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Demand Uncertainty and Airline Network Morphology with Strategic Interactions

by

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Abstract

In this paper, we examine how strategic interactions affect airline network. We develop a three stage duopoly game: At stage 1 airlines determines their network structure (linear versus hub-and-spoke). At stage 2 they decide on their capacities, and at stage 3 firms compete in quantities. The main feature of the model is that firms have to decide on network structure and capacities while facing demand uncertainty. We show that while hubbing is efficient, airlines may choose a linear network for strategic reasons. Furthermore, we show that this structure softens competition by preventing contagion of competition across markets.

Key words: Airlines, Competition, Capacity constraints, Network, Uncertainty.
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I. Introduction

Over the past thirty years, the airline industry has been under close scrutiny by economists, especially after its deregulation. Recently, several studies have examined the determinants of an airline's network morphology in a deregulated environment. A central question of interest has been to explain how airlines choose between two very different network structures, namely hub-and-spoke (h&s) and linear (or point-to-point).[1] Up until now, this has been done by focusing on the link between costs, demand size and network structure. In this paper, we identify another element affecting an airline's choice of network structure, namely the interaction between strategic considerations and demand uncertainty.

In the US, the deregulation of the airline industry in the late seventies led to a major restructuring of carrier networks from a mostly linear to an h&s structure (see Borenstein 1992, Barla 1998). This phenomenon has mainly been explained by invoking the possibility, offered by the h&s, to increase traffic density on each of the direct links served (the spoke). This allows a better exploitation of economies of traffic density as well as the improvement of service quality through increased frequencies. The role of traffic density in the airline industry has been widely studied both theoretically (Bailey *et al.* 1985, Brueckner and Spiller 1991, Hendricks *et al.* 1995) and empirically (Caves *et al.* 1984, Brueckner *et al.* 1992).

These effects of network structure on costs and demand size may also interact with strategic considerations as Oum *et al.* (1995) show. Using a duopoly model with three cities, they show that strategic interactions reinforce the tendency towards hubbing. Indeed, hubbing helps a firm to reduce its marginal cost and increase product quality. This, in turn, commits the firm to producing more, thereby pushing the competitor to reduce its output (outputs being strategic

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[1] In a linear structure city-pairs are linked through direct service, while in an h&s network, cities are linked by direct service only to a few central airports, the hubs. Other city-pairs are linked indirectly, through the hubs.

substitutes). The authors show that, even if hubbing increases total costs, strategic considerations may lead firms to adopt this network structure. Furthermore, they find that rivalry in networking may result in a prisoner's dilemma: if both firms adopt an h&s network, the competitive advantages cancel out and the firms' profits may end up being smaller than in a linear structure.

Although the h&s network structure has become quite popular, some firms such as Southwest Airline in the US, continue to sucessfully operate with linear network structures. Moreover, there is some factual evidence of some US carriers scaling back hubbing by re-introducing non-stop connections in the early nineties.[2] While various other reasons can also have contributed to both the success of Southwest Airline as well as the re-introduction of direct connections, the theoretical model developed here provides some additional insight into explaining these phenomena.

The main feature of our analysis is that airlines have to decide on their network structure and on the capacities they offer before the demand conditions are completely known. Barla and Constantatos (1999) show that when there is uncertainty in demand conditions, hubbing provides another advantage, in addition to those previously identified in the literature. By pooling passengers from several markets into the same plane, an h&s structure offers the airline the flexibility to change the allocation of its capacity across markets as new information about demand is revealed. They show that, in a monopoly setting, this flexibility favors the adoption of an h&s.

In this paper, we show how, despite the flexibility provided by hubbing, a duopolist may choose to adopt a linear network structure. We develop a three stage model in which each firm has a monopoly position in a particular airport, which can also be developed as a hub. There is also a market, between two other airports, for which both airlines compete. In the first stage of the game, each airline decides whether to serve this market directly or through its hub. This decision amounts to choosing between a linear and an h&s network structure. In stage 2, each airline decides on the level of capacity to offer in each of the links it connects. These two decisions are made while facing demand uncertainty. At the beginning of stage 3, demand conditions are revealed and firms compete in quantities.

We show that when capacity costs are low, strategic interactions may induce airlines to adopt a linear network structure. The idea is that by renouncing the hubbing related flexibility, an airline commits itself into a predetermined allocation of its capacity between markets. This commitment has two implications. First, it makes the firm "tough", thereby affecting its rival's capacity choice. Second, it prevents airlines from feeding competition on a market where demand turns out to be high by rationing passengers on other markets. These effects produce the following results: i) there are Nash equilibria where both firms adopt a linear structure; ii) the mutual adoption of linear network structure may make both firms better-off. This last result identifies a disadvantage of hubbing, namely that it may propagate competition across markets.

Interestingly, when we introduce capacity cost differences among carriers, the above identified effects may yield Nash equilibria with asymmetric network structures. For the low cost carrier, the strategic advantage of a linear structure dominates the flexibility advantage of hubbing, while the opposite is true for the high cost airline.

This paper is organized as follows. In section 2, we present the model. In section 3, we find the equilibrium capacities and quantities sold for each possible network structure. In section 4, we find the equilibrium network choice by comparing the expected profits in various network configurations. We conclude in section 5.

II. The Model

In order to examine the interaction between demand uncertainty, capacity constraints and network structure, we develop a three stage game between two firms (labeled 1 and 2) in a simple network such as the one illustrated in figure 1. We assume that firm 1 and firm 2 have exclusive rights in airports at city H1 and H2 respectively and that only these two airports can be developed as hubs. Each firm i has therefore a monopoly position on markets AHi and BHi, i=1,2. There is also a demand for air travel between cities A and B for which both firms are competing.[3] This simple structure is convenient to focus on competition on the AB market. Besides, it is consistent

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[2] See for example "Goodbye Hub and Spoke?", Fortune, December 1993, p 160-161.

[3] To simplify the analysis, we assume that there is no demand for air travel between H1 and H2.

with the observation that markets to and from hubs are, on average, more concentrated than non-hub markets (see Barla, 1998).

Competition between the two airlines is modeled as a three-stage game. In stage 1, both firms simultaneously commit themselves to a network structure by deciding whether to serve market AB directly or indirectly through their hubs. In stage 2, they determine simultaneously their capacities on each of the links they serve. If firm i chooses a linear structure in stage 1, it needs to determine its capacity on three links, namely AHi, BHi and AB. We denote these capacities as $K^i_{AHi}, K^i_{BHi}, K^i_{AB}$ respectively (i=1,2). If firm i chooses an h&s structure, it then has to determine its capacity only on the two links to its hub namely K^i_{AHi}, K^i_{BHi}.[4] Network structure and capacity decisions are made while facing demand uncertainty on market AB. At the beginning of stage 3, this uncertainty is resolved, and firms simultaneously decide their quantities on AB and on the hub markets denoted by $Q^i_{AB}, Q^i_{AHi}, Q^i_{BHi}$ respectively (i=1,2).

The timing of the game reflects some stylized facts of the industry. Typically, in the airline industry, network adjustments are infrequent, especially if one of the endpoint airports is congested and requires gaining access to scarce gates and slots. Capacity can be adjusted more easily. However, last minute capacity adjustments such as switching to a smaller aircraft or even canceling a flight if demand turns out to be low are rather costly, notably in terms of the airline's reputation. For simplicity, we consider such adjustments in network or capacities as being too costly to be undertaken after the demand conditions are revealed.

Probably the most restrictive assumption of this model is that firms compete in quantities at the last stage of the game. Its principal advantage is that it considerably simplifies the analysis since assuming price competition with product homogeneity at the last stage would require that Kreps and Scheinkman’s (1983) seminal paper be extended to a multimarket setting.[5] While extremely useful, such an analysis is however out of the scope of the present paper and is left for future research. Another alternative to quantity competition is to assume price competition with heterogeneous products. However, the introduction of product differentiation would mean adding

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[4] In order to facilitate the presentation, we will hereafter refer to the market AHi, BHi as the *hub markets* and market AB as the *non-hub market* even if AB is served directly and there is therefore no hub *per se* in this case.

[5] With price competition and demand uncertainty, the equilibrium would certainly involve mixed strategies (see Staiger and Wolak 1992). In a multimarket setting, finding these mixed strategies may be quite challenging.

another dimension, and perhaps another stage in the game. This would make the analysis cumbersome and risk obscuring some of the effects we wish to uncover.

We assume that the demand for air travel on market AB is represented by:

$$P_{AB} = a - Q_{AB}$$

where P_{AB} and Q_{AB} respectively stand for price and total number of passengers on route AB.[6]

The uncertainty about demand conditions in stage 1 and 2 is modeled by assuming that a is a random variable with a distribution function given by F_{AB}. To keep the analysis tractable, we assume that a follows a uniform distribution on the support [0,1]. On the hub markets, we suppose that the uncertainty is negligible and demands are similar, given by: $P_m = 1 - Q_m$ (m=AH1, BH1, AH2, BH2). Assuming no uncertainty on the hub markets considerably reduces the number of sub-cases to be studied since it insures that both firms always face symmetric conditions.[7] Note that the demand structure described above implicitly assumes that the non-hub market is relatively smaller than the hub markets. This assumption is consistent with the observation that hub airports are usually located in large cities that generate significant local traffic (see Huston and Butler 1991). In the US in fact, total traffic on links to major hubs is on average composed of about 60% local traffic and 40% connecting traffic to various destinations, therefore implying a significant size asymmetry between the hub and non-hub markets (see Ivy, 1993).

It is worth noting that the structure of the demands we use also implies two other assumptions, namely that: i) passengers on AB are indifferent about flying directly or indirectly and ii) there are no links between the demand across markets, such as complementarities resulting from frequent flyer programs. Both assumptions help keep the analysis in focus by excluding effects that have already been identified elsewhere.

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[6] We do not make any distinction between passengers based on the direction they travel on a route (for example, from A to B or from B to A). The cost structure that we introduce later makes this distinction unnecessary.

[7] We could also argue that airlines have more information on the markets that include their hub.

We further assume that travelers are not allowed to do any arbitrage. If a firm serves AB through its hub, two types of arbitrage are possible: first, AB travelers could, if profitable, buy separate tickets for each sub-route. That is buy a ticket AHi and BHi to travel between A and B. In our model however, given the demand and competitive structure, such an option will never be profitable at the equilibrium.[8] The second possibility of arbitrage is for consumers on the hub markets to buy a ticket on AB and only use the portion corresponding to their actual journey. Following Hendricks *et al.* (1997), we exclude this option since the "carrier can stop this practice by requiring travelers to board their outgoing and return flights at the city designated on the tickets. This is indeed current practice among airlines."

On the cost side, we introduce a very simple structure that allows us to focus on demand and strategic considerations in the choice of network structure. We assume that the capacity costs supported in stage 2 are the only cost, implying zero marginal cost (up to capacity) associated with serving an extra passenger in stage 3. This is consistent with the observation that, in the airline industry, most of the operating costs are associated with offering a seat rather than serving a passenger.[9] Concerning the capacity cost, we assume that: i) both airlines face similar cost; ii) the per unit capacity cost (i.e. the cost of offering one seat) is independent of the number of passengers carried on a route and iii) this cost is c ($0 < c \leq \frac{1}{2}$) on the links that include Hi (AHi, BHi) while it is $2c$ on the link AB (if it is served directly). The first assumption is relaxed later in this paper when we examine the effects of cost asymmetry. The second assumption rules out traditional economies of traffic density, already studied in the literature. By imposing the same cost for carrying a passenger from A to B, whether directly or through the hub, the third assumption eliminates some obvious cost considerations on the choice of network structure.

Developing a hub airport requires fixed investments without which flying passengers from A to B through Hi becomes problematic. For instance, convenient connections imply the development of an efficient system for transferring passengers and their luggage or acquiring time slots that are compatible (see Levine 1987, Oum *et al.* 1995). Rather than introducing these

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[8] A firm will never sell more than $\frac{1}{2}$ in its hub markets implying that the price for two tickets on AHi and BHi will always be higher than 1 (the maximun reservation price for AB travelers). Note that with a different demand structure (for example if we include uncertainty on the hub markets), this form of arbitrage could play a role.

investments explicitly in the model, we simply assume that an overflow on AB cannot be accommodated through the hub if the firm has opted for a linear structure. As it turns out, the main advantage of the latter is its commitment value. Thus, if a firm chooses to fly directly between A and B, it even has an incentive to reduce compatibility between the flights on AHi and BHi (through scheduling for example).

III. Quantity and capacity competition

Before examining the interaction between strategic aspects and demand uncertainty, it is useful to review the effect of demand uncertainty on an airline network choice in a monopoly setting, that is in a situation where there is no strategic considerations. Barla and Constantatos (1999) show that when network structure and capacity levels have to be decided when facing demand uncertainty, hubbing provides an advantage over a linear structure in terms of flexibility. By pooling passengers from several markets into the same plane, the h&s structure allows the firm to adjust the allocation of capacity after the demand conditions are revealed. This flexibility means that if the demand on one market turns out to be low, thereby creating excess capacity, the firm can increase sales in other markets. Moreover, if the demand in one market ends up being high with consequent binding capacity constraints, hubbing helps reduce the opportunity cost of rationing passengers by allowing the firm to first ration low valuation travelers from several markets before rationing higher valuation ones. In the absence of other disadvantages associated with hubbing (passengers are indifferent, hubbing does not involve any extra costs), a monopolist will always adopt an h&s network structure.

Moving from a monopoly to a duopoly, strategic considerations must be added, besides the aforementioned flexibility, to the list of factors that determine an airline's choice of network. The purpose of our paper is, precisely, to analyze the effect of competition in market AB on the two airline network structures.

Four types of network configuration are possible: (1) both firms serve AB directly, (2) both firms serve AB through their respective hubs, (3) firm 1 serves AB directly and firm 2,

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[9] Note that the analysis could easily be extended to include a positive marginal cost in stage 3.

indirectly through H2 and, (4) firm 1 serves AB indirectly through H1 and firm 2, directly. We will refer to these structures as *DD*, *II*, *DI* and *ID* respectively. To find the equilibrium network structure, we need to compare the firms' expected profits in each of the four configurations for capacities and quantities that correspond to a Nash subgame equilibrium. For symmetric network configurations (*i.e.*, *DD* and *II*), we assume that both airlines choose similar capacities in stage 2 and sell the same quantities in stage 3.[10] It is also worth noting that, whatever the network structure, each firm will always choose equal capacities on its two hub links ($K^i_{AHi} = K^i_{BHi}$ for i=1,2) since the corresponding demand conditions are always identical. For the same reason, the quantities $Q^i_{AHi} = Q^i_{BHi}$ (for i=1,2). To simplify the notation, we will refer to these capacities and quantities as K^i_H and Q^i_H, with H=AHi, BHi, respectively. Finally note that the following notational convention used hereafter: the superscript *DD*, *II*, *DI* and *ID* on a variable indicates that it is the Nash equilibrium value for the corresponding network configuration.

(1) *Both Firms Serve AB Directly (DD)*

The following lemma describes the equilibrium capacities and quantities in the *DD* network structure.

Lemma 1: if, in stage 1, both firms decide to serve AB directly: i) they will act as monopolists on their hub markets, choosing $K^i_H = K^{DD}_H = Q^i_H = Q^{DD}_H = \frac{(1-c)}{2}$ (i=1,2); ii) on market AB, for $0 < c \leq \frac{1}{4}$, the symmetric subgame-perfect Nash equilibrium capacities are given by $K^i_{AB} = K^{DD}_{AB} = \frac{1-2\sqrt{c}}{3}$ while the corresponding quantities are:

$$Q^i_{AB} = Q^{DD}_{AB} = \begin{cases} \dfrac{a}{3} & \text{for } a \leq 3K^{DD}_{AB} \\ K^{DD}_{AB} & \text{for } a \geq 3K^{DD}_{AB} \end{cases}$$

For $\frac{1}{4} < c \leq \frac{1}{2}$, market AB is not served.

[10] Since firms face similar conditions, it is therefore legitimate to restrict the analysis to symmetric equilibrium.

Proof: see appendix 1.

Since there is no demand uncertainty on the hub markets, the firms' optimal capacities and quantities on these markets are similar and correspond to the standard monopoly solution. In the subgame-perfect Nash equilibrium, expected profit maximizing capacities on AB are such that i) if the demand is low ($a \leq 3K_{AB}^{DD}$), capacity constraints are not binding, thereby leading to the usual Cournot solution in stage 3; ii) if the demand is high ($a \geq 3K_{AB}^{DD}$), capacity constraints limit the quantities firms can sell on AB in stage 3.

(2) *Both Firms Serve AB Through Their Respective Hub (II)*

With this network structure, both firms have to choose their capacities in stage 2 for the two links, AHi and BHi. In stage 3, after the demand state has been revealed, they have to allocate these capacities between the hub and non-hub markets. The symmetric subgame-perfect Nash equilibrium is described by the following lemma.

Lemma 2: if both firms decide to serve market AB through their hub, the symmetric Nash equilibrium capacity choices in stage 2 are (i=1,2):

$$K_H^i = K_H^{II} = \begin{cases} \dfrac{242 - 7\sqrt{1+1410c}}{282} & \text{for } 0 < c \leq \frac{36}{245} \\[2ex] \dfrac{389 - 7\sqrt{5(576c-43)}}{576} & \text{for } \frac{36}{245} \leq c \leq \frac{1}{2} \end{cases}$$

In stage 3, the Nash equilibrium quantities sold on the different markets are:

i) For $0 < c \leq \frac{36}{245}$:

$$Q_{AB}^i = Q_{AB}^{II} = \begin{cases} \frac{a}{3} & \text{for } 0 \leq a \leq 3(K_H^{II} - \frac{1}{2}) \\ \frac{1}{7}\left(a - 2 + 4K_H^{II}\right) & \text{for } 3(K_H^{II} - \frac{1}{2}) \leq a \leq 1 \end{cases} \qquad Q_H^i = Q_H^{II} = \begin{cases} \frac{1}{2} & \text{for } 0 \leq a \leq 3(K_H^{II} - \frac{1}{2}) \\ K_H^{II} - Q_{AB}^{II} & \text{for } 3(K_H^{II} - \frac{1}{2}) \leq a \leq 1 \end{cases}$$

ii) For $\frac{36}{245} \leq c \leq \frac{1}{2}$:

$$Q_{AB}^i = Q_{AB}^{II} = \begin{cases} 0 & \text{for } 0 \leq a \leq 2 - 4K_H^{II} \\ \frac{1}{7}(a - 2 + 4K_H^{II}) & \text{for } 2 - 4K_H^{II} \leq a \leq 1 \end{cases} \qquad Q_H^i = Q_H^{II} = \begin{cases} K_H^{II} & \text{for } 0 \leq a \leq 2 - 4K_H^{II} \\ K_H^{II} - Q_{AB}^{II} & \text{for } 2 - 4K_H^{II} \leq a \leq 1 \end{cases}$$

Proof: see appendix 2.

For $0 < c \le \frac{36}{245}$, the optimal capacities are higher than $\frac{1}{2}$, implying that for any a, the hub and non-hub markets are always served. In this case, for $0 \le a \le 3(K_H^{II} - \frac{1}{2})$, capacity constraints are not binding, thereby leading to the usual Cournot solution on AB and the standard monopoly revenues maximizing outcome on the hub markets.[11] For $3(K_H^{II} - \frac{1}{2}) \le a \le 1$, capacity constraints are binding and firms have to allocate their limited capacities between the hub and non-hub markets. They will do so in a way that equalizes the marginal revenue of selling to one more traveler on AB to the opportunity cost of this sale, namely having to ration one passenger in AHi and BHi.[12] For $\frac{36}{245} \le c < \frac{1}{2}$, the optimal capacities are less than $\frac{1}{2}$, which implies that for low demand states on AB ($a \le (2 - 4K_H^{II})$), the firms will allocate all of their capacities to the hub markets.[13] For higher demand states, the firms will, once again, allocate their capacities between the hub and non-hub markets in a way that equalizes the marginal revenue to the marginal opportunity cost of an extra sale on AB.

(3) - (4) *One Firm Serves AB Directly and the Other Through Its Hub (DI or ID)*

Since, the network configurations *DI* and *ID* are similar, we only present below the case where firm 1 offers a direct service on AB while firm 2 chooses to serve AB through its hub (*ID* can be derived by symmetry). The next lemma describes the optimal capacities and quantities in this network structure.

Lemma 3: for firm 1, the optimal capacities and quantities on the hub markets correspond to the usual monopoly solution:

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[11] Since capacity costs are sunk in stage 3, the firm objective is then to maximize its revenues.

[12] Recall, that to serve an AB passenger through the hub, the firm needs to free a seat on links AHi and BHi.

[13] This follows given that for low a, the marginal revenue of the first passenger on AB is lower than its opportunity cost.

$$K_H^1 = K_H^{1,DI} = Q_H^1 = Q_H^{1,DI} = \frac{1-c}{2}$$

Firm 1's equilibrium capacity choice on the link AB ($K_{AB}^{1,DI}$) and firm 2's equilibrium capacities on links AH2 and BH2 ($K_H^{2,DI}$) are the solution to the following systems where each equation represents the firms' reaction functions (to save space, the quantities are reproduced in appendix 3):[14]

For $0 < c \leq \frac{41095 - 575\sqrt{649}}{189003}$, we have: $\begin{cases} K_{AB}^1 = \frac{1}{99}(70 - 40K_H^2 - \sqrt{49 + 11880c - 56K_H^2 + 16(K_H^2)^2}) \\ K_H^2 = \frac{1}{38}(44 - 22K_{AB}^1 - 5\sqrt{3(2 + 38c - 2K_{AB}^1 - 9(K_{AB}^1)^2}) \end{cases}$

For $\frac{41095 - 575\sqrt{649}}{189003} \leq c \leq \frac{31}{196}$, we have: $\begin{cases} K_{AB}^1 = \frac{1}{99}(70 - 40K_H^2 - \sqrt{49 + 11880c - 56K_H^2 + 16(K_H^2)^2}) \\ K_H^2 = \frac{1}{488}(269 - 22K_{AB}^1 - 5\sqrt{3(488c - 73 + 148K_{AB}^1 - 192(K_{AB}^1)^2}) \end{cases}$

For $\frac{31}{196} \leq c \leq \frac{9 + \sqrt{6}}{50}$ we have: $\begin{cases} K_{AB}^1 = \frac{1}{21}(14 - 8K_H^2 - \sqrt{91 + 504c - 392K_H^2 + 400(K_H^2)^2}) \\ K_H^2 = \frac{1}{8}(5 + 2K_{AB}^1 - \sqrt{3(8c - 1 + 4K_{AB}^1)}) \end{cases}$

And finally, for $\frac{9 + \sqrt{6}}{50} \leq c < \frac{1}{2}$, firm 2 has a monopoly position on market AB.

Proof: see appendix 3.

For low capacity cost ($c \leq 0.139$), firm 2 chooses capacities higher than $\frac{1}{2}$ on its two links to the hub (AH2,BH2). This means that, for any a, it always serves the hub and non-hub markets. In this case, we can show (see appendix 3) that firm 2 will always be the first to be capacity constrained on market AB (that is $3(K_H^{2,DI} - 0.5) \leq K_{AB}^{1,DI}$). For low demand states, both firms offer the usual Cournot quantities on AB. For intermediate demand states, firm 2 is capacity constrained while its rival is not. Firm 2 allocates its capacity between the hub and non-hub market in a way that equalizes the marginal revenues of an extra sale on AB with its

[14] While these systems have explicit solutions, they are complex and would take up several pages. They are therefore not reported here but are available from the author.

opportunity cost (rationing one passenger in AH2 and BH2) for all possible values of the quantity offered by firm 1 on market AB. Firm 1 determines the quantity to offer on AB using the usual reaction function in a Cournot setting. Finally, for high demand states, both firms are capacity constrained.

For $0.139 \le c \le 0.158$, the optimal capacity choices are such that $K_H^{2,DI} \le \frac{1}{2}$, which implies that for low demand states, firm 2 does not serve market AB. For intermediate demand states, firm 2 serves AB but is constrained by its capacity while firm 1 is not. Finally, for high demand states on AB, both firms are capacity constrained.

For $0.158 \le c \le 0.228$, the capacity choices are such that once again for low demand states, firm 1 has a monopoly position on AB. However in this case, as a increases, firm 1 becomes capacity constrained before firm 2 finds it profitable to serve AB passengers. Finally, for $c \ge 0.228$, firm 1's optimal capacity on AB is zero. Firm 2 therefore has a monopoly position on this market and serves AB passengers only if the demand state is high.

IV. Equilibrium Network Structure

In order to find the equilibrium network structures, let us examine successively two questions: (1) will firm 1 serve market AB directly or indirectly when its competitor serves this market through its hub ? (2) will firm 1 serve AB directly or indirectly when its competitor serves this market directly? Answering the first question implies comparing firm 1's expected profit in the *DI* and *II* network structures, while the same comparison between the *DD* and *ID* structures allows us to answer the second question. Given the symmetry of the game, the answer to these two questions suffices to determine the equilibria of the whole game.

(1) Comparison DI/II

Let us assume that firm 2 chooses to hub. Proposition 1 compares the difference in firm 1's expected profit at the subgame-perfect Nash equilibrium under the *DI* and *II* network configurations. We define $E[\hat{\Delta}(c,a)] \equiv E[\Pi^{1,DI}(c,a) - \Pi^{1,II}(c,a)]$, where $\Pi^{1,DI}(c,a), \Pi^{1,II}(c,a)$ represent firm 1's total profit computed at the subgame-perfect Nash capacity and quantity levels for all possible values of a in *DI* and *II* respectively and *E* is the indicator for expectation over the demand state a .

Proposition 1: *D* is best reply to *I* for intermediate values of *c*, otherwise the best reply to *I* is *I*. More specifically, i) $E[\hat{\Delta}(c,a)] \geq 0, \forall\, c \in [c_1, c_2]$ while ii) $E[\hat{\Delta}(c,a)] < 0, \forall\, c \in (0, c_1) \cup (c_2, \frac{1}{2}]$ where $c_1 = 0.0006524$ and $c_2 = 0.013503$.

Proof: Given the non linearity and the many forms taken by $E[\hat{\Delta}(c,a)]$ depending upon the values of *c*, we prove proposition 1 numerically by calculating $E[\hat{\Delta}(c,a)]$ for all possible values of *c*. The results are presented in figure 2.[15]

Proposition 1 implies that for low (but not too low) values of *c*, offering a direct connection between A and B is the best reply to hubbing by the rival. While this region may appear to be rather small, one should bear in mind that these numerical results are based on specific demand and cost functions. In fact, what matters here is the mechanism that may push an airline to choose a linear network structure.

To understand this mechanism, let us fix *c* at a given value and examine how the difference in *ex post* profits between the structure *DI* and *II i.e.* $\hat{\Delta}(a,c) = \Pi^{1,DI}(c,a) - \Pi^{1,II}(c,a)$ evolves as a function of all possible demand realizations a . The variation of $\hat{\Delta}(a,c)$ is given in figure 3 for two values of *c*, *c*=0.01 and *c*=0.1 corresponding to parts i) and ii) of proposition 1 respectively. $\hat{\Delta}(c,a)$ takes several forms depending upon which capacity constraints are binding. From figure 3, four zones, Z1 to Z4, can be distinguished. [16]

Z1 corresponds to demand states where capacity constraints do not matter for either firm both in *DI* and *II*. In this zone, serving AB through the hub leads to higher profits - *i.e.*

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[15] To enhance readability, we present figure 2 in two parts, a and b, corresponding to low and high values of *c*. Nevertheless, the corresponding curve is continuous.

[16] The mathematical expressions for $\hat{\Delta}(c,a)$ are presented in appendix 4.

$\hat{\Delta}(c,a) < 0, \forall\ a \in Z1$, and this for two reasons. First, given the equilibrium capacity levels, firm 1's total capacity cost is lower in *II* than in *DI*.[17] Second, firm 1's opportunity cost of holding excess capacity is lower in *II* than in *DI* since hubbing allows firm 1 to use part of its excess capacity due to a low demand on AB to increase sales in the hub markets.

In Z2, the demand state is such that in the *DI* network configuration firm 2 is capacity constrained while firm 1 is not. This provides firm 1 with a "cost" advantage on market AB. In stage 3, the marginal cost of increasing sales on AB is zero for firm 1 while it is positive for firm 2, since the latter must reduce sales in the hub markets in order to accommodate an additional passenger between A and B. In the *II* network configuration, firm 1 does not enjoy such a competitive advantage since none of the firms is capacity constrained and thus both have a zero marginal cost associated with increasing sales on AB. This effect increases firm 1's *DI* profit relative to its profit in *II*.

In zone Z3, the capacity constraint conditions are the same as in Z2 for the *DI* network structure and, therefore, firm 1 continues to enjoy a competitive advantage over its rival on market AB. In *II*, both firms are now capacity constrained and both have a positive opportunity cost associated with increasing sales on AB. However, this cost, being similar for both firms, provides firm 1 with no competitive advantage, hence $\hat{\Delta}(a,c) > 0$.

Finally in Z4, firm 1 is now capacity constrained whether it serves AB directly or through its hub. Capacity constraints are also binding for firm 2 under both *II* and *DI* configurations. However, hubbing allows a capacity constrained firm to increase sales in the AB market by reducing sales in the hub markets. This flexibility is not available to firm 1 in the *DI* configuration since firm 1 has no way of increasing sales on AB in case of high a. This may lead to $\hat{\Delta}(a,c)$ becoming negative for high demand states.

To summarize, serving AB directly provides firm 1 with a competitive advantage over its rival in intermediate demand states. Hubbing offers more flexibility resulting in both lower cost of holding excess capacity under low demand, as well as a more efficient allocation of capacity when demand on AB turns out to be high. Firm 1's choice of serving AB directly or indirectly

[17] We have: $c(2K_H^{II}) < 2c\,K_{AB}^{1,DI} + c(2K_H^{1,DI})$.

will depend upon the expected difference in profits $E[\hat{\Delta}(c,a)]$ *i.e.* the integral under the curves in figure 3.[18] This expected difference is positive or negative depending upon the value of *c*. If the capacity cost is low (but not too low), the competitive advantage provided by *DI* may more than compensate the flexibility advantage of hubbing. As the capacity cost increases, the flexibility of hubbing becomes more important than the competitive advantage provided by serving AB directly. Finally, note that for very low capacity costs, the equilibrium capacities are high relative to the maximun demand state on AB, which implies that the range of demands where capacities matter (and thus where *DI* may be more profitable) is very limited. In this case, serving AB through the hub also turns out to be more profitable.

Further insight can be gained by decomposing the expected profit difference $E[\hat{\Delta}(c,a)]$ into:

$$E[\hat{\Delta}(c,a)] = \hat{\Psi}_1 + \hat{\Psi}_2$$

where:

$$\hat{\Psi}_1 = E\left[\Pi^{1,DI}(R_{AB}^{1,DI}(K_H^{II}), K_H^{1,DI}, K_H^{II}) - \Pi^{1,II}(K_H^{II}, K_H^{II})\right]$$
$$\hat{\Psi}_2 = E\left[\Pi^{1,DI}(K_{AB}^{1,DI}, K_H^{1,DI}, K_H^{2,DI}) - \Pi^{1,DI}(R_{AB}^{1,DI}(K_H^{II}), K_H^{1,DI}, K_H^{II})\right]$$

and $R_{AB}^{1,DI}(K_H^{II})$ is firm 1's reaction function in the *DI* network configuration in stage 2.[19]

The first term on the RHS of the above equation ($\hat{\Psi}_1$) is the difference in firm 1's expected profit resulting from the shift in network configuration from *II* to *DI,* holding rival capacity constant, *i.e.*, $K_H^2 = K_H^{II}$. Firm 1 chooses its capacity according to its corresponding reaction function, *i.e.*, $K_{AB}^1 = R_{AB}^{1,DI}(K_H^{II})$. The second term ($\hat{\Psi}_2$) reflects the change in firm 1's expected

[18] The expectation is computed using the assumed uniform distribution for a. Note that one cannot alter the sign of $E[\hat{\Delta}(c,a)]$ in a straightforward manner by simply manipulating the underling probability distribution. Recall that optimal capacities have been computed using a specific (uniform) distribution for a. Hence, changing the probability would alter these optimal capacity choices, thus affecting the shape and position of the curve $\hat{\Delta}(c,a)$.

[19] Of course, the various capacity levels used as arguments to define $\hat{\Psi}_1, \hat{\Psi}_2$ are function of *c* and a.

profit due to firm 2's optimal capacity adjustment from K_H^{II} to $K_H^{2,DI}$. We refer to the first term as the *direct effect* and the second as the *strategic effect*.[20]

Figure 4 and 5 show how these effects evolve as a function of *c*. The *direct effect* is always negative implying that in the absence of a *strategic effect*, firm 1 would never want to serve AB directly. The *strategic effect* is always positive, since serving AB directly makes firm 1 "tough": by committing capacity on AB, firm 1 renounces the possibility of re-allocating its capacity between the hub and AB markets after the demand is revealed.[21] This induces firm 2 to reduce its total capacity which in turn induces an increase in firm 1's AB capacity. The strategic effect therefore extends the range of demand states in *DI* in which firm 2 is capacity constrained while firm 1 is not, that is zones Z2 and Z3. It is precisely in these zones where serving AB directly provides firm 1 with a competitive advantage.

(2) Comparison DD/ID

Next, we compare firm 1's subgame-perfect Nash equilibrium profit under the network configurations *DD* and *ID* in order to determine its optimal reply to firm 2's decision to offer direct service on AB in stage 1 of the game. We define $E[\bar{\Delta}(c,a)] \equiv E[\Pi^{1,DD}(c,a) - \Pi^{1,ID}(c,a)]$, where $\Pi^{1,DD}(c,a), \Pi^{1,ID}(c,a)$ represent firm 1's total profit computed at the Nash subgame perfect capacity and quantity levels for all possible values of a in *DD* and *ID* respectively.

Proposition 2: *D* (*I*) is optimal reply to *D* for low (high) values of *c*. More specifically, i) $E[\bar{\Delta}(c,a)] \geq 0,\ \forall\ c \in (0, c_3]$ while ii) $E[\bar{\Delta}(c,a)] < 0,\ \forall\ c \in (c_3, \frac{1}{2}]$ where $c_3 = 0.03692$.

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[20] We use this terminology by analogy with the traditional decomposition based on total differentiation (see Tirole, 1989). While the decomposition used here is somewhat different, given the discrete nature of the decision we are studying, its purpose is similar: the *direct effect* is the change in profit resulting from the decision to serve AB directly assuming that this decision does not affect the rival's capacity choice, and the *strategic effect* is the change in profits resulting from the rival's reaction. To avoid confusion, note that the notion of *direct effect* is not related to the type of service offered (direct service or service through the hub).

[21] When it serves AB directly, there is only one usage for its capacity $K_{AB}^{1,DI}$,that is, to sell it on market AB

Proof: We proceed numerically by computing $E[\bar{\Delta}(c,a)]$ for all admissible values of *c*. The results are presented in figure 6.

To understand this result, figure 7 reproduces the *ex post* profit difference $\bar{\Delta}(c,a)$ as a function of a holding *c* fixed, for *c*=0.01 and *c*=0.1.[22] Four zones can be distinguished depending upon which capacity constraints are binding.

In Z1, the demand state a is such that capacity constraints do not matter for either firm in both the *DD* and *ID* network configurations. In this zone, $\bar{\Delta}(c,a)<0$ for reasons similar to the corresponding zone in figure 3.

In Z2, capacity constraints remain non binding in *DD*. In *ID*, firm 1 becomes capacity constrained while its rival remains unconstrained. In this network configuration, firm 1 therefore has a competitive disadvantage in market AB since it has a positive marginal cost associated with increasing sales on AB (the opportunity cost of rationing passengers in the hub markets) while its rival has a zero marginal cost. This may lead to $\bar{\Delta}(c,a)>0$.

In Z3, both firms becomes capacity constrained in the *DD* network structure while in *ID*, only firm 1 is constrained. As in Z2, firm 1 is at a competitive disadvantage compared to its rival in *ID* while they are on equal footing in *DD*. There is, however, another effect that also contributes to the increase in the profit difference $\bar{\Delta}(c,a)$: the capacity constraints in *DD* have the effect of limiting the intensity of competition in the third stage of the game.

In Z4, all capacity constraints are binding, including the one that affects firm 2 in the network structure *ID*. It is again worth noting, that by serving AB directly, firm 1 imposes a more rigid capacity constraint on itself than if it serves this market through its hub. This rigidity may once again turn out to be a disadvantage in case of high demand on AB, in which case rationing hub passengers to increase sales on AB is profitable. This may lead to a decline in $\bar{\Delta}(c,a)$ for high demand states.

In conclusion, in intermediate demand states, offering a direct connection on AB when the rival also does so prevents an airline from being at a competitive disadvantage and softens competition as well. Hubbing, on the other hand, helps reduce the cost of excess capacity in low demand states while allowing a more efficient capacity allocation in high demand states. Firm 1's network decision depends upon $E[\bar{\Delta}(c,a)]$ which corresponds to the integral under the curves in figure 7. For low *c*, the advantages of offering a direct connection on AB dominate those associated with hubbing while for higher *c*, the opposite is true. As before, we split firm 1's expected profit difference into a *direct* and a *strategic* component:

$$E[\bar{\Delta}(c,a)] = \bar{\Psi}_1 + \bar{\Psi}_2$$

where:

$$\bar{\Psi}_1 \equiv E\left[\Pi^{1,DD}(R_{AB}^{1,DD}(K_{AB}^{2,ID}), K_H^{DD}, K_{AB}^{2,ID}, K_H^{2,ID}) - \Pi^{1,ID}(K_H^{1,ID} K_{AB}^{2,ID}, K_H^{2,ID})\right]$$

$$\bar{\Psi}_2 \equiv E\left[\Pi^{1,DD}(K_{AB}^{DD}, K_H^{DD}, K_{AB}^{DD}, K_H^{DD}) - \Pi^{1,DD}(R_{AB}^{1,DD}(K_{AB}^{2,ID}), K_H^{DD}, K_{AB}^{2,ID}, K_H^{2,ID})\right]$$

and $R_{AB}^{1,DD}(K_{AB}^{2,ID})$ is firm 1's reaction in the *DD* network configuration at stage 2.[23]

The $\bar{\Psi}_1$ and $\bar{\Psi}_2$ terms correspond to the *direct* and *strategic* effects and bear an interpretation analogous to that of $\hat{\Psi}_1$ and $\hat{\Psi}_2$, respectively. Inspection of figures 8 and 9 shows that the *direct effect* is always negative while the *strategic effect* is positive. The negative *direct effect* results from the loss of the flexibility provided by hubbing. This effect increases with *c*. Firm 2's equilibrium capacity on AB decreases as a result of firm 1's decision to serve AB directly. Indeed in *DD*, firm 2 loses the competitive advantage it has for some demand states in *ID*. In the network structure *DD*, it is interesting to notice that if capacity costs are low and if $K_{AB}^2 > K_{AB}^1$, capacities are strategic complement for firm 1. Hence, firm 2's capacity adjustment leads firm 1, in turn, to reduce its AB capacity.[24] These reductions help firms to soften quantity

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[22] The mathematical expressions for the profit difference are reproduced in appendix 4.

[23] As before, all the capacities in $\bar{\Psi}_1, \bar{\Psi}_2$ are functions of *c* and a .

[24] In *DD*, the derivative of firm 1's stage 2 expected marginal profit with respect to firm 2's capacity is: $-1 + 2K_{AB}^1 + 2K_{AB}^2$. This derivative will be positive when capacities are high (and thus when *c* is low). In fact, when firm 2 decreases its capacity, the range of demand states in which only firm 1 is constrained decreases and moves to lower demand states. Since this zone corresponds to high marginal profits for firm 1 (when capacities are high), a decrease in firm 2's capacity lowers firm 1's expected marginal profit.

competition in the last stage of the game. Strategic considerations may therefore also push firm 1 to serve AB directly.

Given the symmetry of the game, the two comparisons *DI/II* and *DD/ID* allow us to determine equilibria of the whole game. Firm 2 network decisions are indeed symmetric to those of firm 1. Combining propositions 1 and 2 and taking into account the symmetry of the game, one can easily show that:

Proposition 3: For low values of c, *i.e.* $c \le c_3$, the adoption of linear networks by both firms results in a subgame-perfect Nash equilibrium configuration.

In fact, *DD* is a subgame perfect Nash equilibrium network configuration for $c \in (0, c_3]$ and *II* is a subgame-perfect Nash equilibrium network configuration for $c \in (0, c_1] \cup [c_2, \frac{1}{2}]$. Figure 10 illustrates the possible equilibrium network configurations as a function of *c*. Since, for $c \in (0, c_1] \cup [c_2, c_3]$, both *DD* and *II* are equilibrium network structures, it is particularly interesting to compare the equilibrium expected profits corresponding to each of these two network configurations. We define the expected profit difference between these configurations as $E[\Delta^*(c,a)] = E[\Pi^{DD}(c,a) - \Pi^{II}(c,a)]$, where $\Pi^{DD}(c,a), \Pi^{II}(c,a)$ represent firm 1's (or firm 2) total profit computed at the subgame-perfect Nash capacity and quantity levels for all possible values of a in *DD* and *II* respectively.

Proposition 4: Despite the flexibility offered by hubbing, when *c* is low, both airlines make higher profits under the *DD* than under the *II* network configurations. More specifically: i) $E[\Delta^*(c,a)] \ge 0,\ \forall\ c \in (0, c^*]$ while ii) $E[\Delta^*(c,a)] < 0,\ \forall\ c \in (c^*, \frac{1}{2}]$ where $c^* = 0.0268$.

Proof: We proceed numerically by computing $E[\Delta^*(c,a)]$ for all admissible values of *c*. The results are presented in figure 11.

To understand the intuition behind this surprising result, let us examine again the difference in the *ex post* profits $\Delta^*(c,a)$ as a function of the demand state a, holding *c* fixed. We use *c*=0.01 in figure 12. Once again several zones appear depending upon which capacity constraints are binding. When capacities do not matter in either network configuration (Z1), the flexibility of hubbing leads to higher profits in *II* relative to *DD*. Capacity constraints first matter in *II* (Z2). These constraints, by limiting quantity competition in the last stage of the game, further increase the profit advantage of *II*. For higher demand states (Z3), capacity constraints are also binding in *DD*. In this zone, *DD* profits become greater than those in *II*. This is due to the fact that capacity constraints in *DD* are more effective in limiting stage 3 competition than those in *II*. Recall that capacity constraints in *DD* are absolute while in *II*, firms can still increase quantities on AB by rationing passengers on the hub markets. In fact, the flexibility provided by serving AB through the hub feeds competition on AB. In other words, hubbing *propagates* the effects of competition on AB to the hub markets while the network structure *DD* isolates these markets.

The opportunity to *isolate* some markets from the effect of competition on other markets may therefore result in the adoption of linear networks by both competitors. This will be the case when *c* is low and thus when the lack of flexibility of a linear structure is not so important. Clearly, the possibility of isolating some markets from competition in others depends upon the extent to which firms can effectively commit their capacities to specific markets before the demand conditions are revealed. That is, in the *DD* network structure, airlines should not be able to reroute AB passengers through their hub. Such a commitment may be achieved, for example, by making scheduling on AHi and BHi incompatible.

So far, we have assumed that both airlines have the same capacity costs. Introducing cost asymmetry does not fundamentally affect the analysis. The effects that have been uncovered above are still at work. However, cost asymmetry may lead to an asymmetric equilibrium network configuration. Appendix 5 presents an example of asymmetric equilibrium network configuration where the low capacity cost carrier chooses to serve AB directly while the high capacity cost rival chooses to serve AB through its hub. For the low cost carrier, the strategic advantage of serving AB directly dominates the efficiency disadvantage of this structure while for the high cost carrier, the flexibility provided by hubbing turns out to be more important than strategic considerations. This could be a factor explaining the strategy followed by low cost carriers such as Southwest Airline in the US.

V. Conclusion

In this paper, we present a stylized model of airline network and capacity choice in an oligopoly setting. The main feature in our model is that network morphology and capacities have to be decided before the demand conditions are perfectly known. In this setting, hubbing provides airlines with the flexibility to change the allocation of capacity across markets after the demand is revealed. Hence hubbing is always chosen by a monopolist even in the absence of economies of traffic densities.

However, despite this advantage, we show that duopolist may choose to adopt a linear structure. This surprising result is due to the fact that opting for a linear structure and thus renouncing the flexibility provided by hubbing corresponds from a firm's point of view to a commitment to use a specific amount of capacity on a market. On the one hand, this provides an airline company with the advantage of acting "tough", thereby affecting its rival's choice of capacity. On the other hand, this commitment prevents airlines from feeding competition on a market where demand turns out to be high by rationing passengers on other markets. In other words, an h&s network structure may lead to a contagion of competition from one market to others. Hence, a network configuration where both firms adopt a linear structure may, besides providing a Nash equilibrium, also be Pareto optimal in terms of airline profits. We also show that when firms have different capacity costs, asymmetric network configurations may emerge with the low cost carrier adopting a linear structure and the high cost carrier an h&s network. It would be interesting in future research to assess the empirical importance of the effects that have been uncovered in this paper. For example, it would interesting to examine how demand uncertainty, capacity cost and the structure of the competitive environment affect the probability of an airline offering a direct or indirect connection between two cities. Our analysis also suggests several theoretical extensions. For example, it would be interesting (but certainly difficult), to extend our analysis to price competition. It would also be worthwhile studying network choice in more complex competitive environments. For example, what happens if airlines do not have monopoly positions on their hub markets ?

Finally, the results that we derive here for the airline industry are certainly applicable to other multiproduct or multimarket industries. What our analysis suggests is that strategic considerations may lead firms to build several productive units (for example, one for each product or market) even if it would be more efficient to only build one. Besides the existence of demand uncertainty, the occurrence of this effect will depend upon the extent to which firms are able to definitively allocate a productive unit to one product or market before the demands are revealed.

Appendix 1. Proof of Lemma 1.

If both firms choose to serve AB directly, the optimal capacity choice on the hub markets corresponds to the standard monopoly solution. On market AB, there is demand uncertainty. In stage 3, both firms maximize their revenues subject to a potentially binding capacity constraint:

$$\underset{Q^i_{AB}}{Max} \; (a - Q^i_{AB} - Q^j_{AB})Q^i_{AB}$$
$$st: Q^i_{AB} \le K^i_{AB}$$

The firm will either act on the usual Cournot reaction function or sell all of its available capacity. This leads to various Nash equilibrium quantities depending on which capacity constraints are binding (see below).

In stage 2, the firm maximizes the following expected profit function:

$$E(\Pi^i_{AB}(K^1_{AB},K^2_{AB})) = \begin{cases} \int_0^{b_1} R^{i1}da + \int_{b_1}^{b_2} R^{i2}da + \int_{b_2}^{1} R^{i3}da - 2cK^i_{AB} & if \;\; K^i_{AB} \ge K^j_{AB} \\ \int_0^{b_1} R^{i1}da + \int_{b_1}^{b_2} R^{i4}da + \int_{b_2}^{1} R^{i3}da - 2cK^i_{AB} & if \;\; K^i_{AB} \le K^j_{AB} \end{cases}$$

$$with: \; i, j = 1,2 \; and \; i \ne j$$

$$R^{i1} = \frac{a^2}{9}, \; R^{i2} = (a - \tfrac{1}{2}(a - K^j_{AB}) - K^j_{AB})(\tfrac{1}{2}(a - K^j_{AB})), \; R^{i3} = (a - K^i_{AB} - K^j_{AB})K^i_{AB}$$

$$R^{i4} = (a - K^i_{AB} - \tfrac{1}{2}(a - K^i_{AB}))K^i_{AB}$$

$$b_1 = 3Min[K^i_{AB}, K^j_{AB}]$$

$$b_2 = \begin{cases} 2K^i_{AB} + K^j_{AB} & if \;\; K^i_{AB} \ge K^j_{AB} \\ K^i_{AB} + 2K^j_{AB} & if \;\; K^i_{AB} \le K^j_{AB} \end{cases}$$

These expected profit functions are continuous and continuously differentiable once. The first order condition for a maximum with respect to K^i_{AB} are:

$$E_{K^i_{AB}}(\Pi^i_{AB}(K^1_{AB},K^2_{AB})) = \begin{cases} \int_{b_1}^{b_2} R^{i2}_{K^i_{AB}} da + \int_{b_2}^{1} R^{i3}_{K^i_{AB}} da - 2c = 0 & if \;\; K^i_{AB} \ge K^j_{AB} \\ \int_{b_1}^{b_2} R^{i4}_{K^i_{AB}} da + \int_{b_2}^{1} R^{i3}_{K^i_{AB}} da - 2c = 0 & if \;\; K^i_{AB} \le K^j_{AB} \end{cases}$$

where the subscript denotes the derivative with respect to K^i_{AB}.

Assuming symmetry ($K^1_{AB} = K^2_{AB}$), the first order conditions allow us to determine the Nash equilibrium capacity choice in market AB:

$$K^{DD}_{AB} = \frac{1 - 2\sqrt{c}}{3}$$

Note that the second order conditions are met and that this equilibrium is locally stable.

Appendix 2. Proof of Lemma 2.

If both firms serve AB through their hub, airlines have to determine their capacities on the two links to their hub. Let us first note that, if c>0, a firm will never choose capacities such that, given its competitor's choice and the possible demand states, it will never be capacity constrained. If that was the case, this choice would not be optimal since it would imply excess capacities for all possible demand states. Let us examine the firm decision in stage 3.

Stage 3: the firms' objective in stage 3 is to maximize total revenues subject to a potential capacity constraint. That is for firm i (i=1,2):

$$\underset{Q^i_{AB},Q^i_H}{Max.}\ (a - Q^i_{AB} - Q^j_{AB})Q^i_{AB} + 2(1 - Q^i_H)Q^i_H$$

$$st.\ Q^i_{AB} + Q^i_H \le K^i_H$$

Depending on the value of K^i_H and a, this program leads to different reaction functions that can be used to find the Nash equilibrium quantities in stage 3. If $K^i_H \ge \frac{1}{2}$, both markets are always served: i) for a such that the capacity constraint is not binding, the firm sells $\frac{1}{2}$ in the hub markets and acts on the usual Cournot reaction function on market AB ($\frac{1}{2}(a - Q^i_{AB})$), ii) for higher demand states, the capacity constraint is binding, the profit maximization under constraint leads to the following reaction functions: $Q^i_{AB} = \frac{1}{6}(a - 2 + 4K^i_H - Q^j_{AB})$ and $Q^i_H = K^i_H - Q^i_{AB}$. If $K^i_H < \frac{1}{2}$, there are low demand states where $Q^i_{AB} = 0$ and $Q^i_H = K^i_H$. The various subcases are described in the stage 2 expected profit function below.

Stage 2: firm i's expected profit at stage 2 is given by:

$$E(\Pi^i(K^1_H,K^2_H)) = \begin{cases} \int_0^{m_1} S^{i1}da + \int_{m_1}^{m_2} S^{i2}da + \int_{m_2}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \ge \frac{1}{2},\ K^i_H \ge \frac{1}{2}\ and\ K^i_H \ge K^j_H \\ \int_0^{m_3} S^{i1}da + \int_{m_3}^{m_4} S^{i4}da + \int_{m_4}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \ge \frac{1}{2},\ K^i_H \ge \frac{1}{2}\ and\ K^i_H \le K^j_H \\ \int_0^{m_5} S^{i5}da + \int_{m_5}^{m_4} S^{i4}da + \int_{m_4}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \ge \frac{1}{2},\ K^i_H \le \frac{1}{2}\ and\ K^i_H \ge \frac{1}{4}(\frac{5}{2} - K^j_H) \\ \int_0^{m_6} S^{i5}da + \int_{m_6}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \ge \frac{1}{2},\ K^i_H \le \frac{1}{2}\ and\ K^i_H \le \frac{1}{4}(\frac{5}{2} - K^j_H) \\ \int_0^{m_7} S^{i6}da + \int_{m_7}^{m_2} S^{i2}da + \int_{m_2}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \le \frac{1}{2},\ K^i_H \ge \frac{1}{2}\ and\ K^i_H \ge \frac{5}{2} - 4K^j_H \\ \int_0^{m_8} S^{i6}da + \int_{m_8}^{m_9} S^{i7}da + \int_{m_9}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \le \frac{1}{2},\ K^i_H \ge \frac{1}{2},\ K^i_H \le \frac{5}{2} - 4K^j_H \\ \int_0^{m_{10}} S^{i5}da + \int_{m_{10}}^{m_9} S^{i7}da + \int_{m_9}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \le \frac{1}{2},\ K^i_H \le \frac{1}{2}\ and\ K^i_H \ge K^j_H \\ \int_0^{m_6} S^{i5}da + \int_{m_6}^{1} S^{i3}da - c(2K^i_H) & if\ K^j_H \le \frac{1}{2},\ K^i_H \le \frac{1}{2}\ and\ K^i_H \le K^j_H \end{cases}$$

$$with:\ S^{i1} = \frac{a^2}{9} + \tfrac{1}{2}, \quad S^{i2} = \tfrac{1}{2} + (a - Q_{AB}^{i2} - Q_{AB}^{j2})Q_{AB}^{i2}$$
$$Q_{AB}^{i2} = \tfrac{1}{11}(5a + 2 - 4K_H^j), \quad Q_{AB}^{j2} = \tfrac{1}{11}(a - 4 + 8K_H^j)$$
$$S^{i3} = 2(1 - (K_H^i - Q_{AB}^{i3}))(K_H^i - Q_{AB}^{i3}) + (a - Q_{AB}^{i3} - Q_{AB}^{j3})Q_{AB}^{i3}$$
$$Q_{AB}^{i3} = \tfrac{1}{35}(5a - 10 + 24K_H^i - 4K_H^j), \quad Q_{AB}^{j3} = \tfrac{1}{35}(5a - 10 + 24K_H^j - 4K_H^i)$$
$$S^{i4} = (a - Q_{AB}^{i4} - Q_{AB}^{j4})Q_{AB}^{i4} + 2(1 - (K_H^i - Q_{AB}^{i4}))(K_H^i - Q_{AB}^{i4})$$
$$Q_{AB}^{i4} = \tfrac{1}{11}(a - 4 + 8K_H^i), \quad Q_{AB}^{j4} = \tfrac{1}{11}(5a + 2 - 4K_H^i)$$
$$S^{i5} = 2(1 - K_H^i)K_H^i, \quad S^{i6} = \tfrac{1}{2} + \frac{a^2}{4},$$
$$S^{i7} = 2(1 - (K_H^i - Q_{AB}^{i7}))(K_H^i - Q_{AB}^{i7}) + (a - Q_{AB}^{i7})Q_{AB}^{i7}$$
$$Q_{AB}^{i7} = \tfrac{1}{6}(a - 2 + 4K_H^i)$$
$$m_1 = 3(K_H^j - 0.5),\ m_2 = \tfrac{1}{5}(11K_H^i + 4K_H^j - \tfrac{15}{2})$$
$$m_3 = 3(K_H^i - 0.5), \quad m_4 = \tfrac{1}{5}(11K_H^j + 4K_H^i - \tfrac{15}{2}), \quad m_5 = 4 - 8K_H^i$$
$$m_6 = \tfrac{1}{5}(10 - 24K_H^i + 4K_H^j), \quad m_7 = 4 - 8K_H^j$$
$$m_8 = 2(K_H^i - 0.5), \quad m_9 = \tfrac{1}{5}(10 + 4K_H^i - 24K_H^j), \quad m_{10} = 2 - 4K_H^i$$

This function is continuous and continuously differentiable once. After imposing symmetry ($K_H^i = K_H^j = K_H$), the first order conditions for expected profit maximization reduce to:

$$E_{K_H^i}(\Pi^i(K_H^i = K_H, K_H^j = K_H)) = \begin{cases} \displaystyle\int_{3(K_H - \frac{1}{2})}^{1} S^{i3} da - 2c = 0 & if\ K_H \geq \tfrac{1}{2} \\ \displaystyle\int_0^{2-4K_H} S^{i5} da + \int_{2-4K_H}^{1} S^{i3} da - 2c = 0 & if\ K_H \leq \tfrac{1}{2} \end{cases}$$

where the subscript denote the derivative with respect to K_H^i.

Solving for K_H, we obtain:

$$K_H^{II} = \begin{cases} \dfrac{242 - 7\sqrt{1 + 1410c}}{282} & for\ 0 < c \leq \tfrac{36}{245} \\ \dfrac{389 - 7\sqrt{5(576c - 43)}}{576} & for\ \tfrac{36}{245} \leq c \leq \tfrac{1}{2} \end{cases}$$

The second order conditions are respected and the equilibrium is locally stable.

Appendix 3. Proof of Lemma 3.

In cases *DI* or *ID*, firm i (i=1,2) serves AB directly while firm j (j=1,2 and j $\neq$ i) serves this market through its hub. In stage 3, the firms maximize their revenues subject to potential capacity constraints:

Firm i:

On markets AHi, BHi, the standard monopoly solution holds.

On market AB, firm i's stage 3 objective function is:

$$\underset{Q^i_{AB}}{Max} \; (a - Q^i_{AB} - Q^j_{AB})Q^i_{AB}$$

$$st \; Q^i_{AB} \leq K^i_{AB}$$

Firm j's objective function is:

$$\underset{Q^j_{AB}, Q^j_H}{Max} \; (a - Q^i_{AB} - Q^j_{AB})Q^j_{AB} + 2(1 - Q^j_H)Q^j_H$$

$$st \; Q^j_{AB} + Q^j_H \leq K^j_H$$

These optimization problems lead to various reaction functions (and thus equilibrium quantities) depending on the demand state and capacity levels. These various cases are characterized in the firms' expected profit in stage 2 below.

In stage 2, firm i's expected profit on market AB is given by:

$$E(\Pi^i_{AB}(K^i_{AB}, K^j_H)) = \begin{cases} \int_0^{g_1} V^{i1} da + \int_{g_1}^{g_2} V^{i2} da + \int_{g_2}^{1} V^{i3} da - 2cK^i_{AB} & if \;\; K^j_H \geq \frac{1}{2} \; and \; K^i_{AB} \geq K^j_H - \frac{1}{2} \\ \int_0^{g_3} V^{i1} da + \int_{g_3}^{g_4} V^{i4} da + \int_{g_4}^{1} V^{i3} da - 2cK^i_{AB} & if \;\; K^j_H \geq \frac{1}{2} \; and \; K^i_{AB} \leq K^j_H - \frac{1}{2} \\ \int_0^{g_5} V^{i5} da + \int_{g_5}^{g_6} V^{i6} da + \int_{g_6}^{1} V^{i3} da - 2cK^i_{AB} & if \;\; K^j_H \leq \frac{1}{2} \; and \; 2K^i_{AB} \geq 4 - 8K^j_H \\ \int_0^{g_7} V^{i5} da + \int_{g_7}^{g_8} V^{i7} da + \int_{g_8}^{1} V^{i3} da - 2cK^i_{AB} & if \;\; K^j_H \leq \frac{1}{2} \; and \; 2K^i_{AB} \leq 4 - 8K^j_H \end{cases}$$

While firm j's expected profit for its whole network is given by:

$$E(\Pi^j(K^i_{AB},K^j_H))=\begin{cases}\int_0^{g_1}V^{j1}da+\int_{g_1}^{g_2}V^{j2}da+\int_{g_2}^{1}V^{j3}da-c(2K^j_H) & if \quad K^j_H\geq\frac{1}{2}\ and\ K^i_{AB}\geq K^j_H-\frac{1}{2}\\ \int_0^{g_3}V^{j1}da+\int_{g_4}^{g_4}V^{j4}da+\int_{g_4}^{1}V^{j3}da-c(2K^j_H) & if \quad K^j_H>\frac{1}{2}\ and\ K^i_{AB}\leq K^j_H-\frac{1}{2}\\ \int_0^{g_5}V^{j5}da+\int_{g_5}^{g_6}V^{j6}da+\int_{g_6}^{1}V^{j3}da-c(2K^j_H) & if \quad K^j_H\leq\frac{1}{2}\ and\ 2K^i_{AB}\geq 4-8K^j_H\\ \int_0^{g_8}V^{j5}da+\int_{g_8}^{1}V^{j3}da-c(2K^j_H) & if \quad K^j_H\leq\frac{1}{2}\ and\ 2K^i_{AB}\leq 4-8K^j_H\end{cases}$$

with:

$V^{i1}=\frac{1}{9}a^2;\quad V^{i2}=(a-Q^{i2}_{AB}-Q^{j2}_{AB})Q^{i2}_{AB}$

$Q^{i2}_{AB}=\frac{1}{11}(5a+2-4K^j_H);\quad Q^{j2}_{AB}=\frac{1}{11}(a-4+8K^j_H)$

$V^{i3}=(a-K^i_{AB}-Q^{j3}_{AB})K^i_{AB},\quad Q^{j3}_{AB}=\frac{1}{6}(a-2+4K^j_H-K^i_{AB})$

$V^{i4}=(a-K^i_{AB}-\frac{1}{2}(a-K^i_{AB}))K^i_{AB};\quad V^{i5}=\frac{1}{4}a^2;\quad V^{i6}=(a-Q^{i6}_{AB}-Q^{j6}_{AB})Q^{i6}_{AB}$

$Q^{i6}_{AB}=\frac{1}{11}(5a+2-4K^j_H);\quad Q^{j6}_{AB}=\frac{1}{11}(a-4+8K^j_H)$

$V^{i7}=(a-K^i_{AB})K^i_{AB}$

$V^{j1}=\frac{1}{2}+\frac{1}{9}a^2;\quad V^{j2}=2(1-(K^j_H-Q^{j2}_{AB}))(K^j_H-Q^{j2}_{AB})+(a-Q^{i2}_{AB}-Q^{j2}_{AB})Q^{j2}_{AB}$

$V^{j3}=2(1-(K^j_H-Q^{j3}_{AB}))(K^j_H-Q^{j3}_{AB})+(a-K^i_{AB}-Q^{j3}_{AB})Q^{j3}_{AB}$

$V^{j4}=(a-K^i_{AB}-\frac{1}{2}(a-K^i_{AB}))(\frac{1}{2}(a-K^i_{AB}))+\frac{1}{2};\quad V^{j5}=2(1-K^j_H)K^j_H$

$V^{j6}=(a-Q^{i6}_{AB}-Q^{j6}_{AB})Q^{j6}_{AB}+2(1-(K^j_H-Q^{j6}_{AB}))(K^j_H-Q^{j6}_{AB})$

$g_1=3(K^j_H-\frac{1}{2});\quad g_2=\frac{1}{5}(11K^i_{AB}-2+4K^j_H)$

$g_3=3K^i_{AB};\quad g_4=2(K^j_H-\frac{1}{2})+K^i_{AB};\quad g_5=4-8K^j_H$

$g_6=\frac{1}{5}(11K^i_{AB}-2+4K^j_H);\quad g_7=2K^i_{AB};\ g_8=2+K^i_{AB}-4K^j_H$

Let us prove that we cannot have a solution such that $K^j_H>\frac{1}{2}$ and firm i is first constrained by its capacity, *i.e.* $3K^i_{AB}<3(K^j_H-\frac{1}{2})$. Assuming that there is such a solution, it should therefore respect the following first order conditions:

$$\begin{cases}E_{K^i_{AB}}(\Pi^i_{AB}(K^i_{AB},K^j_H))=\int_{g_3}^{g_4}V^{i4}_{K^i_{AB}}da+\int_{g_4}^{1}V^{i3}_{K^i_{AB}}da-2c=0\\ E_{K^j_H}(\Pi^j(K^i_{AB},K^j_H))=\int_{g_4}^{1}V^{j3}_{K^j_H}da-2c=0\end{cases}$$

where the subscripts denote the derivative with respect to the corresponding capacities.

First note that:

$V^{i4}_{K^i_{AB}}=(\frac{1}{2}a-K^i_{AB})>0$ for all $a\in[g_3,g_4]$

Second, since we have: $V^{i3}_{K^i_{AB}}(a = g_4) = \frac{1}{6}(-3 - 5K^i_{AB} + 6K^j_H) > 0$ (indeed $3K^i_{AB} < 3(K^j_H - \frac{1}{2})$)

while $V^{j3}_{K^j_H}(a = g_4) = 0$ and $V^{i3}_{K^i_{AB},a} = \frac{5}{6} > V^{j3}_{K^j_H,a} = \frac{2}{3}$

we have: $V^{i3}_{K^i_{AB}} \geq V^{j3}_{K^j_H}$ for all $a \in [g_4, 1]$.

Once again the subscripts denote the derivative with respect to the corresponding capacities or with respect to the demand state a.

These two points imply that the necessary first order conditions cannot be met.

The three remaining cases give us first order conditions that are quadratic in capacities. We choose the roots for which the second order conditions for a maximun are respected. These roots yield the systems presented in the proposition. Finally, note that the obtained equilibrium is locally stable.

Appendix 4. Difference in Expected Profit

1) DI/II Comparison :

The *ex post* difference in firm 1's equilibrium profit in the two structures (*DI/II*) has three components, $\hat{\Delta}(a,c) = \hat{G}_1 + \hat{G}_2 + \hat{G}_3$ where $\hat{G}_1$ represents the difference in revenues for market AB, $\hat{G}_2$ is the revenues difference for the hub markets and $\hat{G}_3$ is the difference in total capacity costs. Below, we present these three terms for $c \le 0.139$ since it is in this value range that firm 1's expected profit in *DI* may be greater than the expected profit in *II*. First note that in this range, $\hat{G}_3 = 2c(K_{AB}^{1,DI} + K_H^{1,DI} - K_H^{II}) < 0$ for $\forall a$. The expression for $\hat{G}_1, \hat{G}_2$ depends upon which capacity constraints are binding.

- In Z1, $0 \le a \le 3(K_H^{2,DI} - \frac{1}{2})$: $\hat{G}_1 = (\frac{1}{9}a^2 - \frac{1}{9}a^2) = 0$ and $\hat{G}_2 = 2(\frac{1}{4}(1-c)(1+c) - \frac{1}{4})$
- In Z2,

$3(K_H^{2,DI} - \frac{1}{2}) \le a \le 3(K_H^{II} - \frac{1}{2})$: $\hat{G}_1 = [(a - Q_{AB}^{1,DI} - Q_{AB}^{2,DI})Q_{AB}^{1,DI} - \frac{1}{9}a^2]$ and $\hat{G}_2 = 2[\frac{1}{4}(1-c)(1+c) - \frac{1}{4}]$

where: $Q_{AB}^{1,DI} = \frac{1}{11}(5a + 2 - 4K_H^{2,DI})$; $\quad Q_{AB}^{2,DI} = \frac{1}{11}(a - 4 + 8K_H^{2,DI})$

- In Z3, $3(K_H^{II} - \frac{1}{2}) \le a \le \frac{1}{5}(11K_{AB}^{1,DI} - 2 + 4K_H^{2,DI})$:

$\hat{G}_1 = [(a - Q_{AB}^{1,DI} - Q_{AB}^{2,DI})Q_{AB}^{1,DI} - (a - 2Q_{AB}^{II})Q_{AB}^{II}]$ and

$\hat{G}_2 = 2[\frac{1}{4}(1-c)(1+c) - (1 - (K_H^{II} - Q_{AB}^{II}))(K_H^{II} - Q_{AB}^{II})]$

where: $Q_{AB}^{1,DI} = \frac{1}{11}(5a + 2 - 4K_H^{2,DI})$; $\quad Q_{AB}^{2,DI} = \frac{1}{11}(a - 4 + 8K_H^{2,DI})$; $Q_{AB}^{II} = \frac{1}{35}(5a - 10 + 20K_H^{II})$

- In Z4, $\frac{1}{5}(11K_{AB}^{1,DI} - 2 + 4K_H^{2,DI}) \le a \le 1$:

$\hat{G}_1 = [(a - K_{AB}^{1,DI} - Q_{AB}^{2,DI})K_{AB}^{1,DI} - (a - 2Q_{AB}^{II})Q_{AB}^{II}]$ and

$\hat{G}_2 = 2[\frac{1}{4}(1-c)(1+c) - (1 - (K_H^{II} - Q_{AB}^{II}))(K_H^{II} - Q_{AB}^{II})]$

where: $Q_{AB}^{2,DI} = \frac{1}{6}(a - 2 + 4K_H^{2,DI} - K_H^{1,DI})$; $Q_{AB}^{II} = \frac{1}{35}(5a - 10 + 20K_H^{II})$

2) Comparison DD/ID

Similarly, $\bar{\Delta}(a,c) = \bar{G}_1 + \bar{G}_2 + \bar{G}_3$ where the three terms have similar interpretations to those above. These terms take various forms depending upon *c*. Here we present the break down that is valid for *c*=0.01 and *c*=0.1. For these values of *c*, note that $\hat{G}_3 = 2c(K_{AB}^{DD} + K_H^{DD} - K_H^{1,ID}) < 0$ for $\forall a$. The expressions for $\bar{G}_1$ and $\bar{G}_2$ depend upon which capacity constraints are binding:

- Z1: $0 \le a \le 3(K_H^{1,ID} - \frac{1}{2})$: $\bar{G}_1 = (\frac{1}{9}a^2 - \frac{1}{9}a^2) = 0$ and $\bar{G}_2 = 2(\frac{1}{4}(1-c)(1+c) - \frac{1}{4})$
- Z2: $3(K_H^{1,ID} - \frac{1}{2}) \le a \le 3K_{AB}^{DD}$:

$\bar{G}_1 = [\frac{1}{9}a^2 - (a - Q_{AB}^{1,ID} - Q_{AB}^{2,ID})Q_{AB}^{1,ID}]$ and $\bar{G}_2 = 2[\frac{1}{4}(1-c)(1+c) - (1 - (K_H^{1,ID} - Q_{AB}^{1,ID}))(K_H^{1,ID} - Q_{AB}^{1,ID})]$

with: $Q_{AB}^{1,ID} = \frac{1}{11}(a - 4 + 8K_H^{1,ID})$; $\quad Q_{Ab}^{2,ID} = \frac{1}{11}(5a + 2 - 4K_H^{1,ID})$

- Z3: $3K_{AB}^{DD} \le a \le \frac{1}{5}(11K_{AB}^{2,ID} - 2 + 4K_H^{1,ID})$:

$\bar{G}_1 = [(a - 2K_{AB}^{DD})K_{AB}^{DD} - (a - Q_{AB}^{1,ID} - Q_{AB}^{2,ID})Q_{AB}^{1,ID}]$ and

$\bar{G}_2 = 2[\frac{1}{4}(1-c)(1+c) - (1 - (K_H^{1,ID} - Q_{AB}^{1,ID}))(K_H^{1,ID} - Q_{AB}^{1,ID})]$

with: $Q_{AB}^{1,ID} = \frac{1}{11}(5a + 2 - 4K_H^{1,ID})$; $\quad Q_{Ab}^{2,ID} = \frac{1}{11}(5a + 2 - 4K_H^{1,ID})$

• Z4: $\frac{1}{5}(11K_{AB}^{2,ID} - 2 + 4K_H^{1,ID}) \leq a \leq 1$:

$$\bar{G}_1 = [(a - 2K_{AB}^{DD})K_{AB}^{DD} - (a - Q_{AB}^{1,ID} - K_{AB}^{2,ID})Q_{AB}^{1,ID}] \text{ and}$$

$$\bar{G}_2 = 2[\tfrac{1}{4}(1-c)(1+c) - (1 - (K_H^{1,ID} - Q_{AB}^{1,ID}))(K_H^{1,ID} - Q_{AB}^{1,ID})]$$

with: $Q_{AB}^{2,DI} = \frac{1}{6}(a - 2 + 4K_H^{1,ID} - K_H^{2,ID})$

3) Comparison DD/II

Similarly, firm 1 (or firm 2) *ex post* profit difference between the network structures *DD* and *II* can be decomposed into three terms, $\Delta^*(c,a) = G_1^* + G_2^* + G_3^*$. Note that $G_3^* = 2cK_{AB}^{DD} + c(2K_H^{DD}) - c(2K_H^{II})$ is positive or negative depending upon *c* (for *c*=0.01, it is negative). G_1^* and G_2^* take several forms depending upon *c*. We present the expressions for *c*=0.01 below.

• Z1: $a \leq 3(K_H^{II} - \frac{1}{2})$: $G_1^* = (\frac{1}{9}a^2 - \frac{1}{9}a^2) = 0$ and $G_2^* = 2(\frac{1}{4}(1-c)(1+c) - \frac{1}{4})$

• Z2: $3(K_H^{II} - \frac{1}{2}) < a \leq 3K_{AB}^{DD}$:

$$G_1^* = (\tfrac{1}{9}a^2 - (a - 2Q_{Ab}^{II})Q_{AB}^{II}) \text{ and } G_2^* = 2(\tfrac{1}{4}(1-c)(1+c) - (1 - (K_H^{II} - Q_{AB}^{II}))((K_H^{II} - Q_{AB}^{II}))$$

with: $Q_{AB}^{II} = \frac{1}{35}(5a - 10 + 20K_H^{II})$

• Z3: $3K_{AB}^{DD} < a$:

$$G_1^* = ((a - 2K_{AB}^{DD})K_{AB}^{DD} - (a - 2Q_{Ab}^{II})Q_{AB}^{II}) \text{ and } G_2^* = 2(\tfrac{1}{4}(1-c)(1+c) - (1 - (K_H^{II} - Q_{AB}^{II}))((K_H^{II} - Q_{AB}^{II}))$$

with: $Q_{AB}^{II} = \frac{1}{35}(5a - 10 + 20K_H^{II})$

Appendix 5. Example of asymmetric network structure

Suppose the following capacity cost structure:

Firm 1: $c^1 = 0.01$

Firm 2: $c^2 = 0.04$

In stage 2, the following capacity levels represent an equilibrium for each network configuration (note that these capacities are obtained by maximizing the expected profit reproduced in appendices 1,2 and 3) :

Network configuration	Equilibrium capacities
DD	$K_{AB}^{1,DD} = 0.306178$; $K_{AB}^{2,DD} = 0.187643$
	$K_{H}^{1,DD} = 0.495$; $K_{H}^{2,DD} = 0.48$
II	$K_{H}^{1,II} = 0.802534$; $K_{H}^{2,II} = 0.657282$
DI	$K_{AB}^{1,DI} = 0.324181$; $K_{H}^{1,DI} = 0.495$; $K_{H}^{2,DI} = 0.653945$
ID	$K_{H}^{1,ID} = 0.790846$; $K_{AB}^{2,ID} = 0.173359$; $K_{H}^{2,ID} = 0.48$

In stage 1, the expected profit in each network configuration is:

Firm 2 / Firm 1	Direct service on AB	Indirect service on AB
Direct service on AB	0.530137; 0.478623	0.530339; 0.478764
Indirect service on AB	0.531887; 0.477412	0.530212; 0.47893

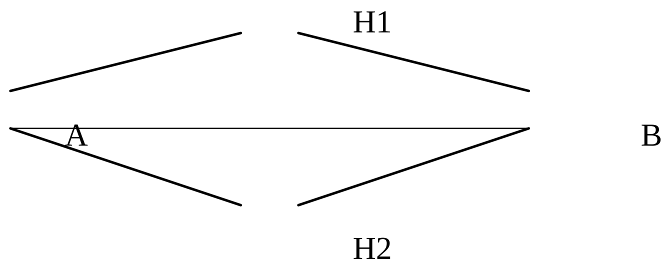


Figure 1. Network structure.

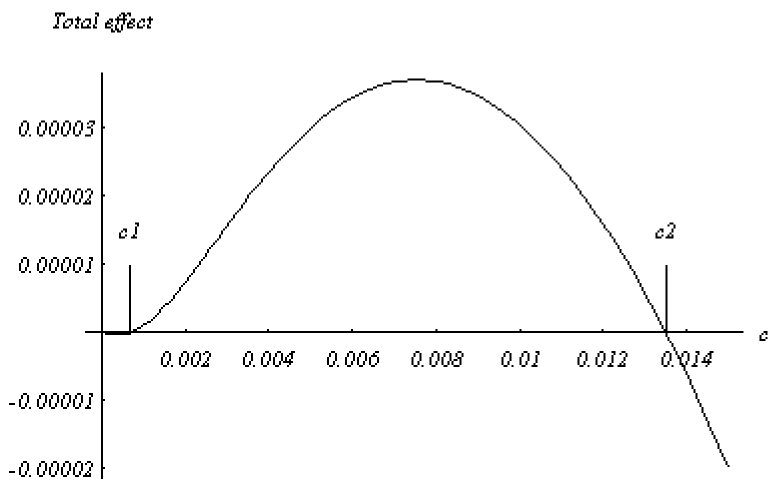


Figure 2a. Firm 1's expected profit in *DI* minus its expected profit in *II* as a function of *c*.

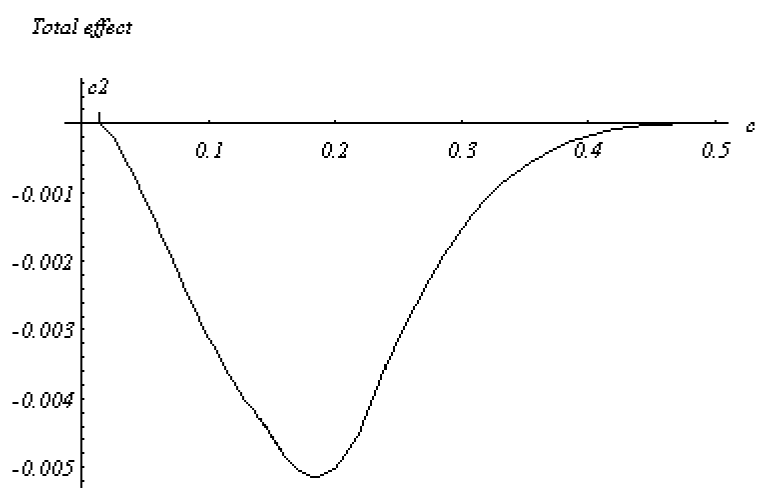
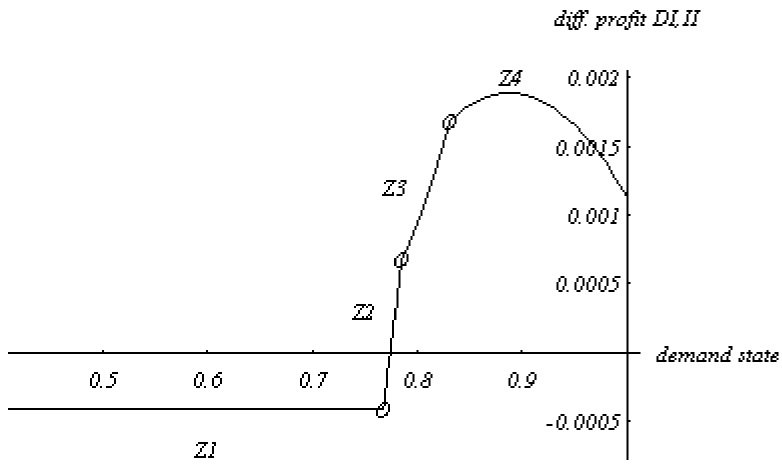
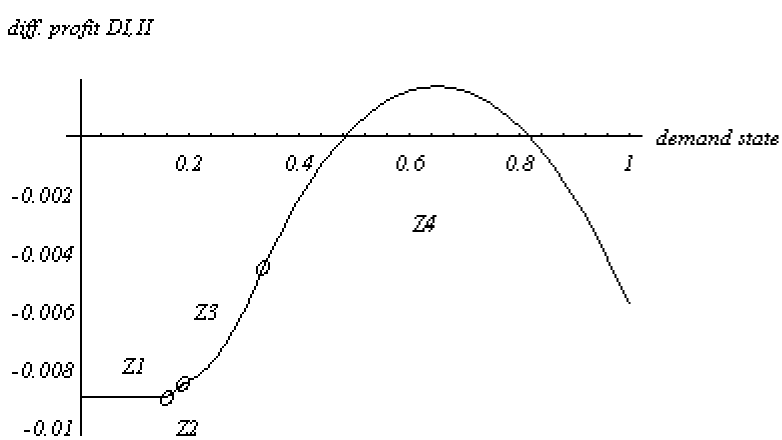


Figure 2b. Firm 1's expected profit in *DI* minus its expected profit in *II* as a function of *c*.



Network structure	Optimal capacities c=0.01	Effect of capacity constraints
DI	$K_{AB}^{1,DI} = 0.284406, K_{H}^{1,DI}\, 0.495$ $K_{H}^{2,DI} = 0.756045$	· *no effect if* $a \le 0.768135$ · *only firm 2 constrained if* $0.768135 \le a \le 0.8305$ · *both firms constrained if* $a \ge 0.8305$
II	$K_{H}^{II} = 0.761698$	· *no effect if* $a \le 0.7850$ · *both firms constrained if* $a \ge 0.7850$

Figure 3a. Firm 1's *ex post* profit difference between the structures *DI* and *II* as a function of a for *c*=0.01.



Network structure	Optimal capacities (c=0.1)	Effect of capacity constraints
DI	$K_{AB}^{1,DI} = 0.132208, K_{H}^{1,DI} = 0.45$ $K_{H}^{2,DI} = 0.552825$	· *no effect if* $a \le 0.158475$ · *only firm 2 constrained if* $0.158475 \le a \le 0.3331176$ · *both firms constrained if* $a \ge 0.3331176$
II	$K_{H}^{II} = 0.562359$	· *no effect if* $a \le 0.187077$ · *both firms constrained if* $a \ge 0.187077$

Figure 3b. Firm 1's *ex post* profit difference between the structures *DI* and *II* as a function of a for *c*=0.1.

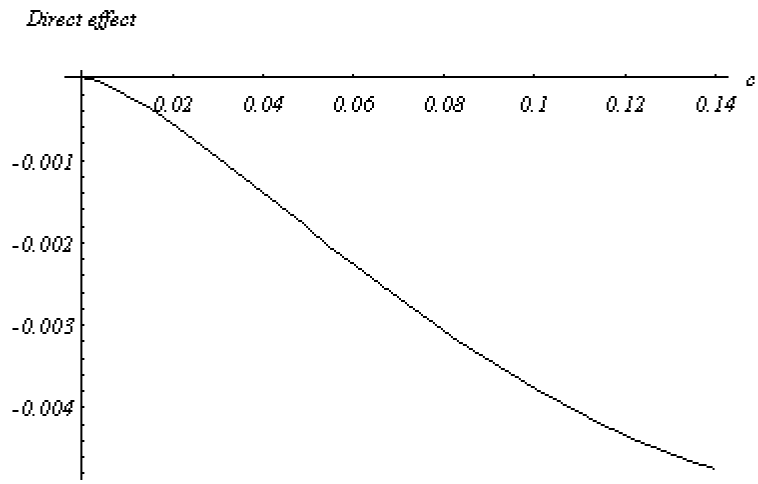


Figure 4. Direct effect associated with serving market AB directly when rival serves it indirectly as function of *c*.

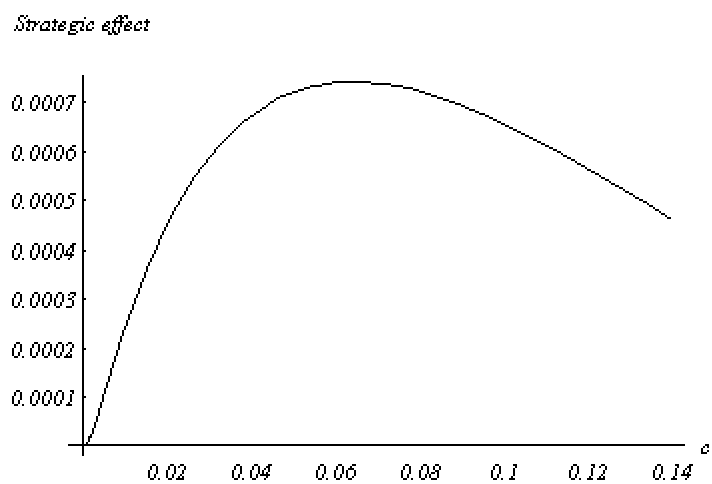


Figure 5. Strategic effect associated with serving market AB directly when rival serves it indirectly as function of *c*.

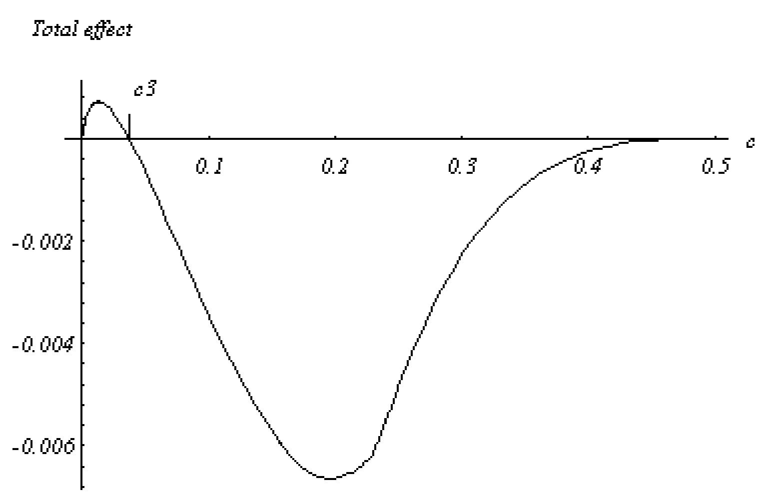
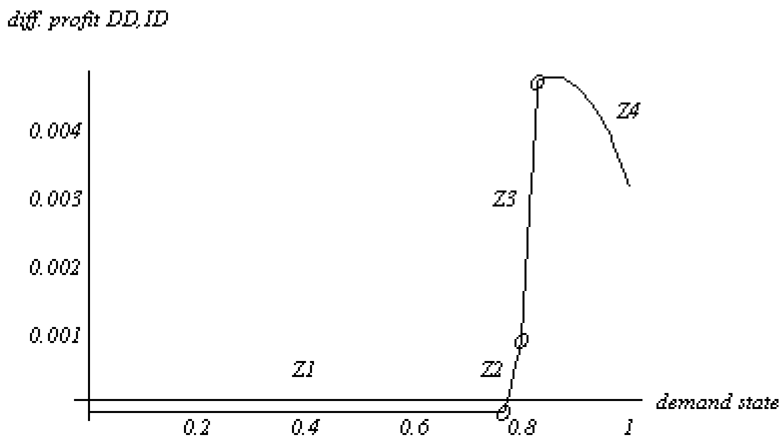
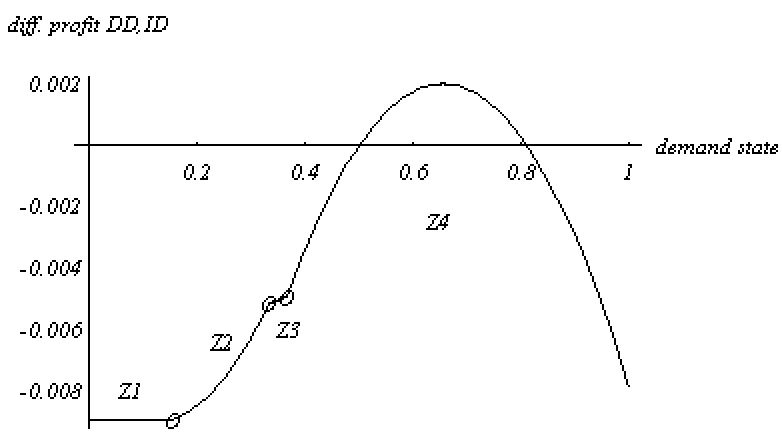


Figure 6. Firm1's expected profit in *DD* minus its expected profit in *ID* as a function of c.



Network structure	Optimal capacities c=0.01	Effect of capacity constraints
ID	$K_H^{1,ID} = 0.756045$ $K_{AB}^{2,ID} = 0.284406, K_H^{2,ID} = 0.495$	· *no effect if* $a \le 0.768135$ · *only firm 2 constrained if* $0.768135 \le a \le 0.8305$ · *both firms constrained if* $a \ge 0.8305$
DD	$K_{AB}^{DD} = 0.26667$ $K_H^{DD} = 0.495$	· *no effect if* $a \le 0.80001$ · *both firms constrained if* $a \ge 0.80001$

Figure 7a. Firm 1's *ex post* profit difference between the structures *DD* and *ID* as a function of a for *c*=0.01.



Network structure	Optimal capacities (c=0.1)	Effect of capacity constraints
DI	$K_H^{1,ID} = 0.552825$ $K_{AB}^{2,ID} = 0.132208, K_H^{2,ID} = 0.45$	· *no effect if* $a \le 0.158475$ · *only firm 2 constrained if* $0.158475 \le a \le 0.3331176$ · *both firms constrained if* $a \ge 0.3331176$
DD	$K_{AB}^{DD} = 0.122515;\ K_H^{DD} = 0.45$	· *no effect if* $a \le 0.367545$ · *both firms constrained if* $a \ge 0.367545$

Figure 7b. Firm 1's *ex post* profit difference between the structures *DD* and *ID* as a function of a for *c*=0.1.

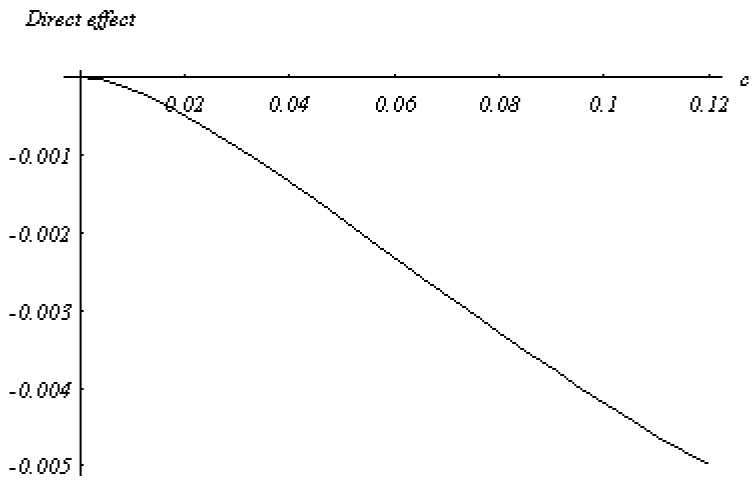


Figure 8. Direct effect associated with serving market AB directly when the rival serves it directly as function of *c*.

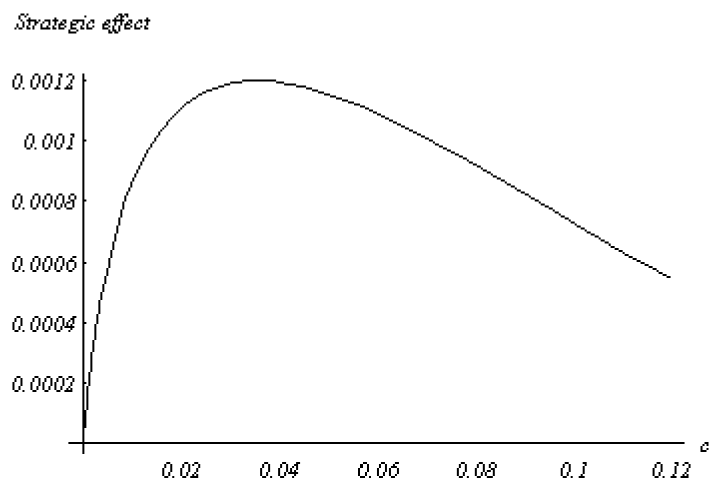


Figure 9. Strategic effect associated with serving market AB directly when rival serves it directly as function of *c*.

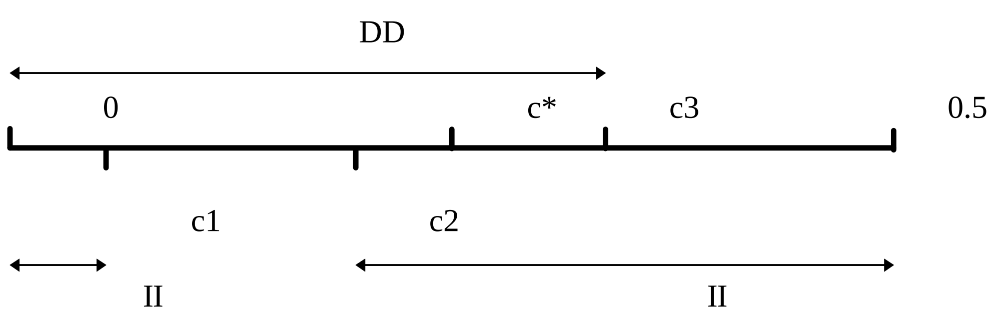


Figure 10. Equilibrium network configurations as a function of *c*.

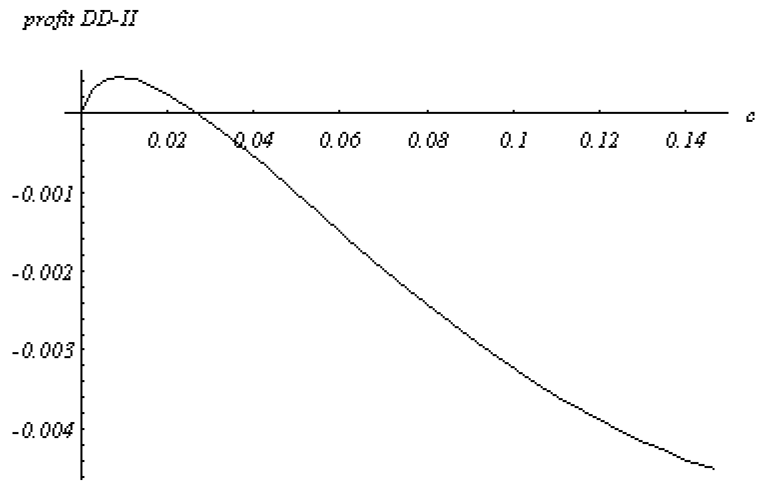
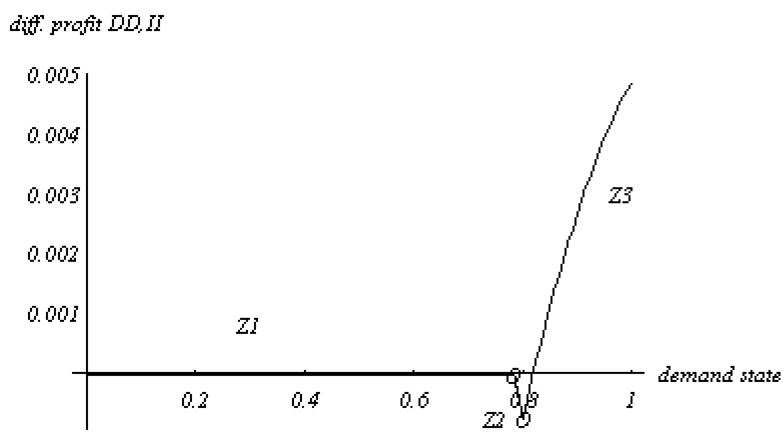


Figure 11. A firm's expected profit in *DD* minus its expected profit in *II* as a function of c.



Network structure	*Optimal capacities (c=0.01)*	*Effect of capacity constraints*
DD	$K_H^{DD} = 0.495$; $K_{AB}^{DD} = 0.266$	· *no effect if* $a \leq 0.7998$
II	$K_H^{DD} = 0.7616$	· *no effect if* $a \leq 0.7850$

Figure 12. A firm's difference in *ex post* profit between the structures *DD* and *II* as a function of a for c=0.01.

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