

# Strategy and Equity: An ERC-analysis of the Güth-van Damme Game

Gary E Bolton  
Smeal College of Business  
Penn State University, USA

Axel Ockenfels  
& Faculty of Economics and Management  
University of Magdeburg, Germany

December 1997

Güth and van Damme's three-person bargaining experiment challenges conventional thinking about how self-interest, as well as fairness, influences behavior. Among other things, the experiment demonstrates that people care about receiving their own fair share, but care far less about how the remainder is divided among the other bargainers. The ERC model posits that, along with pecuniary gain, people are motivated by their own relative payoff standing. Beyond this, ERC employs standard game theoretic concepts. We describe the general ERC model, and show that it predicts many of the key phenomena observed in the experiment.

## *Correspondence*

Bolton: 310 Beam, Penn State University, University Park, PA 16802, USA; (814) 865-0611; fax (814) 863-2381; geb3@psu.edu. Ockenfels: University of Magdeburg, FWW, Postfach 4120, D-39016 Magdeburg, Germany; (+391) 67-12197, fax (+391) 67-12971; axel.ockenfels@ww.uni-magdeburg.de.

## 1 Introduction: Motives and the Güth-van Damme Experiment

*Motives* drive decision making. While most economic and business models posit self-interested material gain as the sole driver, this is of course a modeling abstraction. People are motivated by many things. Some – the drive to procreate, for example – are without a doubt as fundamental as material gain. The question then is whether material gain alone is sufficient to explain the variety of economic activities in which people participate. When confined to casual empiricism, the right answer is hard to judge: People do struggle for profits in highly competitive markets. But they also demand fair treatment in the workplace. People strike mutually beneficial bargains; other times, negotiations collapse in bitter disagreement. People ‘free ride’ on the public domain – and contribute substantially to charity.

The control afforded by the laboratory permits a precision of analysis rarely open to the casual empiricist. And as illustrated by papers in this volume, the variety of behavior suggested by casual empiricism is mirrored in the lab: Experiments featuring market institutions often produce the type of competitive behavior we associate with the struggle for material gain (Hoffman, Liebcap and Shachat, 1998). Experiments featuring simple negotiations yield results suggesting a role for fairness (Güth and van Damme, 1998). Some, but not all, subjects in public good games choose to cooperate more than self-interest would dictate (Croson and Marks, 1998, and Nagel and Tang, 1998). Even when simply given the option of keeping a sum of money or sharing with an anonymous other, many choose to share (Cason and Mui, 1998). While different investigators give these observations different interpretations, we would say that the pattern of evidence compels an investigation of whether economic behavior is motivated by more than just material gain.<sup>1</sup>

Güth and van Damme’s bargaining experiment clarifies some central issues – although in doing so, it deepens the puzzle. The experiment concerns a three-person bargaining game, in which one bargainer, the proposer *X*, proposes a division of 120 points among the three (10 points worth one Dutch guilder, and in some cases worth 2). A minimal amount, 5 points, must be allocated to each player, but otherwise the proposer is free to allocate as he chooses. A second bargainer, the responder *Y*, either accepts or rejects the proposal. If accepted, the money is

---

<sup>1</sup> Of course, one of the main advantages of the laboratory is that we can test competing explanations against one another. This has, and continues to be, done. See Roth (1995) for an overview of hypotheses and experiments concerning bargaining games.

distributed accordingly. If rejected, all receive nothing. The third bargainer, the dummy Z, has no say in the negotiation, and no choice but to accept any agreement set by the other two.

The game was played in three conditions, each distinguished by the information the responder is given about the proposal. In the *xyz*-condition, the responder knows the full proposal at the time of accepting or rejecting. In the *y*-condition, the responder knows only his own allocation. In the *z*-condition, the responder knows only the dummy's allocation. In some treatments, all games played had the same information condition (the *constant* mode). In other treatments, games were rotated through all three conditions (the *cycle* mode).

The prediction of subgame perfection, a standard game theoretic solution based on the self-interested material gain assumption, is invariant to both the information condition and treatment mode: every feasible proposal gives each bargainer a positive amount, so the responder always makes more money accepting than rejecting. The proposer should therefore ask for the maximum allowable. As an alternative prediction, the experimenters consider a hypothesis they call 'strong intrinsic motivation for fairness.' Again, the predictions are invariant to the information condition and treatment mode: each bargainer gets a one-third share. Hence the experimenters pit a hypothesis predicated on the material gain motive against one predicated on fairness.

In the introduction to their paper, Güth and van Damme cite five important regularities that emerge from their experiment. We discuss them later in detail; here is a brief summary: First, proposals depend on the information condition, with the responder sometimes getting a large share. Second, the amount the dummy receives is in all conditions very small. Third, some proposals are rejected, although a smaller proportion than usually observed in two-person versions of the game, where there is just a proposer and a responder. Fourth, there is a learning trend. And fifth, there are some differences across constant and cycle treatment modes.

Most of these observations are inconsistent with one or both hypotheses. We might speculate that the data represent some convex combination of the two. But Güth and van Damme point out that the way proposers and responders treat the dummy is inconsistent with even a moderated concern for fairness, at least if we understand the concept of fairness to be connected in some way to that of altruism (p. 17):

"The experimental data clearly refute the idea that proposers are intrinsically motivated by considerations of fairness: they only allocate marginal amounts to the dummy and they give little to the responder in information condition  $m = z$ . (Also responders don't show concern for the dummy.)"

In sum, conventional understandings of self-interest and fairness, whether taken separately or in combination, appear inadequate to explain the data.

In this paper, we show that the ERC model predicts four of the five regularities cited by Güth and van Damme, not only as the general form stated above, but also in detail; for example, the ERC model accurately predicts the direction proposals move across information conditions. Another paper, Bolton and Ockenfels (1997), demonstrates that the ERC model is consistent with the behavior observed in a wide variety of other laboratory games, including those thought to exhibit behavior reflecting ‘equity,’ ‘reciprocity,’ and ‘competitiveness;’ hence the moniker *ERC*.

The ERC model is constructed from standard game theory, save for the motivational premise: ERC players are motivated by both the monetary payoff from the experiment, as well as by their own ‘relative payoff,’ a measure of how the individual’s monetary payoff compares to that of the rest of the group. Put another way, the model asserts that individuals are motivated by the interaction of two things: *own* absolute (monetary) payoff, and *own* relative payoff. The distribution of payoffs among other players does not enter in the player’s calculation. Hence we see immediately that ERC is consistent with Güth and van Damme’s observation that other players show very little concern for the dummy.<sup>2</sup>

We can say more, and in much greater detail.

## 2 The ERC model

We concern ourselves with  $n$ -player lab games,  $n \geq 1$ , where players are randomly drawn from the population, and anonymously matched. All game payoffs are monetary and non-negative  $y_i$ ,  $i = 1, 2, \dots, n$ . ERC posits that each player  $i$  maximizes the expected value of the *motivation function*,  $v_i(y_i, \mathbf{I}_i)$ . We refer to  $y_i$  as *i*’s *absolute payoff* and  $\mathbf{I}_i$  as *i*’s *relative payoff*, where

$$\mathbf{I}_i(c, y_i) = \begin{cases} \frac{y_i / c}{1/n} = \frac{n}{c} y_i, & \text{if } c > 0 \\ 1, & \text{if } c = 0 \end{cases} \quad \text{is } i\text{'s proportion of the } \textit{social reference share}, 1/n;$$

---

<sup>2</sup> Fehr and Schmidt’s (1997) model of “biased inequity aversion” has some features in common with ERC. One major difference is that the biased inequity model implies that people care about the difference in payoff between self and each of the other individuals.

and  $c = \sum_{j=1}^n y_j$  is the size of the pie that is distributed among all players.

The ‘social reference share’ is the proportion of the total payoff that  $i$  would receive if all players received the same payoff.

The motivation function is characterized as follows:

A0.  $v_i$  is continuous and twice differentiable on  $\mathfrak{R}^+ \times \mathfrak{R}^+$ .

A1(a) Narrow self-interest:  $v_{i1} \geq 0$ ,  $v_{i11} \leq 0$ .

(b) Monotonicity: Fixing a  $\bar{I}_i$ , given two choices where  $v_i(y_i^1, \bar{I}_i) = v_i(y_i^2, \bar{I}_i)$  and  $y_i^1 > y_i^2$  player  $i$  chooses  $(y_i^1, \bar{I}_i)$ .

A2. Comparative effect:  $v_{i2} = 0$  for  $I_i = 1$ , and  $v_{i22} < 0$ .

A0 is posited for mathematical convenience. A1 implies that, fixing the relative payoff,  $i$  has preferences over the absolute payoff like those assumed in traditional economics models. A2 is the main innovation of the ERC model. It implies that, fixing the absolute payoff,  $v_i$  takes its maximum where  $i$  receives the social reference share.

Let  $k = c/n$  be the average absolute payoff. Fixing  $k$ ,  $i$ 's motivation function can be written as  $v_i^k(I_i) \equiv v_i(kI_i, I_i)$ . A3 insures risk aversion with respect to  $I_i$ :

A3. Risk aversion:  $v_i^{k''}(I_i) \leq 0$ .

Define

$$\mathbf{t}_i(k) := \arg \max_{I_i} v_i^k(I_i) \quad \text{and} \quad \mathbf{s}_i(k) : v_i(k\mathbf{s}_i, \mathbf{s}_i) = v_i(0, 1).$$

The value  $\mathbf{t}_i$  is the proportion of the social reference share that  $i$  would ideally assign to self given the average absolute payoff  $k$ . A0-A3 insures that  $\mathbf{t}_i \in [1, \infty)$  and the value is unique up to  $i$  and  $k > 0$ . By definition, player  $i$  is indifferent between a distribution in which  $i$  receives the proportion of the social reference share  $\mathbf{s}_i$  and a distribution in which all players receive nothing. With the addition of A4,  $\mathbf{s}_i \in (0, 1]$  and the value is unique up to  $i$  and  $k > 0$ :

A4. Strong equity effect:  $\mathbf{s}_i \leq 1$ .

In essence, A4 guarantees that  $i$  prefers a distribution in which  $i$  receives more than the social reference share to a distribution in which all players receive nothing.

A5 provides an explicit characterization of the heterogeneity that exists among players, stated in terms of  $\mathbf{t}_i$  and  $\mathbf{s}_i$ :

A5. Heterogeneity: Let  $f$  and  $g$  be density functions and  $k > 0$ . Then  $f(\mathbf{t}_i | k) > 0$  on  $[1, \infty)$  and  $g(\mathbf{s}_i | k) > 0$  on  $(0, 1]$ .

The ERC model presented here is basically equivalent to the ERC model proposed in Bolton and Ockenfels (1997). That paper provides an extensive discussion of the assumptions and their implications. The present model posits three slight modifications that make it easier to apply ERC to the game of Güth and van Damme (GvD game). First, we define the relative payoff of player  $i$  as  $i$ 's proportion of the social reference share rather than as  $i$ 's proportion of the monetary pie  $c$ . These formulations are equivalent when we confine our attention to a fixed number of players  $n$ . The present analysis allows us to do comparative statics across games that have differing numbers of players. Second, in A3 we assume risk aversion rather than a weaker quasiconcavity assumption. These two modifications are used exclusively to derive proposition ERC7 below. Third, we state A4 as a basic assumption, rather than a special one necessary for specific propositions. We emphasize that none of these modifications are inconsistent with any of the results in Bolton and Ockenfels (1997).

## 2.1 Solving the model

We solve the model by applying subgame perfect equilibrium to the class of motivation functions characterized above. Specifically, we derive predictions under the assumption that players choose the strategy that maximizes the expected value of their motivation function given the information they have about their playing partners' motivation functions. Playing partners in the GvD experiment were anonymous to one another, meaning a player could not know the exact characteristics of his partners' motivation functions. We assume that players are sufficiently experienced with one another to know the *distribution* of motivation functions from which the

partners are randomly, and independently selected. In particular, we suppose that proposers know the distribution of  $s_i$ , defined above in A5.

### 3 ERC predictions and the GvD data

In this section, we derive a series of seven ERC predictions and compare them to the GvD game data. We organize the analysis (roughly) around the major observations cited by Güth and van Damme. Following Güth and van Damme, let  $x$  (resp.  $y, z$ ) be the points or "payoffs" received by the proposer  $X$  (resp. responder  $Y$ , dummy  $Z$ ).

#### 3.1 Proposer and responder behavior: fairness and selfishness in the $xyz$ and $y$ -conditions

ERC asserts that individuals are motivated by *their own* absolute and *their own* relative payoff. The distribution of payoffs among other players does not enter the motivation function. The following propositions show that, according to the model, and consistent with the data, neither the proposer  $X$  nor the responder  $Y$  behave altruistically towards the dummy  $Z$  if the information condition is either  $xyz$  or  $y$ :

ERC1: In information conditions  $xyz$  and  $y$ , an offer of the social reference share or more to the responder ( $y \geq 40$ ) is never rejected, *regardless of the dummy-payoff  $z$* .

Proof: Since the pie size is  $c = 120$ , the social reference share  $c/n = 120/3$  is 40. By A1 we have that each player  $i$  prefers  $(40,1)$  to  $(0,1)$  so that  $y = 40$  is never rejected. Moreover, A3 and A4 imply that  $y > 40$  ( $I_i > 1$ ) is never rejected.

Evidence: For information conditions  $xyz$  and  $y$  (constant and cycle modes combined), Güth and van Damme (1994) report a total of 252 offers of  $y$  that are greater than or equal to 40. None of these offers is rejected. Moreover, when the dummy  $Z$  is offered the minimum payoff (5), responders reject in only about 7% of the 88 total cases in conditions  $xyz$  and  $y$  combined. Güth and van Damme conclude from their analysis that, "there is not a single rejection that can be clearly attributed to a low share for the dummy" (p. 5).

The next three predictions of ERC capture some empirical properties of the proposed distributions  $(x, y, z)$  and show that the proposer treats the dummy  $Z$  with substantially less regard than the responder  $Y$ . All of these results make use of the following lemma:

Lemma: The probability that an offer in which  $y < 40$  is rejected, increases as  $y$  decreases.

Proof: Follows directly from the heterogeneity assumption, A5.

ERC2: In the information conditions  $xyz$  and  $y$ , the proposer allocates himself at least the social reference share ( $x \geq 40$ ).

Proof: By A1 and A2 we have that a proposer  $X$  always strictly prefers  $x = y = z = 40$  to any allocation with  $x < 40$ . The proof of ERC1 shows that  $x = y = z = 40$  carries no risk of rejection.

Evidence: True in all but one out of 360 cases.

ERC3: In information conditions  $xyz$  and  $y$ , the dummy never receives more than the social reference share ( $z \leq 40$ ).

Proof: Suppose that  $z > 40$ . Then either  $x < 40$  or  $y < 40$ . If  $x < 40$ , then  $X$  can improve his situation by redistributing some money from  $Z$  to  $X$ . This increases the absolute payoff  $x$  (and increases the value of the proposer's motivation function) without altering the probability of rejection. If  $y < 40$ , then  $X$  can improve his situation by redistributing money from  $Z$  to  $Y$ . This decreases the probability of rejection while holding the absolute payoff  $x$  constant.

Evidence: True in all but two out of 360 cases.

Note that the upper bound for the dummy's payoff, as derived in ERC3, is valid neither empirically nor theoretically for the payoff of the responder  $Y$ . The responder's theoretical upper payoff bound is 75 rather than 40, because  $x$  may be only 40 (ERC2) so that  $y$  can be as large as 75 (recall that the minimum value for  $z$  is 5). In 91 of the 180 cases of the information condition  $xyz$  (constant and cycle modes combined), the responder receives a payoff that is *greater* than the social reference share.

The mechanism underlying the asymmetric treatment of the responder  $Y$  and the dummy  $Z$  becomes even clearer in the next proposition, which states that as long as the probability of rejection is positive, the dummy receives only his minimum payoff. In essence, *the responder is served first*. However, once the probability of rejection is zero, and  $X$  had taken all he wants, any additional amount is, by the theory, allocated indeterminantly:  $Z$  might get more than the minimum payoff, or  $Y$  might get more than the social reference share, or both might happen.



ERC4: In information conditions  $xyz$  and  $y$ : If the proposer offers  $y < 40$ , then  $z = 5$ , the minimum value allowed.

Proof: As long as  $y < 40$  and  $z > 5$ ,  $X$  can redistribute money from  $Z$  to  $Y$ . This redistribution does not change  $X$ 's relative and absolute outcome but increases the probability of acceptance.

Evidence: In the constant mode, ERC4 is true in all but one out of 75 cases with  $y < 40$ . Evidence in the cycle is less conclusive: In 44 out of 108 cases with  $y < 40$  we have  $z > 5$ . In the constant mode data, the responder  $Y$  is clearly served first. While almost none of the dummies receive more than their minimum payoff in the case of  $y < 40$ , a majority of dummies receive a payoff  $z > 5$  in the 69 cases with  $y \geq 40$ .

In essence, ERC4 says that proposers allocate money to where it has the greatest marginal effect. So a proposer who allocates self  $x > 75$ , allocates the remainder to the responder  $Y$  (except the minimum payoff for the dummy) because giving to the dummy only improves the relative standing, while giving to the responder has an *additional* positive effect: it reduces the risk of rejection.

On the other hand, once the proposer is satiated, and the risk of rejection is zero ( $y \geq 40$ ), ERC leaves the distribution of the remaining money indeterminant. In fact, there is evidence in the constant mode that proposers do not much care how the money they distribute to the others is allocated: For proposals with  $y \geq 40$ , the distribution of the adjusted payoffs  $\tilde{y} := y - 40$  and  $\tilde{z} := z - 5$  do not differ significantly (Mann-Whitney  $U$ -test,  $N = 88$ , two-sided  $p$ -value = 0.579; the corresponding test for the cycle mode yields significance). Güth and van Damme observe that a strong intrinsic motivation for fairness would imply that each player receives 40. But this kind of mitigation of payoffs would imply that dummy  $Z$  should receive what is not needed to insure acceptance. The distributions of  $\tilde{y}$  and  $\tilde{z}$  show that proposers do not have a strong tendency to mitigate payoffs. Rather, proposers in the constant mode appear to give arbitrarily once acceptance is insured. (Bolton et al., in press, make a similar observation in the context of the dictator game.)

### 3.2 Proposals are sensitive to the information condition

We now bring the  $z$ -condition into the discussion. Güth and van Damme emphasize that "proposers react systematically and strategically to the information that responders receive about

the proposal." We might speculate that proposers behave strategically by trying to *signal* a generous offer  $y$  in the  $z$ -condition, where the responder  $Y$  receives information solely about the offer  $z$ . But what kind of offer to  $z$  signals that  $y$  is large? There are two possible hypotheses. First, one might speculate that a generous offer to  $Z$  signals that the proposer  $X$  is an altruist, and therefore increases the probability of a generous offer to  $Y$ . We will call this the altruism-signaling hypothesis. It implies a negative correlation between  $z$  and the probability of rejection. In contrast, the ERC-signaling hypothesis suggests that  $z$  is *negatively* correlated with the responder's expectation of  $y$ : Suppose that all proposers want to realize their optimal proportion of the social reference share  $t_x$  in the  $z$ -condition. Then the distribution of  $t_x$  can be associated with a distribution of total offers  $y + z$ . Hence, there is a negative correlation between observed  $z$  and expected  $y$ . And a proposer who wants to signal that  $y$  is large should choose a small  $z$  regardless of her  $t_x$ .

As it happens, the constant mode data exhibits no evidence for signaling of any sort. Specifically, there is no correlation between  $z$  and  $y$  (Spearman rank correlation coefficient of 0.018,  $p = 0.88$ ). There is, however, a significant correlation between  $z$  and  $y$  in the cycle mode (correlation coefficient of 0.37,  $p = 0.00$ ). Because the correlation is positive, we can rule out ERC-signaling. On the other hand, we expect altruism-signaling to be accompanied by a negative correlation between  $z$  and the rejection rate. There is no evidence for this; as Güth and van Damme put it (p. 2), "... responders view high  $z$ -values with suspicion, the percentage of rejected proposals does not decrease with  $z$ ."

In sum, there is no clear evidence for any form of signaling. Therefore, the following propositions are derived under the assumption that signaling does not take place. That is, we assume that the proposal  $z$  does not offer any information that influences the rejection probability. Of course, ERC predicts that very large offers to the dummy, for example  $z = 120$ , are rejected. However,  $z$  is greater than 40 in only three cases and is always smaller or equal to 55. Therefore, we can safely ignore these sorts of offers.

As in Güth and van Damme, let  $p(x)$  (resp.  $p(y)$ ,  $p(z)$ ) be the amount the proposer allocates to player  $P$  ( $P \in \{X, Y, Z\}$ ) in the cycle mode and let  $p(cx)$  (resp.  $p(cy)$ ,  $p(cz)$ ) be the amount the proposer allocates to player  $P$  in the constant mode when the information condition is  $xyz$  (resp.  $y$ ,  $z$ ). Then, the following propositions state the predicted strategic adjustments of the proposals ( $x$ ,  $y$ ,  $z$ ) to the change in the information conditions.

ERC5: The proposer  $X$  demands more in the  $z$ -condition than in the  $xyz$  and  $y$ -conditions ( $x(cx)$ ,  $x(cy) < x(cz)$  and  $x(x)$ ,  $x(y) < x(z)$ ). Likewise, the responder  $Y$  receives less in the  $z$ -condition than in the  $xyz$  and  $y$ -condition ( $y(cx)$ ,  $y(cy) > y(cz)$  and  $y(x)$ ,  $y(y) > y(z)$ ).

Proof: In the  $z$ -condition, the rejection behavior is independent of  $z$  (no signaling) and therefore independent of the decision of  $X$ . Therefore,  $X$  should behave as if he or she is in a role of a dictator faced with two recipients. On the other hand, in the  $xyz$  and  $y$ -conditions, the proposer is in an ultimatum situation. As shown in Bolton and Ockenfels (1997), the ultimatum situation creates an additional strategic incentive to give for all proposers who run the risk of rejection. Hence, ERC predicts lower offers in the  $z$ -condition if proposers are sufficiently selfish:  $t_x > 2$ . Proposers with  $t_x \leq 2$  offer the same total amount in both conditions.

Evidence: ERC5 is strongly supported by the data (Güth and van Damme, p. 9 and 14).

ERC6: Offers  $y$  and demands  $x$  do not differ across the  $xyz$  and  $y$ -condition ( $x(cx) = x(cy)$  and  $x(x) = x(y)$ ;  $y(cx) = y(cy)$  and  $y(x) = y(y)$ ).

Proof: The responder  $Y$  is only interested in  $y$  and  $y/c$  (ERC1). Since  $c$  is common knowledge, the full information condition does not give any additional decision-relevant information to the responder as compared to the  $y$ -information condition. Since the rejection behavior is equivalent in both information conditions, the proposer behavior is equivalent as well.

Evidence: The data clearly supports ERC with respect to the responder's payoff  $y$  (Güth and van Damme, p. 9 and 14). Güth and van Damme find that  $x(cx) < x(cy)$  and  $x(x) < x(y)$ . While these effects are statistically significant, in absolute terms they are "slight" (Güth and van Damme, 1994, p. 3 and 25).

### 3.3 Rejection rates

The overall rejection rate for GvD games in information conditions  $xyz$  and  $y$  is about 4 percent. This is surprisingly low if one compares it with corresponding rates in the 2-person ultimatum game, which typically run in the neighborhood of 15 to 20 percent (see Roth's 1995 survey).

ERC7 offers an explanation. We suppose that the average size of the pie is fixed across games. The underlying idea of the proof is that a 3-person GvD game creates more room to agree on a distribution of relative payoffs between the proposer and the responder than a 2-person game. A proposer with  $t_x \leq 2$  will propose an offer with a zero probability of rejection in the GvD game, but not generally so in the ultimatum game. Risk aversion (A3) implies that the rest of the proposers, those with  $t_x > 2$ , will choose to use some resources to lower the probability of rejection relative to what it would be in the two-person game.

ERC7: Holding the average pie size fixed across games, rejection rates are lower in the 3-person GvD game in the information conditions  $xyz$  and  $y$  than in the 2-person ultimatum game.

Proof: Given that the average pie size,  $k$ , is fixed across games,  $i$ 's motivation function  $v_i^k(I_i)$  can be written as  $v_i(I_i)$  for both  $n = 2$  and 3. Note that if proposer  $X$  offers  $I_Y^2$  to the responder  $Y$  in the 2-person game, then  $X$  receives  $I_X^2 = 2 - I_Y^2$ . If  $X$  offers  $I_Y^3$  in the 3-person game, he or she receives  $I_X^3 = 3 - I_Y^3$ . We also know that the proposer  $X$  always prefers  $I_X^n = 1$  to any  $I_X^n < 1$  (ERC2). On the other hand,  $I_Y^n \geq 1$  is never rejected (ERC1). Therefore, the optimal offer  $I_Y^n$  is smaller or equal to  $n - 1$ . Likewise, since  $I_Y^n = 0$  is always rejected (A1 and A2), the optimal offer  $I_Y^n$  must be strictly positive.

Fixed average pie across games implies  $t_i(k) \equiv t_i \in [1, \infty)$ . Suppose  $t_x = 1$ . Then,  $X$  chooses  $I_X^n = 1$  independent of  $n$ , and by choosing  $I_Y^n \geq 1$ , proposer  $X$ 's offer is never rejected, neither in the 2-person game nor in the 3-person game. Now suppose  $1 < t_x \leq 2$ . Then, in the 3-person game, the proposer can realize his or her optimal proportion of the social reference share  $t_x$  with no risk of rejection by choosing  $I_Y^3 \geq 1$ . However, on average the proposers with  $t_x < 2$  face a *positive* probability of rejection in the 2-person game. (Here we implicitly assume that the population is not too risk averse in the sense that the probability of a proposer with  $1 < t_x < 2$  who demands more than half of the pie in the 2-person game is positive.)

Now, it remains to show that for proposers with  $t_x > 2$ , the rejection rate in the 2-person game is no smaller than in the 3-person game. First, note that by ERC1 and ERC4 the dummy

always receives the minimum payoff from proposers with  $t_x > 2$ . Hence, without loss of generality, we can focus our analysis on the choice of the offer to the responder.

We can write the problem of proposer  $X$  as (normalize  $v_x(0,1) = 0$ ):

$$\max q(I_Y^n)v_x(n - I_Y^n) \text{ with respect to } I_Y^n \in (0, n-1],$$

where  $q(I) = 1 - \int_0^I g(\mathbf{s}_i|k) d\mathbf{s}_i$  is the probability that a randomly chosen responder accepts the offer of  $I$ .

Suppose  $\hat{I}_Y^2$  and  $\hat{I}_Y^3$  are the solutions of the proposer's problem in the 2-person and 3-person game respectively. We show that  $\hat{I}_Y^2 \leq \hat{I}_Y^3$  which implies  $q(\hat{I}_Y^2) \leq q(\hat{I}_Y^3)$ . Suppose to the contrary that  $\hat{I}_Y^2 > \hat{I}_Y^3$ . Necessarily, in the 3-person game,

$$q(\hat{I}_Y^3)v_x(3 - \hat{I}_Y^3) > q(\hat{I}_Y^2)v_x(3 - \hat{I}_Y^2) \quad (1)$$

Define  $\Delta q := q(\hat{I}_Y^3) - q(\hat{I}_Y^2)$ . By A5 and because  $\hat{I}_Y^2 > \hat{I}_Y^3$ ,  $\Delta q < 0$ . Define  $\Delta v := v_x(3 - \hat{I}_Y^3) - v_x(3 - \hat{I}_Y^2)$ . Then  $\Delta v > 0$  because  $I_X^n = n - \hat{I}_Y^n < t_x$  and concavity (A3). Then (1) becomes

$$[q(\hat{I}_Y^2) + \Delta q][v_x(3 - \hat{I}_Y^2) + \Delta v] > q(\hat{I}_Y^2)v_x(3 - \hat{I}_Y^2)$$

or

$$\Delta q v_x(3 - \hat{I}_Y^2) + [q(\hat{I}_Y^2) + \Delta q] \Delta v > 0 \quad (2)$$

In the 2-person game,

$$q(\hat{I}_Y^3)v_x(2 - \hat{I}_Y^3) < q(\hat{I}_Y^2)v_x(2 - \hat{I}_Y^2) \quad (3)$$

Define  $\Delta \tilde{v} := v_x(2 - \hat{I}_Y^3) - v_x(2 - \hat{I}_Y^2)$ . Then  $\Delta \tilde{v} > 0$ . By the same series of substitutions that produce (2) we get

$$\Delta q v_x(2 - \hat{I}_Y^2) + [q(\hat{I}_Y^2) + \Delta q] \Delta \tilde{v} < 0 \quad (4)$$

Concavity (A3) implies that  $\Delta \tilde{v} > \Delta v$ . It follows that l.h.s. (4)  $>$  l.h.s. (2)  $> 0$  which contradicts (4). This completes the proof.

Evidence: ERC 7 shows that, assuming risk aversion, and holding the average pie size fixed, we should expect lower rejection rates in the three-person GvD game than in the two-person ultimatum game. As indicated, rejection rates in the GvD game were substantially lower than those typically reported for the ultimatum game.

### *3.4 Learning*

ERC is an equilibrium theory, with predictions that technically apply only after the learning curve has flattened out. Nevertheless, we can ask whether observed learning is plausibly moving in the direction of equilibrium. The reported learning trend is that, over time,  $x$  increases,  $z$  decreases and  $y$  stays roughly the same. As we have already pointed out, the (stable) offers to the responder are in line with what we expect from ERC. The dynamics are consistent with the idea that proposers begin by offering the dummy more than they would like, out of concern that responders care what the dummy receives. And then as they learn that responders do not care, proposers shift funds from dummy to proposers, and towards equilibrium.

## **4 Summary**

ERC predicts most of what we observe in Güth and van Damme's experiment. For the most part, the model accurately characterizes proposer offers, and responder acceptance thresholds in  $xyz$  and  $y$ -conditions. It correctly predicts how  $z$ -condition proposals will compare to the other two. The model also explains why we should expect rejection rates in the GvD game to be lower than those in the two-person ultimatum game. While ERC is an equilibrium theory and does not predict learning, the dynamic trend observed in the data is moving in the direction we would expect from the equilibrium analysis.

The model exhibits two shortcomings. The first is minor: The model fails to predict the slight, but statistically significant, differences in proposer  $X$ 's share across  $xyz$  and  $y$ -conditions. The other failure is more substantial: while the model fits the constant mode fairly precisely, it fails to predict the higher proposer payoffs in the cycle mode. One potential explanation is that at least some players measure relative payoffs with respect to the entire session rather than the individual game. When the game is held constant for the session, then assuming that relative payoffs are measured with respect to the game, as ERC does, is probably a good approximation, since getting

“the right” outcome for the session in each game is probably a good strategy. But in the cycle mode, a better way of getting the right outcome for the session is to get the absolute payoff in the  $z$  conditions, and tune the relative payoffs in conditions where  $y$  can see what you are up to.

But overall, the ERC model fits the data quite closely. The model’s accuracy demonstrates that when we attribute a relative payoff motive to players, observed behavior is recognizably strategic.

But *why* do people care about relative standing? We speculate that the answer has to do with biology. People have always lived in groups – for most of existence, in small groups. Since individual survival depends in large part on the group’s survival, it seems plausible that evolutionary forces induced a propensity to contribute to the group. The propensity to punish – as exhibited by rejected offers in the ultimatum and GvD games – may be the way that evolution (partially) solves the free riding problem inherent in such an arrangement.

## Literature

- Bolton, Gary E., and Axel Ockenfels (1997): ERC: A Theory of Equity, Reciprocity and Competition, working paper, Penn State University.
- Bolton, Gary E., Elena Katok and Rami Zwick (in press): Dictator Game Giving: Rules of Fairness Versus Acts of Kindness, *International Journal of Game Theory*.
- Cason, Timothy and Vai-Lam Mui (1998): Social Influence in the Sequential Dictator Game, *Journal of Mathematical Psychology*, this issue.
- Croson, Rachel and Melanie Marks (1998), Identifiability of Individual Contributions in a Threshold Public Goods Experiment, *Journal of Mathematical Psychology*, this issue.
- Fehr, Ernst, and Klaus Schmidt (1997), "How to Account for Fair and Unfair Outcomes – A Model of Biased Inequality Aversion," July, presentation at Gerzensee Symposium on Economic Theory.
- Güth, Werner and Eric van Damme (1994): Information, Strategic Behavior and Fairness in Ultimatum Bargaining, CentER Discussion Paper No. 9465.
- Güth, Werner and Eric van Damme (1998): Information, Strategic Behavior and Fairness in Ultimatum Bargaining, An Experimental Study, *Journal of Mathematical Psychology*, this issue.
- Hoffman, Elizabeth, G. Liebcap and Jason Shachat (1998): An Experimental Investigation of the Incentives to Form Agricultural Marketing Pools, *Journal of Mathematical Psychology*, this issue.
- Nagel, Rosemarie and Fang-Fang Tang (1998): Experimental Results on the Centipede Game in Normal Form: An Investigation on Learning, *Journal of Mathematical Psychology*, this issue.
- Roth, Alvin E. (1995): Bargaining Experiments, in *Handbook of Experimental Economics*, J. Kagel and A. E. Roth, eds., Princeton: Princeton University Press.