View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Research Papers in Economics

Federal Reserve Bank of Minneapolis
Research Department Staff Report 380

October 2006

Bankruptcy and Collateral in Debt Constrained Markets*

Timothy J. Kehoe

University of Minnesota,
Federal Reserve Bank of Minneapolis,
and National Bureau of Economic Research

David K. Levine

Washington University in St. Louis
and National Bureau of Economic Research

ABSTRACT

Typical models of bankruptcy and collateral rely on incomplete asset markets. In fact, bankruptcy and collateral add contingencies to asset markets. In some models, these contingencies can be used by consumers to achieve the same equilibrium allocations as in models with complete markets. In particular, the equilibrium allocation in the debt constrained model of Kehoe and Levine (2001) can be implemented in a model with bankruptcy and collateral. The equilibrium allocation is constrained efficient. Bankruptcy occurs when consumers receive low income shocks. The implementation of the debt constrained allocation in a model with bankruptcy and collateral is fragile in the sense of Leijonhufvud's "corridor of stability," however: If the environment changes, the equilibrium allocation is no longer constrained efficient.

*This paper was prepared for presentation at the Conference in Honor of Axel Leijonhufvud held at UCLA, 30–31 August 2006. The authors are grateful for financial support from the National Science Foundation. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

> The absence of private information implies that no consumer actually goes bankrupt in equilibrium: the credit agency will never lend so much to consumers that they will choose bankruptcy. This is very unlike … incomplete markets bankruptcy models. (Kehoe and Levine 2001)

1. Introduction: "A foolish consistency is the hobgoblin of little minds"[1]

General equilibrium models of bankruptcy have generally taken the perspective that bankruptcy is observed in the world, and so general equilibrium models should attempt to account for it. This point of view is very much in the spirit of the incomplete markets models on which these models are based. The theoretical literature on equilibria with incomplete markets and bankruptcy includes Araujo, Páscoa, and Torres-Martínez (2002), Dubey, Geanakoplos, and Zame (1995), Dubey, Geanakoplos and Shubik (1989), Geanakoplos and Zame (2002), Kubler and Schmedders, (2003), Orrillo (2002), and Zame (1993). Recently papers by Chatterjee, Corbae, Nakajima, and Ríos-Rull (2004) and Livshits, MacGee, and Tertilt (2003) have constructed models with incomplete markets and bankruptcy, calibrated them to data, and used them to address policy issues. These models only partially address some fundamental questions: Why should bankruptcy be allowed? What underlying economic fundamentals lead to particular types of bankruptcy?

The enforcement constraint models of Kehoe and Levine (1993, 2001) and others attempt to answer the question of why we observe incomplete markets for insurance. The answer given is that not all profitable transactions can be carried out because some would violate the individual rationality constraint that under some circumstances it would be better to "run away" than to pay an existing debt. This links insurance possibilities to economic fundamentals.

This paper is an approach to bankruptcy and collateral based on these enforcement constraint models. Although, as the authors observed in the quotation above, no consumer actually runs away in equilibrium, we argue here that "running away" is not the proper interpretation of bankruptcy. Rather, the Kehoe-Levine enforcement constraint model requires complete contingent claims, and, in practice, these claims are implemented not through Arrow securities, but rather through a combination of non-contingent assets and bankruptcy. With this in mind, we reexamine the example of Kehoe and Levine (2001) and show how the efficient —

[1] With the exception of this opening quotation from Ralph Waldo Emerson's "Self Reliance," the quotations at the beginning of sections are all taken from Axel Leijonhufvud's (1973b) "Life among the Econ."

that is, second best — stationary equilibrium allocation can be implemented in an equilibrium without contingent claims, but with bankruptcy and collateral.

This reinterpretation brings new economic insight. If the model has consequences for unanticipated shocks, then the institution of bankruptcy and collateral that may be well suited for "ordinary" shocks may break down when subject to unusual shocks. This is closely related to Leijonhufvud's (1973a) "corridor of stability." Our perspective, then, is quite different from that in the incomplete markets literature or that in the work of Kiyotaki and Moore (1997). In those models, it is hypothesized that bankruptcy and collateral are an inefficient solution to a not completely well-specified economic problem. Here we view bankruptcy and collateral as an efficient solution to the problems posed by ordinary transactions. We also recognize that solutions that may suit ordinary events well, however, may be fragile when exposed to less ordinary events.

2. A modl "finely carved from the bones of Walras"

We start by summarizing the model of Kehoe and Levine (2001). There are an infinite number of discrete time periods $t = 0,1,\ldots$. In each period there are two types of consumers, $i = 1, 2$, and a continuum of each type of consumer. At each moment of time, one consumer has high productivity and one has low productivity. The state $\eta_t \in \{1,2\}$ at time t is the index of the consumer who has high productivity at that time. This random variable follows a Markov process characterized by a single number $0 < \pi < 1$, the probability of a *reversal,* that is, a transition from the state where type 1 has good productivity to the state where type 2 has good productivity, or vice versa.

Uncertainty evolves over an uncertainty tree. The root of the tree is determined by the fixed initial state η_0. A *state history* is a finite list $s = (\eta_1,\ldots,\eta_t)$ of events that have taken place through time $t(s)$, where $t(s)$ is the length of the vector s, the time at which s occurs. The history immediately prior to s is denoted $s-1$, and if the node σ follows s on the uncertainty tree, we write $\sigma > s$. The countable set of all state histories is denoted S. The probability of a state history is computed from the Markov transition probabilities

$$\pi_s = pr(\eta_{t(s)} \mid \eta_{t(s)-1})\, pr(\eta_{t(s)-1} \mid \eta_{t(s)-2}) \cdots pr(\eta_1 \mid \eta_0)\,. \tag{1}$$

There is a single consumption good c; the representative consumer of type i consumes c_s^i if the state history is s. Both consumers have the common stationary additively separable expected utility function

$$(1-\beta)\sum_{s\in S}\beta^{t(s)}\pi_s u(c_s^i). \tag{2}$$

The period utility function u is twice continuously differentiable with $Du(c)>0$, satisfies the boundary condition $Du(c)\to\infty$ as $c\to 0$, and has $D^2u(c)<0$. The common discount factor β satisfies $0<\beta<1$.

There are two types of capital: human capital (or labor) and physical capital (or trees). The services of the one unit of human capital held by type i consumer in state η are denoted $w^i(\eta)$. These services take on one of two values, ω^b and ω^g, with $\omega^b<\omega^g$, corresponding to low and high productivity, respectively. Moreover, if one consumer has high productivity, then the other consumer has low productivity, so if $w_t^i=\omega^b$ then $w_t^{-i}=\omega^g$, where $-i$ is the type of consumer who is not type i. Finally, the state indexes which consumer has high productivity, so $\omega^\eta(\eta)=\omega^g$, $\omega^{-\eta}(\eta)=\omega^b$.

There is one unit of physical capital in the economy. This capital is durable and returns $r>0$ of the consumption good in every period. We can interpret this physical capital as trees, with r being the amount of consumption good produced every period by the trees. A consumer of type i holds a share θ_s^i of the capital stock contingent on the state history s. Initial physical capital holdings are θ_0^i.

The total supply of the consumption good in this economy is the sum of the individuals' productivities plus the return on the single unit of physical capital, $\omega=\omega^g+\omega^b+r$. The social feasibility conditions for this economy in each state are

$$c_s^1+c_s^2\le\omega^g+\omega^b+r=\omega \tag{3}$$

$$\theta_s^1+\theta_s^2\le 1. \tag{4}$$

3. The debt constrained economy: "Venturing stark naked out into the chill winds of abstraction"

Our first model of intertemporal trade is the debt constrained economy. Borrowing, lending, and the sale and purchase of insurance contracts are possible. There are, however, debt constraints. These come about because consumers have the option of opting out of intertemporal trade. If they choose to do this, they renege on all existing debts. They are excluded from all further participation in intertemporal trade, however, and their physical capital is seized. The endowment of human capital is assumed to be inalienable: it cannot be taken away, nor can consumers be prevented from consuming its returns.

Formally, this is a model in which consumers face the individual rationality constraint

$$(1-\beta)\sum_{\sigma\geq s}\beta^{t(\sigma)-t(s)}(\pi_\sigma/\pi_s)u(c_\sigma^i)\geq(1-\beta)\sum_{\sigma\geq s}\beta^{t(\sigma)-t(s)}(\pi_\sigma/\pi_s)u(w^i(\eta_\sigma)). \quad (5)$$

This constraint says that, in every state history, the value of continuing to participate in the economy is no less than the value of dropping out.

In this debt constrained economy, since markets are complete, consumers purchase contingent consumption for the state history s for the present value price p_s and they sell the return on their capital $w^i(\eta_s)+r\theta_0^i$ at the same price. The corresponding optimization problem is

$$\max\,(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t^i)$$

subject to

$$\sum_{s\in S}p_s c_s^i\leq\sum_{s\in S}p_s(w^i(\eta_s)+\theta_0^i r) \quad (6)$$

$$(1-\beta)\sum_{\sigma\geq s}\beta^{t(\sigma)-t(s)}(\pi_\sigma/\pi_s)u(c_\sigma^i)\geq(1-\beta)\sum_{\sigma\geq s}\beta^{t(\sigma)-t(s)}(\pi_\sigma/\pi_s)u(w^i(\eta_\sigma)).$$

Notice that we have written the budget constraint in the Arrow-Debreu form. As is usual in this sort of model, we can equally well formulate the budget constraint as a sequence of budget constraints in complete securities markets,

$$c_s^i+q_{(s,1)}\theta_{(s,1)}^i+q_{(s,2)}\theta_{(s,2)}^i\leq w^i(\eta_s)+(v_s+r)\theta_s^i \quad (7)$$

$$\theta_s^i\geq-\Theta,\ \theta_0^i\text{ fixed,}$$

where $q_{(s,\eta)}$ is the price of the Arrow security traded in state history *s* that promises a unit of physical capital to be delivered at state history (s,η). A standard arbitrage argument implies that $q_{(s,1)} + q_{(s,2)} = v_s$. The constraint $\theta_s^i \geq -\Theta$ rules out Ponzi schemes, but Θ is a positive constant chosen large enough not to otherwise constrain borrowing in equilibrium.

An *equilibrium* of the debt constrained economy is an infinite sequence of consumption levels and consumption prices such that consumers maximize utility given their constraints and such that the social feasibility condition for consumption is satisfied.

A *symmetric stochastic steady state* satisfies the equilibrium conditions for an appropriate choice of initial capital holdings $\bar{\theta}_0^1$ and $\bar{\theta}_0^2$ and is specified by consumption c^g when productivity is high, c^b when productivity is low, and the rule

$$c_s^i = \begin{cases} c^g & \text{if } w_s^i = \omega^g \\ c^b & \text{if } w_s^i = \omega^b \end{cases}. \tag{8}$$

Kehoe and Levine (2001) prove that every stochastic steady state in which the individual rationality constraint binds on at least one consumer type is symmetric. They also analyze transition paths and prove that the equilibrium reaches the stochastic steady state as soon as a reversal has taken place.

4. Solution of the debt constrained model

We find the symmetric stochastic steady state by decreasing c^g from ω^g until we either achieve the symmetric first best at $x^g = \omega/2$ or until the individual rationality constraint binds. We define a function proportional to the difference between the utility from the steady state consumption plan and consumption in autarky. A recursive calculation shows that this function is

$$f^D(c^g) = \big(1-\beta(1-\pi)\big)\big(u(c^g) - u(\omega^g)\big) + \beta\pi\big(u(\omega - c^g) - u(\omega^b)\big), \tag{9}$$

where $c^b = \omega - x^g$.

Proposition 1: *A symmetric stochastic steady state* c^g *of the debt constrained economy is characterized by* $f^D(\omega/2) \geq 0$ *and* $c^g = \omega/2$ *or by* $\omega^g > \omega/2$ *,* $f^D(c^g) = 0$ *, and* $c^g \in (\omega/2, \omega^g)$ *.*

The function f^D is concave and satisfies $f^D(\omega^g) > 0$. Observe first that $\omega^g \leq \omega/2$ implies that $f^D(\omega/2) > 0$. Either $f^D(\omega/2) \geq 0$ or $f^D(\omega/2) < 0$. If $f^D(\omega/2) \geq 0$, then $f^D(\omega^g) > 0$ and the concavity of f^D imply that $f^D(c) > 0$ for all $c \in [\omega/2, \omega^g]$. Consequently, the unique steady state is characterized by $c^g = \omega/2$. If, instead, $f^D(\omega/2) < 0$, then $\omega^g > \omega/2$. In this case, $f^D(\omega^g) > 0$ and the concavity of f^D imply that $f^D(c^g) = 0$ for a unique $c^g \in (\omega/2, \omega^g)$.

Proposition 2: *A symmetric stochastic steady state exists in the debt constrained economy. There is only one symmetric stochastic steady state.*

An interesting question is how the steady state level of consumption depends on the parameter $1-\pi$ measuring the persistence of the shock. From the implicit function theorem, in the case where the debt constraint binds, we can compute

$$\frac{dc^g}{d(1-\pi)} = \frac{\partial f^D / \partial \pi}{\partial f^D / \partial c^g}. \tag{10}$$

At an interior steady state f^D must intersect the axis from below, so $\partial f^D / \partial c^g$ is positive. We can also rewrite f^D as

$$f^D(c^g) = (1-\beta)\left(u(c^g) - u(\omega^g)\right) + \beta\pi\left(u(\omega - c^g) - u(\omega^b) + u(c^g) - u(\omega^g)\right). \tag{11}$$

When $f^D(c^g) = 0$, since the first term is negative, the second term is positive, and since $\partial f^D / \partial \pi$ is proportional to the second term, it is also positive. We conclude that

$$\frac{dc^g}{d(1-\pi)} > 0, \tag{12}$$

implying that a more persistent shock results in greater consumption by the consumer with the high endowment, or, equivalently, less risk sharing between the two consumers.

5. A numerical example

To see how the equilibrium works in more detail, we examine a numerical example. We suppose that the discount factor is $\beta = 4/5$, that the probability of reversal is $\pi = 1/8$, and that the endowments are

$$w_s^i = \begin{cases} 54 & \text{if } \eta_t = i \\ 8 & \text{if } \eta_t \neq i \end{cases}. \tag{13}$$

There is a single unit of physical capital that produces $r = 1$ unit of the good every period.

The first-order conditions for the consumer's problem are

$$\beta^{t(s)}\pi_s \frac{1}{c_s^i} - \lambda_s^i + \beta^{t(s)}\pi_s \frac{1}{c_s^i}\sum\nolimits_{\sigma\le s}\mu_\sigma^i = 0 \tag{14}$$

and

$$-\lambda_s^i q_{(s,\eta')} + \lambda_{(s,\eta')}^i (v_{(s,\eta')} + r) = 0\,, \tag{15}$$

where λ_s^i is the Lagrange multiplier for the sequential markets budget constraint (7) for consumer type i in state history s and μ_s^i is the Lagrange multiplier for the individual rationality constraint (5).

If $c_{(s,\eta')}^i = c^b$, then the individual rationality constraint does not bind and $\mu_{(s,\eta')}^i = 0$. First, consider the case where $c_s^i = c^g$ and $c_{(s,\eta')}^i$ is c^b. Then, since $\mu_{(s,\eta')}^i = 0$, we can write the first-order condition for $c_{(s,\eta)}^i$ as

$$\beta^{t(s)+1}\pi_s\pi \frac{1}{c^b} - \lambda_{(s,\eta')}^i + \beta^{t(s)+1}\pi_s\pi \frac{1}{c^b}\sum\nolimits_{\sigma\le s}\mu_\sigma^i = 0\,. \tag{16}$$

Combining this with the first-order condition for c_s^i, (14), we obtain

$$\frac{u'(c^g)}{\delta\pi u'(c^b)} = \frac{v_{(s,\eta')} + r}{q_{(s,\eta')}}\,. \tag{17}$$

We construct an equilibrium assuming that capital prices are constant, $v_{(s,\eta')} = v$. Kehoe and Levine (2001) prove that this is the only possibility.

The first-order condition (15) becomes

$$q_{(s,\eta')} = q_r = \frac{\beta\pi u'(c^b)}{u'(c^g)}\left(v+r\right). \tag{18}$$

Here q_r is the price paid for an Arrow security to purchase one unit of physical capital in the case of reversal — where $\eta_s = 1$, for example, but $\eta' = 2$.

Consider now the case where $c_s^i = c^b$ and $c_{(s,\eta')}^i$ is c^b. (We can think of this as the same state history s; we are just looking at the other consumer type's first-order conditions.) We obtain

$$\frac{u'(c^b)}{\delta\left(1-\pi\right)u'(c^b)} = \frac{v+r}{q_n} \tag{19}$$

where

$$q_n = \beta\left(1-\pi\right)\left(v+r\right). \tag{20}$$

Here q_n is the price paid for an Arrow security to purchase one unit of physical capital in the case of no reversal.

Consider now that the function

$$f^D(c^g) = \left(1-\beta(1-\pi)\right)\left(\log c^g - \log 54\right) + \beta\pi\left(\log(63-c^g) - \log 8\right) \tag{21}$$

Setting $f^D(c^g) = 0$, where $\beta = 4/5$ and $\pi = 1/8$, we obtain $c^g = 36$. We want to find values of c^b, θ^g, θ^b, q_r, q_n, and v such that these variables constitute a symmetric steady state. Obviously, $c^b = 63-36 = 27$. Plugging these values into the first-order conditions (18) and (20), we find that

$$q_r = \frac{2}{15}\left(v+1\right) \tag{22}$$

$$q_n = \frac{7}{10}\left(v+1\right). \tag{23}$$

Notice that we can combine these two conditions to obtain

$$q_r + q_n = v = \frac{5}{6}\left(v+1\right), \tag{24}$$

which implies that $v = 5$, $q_r = 4/5 = 0.8$, $q_n = 21/5 = 4.2$. We can plug this into the budget constraint for the consumer with the high endowment,

$$c^g + q_n\theta^g + q_r\theta^b = \omega^g + (v + r)\theta^g \tag{25}$$

$$36 + \frac{21}{5}\theta^g + \frac{4}{5}\left(1 - \theta^g\right) = 54 + 6\theta^g , \tag{26}$$

to solve for $\theta^g = -86/13 = -6.6154$, $\theta^b = 99/13 = 7.6154$.

To implement this steady state as an equilibrium, we can now go back and verify that all of the equilibrium conditions are satisfied for the right choice of $\bar{\theta}_0^1$ and $\bar{\theta}_0^2$.

The comparative statics of this example are of some interest. Suppose that we increase the variance of shocks by increasing (ω^g, ω^b) from (54, 8) to say (56, 6), and then to (58, 4). A computation along the lines above shows that the equilibrium risk sharing increases as the variance of the shocks becomes larger: (c^g, c^b) goes from (36, 27) to (32.6074, 30.3926) and then to (31.5, 31.5), where there is complete risk sharing. As we decrease the variance of shocks, equilibrium risk sharing decreases: as (ω^g, ω^b) goes to (52, 10) and then to (50, 12), (c^g, c^b) goes to (38.6539, 24.3461) and then to (40.6209, 22.3791). Notice that increasing the variance of the shock reduces the attractiveness of running away and increases the desirability of trade. That is, we should not interpret this as meaning the economy as a whole has become more risky, but rather that the economy as a whole has become more specialized and interdependent. Because it is less attractive to run away, it becomes possible to enforce more efficient risk sharing.

This negative relation between the variance of income shocks and the level of risk sharing in equilibrium is a general feature of debt constrained models. Krueger and Perri (2006) study the empirical significance of this relation.

6. The economy with bankruptcy and collateral: "English words that have crept into their language are often used in senses that we would not recognize"

In this section, we show that, when the individual rationality constraint (5) holds, we can support the equilibrium allocation in the debt constrained economy by a combination of bankruptcy and collateral. The possibility of bankruptcy provides a state contingency. The basic idea is that in every period each type makes a loan to the other type. Then the consumers of whichever type has low productivity in the next period default on their loans — that is, they collect the promised payment from the other type, but they do not pay back their own loan. Bankruptcy comes with a penalty: a consumer who defaults loses any holdings of physical capital and — to prevent consumers who have high productivity from defaulting — loses the returns to labor in excess of ω^b. We impose a constraint on borrowing to ensure that consumers do not borrow so much that they violate the individual rationality constraint (5). Notice that the imposition of this constraint makes it possible to impose the bankruptcy penalty of garnishing wages up to the level of ω^b: the choices faced by the high productivity type are to not declare bankruptcy; to declare bankruptcy and pay the penalty; or to run away. In equilibrium, the optimum among these three choices is to not declare bankruptcy.

Let b_s^i denote borrowing by type i in state history s and let a_s^i denote lending. Because the two types have different probabilities of future default, borrowing and lending need not trade at the same price, so we let q_s^i denote the price of the asset corresponding to borrowing by type i in state history s.

Consumers of type i now face the problem

$$(1-\beta)\sum_{s\in S}\beta^{t(s)}\pi_s u(c_s^i)$$

subject to

$$c_s^i + q_s^{i'}a_s^i - q_s^i b_s^i + v_s\theta_s^i \le w^i(\eta_s) + \delta_s^{-i}a_{s-1}^i + \max[-b_{s-1}^i + (v_s + r)\theta_{s-1}^i, \omega^b - w^i(\eta_s)] \tag{27}$$

$$a_s^i \ge 0\,,\ \bar{b} \ge b_s^i \ge 0\,,\ \theta_s^i \ge 0\,,\ \theta_0^i \text{ fixed.}$$

There are two new market clearing conditions:

$$a_s^1 - b_s^2 = 0 \tag{28}$$

$$a_s^2 - b_s^1 = 0\,. \tag{29}$$

The price of a claim to one unit of the income of consumer i in state history s is determined in $s-1$, $q_{s-1}^{i'}$. The return on this claim depends on whether or not consumer i defaults:

$$\delta_s^i = \begin{cases} 1 & \text{if } -b_s^i + (v_{(s,\eta)} + r)\theta_s^i \geq \omega^b - w^i(\eta) \\ \left((v_{(s,\eta)} + r)\theta_s^i + w^i(\eta) - \omega^b\right)/b_s^i & \text{if } -b_s^i + (v_{(s,\eta)} + r)\theta_s^i \leq \omega^b - w^i(\eta) \end{cases}. \tag{30}$$

The concepts of equilibrium and of symmetric stochastic steady state for this economy with bankruptcy and collateral are defined analogously to their counterparts for the debt constrained economy.

Proposition 3: *Given any symmetric stochastic steady state of the debt constrained economy, there exists a borrowing constraint $\bar{b} > 0$ such that there is a symmetric stochastic steady state of the economy with bankruptcy and collateral with the same consumption allocation.*

Proof: We explicitly construct the equilibrium. In this equilibrium, consumers who have low productivity always declare bankruptcy and consumers who have high productivity never do. We use the first-order conditions for the consumer's problem (27) along with the budget constraints and feasibility conditions to construct an equilibrium with these properties.

We first need to determine which consumer purchases the capital. If the consumer with the high productivity purchases the capital, the first-order condition is

$$-v + \beta(1-\pi)(v+r) = 0, \tag{31}$$

which implies that

$$v = \frac{\beta(1-\pi)r}{1-\beta(1-\pi)}. \tag{32}$$

The first-order condition for the consumer with low productivity is

$$-\frac{v}{c^b} + \frac{\beta\pi(v+r)}{c^g} \leq 0, \tag{33}$$

which holds if and only if

$$\frac{c^g}{c^b} \leq \frac{1-\pi}{\pi}. \tag{34}$$

If, on the other hand, the consumer with low productivity purchases the capital, the first-order condition in (33) holds with equality, which implies that

$$v = \frac{\beta\pi c^g r}{c^b - \beta\pi c^g}. \tag{35}$$

In this case, the first-order condition for the consumer with high productivity is

$$-v + \beta(1-\pi)(v+r) \leq 0, \tag{36}$$

which holds if and only if the direction of the inequality (34) is reversed. Consequently, we can divide equilibria into two types, along with a borderline case. In the first type, condition (34) holds and the consumer with high productivity purchases all of the capital. In the first type, condition (34) is violated and the consumer with low productivity purchases all of the capital. If condition (34) holds with equality, it turns out that the two consumers can purchase capital in arbitrary amounts θ^g, θ^b where $\theta^i \geq 0$, $\theta^g + \theta^b = 1$, without affecting the equilibrium allocation. Notice that, in this borderline case, the two calculations of v, (32) and (35), coincide.

To keep the exposition simple, we first consider the case where condition (34) holds. We start by writing the budget constraints as

$$c^g + q^b a^g - q^g b^g + v = \omega^g - b^g + (v+r) = \omega^g - b^b + \delta^g a^b \tag{37}$$

$$c^b + q^g a^b - q^b b^b = \omega^b + a^b = \omega^b + a^g. \tag{38}$$

Notice that, although consumers' consumption and asset accumulation depend only on the state in which they are, there are two ways to get to each state: either a reversal has taken place or not.

To construct the steady state equilibrium, we need to compute the asset prices q^g and q^b, the lending levels a^g and a^b, and the borrowing levels b^g and b^b. Notice that $\delta^g a^b = v + r$ implies that $\delta^g = (v+r)/a^b$.

A consumer who has high productivity pays $q^b a^g$ for a return of a^g if a reversal takes place. The corresponding first-order condition is

$$\frac{q^b}{c^g} = \frac{\beta\pi}{c^b}, \tag{39}$$

which implies that

$$q^b = \frac{\beta\pi c^g}{c^b}. \tag{40}$$

A consumer who has low productivity lends $q^g a^b$ for a return of a^b if no reversal takes place and $\delta^g a^b = (v+r)$ if a reversal takes place. The corresponding first-order condition is

$$\frac{q^g}{c^b} = \frac{\beta(1-\pi)}{c^b} + \frac{\beta\pi\delta^g}{c^g}, \tag{41}$$

which implies that

$$q^g = \beta(1-\pi) + \frac{\beta\pi\delta^g c^b}{c^g}. \tag{42}$$

Notice that the first-order condition for borrowing becomes

$$\frac{q^b}{c^b} - \frac{\beta\pi}{c^g} = \frac{\beta\pi c^g}{(c^b)^2} - \frac{\beta\pi}{c^g} > 0 \tag{43}$$

when the consumer has low productivity and

$$\frac{q^g}{c^g} - \frac{\beta(1-\pi)}{c^g} = \frac{\beta\pi\delta^g c^b}{(c^g)^2} > 0, \tag{44}$$

when the consumer has high productivity. These conditions imply that the borrowing constraints $\overline{b} \geq b_s^i$ bind.

Combining the budget constraints (37) and (38) with the market clearing conditions for borrowing and lending (28) and (29), we find that

$$a^g = b^g = a^b = b^b. \tag{45}$$

We can easily calculate b and set the borrowing constraint $\overline{b} = b$ so that the budget constraints (37) and (38) are satisfied:

$$(1 + q^b - q^g)b = c^b - \omega^b \tag{46}$$

$$\left(1 + \frac{\beta\pi c^g}{c^b} - \beta(1-\pi)\right)b - \frac{\beta\pi(v+r)c^b}{c^g} = c^b - \omega^b \tag{47}$$

$$b = \left(\frac{c^b}{\left(1-\beta(1-\pi)\right)c^b + \beta\pi c^g} \right)\left(c^b - \omega^b + \frac{\beta\pi r c^b}{\left(1-\beta(1-\pi)\right)c^g} \right). \tag{48}$$

It is straightforward, but tedious, to verify that a consumer with low productivity always chooses to default but that a consumer with high productivity never does.

The construction where condition (34) is reversed is similar. The budget constraints become

$$c^g + q^b a^g - q^g b^g + v\theta^g = \omega^g - b^g + (v+r)\theta^g + \delta^b a^g = \omega^g - b^b + (v+r)\theta^b + \delta^g a^b \tag{49}$$

$$c^b + q^g a^b - q^b b^b + v\theta^b = \omega^b + a^b = \omega^b + a^g . \tag{50}$$

Here we treat the general case. If $\pi c^g < (1-\pi)c^b$, then $\theta^g = 1$; if $\pi c^g < (1-\pi)c^b$, then $\theta^b = 1$; and $\pi c^g = (1-\pi)c^b$, then θ^g is arbitrary. Of course, $\delta^b a^g = (v+r)\theta^b$ and $\delta^g a^b = (v+r)\theta^g$. The asset prices q^b become

$$q^b = \frac{\beta\pi c^g}{c^b} + \beta(1-\pi)\delta^b , \tag{51}$$

$$q^g = \beta(1-\pi) + \frac{\beta\pi\delta^g c^b}{c^g} . \tag{52}$$

Once again, the first-order conditions for borrowing hold and $a^g = b^g = a^b = b^b$. The calculation of b becomes

$$(1 + q^b - q^g)b = c^b - \omega^b + v\theta^b \tag{53}$$

$$\left(1 + \frac{\beta\pi c^g}{c^b} - \beta(1-\pi)\right)b + \frac{\beta\left((1-\pi)\theta^b c^g - \pi\theta^g c^b\right)(v+r)}{c^g} = c^b - \omega^b + v\theta^b \tag{54}$$

$$b = \left(\frac{c^b}{\left(1-\beta(1-\pi)\right)c^b + \beta\pi c^g} \right)\left(c^b - \omega^b + v\theta^b - \frac{\beta\left((1-\pi)\theta^b c^g - \pi\theta^g c^b\right)(v+r)}{c^g} \right), \tag{55}$$

where v is determined by equation (32) or equation (35) depending on the case that we are in.

When $\pi c^g = (1-\pi)c^b$, this expression becomes

$$b = c^b - \omega^b + \frac{\beta\pi\theta^g c^b r}{\left(1-\beta(1-\pi)\right)c^g}. \tag{56}$$

Notice that, in this borderline case, the asset prices q^g and q^b, the lending levels a^g and a^b, and the borrowing levels b^g and b^b all vary with $\theta^g = 1-\theta^b$, but the consumption levels c^g and c^b are fixed at their levels in the debt constrained equilibrium. ■

7. The numerical example revisited: "The ability to say the same thing in several different tongues is a highly esteemed talent among them"

We now apply proposition 3 to show how the equilibrium allocation of the debt constrained economy in our numerical example can be implemented as an equilibrium allocation in the economy with bankruptcy and collateral. We first use (48) to calculate the levels of borrowing and lending:

$$b = \frac{1155}{26} = 44.4231. \tag{57}$$

Next, we use (32) to calculate the price of capital:

$$v = \frac{7}{3} = 2.3333. \tag{58}$$

Notice that this implies that

$$\delta = \frac{v+r}{b} = \frac{52}{693} = 0.0750. \tag{59}$$

We can now use (40) and (42) to calculate the prices of claims on next period's income:

$$q^b = \frac{2}{15} = 0.1333 \tag{60}$$

$$q^g = \frac{163}{231} = 0.7056. \tag{61}$$

Kehoe and Levine (2001) provide a simple argument that demonstrates that the equilibrium allocation in the debt constrained model — and consequently the equilibrium allocation in this model with bankruptcy and collateral — is Pareto efficient among allocations that satisfy the individual rationality constraint. Notice how this allocation is supported by borrowing and lending assets with different returns. Proposition 3, which shows that consumers can exploit the contingencies provided by collateral to achieve an efficient allocation, is reminiscent of results in finance, like those of Duffie and Huang (1985) and Kreps (1982), that show that a small number of assets can span the uncertainty facing investors. What is important in our model is that the consumers can go long in some assets and short in others. The efficient nature of the outcome turns the advice of Polonius to Laertes in Shakespeare's *Hamlet* on its head: "Both a borrower and a lender be."

Now consider the comparative static experiment of increasing specialization in the sense that the variance of shocks increases by changing (ω^g, ω^b) from (54, 8) to (56, 6) and then to (58, 4), as we did in the market with complete contingent claims. With complete contingent claims, we saw that equilibrium risk sharing increased as the variance of income shocks increased. This cannot be the case with the model of bankruptcy and collateral: the borrowing limit is calibrated to the old equilibrium, not the new, so it is impossible for the equilibrium to adjust in the short run. In fact, equilibrium risk sharing goes down, as (c^g, c^b) goes from (36, 27) to (37.4078, 25.5922) and then to (38.7722, 24.2278). To achieve the same equilibrium allocation as in the debt constrained model, we would need to loosen the borrowing constraint $\bar{b}$ from 44.4231 to 60.6534 and then to 69.5833.

More surprising perhaps is what happens if we decrease specialization in the sense that the variance of the shocks decreases by changing (ω^g, ω^b) to (52, 10). In this case the consumers with high productivity want to run away, and the equilibrium collapses to autarky. Even if we devise a scheme to keep consumers from running away in the Kehoe-Levine (2001) sense, we run into trouble as we decrease the variance of shocks still further by setting (ω^g, ω^b) to (50, 12). In this case, even consumers with high productivity choose to default, and the equilibrium collapses to autarky.

8. Leijonhufvudian Economics and the Economics of Leijonhufvud

The literature on bankruptcy in general equilibrium typically takes the incomplete markets model as its point of departure. In this model, bankruptcy — like the incomplete markets themselves — is a pathology. Bankruptcy serves to solve no substantive economic problem, and serves only to hinder the proper working of the economy. The only conclusion we can sensibly reach from this literature is that the economy does not work well.

The idea that on a day-to-day basis the economy works poorly is deeply anti-Leijonhufvudian in spirit. Leijonhufvud's deepest insight is his (1973a) notion of the "corridor of stability." On a day-to-day basis in modern economies, things work well. It is not plausible that we could all be much better off if not for the nasty facts of market incompleteness, bankruptcy, and collateral.

This paper takes a point of view more consistent with Leijonhufvud's corridor of stability. Here borrowing limits, bankruptcy, and collateral arise to solve a real economic problem, that of providing insurance in the presence of individual rationality constraints. In our account, this economy is second best: given the underlying individual rationality constraints, the equilibrium is the best possible.

Having given a description of the corridor of stability where the economy responds efficiently to ordinary shocks, we are now free to ask the deeper Leijonhufvudian question: How robust are the institutions of bankruptcy and collateral in responding to a shock for which they are not designed? The answer is that these institutions are quite fragile. While the debt constrained complete market economy responds to changes in the variance of the shocks by adjusting the amount of risk sharing, the collateralized economy cannot adjust the risk sharing upwards in response to increased variance of shocks — and collapses completely in the face of decreased variance to shocks. This latter point is of some interest: our general intuition is that reducing the variance of shocks should be a good thing.

References

A. Araujo, M. Páscoa, and J. Torres-Martínez (2002), "Collateral Avoids Ponzi Schemes in Incomplete Markets," *Econometrica* 70: 1613–1638.

S. Chatterjee, D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2004), "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," University of Pennsylvania.

P. Dubey, J. Geanakoplos and M. Shubik (1989), "Default and Efficiency in a General Equilibrium Model with Incomplete Markets," Cowles Foundation Discussion Paper 900.

P. Dubey, J. Geanakoplos, and W. R. Zame (1995), "Default, Collateral, and Derivatives," Yale University.

D. Duffie and C. F. Huang (1985), "Implementing Arrow-Debreu Equilibria by Continuous Trading of Few Long-Lived Securities," *Econometrica* 53: 1337–1356.

J. Geanakoplos and W. R. Zame (2002), "Collateral and the Enforcement of Intertemporal Contracts," UCLA.

T. J. Kehoe and D. K. Levine (1993), "Debt Constrained Asset Markets," *Review of Economic Studies* 60: 865–888.

T. J. Kehoe and D. K. Levine (2001), "Liquidity Constrained Markets versus Debt Constrained Markets," *Econometrica* 69: 575–598.

N. Kiyotaki and J. Moore (1997), "Credit Cycles," *Journal of Political Economy* 105: 211–248.

D. M. Kreps (1982), "Multiperiod Securities and the Efficient Allocation of Risk: A Comment on the Black-Scholes Option Pricing Model," in J. J. McCall, editor, *The Economics of Uncertainty and Information*, University of Chicago Press, 203–232.

D. Krueger and F. Perri (2006), "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies* 73: 163–193.

F. Kubler and K. Schmedders, (2003), "Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral," *Econometrica* 71: 1767–1793.

A. Leijonhufvud (1973a), "Effective Demand Failures," *Swedish Journal of Economics* 75: 27–48.

A. Leijonhufvud (1973b), "Life among the Econ," *Western Economic Journal* 11: 327–337.

I. Livshits, J. MacGee, and M. Tertilt (2003), "Consumer Bankruptcy: A Fresh Start," Federal Reserve Bank of Minneapolis Working Paper 617.

J. Orrillo (2002), “Default and Exogenous Collateral in Incomplete Markets with a Continuum of States,” *Journal of Mathematical Economics* 35: 151–165.

W. R. Zame (1993), “Efficiency and the Role of Default When Security Markets Are Incomplete,” *American Economic Review* 83: 1142–1164.