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Neuro-fuzzy Knowledge Processing in Intelligent Learning Environments for Improved Student Diagnosis

¹Regina Stathacopoulou^{*}, George D. Magoulas^{**}, Maria Grigoriadou^{*}
and Maria Samarakou^{***}

^{*}Department of Informatics and Telecommunications, University of Athens, Panepistimiopolis,
GR-15784 Athens, Greece
{sreg@di.uoa.gr, gregor}@di.uoa.gr
Phone: +30-210 7275230; Fax: +30-210 7275214

^{**}School of Computer Science and Information Systems, Birkbeck College, University of London
Malet Street, London WC1E 7HX, United Kingdom
g.magoulas@bbk.ac.uk

^{***}Department of Energy Technology, Technological Education Institute of Athens,
Ag. Spyridonos Str. GR 12210, Egaleo, Athens, Greece
marsam@teiath.gr
Phone: +30-210-5385322, Fax: +30-210-5385306

¹Corresponding author

Abstract. In this paper, a neural network implementation for a fuzzy logic-based model of the diagnostic process is proposed as a means to achieve accurate student diagnosis and updates of the student model in Intelligent Learning Environments. The neuro-fuzzy synergy allows the diagnostic model to some extent “imitate” teachers in diagnosing students’ characteristics, and equips the intelligent learning environment with reasoning capabilities that can be further used to drive pedagogical decisions depending on the student learning style. The neuro-fuzzy implementation helps to encode both structured and non-structured teachers’ knowledge: when teachers’ reasoning is available and well defined, it can be encoded in the form of fuzzy rules; when teachers’ reasoning is not well defined but is available through practical examples illustrating their experience, then the networks can be trained to represent this experience. The proposed approach has been tested in diagnosing aspects of student’s learning style in a discovery-learning environment that aims to help students to construct the concepts of vectors in physics and mathematics. The diagnosis outcomes of the model have been compared against the recommendations of a group of five experienced teachers, and the results produced by two alternative soft computing methods. The results of our pilot study show that the neuro-fuzzy model successfully manages the inherent uncertainty of the diagnostic process; especially for marginal cases, i.e. where it is very difficult, even for human tutors, to diagnose and accurately evaluate students by directly synthesizing subjective and, some times, conflicting judgments.

Keywords. Student Diagnosis, Uncertainty Management, Fuzzy Logic, Neural Networks, Student modelling, Intelligent Learning Environments, Intelligent Tutoring Systems, Discovery Learning Environments, Learning Styles.

1. Introduction

User and student modeling is a fundamental mechanism to achieve individualized interaction between computer systems and humans [41]. It is usually concerned with modelling several user related issues, such as goals, plans, preferences, attitudes, knowledge or beliefs. The most difficult task in this context is the process of interpreting the information gathered during interaction in order to generate hypotheses about users and students behaviour [41], and involves managing a good deal of uncertainty. Interactive computer systems deal in general with more meagre and haphazardly collected users' data than it usually happens when humans are engaged in face-to-face interaction [26]. Thus, the gap between the nature of the available evidence and the conclusions that are to be drawn is often much greater [26]. Numerical techniques have been employed in several cases in order to manage uncertainty, [3] [13] [22] [23] [24] [26] [27] [30] [42] [59], and neural networks have been used in order to add learning and generalization abilities in user models and draw conclusions from existing user profiles [10] [19] [21] [32] [36] [37] [43] [46] [53] [61].

According to Self, [50], student modelling is the process of creating and maintaining student models. It is divided into the design of two different but tightly interwoven components [55]: (i) the *student model* which, in its simplest form, is a data structure that stores information about the student; (ii) the *diagnostic module* which performs the diagnostic process that updates the student model. Student models are distinguishing features of Artificial Intelligence, (AI), based computer-based instructional systems.

This work focuses on an application of student modelling in Intelligent Learning Environments (ILE). ILEs are considered as generalization of traditional Intelligent Tutoring

systems (ITS), which are based on objectivist epistemology, and embrace instructional environments that make use of theories on constructivism and situated cognition [1]. Naturally, a good background for building student models for ILEs is provided by research conducted in the area of ITSs [8]. ITSs make use of AI techniques to represent and process knowledge about the domain and the student, and usually follow a natural division of the task of knowledge communication into four distinct components: *domain expertise*, *model of the student*, *communication strategies* or *pedagogical expertise*, and *interface with the student* [60]. The student model-centered architecture is also proposed for ILEs in order to support student-driven learning and knowledge acquisition [8].

Ideally, the student model should include all the aspects of student's behaviour and knowledge that have repercussions for their performance and learning [60]. In practice, the contents of the student model depend on the application. It includes learner goals and plans, capabilities, attitudes and/or knowledge or beliefs, and is used as a tool to adapt ILE's behaviour to the individual student [25][50]. Inferring a student model is called *diagnosis* because it is much like a medical task of inferring a hidden physiological state from observable signs [55], i.e. the ILE uncovers the hidden cognitive state (student characteristics) from observable behavior.

Researchers in student modelling area have used AI techniques in order to develop models that provide detailed diagnosis of student's knowledge, bugs and misconceptions, and/or simulate the cognitive behaviour of a student during learning and problem solving activities (see [39] for reports on various approaches, and [49][51][55][60] for reviews).

Along these lines, the model of the diagnostic process that is proposed in this paper aims to diagnose student behaviour based on teachers' expertise for the purpose of adapting pedagogical decisions to the individual student. Evidence shows that human teaching is not based on fine-grained diagnostic behaviour [48]. In particular, studies in human tutoring have

found little evidence to suggest that human tutors build detailed cognitive models as a basis for understanding student performance and adapting their tutoring strategy [35][47]. More recently, researchers have tried to identify the constructs that tutors use to classify and discriminate among different students states for the purpose of adapting tutoring to student individual differences [15]. Their results have been based on the assumption that, during tutoring, the expert tutor gathers evidence and forms relatively general ideas of the kind of tutoring that might work better for each student. According to these findings, all tutors judged and classified students in terms of two underlying dimensions that were similarly defined, through not exactly alike, across tutors: motivation and intellectual ability.

The neural network-based fuzzy model presented in this paper aims to “imitate” teacher's knowledge acquisition procedure in evaluating student's learning characteristics, such as capabilities, attitudes, knowledge level, motivation and learning style. Fuzzy logic is used to provide a mode of qualitative reasoning, which is closer to human decision making since it handles imprecision and vagueness by combining fuzzy facts and fuzzy relations, whilst neural networks provide a convenient way to achieve adaptability of the diagnostic process to teacher's subjective reasoning and judgments. Thus, a neuro-fuzzy implementation helps the system to encode both structured and unstructured knowledge, e.g. fuzzy rules and learning from examples, respectively.

The paper is organized as follows. In Section 2 we give a brief overview of fuzzy logic and neural network techniques in user and student modelling, and provide a general description of our approach explaining its differences from existing techniques. Section 3 covers several aspects of our model: data gathering, knowledge representation and implementation details of the neural-network based fuzzy model. Section 4 presents an application of the proposed model in a discovery learning environment, giving details on the environment, the aspects of

the students' learning style diagnosed by our model, and comparative evaluation results. Lastly, conclusions are drawn and directions for future work are presented.

2. Fuzzy and neural approaches to user and student modelling

As already mentioned a variety of numerical techniques have been employed in user and student modelling systems in order to handle the imprecise information provided by the users, and reason under vagueness and uncertainty; a comparative review of techniques can be found in [26]. For example, Bayesian networks have been successfully used to relate in a probabilistic way user's knowledge and characteristics with user's observable behaviour. The key to success with all Bayesian network models lies in accurately representing the probabilistic dependencies in the task domain [13]. Fuzzy logic techniques have also been used for this task effectively. When considering the use of such techniques in a user or student modelling system, the addressed arguments do not concern in principle the question of whether or not fuzzy logic provides accurate or useful results by rather the usability of fuzzy logic techniques in the design of the specific system, in terms of knowledge engineering requirements, programming effort, empirical model adjustment, computational complexity, human-likeness, interpretability and justifiability [26]. Fuzzy logic can claim advantages with respect to other alternatives in several of these issues [26], as for example in computational complexity. In addition reasoning of a fuzzy logic system is considered easy for designers and users to understand and/or to modify [26]. One of the factors for this consideration is human-likeness. Although, the gap between human and Bayesian inference is not as wide as is commonly believed, human-likeness is much stronger associated with fuzzy logic since it can provide human-like descriptions of knowledge and imitate a "human" style of reasoning with vague concepts [26]. These are of particular interest when trying to design an interpretable student modelling system based on teacher's reasoning and conceptualization of the learner,

as in our approach. In addition, the Bayesian approach requires the determination of probabilities from experts' judgments, whilst fuzzy logic provides a convenient method to elicit the necessary knowledge from domain experts, thus expert teachers in case of student modeling, to implement the system. It is easier and more reliable to extract knowledge from experts in linguistic form rather than in numbers representing this knowledge since experts feel most comfortable giving the original linguistic data [28].

One of the first attempts in using fuzzy student modelling has been made by Hawkes *et al.* [23]. In this context fuzzy logic has been proposed as a flexible and realistic method to easily capture the way human tutors might evaluate a student and handle tutoring decisions, which are not clear-cut ones. Clearly, the capability to deal with such imprecision is a definite enhancement to both ITSs and ILEs. This approach, which has been revised some years later [22], was used to evaluate students in a system called TAPS, and applied degrees of membership to linguistic labels that match student's solutions to "acceptable" solutions with the use of informal fuzzy reasoning.

Towards this direction, several other attempts have been proposed in the literature. In Sherlock II [27] and in the MDF tutor [1] the uncertainty in student's performance was managed using fuzzy distributions and a set of rules for their formulation and update. Several other systems have been employed based on fuzzy logic concepts. In an ITS for the physics domain, the, so called, "Knowledge and Learning Student Model" [42] has been proposed to infer student's knowledge level and cognitive abilities through processing and aggregating membership functions that represent teacher's assessments. Fuzzy rules have been proposed in the BSS1 tutoring system [59] to implement a general fuzzy logic engine that can better manage student's learning, and in SYPROS [24] to help determine student's plans. A fuzzy algebraic structure has been proposed as a dynamic model of user's states during navigation to monitor cognitive variables of the user model in a multimedia tutoring system [30].

The development of fuzzy logic in user or student modelling systems was motivated largely by the desire to make the arbitrary specification of precise numbers unnecessary [26]. However, the fuzzy approach translates and process knowledge in a numerical framework. In addition, although fuzzy logic allows knowledge engineers to acquire knowledge from experts in linguistic form, experts rarely can articulate the propositional or mathematical rules that describe their expert behaviour [28]. A complementary strategy is to employ machine learning techniques for implementing the system and acquiring the necessary numbers [26]. Neural networks can serve this purpose. Both neural networks and fuzzy systems are model-free estimators. Unlike statistical estimators, they estimate a function without a mathematical model/assumption of how outputs depend on inputs [28]. They can “learn from experience” expert’s knowledge with linguistic or numerical sample data by means of specialised learning procedures, and provide a robust approach to approximating real-valued, discrete-valued, and vector-valued target functions. For certain types of problems, such as learning to interpret complex real-world sensor data, neural networks are among the most effective learning methods currently known [38]. In the user or student modelling field, neural networks have been proposed in the literature mainly due to their ability to learn from noisy or incomplete patterns of users’ or students’ behaviour, generalize over similar cases, and then use this generalized knowledge to recognize unknown sequences [10] [61]. Particularly in student modelling, neural networks have been originally proposed to simulate student’s cognitive process of performing subtraction with the aim to predict student's responses and errors [36].

A problem, which comes up when trying to apply a neural network in modelling human behaviour, is knowledge representation [61]. The fact that student models need to be inspectable, [60], explains the small number neural network-based student models as opposed to symbolic approaches [51]. Neural networks and other numeric-based AI methods have been criticized as unable to support learning interactions because they only allow for implicit

understanding [49]. However, several attempts have been made to incorporate the powerful learning abilities of neural networks in existing student modelling systems taking advantage of synergies with other AI methods. A hybrid approach, where each node and connection has symbolic meaning, has been proposed in TAPS [46]. The back-propagation algorithm has been used to modify weights that represent importance measures of attributes associated with student's performance, in order to refine and expand incomplete expert knowledge. Another approach combining ideas from neuro-fuzzy systems has been proposed [19]. In [32], the model of [19] has been expanded to incorporate evaluation mechanisms that used multi-attribute decision making for synthesizing various judgments to estimate student's knowledge levels and personal characteristics in order to plan the content of a Web based course.

This paper makes use of neuro-fuzzy synergism in order to infer the learning characteristics of the student in an ILE, and to create and update the student model taking into consideration teacher's personal opinion/judgment. Fuzzy logic is used to handle uncertainty and to express teacher's qualitative knowledge in a clearly interpretable way. The fuzzy model represents teacher's knowledge in linguistic form and infers student's characteristics through a set of fuzzy systems, realizing in this way a human-like diagnostic process, i.e. a decision is made by combining fuzzy facts, each one contributing to some degree to a fuzzy relation and to the final decision. Neural networks are used to equip the fuzzy model with learning and generalization abilities, which are eminently useful when teacher's reasoning process cannot be defined explicitly.

The new approach aims to represent human teacher's conceptualization of student during instruction by modelling their reasoning process in diagnosing unobservable student's characteristics. To this end, teacher's evaluation procedure is decomposed into three meaningful stages: *gathering* evidence during interaction; *evaluating* the student; *reaching* a decision. Information of student's observable behaviour is described and processed

qualitatively with the use of fuzzy logic variables and operators. Thus, a more accurate and more natural modelling of human's tutor diagnostic process is achieved. This form of modelling permits to determine the specific characteristics of the diagnostic process, such as the types of evidences that must be used to discriminate among students, the characteristics of students that lead to pedagogical decisions, and the rules underlying the inference process. Furthermore, it is able to cope with subjectivity incorporated in knowledge acquisition and reasoning; thus, it can be easily adapted to the lesson content according to teacher's subjective inferences and decisions.

The proposed model allows exploiting and efficiently processing structured knowledge in the form of linguistic rules. Of course it is not always possible to elicit this knowledge from the teachers. Teachers, sometimes, although they can easily classify students by observing their actions, they cannot articulate rules that reproduce their decisions. In addition, teachers are able to classify students with respect to specific characteristics, whilst in the case of ILE-supported learning students' behaviour cannot be defined accurately. To alleviate these problems, a neural network-based implementation of the diagnostic process is adopted. Specialized neural networks are trained through examples of existing students' profiles, or using examples that represent teacher's experience. Knowledge is represented by developing association of student's behaviour patterns with particular characteristics through neural network learning and is expressed, if necessary, with fuzzy if-then rules. Thus, it is possible to encode structured and non-structured knowledge.

3. Fuzzy modelling of the diagnostic process

3.1. Collecting and processing information

Student's observable behaviour is considered important source of diagnostic evidence to both human tutors and ILEs. In the terminology of ILEs, *student's behaviour* refers to a

student's observable response to a particular stimulus in a given domain. The response, together with the stimulus, serves as the primary input to the student modelling system [51]. The input can be an action or the result of that action, and can also include intermediate results [51]. However, it is not generally clear what type of information is available during interaction, and which features of student's behaviour should be selected as inputs to the diagnostic process. Human tutors obtain diagnostic information from observing what students would say and do, and how something is said and done, i.e. tone of voice, inflection, hesitancy, etc. [15]. Studies in human tutoring found that tutors use as diagnostic evidence for adapting their tutoring not only errors and student's responses to queries, but also features of interaction, e.g. the timing of student responses, the way of delivering a response and others [15]. ILEs are handicapped in this regard, since the communication channel between student and computer is very restricted (usually a keyboard and a mouse) [60]. However, some indirect information that approximates student's unobservable behaviour can be obtained [55][60]. In addition, an appropriately designed interface can facilitate the process of collecting the best available information about what the student is doing (e.g. timing each keystroke) to make diagnosis both computationally tractable and more accurate [60].

In order to alleviate the problem of limited information that is caused by the restricted communication channel between student and ILE, our system implements a close monitoring mechanism of student's actions over time, where each response such as keystroke, mouse move or drag can be timed and recorded. In this way various data can be extracted from student's records: (i) *knowledge data*, such as the number of correct, incorrect or almost correct answers in separate tests, and the number of student's conceptual errors; (ii) *chronometric data*, such as the time spent to read the theory, a page or a line, the time to find the correct answers in a test, the total time on task, the time of idle intervals; (iii) *try data*, such as the number of attempts to find the correct solution, the number of times needed to

review the theory; (iv) *navigation data*, such as the number of times a topic, activity, tool, or exercise has been selected, frequency that specific student selections occurred, the number of times the student moves to another topic without achieving a previously set goal. In this manner student's observable responses are summarized into k groups. Each group contains information about student's behaviour of a specific type of knowledge data, chronometric data, try data or navigation data. A teacher usually defines specific types of responses that enable him or her to discriminate among students with regards to a particular characteristic.

The set $B = \{B_1, B_2, \dots, B_i, \dots, B_k\}$, where B_i ($i=1, 2, \dots, k$) is a word or a sentence describing the i -th type of response that is observed, describes linguistically the k aspects of student's observable behaviour that will serve as inputs to the diagnostic process. The term observable, here, stands for measurable. The k measured responses constitute a set of numeric information that represents student's behaviour. Each type i ($i= 1, 2, \dots, k$) takes its values in a set of positive numbers U_i . The numerical input $X = \{x_1, \dots, x_i, \dots, x_k\}$, where $x_i \in U_i$ and U_i is the universe of discourse of the i -th input; each $U_i \subset \mathfrak{R}^+$ ($i= 1, 2, \dots, k$) represents the measured values of B_i and formulates an input to the diagnostic process.

The output of the diagnostic process updates the student model regarding L different student learning characteristics C_1, C_2, \dots, C_L , such as student's abilities, motivation or learning style. Student's evaluation regarding each characteristic C_j ($j=1, 2, \dots, L$) is described qualitatively with the use of linguistic values. Depending on the j -th characteristic we use a different number m_j of linguistic values that describe C_j ($j=1, 2, \dots, L$).

Student's evaluation regarding each characteristic is assessed by processing the numerical input $X = \{x_1, \dots, x_i, \dots, x_k\}$, of student's behaviour. The process consists of three stages: fuzzification, inference, and defuzzification (see Figure 1). In the first stage a qualitative description of student behaviour is obtained by transforming the numeric input data into

linguistic terms. The i -th fuzzifier ($i=1,2,\dots,k$) transforms the numeric input x_i into membership degrees of the linguistic values that describe B_i . In the second stage, the inference process provides a fuzzy assessment of student's characteristics, C_1, C_2, \dots, C_L , by assessing membership degrees to the linguistic terms that describe each characteristic C_j . To this end, an ensemble of specialized fuzzy systems, where each system infers about a particular characteristic C_j is used to make a fuzzy assessment from a fuzzy precondition. A fuzzy system of this type combines linguistic values and realizes fuzzy relations operated with the *max-min* composition. These relations represent the estimation of a human tutor to the degree of association between an observed input $X = \{x_1, \dots, x_i, \dots, x_k\}$, and a fuzzy assessment of a particular student characteristic C_j ($j=1,2,\dots,L$). Finally, in the third stage, the fuzzy assessments are defuzzified to non-fuzzy values, i.e. evaluation decisions for the characteristics C_1, \dots, C_L by using a defuzzifier from the ensemble of the M defuzzifiers. Each defuzzifier has a different number of inputs. Therefore, depending on the number of linguistic values m_j of each characteristic C_j ($j=1,2,\dots,L$) a different defuzzifier M is used in order to evaluate student's characteristic.

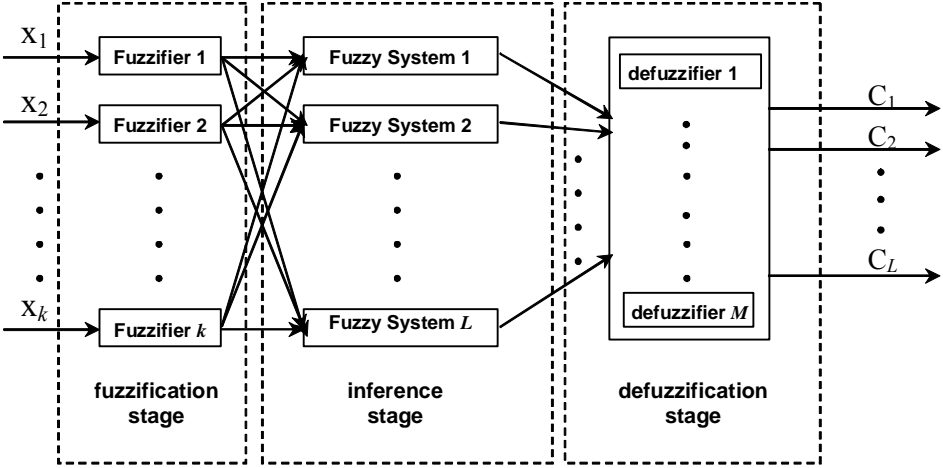


Figure 1. Schematic of the diagnostic model.

3.2. A scheme for fuzzy knowledge representation

3.2.1 Fuzzification stage

This stage represents in linguistic form teacher's subjective description of student's responses when acting face-to-face communication during instruction (e.g. the time needed to solve the exercises was *short*; the student answered *enough* questions during instruction). The types of responses $B_1, \dots, B_i, \dots, B_k$ are treated as linguistic variables. Each variable B_i ($i=1, 2, \dots, k$) can take a different number of linguistic values f_i . The number f_i of the linguistic values and their names V_1, V_2, \dots, V_{f_i} are defined by the developer with the help of experts, and depend on each variable. The set $T(B_i) = \{V_{i1}, V_{i2}, \dots, V_{if_i}\}$ is the term set of B_i . For example, let us consider the linguistic variable $B_i = \text{"time on task"}$. The corresponding term set could be $T(B_i) = T(\text{time on task}) = \{\text{Short, Normal, Long}\}$ including three ($f_i=3$) linguistic values, or any classification such as $T(B_i) = T(\text{time on task}) = \{\text{Very Short, Short, Normal, Long, Very Long}\}$ including five ($f_i=5$) linguistic values, depending on the required resolution. $T = \{T(B_1), \dots, T(B_i), \dots, T(B_k)\}$ is the set of all term sets that represent the overall observable behaviour B (for all B_i , $i=1, 2, \dots, k$). Thus, the numeric input $X = \{x_1, \dots, x_i, \dots, x_k\}$, that represents the measured values of $B_1, \dots, B_i, \dots, B_k$ is fuzzified by means of linguistic values $V_{11}, V_{12}, \dots, V_{1f_1}$; $V_{i1}, V_{i2}, \dots, V_{if_i}$; $V_{k1}, V_{k2}, \dots, V_{kf_k}$. Thus, the student behaviour B is represented as a set of numeric values $Y = \{(y_{11}, y_{12}, \dots, y_{1f_1}), \dots, (y_{i1}, y_{i2}, \dots, y_{if_i}), \dots, (y_{k1}, y_{k2}, \dots, y_{kf_k})\}$ in $[0, 1]$, which represent the degree of membership of each numeric value x_i ($i=1, \dots, k$) into the term set of B_i with linguistic values $V_{i1}, V_{i2}, \dots, V_{if_i}$.

3.2.2 Inference stage

This stage represents teacher's reasoning in categorizing students qualitatively according to their abilities and personal characteristics, such as attentive, rather slow, good, etc. Teachers' can provide a series of IF-THEN rules that approximates their reasoning. For example, *if the*

time spent to read the theory is short and the number of correct answers is high, and few attempts to find the correct answers have been made then the student learning rate is fast.

In our model, a qualitative description of student's characteristics C_1, C_2, \dots, C_L is performed by treating student's characteristics as linguistic variables. Each linguistic variable C_j can take a different number of linguistic values m_j . $T(C_j) = \{C_{j1}, C_{j2}, \dots, C_{jm_j}\}$ is the term set of C_j . The expert-teachers set the number m_j of the linguistic values and their names $C_{j1}, C_{j2}, \dots, C_{jm_j}$ for each characteristic C_j according to their personal judgement. For example, if we treat the linguistic variable $C_j = \text{"learning rate of the student"}$ using five linguistic values ($m_j = 5$) then the term set could be: $T(C_j) = T(\text{learning rate}) = \{\text{Slow, Rather Slow, Normal, Almost Fast, Fast}\}$. In this way, a mode of qualitative reasoning, in which the preconditions and the consequents of the IF-THEN rules involve fuzzy variables [64], is used to provide an imprecise description of teacher's reasoning:

“IF B_1 is V_{1I_1} AND B_2 is $V_{2I_2} \dots$ AND B_k is V_{kI_k} THEN C_1 is C_{1J_1} AND C_2 is $C_{2J_2} \dots$ AND C_L is C_{LJ_L} .”

where $I_1 = 1, 2, \dots, f_1$; $I_2 = 1, 2, \dots, f_2$; $I_k = 1, 2, \dots, f_k$; $J_1 = 1, 2, \dots, m_1$; $J_2 = 1, 2, \dots, m_2$; $J_L = 1, 2, \dots, m_L$.

All possible combinations in the preconditions, denoted as PCP below, are represented by the Cartesian product of the sets in $T = \{T(B_1), T(B_2), \dots, T(B_k)\}$: $PCP = T(B_1) \times T(B_2) \times \dots \times T(B_k)$, and the number $n = f_1 \times f_2 \times \dots \times f_k$ of possible cases in the preconditions equals to the number n of elements of PCP . Each fuzzy system j (see Fig. 1) infers a fuzzy assessment of a different characteristic C_j ($j = 1, 2, \dots, L$). Within each fuzzy system, the intersection (corresponding to the logical AND) between the membership functions associated with the linguistics values of each precondition is the *min* operation, and results in the numerical truth-value p_n of the precondition. Thus, student's current behaviour is described by a vector $P = (p_1, p_2, \dots, p_n)$, where p_1, p_2, \dots, p_n are in the interval $[0, 1]$, representing degrees of fulfilment of preconditions. By means of a fuzzy relation, [44] [45], as described below, P is translated into fuzzy

assessments by exploiting teacher's subjective judgments (denoted by the symbol R_j in the relation right below) with respect to a characteristic C_j

$$P \circ R_j = C_j,$$

where C_j is an m -dimensional vector $C_j = [c_{j1}, c_{j2}, \dots, c_{jm}]$ with c_{j1}, c_{j2}, c_{jm} in $[0,1]$ representing the fuzzy assessment of student's characteristic C_j , i.e. an assessment with membership degrees $c_{j1}, c_{j2}, \dots, c_{jm}$ on each linguistic value ($C_{j1}, C_{j2}, \dots, C_{jm}$) of the linguistic variable for the characteristic C_j ; R_j is a $n \times m_j$ weight matrix representing teachers' estimations of the degree of association between precondition P and the linguistic values of student's characteristic C_j ; the symbol \circ denotes the *max-min* composition operator.

3.2.3 Defuzzification stage

This stage represents teacher's final decision in classifying a student in one of the predefined linguistic values $C_{j1}, C_{j2}, \dots, C_{jm}$ of the characteristic C_j . This process is performed by weighting the fuzzy assessment. Depending on the number of linguistic values m_j of each characteristic C_j , we use an appropriate defuzzifier from the ensemble, i.e. implementing a different defuzzification procedure that "imitates" a teacher's subjective decisions. Teacher's decisions may be clear-cut or marginal. Decisions in marginal cases are highly subjective and, usually, teachers are reserving the best or the worst qualification of their students. Thus, we have used a neural network-based implementation, which allows the system to adapt the defuzzification procedure to individual user's (teacher) opinion by training, as will be explained in the next section.

3.3. Neural-network based implementation of the fuzzy model

3.3.1 Fuzzification

Depending on the linguistic variable B_i and the linguistic value $V_{i1}, V_{i2}, \dots, V_{ifi}$, we subjectively define different membership functions, which assign to each element x_i of the universe of discourse U_i ($i=1, \dots, k$) a degree of membership $y_{if_i}(x_i)$ to the linguistic value V_{if_i} of B_i . In this way they contribute to the semantic rule that associates each linguistic value V_{if_i} of B_i with its meaning [63]. In general, the form of a membership function depends experts opinions [62]. In our case, we have adopted an approach that simplifies the implementation by approximating the membership functions using a library of regular shapes and implementing the fuzzifier stage as a group of fixed weight neural networks that calculate such regular shapes. Since membership functions are subjective and generally context-dependent, [63], a set $M = \{m_1, m_2, \dots, m_k\}$ of parameters that adjust the membership functions [53] is defined to allow a range of adaptations to teacher's subjective judgments. Thus, for each one of the linguistic values of the set $T = \{T(B_1), T(B_2), \dots, T(B_k)\}$, the fuzzifier stage calculates the output Y of numeric values in $[0,1]$ based on the input vectors $X = \{x_1, \dots, x_i, \dots, x_k\}$, and $M = \{m_1, m_2, \dots, m_k\}$:

$$Y = \left\{ \{y_{11}(x_1, m_1), y_{12}(x_1, m_1), \dots, y_{1f_1}(x_1, m_1)\}, \right. \\ \left. \{y_{21}(x_2, m_2), y_{22}(x_2, m_2), \dots, y_{2f_2}(x_2, m_2)\}, \dots, \{y_{k1}(x_k, m_k), y_{k2}(x_k, m_k), \dots, y_{kf_k}(x_k, m_k)\} \right\}$$

Thus, in our implementation, shown in Figure 2, we have used sigmoid functions as membership functions for the extreme linguistic values V_1, V_{f_i} , and the pseudotrapezoidal function (composed of two sigmoid functions) for the intermediate values, V_2, \dots, V_{f_i-1} ; the adjusting parameter m_i is the expected mean value of a measured value x_i , as estimated by the teacher of the specific teaching subject.

Each fuzzifier i ($i=1, 2, \dots, k$) of Figure 1 is implemented with a network of the type shown in Figure 2. The network of Figure 2 is used to calculate the membership grades of the

linguistic values f_i , when $x_i=x$ and $m_i=m$ (see Figure 3 for a sample of membership functions used in our system).

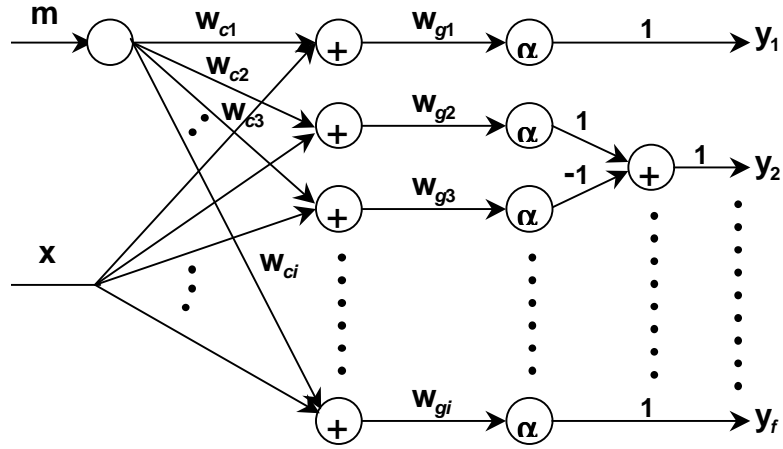


Figure 2. The implementation of a fuzzifier.

The left and the right extreme fuzzy sets are given by

$$y_1(x, m) = \frac{1}{1 + \exp(-w_{g1}(x + w_{c1}m))}, w_{g1} < 0;$$

$$y_f(x, m) = \frac{1}{1 + \exp(-w_{gf}(x + w_{cf}m))}, w_{gf} > 0;$$

where $i = 2(f-1)$. An intermediate set j is given by

$$y_j(x, m) = \frac{1}{1 + \exp(-w_{gj}(x + w_{cj}m))} - \frac{1}{1 + \exp(-w_{gj'}(x + w_{cj'}m))},$$

where $j = 2, \dots, f-1$, $w_{gj} > 0$, $w_{gj'} > 0$ ($i = 2(j-1)$; $i' = i+1$).

In the above relations, x indicates the current measurement of the observed response; w_{ci} and w_{gi} , are defined in advance according to human teachers opinions; $w_{ci} \cdot m$ ($i = 1, \dots, 2(f-1)$), is the central position of the sigmoid function; w_{gi} , ($i = 1, \dots, 2(f-1)$) is the gradient of the sigmoid function.

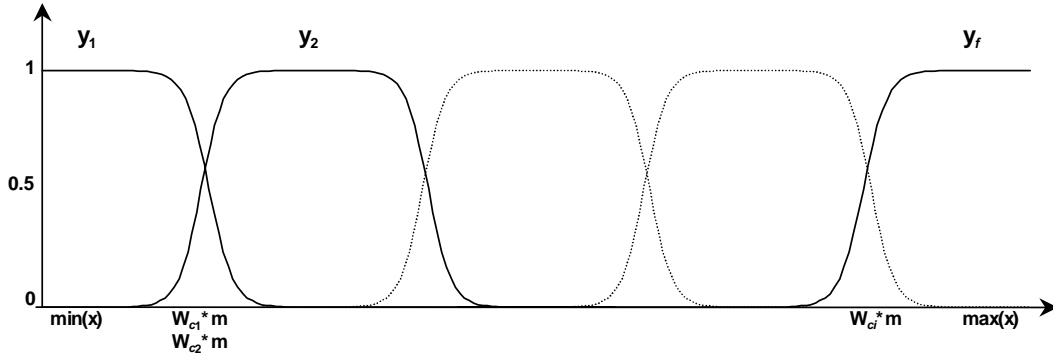


Figure 3. Sample of membership functions.

3.3.2 Inference stage

The preconditions $P = [p_1, p_2, \dots, p_n]$ are produced by a single layer of n , $n = f_1 \times f_2 \times \dots \times f_k$, nodes. The network realizes the intersection by performing the *min* operation on the membership functions ending at each node. Thus, each node is activated to the degree of the numerical truth value p_n of the precondition in $[0,1]$.

Each fuzzy system j (see Figure 1) contains a precondition layer and realizes a fuzzy relation $P \circ R_j = C_j$ which is implemented by a two layer network with n , $n = f_1 \times f_2 \times \dots \times f_k$, input nodes and m_j output nodes as shown in Figure 4. The output nodes perform the *max-min* composition and the synaptic weights r_{il} ($i = 1, \dots, n; l = 1, \dots, m_j$) are the elements of the R_j matrix.

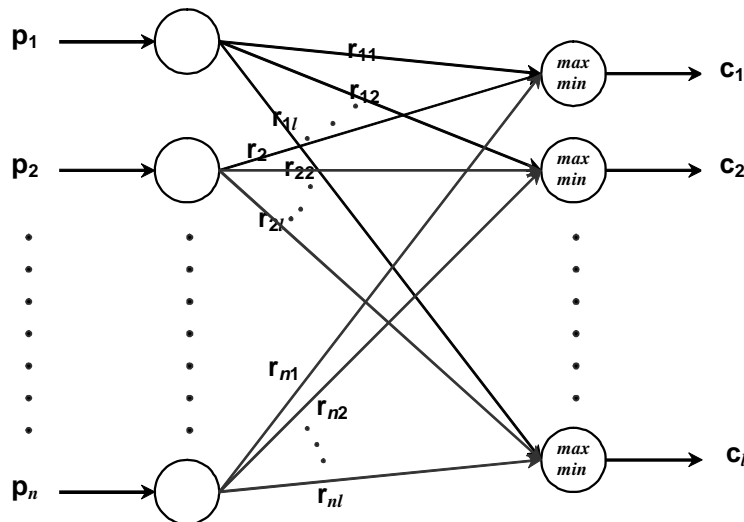


Figure 4: Network architecture for implementing the fuzzy relation.

3.3.3 Defuzzification

We have used a neural network-based approach, which allows the system to adapt the defuzzification to individual teacher's opinion by training. A three-layer neural network with m_j input and m_j output nodes and a hidden layer was trained with a modified backpropagation algorithm that uses variable stepsize, called BPVS [33]. Training the network results in encoding teachers' unstructured knowledge, and during operation the network acts as a "generaliser" that defuzzifies in a way that imitates teachers' decision procedure.

In our application, reported in the next section, the network used for defuzzification was trained using the population of 200 simulated student cases and desired outputs as specified by a group of five expert teachers, as described in [53]. This approach allows us to capture some "rules" in teachers' judgements that cannot easily be captured when using a standard defuzzification procedure, such as the Center-Of-Area (COA) that was used in [42]. For example, we have found that students were classified according to the best fuzzy assessment if this is a clear decision (a fuzzy value 30% larger than all others). If this is not the case, then the student is classified into an intermediate or into a more "conservative" category between two of "approximately equal" values (e.g. when the difference between two fuzzy values is less than 20% they could be considered approximately equal) for a particular student characteristic.

3.4. Encoding teacher's knowledge of evaluating student's characteristics

Depending on the characteristic that is evaluated and the lesson content, teacher's subjective reasoning is encoded in the fuzzy relation network (Figure 4). The weights r_{il} ($i=1,2,\dots,n; l=1,2,\dots,m_j$) are adjusted in order to relate the precondition with the consequents of teacher's reasoning. This form of modelling allows us to simplify the determination of the set of $n \times m_j$ linguistic rules that describe the fuzzy system [9] to the estimation of a matrix. A

weight r_{il} can be considered as measure of possibility of a linguistic rule relating a fuzzy input with a fuzzy output [44], as a confidence measure of that rule [14], or as measure of contribution of that rule in the output [9]. We interpret these weights as the degree of confidence of teacher's rules. This connectionist implementation provides the ability to encode teacher's structured or unstructured knowledge, as will be explained below.

3.4.1 Case 1: Teacher's diagnostic knowledge is available in the form of rules

In the simple case, where teacher's reasoning is well defined and available in the form of IF-THEN rules, these rules can be encoded in the network of Figure 4. If the rules are provided with certainty, denoting that the numerical truth-values of the preconditions and consequents are equal to 1, a weight r_{il} associated with a rule takes the value of 1. If consequents are provided with some degree of confidence, then the weight r_{il} is replaced with this degree i.e. with the numerical truth values of the consequents. Connections, which are not associated with rules, can be pruned.

3.4.2 Case 2: Teacher's diagnostic knowledge is available by means of examples

In case teacher's reasoning cannot be exactly described but is available in the form of examples, or in case labelled patterns of students observable behaviour are available, weights are adjusted through learning by examples. The numeric data X of student's behaviour are fuzzified and combined in the precondition layer to produce the learning vectors. A variety of methods have been proposed to train networks that implement fuzzy relations [45][31][14], by replacing the product operation with the minimum operation and the addition operation with the maximum operation. In our implementation a Hebbian-style learning approach is adopted, as suggested in [14]. Thus, the weights update equation at the presentation of t example is

$$r_{il}(t) = r_{il}(t-1) \oplus \rho \cdot p_i(t) \otimes c_l(t),$$

where ρ is a positive stepsize, \oplus represents a maximum operator and \otimes represents a minimum operator. Thus, unknown rules are encoded and the weights $r_{il}(i=1,\dots,n;l=1,\dots,m_j)$ are replaced with degrees of confidence of the rules that represent teacher's inference.

4. Application Example

4.1. The Learning Environment

The Intelligent Learning Environment consists of the educational software “*Vectors in Physics and Mathematics*” [20], and the neuro-fuzzy model that we have already described in Section 3. The introductory menu of the educational software “*Vectors in Physics and Mathematics*” is shown in Figure 5. This is a discovery (exploratory) learning environment that has been designed and developed according to constructivist theory of learning [20]. Within this framework, the design is based on a series of principles, which emphasize the student's active involvement in authentic activities, which correspond to real world processes (situated/anchored learning) [7][58]. Moreover, the software supports students' creative activities, allowing them to control their own learning procedure, and providing them with help and guidance when this is necessary [16].

The educational software aims to help teachers to instruct, and students to construct the concepts of vectors in physics and mathematics in the secondary school. The difficulties students encounter with the conceptualisation of the various phenomena that correspond to physical entities, and which can cause misconceptions and inert knowledge, [1] [17] [52], have been taken into consideration during the design of the software.



Figure 5. Introductory screen of the learning environment “Vectors in Physics and Mathematics”.

The thematic units of the software are: *Position and Displacement*; *Motion*; *Forces and Equilibrium*; *Forces and Motion*; *Forces and Momentum*. Each one of these units contains several *scenarios*, which refer to real-life situations. The students carry out selected *activities* within these scenarios. Examples of such scenarios are: “Going fishing”, “planning a journey”, “which ship moves faster?”, “travelling in the islands”, “playing golf”, “bodies in equilibrium” (see Fig. 6), “imaginary climbing”, “falling objects”, “away from the earth”, etc. The environment also includes a short presentation of the theory and a dictionary of useful terms and concepts.

The neural network-based fuzzy model was tested in the scenario “bodies in equilibrium” (see Fig. 6) of the unit “Forces and Equilibrium”. The environment resembles a simple mechanics-laboratory. A table appears on the screen and several objects such as boxes, cords, a spring and a pulley are available for use by the students. The students can drag and drop these objects and then use the available tools that manipulate vectors representing forces, carry out measurements, etc. to compose an equilibrium experiment. In this way, student is

allowed to give their own Newtonian model by drawing the vectors that compose this model, observe the behaviour of this model, and compare to the scientific model.



Figure 6. Scenario “Bodies in equilibrium”.

Within this scenario the students have the opportunity to carry out a set of 16 different activities (equilibrium experiments) by selecting one or two from the available objects from the object box (see Fig. 6). For example he/she can place a single box of 20N weight or 40N weight on the table or he/she can select a box and the spring or a rope and hang the box from the ceiling through them, or s/he can place a box of 20N or 40N on the table and then place another box on top. S/he can also select different worktops for the table (i.e. with different static friction coefficients) in case of experiments with the pulley and a box. Then, he has to decide about the kind (gravitational/contact) and the properties (magnitude and direction) of the forces acting upon each object and draw them according to his/her conception.

In Figure 7, an example activity with two boxes on the table is shown. The student draws the forces acting on the top box, according to his/her opinion. The student can then use the “Test” button to observe the behaviour of the model. For example, if the resultant force is not

equal to zero, the box will move towards the direction of this force. The student can also check the “Reality” radio button, in order to observe the scientific model in action, i.e. the effect of the correct forces acting on the box. Afterwards, either s/he can correct the forces acting on the box and maybe test again the effect, or s/he can clear the screen and conduct a new equilibrium experiment.

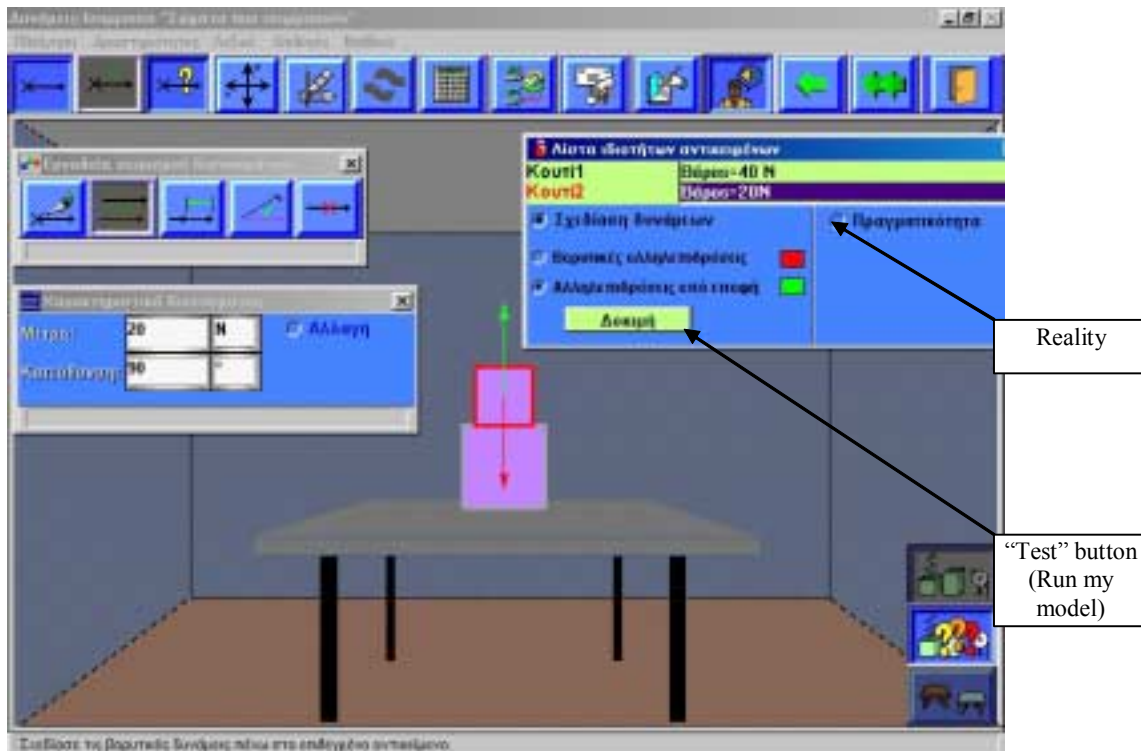


Figure 7. Activity with two boxes on the table.

Students’ lack of knowledge and misconceptions associated with this scenario have been identified on the basis of findings from studies in physics problem solving related to Newton’s third Law [1][11][16][17]. For example, a student may believe that lack of motion implies no force is applied on the object; s/he may be unfamiliar with contact forces or unfamiliar with gravitational force; s/he may confusing gravitational force with contact force; s/he may ignore that action-reaction pairs are opposite in direction or equal in magnitude. Student’s actions during task execution help us to estimate student’s lack of knowledge or

misconceptions by comparing the number, the kind (gravitational/contact) and the direction of the forces acting upon an object the student chooses to draw with the respective parameters of the scientific model. For example, if the student tests a model without having drawn contact forces we can suppose that s/he is unfamiliar with contact forces.

In our experiments, an aspect of the surface/deep approach [6] of student's learning style, [6], has been evaluated in order to provide an intelligent help to the student during learning interaction. Deep learners often prefer self-regulated learning; conversely, surface learners often prefer externally regulated learning [4]. In the learning environment "*Vectors in Physics and Mathematics*" diagnosing a student as deep or surface is used to sequencing the educational material.

In order to acquire teachers' knowledge in evaluating student's learning style, needed to implement our approach, a group of five experts in teaching the subject content has been used: three of them were experienced in teaching physics in secondary education, one of them was expert in didactics of physics, and the last one was an expert in the design of educational software. The group has been asked, taking into account their individual experiences in evaluating real students interacting with the learning environment, to reach consensus on the following aspects of student's learning style relating to our approach: the parameter k ; the names B_i ($i=1,2,\dots,k$); the universes of discourse U_i (for each $i=1,2,\dots,k$), and the association between the universes of discourse and the linguistic values of the linguistic variables of student's observable behaviour B that will serve as input for the diagnosis. A detailed description of the group's suggestion is given below. The group was also asked to agree on a set of IF-THEN rules (cf. with Section 3.4.1) describing their experiences of evaluating real students when they interact with the learning environment, as well as to agree on the labelling of a set of simulated students that were used for off line training of the networks (cf. with

Section 3.4.2) and for testing our approach. (The procedure to generate the simulated students, and the training and testing of the neuro-fuzzy system are described in the next subsections.)

In addition, the Learning Environment stores on a log file all the available information on what a student is doing, recording each student action with a time stamp. Typical examples of student actions include: selection of objects for experimentation, selection of available tools, mouse moves, mouse drags or clicks on tools or objects or mouse drags when he/she is trying to draw a vector, details about the vectors (forces) that the user draws, i.e. magnitude direction and kind, as well as the time the action was performed. The coding of the neural network-based fuzzy model and the pre-processing of the log files were developed in MATLAB software.

4.2. The deep/surface approach to learning

A lot of work has been done in defining student's deep or surface learning style [6] [18] [34] and constructing inventories [5] [57] to identify them. All these research efforts aim to identify the defining characteristics of these different approaches to learning, and to scale through questionnaires, which assess these characteristics, student's deep or surface learning style. The deep approach to learning is characterised by the following defining features: intention to understand vigorous interaction with content, relating new ideas to previous knowledge, relating concepts to everyday experience, relating evidence to conclusions, and examining the logic of the argument [18]. In contrast, the surface approach includes: intention to complete task requirements, memorising information needed for assessments, failure to distinguish principles from examples, treating task as an external imposition, focus on discrete elements without integrating, unreflectiveness about the purpose or strategies [18]. All the above features cannot be evaluated easily through tracking of student's activities during instruction. Study strategies are more easily estimated from student's activities. Study

strategies are closely related to student's learning style, since student's learning style is defined as “a predisposition on the part of some students to adopt a particular learning strategy regardless of the specific demands of the learning task” [4]. Recently, study strategies of students with deep or surface learning style have been evaluated and compared with the aid of a computer assisted study environment for learning from text [4]. For the purpose of this research, students were classified using the Inventory of learning styles (ILS) as deep or surface and pre-tested before the learning task. The study environment recorded all users’ actions, together with a time-stamp, as well as student’s reading speed, in order to identify study activities in relation to student's deep or surface learning style. According to the results of this research, deep learning students know more about the diagnostic study task and develop increased reading speed.

Learning by discovery is quite different from learning by textbook; therefore, the work supported by the computer-assisted study environment cannot be easily transferred to a discovery learning environment. The educational software “*Vectors in Physics and Mathematics*” is designed on the basis of student’s active engagement during the learning process, allowing students to control and observe the evolution of real world phenomena, take measurements, change various parameters, examine “what if” scenarios etc. Within this framework, students’ intention to understand and their vigorous interaction with the content (as opposed to their intention to complete task requirement and treating the task as an external imposition) were suggested by our group of experts, as fundamental characteristics of learning style to be evaluated. For the purpose of this experiment, the two characteristics were labelled as “*student's tendency to learn by discovery in a deep or surface way*” and assessed as one characteristic by the neuro-fuzzy model. The students were classified as *shallow* or *deep* with respect to their processing activities during learning by discovery.

Another important step is to decide what events of student's performance must be tracked and evaluated in order to assess this characteristic. The study activities that could help evaluating the learning style were suggested by the group of experts based on studies in cognitive psychology. Since the outcome of the deep approach to learning is a deep level of understanding of the subject matter, which is one of the evidences of expert-novice difference in physics, the group used information from research in expert-novice differences in physics in order to suggest the study activities. For example, experts tend to work forwards to a solution whereas novices tend to work backwards [29]. When experts have analyzed a problem, they apply the principles they have selected to the given quantities of the problem. In that sense, the number of times a student tested his/her ideas, or compared his/her ideas with the reality is taken into account in order to identify if the student is using *trial and error* strategies. Student's activities when trying to find the correct forces, or after testing a correct or incorrect idea were also taken into consideration. In addition, students' problem solving speed has also been taken into account. Research discovered that even though experts solve problems four times faster than novices, they spent more time than novices analyzing and understanding the problems [12].

4.3. Implementing the neural network-based fuzzy model

4.3.1 Tailoring the model

Following the discussion above, the group of experts suggested three linguistic variables B_1 , B_2 , B_3 associated with student's actions within the 16 different activities (equilibrium experiments) of the scenario "Bodies in equilibrium" that describe a subset of student's observable behaviour B to be used in the diagnosis of *student's tendency to learn by discovery in a deep or surface way*. In addition the group also suggested the number and the names of the linguistic values of each linguistic variable. Student's actions *before trying to solve the*

problem or after making an incorrect attempt have been taken into account in B_1 = “*the number of times a student tests their ideas or compared their ideas with the reality*”, described by the term set $T(B_1) = \{Seldom, Sometimes, Frequently\}$. Student's study activities *during problem solving, or after testing an incorrect idea* have been taken into account in B_2 = “*the number of times the student consults the dictionary or reviews the theory or temporarily stops to think*”, expressed with the term set $T(B_2) = \{Sometimes, Frequently, Always\}$. The linguistic variable B_3 = “*problem solving speed*” was described by the term set $T(B_3) = \{Slow, Medium, Fast\}$.

The experts took into consideration observations of students interacting with the learning environment and agreed on the ranges of the universe of discourses U_k ($k=1,2,3$) for each input x_1, x_2, x_3 representing the measured values of B_1, B_2, B_3 , respectively, as well as on the associations between the linguistic values of each linguistic variable B_k and the universe of discourse U_k . For example, student's action “*temporarily stops in order to think*”, which is used in the calculations of x_2 , is measured from the *student's idle interval between tries*. For the universe of discourse U_2 of B_2 = “*the number of times the student consults the dictionary or reviews the theory or temporarily stops in order to think*”, a time percentage of this interval is used, since it depends on *the total time the student used the learning environment*. The linguistic variable B_3 = “*problem solving speed*” is determined by computing the average percentage of time needed to find the correct forces of each experiment [20]. The time needed to find the forces applied to an object was compared against the time the group of experts defined as the average time multiplied by two; thus the universe of discourse was set to $[0, 100]$. In addition the group of experts also suggested to take into account student's prior experience with the interface of the educational environment, as from their observations of students interacting with the software it was realised that the time a student needs to find the correct forces may also include the time needed to use the available tools that manipulate

vectors and draw these forces. Thus, the ranges can be adjusted for students with more prior experience than ever expected. In the fuzzification stage, sigmoid functions for the extreme values and pseudo trapezoidal functions for the intermediate values have been used. Figure 8 illustrates the membership functions (continuous lines) and the adjusted membership functions (dotted lines) for the linguistic variable “*problem solving speed*”.

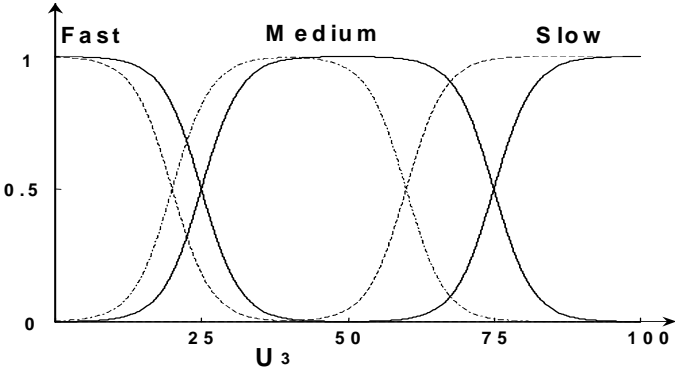


Figure 8. Membership functions for the three linguistic terms of the linguistic variable “*problem solving speed*”.

The three linguistic variables provide 27 (i.e. 3×3×3) possible combinations of the linguistic values in the preconditions of the IF-THEN rules. The output of the diagnostic process was described with five linguistic values ($m_j = 5$) in the term set $T(C_j) = \{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$. The implemented neural network-based fuzzy model is shown in Figure 9. It associates student’s observable behaviour B with student’s “deep” or “shallow” *tendency to learn by discovery* by processing numerical input X (see Figure1) through a set of stages corresponding to fuzzification, inference and defuzzification, as described in the previous section.

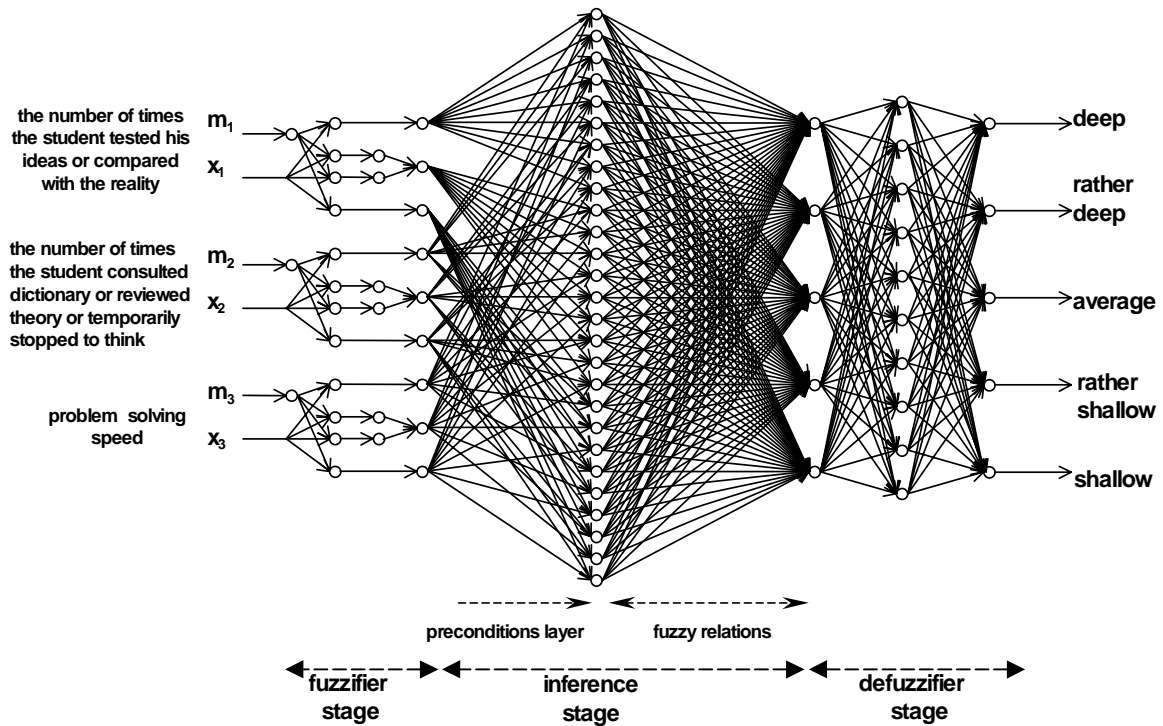


Figure 9. Network implemented to assess student’s tendency to learn by discovery in a deep or surface way.

4.3.2 Generating the simulated students

A set of simulated student has been generated in order to test our approach in case rule-based diagnostic knowledge is available (Case 1; Section 3.4.1), and to represent in the neural network teachers’ diagnostic reasoning available by means of examples (Case 2; Section 3.4.2).

Simulated students have been used in several ITS studies (see for example [21] [54] [56]). Since formative evaluation with real students is expensive, simulated students can help teachers and instructional developers to practice and evaluate the proposed instruction and can provide an early feedback to developers in order to troubleshoot with their designs early in the design process [56].

In the approach presented in this paper, we are interested to propose a convenient method to encode teacher’s reasoning in evaluating general student’s learning characteristics such as “*student's tendency to learn by discovery in a deep or surface way*”. The simulated students

can provide a convenient way to obtain the large number of labelled patterns of students behaviour needed to test the proposed approach in case of IF-THEN rules, or to train and test the networks of the proposed approach in case where teacher's knowledge is available by means of examples, at an early stage of development.

In order to construct simulated students' patterns of interaction with the learning environment that are "close" to real students' behaviour patterns, we modified the underlying elements of patterns of a small set of real students. The real students' interaction patterns have been provided during an experiment which was carried out with the assistance of the group of experts. In particular, the group identified 10 students to participate in the experiment; two from each of the five learning style categories considered in our model. During the experiment participants were asked to perform the 16 different activities (equilibrium experiments) of the scenario "bodies in equilibrium", and their interactions were recorded in the log file.

The interactions data are organised in the following way: student's actions until s/he quits an activity are decomposed in terms of episodes. Each episode includes a series of actions which begins or ends when the student clears the screen in order to start a new attempt on the same activity, or a new equilibrium activity. Within each episode the student conducts, successfully or unsuccessfully, an equilibrium experiment.

In the experiment, students of different learning style categories exhibited different interactive behaviour, giving different linguistic values for the linguistic variables B_1 , B_2 , B_3 of their observable behaviour B and the respective measured values of the inputs $\{x_1, x_2, x_3\}$. For example, in case students patterns were classified as "deep", $B_1 = \{The\ number\ of\ times\ the\ student\ tests\ or\ compares\ ideas\ with\ the\ reality\ before\ trying\ to\ solve\ a\ problem,\ or\ after\ making\ an\ incorrect\ attempt\}$ was described with the linguistic value *seldom*, $B_2 = \{the\ number\ of\ times\ the\ student\ consults\ the\ dictionary\ or\ reviews\ the\ theory\ or\ temporarily\ stops$

to think} with *always*, and $B_3 = \{\textit{problem solving speed}\}$ with *fast*. In contrast, for student cases classified as “shallow”, B_1 was described with *frequently*, B_2 with *sometimes*, and B_3 with *slow*.

The simulated students’ records have been produced by modifying the number of episodes and by inserting, deleting or changing, at the appropriate position within each episode or between episodes, actions that are used to calculate the values of the input $X = \{x_1, x_2, x_3\}$ which represents the measured values of B_1, B_2, B_3 . For example, inserting an action, such as the use of the “Test” button after an incorrect attempt, will cause an increase to the value of x_1 , which gives the measured value of B_1 . Deleting idle intervals between attempts will cause a decrease to the value of x_2 , which gives the measured value of B_2 . Thus, starting with 10 real students’ records we can generate simulated students, altering the values of x_1, x_2, x_3 in the students’ patterns by giving appropriate values within their universes of discourse U_1, U_2, U_3 .

The first episode, showing an unsuccessful equilibrium experiment, from a series of episodes of a “shallow” real-student record is presented in tabular form in Figure 10. Each entry of the record corresponds to an action of the student together with a time-stamp showing minutes and seconds elapsed from the start of the activity. Words in quotes refer to tools/buttons available, and pairs of unquoted numbers refer to mouse cursor positions. Entries in standard font refer to mouse moves or idle mouse states (e.g. the entry <“test” 3min, 0sec> denotes that the user moves the mouse over the button “Test” but s/he does not click it). Entries in bold refer to particular mouse events, i.e. selecting/clicking buttons (e.g. the entry <“**test**” 3min, 0sec> denotes that the user clicks the button “Test”), dragging objects (as for example when the student moves an object or he/she draws a vector- a typical example of drawing action is shown in the third column of the table <“**create vector**” 1min, 10sec>. The process involves mouse drag, starting in row <7335 5010 1min, 16sec> and ending in

row <7440 6300 1min, 19sec>). Entries in bracket provide a short description of the actions of the real student (e.g. {Creates a gravitational force on box 40 magnitude 24 and direction -90} is the result of the drawing action <“create vector” 1min, 10sec>). The record of the overall episode also reveals a misconception of the real student regarding the number of forces acting on the box of 40N. This is an indication of unfamiliarity with contact forces, as the student draws only one contact force acting on the box of 40N whilst two forces are actually needed.

In this particular episode, the student *frequently* ($x_1=8$) uses the “Test” or “Reality” button before trying to solve a problem or after an incorrect attempt. No idle intervals or dictionary consults ($x_2=0$) were found on this record, regardless of student’s inability to achieve a successful equilibrium experiment. In addition the student at the end of the episode observes the effect of his/her choices on the Reality and decides to clear the screen, although the results obtained shown his/her actions went wrong.

In order to generate simulated students, the episode can be altered in different ways: deleting some of the “Test” or “Reality” button selections, e.g. changing x_1 values in the interval [0,8] results in changing B_1 to a predefined membership degree of the linguistic values *seldom* and *sometimes*; adding idle intervals and/or “dictionary” selections before drawing forces, or after an incorrect “Test”, or at the end of the unsuccessful episode, e.g. changing x_2 values and membership degrees of the values of B_2 . Adding idle intervals will also increase x_2 , i.e. the problem solving speed. In addition to the above alterations, we can also reduce the problem solving speed of the generated simulated students by reducing the number of episodes needed to find the correct forces of a successful equilibrium experiment. For example, the particular student needed 5 episodes and 18 minutes overall to produce a correct solution in this activity, i.e. the episode presented in Figure 10 lasts 5 minutes and is just one out of the 5 episodes needed for a successful equilibrium experiment (an overall time

of 18 minutes). As described in Subsection 4.3.1, B_3 = “*problem solving speed*” is defined as a percentage of time, and the value of x_3 is calculated by comparing the time a student needs in order to find the correct forces in each activity with the group’s average time for finding the correct forces multiplied by two. For the particular activity that the student of Figure 2 is performing using the two boxes, the group’s estimated average time is 10 minutes. Thus, calculating the percentage that corresponds to 10 minutes multiplied by 2 (i.e. 20 minutes), for this student $x_3 = 90\%$ which corresponds to the linguistic value “*Slow*” with membership degree very close to 1 (see Figure 8). By reducing the number of episodes of this activity to 4, the total time of the episodes needed to find the correct forces will be 15 minutes; this corresponds to a value of $x_3 = 75\%$, and the linguistic value for problem solving speed is now “*slow*” with a membership degree 0.5 and “*Medium*” with a membership degree 0.5 (see Figure 8).

<i>{Begin:}</i>	7455 4695 0min, 42sec	{create vector” 1min, 10sec	{move vector” 2min, 52sec	9390 3930 3min, 48sec
<i>{May 21 2001 12:15:15 PM}</i>	9465 4725 0min, 43sec	“create vector” 1min, 10sec	“create vector” 2min, 52sec	7830 1785 3min, 48sec
2280 2685 0min, 0sec	7305 3420 0min, 44sec	7335 5010 1min, 16sec	“create vector” 2min, 52sec	“reality” 3min, 49sec
4410 3420 0min, 0sec	5295 3255 0min, 45sec	7335 5070 1min, 16sec	7410 4365 2min, 56sec	“reality” 3min, 49sec
6480 3840 0min, 0sec	4005 1245 0min, 46sec	7410 4425 2min, 56sec	5775 1515 3min, 50sec
8535 3915 0min, 0sec	“useful tools” 0min, 49sec	(continued)	7410 4500 2min, 56sec	“delete vector” 3min, 50sec
10185 6000 0min, 3sec	“useful tools” 0min, 50sec	7410 4560 2min, 56sec	“attribute list” 3min, 51sec
“attribute list” 0min, 3sec	<i>{opens the useful tools}</i>	7470 6180 1min, 18sec	7410 4620 2min, 56sec	<i>{closes the attribute list}</i>
“select table” 0min, 3sec	“goniometer” 0min, 51sec	7470 6240 1min, 19sec	7410 4680 2min, 56sec	8280-210 3min, 51sec
11790 8130 0min, 3sec	“assistant line” 0min, 51sec	7440 6300 1min, 19sec	7410 4755 2min, 56sec	“memo” 3min, 52sec
“select table” 0min, 3sec	“spots” 0min, 51sec	<i>{Creates a gravitational force</i>	7410 4815 2min, 56sec	8130 1890 3min, 52sec
“attribute list” 0min, 3sec	“axes” 0min, 52sec	<i>on box 40 magnitude 24 and</i>	7410 4875 2min, 57sec	8130 4065 3min, 53sec
“object list” 0min, 3sec	“spots” 0min, 52sec	<i>direction -90}</i>	7425 4935 2min, 57sec
11775 6090 0min, 3sec	“clear screen” 0min, 52sec	8460 4140 1min, 24sec	7440 4995 2min, 57sec	(continued)
“object list” 0min, 4sec	“useful tools” 0min, 53sec	6420 3285 1min, 25sec	7440 5055 2min, 57sec
<i>{opens the object list}</i>	<i>{closes the useful tools}</i>	7425 4995 2min, 58sec	“box 2 4min, 7sec
11310 4065 0min, 5sec	“clear screen” 0min, 53sec	(continued)	<i>{Creates a gravitational force</i>	“test” 4min, 7sec
11445 2025 0min, 7sec	“useful tools” 0min, 54sec	7485 3060 2min, 39sec	<i>on box 20 magnitude 15 and</i>	“test” 4min, 8sec
11415 4035 0min, 17sec	“axes” 0min, 55sec	7425 3105 2min, 40sec	<i>direction -88 }</i>	“reality” 4min, 15sec
9180 3195 0min, 25sec	6300 2025 0min, 57sec	<i>{Creates a contact force on box</i>	“test” 3min, 0sec	“reality” 4min, 16sec
7140 3060 0min, 26sec	8460 2760 0min, 58sec	<i>20 magnitude 17 and direction</i>	“test” 3min, 0sec	“sound” 4min, 21sec
9255 2625 0min, 29sec	10530 3435 0min, 58sec	<i>90}</i>	7815 2805 3min, 5sec	“attribute list” 4min, 23sec
11340 3375 0min, 29sec	10800 5535 0min, 58sec	“object list” 0min, 58sec	5220 1140 3min, 5sec	<i>{closes the attribute list}</i>
9285 3420 0min, 31sec	“object list” 0min, 58sec	“attribute list” 0min, 59sec	“attribute list” 3min, 9sec	8340 -195 4min, 23sec
11295 4275 0min, 32sec	“attribute list” 1min, 0sec	“test” 2min, 42sec	<i>{closes the attribute list}</i>	“memo” 4min, 23sec
11190 2250 0min, 33sec	<i>{opens the attribute list}</i>	5265 1950 2min, 46sec	8100 2070 4min, 23sec
“box 40 N” 0min, 36sec	“box 1” 1min, 5sec	3210 1920 2min, 47sec	(continued)	8970 4365 4min, 24sec
9795 2070 0min, 36sec	“draw forces” 1min, 7sec	1185 1785 2min, 48sec	11010 2550 4min, 57sec
7785 3435 0min, 36sec	“gravitational forces” 1min,	3585 1755 2min, 48sec	7365 3495 3min, 39sec	“EXIT” 4min, 57sec
<i>{Puts on table box 40 N}</i>	8sec	5745 1830 2min, 48sec	7365 3435 3min, 40sec	11985 435 4min, 57sec
7395 5160 0min, 36sec	“gravitational forces” 1min,	“gravitational forces” 2min,	7380 3375 3min, 40sec	“EXIT” 4min, 58sec
9630 3570 0min, 36sec	9sec	50sec	<i>{Creates a contact force on box</i>	“clear screen” 5min, 0sec
“box 20 N” 0min, 42sec	“shift direction” 1min, 9sec	“shift direction” 2min, 50sec	<i>40 magnitude 28 and direction</i>	<i>{End:12:20:15}</i>
8895 3585 0min, 42sec	“change length” 1min, 9sec	“delete vector” 2min, 50sec	<i>91}</i>	<i>{Duration : 5 min and 0 sec}</i>
6795 4800 0min, 42sec	“move vector” 1min, 10sec	“shift direction” 2min, 52sec	“test” 3min, 41sec	
<i>{Puts on box 40 N box 20N}</i>		“change length” 2min, 52sec	“test” 3min, 41sec	

Figure 10. An episode from a “shallow” real-student record.

4.3.3 Encoding rule-based diagnostic knowledge (Case 1).

The group of experts provided us with a series of IF-THEN rules that describe their reasoning in evaluating students’ tendency to learn by discovery in a deep or surface way when working with the 16 different activities (equilibrium experiments) of the scenario “bodies in equilibrium”. The group’s experience has been acquired through observing real-students interacting with the learning environment. Students’ observable behaviour has been described linguistically using the universes of discourse U_k , $k=1,2,3$, as described in previous sections. In order to obtain the set of rules, the groups was asked to classify students’ behaviour in one of the predefined linguistic values of the term set $\{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$, using with a combination of linguistic values of the linguistic variables (this results in 27 different cases that correspond to preconditions of 27 rules). This allows the group to agree on a linguistic representation of the experts’ individual reasoning (e.g. [if] the student *seldom* tests or compares ideas with activities of the real world before trying to solve a problem, or after an incorrect attempt, and *always* consults the dictionary or reviews the theory or temporarily stops in order to think after testing an incorrect idea, and their problem solving speed is *high* [then] the student tends to learn in a *deep* way). In order to obtain the degree of confidence of each rule and implement the neural network that realizes the fuzzy relation, as has been described in Subsection 3.4.1, the group was also asked to rate the confidence of their judgments using the rating scale: *absolutely clear, very strong, strong, rather strong, doubtful*. We arbitrary adjusted the following values, dc_l ($l = 1,2,\dots,5$), to each judgment $dc_l = \{1, 0.9, 0.8, 0.7, 0.6\}$. In the first case, i.e. where the rule was provided with the highest degree of confidence, a value of 1 was used. In all other cases, i.e.

$l=2, \dots, 5$, the value $(1-dc_l)$ was heuristically split over the two closest judgments (represented by neighbouring nodes in the network). Thus, group's diagnostic knowledge was encoded in the network that realize the fuzzy relation and the following weights r_{ij} ($i=1,2, \dots, 27, j=1,2, \dots, 5$) of the matrix R_α were adjusted in the network:

$$R_\alpha = \begin{pmatrix} 0.1000 & 0.8000 & 0.1000 & 0 & 0 \\ 0.0500 & 0.9000 & 0.0500 & 0 & 0 \\ 0.8000 & 0.1000 & 0.0500 & 0.0500 & 0 \\ 0.1000 & 0.8000 & 0.1000 & 0 & 0 \\ 0.0500 & 0.9000 & 0.0500 & 0 & 0 \\ 0.8000 & 0.1000 & 0.0500 & 0.0500 & 0 \\ 0.7000 & 0.1500 & 0.1000 & 0.0500 & 0 \\ 0.8000 & 0.1000 & 0.0500 & 0.0500 & 0 \\ 0.9000 & 0.0500 & 0.0500 & 0 & 0 \\ 0 & 0 & 0.0500 & 0.9000 & 0.0500 \\ 0 & 0 & 0.1000 & 0.8000 & 0.1000 \\ 0.0500 & 0.1000 & 0.7000 & 0.1000 & 0.0500 \\ 0 & 0 & 0.1000 & 0.8000 & 0.1000 \\ 0 & 0.0500 & 0.9000 & 0.0500 & 0 \\ 0.1000 & 0.7000 & 0.1000 & 0.1000 & 0 \\ 0 & 0.1000 & 0.8000 & 0.1000 & 0 \\ 0.0500 & 0.9000 & 0.0500 & 0 & 0 \\ 0.1000 & 0.8000 & 0.1000 & 0 & 0 \\ 0 & 0 & 0.0500 & 0.0500 & 0.9000 \\ 0 & 0.0500 & 0.0500 & 0.1000 & 0.8000 \\ 0.0500 & 0.0500 & 0.1000 & 0.7000 & 0.1000 \\ 0 & 0 & 0.1000 & 0.2000 & 0.7000 \\ 0 & 0 & 0.1000 & 0.8000 & 0.1000 \\ 0 & 0.1000 & 0.8000 & 0.1000 & 0 \\ 0 & 0 & 0.0500 & 0.9000 & 0.0500 \\ 0.0500 & 0.1000 & 0.7000 & 0.1000 & 0.0500 \\ 0 & 0.0500 & 0.9000 & 0.0500 & 0 \end{pmatrix}$$

Each row of matrix R_α corresponds to one of the 27 preconditions. Each column represents one of the five learning style characterizations. Notice, for example that the last two rows of matrix R_α represent in the network two, but relatively close, cases identified by the group of teachers: student's different learning style may be "classified" as *Average* using different degrees of confidence, i.e. 0.7 and 0.9 for the corresponding central nodes. Thus, four neighbouring nodes can be activated in the first case, and two nodes in the second.

4.3.4 Encoding example-based diagnostic knowledge (Case 2)

The group of expert teachers was asked to label patterns of simulated students performing the 16 different activities (equilibrium experiments) of the scenario "bodies in equilibrium", with respect to their "tendency to learn in a deep or surface way". A set of 54 simulated students has been generated to this end. The set included two simulated student for each one of the 27 combinations of linguistic values of the linguistic variables representing student's behaviour, in accordance with the preconditions of the 27 rules. In addition almost clear-cut simulated

students cases were generated, i.e. simulated students with membership degrees to each linguistic value greater than 0.7. The group classified the set of 54 (27×2) simulated students in one of the linguistic values of the term set $\{Deep, Rather\ Deep, Average, Rather\ Shallow, Shallow\}$. The particular input values $X=\{x_1, x_2, x_3\}$ of each simulated student pattern were processed through the fuzzifier stage and the preconditions layer, in order to form together with experts' classifications the input-output vectors to train the fuzzy relations network, as described in Subsection 3.4.2. A positive stepsize $\rho=1$ was used for training. The following matrix R_l was produced:

$$R_l = \begin{pmatrix} 0 & 0.8800 & 0 & 0 & 0 \\ 0.0200 & 0.8800 & 0 & 0 & 0 \\ 0.8800 & 0 & 0 & 0 & 0 \\ 0.1200 & 0.8800 & 0 & 0 & 0 \\ 0.1200 & 1.0000 & 0 & 0 & 0 \\ 0.8800 & 0.0200 & 0 & 0 & 0 \\ 0.8300 & 0 & 0 & 0 & 0 \\ 0.9800 & 0 & 0 & 0 & 0 \\ 0.8800 & 0.0200 & 0 & 0 & 0 \\ 0 & 0.2300 & 0 & 0.9800 & 0.0200 \\ 0.0200 & 0.1200 & 0.0200 & 1.0000 & 0.1200 \\ 0.1200 & 0 & 1.0000 & 0.1200 & 0 \\ 0.0200 & 0.1200 & 0.1200 & 0.9800 & 0.1200 \\ 0.0200 & 0.1200 & 0.9800 & 0.1200 & 0.0200 \\ 0.1700 & 0.9800 & 0.1200 & 0.1200 & 0 \\ 0.1700 & 0 & 1.0000 & 0.1200 & 0 \\ 0.0200 & 0.9800 & 0.1200 & 0.1200 & 0 \\ 0.2300 & 0.8800 & 0.1200 & 0.1200 & 0 \\ 0 & 0 & 0 & 0 & 0.9800 \\ 0 & 0 & 0 & 0.1200 & 1.0000 \\ 0 & 0 & 0 & 0.8800 & 0.0200 \\ 0 & 0 & 0.0200 & 0.1200 & 0.9800 \\ 0 & 0.0200 & 0.1200 & 1.0000 & 0.1200 \\ 0 & 0.0200 & 1.0000 & 0.1700 & 0.0200 \\ 0 & 0 & 0.0200 & 0.8800 & 0.0200 \\ 0 & 0.0200 & 1.0000 & 0.1700 & 0.0200 \\ 0 & 0.0200 & 0.9800 & 0.1200 & 0 \end{pmatrix}$$

The weights learned, i.e. the elements of matrix R_l , represent the degree of confidence of the rules. We can find similarities between matrix R_l and matrix R_a , since the same group of experts participated in both experiments. For example, the same network connections have weights greater than 0.8 in both matrices. In addition, connections of neighbouring nodes have weights less than 0.3; thus only slightly activating the neighbour nodes, but contributing to the final classification. The defuzzifier was trained to produce the final decision, as described in the previous section.

4.4. Evaluating the neuro-fuzzy diagnostic model

4.4.1 Testing the rule-based diagnostic model (Case 1)

In order to evaluate the performance of the rule-based neuro-fuzzy model, three test sets each one having 62 simulated students with predefined linguistic values in the linguistic variables of their observable behaviour, and predefined membership degrees to these values as well, have been generated. The first set contains patterns with clear-cut descriptions of students' observable behaviour, i.e. their membership degrees in the linguistic values of each linguistic variable are close to 1. The second set involves a lot of uncertainty; there are no clear-cut cases due to lack of well-defined boundaries in evaluating students' observable behaviour. This set includes marginal cases, i.e. patterns that contain membership degrees close to 0.5 in two linguistic values of one or more than one linguistic variables. This data set was used to test the capability of the model in the handling of uncertainty incorporated in the marginal cases of students' observable behaviour. This capability is usually not supported in a non-fuzzy rule-based environment. The third set consists of special marginal cases, which are possible to cause conflicting judgments if they processed by classic IF-THEN rules. A typical example is when two IF-THEN rules with close precondition categorize the student into two different non-adjointing categories, as will be described below.

The patterns of these data sets formulate the input values $X=\{x_1, x_2, x_3\}$ of the rule-based neuro-fuzzy model, and are classified in one out of the five categories $\{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$. The three set of simulated student cases have been presented to the group of expert, in order to be labelled according to the term set $\{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$. The classifications of the neuro-fuzzy model were compared with experts' classifications of the same simulated students. The average success in diagnosis for the first test set reached 100%. The model also provided an excellent average performance, 90%, in evaluating marginal cases (second test set), in

accordance to group's judgements. In the third test set (special marginal cases) an average performance of 85% was achieved. The neuro-fuzzy model showed that it is indeed capable to handle these special marginal cases by fine-tuning the rules encoded in the fuzzy system through the neural network-based defuzzification procedure.

At this point is useful to illustrate the behaviour of our model with some examples. Let us consider a student who "*frequently* tests or compares ideas with activities of the real world before trying to solve a problem, or after an incorrect attempt, who *sometimes* consults the dictionary or reviews the theory or *temporarily* stops after testing an incorrect idea in order to think, and has *slow* problem solving speed", and another student who "*sometimes* tests or compares ideas with activities of the real world, and *frequently* consults or reviews or thinks, and has *medium* problem solving speed". The first student's learning style has been evaluated by the group as "*Shallow*", and the second student's style as "*Average*"; group's confidence in their judgments is in both cases "*Very strong*". In our model, when the membership degrees to the above linguistic values of student's observable behaviour are equal to 1, these two evaluation decisions provide at the output of the inference stage the following fuzzy assessments vectors: [0, 0, 0.05, 0.05, 0.9] and [0, 0.05, 0.9, 0.05, 0], for the first and the second case respectively. Finally, after defuzzification, the students are classified into two quite different non-adjointing categories.

Let us now consider a special marginal case where student's observable behaviour causes two rules to fire. This may be the case of a student who tests or compares ideas with activities of the real world *frequently* with a membership degree of 0.4 and *sometimes* with a membership degree of 0.59. The student also *sometimes* with a membership degree of 0.4 and *frequently* with a degree of 0.59 consults the dictionary or reviews the theory or *temporarily* stops to think after testing an incorrect idea. The same student also has problem solving speed that is *slow* with a membership degree of 0.4 and *medium* with a degree of 0.59. This

complicate case will provide at the end of the inference stage the following fuzzy assessment vector: [0, 0.05, 0.59, 0.4, 0.4]. The final decision at the output of the defuzzifier is that this student's learning style is "*Rather Shallow*" a decision between the two categories, which is indeed consistent with group's judgments when classifying similar marginal cases of real or simulated students.

One of the goals in our implementation is to propose a model that can be tailored to individual teacher's experiences or judgment. The neuro-fuzzy implementation of the model can easily handle subjectivity of teachers' suggestions and reasoning, because it allows teacher's rules to be encoded directly from teacher's linguistic description, creating that way a model tailored to the needs of a particular teacher in case of disagreement. That was indeed very useful in our case because one of the teachers in our group of experts wanted to use the learning environment in a primary school, i.e. with students of smaller age and different knowledge level and experience than the ones used so far. That gave us the opportunity to evaluate the adaptability of our model to teacher's subjective judgements following his suggestions, and additional experiments have been performed.

We used the same linguistic variables and the same linguistic values for student's observable behaviour, as well as the same linguistic values that were suggested by our group of experts. To tailor the model to the teacher's suggestions, adjustments have been made in the association between the linguistic values and the universes of discourse by changing the adjusting parameters m_1, m_2, m_3 that represent the expected mean value of the numerical input $X = \{x_1, x_2, x_3\}$ (thus, slightly altering the shape of the membership functions, i.e. the degree of membership to each linguistic value), as well as in the IF-THEN rules by changing the weights in the fuzzy relations network that realizes the inference stage. Additional experiments with simulated students have been performed to test the tailored model. Students' classifications by the neuro-fuzzy model were compared with teacher's classifications for the

same simulated students. The model showed was successfully adapted, classifying the students according teacher's classifications with constant classification success for each case.

4.4.2 Testing the example-based diagnostic model (Case 2)

In order to test our approach when the diagnostic knowledge is available by means of examples, we used the same three test data sets. The input values $X=\{x_1, x_2, x_3\}$ of each pattern of the three test data sets has been processed by the trained neuro-fuzzy model and classified in one of the linguistic values of the term set $\{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$. In addition, the three set of simulated student were classified in one of the linguistic values of the term set $\{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$ by the group of experts, and groups' classifications were compared against the neuro-fuzzy model classifications. The overall average success in diagnosis reached 94%, i.e. 100%, 96%, 86% for each of the three data sets respectively; practically the same levels as in case of IF-THEN rules.

We conducted additional experiments in order to compare the neural-network based fuzzy model proposed in this paper against two other approaches, namely a classic multilayer Neural Network (NN) with 3 input-10 hidden-5 output nodes trained with the backpropagation algorithm with variable stepsize [33], and a Fuzzified Neural Network (FNN) that is based on the ANFIS architecture, [40], with pseudotrapezoidal fuzzy sets, 27 rules and outputs corresponding to the categories $\{Deep, Rather Deep, Average, Rather Shallow, Shallow\}$. All methods used the same simulated students for training and were tested on the same testing data sets (test set 1 contains clear-cut cases of simulated students; test set 2 marginal cases; test set 3 special marginal cases)

Figure 11 shows the best available performance in classification achieved by each model. The classic NN approach provides a diagnostic success of 84%, 82%, and 80% in the three data sets. The diagnostic success of the FNN was 100%, 98%, and 63%. When we compare

these results with the corresponding results of the neuro-fuzzy model, which represents knowledge with the use of fuzzy relations in addition to the fuzzified inputs and the precondition layer, it is clear that the neuro-fuzzy model provides improved performance in classifying the third test data set (special marginal cases).

Test set	NN	FNN	neuro-fuzzy
No 1	84%	100%	100%
No 2	82%	98%	96%
No 3	80%	63%	86%

Figure 11: Comparative results in the three test data.

We have further analyzed the average behaviour of the three models as they all incorporate training networks. 30 instances of each model were trained and tested on the three test sets. The average classification success and standard deviation, for the three models are shown in Table 1.

Test set	NN	FNN	neuro-fuzzy
No 1	78 ± 3	99.7 ± 0.6	99 ± 3.0
No 2	76 ± 3	96 ± 2	93.0 ± 3
No 3	74 ± 3.5	57 ± 2	84.4 ± 0.8

Table 1. Average classification success and standard deviation for the three models.

The performance results were checked for statistical significance using the *t*-test. All differences found to be statistically significant with *t* values greater than 10. As we can see in Table 1, the FNN shows a better performance than the neuro-fuzzy model in the test set 1 (clear-cut cases) and the test set 2 (marginal cases). This performance of the neuro-fuzzy model is compensated from its performance in the test data 3 (special marginal cases).

We have also analyzed the types of classification errors the three models can produce. This is particularly important as the outcome of the diagnosis has an impact on the pedagogical strategy adopted for each student. In our tests, we have identified three types of errors. The type 1 error happens when a student has been incorrectly classified in an adjoining category,

i.e. with rank difference of one. For example, this type of error occurs when a student is evaluated by the group of experts as *rather shallow* (regarding his tendency to learn by discovery in a deep or surface way), but a model classifies him/her in the category *shallow* or *average*. On the other hand, when the student is classified as *rather deep* (rank difference of two), a type 2 error occurs. When the student is classified as *deep* (rank difference of three), a type 3 error occurs. All misclassifications of the rule-based neuro-fuzzy model (Case 1), produced type 1 errors, i.e. students were classified into an adjoining category compared with the groups' classification. The same behaviour has been exhibited by the example-based neuro-fuzzy model (Case 2). This was also a significant improvement over previous work [53]. In contrast, as shown in Figure 12, the other models produce misclassifications of types 2 and 3; the FNN exhibits a 4% of type 2 errors and 1% of type 3 errors; whilst the NN exhibits 6% of type 2 errors, and 1% of type 3 errors.

Error type	NN	FNN	neuro-fuzzy
type 1	93%	95%	100%
type 2	6%	4%	0%
type 3	1%	1%	0%

Figure 12. Percentage of type of errors for the three models.

5. Conclusions

In this paper a neuro-fuzzy model of the diagnostic process was proposed for inferring student characteristics. A main advantage of the new approach is that the neuro-fuzzy model allows creating an interpretable knowledge representation, which can be developed on the basis of rules when reasoning is well defined, as well as it can be trained when the reasoning strategy is purely intuitive and ill-defined. In addition the model can be easily tailored to a teacher's personal view. This approach can be used to implement an open student model, which will be interactively adjusted by the teacher.

Experimental results from testing the new model in a discovery learning environment were particularly encouraging, showing that this method is capable of handling uncertainty better than other soft computing methods. The experiment has shown the potential of neuro-fuzzy synergism, but it was only a small-scale study. Further work needs to be undertaken to fully explore the benefits and limitations of this approach. Our current work targets the extraction of knowledge from existing student profiles to drive model's adaptation during operation with the aim to adapt the feedback and pedagogical strategy to students' learning style.

6. References

- [1] F. N. Akhras, J. A. Self, Beyond intelligent tutoring systems: Situations, interaction, process and affordances. *Instructional Science* 30 (2002) 1-30.
- [2] A. B. Arons, *A guide to introductory physics teaching*. Washington, John Wiley and Sons Inc, 1990.
- [3] J. Beck, M. Stern, B. P. Woolf, Using the Student Model to Control Problem Difficulty, in: *Proceedings of the Sixth International Conference on User Modeling* (1997), pp. 277-289, available on-line: <http://www.cs.umass.edu/~beck/publications.html>, accessed at 31/11/2000.
- [4] J. J. Beshuizen, E. T. Stoutjesdijk, Study strategies in a computer assisted study environment, *Learning and Instruction* 9 (1999) 281-301.
- [5] J. Biggs, *The Study Process Questionnaire (SPQ) Manual*, Australian Council for Educational Research, Melbourne, 1987.
- [6] J. Biggs, *Student approaches to learning and studying*, Australian Council for Educational Research, Hawthorn Victoria, 1987.
- [7] J. S. Brown, A. Collins, P. Duguid, Situated cognition and the culture of learning, *Educational researcher* 18 (1989) 32-34.
- [8] P. Brusilovski, Student model centered architecture for intelligent learning environments. In *Proc. of Fourth International conference on User Modeling*, 15-19 August, Hyannis, MA, Usa, User Modeling Inc, (1994), pp. 31-36.
- [9] R. J. G. B. Campello, W. C Amaral, Modeling and linguistic knowledge extraction from systems using relational models, *Fuzzy Sets and Systems* 121 (2001) 113-126.

- [10] Q. Chen, A. F. Norcio, J. Wang, Neural Network Based Stereotyping for User Profiles, *Neural Computing and Applications* 9 (2000) 259-265.
- [11] M.T.H. Chi, R. Glaser, E. Rees, Expertise in problem solving, In Sternberg R. J. (Ed): *Advances in the psychology of human intelligence*, Vol 1, Hillsdale, NJ: Erlbaum, 1982.
- [12] M. T. H. Chi, P. J. Feltovich, R. Glaser, Categorization and representation of physics problems by experts and novices, *Cognitive Science* 5 (1981) 121-152
- [13] C. Conati, A. Gertner, K Vanlehn, Using Bayesian Networks to Manage Uncertainty in Student Modeling, *User Modeling and User-Adapted Interaction* 12 (2002) 371 -417.
- [14] J. Dae-Sik, C Hyung-Il, Fuzzy Inference System Based on Fuzzy Associative Memory, *Journal of Intelligent and Fuzzy Systems* 5 (1997) 271-284.
- [15] S. J. Derry, M. K. Potts, How Tutors Model Students: A Study of Personal Constructs in Adaptive Tutoring, *American Educational Research Journal* 35 (1) (1998) 65-99.
- [16] R. Driver, *The pupil as a scientist?*, Milton Keynes: Open University Press, 1983.
- [17] R. Driver, E. Guesne, A. Tiberghien, *Children's Ideas in Science*, Philadelphia: University Press, 1985.
- [18] N. Entwistle, A model of the teaching-learning process, in: J. T. E. Richardson M. W. Eysenck and D. Warren Piper (Ed), *Student Learning : Research in Education and Cognitive Psychology*, Milton Keynes: SRHE; Open University Press, 1987, pp. 13-28
- [19] M. Grigoriadou, G. D. Magoulas, M. Panagiotou. A hybrid decision making model for intelligent tutoring systems, in: *Proceedings of the 5th International Conference of the Decision Sciences Institute*, Athens, Greece, 1999, pp. 195-197.
- [20] M. Grigoriadou, D. Mitropoulos, M. Samarakou, C. Solomonidou, E. Stavridou, Methodology for the Design of Educational Software in Mathematics and Physics for Secondary Education, in: *Conference Proceedings of Computer Based Learning in Science*, 1999, pB3.
- [21] S. A. Harp, T. Samad, M. Villano, Modeling student knowledge with self-organizing feature maps, *IEEE Transactions on Systems Man and Cybernetics* 25 (5) (1995) 727-737.
- [22] L. W. Hawkes, S. J. Derry, Advances in Local Student Modeling Using Informal Fuzzy Reasoning, *International Journal of Human-Computer Studies* 45 (1996) 697-722.
- [23] L. W. Hawkes, S. J Derry, E. A. Rundensteiner, Individualized tutoring using an intelligent fuzzy temporal relational database, *International Journal of Man-Machines Studies* 33 (1990) 409-429

- [24] C. Herzog, Fuzzy-Techniken für das Verstehen von Studentenlösungen in intelligenten Lehrsystemen [Fuzzy techniques for understanding student solutions in intelligent tutoring systems], in: R. Gunzenhauser, C. Mobus, and D. Rosner (Ed), Beiträge zum 7. Arbeitstreffen der GI-Fachgruppe 1.1.5/7.0.1, "Intelligente Lehr-/Lernsysteme" [Papers for the Seventh Meeting of GI Section 1.1.5/7.0.1, "Intelligent Tutoring Systems"], Research Institute for Application-Oriented Knowledge Processing (FAW), Germany, 1994.
- [25] P. Holt, S. Dubs, M. Jones, J. Greer, The state of student modeling, in: Proceeding of NATO advanced research workshop on "Student Modeling: The Key to Individualized Knowledge-Based Instruction", Quebec, Canada, Springer-Verlag, 1991, pp 5-35.
- [26] A. Jameson, Numerical Uncertainty Management in User and Student Modeling: An Overview of Systems and Issues, *User Modeling and User-Adapted Interaction* 5 (1996) 193-251, available on-line: <http://www.w5.cs.uni-sb.de/amp/home-page.html>, accessed at 20/01/2001.
- [27] S. Katz, A. Lesgold, G. Eggan, M. Gordin, Modelling the student in Sherlock II, *Journal of Artificial Intelligence in Education* 3 (4) (1992) 495-518.
- [28] B. Kosko, *Neural Networks and Fuzzy systems: A Dynamical Systems Approach to Machine Intelligence*, Prentice-Hall, 1992.
- [29] J. H. Larkin, J. McDermott, D. Simon, H. A. Simon, Expert and novice performance in solving physics problems, *Science* 208 (1980) 1335-1342.
- [30] L. D. Lascio, A. Gisolfi, V. Loia, Uncertainty processing in user-modeling activity, *Information Sciences* 106 (1998) 25-47.
- [31] X. Li, D. Ruan, Novel neural algorithms based on fuzzy δ rules for solving fuzzy relation equations: Part I, *Fuzzy Sets and Systems* 90 (1997) 11-23.
- [32] G. D. Magoulas, K. A. Papanikolaou, M. Grigoriadou, Neuro-fuzzy Synergism for Planning the Content in a Web-based Course, *Informatica* 25 (2001) 39-48.
- [33] G. D. Magoulas, M. N. Vrahatis, G. S. Androulakis, Effective back-propagation training with variable stepsize, *Neural Networks* 10 (1997) 69-82.
- [34] F. Marton, R. Saljo, Approaches to learning, in: F. Marton, D. Hounsell and N. Entwistle (Ed.), *The experience of learning*, Scottish Academic Press, Edinburgh, 1984, pp. 36-55.
- [35] D. McArthur, C. Stasz, M. Zmuidzinis, Tutoring techniques in algebra, *Cognition and Instruction* 7 (1990) 197-244.

- [36] S Mengel, W Lively, Using a neural network to predict student responses, in: Proceedings of the 1992 ACM/SIGAPP Symposium on Applied Computing, 2, March 1-3, Kansas City, MO, 1992, pp. 669-676.
- [37] S. Mengel, W. Lively, On the use of neural networks in intelligent tutoring systems, *Journal of Artificial Intelligence in Education* 2 (1) (1990) 43-56.
- [38] T., M., Mitchell. *Machine learning*, McGraw-Hill, 1997.
- [39] NATO ASI Series, Proceedings of NATO advanced research workshop on "Student Modeling: The Key to Individualized Knowledge-Based Instruction" Quebec, Canada, Springer-Verlag, 1991.
- [40] M. Negnevitsky, *Artificial Intelligence: a guide to intelligent systems*, Addison Wesley, 2002.
- [41] A. Paiva, About User and Learner Modeling- an Overview (1995), available on-line: <http://cbl.leeds.ac.uk/~euroaied/papers/Errico>, accessed at 25/01/2001.
- [42] M. Panagiotou, M. Grigoriadou, An Application of Fuzzy Logic to Student Modeling, in: Proceedings of the IFIP World conference on Computer in Education (WCCE95), Birmigham, 1995.
- [43] K. A. Papanikolaou, K. A. Magoulas, M. Grigoriadou, Computational intelligence in adaptive educational hypermedia, in: Proceedings of Int. Join Conf. on Neural Networks 2000 Como Italy, IEEE Press, Los Altamitos, 2000, pp. VI-629
- [44] W. Pedrycz, Identification in Fuzzy Systems, *IEEE Trans. on Systems Mans and Cybernetics* 14 (2) (1984) 361-366.
- [45] W. Pedrycz, Neurocomputations in Relational Systems, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 13 (3) (1991) 289-296.
- [46] C. L. Posey, L. W. Hawkes, Neural Networks Applied to Knowledge Acquisition in the Student Model, *Information Sciences* 88 (1996) 275-298.
- [47] R. T. Putman, Structuring and adjusting content for students: A study of live and simulated tutoring in addition, *American Educational Research Journal* 24 (1987) 13-48.
- [48] K. Reusser, From Cognitive Modeling to the Design of Pedagogical Tools, in: S. Vosniadou E. De Corte R.Glaser H. Mandl (Ed), *International Perspectives on the Design of Technology-Supported Learning Environments*, 1996, pp. 81-103
- [49] J. Self, Computational Mathetics: Towards a Science of Learning Systems Design, Draft www document, 1995, available on-line: <http://cbl.leeds.ac.uk/~jas/cm.html>, accessed at 20/02/2002.

- [50] J. Self, Formal Approaches to Student Modelling, in: Proceedings of NATO advanced research workshop on "Student Modeling: The Key to Individualized Knowledge-Based Instruction", Quebec, Canada, Springer-Verlag, 1991, pp. 295-352.
- [51] R. Sison, M. Shimura, Student Modeling and Machine Learning, *International Journal of Artificial Intelligence in Education* 9 (1998) 128-158.
- [52] C. Solomonidou, E. Stavridou, T. Christidis, The history of ideas and the students' learning difficulties about force and motion as a guide for the didactic use of the software "Interactive Physics" (Published in Greek), *Paidagogiki Epitheorisi* 26 (1997) 77-112.
- [53] R. Stathacopoulou, G. D. Magoulas, M. Grigoriadou, Neural Network-based Fuzzy Modeling of the Student in Intelligent Tutoring Systems, in: Proceedings of Inter. Joint Conference on Neural Networks 1999 Washington, CD-ROM proceeding, IEEE Catalog Number: 99CH36339C, 1999, paper #679.
- [54] S. Ur, K. VanLehn, STEPS: A simulated, tutable physics student, *Journal of Artificial Intelligence in Education* 6 (4) (1995) 405-437.
- [55] K. VanLehn, Student Modeling, in: M. C. Polson and J. J. Richardson (Ed), *Foundations of Intelligent Tutoring Systems*, Lawrence Elbaum, 1988, pp.55-78.
- [56] K. Vanlehn, S. Ohlsson, R. Nason, Applications of Simulated Students: An Exploration. *Journal of Artificial Intelligence in Education* 5(2) (1994) 135-175.
- [57] J. D. H. M. Vermunt, F. A. W. M. Van Rijswijk, *Inventaris Leerstijlen voor het hoger onderwijs [Inventory of learning styles for higher education]*, The Netherlands: Brabant University, Tilburg, 1987.
- [58] S. Vosniadou, From cognitive theory to educational technology, In: Vosniadou S., De Corte E., Mandl H. (Ed), *Technology-Based Learning Environments, Psychological and Educational Foundations*. NATO ASI Series F, vol. 137, Berlin: Springer-Verlag, 1994, pp. 11-17.
- [59] K. Warendorf, S. J. Tsao, Application of Fuzzy Logic Techniques in the BSS1 Tutoring System, *Journal of Artificial Intelligence in Education* 8 (1) (1997) 113-146
- [60] E. Wenger, *Artificial Intelligence and Tutoring Systems: Computational and Cognitive Approaches to the Communication of Knowledge*, Morgan Kaufmann, 1987.
- [61] R. Yasdi, A Literature Survey on Applications of Neural Networks for Human-Computer Interaction, *Neural Computing and Applications* 9 (2000) 245-258.

- [62] L. A. Zadeh, Fuzzy sets and systems (1965), in: G. J. Klir and B. Yuan (Ed), Fuzzy Sets Fuzzy Logic and Fuzzy Systems: Selected Papers by Lofti A. Zadeh, Advances in Fuzzy Systems-Application and Theory Vol 6, World Scientific, 1996, pp. 35-43.
- [63] L. A. Zadeh, Fuzzy Languages and their Relation to Human and Machine Intelligence(1972), in: G. J. Klir and B. Yuan (Ed), Fuzzy Sets Fuzzy Logic and Fuzzy Systems: Selected Papers by Lofti A. Zadeh, Advances in Fuzzy Systems-Application and Theory Vol 6, World Scientific, 1996, pp. 148-179.
- [64] L. A. Zadeh, Knowledge Representation in Fuzzy Logic(1989), in: G. J. Klir and B. Yuan (Ed), Fuzzy Sets Fuzzy Logic and Fuzzy Systems: Selected Papers by Lofti A. Zadeh, Advances in Fuzzy Systems-Application and Theory Vol 6, World Scientific, 1996, pp. 764-774.

Figure 1: Schematic of the diagnostic model.

Figure 2: The implementation of a fuzzifier

Figure 3: Sample of membership functions.

Figure 4: Network architecture for implementing the fuzzy relation.

Figure 5: Introductory screen of the learning environment "Vectors in Physics and Mathematics"

Figure 6: Scenario "Bodies in equilibrium"

Figure 7: Activity with two boxes on the table.

Figure 8: Membership functions for the three linguistic terms of the linguistic variable "*problem solving speed*".

Figure 9: Network implemented to assess student's tendency to learn by discovery in a deep or surface way.

Figure 10. An episode from a "shallow" real-student record

Figure 11: Comparative results for the three test data.

Figure 12. Percentage of type of errors for the three models.

Table 1. Average classification success and standard deviation for the three models.