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Summary. In this paper we explore the link between wealth inequality and stability in a two-sector neoclassical growth model with heterogeneous agents. The stability of the steady state depends on the various parameters of the model and in particular on individual preferences. We show that when consumers have identical preferences and the inverse of absolute risk aversion (or risk tolerance) is a strictly convex function, inequality is a factor that favors instability. In the opposite case, inequality favors stability. Our characterization also shows that whenever absolute risk tolerance is linear, as when preferences exhibit hyperbolic absolute risk aversion (HARA), wealth heterogeneity is neutral. As there is not yet evidence on the concavity of absolute risk tolerance, our results unfortunately do not lead to a unique conclusion on the sign of the effect of wealth inequality on stability.

Economic growth, Heterogeneity, Wealth and Income Inequality, Instability.

JEL-classification numbers: D30, D50, D90, O41.

1 Introduction

The relation between wealth or income inequality and growth has been explored in a large number of theoretical and empirical studies (recent surveys are Aghion, Caroli and Garcia-Penalosa (1999) and Benabou (2000)). In the present paper we focus on how inequality affects the dynamics of a two-sector neoclassical growth model. Dynamic considerations are relevant for this issue as there is no a priori reason to believe that economies are on their balanced growth path. The channel we consider is rather straightforward. Wealth heterogeneity affects the "social" utility function and conesquently the stability properties of the equilibrium even in the absence of heterogeneity in preferences. The issue is then to relate plausible specifications of preferences with the direction of the effect of heterogeneity.

We adopt the simple version of the neoclassical two-sector growth model with a single consumption good considered by Boldrin and Deneckere (1990) but we abandon the representative agent assumption and admit non-linear utility functions. Agents may be heterogeneous in respect to the share of the initial stock of capital and in labor endowments, as well as in preferences. As labor is provided inelastically labor endowments are considered as exogenous parameters. Furthermore, due to the structure of the model individual characteristics and heterogeneity do not affect the steady state values of the aggregate variables.

The analysis is standard for economies with heterogenous agents. First, we focus on the properties of the Pareto optimal allocations. These are obtained as solutions to a social planner's problem characterized by a utility function depending on the welfare weights. In the model, these weights are continuous functions of the initial conditions (see Ghiglino and Olszak-Duquenne (2001) and Ghiglino (2002)). Consequently, the local dynamic properties of the general equilibrium model with heterogeneous agents and those of the planner's problem with the welfare weights fixed at their steady state values are identical. We are then able to obtain the dynamic properties of the equilibrium path in the aggregate variables for exogenously fixed welfare weights. Decentralization of these equilibria only occurs at a second stage of our analysis where we characterize the effect of agent's heterogeneity on dynamics.

Ii is known that for some plausible specification of technology and preferences this model exhibit instability and fluctuations (Boldrin and Deneckere (1990) and Ghiglino and Olszak-Duquenne (2001)). Here we give the conditions on the individual utility functions such that wealth heterogeneity favors instability and the conditions such that the opposite occurs. We find that when the inverse of absolute risk aversion is a concave function heterogeneity favors stability. The result is driven by the fact that stability depends monotonously on absolute risk tolerance, at least over the relevant range. Consequently, the characterization involves the concavity of absolute risk tolerance, i.e. the derivatives of the utility function as high as the fourth order. Unfortunately there is little direct empirical evidence concerning their value and sign. Some weak and indirect evidence in support of the concavity of absolute risk tolerance can be found (see Gollier (2001)). According to the present paper, this evidence would suggest that agent's heterogeneity favors stability. On the other hand, for the class of preferences exhibiting hyperbolic absolute risk aversion (HARA) income heterogeneity is neutral.

The tractability of the model is based on some simplifying assumptions. First, there is only one consumption good and one capital good. Second, as the production functions are analytically specified the technology is implicitly restricted to belong to some class. Finally, labor is provided inelastically. The values of the parameters giving rise to fluctuations in the present model are not particularly plausible (see for ex. Boldrin and Deneckere (1990)). However, we don't think this is a major weakness as many subsequent papers have shown that fluctuations are possible with reasonable parameter values in two-sector growth model. We also believe that our results hold in more general frameworks. However, more research is needed to quantify this statement.

Beside Ghiglino and Olszak-Duquenne (2001) the present paper is related to Ghiglino and Sorger (2002). In that paper, indeterminacy is shown to occur in a continuous time, endogenous growth model with an externality and heterogeneous agents. However, their analysis fail to qualify the effects of redistributions on the occurrence of indeterminacy because the welfare weights cannot be proven to be continuous functions of the initial conditions.

The paper is organized as follows: In section 2 the model is introduced while the equilibria are defined in Section 3. Section 4 focuses on the relationship between endowment distribution and instability. In section 5 the occurrence of instability is related to heterogeneity. Section 6 concludes.

2 The model

In the present paper we consider a competitive two-sector economy with heterogenous agents. The technology is formalized as in Boldrin and Deneckere (1990). There is no joint-production and firms produce according to constant returns production functions so that at the optimum, profits are zero. There are two produced goods, a consumption good and a capital good. The consumption good cannot be used as capital so it is entirely consumed. The capital good cannot be consumed. There are two inputs, capital and labor. We also suppose that there is instantaneous capital depreciation and that labor is inelastically used in production.

There are two firms, one for each sector. The firm in the first sector produces a consumption good from capital and labor according to a production function $F^1(k^1, l^1)$. We assume that $F^1(k^1, l^1) = (l^1)^\alpha (k^1)^{1-\alpha}$ with $\alpha \in (0,1)$ where l^1, k^1 are the amount of capital and labor used by the firm of the consumption sector. In a decentralized economy, the firm maximizes profit

$$Max\ p_t^1\ F^1(k_t^1, l_t^1) - p_{t-1}^2 k_t^1 - w_t l_t^1$$

where p_t^1 is the present price of the consumption good at period t, p_{t-1}^2 is the present price of the capital good bought at period $t-1$ and w_t the present price of labor at period t. In the second sector, the representative firm produces a capital good according to a Leontief function $F^2(k^2, l^2) = Min\ (l^2, \frac{k^2}{\gamma})$ with $\gamma \in (0,1)$. The optimal production plan for this firm is

$$l_t^2 = \frac{k_t^2}{\gamma}$$

There are n agents. In each period consumers provide inelastically a constant amount of labor ω_i, $i = 1, ..., n$ with $\sum_{i=1}^n \omega_i = 1$. A model in which the amount of labor provided is endogenously determined could be analyzed but at a much higher cost. At the beginning of the economy, each agent i is endowed with a fixed share θ_i of the initial stock k_0 of capital, with $\sum_{i=1}^n \theta_i = 1$. Consumer's preferences are characterized by a discounted utility function of the form

$$U^i(x^i) = \sum_{t=0}^{\infty} \delta^t \quad u_i(x_{it})$$

where x_{it} is the consumption of agent i at time t and x^i is its intertemporal consumption stream. We assume $\delta > \gamma$. The instantaneous utility function fulfills the Inada condition

$$\lim_{x_{it} \to 0} u_i'(x_{it}) = +\infty.$$

In a decentralized economy, an agent i maximizes his utility function subject to a single budget constraint

$$\sum_{t=0}^{\infty} p_t^1 x_{it} = \sum_{t=0}^{\infty} w_t \omega_i + \theta_i k_0 \quad \text{with } i = 1, ..., n.$$

where we have normalized the price of k_0 to unity.

3 Competitive equilibria and the path of capital

In the present economy the first welfare theorem holds. Every competitive equilibrium obtained in the decentralized economy is a Pareto optimum in the sense that it is the solution to the maximization of a social welfare function. In the current section we first define competitive equilibria and then characterize the set of Pareto optima.

Definition 1 *A competitive equilibrium is a sequence of prices* $(p_t^1, p_t^2, w_t)_{t=0}^{\infty}$ *such that markets clear for every* $t \geq 0$

- $l_t^1 + l_t^2 = \sum_{i=1}^{n} \omega_i = 1$
- $k_{t+1}^1 + k_{t+1}^2 = F^2(k_t^2, l_t^2)$
- $\sum_{i=1}^{n} x_{it} = F^1(k_t^1, lx_t^1)$
- $k_0^1 + k_0^2 = k_0$ *with* k_0 *given*

where

- (x_{it}) *is a solution to the individual maximization program of agent* i, $i = 1, ..., n$ *for* $(p_t^1, p_t^2, w_t)_{t=0}^{\infty}$.

- (k_t^j, l_t^j) *is a solution to profit maximization for firm* j, $j = 1, 2$ *for* $(p_t^1, p_t^2, w_t)_{t=0}^{\infty}$.

Every competitive equilibrium is a Pareto optimal allocation. A Pareto optimal allocation is a solution to the planner's problem for a given vector of welfare weights $\mu \in [0,1]^{n-1}$:

$$\begin{aligned} Max \quad & \sum_{i=1}^{n-1} (\mu_i \sum_{t=0}^{\infty} \delta^t \ u_i(x_{it})) + (1 - \sum_{i=1}^{n-1} \mu_i) \sum_{t=0}^{\infty} \delta^t \ u_n(x_{nt}) \\ s.t. \quad & \sum_{i=1}^{n} x_{it} = F^1(k_t^1, l_t^1) \ \text{ for all } t \\ & k_{t+1}^1 + k_{t+1}^2 = F^2(k_t^2, l_t^2) \ \text{ for all } t \\ & l_t^1 + l_t^2 = 1 \ \text{ for all } t \\ & k_0 \ \text{ given} \end{aligned}$$

The solution to the above program depends on the vector μ and on k_0. The set of Pareto optima is obtained when μ spans $[0,1]^{n-1}$ with $\sum_{i=1}^{n-1} \mu_i \leq 1$. A given competitive equilibrium is obtained for a μ such that the associated allocations saturate the budget constraint of all the consumers.

Note also that the solutions are interior as soon as $\omega_i \neq 0$ or $\theta_i \neq 0$ for $i = 1, ...$ As shown in Ghiglino and Olszak-Duquenne (2001) this is a consequence of the Inada conditions on preferences and technology.

Let u_μ be a social utility function defined by

$$\begin{aligned} u_\mu(x) \ = \ & Max \sum_{i=1}^{n-1} \mu_i u_i(x_{it}) + (1 - \sum_{i=1}^{n-1} \mu_i) \ u_n(x_{nt}) \\ & s.t \ \sum_{i=1}^{n} x_{it} = x \end{aligned}$$

Let $T(k,y)$ be the usual transformation function giving the maximal output in the capital good compatible with total capital input k and to consumption output at least equal to y. With the specification of production adopted through the paper $T(k,y) = (1-y)^{\alpha}(k-\gamma\ y)^{1-\alpha}$. Let the return function be $V : R_+ \times R_+ \rightarrow R$ defined by $V(k,y) = u_\mu(T(k,y))$. Then the planner's problem is equivalent to

$$\begin{aligned} Max \quad & \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t+1}) \\ s.t. \quad & F^2(k_t, 1) \geq k_{t+1} \\ & k_0 \ \text{ given} \end{aligned}$$

Note that the solution depends on k_0.

In the present framework it is a standard result that the set of interior Pareto optima are the set of $\{k_t\}_t$ that satisfies the transversality condition $\lim_{t\longrightarrow\infty} \delta^t V_1(k_t, k_{t+1})k_t = 0$ and are solutions to the system

$$V_2(k_t, k_{t+1}) + \delta V_1(k_{t+1}, k_{t+2}) = 0 \quad \forall t \geq 0$$

where V_j represents the first order derivative in respect to the jth argument.

An interior aggregate steady state is a sequence $k_t = k^*, \forall t \geq 0$, that solves the set of Euler equation. The steady state capital k^* can be expressed as a function of the discount factor and the technology parameters only

$$k^* = \frac{(1-\alpha)(\gamma-\delta)}{\gamma-\alpha-\delta(1-\alpha)}.$$

The aggregate consumption x^* can also be obtained

$$x^* = T(k^*, k^*) = k^*(k^{*-1} - 1)^{\alpha}(1-\gamma)^{1-\alpha}.$$

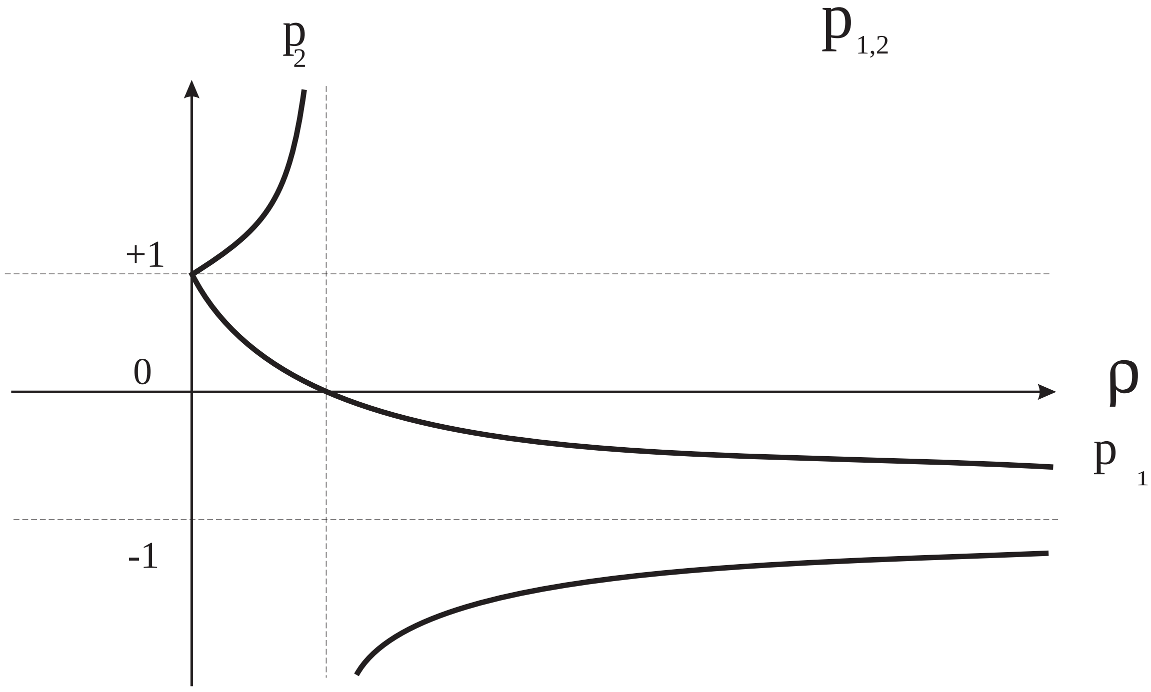
Note that at the steady state, aggregate capital and consumption depend only on the total labor supply. This property is a consequence of the fact that at the steady state the return function can be eliminated from the Euler equation.

Near the steady state the behavior of the dynamic system is equivalent to the behavior of the linearized system. The dynamic properties of the steady state are then related to the eigenvalues of the matrix associated to the linearized system. In particular, the stability property of the steady state depends on how the modulus of the two eigenvalues compare to one. In fact, these can be shown to depend on the first and second order derivatives of the instantaneous utility function.

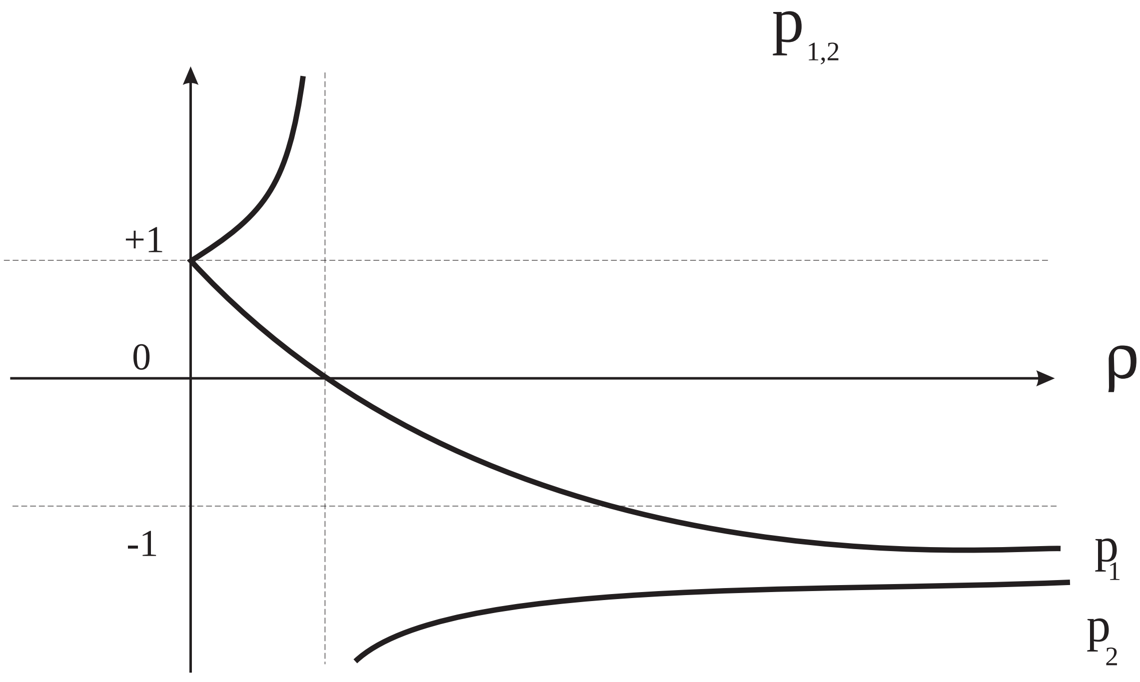
Definition 2 *Let* u *be the social utility function,* $u : R_+ \rightarrow R$*. Let* $\rho(x) = -\frac{u'(x)}{u''(x)}$ *be the inverse of the absolute risk aversion, also called absolute risk tolerance..*

For a given discount factor and technology parameters, the eigenvalues depend on ρ. The relationship is represented in Fig. 1 where p_1 is the eigenvalue with the smallest modulus, i.e. $| p_1 |<| p_2 |$. Note that ρ is positive and that ρ close to zero indicates a high degree of curvature of the utility function. A property which plays an important role in the subsequent developments is that at most one of the two graphs $p_1(\rho)$ and $p_2(\rho)$ intersects the horizontal line drawn at -1. This is a consequence of the fact that the branches of $p_i(\rho)$ are monotonous for large values of ρ.

The graphs $p_i(\rho), i = 1, 2$, depend on the parameters (α, γ, δ). A change in one of the parameters modify the graphs. The following Lemma gives the stability properties of the steady state as a function of the technology parameters, the aggregate endowments, the discount factor and absolute risk tolerance ρ.



a) Stability



b) Instability

Fig. 1

Lemma 1 *Let* $\delta_c = \frac{\gamma}{1-2\alpha}$, $\delta_{cc} = \frac{\alpha+\gamma}{1-\alpha}$. *Then,*

1. *If* $\alpha \geq 0.5(1-\gamma)$ *then the stability of the steady state is independent of* ρ.

2. *If* $\alpha < 0.5(1-\gamma)$ *and* $\delta \notin]\delta_c, \delta_{cc}[$ *then the steady state is (saddle-path) stable.*

3. *If* $\alpha < 0.5(1-\gamma)$ *and* $\delta \in]\delta_c, \delta_{cc}[$ *then*

$$\begin{array}{lcl} \rho \geq \rho_c & \Leftrightarrow & \textit{Unstable steady state} \\ \rho \in [0, \rho_c[& \Leftrightarrow & \textit{Stable steady state} \end{array}$$

with

$$\rho_c = \frac{2\alpha(\alpha-1)(1+\delta)\delta}{\delta^2(2\alpha-1)(\alpha-1)+\delta(2\alpha^2-\alpha(1-3\gamma)-2\gamma)+\gamma(\alpha+\gamma)} k^*(k^{*^{-1}}-1)^{\alpha}(1-\gamma)^{1-\alpha}.$$

Proof: The issue is to find ρ_c such that $\lambda_1 = -1$. For a proof see Ghiglino and Olszak-Duquenne [11]. Q.E.D

Remark: For any given set of admissible parameters (α, γ, δ), there exists a value of the absolute risk aversion of the social utility function, R_0, such that for all economies with a higher curvature, $R > R_0$, the steady state of the reduced model is stable.

4 Instability in the heterogenous agents economy

The steady state value of individual consumption depends on the individual characteristics because the return function depends on the welfare weights. The exact relationship is provided by the following Lemma.

Lemma 2 *At a steady state k^* the individual allocations are*

$$x_i{}^*(\theta_i, \omega_i) = \frac{x^*}{1-\gamma} [\, (\delta(1-\alpha) + \alpha - \gamma)\omega_i + (1-\delta)(1-\alpha)\theta_i \,]$$

where $x^* = k^*(k^{*^{-1}} - 1)^\alpha (1-\gamma)^{1-\alpha}$.

Proof: See Ghiglino and Olszak-Duquenne (2001).

The curvature of the social utility function can now be expressed as a function of the individual consumptions and therefore of the individual shares of capital and labor endowments. We have the following result

Lemma 3 *The absolute risk tolerance of the social utility function computed at the steady state is given by*

$$\rho((\theta_i, \omega_i)_{i=1}^n) = - \sum_{i=1}^n \frac{u_i'}{u_i''}(x_i^*(\theta_i, \omega_i))$$

Proof: See appendix.

In the present general equilibrium model the social utility function depends on the welfare weights. Furthermore, these depend on the equilibrium allocations which in turn depend on the initial conditions and on the distribution of individual endowments. As a consequence the characterization of the dynamic properties of the general equilibrium model is hard to obtain, even when restricted to a neighborhood of the steady state. However, in Ghiglino and Olszak-Duquenne (2001) it is shown that the local dynamic properties of the general equilibrium model are related to the dynamic properties of an appropriate "optimal growth" model.

Lemma 4 *The local stability properties of the general equilibrium model with endogenous weights and of the model with the welfare weights fixed at their steady state values are equivalent.*

Proof: See Ghiglino and Olszak-Duquenne (2001). The proof is based on Kehoe, Levine and Romer (1990) and Santos (1992). Q.E.D

The following Proposition gives the conditions for which heterogeneity matters. It is the main result of this section.

Proposition 1 *Let* $\rho_{\min} = \min_{((\theta_i,\omega_i)_{i=1}^n} \rho((\theta_i,\omega_i)_{i=1}^n)$ *and* $\rho_{\max} = \max_{(\theta_i,\omega_i)_{i=1}^n} \rho((\theta_i,\omega_i)_{i=1}^n)$. *Let also* ρ_c, δ_c *and* δ_{cc} *as defined in Lemma 1. Then*

1. *If* $\delta \notin]\delta_c, \delta_{cc}[$ *then the steady state is (saddle-path) stable.*

2. *If* $\delta \in]\delta_c, \delta_{cc}[$ *and* $\rho_c \in]\rho_{\min}, \rho_{\max}[$ *then the distribution of shares and/or labor endowments matters, i.e. the stability of the steady state is affected by the distribution of wealth. For* $\rho > \rho_c$ *the steady state is unstable while it is stable for* $\rho \leq \rho_c$.

3. *If* $\delta \in]\delta_c, \delta_{cc}[$ *and* $\rho_c \notin]\rho_{\min}, \rho_{\max}[$ *then the distribution of shares and/or labor endowments don't affect stability.*

Proof: Obvious considering Lemma 1, Lemma 3 and Lemma 4. Q.E.D

The previous result gives the conditions for which wealth heterogeneity matters for stability. In these, as we will see, preferences play a crucial role. In some cases, for example when all consumers are characterized by identical CES utility function, $]\rho_{\min}, \rho_{\max}[$ is empty and Case 2 in Proposition 1 never occurs.

5 On the effects of inequality on stability

In this section we establish the link between dynamic instability and agents heterogeneity. When agents have identical preferences, the spread in individual wealth, i.e. in shares of capital and/or labor endowments, gives a good indication of the level of heterogeneity of the economy. Indeed, in this case the agents can be distributed on the real line according to their wealth. In a purely homogeneous economy all consumers have the same wealth while in an heterogenous economy actual individual wealth is spread over some interval. There are several possible formal definitions. We chose the following.

Definition 3 *Assume there are N types of consumers ordered according to their steady state allocation, i.e. $x_i \leq x_j$ for $i < j$. Let $n_i(J)$ be the number of consumers of type i in economy J and let $n(J)$ be the corresponding distribution. Furthermore, assume that the mean of the distribution $\sum_{i=1}^{N} n_i(J)x_i$* is *independent of J. Then Economy B is said to be more heterogenous, or unequal, than Economy A iff B has more weight in the tails than A, i.e. $n(A) \preceq_I n(B)$* where *the ordering $\preceq_I$ is formally defined in Rothschild and Stiglitz (1970).*

Note that when considering the effect of a redistribution at most $N = 2n$ types need to be considered as there are at most n types in the initial configuration and at most n types in the final configuration.

Rothschild and Stiglitz (1970) have shown the equivalence among a class of intuitive notions of spread. In particular, they show that the notion in Definition 3 is equivalent to the property that a spread in the distribution of consumer's type decreases the expected value of $f(x)$ for any f continuous and concave, i.e. $\sum_{i=1}^{2n} n_i(A)f(x_i) \geq \sum_{i=1}^{2n} n_i(B)f(x_i)$

for all continuous concave functions f.

That heterogeneity may have an effect on stability is a consequence of Proposition 1. In fact this link is expected to hold under general conditions. However, it may also be seen that the usual fundamental axioms on preferences don't limit the sign of the effect. The reason is that the occurrence of instability depends on the third and fourth order derivatives of the utility functions. Standard assumptions on preferences do not put any limitation on these and direct empirical data is also lacking. However, some indirect evidence on the properties of absolute risk aversion and its inverse, sometimes called *absolute risk tolerance*, exist or could be obtained soon. Our condition therefore involves absolute risk tolerance.

Proposition 2(i) When inequality is good for stability *Assume that the inverse of absolute risk aversion is a strictly concave function. Provided heterogeneity affects stability (Case 2 of Proposition 1 occurs) there exists a distribution $n(0)$ such that the steady state is locally stable for any economy J with $n(0) \preceq_I n(J)$ and is unstable otherwise.*

Proposition 2(ii) When equality is good for stability *Assume that the inverse of absolute risk aversion is a strictly convex function. Provided heterogeneity affects stability (Case 2 of Proposition 1 occurs) there exists a distribution $n(0)$ such that the steady state is locally stable for any economy J with $n(J) \preceq_I n(0)$ and is unstable otherwise.*

Proof: Let $u_i(x) = v(x)$. Let $i = 1, ..., N$ be the subscript indicating the type and let n_i indicate the number of consumers of type i. Lemma 3 gives

$$\rho((\theta_i, \omega_i)_{i=1}^N) = -\sum_{i=1}^N n_i \frac{v'}{v''}(x_i^*(\theta_i, \omega_i))$$

Provided $W(x) = -\frac{v'(x)}{v''(x)}$ is a concave function, Definition 3 and the discussion thereafter implies that B is more heterogeneous than A iff $\sum_{i=1}^N n_i(A)W(x_i) \geq \sum_{i=1}^N n_i(B)W(x_i)$.

If we define $\rho(J)$ as the value of $\rho((\theta_i, \omega_i)_{i=1}^N)$ associated to the distribution $n_i(J)$ the previous condition becomes iff $\rho(A) \geq \rho(B)$. On the other hand, according to Lemma 1 an increase in ρ favors instability. Therefore, when individual absolute risk tolerance $W(x)$ is a concave function heterogeneity favors stability (Proposition 2(i)). The second part of the result is proven similarly. Q.E.D.

The traditional theory of precautionary saving requires the third derivative to be positive while the fourth derivative is unconstrained. Recent research suggests that a positive third order derivative is not sufficient for the expected wealth accumulation to be increasing with the earning risks (see Huggett and Vidon (2002)). A sufficient condition is that $\frac{v'(x)v'''(x)}{(v''(x))^2}$ is a constant k with $k > 0$, implying that the utility function belongs to a subset of the HARA class (see Caroll and Kimball (1996)). Note that this class include most of the commonly used specifications, as the CARA and CRRA. As $(R^{-1}(x))'' = (\frac{v'(x)}{v''(x)})''$ $= (\frac{(v''(x))^2 - (v'(x)v'''(x)}{(v''(x))^2})' = (-\frac{v'(x)v'''(x)}{(v''(x))^2})'$ it is straightforward to realize that in all these cases heterogeneity doesn't affect stability.

Corollary 1 HARA preferences *Assume that individual preferences can be represented by a utility function of the HARA class, i.e.* $v(x) = \frac{1-\gamma}{\gamma}(\frac{ax}{1-\gamma} + b)^\gamma$ *with* a, b *and* γ *as parameters. Then wealth inequality plays no role in the stability of the steady state.*

Nothing in the data indicates that the analysis should be confined to the HARA class. However, although there are good reasons to believe that absolute risk aversion is convex the evidence on the concavity of its inverse is not conclusive (see Gollier (2001)). Clearly more research is needed before we can apply our results to conclude on the sign of the effect of wealth heterogeneity on stability.

Remark: The conditions in Proposition 2 can be expressed in terms of the derivatives of absolute risk aversion $R(x)$. Indeed, since $(v'(x)/v''(x))' = (-1/R(x))' = -(R^2(x))^{-1}R'(x)$

we obtain $(-v'(x)/v''(x))'' = ((R^2(x))^{-1}R'(x))' = R^2(x)^{-1}[R''(x) - 2R(x)^{-1}R'(x)^2]$. Therefore, if $2R'^2(x)/R(x) < R''(x)$ then $W(x) = -\frac{v'(x)}{v''(x)}$ is strictly convex. Of course this condition may be fulfilled only if absolute risk aversion is strongly strictly convex.

6 Conclusion

The present paper identifies within the chosen model the conditions on consumer's preferences such that wealth inequality favor instability and those that favor stability. The paper also shows that there is a large set of economies such that heterogeneity is neutral, and this set include all preferences satisfying the HARA property. The characterization involves the concavity of absolute risk tolerance, i.e. the inverse of absolute risk aversion. As reviewed by Gollier (2001), properties of the absolute risk tolerance play a crucial role also in asset pricing theory and some effort is being devoted to find empirical evidence. However, these findings do not lead to a conclusive evidence on the concavity of absolute risk tolerance and therefore on the sign of the effect of wealth heterogeneity on stability.

It is an open question whether the results can be extended to a more general framework. Two properties happened to be crucial. First, the welfare weights need to be continuous functions of the initial conditions. Second, at most one of the two graphs representing the eigenvalues of the dynamical system as a function of absolute risk tolerance, should intersect the horizontal line drawn at -1. And this should occur only once. Provided these two properties hold, the results can be extended to a completely general two-sector economy. Similar conditions can be specified so that the result would hold in general multi-sector models. It should also be pointed out that in our model heterogeneity in individual productivity are not explicitly taken into account. However, as we allow for heterogeneity in labor endowments, our model can be reinterpreted as to include different levels of individual labor productivity.

7 References

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8 Appendix

8.1 Proof of Lemma 3.

Without loss of generality assume there are three types of consumers. Then the social utility function is defined by

$$u(x) = Max\ \mu_a n_a\ u_a(x_a) + \mu_b\ u_b n_b(x_b) + (1-\mu_a-\mu_b)n_c\ u_c((x - n_a x_a + n_b x_b)/n_c)$$

The first and second order derivatives of the social utility function can be related to the derivatives of the individual utility function of the agents. Indeed, the first order conditions associated to the maximization problem that define the social utility function give

$$\begin{aligned}
\Psi^1(x_a, x_b, x; \mu_a, \mu_b) &= \mu_a n_a\ u'_a(x_a) - (1-\mu_a-\mu_b)n_a\ u'_c((x - n_a x_a + n_b x_b)/n_c) = 0 \\
\Psi^2(x_a, x_b, x; \mu_a, \mu_b) &= \mu_b n_b\ u'_b(x_b) - (1-\mu_a-\mu_b)n_b\ u'_c((x - n_a x_a + n_b x_b)/n_c) = 0
\end{aligned}$$

Then the following expressions are easily obtained

$$\begin{array}{ll} u'(x) & = (1-\mu_a-\mu_b)\; u_c'((x-n_a x_a+n_b x_b)/n_c) = \mu_a n_a\; u_a'(x_a) \\ u''(x) & = \mu_a n_a\; u_a''(x_a)\frac{\partial x_a}{\partial x} \end{array}$$

where x represents the aggregate consumption. The implicit function theorem applied to Ψ allows to express x_a as a function of x near the steady state (x_a^*, x_b^*, x^*). In matrix form we can write,

$$\left(\begin{array}{c} \frac{\partial x_a}{\partial x} \\ \frac{\partial x_b}{\partial x} \end{array} \right) = \left(\begin{array}{cc} \frac{\partial \Psi^1}{\partial x_a} & \frac{\partial \Psi^1}{\partial x_b} \\ \frac{\partial \Psi^2}{\partial x_a} & \frac{\partial \Psi^2}{\partial x_b} \end{array} \right)^{-1} \left(\begin{array}{c} \frac{\partial \Psi^1}{\partial x} \\ \frac{\partial \Psi^2}{\partial x} \end{array} \right)$$

Some straightforward computations give

$$x_a'(x^*) = \frac{\partial x_a^*}{\partial x} = \frac{\mu_c \mu_b u_b''(x_b^*) u_c''(x_c^*)}{\mu_a \mu_b n_c u_a''(x_a^*) u_a''(x_a^*) + \mu_a \mu_c n_b u_a''(x_a^*) u_c''(x_c^*) + \mu_b \mu_c n_a u_b''(x_b^*) u_b''(x_b^*)}$$

where $\mu_c = 1 - \mu_a - \mu_b$.

The result then follows from the definition of ρ.