while sellers face transportation cost $t > 0$. In case of price tie(s), the market(s) are shared evenly. It is easy to see that the ask price a will not be underbid by the forward contracting competitor whenever $\left(a - \frac{1}{4}t\right)\frac{1}{4} \geq \left(a - \frac{1}{2}t\right)\frac{1}{2} \Leftrightarrow a \leq \frac{3}{4}t$ holds. Thus, when intermediaries sell forward contracts, there is a symmetric equilibrium with $a^* = \frac{3}{4}t$ and $b^* = \frac{1}{4}t$, implying an equilibrium profit of $\frac{1}{8}t$ for each market maker, just like in the model without forward contracts. Regardless of whether there are forward contracts or not, the equilibrium thus involves positive profits as long as there is differentiation on at least one side. This observation contrasts with, and complements, Stahl (1988), who finds that the equilibrium profits of two forward contracting intermediaries are zero when goods are homogenous.

[13] This observation is consistent with the experience of an anonymous job market intermediary (personnel communication) who says that their profits are smallest in those sectors where the supply of labor is largest.

4 Equilibrium with Search Markets

A number of papers have started to study market microstructures when sellers and buyers have the additional option of meeting bilaterally.[14] This motivates to see how equilibrium behavior is affected if competing, horizontally differentiated intermediaries face competition from search markets. This is what I analyze in this section.

4.1 Assumptions

I now assume that there is a search market in W and one in E. As before, buyers and sellers have a cost per unit of transportation $t > 0$. I assume that in a given search market, buyers and sellers are uniformly randomly matched. If there are, say, more buyers than sellers in a search market, sellers are matched with probability one, while buyers are matched with a probability proportional to the ratio of the number of sellers over the number of buyers. If matched, a buyer and a seller share the gains from trade evenly. Again, transportation costs are incurred only when trade occurs.

The assumptions underlying the organization of search markets are similar to those of Gehrig (1993). A difference is that he assumes take-it-or-leave-it offers, whereas here sellers and buyers share gains from trade evenly. Because here there are two search markets whereas in his model there is but one, the equilibrium outcome will be somewhat different in my model. Another difference to Gehrig's model that has already been mentioned is that there are no inactive buyers and sellers because all possible buyer-seller matches generate positive surplus. It seems also possible to model the search market as a dynamic matching market à la Spulber (1996) and Rust and Hall (2003), where buyers and sellers search for an opportunity to trade with middlemen. As in the homogenous goods model I expect both models to yield very similar inverse demand and supply functions facing intermediaries.

4.2 Equilibrium Without Market Makers

For this subsection, I assume that there are no market makers. Let $\underline{v}_c$ and $\overline{v}_c$ be, respectively, the lowest and highest net valuations of consumers present in a given search market. Similarly, I denote by $\underline{v}_p$ and $\overline{v}_p$, respectively, the lowest and highest net cost of producers present in a given search market. Notice that

[14] See Rubinstein and Wolinksy (1987), Gehrig (1993), Spulber (1996, 1999), Fingleton (1997), Rust and Hall (2003), Ju et al. (2004), and Shevchenko (2004).

the maximal distance a producer travels is no larger than $\frac{1}{2}$. Consequently,

$$\overline{v}_p \leq \frac{1}{2}t$$

will hold. Similarly, no buyer will travel more than $\frac{1}{2}$. Thus, $\underline{v}_c \geq v - \frac{1}{2}t > t$, where the strict inequality follows from the assumption $v > \frac{3}{2}t$. Therefore,

$$\underline{v}_c > t > \overline{v}_p$$

will hold in any search market.[15]

Let $V_c^W(x)$ and $V_c^E(x)$ denote the expected utility of a buyer at location x when participating in the search market in W and E, respectively. Similarly, let $V_p^W(y)$ and $V_p^E(y)$ denote the expected utility of a seller at location y when participating in the search market in W and E.

Lemma 2 *(Single-crossing) In any equilibrium with search market participation in both cities, the following holds: If $V_c^W(x') \gtreqqless V_c^E(x')$ for some x', then $V_c^W(x) \gtrless V_c^E(x)$ for all $x \lesseqgtr x'$. Analogously, if $V_p^W(y') \gtreqqless V_p^E(y')$ for some y', then $V_p^W(y) \gtrless V_p^E(y)$ for all $y \lesseqgtr y'$.*

Proof: Let $F_p^k(y)$ be the non-degenerate equilibrium distribution of sellers in the search market in k, $k = E, W$, and let $\gamma_c^k \leq 1$ be the probability that a buyer is matched in market k. Then,

$$V_c^W(x') = \frac{\gamma_c^W}{2} \int_0^{\frac{1}{2}} [v - tx' - ty] dF_p^W(y) \tag{16}$$

and

$$V_c^E(x') = \frac{\gamma_c^E}{2} \int_0^{\frac{1}{2}} \left[v - t\left(\frac{1}{2} - x'\right) - t\left(\frac{1}{2} - y\right)\right] dF_p^E(y). \tag{17}$$

Note that because of the assumption that there is a continuum of buyers, whether a particular buyer joins market k has no effect on γ_c^k. Therefore, $\frac{\partial V_c^W(x')}{\partial x'} < 0$ and $\frac{\partial V_c^E(x')}{\partial x'} > 0$ follows for any non-degenerate distributions F_p^W and F_p^E. Consequently, if $V_c^W(x') \geq V_c^E(x')$ holds, then $V_c^W(x) > V_c^E(x)$ holds for $x < x'$, and similarly, if $V_c^W(x') \leq V_c^E(x')$ holds, then $V_c^W(x) < V_c^E(x)$ holds for $x > x'$. The proof for sellers is completely symmetric and is therefore omitted. ■

The result reported in Lemma 2 has first been stated by Gehrig (1993). The result implies that the sets of buyers and sellers joining a search market will be convex sets and will include the buyer with the highest net valuation and

[15] Using the terminology of Spulber (2005), all possible matches are viable for $\frac{v}{t} > 1$.

the lowest cost seller. That is, buyer and seller with location 0 will join the search market in W and the buyer and seller at $\frac{1}{2}$ will join the search market in E. Moreover, because all buyers and all sellers in these sets will join the same search market, the distribution of buyers and sellers active in a search market will be uniform.

Let v_c be the net valuation of a buyer joining a search market, where the highest and lowest cost sellers have costs $\overline{v}_p$ and $\underline{v}_p$ and where the probability of a match for a buyer is γ_c. Then, the expected utility of the buyer v_c is[16]

$$V_c(v_c) = \frac{\gamma_c}{2} \int_{\underline{v}_p}^{\overline{v}_p} (v_c - v_p) \frac{1}{\overline{v}_p - \underline{v}_p} dv_p, \tag{18}$$

where $\frac{1}{\overline{v}_p - \underline{v}_p}$ is the density for the uniform distribution from which sellers' costs are drawn. Integrating out and simplifying yields

$$V_c(v_c) = \frac{\gamma_c}{2} \left[v_c - \frac{1}{2}(\overline{v}_p + \underline{v}_p) \right]. \tag{19}$$

This formula has a neat interpretation. Consider the second term inside the bracket, $\frac{1}{2}(\overline{v}_p + \underline{v}_p)$. This is the expected cost of the producer to whom the buyer will be matched. Consequently, the difference between v_c and the expected cost is the aggregate surplus buyer v_c expects to generate. If matched, the buyer just gets one half of this surplus because of the even sharing assumption, which explains the fraction $\frac{1}{2}$ that pre-multiplies the bracket.

Similarly, consider a seller with cost v_p who joins a search market where the highest and lowest valuations of buyers are $\overline{v}_c$ and $\underline{v}_c$ and where the probability of being matched is γ_p. Then,

$$V_p(v_p) = \frac{\gamma_p}{2} \int_{\underline{v}_c}^{\overline{v}_c} (v_c - v_p) \frac{1}{\overline{v}_c - \underline{v}_c} dv_c = \frac{\gamma_p}{2} \left[\frac{1}{2}(\overline{v}_c + \underline{v}_c) - v_p \right], \tag{20}$$

where the second equality follows after integrating and simplifying.

Stable Equilibria Clearly, not joining a given search market if no one else joins it is a best response for every buyer and seller. Therefore, it is always an equilibrium that one or both search markets are inactive. However, these equilibria are easily seen to be unstable: If an arbitrarily small number of agents (a set of agents with positive measure) were to deviate and to join a hitherto

[16] I make a slight abuse of notation by using $V_i(.), i = c, p$ to denote both the expected utility from search market participation as a function of the net valuation (e.g., v_c) *and* as a function of the location (e.g., x). Though the two things are clearly connected, e.g. for a buyer at x who joins the search market in W $v_c \equiv v - tx$, the two functions are not exactly the same.

inactive search market, then it would be a best response for many other agents to join this search market, too. On the other hand, equilibria where both search markets are open are stable. Even if a small number of agents deviates and becomes either inactive or joins the other search market, it will still be optimal for the other agents who in equilibrium join this search market to stay in this search market. Therefore, equilibria where one or both search markets are inactive are unstable, whereas equilibria where both search markets are open are stable.

I now turn to a characterization of all equilibria.

Lemma 3 *There is no equilibrium where both search markets are balanced, (i.e., attract the same number of buyers and sellers) but where one is larger than the other one.*

Proof: Notice first that Lemma 2 implies that in any stable equilibrium, the buyer and seller at 0 will join the search market in W and the buyer and seller at $\frac{1}{2}$ will join the one in E. Therefore, $\underline{v}_p^k = 0$ and $\overline{v}_c^k = v$ for $k = E, W$.

Next, assume without loss that the larger search market is in W. Let $q \geq \frac{1}{4}$ be the number of buyers joining W in equilibrium. Since by hypothesis search markets are balanced, q is also the number of sellers joining W, and all buyers and sellers will be matched in either market. Then, the buyer at location q must get at least the level of utility from the search market in W as from the one in E. That is,

$$V_c^W(q) = \frac{1}{2}\left[v - tq - \frac{1}{2}tq\right] \geq \frac{1}{2}\left[v - t\left(\frac{1}{2} - q\right) - \frac{1}{2}t\left(\frac{1}{2} - q\right)\right] = V_c^E(q),$$

where $tq = \overline{v}_p^W$ and $t\left(\frac{1}{2} - q\right) = \overline{v}_p^E$. Solving for q yields $q \leq \frac{1}{4}$, implying together with the assumption $q \geq \frac{1}{4}$ that $q = \frac{1}{4}$. Thus, both markets must be of equal size. ■

I now show first that there are no equilibria with unbalanced search markets, and second that in the unique (stable) equilibrium, buyers and sellers with $x \leq \frac{1}{4}$ and $y \leq \frac{1}{4}$ join the market in W and all other buyers and sellers join the market in E.

Lemma 4 *There are no equilibria with unbalanced search markets.*

Proof: Without loss, assume $\tilde{x} < \tilde{y}$, so that buyers are rationed in market E and sellers in market W, and assume that this is an equilibrium. When joining the search market in W, buyer $\tilde{x}$ gets utility

$$V_c^W(\tilde{x}) = \frac{1}{2}\left[v - t\tilde{x} - \frac{1}{2}t\tilde{y}\right],$$

while when joining the market in E he gets

$$V_c^E(\tilde{x}) = \frac{\gamma_c^E}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{y}\right)\right].$$

Since $\gamma_c^E < 1$, $V_c^E(\tilde{x}) < \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{y}\right)\right]$ holds. In equilibrium, $\tilde{x}$ must be indifferent between the two markets, so

$$\frac{1}{2}\left[v - t\tilde{x} - \frac{1}{2}t\tilde{y}\right] < \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{y}\right)\right]$$

must hold. Re-arranging and simplifying yields

$$\tilde{x} > \frac{3}{8} - \frac{1}{2}\tilde{y}. \tag{21}$$

On the other hand, the seller at $\tilde{y}$ gets utility

$$V_p^W(\tilde{y}) = \frac{\gamma_p^W}{2}\left[v - \frac{1}{2}t\tilde{x} - t\tilde{y}\right]$$

when joining the search market in W and utility

$$V_p^E(\tilde{y}) = \frac{1}{2}\left[v - \frac{1}{2}t\left(\frac{1}{2} - \tilde{x}\right) - t\left(\frac{1}{2} - \tilde{y}\right)\right]$$

when going to E. Since $\gamma_p^W < 1$, $V_p^W(\tilde{y}) < \frac{1}{2}\left[v - \frac{1}{2}t\tilde{x} - t\tilde{y}\right]$ holds. In equilibrium, $\tilde{y}$ must be indifferent between the two markets. A necessary condition for this is

$$\frac{1}{2}\left[v - \frac{1}{2}t\tilde{x} - t\tilde{y}\right] > \frac{1}{2}\left[v - \frac{1}{2}t\left(\frac{1}{2} - \tilde{x}\right) - t\left(\frac{1}{2} - \tilde{y}\right)\right].$$

Re-arranging and simplifying yields

$$\frac{3}{4} - 2\tilde{y} > \tilde{x}. \tag{22}$$

Taken together, the equilibrium conditions are thus

$$\frac{3}{4} - 2\tilde{y} > \tilde{x} > \frac{3}{8} - \frac{1}{2}\tilde{y} \tag{23}$$

and by assumption

$$\tilde{y} > \tilde{x}. \tag{24}$$

The final step is to show that conditions (23), (24), and $\tilde{x} \neq \frac{1}{4}$ are not compatible. To see this, assume first $\tilde{x} < \frac{1}{4}$. The second inequality in (23) can then be satisfied only if $\tilde{y} > \frac{1}{4}$. However, the conditions in (23) require also

$$\begin{aligned} \frac{3}{4} - 2\tilde{y} &> \frac{3}{8} - \frac{1}{2}\tilde{y} \\ &\Leftrightarrow \\ \tilde{y} &< \frac{1}{4}, \end{aligned}$$

which is the desired contradiction. On the other hand, if $\tilde{x} > \frac{1}{4}$, the first inequality in (23) requires $\tilde{y} < \frac{1}{4}$, which contradicts (24). Hence, there is no equilibrium with unbalanced search markets. ■

Proposition 3 *In the unique stable equilibrium, all buyers and sellers with locations $x, y \leq \frac{1}{4}$ join the market in W, and all buyers and sellers with $x, y > \frac{1}{4}$ join the market in E.*

Proof: Consider the constraints in the previous lemma and replace all strict inequalities with weak inequalities. The only case when all constraints are satisfied is when $\tilde{x} = \tilde{y} = \frac{1}{4}$.■

4.3 Equilibrium with Market Makers

I now analyze equilibrium when there are market makers. To see the potential for market making, observe from equation (19) that the expected utility of search market participation for a buyer with net valuation v_c is less than $\frac{1}{2}v_c$ with $\overline{v}_p > 0$. Thus, there is a positive ask price a such that this buyer would be indifferent between buying at this price and joining the search market. That is, there exists an ask price a such that

$$v_c - a = V_c(v_c). \tag{25}$$

In other words, for every buyer v_c there is a reservation price $a(v_c) \equiv v_v - V_c(v_c)$ such that he is indifferent between buying from the intermediary at the ask price $a = a(v_c)$ and joining the search market. Substituting the expression for $V_c(v_c)$ given in equation (19) above yields

$$v_c - a = \frac{1}{2}\left[v_c - \frac{1}{2}(\overline{v}_p + \underline{v}_p)\right].$$

Of particular interest is the reservation price of the buyer with the highest net valuation active in the search market, i.e.,

$$a(\overline{v}_c) \equiv \frac{1}{2}\overline{v}_c + \frac{1}{4}(\overline{v}_p + \underline{v}_p), \tag{26}$$

since this is the reservation price that is relevant for the market maker - if it sets $a = a(\overline{v}_c)$, all buyers with greater net valuations will prefer buying from the market maker to search market participation, and all buyers with smaller net valuations will prefer participating in the search market.[17]

Analogously, one can derive reservation prices $b(v_p)$ for sellers with net cost v_p such that seller v_p is indifferent between participating in the search market and selling to the intermediary, i.e.,

$$b(v_p) - v_p = V_p(v_p). \tag{27}$$

For reasons analogous to those for buyers, the bid price relevant for an intermediary will be the one that makes the most efficient seller in the search market, $\underline{v}_p$, indifferent between participating in the search market and selling to the intermediary, which is given by

$$b(\underline{v}_p) = \underline{v}_p + V_p(\underline{v}_p). \tag{28}$$

Indifference Between Search Markets Before deriving the inverse demand and supply function facing the intermediaries, the location of the buyer and seller who are indifferent between the search market in W and E have to be determined. The reason for that is that the expected utility of search market participation of the buyer and seller at q^W (who are indifferent between trading with the intermediary in W and joining the search market in W) depend on the net cost and the net valuation of the seller and buyer who are indifferent between the search markets, as can be seen from equations (19) and (20).

As a function of q^W and q^E, there will be a buyer $\tilde{x}(q^W, q^E)$ who is indifferent between the two search markets. Similarly, denote the location of the seller who is indifferent between the two markets by $\tilde{y}(q^W, q^E)$. The derivation of these indifferent agents is very similar to the model of market making without search markets. The difference is that they are now not indifferent between trading with the two intermediaries, but only between joining the two search markets. Nonetheless, it is via these agents that the decisions of the market maker in W and E have an impact on the other one's payoff.

So, let the sellers at q^W (and $\frac{1}{2} - q^E$) and $\tilde{y}$ be the sellers in the search markets in W (and E), respectively, with the lowest and highest cost. Then, the expected utility of the buyer at $\tilde{x}$, who is indifferent between the two search

[17] To see this, differentiate both sides of (25) with respect to v_c. The derivative of the left-hand side is one, while the right-hand side increases by less as v_c increases. Therefore, if for a given a, $\tilde{v}_c - a = V_c(\tilde{v}_c)$ holds for some $\tilde{v}_c$, then $v_c - a \gtreqless V_c(v_c) \Leftrightarrow v_c \gtreqless \tilde{v}_c$, whence $\tilde{v}_c = \overline{v}_c$ follows.

markets and who has therefore the lowest net valuation in either of them, satisfies

$$V_c^W(\tilde{x}) = \frac{1}{2}\left[v - t\tilde{x} - \frac{1}{2}(tq^W + t\tilde{y})\right] = \frac{1}{2}(v - t\tilde{x}) - \frac{1}{4}(tq^W + t\tilde{y}) \tag{29}$$

and

$$V_c^E(\tilde{x}) = \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right)\right] - \frac{1}{4}\left[tq^E + t\left(\frac{1}{2} - \tilde{y}\right)\right]. \tag{30}$$

Solving $V_c^W(\tilde{x}) = V_c^E(\tilde{x})$ for $\tilde{x}$ yields

$$\tilde{x} = \frac{3}{8} - \frac{1}{2}\tilde{y} + \frac{1}{4}\left(q^E - q^W\right). \tag{31}$$

Analogously, the expected utilities from search in W and E for seller $\tilde{y}$ satisfy

$$V_p^W(\tilde{y}) = \frac{1}{2}\left[\frac{1}{2}(v - tq^W + v - t\tilde{x}) - t\tilde{y}\right] = \frac{1}{2}(v - t\tilde{y}) - \frac{1}{4}(tq^W + t\tilde{x}) \tag{32}$$

and

$$V_p^E(\tilde{y}) = \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{y}\right)\right] - \frac{1}{4}\left[tq^E + t\left(\frac{1}{2} - \tilde{x}\right)\right]. \tag{33}$$

Solving $V_p^W(\tilde{x}) = V_p^E(\tilde{x})$ for $\tilde{y}$ yields

$$\tilde{y} = \frac{3}{8} - \frac{1}{2}\tilde{x} + \frac{1}{4}\left(q^E - q^W\right). \tag{34}$$

Finally, solving equations (31) and (34) for $\tilde{x}$ and $\tilde{y}$ yields

$$\tilde{x}(q^W, q^E) = \tilde{y}(q^W, q^E) = \frac{1}{4} + \frac{1}{6}(q^E - q^W). \tag{35}$$

The following is noteworthy. As, say, q^W increases, $\tilde{x}(q^W, q^E)$ and $\tilde{y}(q^W, q^E)$ decrease. Due to the increased trade by the intermediary in W, the highest net valuation of a buyer in the search market in W decreases and the lowest cost of a seller in the search market in W increases. As a consequence, the search market in W becomes less attractive for buyers close to $\tilde{x}$, and thus, more buyers and sellers want to join the search market in E. Observe that $\underline{v}_c^E$ decreases and $\overline{v}_p^E$ increases as $\tilde{x}$ and $\tilde{y}$ increase. Consequently, from the perspective of the buyers and sellers close to E (i.e., close to $\frac{1}{2}$), search market participation becomes less attractive. This has some interesting implications that will be discussed below.

Indifference Between Intermediated Trade and Search Market Having determined the location of the buyer and seller who are indifferent between search markets, I can now derive the expected utility from search market participation for the buyer and seller who are indifferent between search market participation and trading with an intermediary. This will then allow me to compute the reservation price of this buyer and seller for trading with the intermediary.

The expected utility of the buyer at q^W from search market participation is

$$V_c^W(q^W) = \frac{1}{2}v - \frac{1}{16}t - \frac{17}{24}tq^W - \frac{1}{24}tq^E. \tag{36}$$

As this buyer is indifferent between search market participation and buying from the intermediary in W at ask price a^W, whence he derives a surplus of

$$U_c^W(q^W) \quad = \quad v - tq^W - a^W,$$

it has to be true that

$$U_c^W(q^W) = V_c^W(q^W). \tag{37}$$

Solving equation (37) for a^W yields the reservation price of the indifferent buyer for buying from the intermediary in W, and thus the inverse demand function this intermediary faces. Let $A^W(q^W, q^E)$ denote this solution. It is given by

$$A^W(q^W, q^E) = \frac{v}{2} + \frac{1}{16}t + \frac{1}{24}tq^E - \frac{7}{24}tq^W. \tag{38}$$

Analogously, the seller at q^W expects utility

$$V_p^W(q^W) = \frac{1}{2}v - \frac{1}{16}t - \frac{17}{24}tq^W - \frac{1}{24}tq^E \tag{39}$$

from participating in the search market in W. If he sells to the intermediary at bid price b^W, his surplus is

$$U_p^W(q^W) \quad = \quad b^W - tq^W.$$

He is indifferent between search and selling to the market maker if and only if

$$U_p^W(q^W) = V_p^W(q^W). \tag{40}$$

Solving (40) for b^W yields the reservation price of the seller at q^W for selling to the intermediary, and thus the inverse supply function facing the intermediary. Denote this solution by $B^W(q^W, q^E)$. It is

$$B^W(q^W, q^E) = \frac{v}{2} - \frac{1}{16}t - \frac{1}{24}tq^E + \frac{7}{24}tq^W. \tag{41}$$

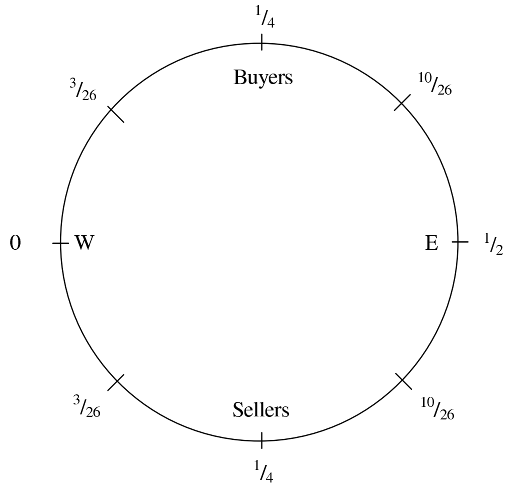

Figure 3: Equilibrium with Search and Intermediated Markets.

Profit Maximization by Market Makers Given the inverse demand and supply functions (38) and (41), the profit of the market maker in W is

$$\Pi^W(q^W, q^E) \quad = \quad Z^W(q^W, q^E)q^W, \tag{42}$$

where

$$Z^W(q^W, q^E) \equiv A^W(q^W, q^E) - B^W(q^W, q^E) = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{7}{12}q^W\right) \tag{43}$$

is the spread function the intermediary in W faces.

As in the case without search markets, the profit of an intermediary is thus independent of buyers' gross utility v. Maximizing $\Pi^W(q^W, q^E)$ over q^W, taking q^E as given, yields

$$0 = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{14}{12}q^W\right). \tag{44}$$

Thus, the reaction function is

$$q^W(q^E) = \frac{3}{28} + \frac{1}{14}q^E. \tag{45}$$

By symmetry, $q^E = q^W = q^*$ will hold in equilibrium. Thus, equilibrium quantity traded by an intermediary will be

$$q^* = \frac{3}{26}. \tag{46}$$

Inserting $q^E = q^W = q^*$ into the inverse demand and supply functions (38) and (41) yields the equilibrium ask and bid prices

$$a^* = \frac{v}{2} + \frac{7}{208}t \quad \text{and} \quad b^* = \frac{v}{2} - \frac{7}{208}t, \tag{47}$$

so that the equilibrium spread is $z^* = t\frac{7}{104}$. Equilibrium profit is

$$\Pi^* = t\frac{21}{2704}, \tag{48}$$

which is approximately $0.0078t$. Witness the substantial reduction of equilibrium profit compared to the case without search markets, when equilibrium profit is $\frac{1}{4}t$, which is more than thirty times larger than $0.0078t$.

Proposition 4 *The model has a unique equilibrium with two market makers and two active search markets. In this equilibrium, market makers set* $a^* = \frac{v}{2} + \frac{7}{208}t$ *and* $b^* = \frac{v}{2} - \frac{7}{208}t$. *The buyers and sellers with locations* $x, y \in \left[0, \frac{3}{26}\right]$ *trade with the intermediary in* W, *the buyers and sellers with* $x, y \in \left(\frac{3}{26}, \frac{1}{4}\right]$ *join the search market in* W, *the buyers and sellers with* $x, y \in \left(\frac{1}{4}, \frac{10}{26}\right)$ *join the search market in* E *and the buyers and sellers with* $x, y \in \left[\frac{10}{26}, \frac{1}{2}\right]$ *trade with the intermediary in* E.

Proof: The proof is in Appendix C.■

Note that the claim of Proposition 4 is not that there is a unique equilibrium, but that there is a unique equilibrium where the two search markets and the two market makers are active. Nonetheless, the uniqueness of equilibrium in the presence of search markets contrasts with the continuum of equilibria in the model without search markets.[18] Observe also from (45) that the best response quantity of the intermediary in W is upward-sloping in q^E, implying that quantities traded are strategic complements. Were the two intermediaries able to collude, they would choose q^W and q^E to maximize

$$\Pi^W + \Pi^E = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{7}{12}q^W\right)q^W + t\left(\frac{1}{8} + \frac{1}{12}q^W - \frac{7}{12}q^E\right)q^E,$$

yielding as collusive quantities $\tilde{q}^W = \tilde{q}^E = \frac{5}{26} > \frac{3}{26} = q^*$.

Discussion Much like in the models of Gehrig (1993) and Rust and Hall (2003), equilibrium is characterized by a partition of the sets of sellers and buyers. High valuation buyers and low cost sellers trade with intermediaries, and less efficient producers and lower valuation consumers participate in search markets.[19] In contrast to these models, this happens here in the presence of competition between intermediaries, and these intermediaries net positive profits in equilibrium. Moreover, the presence of search markets has an effect on

[18] This point is similar to Yavas (1995), who observes that in a model with endogenous search intensities the presence of a broker can reduce the set of equilibria and may even induce a unique equilibrium.

[19] Similar equilibrium behavior obtains in the model of Fingleton (1997).

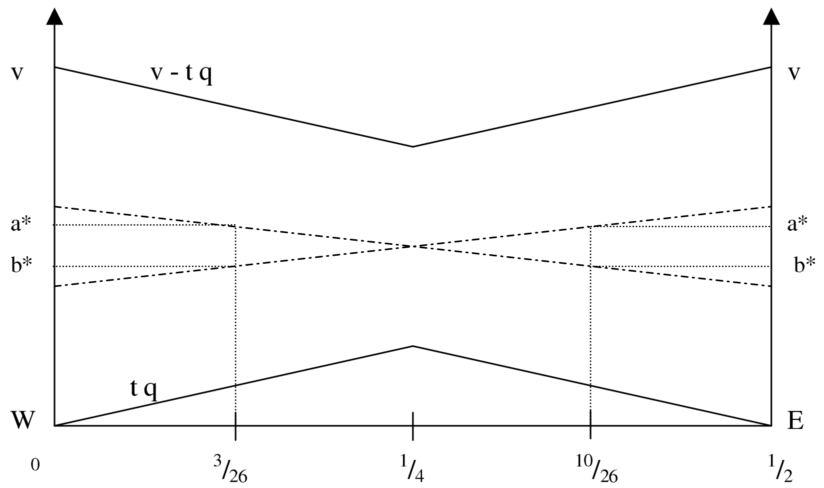

Figure 4: Equilibrium Supply and Demand Functions and Equilibrium Prices.

the equilibrium quantities traded by intermediaries in my model: An intermediary's quantity traded decreases from $\frac{1}{4}$ in the game without to $\frac{3}{26}$ in the game with search markets. This contrasts with the above mentioned models, where the presence or absence of search markets only has an impact on the spread functions the intermediaries face, but not on the quantities traded in equilibrium. To see this, note that the equilibrium spread a monopolistic intermediary in the models of Gehrig (1993) and Rust and Hall (2003) faces can be written as

$$Z(q) = \Delta(1 - 2q), \tag{49}$$

where q is the quantity traded by the intermediary and $\Delta \in [0, 2]$ is a parameter measuring the effectiveness of the search market.[20] It takes the value one if the search market is shut down (or completely ineffective) and smaller values the more efficient the search market. Assuming zero costs for market making, the profit function is

$$\Pi(q) = \Delta(1 - 2q)q. \tag{50}$$

It is easy to see that this function is maximized with

$$q^* = \frac{1}{4},$$

which obviously is independent of Δ. In other words, in these models the equilibrium quantity traded (though, of course, not the equilibrium profit) is

[20]More precisely, $\Delta = \frac{2-\lambda}{2} \in [\frac{1}{2}, 1]$ for the Gehrig model, where $\Delta = \frac{1}{2}$ occurs if and only if the search market is fully efficient, i.e., for $\lambda = 1$. In contrast, in the model of Rust and Hall, $\Delta = \frac{4\delta}{1+4\delta}$ can take any value between zero and one, where $\delta \in (0, \infty)$ is a discount factor that is adjusted for the probability of exit. A similar representation can be obtained for the model studied by Fingleton (1997). A derivation of the equilibrium spread function for these models is available upon request.

independent of whether there is a search market or not, or how efficient the matching process is.

5 Asymmetric Market Structures

In this section, I briefly investigate asymmetric market structures. In part, this is motivated by the observation of Ju et al. (2004) that in the aftermath of the exit of Enron as a market maker in the North-American natural gas market, price dispersion in the search market increased dramatically. They show also that this increase in price dispersion is consistent with a model where market makers are homogenous. So, it is interesting to see whether a similar prediction obtains in a model where market makers are horizontally differentiated.

For that purpose, I assume now that the market maker in E exits, and I derive the equilibrium for the situation when there are two search markets (one in W and one in E) and a monopoly market maker in W. I then compare the equilibrium with the one derived in the previous section.

When there is but one intermediary in W, but two search markets, buyers and sellers that are close to the intermediary will trade with the intermediary, while buyers and sellers who are a bit further away will trade with one another in the search market in W, and buyers and sellers closest to E will trade in the search market in E. As before, denote by $\tilde{x}$ and $\tilde{y}$ the locations of the buyer and seller who are indifferent between joining either search market. For notational ease, denote by q the quantity traded by the intermediary. In equilibrium, $\tilde{x} = \tilde{y}$. These values will be functions of q and are derived as solution to

$$V_c^W(\tilde{x}) = \frac{1}{2}\left[v - t\tilde{x} - \frac{1}{2}t(q+\tilde{x})\right] = \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{x} + 0\right)\right] = V_c^E(\tilde{x}),$$

which is

$$\tilde{x} = \frac{1}{4} - \frac{1}{6}q.$$

It is re-assuring that this is the same as the result when setting $q^E = 0$ in (35).

Plugging in these values into the utility functions for the buyer and seller at locations q yields

$$V_c^W(q) = \frac{1}{2}v - \frac{1}{16}t - \frac{17}{24}tq$$

and

$$V_p^W(q) = \frac{1}{2}v + \frac{1}{16}t + \frac{17}{24}tq.$$

Thus, the reservation prices for trade with the intermediary are

$$A(q) = \frac{1}{2}v + \frac{1}{16}t - \frac{7}{24}tq$$

and

$$B(q) = \frac{1}{2}v - \frac{1}{16}t + \frac{7}{24}tq.$$

Consequently, the intermediary's profit is

$$\Pi(q) = t\left(\frac{1}{8} - \frac{7}{12}q\right)q,$$

yielding the monopoly quantity

$$q^{M*} = \frac{3}{28}.$$

Again, it is re-assuring that this is the same value that is derived when setting $q^E = 0$ in the reaction function (45). It is noteworthy that q^{M*} is smaller than an individual market maker's equilibrium quantity under duopoly, $q^* = \frac{3}{26}$. The equilibrium ask and bid prices of the monopoly are

$$a^{M*} = \frac{v}{2} + \frac{t}{32} \quad \text{and} \quad b^{M*} = \frac{v}{2} - \frac{t}{32}.$$

Notice that $a^{M*} < a^*$ and $b^{M*} > b^*$. As both the spread the intermediary earns and the quantity it trades are smaller under monopoly than under duopoly, its equilibrium profit will also be smaller. This profit is

$$\Pi^{M*} = t\left(\frac{1}{8} - \frac{3}{48}\right)\frac{3}{28} = \frac{3}{448}t,$$

which is less than $\Pi^* = \frac{21}{2704}t$. Also note that $\tilde{x} = \frac{13}{56} < \frac{1}{4}$.

So as to show that there is no profitable deviation for the monopolistic intermediary, I first show that it is not profitable for the monopolistic intermediary in W to extinguish only the search market in W while keeping the one in E alive. The issue here is that if the quantity q^W traded by the intermediary in W is large enough, then no agent with location $\tilde{x} > q^W$ will be indifferent between the two search markets, but will rather prefer search market E to the search market W. This is easiest to see by inspection of (35) when setting $q^E = 0$. For $\tilde{x} \geq q^W$ to hold,

$$\frac{1}{4} - \frac{1}{6}q^W \geq q^W \Leftrightarrow q^W \leq \frac{3}{14}.$$

For larger q^W's, the agents at q^W will be indifferent between joining the intermediary in W and the search market in E.[21] Consequently, the reservation

[21] An alternative way to see this is as follows. Let $\tilde{q}$ be the location of the buyer and seller who are the only agents in the search market in W and who would consequently be the agents with the lowest valuation and the highest cost in the search market in E. Then, their utility from the search market in W is

$$V_c^W(\tilde{q}) = \frac{v}{2} - t\tilde{q} = V_p^W(\tilde{q}),$$

prices for trading the quantity $\tilde{q}$ with the intermediary in W will be an a^W and b^W such that

$$v - a^W - t\tilde{q} = V_c^E(\tilde{q})$$

and

$$b^W - t\tilde{q} = V_p^E(\tilde{q}),$$

where $V_c^E(\tilde{q})$ and $V_c^E(\tilde{q})$ are as defined in the previous footnote. Solving for a^W and b^W yields the inverse demand and supply functions

$$A^W(\tilde{q}) = \frac{v}{2} + \frac{3}{8}t - \frac{7}{4}t\tilde{q}$$

and

$$B^W(\tilde{q}) = \frac{v}{2} - \frac{3}{8}t + \frac{7}{4}t\tilde{q},$$

so that the intermediary's profit is

$$\Pi^W(\tilde{q}) = \frac{3}{4}t(3 - 14\tilde{q})\tilde{q},$$

which is maximized at $\tilde{q}^* = \frac{3}{28}$. However, this violates the restriction that $\tilde{q}$ must be larger than $\frac{3}{14}$ for these inverse demand and supply functions to be valid in the first place. Consequently, the optimal quantity will be as small as necessary, i.e., will equal $\frac{3}{14}$. Inserting this value into the profit function reveals immediately that the profit will be zero. Hence, this deviation does not pay.

Second, the intermediary may want to take over the whole market by extinguishing the search market in E, too. However, so as to attract the buyer and seller at $\frac{1}{2}$, these agents must be offered an ask price below $\frac{v}{2}$ and a bid price above $\frac{v}{2}$, where $\frac{v}{2}$ is the price at which they would trade in the search market in E if only these two agents join the search market in E. Clearly, this deviation entails a negative profit and is therefore not profitable. Thus, I have shown:

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while their expected utility from participating in E is

$$V_c^E(\tilde{q}) = \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{q}\right) - \frac{1}{2}t\left(0 + \frac{1}{2} - \tilde{q}\right)\right] = \frac{v}{2} - \frac{3}{8}t + \frac{3}{4}t\tilde{q}$$

and

$$V_p^E(\tilde{q}) = \frac{1}{2}\left[\frac{1}{2}\left(v + v - t\left(\frac{1}{2} - \tilde{q}\right)\right) - t\left(\frac{1}{2} - \tilde{q}\right)\right] = \frac{v}{2} - \frac{3}{8}t + \frac{3}{4}t\tilde{q},$$

where the fact has been used that the most efficient seller in E has cost equal to zero and the highest valuation buyer a valuation of v. Notice that these utilities increase in $\tilde{q}$ because the search market in E becomes more attractive the closer to $\frac{1}{2}$ the marginal buyers and sellers are. It is easy to see that

$$V_c^W(\tilde{q}) \geq V_c^E(\tilde{q}) \Leftrightarrow \tilde{q} \leq \frac{3}{14}.$$

Proposition 5 *With active search markets, the quantity a market maker trades, the spread it sets and the profit it earns are smaller in equilibrium under monopoly than under duopoly.*

Discussion It is well known that for some values of v and t, the standard Hotelling model of horizontal differentiation in product markets exhibits the feature that equilibrium prices rise as a second firm enters the market.[22] The reason for this counterintuitive effect is that the location of the marginal consumer is closer to the firm under duopoly than under monopoly. Consequently, the willingness to pay of this consumer is larger under duopoly than under monopoly, which is why equilibrium prices increase as entry occurs. However, though prices may increase with entry in the standard Hotelling model, profits unambiguously decrease when a second firm enters.[23] In the model where market makers face competition from search markets, the mechanism at work is very much the same. To see this, note that with $q^E = 0$, $\tilde{x} < \frac{1}{4}$ for any positive quantity traded by the intermediary in W, where $\tilde{x}$ corresponds to the marginal consumer who is closer in the standard Hotelling model. The twist in the present model is that the smaller $\tilde{x}$, the *more* attractive is search market participation in W from the perspective of the marginal buyer and seller at q^W. That is, utility from search increases as $\tilde{x}$ decreases, as a consequence of which the willingness to pay (and the reservation price to sell) for intermediated trade decreases (increases). This contrasts with the equilibrium with two market makers, where $\tilde{x} = \frac{1}{4}$. Consequently, the buyer and seller at q^W will derive less utility from search market participation when there are two intermediaries. Thus, the buyer and the seller at q^W will have a higher willingness to pay for the intermediary and a lower reservation price to sell to the intermediary than if there is no intermediary in E. Therefore, the profit of the intermediary in W increases, as an intermediary enters in E.[24]

The Effect on Equilibrium Price Dispersion I only characterize the equilibrium support of prices in the search markets, i.e., I do not pay attention to the distribution of these prices. As a buyer with valuation v_c and a seller with costs v_p share the aggregate surplus evenly, the price at which they trade is given as $p = \frac{v_c+v_p}{2}$. Thus, the highest price, $\overline{p}$, occurs when the seller with

[22]If the Hotelling line is of length 1, this happens for values of $\frac{v}{t} \in \left[\frac{3}{2}, 2\right]$.

[23]See Chen and Riordan (2005) for a model where for some parameters, profits increase when the number of firms increases.

[24]Notice that in contrast to the model of Chen and Riordan (2005) and the Hotelling model, higher profits and higher ask prices (and lower bid prices) obtain for a very wide range of parameters, i.e., at least for all $\frac{v}{t} > \frac{3}{2}$.

the highest cost $\overline{v}_p$ is matched to the buyer with the greatest valuation $\overline{v}_p$. Similarly, the lowest price, denoted as $\underline{p}$, in the support occurs when the most efficient seller $\underline{v}_p$ active in a search market is matched to the buyer with the lowest valuation $\underline{v}_c$.

When two market makers are active, each of them trades the quantity $q = \frac{3}{26}$ and $\tilde{x} = \tilde{y} = \frac{1}{4}$. Thus, $\overline{v}_p = \frac{1}{4}t$ and $\overline{v}_c = v - \frac{3}{26}t$, so that

$$\overline{p} = \frac{v}{2} + \frac{7}{104}t.$$

As for the lowest price, $\underline{v}_p = \frac{3}{26}t$ and $\underline{v}_c = v - \frac{1}{4}t$, so that

$$\underline{p} = \frac{v}{2} - \frac{7}{104}t.$$

When there is but one market maker, the two search markets differ. Recall that $\tilde{x} = \frac{13}{56}$. Therefore, $\overline{v}_c^E = v$ and $\overline{v}_p^E = t\frac{15}{56}$. This implies

$$\overline{p}^E = \frac{v}{2} + \frac{15}{112}t.$$

The lowest price in E is given by

$$\underline{p}^E = \frac{v}{2} - \frac{15}{112}t,$$

since $\underline{v}_p^E = 0$ and $\underline{v}_c^E = v - t\frac{15}{56}$. In the search market in W, $\overline{v}_p^W = \frac{13}{56}t$, $\overline{v}_c^W = v - \frac{3}{28}t$, $\underline{v}_p^W = \frac{3}{28}t$ and $\underline{v}_c^W = v - \frac{13}{56}t$. Consequently,

$$\overline{p}^W = \frac{v}{2} + \frac{7}{112}t$$

and

$$\underline{p}^W = \frac{v}{2} - \frac{7}{112}t$$

follows. Thus,

$$\underline{p}^E < \underline{p} < \underline{p}^W < \overline{p}^W < \overline{p} < \overline{p}^E.$$

Thus, measured by the breadth of the support in both markets, equilibrium price dispersion increases as a market maker exits. However, if one considers only the market in W, price dispersion decreases.

Equilibrium with one intermediary and one search market Finally, I look at the case where the intermediary in W faces competition from only one search market that is located in W as well. Then,

$$V_c(q) = \frac{1}{2}\left[v - tq - \frac{1}{2}\left(tq + \frac{1}{2}t\right)\right]$$

and

$$V_p(q) = \frac{1}{2}\left[\frac{1}{2}\left(v - tq + v - \frac{1}{2}t\right) - tq\right],$$

yielding

$$A(q) = \frac{1}{2}v + \frac{1}{8}t - \frac{1}{4}tq$$

and

$$B(q) = \frac{1}{2}v - \frac{1}{8}t + \frac{1}{4}tq.$$

So, profit of the intermediary is

$$\Pi(q) = \frac{1}{4}t\left(1 - 2q\right)q,$$

which is maximized at $q^* = \frac{1}{4}$. Hence, the maximal profit of the intermediary is $\Pi^* = \frac{1}{32}t$.

So as to check the validity of my claim above that in contrast to Gehrig and Rust and Hall's models the equilibrium quantity traded is affected by the presence or absence of search markets in my model, I also analyze the case with a monopolistic intermediary and no search market.

The unconstrained intermediary's profit as a function of q is then

$$\Pi(q) = (v - 2tq)q,$$

which is maximized by

$$q^* = \frac{v}{4t}.$$

However, the additional constraint $q^* \leq \frac{1}{2}$ has to be taken care of, so $\frac{v}{t} \leq 2$ has to hold for this solution to be valid. Otherwise, the equilibrium quantity is simply $\frac{1}{2}$. To see that $q^* > \frac{1}{4}$, note that

$$\frac{v}{4t} = \frac{1}{4} \Leftrightarrow \frac{v}{t} = 1,$$

which contradicts the assumption $v > \frac{3}{2}t$. Thus, the claim made above is indeed valid.

6 Market Makers vs. Matchmakers

Up to now, the assumption was maintained that sellers are paid for providing the good, regardless of whether a market maker attracts any buyers at all. This "standing ready to buy" is a defining element of market makers, as witnessed by the definition from the SEC quote above. On the up side, the assumption has the advantage of eliminating coordination problems of buyers and sellers. The down side, though, is that it precludes many interesting applications. For example, intermediaries in many markets do not actually buy goods to stock, but rather make payments only when the transaction is completed. For example, a job market intermediary typically only pays wages once a worker is matched to a firm. Similarly, intermediaries in housing markets do typically not buy or sell houses and apartments, but rather require a fee from buyers and/or sellers, which becomes due only when a contract is written.[25] Clearly, any customer of such an intermediary will not only be concerned about the fees it charges, but also about the number (and quality) of sellers and buyers it attracts. This is why these types of intermediaries can be called *matchmakers* rather than market makers. Recently, considerable research efforts have been devoted to the study of matchmakers and platforms (see, e.g., Armstrong, 2004; Caillaud and Jullien, 2001, 2003; McCabe and Snyder, 2004; Rochet and Tirole, 2003, 2004).

Though there are substantial differences between matchmakers and market makers, there is also a great degree of similarity between the two types of intermediary. In particular, I will show that every equilibrium in a game with market makers is also an equilibrium in an "appropriately defined" game with matchmakers. I begin by introducing the game with matchmakers, whence it should become clear what I mean by appropriately defined.

The Matchmaker Game Matchmakers charge fees $\phi_c \geq 0$ and $\phi_p \geq 0$ to consumers and producers for joining their platform. I assume that these fees are due when joining the matchmaker (or platform), i.e., before eventually being matched. This assumption is made mainly for convenience, though it is not completely innocuous.[26] I also assume that each matchmaker sets an internal transaction price of $p = \frac{v}{2}$ at which a buyer and a seller on the platform exchange the good. This is a simple way to make sure that a matchmaker generates the same gross utility for buyers and sellers as a market maker if it attracts the same buyers and sellers. In case the number of agents of one type (buyers or sellers) exceeds the number of the other type, the agents on the short side are matched with probability one, and those on the long side are matched with a probability less than one.[27] Buyers and sellers are uniformly distributed on

[25] An additional fee may also be charged to the seller to be admitted to the database of the intermediary.

[26] If fees are only due in case a match occurs, then not joining a platform may be weakly dominated by joining it, whereas this is not true when fees are charged upon joining a platform.

[27] For example, if there are 5 buyers and 3 sellers, each buyer is matched with probability 0.6.

semicircles of length $\frac{1}{2}$ and have transportation costs t. Gross utility is v for every buyer, and matchmakers are located at W and E. There may or may not be search markets in W and E as well. I assume also that in addition to joining either search market and either intermediated market, all buyers and sellers may remain inactive.

Before stating and proving the proposition, it is useful to note the connection between the fees ϕ_c and ϕ_p set by matchmakers and ask and bid prices a and b set by market makers. Let $a > \frac{v}{2}$ and $b < \frac{v}{2}$ be the prices set by a market maker. Then, the fees

$$\phi_c(a) \equiv a - \frac{v}{2} \quad \text{and} \quad \phi_p(b) \equiv \frac{v}{2} - b$$

are equivalent in the sense that they generate the same utility for buyers and sellers, conditional on being matched with probability one. To see this, note that the net utility of a buyer with utility v_c from being served by the market maker is

$$v_c - a,$$

which equals the surplus of joining the matchmaker,

$$v_c - \phi_c - \frac{v}{2},$$

if $\phi_c = \phi_c(a)$. Similarly, a seller with cost v_p expects $b - v_p$ from a market maker and $\frac{v}{2} - \phi_p - v_p$ from a matchmaker if matched with certainty. The two expressions are the same if $\phi_p = \phi_p(b)$.

Denote by (a^*, b^*) the ask and bid prices in an equilibrium of the game with market makers, and let $I^{W*} = (I_p^{W*}, I_c^{W*})$ and $I^{E*} = (I_p^{E*}, I_c^{E*})$ be the sets of sellers and buyers joining the intermediary in W and E in this equilibrium. Also, let $S^{W*} = (S_p^{W*}, S_c^{W*})$ and $S^{E*} = (S_p^{E*}, S_c^{E*})$ be the sets of sellers and buyers joining the search market W and E in equilibrium. Note that the sets S^{W*} and S^{E*} can be empty.

Proposition 6 *The matchmaker game has a subgame perfect equilibrium, where both matchmakers set*

$$\phi_c = \phi_c(a^*) \quad \textit{and} \quad \phi_p = \phi_p(b^*),$$

and where on the equilibrium path the buyers and sellers in I^{W*} and I^{E*} join the matchmaker in W and E and the sellers and buyers in S^{W*} and S^{E*} join the search markets in W and E.

Proof: Note first that in any equilibrium, market makers make positive profits. If buyers and sellers behave as stated in the proposition, then matchmakers will thus make positive profits when setting $\phi_c(a^*)$ and $\phi_p(b^*)$. Second, from the fact that the strategies of the buyers and sellers are an equilibrium in the game with market makers, it follows that the actions of buyers and sellers in the proposition are optimal in the game with matchmakers, given that all the other buyers and sellers behave as stated. What therefore remains to be shown is that the matchmakers cannot increase profits given the strategies played by buyers and consumers. To see that buyers and sellers can deter any deviation by a matchmaker, note that not joining any matchmaker if no one else joins a matchmaker is a best response for every buyer and seller. Thus, if buyers and sellers join the matchmakers if and only if these set $\phi_c = \phi_c(a^*)$ and $\phi_p = \phi_p(b^*)$, and otherwise remain inactive or go to the search markets, deviation does not pay for matchmakers either: A deviation yields zero profits, while the fees $\phi_c = \phi_c(a^*)$ and $\phi_p = \phi_p(b^*)$ generate positive profits. ■

7 Conclusions

In this paper, I analyzed duopolistic competition between market makers who are horizontally differentiated and who may face competition from simultaneously active search markets. In equilibrium, market makers net positive profits, which contrasts with competition between homogenous intermediaries (see, e.g., Stahl, 1988; Gehrig, 1993; Fingleton, 1997; Rust and Hall, 2003). Nonetheless, the presence of active search markets reduces equilibrium profits of market makers substantially.

Moreover, with active search markets, each intermediary nets a larger profit and trades a larger quantity if there are two intermediaries than if there is a single one. This result is similar to models of differentiated Bertrand competition in product markets, where equilibrium prices can increase as a second firm enters. The reason is that competition increases the marginal willingness to pay of buyers. In contrast to the standard product market model, though, in the model with market makers and search markets, not only (consumer) prices but also profits are larger with two intermediaries. This suggests that there may be the potential for contagion insofar as the profit of a market maker decreases if the other one goes bankrupt.

Appendix

A Generalization for all values of v and t

In the text, I focussed on the case with $\frac{v}{t} > \frac{3}{2}$. I now extend the model to all values of v and t. First, I analyze the case when there are only market makers. I show that the results from the product market models (Hotelling, 1929; Salop, 1979) carry over. Second, I introduce competition from search markets. Though the case with intermediate values of v/t adds a few computational complications, I show that the basic results from the main text are valid for all values of v/t.

A.1 Equilibrium without Search Markets

When there are no search markets, three classes of oligopolistic competition exist as function of v and t: (i) *local monopolies* for small values of v/t, (ii) *constrained duopoly* for intermediate values of v/t and (iii) *unconstrained duopoly* for large v/t. The unconstrained duopoly corresponds to the case analyzed in the paper with "large" meaning $\frac{v}{t} > \frac{3}{2}$.

A.1.1 Local Monopolies

As local monopoly, each market maker has an equilibrium market coverage q smaller than $\frac{1}{4}$. Consequently, buyers and sellers in $[q, \frac{1}{2}]$ will be inactive, and the two market makers will act independently of one another. Consider the market maker in W. Its profit function under local monopoly is

$$\Pi^W(q) = (v - 2tq)q,$$

which is maximized with $q^* = \frac{v}{4t}$. Since $q^* < \frac{1}{4}$ is required, $\frac{v}{t} < 1$ must hold. Hence, for

$$0 < \frac{v}{t} < 1$$

the local monopoly case is relevant. Under local monopoly, equilibrium ask and bid prices are

$$a^{LM*} = \frac{3}{4}v \quad \text{and} \quad b^{LM*} = \frac{v}{4}.$$

A.1.2 Constrained Duopoly

In the case of constrained duopoly, the buyer and seller at $\frac{1}{4}$ are indifferent between the intermediary in E and W, from which both get zero net utility (see also Salop, 1979). Hence, equilibrium prices are such that

$$v - t\frac{1}{4} - a^{CD*} = 0 \quad \text{and} \quad b^{CD*} - t\frac{1}{4} = 0.$$

Thus,

$$a^{CD*} = v - t\frac{1}{4} \quad \text{and} \quad b^{CD*} = t\frac{1}{4}.$$

For $a^{CD*} < a^{LM*}$ and $b^{CD*} > b^{LM*}$ to hold, it must be true that

$$\frac{v}{t} > 1.$$

Summarizing, we therefore have that for $0 < \frac{v}{t} < 1$, market makers are local monopolies, for $0 < \frac{v}{t} < \frac{3}{2}$, they are a constrained duopoly, and for $\frac{3}{2} < \frac{v}{t}$, they are an unconstrained duopoly. Note that this result replicates the result for two imperfectly firms that interact only on a product market.

A.2 Equilibrium with Search Markets

Case 1: $1 < \frac{v}{t} < \frac{3}{2}$.

Note that $1 < \frac{v}{t}$ implies that a buyer and a seller who are both located at $\frac{1}{2}$ would be willing to trade with one another even if they have to join the search market in W. Consequently, the indifference constraint $\tilde{x}(q^W, q^E)$ will be the same as in the text, and thus the equilibrium with search markets will be the same as for $\frac{v}{t} > \frac{3}{2}$. Hence, in this model search markets do not only impose additional constraints to eliminate a continuum of equilibria, but they also eliminate the case of the constrained duopoly.

Case 2: $\frac{1}{3} < \frac{v}{t} < 1$.

In this case, the search markets are still connected and no buyers and sellers are inactive in equilibrium. However, there are some substantial complications because utilities of the buyers and sellers who are indifferent between the two search markets are now complicated, polynomial expressions. Nevertheless, I can show that an equilibrium with active search markets exists, and I derive the equilibrium quantity traded.

Because of symmetry, it is safe to assume that $\tilde{x} = \tilde{y}$ will hold in equilibrium. The expected utility of the buyer in the middle from joining the search markets in W and E are

$$V_c^W(\tilde{x}) = \frac{1}{4}\frac{1}{\tilde{x} - q^W}\left(v - t\tilde{x} - tq^W\right)^2$$

and

$$V_c^E(\tilde{x}) = \frac{1}{4}\frac{1}{\frac{1}{2} - \tilde{x} - q^E}\left(v - t(\frac{1}{2} - \tilde{x}) - tq^E\right)^2.$$

I first show that on the interval $[q^W, \frac{1}{2} - q^E]$ a unique fix point $\tilde{x}(q^W, q^E)$ exists such that

$$V_c^W(\tilde{x}(q^W, q^E)) = V_c^E(\tilde{x}(q^W, q^E))$$

holds. To see this, note that $V_c^W(\tilde{x})$ approaches infinity as $\tilde{x}$ approaches q^W from the right. Moreover, $V_c^W(\tilde{x})$ is continuous and decreasing in $\tilde{x}$ and takes a finite value at $\tilde{x} = \frac{1}{2} - q^E$. Similarly, $V_c^E(\tilde{x})$ takes a finite value at $\tilde{x} = q^W$ and continuously increases in $\tilde{x}$ on $[q^W, \frac{1}{2} - q^E]$ and approaches infinity as $\tilde{x}$ approaches $\frac{1}{2} - q^E$ from the left. Thus, a unique fix point $\tilde{x}(q^W, q^E) \in [q^W, \frac{1}{2} - q^E]$ exists.

There is no nice explicit solution for this fix point $\tilde{x}(q^W, q^E)$. However, this is not required since in equilibrium, $q^E = q^W < \frac{1}{4}$ will hold. It is easy to see that $\tilde{x}(q, q) = \frac{1}{4}$ is the desired fix point.[28] What is needed in addition to that is knowledge of the derivative $\frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W} |_{q^W = q^E}$, which can be derived using the implicit function theorem. For that purpose, some additional notation is useful. Write $V_c^W(q^W, q^E, \tilde{x})$ and $V_c^E(q^E, q^W, \tilde{x})$ for the expected utility from search in W and E, respectively, for the buyer at $\tilde{x}$, and denote the derivative after the i-th argument by V_{ci}^W and V_{ci}^E, $i = 1, 2, 3$. Note that $V_{c2}^W = V_{c2}^E = 0$. By the implicit function theorem, the partial derivative $\frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W}$ satisfies

$$V_{c1}^W + V_{c3}^W \frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W} = V_{c3}^E \frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W}.$$

Solving for $\frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W}$ yields

$$\frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W} = \frac{V_{c1}^W}{V_{c3}^E - V_{c3}^W}.$$

Though this is a rather cumbersome expression in general, it simplifies to

$$\frac{\partial \tilde{x}(q^W, q^E)}{\partial q^W} = \frac{4tq^W + 4v - 3t}{2(4v + t - 12q^W)} \tag{51}$$

at $q^W = q^E$, which also implies $\tilde{x} = \frac{1}{4}$.

The expected utility from search market participation of the buyer at q^W is

$$V_c^W(q^W) = \frac{1}{2}\left[v - tq^W - \frac{1}{2}(tq^W + t\tilde{x}(q^W, q^E))\right],$$

so that the reservation price for buying from the intermediary is

$$A^W(q^W) = v - tq^W - V_c^W(q^W).$$

[28] It is easy to see that it is a fix point. That it is the desired one follows then immediately from its uniqueness.

Similarly, for the seller at q^W, expected utility from search is $V_p^W(q^W) = \frac{1}{2}\left[\frac{1}{2}(v - tq^W + v - t\tilde{x}(q^W, q^E)) - tq^W\right]$, so that the inverse supply function facing the intermediary in W is

$$B^W(q^W, q^E) = V_p^W(q^W) + tq^W.$$

Consequently, the intermediary's profit is

$$\Pi^W(q^W, q^E) = \left(v - 2tq^W - V_c^W(q^W) - V_p^W(q^W)\right) q^W.$$

The condition for a profit maximum for the intermediary in W is

$$0 = v - 4tq^W - \frac{\partial\left(V_c^W(q^W) + V_p^W(q^W)\right)}{\partial q^W} q^W - V_c^W(q^W) - V_p^W(q^W).$$

Notice that

$$\frac{\partial\left(V_c^W(q^W) + V_p^W(q^W)\right)}{\partial q^W} = -\frac{t}{2}\frac{\partial\tilde{x}(q^W, q^E)}{\partial q^W}.$$

Imposing symmetry, i.e., $q^W = q^E$, replacing $\tilde{x}$ by $\frac{1}{4}$ and $\frac{\partial\tilde{x}(q^W,q^E)}{\partial q^W}$ by $\frac{4tq^W+4v-3t}{2(4v+t-12q^W)}$, the first order condition thus becomes

$$0 = -\frac{5}{2}tq^W + \frac{1}{8}t + \frac{4tq^W + 4v - 3t}{4(4v + t - 12q^W)}tq^W.$$

The two solutions to this quadratic equation are

$$q_{1,2}^{W*} = \frac{38t + 72v \pm 2\sqrt{113t^2 + 376vt + 1296v^2}}{496t}.$$

It is easy to check that the relevant solution is the smaller one, i.e.,

$$q^{W*} = \frac{38t + 72v - 2\sqrt{113t^2 + 376vt + 1296v^2}}{496t},$$

which takes values between 0.046 and 0.051 for $\frac{v}{t} \in \left[\frac{1}{3}, 1\right]$.

To verify that the second order conditions is satisfied at $q^{W*} = q^{E*}$, observe that the second order condition reads

$$0 \geq -4t - 2\frac{\partial\left(V_c^W(q^W) + V_p^W(q^W)\right)}{\partial q^W} - \frac{\partial^2\tilde{x}}{\partial {q^W}^2}q^W = -4t + t\frac{\partial\tilde{x}}{\partial q^W} + \frac{t}{2}\frac{\partial^2\tilde{x}}{\partial {q^W}^2}q^W.$$

The only way this condition could be violated is that

$$\frac{\partial\tilde{x}}{\partial q^W} + \frac{1}{2}\frac{\partial^2\tilde{x}}{\partial {q^W}^2}q^W > 4.$$

However, it can be checked that the left-hand side is strictly less than $\frac{1}{4}$ for $\frac{1}{3} < \frac{v}{t} < 1$. Thus, the second order condition is satisfied.

Case 3: $0 < \frac{v}{t} < \frac{1}{3}$.

Consider now the case with $\frac{v}{t} < \frac{1}{3}$. I am going to show that under this condition there is an equilibrium such that the two search markets and consequently the two intermediated markets are disconnected. In this equilibrium, each market maker acts as a local monopolist who is constrained by the presence of a simultaneously open search market. This case is therefore analogous to those analyzed by Gehrig (1993) and Rust and Hall (2003).

The search markets are disconnected if, say, the buyer at location $\overline{x} < \frac{1}{4}$ is indifferent between joining the search market in W and being inactive. Similarly, if search markets are disconnected, a seller at some location $\overline{y} < \frac{1}{4}$ will be indifferent between joining the search market in W and being inactive. Let q^W be the quantity traded by the intermediary in W. The seller with location q^W will be indifferent between joining the search market and trading with the intermediary in W. Therefore, all sellers active in the search market will have costs larger than tq^W, so that no buyer at a location x, with $v - tx \leq tq^W$ can expect positive utility from search and will thus be inactive.[29] Thus, the buyer who is indifferent between joining the search market in W and being inactive has location

$$\overline{x} = \frac{v}{t} - q^W.$$

Similarly, the seller with

$$\overline{y} = \frac{v}{t} - q^W$$

just breaks even when entering the search market.

Thus, the buyer at q^W has an expected utility when joining the search market in W of

$$V_c^W(q^W) = \frac{1}{2}\left[v - tq^W - \frac{1}{2}\left(tq^W + v - tq^W\right)\right] = \frac{1}{4}\left(v - 2tq^W\right),$$

so that the inverse demand function facing intermediary in W is

$$A^W(q^W) = \frac{3}{4}v - \frac{1}{2}tq^W.$$

Similarly, the expected utility of search for seller q^W is

$$V_p^W(q^W) = \frac{1}{2}\left[v - tq^W - \frac{1}{2}\left(tq^W + v - tq^W\right)\right] = \frac{1}{4}\left(v - 2tq^W\right),$$

so that the inverse supply function facing the intermediary is

$$B^W(q^W) = \frac{1}{4}v + \frac{1}{2}tq^W.$$

[29] Like Gehrig (1993), I assume that buyers and sellers do not join a search market if their expected utility of doing so is not positive. Alternatively, one could introduce a small, positive fix cost of entering a search market.

The profit of the intermediary W is

$$\Pi^W(q^W) = \left(\frac{1}{2}v - tq^W\right) q^W,$$

which is maximized at $q^{W*} = \frac{v}{4t}$. As $\overline{x} = \frac{v}{t} - q^{W*}$ must be smaller than $\frac{1}{4}$,

$$\frac{3}{4}\frac{v}{t} < \frac{1}{4} \Leftrightarrow \frac{v}{t} < \frac{1}{3}$$

has to hold.

B Proportional Rationing

I am now going to show that in the model with homogenous sellers and heterogenous buyers of section 3, the equilibrium ask price is $a = v - \frac{1}{4}t$ if both intermediaries have a capacity of $\frac{1}{4}$ and if rationing is proportional. Note first that if the other intermediary sets this ask price, there is no incentive for the other one to set a lower price since the lower price would not increase quantity sold. On the other hand, higher prices cannot be optimal either because demand is price elastic. Thus, $a = v - \frac{1}{4}t$ is a best response to itself. The remainder of the argument is therefore only needed to show that no other prices can be mutually best responses. The line of reasoning is as follows. Prices such that $a^k < a^{-k} \leq v - \frac{1}{4}t$ cannot be set in equilibrium because intermediary k would have an incentive to raise its price since for any price no larger than a^{-k} the quantity sold is $\frac{1}{4}$. Therefore, the only candidate prices for another equilibrium are prices such that $a^W = a^E < v - \frac{1}{4}t$. As I will now show, such prices are not consistent with equilibrium because each intermediary has an incentive to raise its price. To see that, recall from footnote 10 that if there is random rationing (and if a rationed buyer can join the other intermediary), the buyer who at ask prices a^W and a^E is indifferent between the two intermediaries is the same as the indifferent buyer if there is no rationing, i.e., $\tilde{x} = \frac{1}{4} + \frac{a^E - a^W}{2t}$ is indifferent. Without loss, assume $a^E \leq a^W$. Then, the quantity demanded for intermediary E is $\frac{1}{2} - \tilde{x} = \frac{1}{4} + \frac{a^W - a^E}{2t} \geq \frac{1}{4}$, where $\frac{1}{4}$ is the capacity constraint or stock of intermediary E. Consequently,

$$\alpha = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{a^W - a^E}{2t}} = \frac{t}{t + 2(a^W - a^E)},$$

which is weakly smaller than one.

With $a^W \geq a^E$, the demand the intermediary in W faces consists of two parts. First, all buyers with $x \leq \tilde{x}$ will buy directly from W. Second, some consumers who get rationed at E are willing to buy from W at ask price a^W. Denote by $\hat{x}$ the buyer who is indifferent between buying from W and not buying at at all, i.e.,

$$\hat{x} = \frac{v - a^W}{t},$$

which is larger than $\tilde{x}$ for $a^W \leq v - \frac{1}{4}t$. Consequently, the fraction $(1-\alpha)$ of those consumers with $x \in [\tilde{x}, \hat{x}]$ will be rationed in E and will subsequently be willing to buy from the market maker in W. Therefore, the demand function it faces is $\tilde{x} + (1-\alpha)(\hat{x} - \tilde{x}) = \alpha\tilde{x} + (1-\alpha)\hat{x}$.

The revenue of the intermediary in W is

$$R^W(a^W, a^E) = a^W(\alpha\tilde{x} + (1-\alpha)\hat{x}).$$

Differentiating $R^W(a^W, a^E)$ with respect to a^W when $a^W = a^E \equiv a$ yields

$$\frac{\partial R^W(a^W, a^E)}{\partial a^W} |_{a^W = a^E = a} = \frac{t^2 + 8a(v-a)}{4t^2}.$$

Obviously, this is greater than zero for all $a \leq v$, and therefore for all $a < v - \frac{1}{4}t$. Hence, it follows that for any $a^W = a^E < v - \frac{1}{4}t$ either market maker has an incentive to raise its price. Consequently, no prices other than $a^W = a^E = v - \frac{1}{4}t$ can be set in equilibrium.

C Proof of Proposition 4

Existence. I first show that the strategies mentioned in the proposition constitute an equilibrium. It has already been shown in the text that given that search markets exist where all buyers and sellers behave as stated, market makers' prices are mutually best responses. It has also been shown that given these prices and given the behavior of all other buyers and sellers, every buyer and seller is best off playing the strategy assigned to him in the proposition. What remains to be shown in order to prove existence is therefore that no market maker has an incentive to unilaterally deviate and to extinguish the search markets.

Consider the market maker in W who can "kill" the search market by attracting all buyers and sellers who are not attracted by the intermediary in E. That is, intermediary W could set prices $\tilde{b}^W$ and $\tilde{a}^W$ such that

$$\tilde{b}^W - \frac{10}{26}t = \overbrace{\frac{1}{2}v - \frac{7}{208}t}^{=b^*} - \frac{3}{26}t \tag{52}$$

and

$$v - \tilde{a}^W - \frac{10}{26}t = v - \overbrace{\left(\frac{1}{2}v + \frac{7}{208}t\right)}^{=a^*} - \frac{3}{26}t, \tag{53}$$

where the right-hand side of both equations is the net utility of the seller and buyer at $\frac{10}{26}$ of patronizing the market maker in E. Solving for $\tilde{b}^W$ and $\tilde{a}^W$ reveals that

$$\tilde{b}^W = \frac{1}{2}v + \frac{49}{208}t > \frac{1}{2}v - \frac{49}{208}t = \tilde{a}^W.$$

In other words, the bid price required by this deviation exceeds the ask price. Therefore, this deviation cannot be profitable.

However, there is a priori no reason why the intermediary should contend itself with symmetric strategies when attempting to extinguish the much disliked search market. Similar to the model of Stahl (1988), it is in principle enough to buy the whole stock in order to corner the market. Depending on the elasticity of the demand function, it may then be in its best interest to sell less than it bought. I am going to show now that even this asymmetric deviation strategy is not profitable.

So as to see this, note first that the buyer with location $q < \frac{10}{26}$ is indifferent between buying from the intermediary in W at a^W and the intermediary in E at a^* if and only if

$$v - a^W - tq = v - a^* - t\left(\frac{1}{2} - q\right),$$

which after replacing a^* by $\frac{1}{2}v + \frac{7}{208}t$ and re-arranging is seen to imply

$$\tilde{A}^W(q) = \frac{1}{2}v + \frac{111}{208}t - 2tq$$

as the inverse demand function. The revenue maximizing quantity $\tilde{q}^* \equiv \arg\max_q A^W(q)q$ is therefore

$$\tilde{q}^* = \frac{104v + 111t}{832t}.$$

The maximal revenue is

$$\frac{(104v + 111t)^2}{346112t}.$$

On the other hand, the expenditure required to buy the quantity $\frac{10}{26}$ is

$$\tilde{b}^W \cdot \frac{10}{26} = \frac{5}{26}v + \frac{245}{2704}t.$$

The deviation has already been shown not to pay for $\tilde{q}^* = \frac{10}{26}$, so only the case with $\tilde{q}^* < \frac{10}{26}$ needs to be considered further. But $\tilde{q}^* < \frac{10}{26}$ holds if and only if $v < \frac{209}{104}t$, whereas the deviation is profitable if and only if

$$\frac{(104v + 111t)^2}{346112t} - \left(\frac{5}{26}v + \frac{245}{2704}t\right) \geq \frac{27}{2704}t.$$

This in turn requires v to be larger than $4.4t$. Thus, there is no feasible, profitable deviation.

Uniqueness. The proof of uniqueness consists of three parts. I first show that for given symmetric ask and bid prices $\hat{a}$ and $\hat{b}$ that are the same in E and W, where symmetry means that for any spread $z \geq 0$

$$\hat{a} = \frac{v+z}{2} \quad \text{and} \quad \hat{b} = \frac{v-z}{2}$$

holds, there is a unique quantity $q^*(z)$ traded by each intermediary. (Note also that the prices a^* and b^* are symmetric.) Second, I show that there are no equilibria where ask and bid prices are symmetric in E and W, but where $a^W \neq a^E$ (and $b^W \neq b^E$). Third, I show that there are no equilibria where an intermediary sets asymmetric prices.

Claim I: For any spread z that is symmetric around $\frac{v}{2}$, there is a unique quantity $q^*(z)$ of sellers and buyers joining each intermediary.

Note: The concern here is that in principle there may be a coordination problem between sellers and buyers because if more (high valuation) buyers are active in the search market, search markets are more attractive for sellers. I will show that because of the market maker's commitment to buy any alternative candidate equilibrium unravels.

Proof: For analytical ease, I assume that if there is rationing at an intermediary, the efficient rationing rule applies. That is, buyers and sellers who are closer to an intermediary have priority over agents who are further away. This assumption is not implausible if agents are served on a first come first serve basis and if agents who are closer to an intermediary reach the intermediary before others do. Buyers and sellers who are rationed can go back to the search market.[30]

Assume $q^W = q^E = q^*$ and invert equation (43) to get quantity $q^*(z)$ as a function of the spread z (replacing Z^W by z). It is easy to see that

$$q^*(z) = \frac{1}{4} - 2\frac{z}{t}.$$

Step 1a: Given $\hat{b} = \frac{v-z}{2} > 0$, there is a unique $y_1 \in (0, q^*(z)]$ such that all $y \leq y_1$ and all $y \geq \frac{1}{2} - y_1$ join the intermediaries in W and E even if all buyers are active in the search market. The remainder of the argument applies repeatedly the same two steps.

[30] If proportional (or random) rationing were assumed, then it is almost indispensable to assume that rationed agents cannot go back to the search market because otherwise, expected utility from search market participation is very hard to compute.

Proof: Solve

$$V_p^W(y_1) = \frac{1}{2}\left[\frac{1}{2}(v + v - t\frac{1}{4}) - ty_1\right] = \hat{b} - ty_1$$

for y_1 to get

$$y_1 = \frac{1}{8} - \frac{z}{t} = \frac{1}{2}q^*(z).$$

Note that the assumption $\tilde{x} = \frac{1}{4}$ has been implicitly used. Since initially, i.e., before the set of sellers in $[0, y_1]$ and $[\frac{1}{2} - y_1]$ joined the intermediaries in W and E, the two search markets are symmetric, the assumption is indeed correct. Moreover, the search markets being symmetric after step 1a, the assumption that $\tilde{y} = \frac{1}{4}$ is correct and can be used in step 1b. For the same reasons, $\tilde{x} = \tilde{y} = \frac{1}{4}$ will hold in any subsequent step.

Step 1b: Given $\hat{a}$ and that the y_1 most efficient sellers leave the search markets, there is a $x_1 \in (0, q^*(z)]$ such that all buyers with $x \leq x_1$ and all $x \geq \frac{1}{2} - x_1$ join the intermediaries in W and E.

Proof: Here the efficient rationing rule comes to play a role. Solve

$$V_c^W(x_1') = \frac{1}{2}\left[v - tx_1' - \frac{1}{2}(ty_1 + t\frac{1}{4})\right] = v - tx_1' - \hat{a}$$

for x_1' to get

$$x_1' = \frac{1}{8} - \frac{z}{t} + \frac{1}{4}q^*(z) = \frac{3}{4}q^*(z) > y_1.$$

However, since only the $y_1 < x_1'$ closest buyers will be served, only the y_1 closest buyers will leave the search market. So, let

$$x_1 = y_1.$$

Step 2a: Given steps 1a and 1b, there is a $y_2 \in (y_1, q^*(z)]$ such that all $y \leq y_2$ and all $y \geq \frac{1}{2} - y_2$ join the intermediaries in W and E even if all remaining buyers are active in the search market.

Proof: Solve

$$V_p^W(y_2) = \frac{1}{2}\left[\frac{1}{2}(v - tx_1 + v - t\frac{1}{4}) - ty_2\right] = \hat{b} - ty_2$$

for y_2 to get

$$y_1 = \frac{3}{16} - \frac{3}{2}\frac{z}{t} = \frac{3}{4}q^*(z).$$

Step 2b: Apply the same reasoning as in step 1b to establish that

$$x_2 = y_2.$$

Step k: In general, after the k-th step, the buyers and sellers attracted by an intermediary will be

$$y_k = \sum_{i=1}^{k} \left(\frac{1}{2}\right)^i q^*(z).$$

Let k go to infinity and use the formula for a geometric sum to see that

$$y_\infty \equiv \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i q^*(z) = q^*(z).$$

Thus, there is a unique equilibrium with active search markets given symmetric prices $\hat{a}, \hat{b}$. This completes the proof of claim I. ■

Claim II: There are no equilibria where spreads are symmetric but different in E and W. That is, there are no equilibria with $z^W \neq z^E$, z^k being such that $a^k = \frac{v+z^k}{2}$ and $b^k = \frac{v-z^k}{2}$, $k = E, W$.

Proof: Consider equation (43) and the corresponding equation for E, which is

$$Z^E(q^E, q^W) = t\left(\frac{1}{8} + \frac{1}{12}q^W - \frac{7}{12}q^E\right).$$

The two first order conditions

$$0 = Z^{k'}(q^k, k^{-k})q^k + Z^k(q^k, k^{-k})$$

for $k = E, W$ have a unique solution, with $q^k = q^{-k}$. Accordingly, $z^W = z^E$ will hold. This completes the proof of claim II. ■

Claim III: There are no equilibria with asymmetric spread(s).

Proof: Let $y(b, a)$ and $x(a, b)$ be the sellers and buyers joining the intermediary in W who sets price a and b. Assume first that its prices a and b are such that $y(b, a) < x(a, b)$. Since the rationed buyers (who are located in $[y(b, a), x(a, b)]$) will join the search market, increasing a until $x(a, b) = y(b, a)$ will unambiguously increase the profit of the intermediary.

So, assume that $y(b, a) > x(a, b)$. Ruling out this case is not straightforward because increasing b and thereby decreasing y will make search market participation more attractive for buyers, thus reducing their willingness to pay. Put differently, one reason why an intermediary might choose prices a and b to induce $y(b, a) > x(a, b)$ is that this increases buyers' reservation prices to buy from the intermediary because search market participation is less attractive. However, as I will show now, such a policy will never be optimal because it will be in the interest of the intermediary to sell all the quantity it buys.

Denote by $\hat{A}^W(x)$ the inverse demand function facing the intermediary in W when inducing an unbalanced search market in W. Clearly, a necessary condition for the policy to be profitable is that it results in an outward shift of this inverse demand function, i.e.,

$$\hat{A}^W(x) > A^W(q^W, q^E)$$

must hold for $x = q^W$, where $A^W(q^W, q^E)$ is as defined in (38). Denote $q_0 \equiv \arg\max_{q^W} A^W(q^W, q^E)q^W$. It is easy to see that

$$q_0 = \frac{6}{7}\frac{v}{t} + \frac{3}{28} + \frac{1}{14}q^E.$$

Since $\frac{v}{t} > \frac{3}{2}$, $q_0 > \frac{1}{2}$ follows (which of course is not feasible, but that is immaterial for the present argument). In other words, the intermediary would like to sell more than its stock if there were no costs of acquiring stock (and neglecting any other constraints). Now, because $\hat{A}^W(x) > A^W(q^W, q^E)$,

$$x_0 \equiv \arg\max_x \tilde{A}^W(x)x > q_0$$

follows. Since $y(b, a) > x(a, b)$ holds by hypothesis, it will be both possible and desirable to sell more than $x(a, b)$ by adjusting prices to a', b' such that $y(b', a') = x(a', b')$ holds. Thus, the strategy (a, b) such that $y(b, a) > x(a, b)$ cannot be optimal.

The final thing to show is that $y(b, a) = x(a, b)$ can only be achieved by symmetric prices $a = \frac{v+z}{2}$ and $b = \frac{v-z}{2}$. But the seller at location q^W will be indifferent between selling to W at b^W and joining the search market in W if and only if

$$b^W = B^W(q^W, q^E) = \frac{v}{2} - \frac{1}{16}t - \frac{1}{24}tq^E + \frac{7}{24}tq^W,$$

where $B^W(q^W, q^E)$ is the inverse supply function from equation (41). Similarly, given ask price a^W the buyer at q^W is indifferent to buy from the intermediary and participating in the search market if and only if

$$a^W = A^W(q^W, q^E) = \frac{v}{2} + \frac{1}{16}t + \frac{1}{24}tq^E - \frac{7}{24}tq^W,$$

where $A^W(q^W, q^E)$ is the inverse demand function in (38). Clearly, $a^W = \frac{v+z}{2}$ and $b^W = \frac{v-z}{2}$ with $z = \frac{1}{8}t + \frac{1}{12}tq^E - \frac{7}{12}tq^W$. Thus, given that the other intermediary attracts the same number of buyers and sellers (i.e., q^E), an intermediary can attract the same number of buyers and sellers q^W if and only if it sets symmetric prices. ■

D Less Than Fully Efficient Search Markets

Throughout the paper, I have maintained the assumption that search markets are fully efficient. Full efficiency obtained because (i) all agents are matched with probability one if the search markets are balanced and (ii) there are no mismatches, i.e., all buyers and sellers exchange the good to the benefit of both if matched. I now replace (i) and assume instead that in a balanced search market, each agent is matched to an agent of the opposite type with probability

$$\lambda \in [0, 1].$$

If the search market is not balanced, say, because there are more buyers than sellers, buyers are matched with probability $\gamma_c \lambda < \lambda$, while sellers are matched with probability λ. This exercise is motivated by the observation that search markets need not be as efficient as intermediated markets. It is also of interest to see how changes in the search parameter λ affect equilibrium behavior. Apart from capturing search frictions, the parameter λ can be interpreted as capturing effects of legal and regulatory institutions. For example, one main reason why in many countries firms have a preference to hire workers temporarily via intermediaries is that hire (and fire) is much easier in this way than if they employed workers themselves. Therefore, more restrictive labor laws would correspond to a lower value of λ.

Reservation Prices of Buyers and Sellers It is easy to see that the only relevant change of introducing the search friction $\lambda \in [0, 1]$ is that the terms in brackets in equations (19) and (20) have to be multiplied by λ. Thus, the buyer and seller at q^W who are indifferent between search market participation and trading with the intermediary in W expect utility

$$V_c(q^W) = \frac{\lambda}{2}\left[v - tq^W - \frac{1}{2}(tq^W + t\tilde{x}(q^W, q^E))\right] = \lambda\frac{1}{2}v - \lambda t\frac{17}{24}q^W - \lambda t\frac{1}{16} + \lambda t\frac{1}{24}q^E$$

and

$$V_p(q^W) = \frac{\lambda}{2}\left[v - tq^W - \frac{1}{2}(tq^W + t\tilde{x}(q^W, q^E))\right] = \lambda\frac{1}{2}v - \lambda t\frac{17}{24}q^W - \lambda t\frac{1}{16} + \lambda t\frac{1}{24}q^E$$

from search market participation.

The indifference conditions are

$$v - tq^W - a^W = V_c(q^W)$$

and

$$b^W - tq^W = V_p(q^W),$$

yielding the inverse demand and supply functions

$$A^W(q^W, q^E) = \frac{2-\lambda}{2}v - \left(1 - \frac{17}{24}\lambda\right) tq^W + \frac{1}{16}\lambda t + \frac{1}{24}\lambda t q^E \quad (54)$$

and

$$B^W(q^W, q^E) = \frac{\lambda}{2}v + \left(1 - \frac{17}{24}\lambda\right) tq^W - \frac{1}{16}\lambda t - \frac{1}{24}\lambda t q^E. \quad (55)$$

Notice that $A^W(.)$ and $B^W(.)$ are not symmetric around $\frac{v}{2}$ for $\lambda < 1$. As will be seen shortly, this is of some importance for the model with less than fully efficient search markets.

The profit of the intermediary in W is thus

$$\Pi^W(q^W, q^E) = \left[(1-\lambda)v - 2\left(1 - \frac{17}{24}\lambda\right) tq^W + \frac{1}{8}\lambda t + \frac{1}{12}\lambda t q^E\right] q^W.$$

The first order condition is

$$0 = (1-\lambda)v - 4\left(1 - \frac{17}{24}\lambda\right) tq^W + \frac{1}{8}\lambda t + \frac{1}{12}\lambda t q^E,$$

which can be solved for q^{W*} using the symmetry $q^{W*} = q^{E*} = q^*(\lambda)$ to get

$$q^*(\lambda) = \frac{12}{48 - 35\lambda}\left[(1-\lambda)\frac{v}{t} + \frac{\lambda}{8}\right]. \quad (56)$$

Note that for $\lambda = 1$, $q^*(\lambda) = \frac{3}{26}$, which is as it ought to be.

The implied equilibrium prices are

$$a^*(\lambda) = \frac{576v - 608v\lambda + 136v\lambda^2 + 24\lambda t - 17\lambda^2 t}{16(48 - 35\lambda)}$$

and

$$b^*(\lambda) = \frac{192v + 48v\lambda - 136v\lambda^2 - 24\lambda t + 17\lambda^2 t}{16(48 - 35\lambda)},$$

so that equilibrium profits are

$$\Pi^*(\lambda) = \frac{3\left(192v - 328v\lambda + 136v\lambda^2 + 24\lambda t - 17\lambda^2 t\right)\left(8v(1-\lambda) + \lambda t\right)}{16t(48 - 35\lambda)^2}.$$

It is easy to check that for $\lambda = 1$, $a^*(\lambda) = \frac{v}{2} + \frac{7}{208}t$, $b^*(\lambda) = \frac{v}{2} - \frac{7}{208}t$ and $\Pi^*(\lambda) = \frac{21}{2704}t$.

Notice though that for the search markets to be active,

$$q^*(\lambda) < \frac{1}{4}$$

must hold. Solving $q^*(\lambda) = \frac{1}{4}$ for λ yields the critical threshold value

$$\lambda^*(v/t) \equiv \frac{\frac{v}{t} - 1}{\frac{v}{t} - \frac{41}{48}}. \quad (57)$$

Only for $\lambda > \lambda^*$ can the search markets be active. Note that because $v/t > 3/2$ is assumed, $\lambda > \frac{24}{31} \approx 0.77$ must hold for the search markets to be active.

In addition, it must also be the case that no intermediary has an incentive to "kill" the search market by attracting all the sellers $\frac{1}{2} - q^*(\lambda)$ not attracted by the other one. The relative attractiveness of this option depends on v in a way that is not clear a priori. On the one hand, as v increases, $q^*(\lambda)$ increases, so that a smaller gap has to be bridged in order to extinguish the search market. On the other hand, the profit $\Pi^*(\lambda)$ also depends positively on v, which all else equal runs counter the profitability of the deviation.

This deviation, say, by the intermediary in W, requires setting ask and bid prices $\tilde{a}$ and $\tilde{b}$ such that

$$v - t\left(\frac{1}{2} - q^*(\lambda)\right) - \tilde{a} = v - tq^*(\lambda) - a^*(\lambda)$$

and

$$\tilde{b} - t\left(\frac{1}{2} - q^*(\lambda)\right) = b^*(\lambda) - tq^*(\lambda).$$

Solving yields the deviation prices

$$\tilde{a} = \frac{960v - 992v\lambda + 136v\lambda^2 + 352\lambda t - 17\lambda^2 t - 384t}{16(48 - 35\lambda)}$$

and

$$\tilde{b} = \frac{192v - 432v\lambda + 136v\lambda^2 + 352\lambda t - 17\lambda^2 t - 384t}{16(48 - 35\lambda)}.$$

Therefore, the deviation profit $\tilde{\Pi}$ is

$$\begin{aligned}\tilde{\Pi} &= (\tilde{a} - \tilde{b})\left(\frac{1}{2} - q^*(\lambda)\right) \\ &= \frac{(-384t + 352\lambda t + 576v - 712v\lambda + 136v\lambda^2 - 17\lambda^2 t)(24t - 19\lambda t - 12v + 12v\lambda)}{8(48 - 35\lambda)^2 t}.\end{aligned}$$

The equation $\Pi^*(\lambda) = \tilde{\Pi}$ has two solutions in v/t,

$$\frac{v}{t} = \frac{48 - 41\lambda}{48(1 - \lambda)} \quad \text{and} \quad \frac{v}{t} = \frac{17\lambda^2 - 328\lambda + 384}{8(17\lambda^2 - 65\lambda + 48)}.$$

This confirms that increases in v/t have ambiguous effects on the profitability of deviation. For all values in between, $\Pi^*(\lambda) < \tilde{\Pi}$ holds, so that deviation pays for all

$$\frac{v}{t} \in \left[\frac{48 - 41\lambda}{48(1 - \lambda)}, \frac{17\lambda^2 - 328\lambda + 384}{8(17\lambda^2 - 65\lambda + 48)}\right],$$

while for all other values, $\Pi^*(\lambda) > \tilde{\Pi}$ holds, so that deviation does not pay. However, notice that $\frac{v}{t} = \frac{48-41\lambda}{48(1-\lambda)}$ is the inverse of $\lambda^*(v/t)$. So, the prices $a^*(\lambda)$

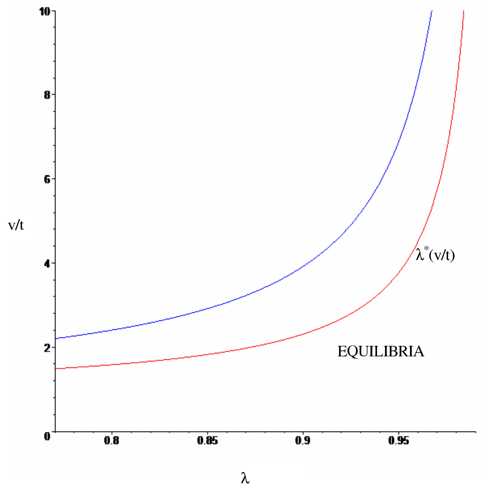

Figure 5: Equilibrium with Active Search Markets Exists for all $\lambda \in [\lambda^*, 1]$.

and $b^*(\lambda)$ are an equilibrium if and only if

$$\lambda > \lambda^* \left(\frac{v}{t}\right) \Leftrightarrow \frac{v}{t} < \frac{48 - 41\lambda}{48(1-\lambda)}.$$

Thus, the deviation is not profitable for those values of λ and $\frac{v}{t}$ for which $q^*(\lambda) < \frac{1}{4}$ holds; see Figure 5.

Finally, I have to check for asymmetric deviations where an intermediary deviates to buy the whole stock $\frac{1}{2} - q^*(\lambda)$ but sells less. Let W contemplate this type of deviation and set a'. When the intermediary in E sets the ask price $a^*(\lambda)$, the buyer at the position x is indifferent between E and W whenever

$$v - tx - a' = v - t\left(\frac{1}{2} - x\right) - a^*(\lambda).$$

Thus, the inverse demand function the deviating intermediary in W faces is

$$A'(x) = a^*(\lambda) + \frac{1}{2}t - 2tx.$$

Maximizing $A'(x)x$ over x yields as optimal quantity $x^* = \frac{a^*(\lambda)}{4t} + \frac{1}{8}$. It is easy to check that $x^* > \frac{1}{2} - q^*$. Thus, it will never be optimal not to sell all the quantity the intermediary buys when deviating in this manner. Therefore, no such deviation will be profitable.

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