Agreeing to disagree without the countable additivity axiom*

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Abstract. An example is given in which agents "agree to disagree". It relies on the absence of the countable additivity axiom.

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Ann and Bob face a countable set of possible states of nature, that we denote by $\Omega = \{1, 2, ...\}$. Knowing nothing about probability theory, they consider all states to be equally probable.

Ann and Bob have different observation capabilities, described by partitions of Ω. These are such that what is observed is the element of the partition that contains the actual state of nature. Their private information partitions are, respectively:

$$P_A = \{\{1, 2, 3\}, \{3 + 1, ..., 2^2 \cdot 3\}, ..., \{2^{2k} \cdot 3 + 1, ..., 2^{2k+2} \cdot 3\}, ...\};$$

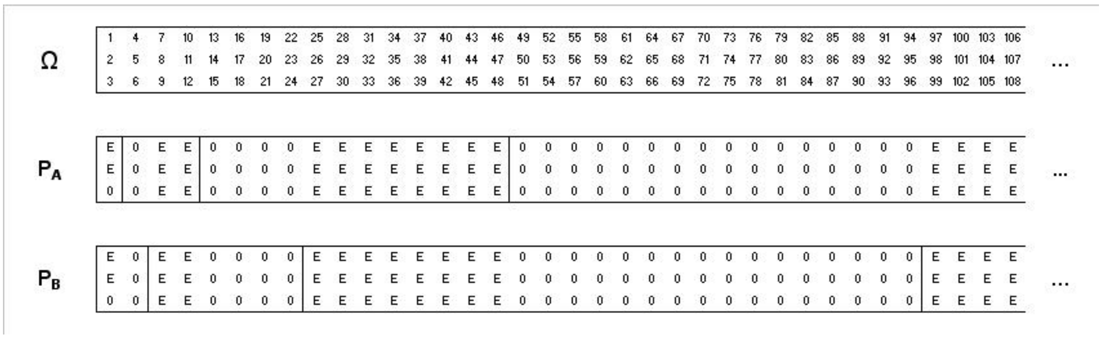
$$P_B = \{\{1, ..., 2 \cdot 3\}, \{2 \cdot 3 + 1, ..., 2^3 \cdot 3\}, ..., \{2^{2k-1} \cdot 3 + 1, ..., 2^{2k+1} \cdot 3\}, ...\}.$$

Ann and Bob are curious about whether the event described as follows occurred:

$$E = \{1, 2\} \cup \{2 \cdot 3 + 1, 2^2 \cdot 3\} \cup ... \cup \{2^{(2k+1)} \cdot 3 + 1, 2^{(2k+2)} \cdot 3\} \cup ... \, ;$$

$$\sim E = \{3, 2 \cdot 3\} \cup \{2^2 \cdot 3 + 1, 2^3 \cdot 3\} \cup ... \cup \{2^{(2k)} \cdot 3 + 1, 2^{(2k+1)} \cdot 3\} \cup ... \, .$$

All this is common information. The space of states, the information structures, and the event are described in the picture below. Notice that in every set of Ann's information structure, the event E occurs in two thirds of the states of nature; while in every set of Bob's information structure, E occurs in only one third of the states.



Having received their private information (that is, knowing in which set of their partition of information is the actual state of nature), Ann will estimate that the probability of occurrence of E is $2/3$, and Bob will estimate that it is $1/3$. This is common knowledge, since it is independent of the state of nature that occurs. For the same reason, truthful exchange of information about their estimates does not lead to any update of their posteriors.

Pooling their information, Ann and Bob are always able to find out whether E occurred or not. Being restricted to truthful communication of their estimates, Ann and Bob always agree to disagree on the probability of occurrence of E.

References

Aumann, R.J. (1976), “Agreeing to Disagree”, *Annals of Statistics*, 4, pp. 1236-1239.