Linear Inequality Measures and the Redistribution of Income

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Abstract

A class of inequality measures that is a natural companion to the popular Lorenz curve is the class of measures that are linear in incomes. These measures, which include the Gini and S-Gini coefficients, can be interpreted as ethical means of relative deprivation feelings. Their change through the tax and benefit system can be decomposed simply as a sum of progressivity indices for individual taxes and benefits, minus an index of horizontal inequity measured by the extent of reranking in the population. These progressivity and horizontal inequity indices can also be interpreted as ethical means of perceptions of fiscal harshness and ill-performance. We furthermore derive the asymptotic sampling distribution of these classes of indices of redistribution, progressivity, and horizontal inequity, which enables their use with micro-data on a population. We illustrate the theoretical and statistical results through an application on the distribution and redistribution of income in Canada in 1981 and in 1990.

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I. Introduction

A class of inequality measures that is a natural companion to the popular Lorenz curve is the class of measures that are linear in incomes. These inequality indices, which include the Gini and the S-Gini coefficients, are weighted areas of the gap between the line of perfect income equality and the Lorenz curve for a distribution of income. They are thus straightforward to interpret graphically, and they differ simply in the set of weights which each measure uses to weight the distance between the cumulative population share and the cumulative income shares at various points of the income distribution. This class flexibility in the use of weights at different points of the Lorenz curve also generates an infinity of possible ethical attitudes to the measurement of inequality and social welfare.

We will see how this simple structure gives a subclass of linear inequality measures an intuitive and interesting interpretation as ethical means of relative deprivation feelings across a population. This interpretation as aggregates of individual welfare (or ill-fare) demarcates these inequality measures from other measures that cannot be so clearly understood. Because of their linearity in income, the class of measures can easily be decomposed into the contribution of various income components (including taxes, benefits, evaded taxes, unearned income, income from the underground economy, etc.). This allows the definition of associated classes of indices of tax progressivity, income redistribution operated by the tax and benefit system, and reranking, reranking being one of the manifestations of horizontal inequity. These indices of progressivity, income redistribution and horizontal inequity can also be interpreted, respectively, as ethical means of individual perceptions of fiscal harshness on the rich, as ethical means of declines in individual feelings of relative deprivation, and as ethical means of individual feelings of relative ill-performance in the allocation of taxes and benefits.

To apply these general classes of indices to the empirical measurement of inequality, progressivity, redistribution and horizontal inequity, we must derive the sampling distribution of their estimators when only sample (and not the entire population) micro-data are available to the empirical analyst. This is done using recently derived results on the joint sampling distribution of possibly dependent Lorenz and concentration curves. The methodology takes into account the classical sampling variability of estimates of conditional means as well as the sampling variability of quantile estimates, which are both needed for the computation of Lorenz and concentration curves. The sampling distribution of the estimators of the classes of linear inequality measures and of the associated classes of indices of progressivity, income redistribution and horizontal inequity makes it possible to infer statistically, for instance, whether inequality or income redistribution has increased over time, or whether tax progressivity or horizontal inequity is greater in one country or for one tax system than for another, or which component of the tax and benefit system appears to be the most progressive. As a corollary, these results characterise the sampling distribution of a number of well-known particular indices of redistribution, progressivity, and horizontal inequity[1].

We apply these theoretical and statistical results to the distribution and redistribution of income in Canada in 1981 and in 1990. A major finding is that inequality and feelings of relative deprivation increased significantly between 1981 and 1990 for the distribution of gross incomes, but decreased between these two years for the distribution of net incomes for most values of the ethical parameter. The 1990 tax and benefit system is furthermore unambiguously more redistributive than the 1981 system. Old age transfers account for more than a third of the total redistribution exerted by the tax and benefit system. With social assistance and unemployment benefits, they also account for most of the fall in the feelings of relative deprivation between 1981 and 1990.

II. <u>Linear Inequality Indices</u>

Denote gross incomes by *X*, taxes (which can be negative) by *T(X)*, and net incomes by *N(X)=X-T(X)*. The Lorenz curve $L_X(p)$ for a distribution $F_X(x)$ of gross incomes is defined as:

$$L_X(p) = \frac{1}{\mu_X} \int_0^y x \, dF_X(x) \ , \text{ with } p = F_X(y) \tag{1}$$

where μ_X is mean gross income. $L_X(p)$ thus shows the percentage of total income held by the p•100% poorest individuals. The Lorenz curve $L_N(p)$ for net incomes can be expressed analogously using the distribution of net income $F_N(x)$. A general class *I* of linear inequality measures [Mehran (1976)] can then be defined as:

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[1] See, e.g., Musgrave and Thin (1948), Kakwani (1977), Reynolds and Smolensky (1977), Atkinson (1979), Plotnick (1981), and Pfähler (1987).

$$I = \int_0^1 \left| p - L(p) \right| w(p) \; dp \qquad (2)$$

with w(p) being a positive (or at least non-negative) weight that can vary along the distribution of incomes. Let $X(p) \equiv F^{-1}(p)$ be the inverse distribution function. We can then interpret the difference $p\text{-}L_X(p)$ as the difference between $(1\text{-}L_X(p))$ and $(1\text{-}p)$. $(1\text{-}L_X(p))$ indicates the proportion of total income which the richer than $X(p)$ hold in the distribution of income. $(1\text{-}p)$ indicates the share of the population which these richer individuals represent; it also measures the proportion of total income that they would have held if income had been distributed equally. $p\text{-}L_X(p)$ is thus the income share of the rich (richer than $X(p)$) in excess of their "more equitable" share in an equal distribution of income. I weights with $w(p)$ these excess shares at different points p of the income distribution.

A well-known special case of (2) is the Gini coefficient, for which $w(p) \equiv 2$. By integration by parts, we can also show that (2) is equivalent to

$$I = \int_0^1 \left[\frac{X(p)}{\mu} - 1 \right] W(p) \; dp \qquad (3)$$

where we may define $W(p)$ as

$$W(p) = \int_0^p w(q) \; dq \qquad (4)$$

Equation (3) shows clearly why I is linear in incomes X(p); in fact, I is an "ethically" weighted sum of the distance between $X(p)/\mu$ and unity, which is the ratio of incomes to mean income under a perfectly equal distribution of incomes. A transfer a of incomes from a rich to a poor will have an impact on I proportional to the integral of w(p) over the area of the distribution between the two individuals. For the Gini coefficient, this impact will thus be proportional to twice the difference between the rank of the rich and that of the poor in the income distribution.

It is well-known that if $L_X(p) \leq L_Y(p)$ for all $p \in \,]0,1[$, with the inequality holding strictly for at least some p in the interval, then inequality in the distribution of income X is necessarily greater than the inequality in the distribution of Y for all strictly

S-convex inequality measures [Dasgupta et al. (1973)]. These measures include, among others, the linear class of (2) as well as the Atkinson (1970) inequality index and the class of generalised entropy measures [e.g., Bourguignon (1979) and Cowell (1980)]. Using the statistical inference results of Beach and Davidson (1983), a number of recent studies have attempted to determine whether inequality dominance could be inferred for that general class of S-convex measures. Such general inequality dominance cannot always be established, however, sometimes because the two Lorenz curves cross, sometimes because the curves are simply statistically not distinguishable for some values of *p*, and therefore cannot be ordered unambiguously.

We will focus here on one particular parametric class of linear inequality measures. As demonstrated above, linear inequality measures have a straightforward graphical interpretation, and, as we will see below, some of them can be interpreted and explained nicely and intuitively, a property not exhibited by somewhat more abstruse statistical measures of income dispersion. When Lorenz curves cross or are not statistically distinguishable, the use of these linear measures helps understand how and why the distributions differ and whether, for a plausible range of parameter values, inequality under one distribution is nevertheless unambiguously greater or lesser than under another distribution.

Let

$$k^{*}(p) = \frac{v\,(v+1)\,|q-p|^{(v-1)}}{q^{(v+1)} + (v+q)\,(1-q)^{v}} \tag{5}$$

with v>0 and 0≤q≤1. We can then define the following two-parameter class of linear inequality measures, *$G^{*}(v,q)$*:

$$G_X^{*}(v,q) = \int_0^1 \left[p - L_X(p)\right] k^{*}(p)\, dp \tag{6}$$

The functional form $k^{*}(p)$ is a two-parameter specification of w(p); the parameters v and q can be set to give different weights to different regions of the income distribution in which inequalities can be more or less ethically important. We can check that the Gini coefficient is a special case of $k^{*}(p)$ obtained when v=1. For q=1, $k^{*}(p)$ yields the Single-Parameter Gini (or S-Gini) of Donaldson and Weymark (1980) and Yitzhaki (1983), to which we return later. $G^*(v,q)$ has a minimum of 0 when the distribution of income is perfectly equal and has a maximum of 1 when the richest individual enjoys all of a society's income[2].

Given our interpretation of the distance $p\text{-}L_X(p)$, the excess incomes of the richer than X(p) (for different points *p* of the income distribution) are attributed ethical weights $k^*(p)$ that vary with v and q. These weights are symmetric around the value of q; for q=0.5, for instance, the weights are centered around the middle (median) of the income distribution. For v between 0 and 1, the ethical weights are large (low) at values of p close to (far from) q. For q=0.5, and v between 0 and 1, our ethical focus would therefore mostly be on the excess income shares of the more than median class. When, at the limit, v tends to 0, $k^*(p)$ is concentrated exclusively at q. For v=1, the ethical weights are uniform across the distribution. When v exceeds 1, the weights are large (small) when p is far from (close to) q. For v tending to infinity, all of the ethical weight in averaging the excess incomes $p\text{-}L_X(p)$ is put at one of the two extremes (p=0 or p=1) of the distribution, depending on the value of q. Table 1 summarises the impact of v and q on the valuation of $k^*(p)$ and $G^*(v,q)$.

A special case of $G^*(v,q)$ which has received some attention in the past occurs when q=1; this yields the S-Gini of Donaldson and Weymark (1980) and Yitzhaki (1983), as mentioned above. Define $G(v)\equiv G^*(v,1)$ and $k(p)=v(v+1)(1-p)^{(v-1)}$. As indicated in Table 1, for v<1, more ethical weight is then put on the higher portion of the distribution, for v=1, equal weight applies to all p, and for v>1 greater weight is applied to areas with low values of p. We can also interpret k(p) as (v+1) times the density of a minimum income X(p) in a sample of v incomes randomly and independently drawn from F_X [see, for instance, Lambert (1993), p.129]. To see this, note that $K(p)=(v+1)\bullet[1-(1-p)^v]$ is (v+1) times the probability that the minimum income in a sample of v incomes fall below X(p), and thus that k(p)=dK(p)/dp is (v+1) times the density of that minimum income. For integer values of v, we may therefore interpret the weights k(p) as the frequency with which an individual with income X(p) finds himself the poorest in randomly and independently sampled groups of v individuals. The greater the value of v, the greater the density of the poorer relative to the richer among those who find themselves the poorest in the random groups of v individuals.

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[2] $G^*(v,q)$ can nevertheless exceed 1 if some incomes are negative.

With this in mind, we can show how G(v) is an ethically weighted average of relative deprivation feelings[3]. Let the relative deprivation feeling $\delta(p_i,p_j)$ of an individual *i* (with rank p_i and income X_i) who compares himself to an individual *j* (of rank p_j with income X_j) be given by

$$\delta(p_i,p_j) = \begin{cases} X_j - X_i \ , \ \text{if } p_j > p_i \\ 0 \ , \ \text{otherwise} \end{cases} \tag{7}$$

This posits that individual i does not feel relatively deprived when he compares himself to a poorer individual, and that the intensity of his feeling of relative deprivation when comparing himself to a richer individual is equal to the gap between the income levels of the two individuals. The relative deprivation feeling of individual i, averaged over all individuals *j*, then equals $d(p_i)$:

$$d(p_i) = \int_0^1 \delta(p_i,p) \, dp \tag{8}$$

$d(p_i)/\mu_X$ is the difference between the total income share held by the richer than i and the total income share that would accrue to these richer individuals if they held individual i's income instead. We can then show that G(v) is the average feeling $d(p_i)$ of relative deprivation, weighted by the ethical weight k(p) and normalised by (v+1) times average income:

$$G(v) = \frac{1}{(v+1) \cdot \mu_X} \int_0^1 d(p) \, k(p) \, dp \tag{9}$$

G(v) is thus the expected relative deprivation feeling of the most deprived individual within a group of v individuals randomly and independently drawn from F_X. For v=1, all feelings of relative deprivation across the population are weighted equally. The greater the number v of individuals among whom we seek a most deprived individual, the greater the expected relative deprivation feeling of this most deprived individual.

[3] For the full demonstration, see Duclos (1995).

We will need later to focus our attention on a plausible range of values for v for our applied discussion of income redistribution, tax progressivity and horizontal inequity. To this end, we note that to the index of relative inequality G(v) corresponds a class of homothetic social evaluation functions[4] whose *equally distributed equivalent income function* E(v) is given by:

$$E(v) = \mu \, (1 - G(v)) = (v + 1) \int_0^1 X(p) \, (1 - p)^v \, dp \tag{10}$$

where the last equality is obtained by integration by parts[5]. Using the leaking bucket experiment of Okun (1975), it is then possible to assess which range of v values is ethically sensible. Suppose that, with no effect on individual ranking, a tax of \$a is enforced onto an individual with rank p_j in the income distribution, so that a transfer of \$a(1-α) can be made to a poorer individual of rank p_i in the distribution, where α is the size of the bucket leak in making that transfer ($0 \leq \alpha \leq 1$). This leak reflects the feature that tax and benefit programmes often generate *efficiency* losses which are nevertheless tolerated because these programmes can enhance the *equity* of the income distribution by making it less unequal. With α=0, making the transfer will decrease inequality and leave mean income unchanged, so that E(v) and social welfare would then necessarily increase. With α=1, the tax of \$a makes individual j worse off without making the poor better off, so E(v) falls. Agreeing on an intermediate α value which is socially tolerable, at the limit, will also determine a value for the ethical parameter v. A tax of \$a is indeed just socially acceptable if dE(v)/da=0, that is, when

$$a(1 - \alpha) \, (1 - p_i)^v - a \, (1 - p_j)^v = 0 \tag{11}$$

This leads to the following relationship between the ethical parameter v and a socially tolerable limit value of α:

[4] For the link between relative inequality indices and social evaluation functions, see Blackorby and Donaldson (1978).

[5] In a manner analogous to the discussion of the weights k(p) on the relative deprivation feelings of (7), we may interpret E(v) as the expected income of the poorest individual in a group of (v+1) individuals randomly drawn from F_X.

$$1 - \alpha = \left[\frac{1 - p_j}{1 - p_i}\right]^v \qquad (12)$$

Table 2 displays the socially tolerable limit values of α for values of v ranging from 0.0 to 5.0 and for different tax paying p_j and transfer receiving p_i. For p_i=0.2 and p_j=0.8, for instance, specifying v=1 amounts to a social tolerance of efficiency leaks of up to $0.75 for each dollar of tax on individual j. The other two columns indicate that, for a given value of v, we become less tolerant of efficiency losses and "bucket leaks" if the p_i of the transfer-receiving and the p_j of the tax-paying individuals are closer to one another. For v=3, the limit to the tolerable efficiency loss rises everywhere to above 85%, and to 98.4% in the first column; these are rather large limit values by most ethical standards. Allowing for larger values of v would imply an ethical acceptance of redistributive transfers which would be almost completely wasteful. In our applications later, we will thus limit our discussion to the results for ethical values of v ranging from 0 to 3.

III. Progressivity, Horizontal Inequity and Redistribution

Define the concentration curve for taxes T(X) as $C_T(p)$:

$$C_T(p) = \frac{1}{\mu_T} \int_0^y T(x)\, dF_X(x) \;, \text{ with } p = F_X(y) \qquad (13)$$

with μ_T being mean taxes. $C_T(p)$ thus shows the proportion of total taxes that is paid by the p•100% poorest individuals in the population. Note that these individuals are ordered according to the size of their gross incomes X. The concentration curve $C_N(p)$ for net incomes is defined similarly to (13) by replacing T(x) with N(x). Unlike the Lorenz curve $L_N(p)$, $C_N(p)$ ranks individuals by the size of their gross incomes; we then have that $C_N(p) \geq L_N(p)$ for all p between 0 and 1, with strict inequality somewhere if and only if the tax system reranks individuals. The greater the extent of reranking, the farther is $C_N(p)$ from $L_N(p)$. The difference $C_N(p)$-$L_N(p)$ can thus be used to assess the horizontal inequity exerted by the tax T(X) since a horizontally equitable tax, besides treating equals equally, should not change the rank of individuals in the income distribution [see, for instance, Feldstein (1976)].

We can measure the progressivity of the tax T(X) along either of two views: the Tax Redistribution (TR) view, and the Income Redistribution (IR) view. A non-negative tax is said to be TR progressive if $C_T(p) \leq L_X(p)$ for all $p \in [0,1]$, with the strict inequality holding somewhere[6], and IR progressive if $C_N(p) \geq L_X(p)$ for all $p \in [0,1]$, with the strict inequality holding for at least some p. As for the class of linear inequality measures *I* defined in (2), it is possible to define classes of aggregate TR and IR progressivity indices [see Kakwani (1986) and Pfähler (1987)] by weighting the differences $L_X(p)$-$C_T(p)$ and $C_N(p)$-$L_X(p)$ with the weights w(p).

Here, we focus, again, on the class k(p) of ethical weights. We define [Kakwani (1984)] the following classes $\pi(v)$ and $\rho(v)$ of TR and IR progressivity measures:

$$\pi(v) = \int_0^1 \left[L_X(p) - C_T(p) \right] k(p) \, dp \tag{14}$$

$$\rho(v) = \int_0^1 \left[C_N(p) - L_X(p) \right] k(p) \, dp \tag{15}$$

As for G(v), we can interpret $\pi(v)$ and $\rho(v)$ as ethically weighted averages of the differences between 1-$C_T(p)$ and 1-$L_X(p)$, and between 1-$L_X(p)$ and 1-$C_N(p)$, respectively. 1-$L_X(p)$ is the total gross income share of those individuals richer than X(p); it should also be their share of total taxes (1-$C_T(p)$) and total net income (1-$C_N(p)$) if the tax was proportional to income and thus distributionally neutral. $\pi(v)$ and $\rho(v)$ weight the departures from tax proportionality $L_X(p)$-$C_T(p)$ and from distributional neutrality $C_N(p)$-$L_X(p)$ with the density k(p) of the poorest individual in a comparison group of v random individuals.

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[6] A transfer (a negative tax) is TR progressive if $C_T(p) \geq L_X(p)$ for all $p \in [0,1]$, with the strict inequality holding for at least some p in that interval.

By integration by parts, we can also show that

$$\pi(v) = \frac{1}{\mu_T} \int_0^1 \left[tX(p) - T(X(p)) \right] (v+1)\,(1-p)^v \, dp \tag{16}$$

$$\rho(v) = \frac{1}{\mu_N} \int_0^1 \left[N(X(p)) - (1-t)\cdot X(p) \right] (v+1)\,(1-p)^v \, dp \tag{17}$$

$\pi(v)$ and $\rho(v)$ can thus also be interpreted as ethically weighted averages of differences between proportional taxes and actual taxes, and between net incomes under proportional taxation and actual net incomes, respectively. The weights are equal to the density of minimum incomes in a sample of (v+1) observations randomly and independently drawn from F_X, and they increase with p.

Analogously to the relative deprivation feeling of (7), now define individual i's perception of relative fiscal harshness on a richer individual j as:

$$\phi(p_i,p_j) = \begin{cases} T[X(p_j)] - T[X(p_i)] \ , \ \text{if } p_j > p_i \\ 0 \ , \ \text{otherwise} \end{cases} \tag{18}$$

Perceived fiscal harshness $\phi(p_i,p_j)$ is negative when a richer individual j pays less tax than i. Individual i's average perception of relative fiscal harshness on the richer in the population then equals $f(p_i)$:

$$f(p_i) = \int_0^1 \phi(p_i,p)\, dp \tag{19}$$

We can then demonstrate that the class $\pi(v)$ of indices of tax progressivity is the (mean-normalised) difference between the average perception of fiscal harshness and the average feeling of relative deprivation in the population:

$$\pi(v) = \frac{1}{v+1} \int_0^1 \left[\frac{f(p)}{\mu_T} - \frac{d(p)}{\mu_X} \right] k(p)\, dp \tag{20}$$

If the perceived fiscal harshness exceeds the feelings of relative deprivation, the tax is considered progressive [$\pi(v)>0$]; if the two are equal, the effect of the tax is deemed equivalent to that of a proportional tax; if fiscal harshness falls below the feelings of relative deprivation, the tax is judged regressive. For v=1, k(p) gives equal weight, across the population, to all perceptions of fiscal harshness and to all feelings of relative deprivation. An analogous construction can be made for the class $\rho(v)$ of IR progressivity measures; this involves the difference between the average feeling of relative deprivation with and without a tax, when pre-tax income is used to rank units. A tax is then IR progressive if the tax succeeds in lowering relative deprivation feelings.

The linearity of the $\pi(v)$ and $\rho(v)$ measures makes it straightforward to decompose total TR and IR progressivity into the sum of $\pi(v)$ and $\rho(v)$ indices for individual taxes and benefits. Denote $t=\mu_T/\mu_X$ as the average global rate of gross income taxation, t_m, m=1,...,M, as the average rate[7] of gross income taxation of tax or benefit T_m, with $t=\Sigma^M_{m=1}t_m$, and $\pi_m(v)$ and $\rho_m(v)$ as the generalised Kakwani and Reynolds-Smolensky indices for tax or benefit m. Also define a transformed Reynolds-Smolensky index $\rho^*_m(v)$ as $\rho^*_m(v)=(1-t_m)/(1-t)\bullet\rho_m(v)$. We can verify that:

$$\pi(v) = \sum_{m=1}^{M} \frac{t_m}{t}\, \pi_m(v) \tag{21}$$

$$\rho(v) = \sum_{m=1}^{M} \left(\frac{1-t_m}{1-t}\right) \rho_m(v) \equiv \sum_{m=1}^{M} \rho^*_m(v) \tag{22}$$

and

$$\rho^*_m(v) = \frac{t_m}{1-t}\, \pi_m(v) \tag{23}$$

[7] Both t and t_m can be negative.

The class of horizontal inequity indices associated with $\pi(v)$ and $\rho(v)$ is $\eta(v)$:

$$\eta(v) = \int_0^1 \left| C_N(p) - L_N(p) \right| k(p)\, dp \tag{24}$$

The greater the extent of reranking by the tax and benefit system, the greater these indices of horizontal inequity. Duclos (1995) shows that $\eta(v)$ can be understood as an ethically weighted average of individual feelings of relative ill-performance in the allocation of taxes and benefits. Assume that any individual i can determine whether a random individual j has jumped up (or below) i's net income position, because of a particularly favourable (or unfavourable) tax treatment. Suppose, moreover, that the intensity of that feeling of relative ill-performance is measured by individual j's income after the tax allocation, and that it is positive for a jump above (N_j) -- and negative for a jump below ($-N_j$) -- i's net income position. Then $\eta(v)$ is the expected feeling of ill-performance of the poorest individual in a group of v randomly and independently drawn individuals. Again, for v=1, equal ethical weight is applied on all feelings of relative ill-performance.

We can finally show that the redistributive change in inequality caused by the tax and benefit system is accounted for by the sum of the progressivity and horizontal inequity indices:

$$\Delta(v) \equiv G_X(v) - G_N(v) \equiv \frac{t}{1-t}\pi(v) - \eta(v) \equiv \rho(v) - \eta(v) \tag{25}$$

As shown in equations (21) and (22), $\pi(v)$ and $\rho(v)$ can also themselves be decomposed into the sum of progressivity indices for separate taxes and benefits.

For v=1, these classes of redistribution, progressivity and horizontal inequity measures reduce to familiar and popular indices. $\Delta(1)=G_X(1)-G_N(1)$ measures the change in the Gini coefficient induced by the tax [the Musgrave and Thin (1948) index of tax progressivity], $\pi(1)$ is the Kakwani (1977) index of progressivity, $\rho(1)$ is the Reynolds-Smolensky (1977) index of vertical equity, and $\eta(1)$ is the Atkinson (1979) - Plotnick (1981) index of horizontal inequity.

IV. Statistical Inference

To infer from a sample of observations whether a tax is TR or IR progressive, redistributive, or horizontally equitable over the whole population, we need to establish the sampling distribution of the estimators of the various indices involved. The computation of the indices described above necessarily involves estimates and comparisons of Lorenz and concentration curves that are typically not distributed independently one from the other. Because the structure of the indices and of their estimators is statistically similar, we can establish their asymptotic sampling distribution within a rather general framework.

Consider two jointly distributed random variables Y and Z, with F being the marginal distribution of Z and G(p) being its inverse distribution function. Suppose that H independent drawings have been made from this joint distribution. Define an indicator function $I_{[0,y]}(Y)$ as follows:

$$I_{[0,y]}(Y) = \begin{cases} 1 \text{ if } Y \in [0,y] \\ 0 \text{ otherwise} \end{cases} \tag{26}$$

We are interested in conditional expectations of the form $\gamma_p \equiv E[Y | Z \leq G(p)]$ since $p\gamma_p/\gamma_1$ will be a concentration curve for taxes if the variable Y is taxes T(X) and Z is gross incomes X, $p\gamma_p/\gamma_1$ will be a concentration curve for net incomes if the variable Y is net incomes N(X) and Z is gross incomes X, and $p\gamma_p/\gamma_1$ will be a Lorenz curve for variable Y if $Y \equiv Z$.

A natural estimator for $p\gamma_p$ is

$$p\hat{\gamma}_p = \frac{1}{H} \sum_{i=1}^{H} Y_i \, I_{[0,\hat{G}(p)]}(Z_i) \tag{27}$$

where $\hat{G}(p)$ is the sample estimate of the p-quantile of Z. Consider as well a second set of jointly distributed random variables V and W, with F^* and $G^*(p)$ being the marginal distribution and the inverse distribution functions of W. Assume also that a sample of H independent observations on V and W has also been drawn. A natural estimator for $p\lambda_p \equiv p \bullet E[V | W \leq G^*(p)]$ can be defined in a manner analogous to (27):

$$p\hat{\lambda}_p = \frac{1}{H} \sum_{i=1}^{H} V_i \; I_{[0,\hat{G}^*(p)]}(W_i) \tag{28}$$

Denote the following vector of estimators as $\hat{\Theta}$:

$$\hat{\Theta} = \left[p_1\hat{\gamma}_{p_1},\ \ldots,\ p_{K-1}\hat{\gamma}_{p_{K-1}},\ \hat{\gamma}_1,\ p_1\hat{\lambda}_{p_1},\ \ldots,\ p_{K-1}\hat{\lambda}_{p_{K-1}},\ \hat{\lambda}_1\right]' \tag{29}$$

where K is the number of quantiles at which the estimators $\hat{\gamma}_p$ and $\hat{\lambda}_p$ are to be computed, with γ_1 and λ_1 being the mean of Y and V, respectively. If K=100, for instance, the p's will denote centiles. Theorem 1 of Davidson and Duclos (1995) then shows that, under suitable regularity conditions, $\hat{\Theta}$ is consistent and asymptotically normal; it also has an asymptotic variance-covariance matrix Ω whose typical element $\mathrm{cov}(p\hat{\gamma}_p\ ,\ p'\hat{\lambda}_{p'})$ is (for arbitrary p and p', both between 0 and 1):

$$\begin{aligned}
\lim_{H\to\infty} H\ \mathrm{cov}(p\hat{\gamma}_p,\ p'\hat{\lambda}_{p'}) &= E\left[YV\ I_{[0,G(p)]}(Z)\ I_{[0,G^*(p')]}(W)\right] \\
&- E\left[Y \mid Z=G(p)\right]\ E\left[V\ I_{[0,G(p)]}(Z)\ I_{[0,G^*(p')]}(W)\right] \\
&- E\left[V \mid W=G^*(p')\right]\ E\left[Y\ I_{[0,G(p)]}(Z)\ I_{[0,G^*(p')]}(W)\right] \\
&+ E\left[Y \mid Z=G(p)\right]\ E\left[V \mid W=G^*(p')\right]\ E\left[I_{[0,G(p)]}(Z)\ I_{[0,G^*(p')]}(W)\right] \\
&- pp'\left[\left(\gamma_p - E\left[Y \mid Z=G(p)\right]\right)\left(\lambda_{p'} - E\left[V \mid W=G^*(p')\right]\right)\right]
\end{aligned} \tag{30}$$

Everything in (30) can be estimated consistently in a distribution-free manner, that is, without specifying an *a priori* distributional form for the distribution of V, W, Y or Z. Kernel estimation can, in particular, be used to estimate under weak regularity conditions the conditional expectation of the form $E[Y|Z=G(p)]$ or $E[V|W=G^*(p')]$.

For the $\Delta(v)$, $\pi(v)$, $\rho(v)$, and $\eta(v)$ classes of measures, we are interested in the differences $\Gamma_p=p\gamma_p/\gamma_1-p\lambda_p/\lambda_1$, $0<p<1$, with $\hat{\Gamma}_p$ being its obvious estimator:

$$\hat{\Gamma}_p = \frac{p\hat{\gamma}_p}{\hat{\gamma}_1} - \frac{p\hat{\lambda}_p}{\hat{\lambda}_1} \tag{31}$$

Let J be the Kx2K Jacobian of the K-vector $\Gamma=[\Gamma_{1/K},....,\Gamma_1]$ with respect to the 2K vector Θ. A standard statistical result [see Rao (1973), pp.388-9] indicates that $\hat{\Gamma}_p$ is then asymptotically normally distributed with mean Γ_p and covariance matrix $J\Omega J'$. Finally, the general indices Ξ_p defined as

$$\Xi(v) = \frac{1}{K} \sum_{i=1}^{K} \Gamma_{\left(\frac{i}{K}\right)} w\left(\frac{i}{K}\right) \tag{32}$$

are a discrete approximation to the form of $\Delta(v)$, $\pi(v)$, $\rho(v)$, and $\eta(v)$ when Γ represents (L_N-L_X), (L_X-C_T), (C_N-L_X) and (C_N-L_N), respectively, and when w(p) takes the particular functional form $k(p)=v(v+1)(1-p)^{v-1}$. For $\Xi(v)$, we can use the estimator

$$\hat{\Xi}(v) = \frac{1}{K} \sum_{i=1}^{K} \hat{\Gamma}_{\left(\frac{i}{K}\right)} w\left(\frac{i}{K}\right) \tag{33}$$

Defining a K vector ξ as $\xi=[w(1/K), ..., w((K-1)/K), w(1)]/K$, we can then state that the estimator $\hat{\Xi}(v)$ of the general index $\Xi(v)$ is asymptotically normally distributed with mean $\Xi(v)$ and covariance matrix $\xi J\Omega J'\xi'$. This result establishes, *inter alia*, the sampling distribution of the estimators of the class of measures $\Delta(v)$, $\pi(v)$, $\rho(v)$, and $\eta(v)$.

V. <u>Income Distribution and Redistribution in Canada</u>

The distribution of income has been subjected to important disturbances in the 1980's in many countries around the world. Canada was no exception, having witnessed a severe recession between 1981 and 1983, followed by a significant recovery with relatively high growth rates until the end of 1988, and with the beginning of another recession thereafter. To this were combined important labour market, demographic and technological changes. The last decade was also the decade of major tax reforms; in Canada, taxation was particularly altered by the 1987 revision of personal income taxation, which decreased the number of tax brackets, trimmed the top marginal tax rates, replaced a number of tax allowances by tax credits, broadened the tax base, and aimed, generally, to improve the perceived "fairness" of the tax system. The social security system (including the unemployment insurance, public pension, and social assistance schemes) also evolved significantly with changes in public policy and in the socio-demographic environment.

To illustrate the application of the above conceptual and statistical results, we thus use the Canadian Surveys of Consumer Finances for 1981 and 1990. These surveys contain, respectively, 38,000 and 45,000 observations on the distribution of pre-tax and pre-benefit family income, on the amount of personal taxes paid, and on various cash transfers received from the provincial and federal governments. To adjust income as well as tax and benefit data for heterogeneity in the size and the composition of families, we use the OECD equivalence scale; all monetary variables are thus in an "equivalised" form. For convenience, we have removed those families who reported negative gross or net incomes. The definition of the monetary variables is as follows:

Gross income (pre-tax and pre-benefit): Includes wages and salaries, self-employment income, private pensions, and total investment income;

TAX: Total federal and provincial income tax;

FAAL: Federal and Québec family and youth allowances;

CHILD: Child Tax Credit;

OLD: Old Age Security Pensions and Guaranteed Income Supplement;

PEN: Canada/Québec Pension Plan Benefits;

UNEMP: Unemployment Insurance Benefits;

SOCASS: Social Assistance Benefits and provincial income supplements;

OTHER: Various tax credits and grants to individuals, veterans' pensions, pensions to widows, workers' compensation, etc..

Table 3 shows the Kakwani and Reynolds-Smolensky indices for various components of the 1981 and 1990 tax and benefit systems [$\pi(1)$ and $\rho_m^*(1)$]. These are obtained when equal ethical weight is granted to feelings of relative deprivation and fiscal harshness across the population, as shown above. To compare the Kakwani indices across negative (transfers) and positive taxes, we must consider the negative of their values for negative taxes. The highest Kakwani index is then found for SOCial ASSistance benefits, followed by OLD age pensions and CHILD tax credit. As can be checked, these comparisons are statistically significant. The Kakwani index value for OTHER benefits and PENsions cannot be distinguished, but they are significantly greater, statistically, than the index values for UNEMPloyment benefits, FAmily ALowances and income TAXation.

The second panel of Table 3 presents the evolution of the average tax rate (as a percentage of gross income) for each group of taxes and benefits between 1981 and 1990. The most significant changes occur for income TAXation, whose average rate increases from 16.7% to 22.1%, and for PENsions (1.1% to 2.1%), SOCial ASSistance (1.0% to 1.4%), and UNEMPloyment benefits (1.9% to 2.7%). The only benefit to witness a decrease in the average benefit is FAmily ALlowances (1.0% to 0.8%). The variation of these rates reflects both deliberate changes in tax and benefit policy and the changing structure of the society (e.g., growing numbers of unemployment and welfare recipients, ageing of the population).

As equation (23) indicates, the Reynolds-Smolensky index [$\rho^*_m(v)$] of IR progressivity is a simple product of the Kakwani index of TR progressivity and of the average rate of taxation as a proportion of *net* income. Indeed, because IR progressivity takes into account the importance of the average rate of taxation in the redistribution of income, it is a better indicator of the impact of taxes and benefits in reducing inequality than TR progressivity. The last panel of Table 3 reveals that, in 1981, OLD age benefits have the highest $\rho^*_m(1)$ index value, followed by income TAXes, SOCial ASSistance, UNEMPloyment benefits, and public PENsions. These rankings are all statistically significant. OTHER benefits and FAmily ALlowances have the same index value, and the least progressive is CHILD tax credits. Those with high Reynolds-Smolensky indices have either high Kakwani indices (such as OLD age pensions and SOCial ASSistance) or large taxation rates (income TAXes). The $\rho^*_m(1)$ ranking is the same for 1990, with the exception of public PENsions which become significantly greater than SOCial ASSistance, and FAmily ALlowances which fall at the bottom of the list, even below CHILD tax credits.

Table 4 exhibits, for 1990, the dependence of the IR progressivity ranking [$\rho^*_m(v)$] upon the valuation of the ethical parameter v. Recall that a rise in v increases the ethical weights granted to the relative deprivation of the poorer in the population; it also renders indices of redistribution and of IR progressivity more dependent upon the change in the deprivation of those poorer individuals. Table 4 confirms this by indicating that the generalised Reynolds-Smolensky indices increase with v for all groups of taxes and benefits; the average fall in relative deprivation feelings thus increases as we focus more and more on the feelings of the poorer. This rise is particularly fast for the benefits (such as OLD age pensions and SOCial ASSistance, for instance) that are most directed to the poorest individuals in the population. Note also that the indices are quite precisely estimated, with very small standard errors relative to the size of the estimates.

Figure 1 makes it easier to see how the IR progressivity ranking of the various groups of taxes and benefits varies with values of v, and Table 5 summarises the statistically significant results. The most sensitive group is OLD age pensions, which quickly (once v is greater than 0.4) becomes statistically more IR progressive than income TAXes and all other groups of transfers. Income TAXes start from being the most progressive to being significantly less progressive than OLD age pensions, SOCial ASSistance benefits, and public PENsions (as soon as v lies above 1.8). Hence, for ethical parameters that grant sufficient weight to the relative deprivation of the poorer, income taxation, notwithstanding its relatively large rate of taxation, is significantly less progressive and redistributive than some fairly well targeted groups of benefits. FAmily ALlowances are generally the least IR progressive of all groups, followed by CHILD tax credits and OTHER benefits. These last results hold for a wide range of v and are generally statistically very significant.

Analogous rankings are shown in Table 6 for 1981. The most significant differences with the rankings of 1990 are that income TAXes are in 1981 everywhere more progressive than public PENsions and UNEMPloyment benefits, that the progressivity of FAmily ALlowances relative to other groups has increased slightly (especially compared to CHILD tax credit), that the top ranking of OLD age pensions is even stronger in 1981, that public PENsions are then much less progressive, especially when compared to UNEMPloyment benefits and SOCial ASSistance, and that SOCial ASSistance, when compared to income TAXation, is less progressive in 1981, but more progressive relative to public PENsions and UNEMPloyment insurance. These changes are consistent with the ageing of the population between 1981 and 1990, with increased support from the state to the retired population, and with increased unemployment and social assistance dependence during a decade of important cyclical and structural turbulences.

Figure 2 depicts the contribution of each group of taxes and benefits to the total 1990 Reynolds-Smolensky index, according to the decomposition of equation (22).

As a percentage of total IR progressivity, IR progressivity for income TAXation falls rapidly with increases in the ethical parameter v. The percentage contribution of most other groups is relatively constant, except for SOCial ASSistance and OLD age pensions whose relative importance in the redistributive process rises steadily. For larger v, almost a third of the total IR progressivity is exerted by OLD age pensions.

The change in the generalised Reynolds-Smolensky indices between 1981 and 1990 is exhibited on Figure 3. All groups of taxes and benefits have shown a statistically significant increase in progressivity for all values of v, except for FAmily ALlowances, whose IR progressivity witnessed a statistically significant fall for all v greater than 0.4. Public PENsions witnessed the greatest and most statistically significant increase in progressivity for any value of v; for v=1, for instance, this additional progressivity alone implies an additional downward pressure on the Gini coefficient of more than 0.01. As v increases, the increase in IR progressivity also becomes most notable for SOCial ASSistance, OTHER benefits, UNEMPloyment benefits, and OLD age pensions.

Figure 4 indicates how the generalised Gini coefficients for the distribution of gross and net incomes evolved between 1981 and 1990, for various values of v. These generalised Gini coefficients represent ethically-weighted average feelings of relative deprivation in the population, and we note that they naturally increase with v. The statistical estimates with their standard errors are shown in Table 7. Gross income inequality witnessed a statistically significant increase between 1981 and 1990, for all values of v; for v=2, for instance, gross income inequality increased by 0.025, from 0.384 to 0.409, implying an increase of 7% in the average feeling of relative deprivation. Interestingly, however, net income inequality *fell* significantly between 1981 and 1990, for all values of v equal to or greater than 1.

Figure 4 and Table 7 also portray the values of the indices of total redistribution $\Delta(v)$ for the two years. As equations (22) and (25) show, total redistribution is a function of the sum of IR progressivity for all groups of taxes and transfers, minus the index of horizontal inequity. In the light of the previous results on the change in inequality between 1981 and 1990 and on the change in the progressivity indices between these two years, it is therefore not surprising to note that the overall redistributive impact of the tax and benefit system increased significantly between 1981 and 1990, regardless of the value of v. The redistributive fall in the average relative deprivation feeling increases as our ethical focus on the poor rises; relative to 1981, the fall in 1990 is also larger as v increases. For v=1, only 77% of the relative deprivation feeling in the 1981 gross income distribution remains after the income redistribution operated by the tax and benefit system; for 1990, the figure is 71%. These percentage falls do not vary much across the different values of v.

We also find in Table 7 the estimates of the $\eta(v)$ indices of reranking and horizontal inequity. These indices are ethically-weighted average feelings of relative ill-performance; for v=1, for instance, the equally-weighted average feeling of ill-performance equals 1.1% of mean income in 1981, and 1.7% in 1990. This rises, respectively, to 2.8% and 4.2% of per capita income when we estimate the expected ill-performance feeling of the most deprived individual in random groups of 3 individuals (v=3).

It is clear from Table 7 that horizontal inequity has increased between 1981 and 1990 for all v equal to or greater that 0.6. The increase becomes more and more statistically significant as v rises, indicating that it is mostly the feelings of relative ill-performance of the poorer which have been affected. Finally, as equation (25) indicates, horizontal inequity lessens the $\rho(v)$ progressivity impact on the reduction in generalised Gini coefficients. We can check that, as a proportion of $\rho(v)$, horizontal inequity increases with v, and increases as well between 1981 and 1990. This suggests that, in mitigating income redistribution, horizontal inequity is deemed more costly in 1990 than in 1981, and deemed more costly too if we increase our ethical focus on the poor.

VI. <u>Conclusion</u>

We describe a class of inequality measures that are simple to visualise and easy to interpret as average feelings of relative deprivation across a population. The ethical parameter v weights these feelings with the probability of being the most deprived individual in a random group of v individuals. The linearity in incomes of the class of inequality measures makes it simple to decompose the redistributive change in inequality into a combination of indices of progressivity for individual taxes and benefits minus an index of horizontal inequity. These indices can be intuitively understood as ethical means of perceptions of fiscal harshness on the rich, and as ethical means of ill-performance feelings in the reranking of units by the tax and benefit system. The classes of indices that we discuss include very popular particular indices of inequality, redistribution, progressivity, and horizontal inequity. We also derive the asymptotic sampling distribution of the estimates of these classes of indices, a derivation which enables us to test the value of these indices across time, societies, and tax and benefit regimes, including under different scenarios of tax reforms.

We illustrate these results using microdata on the distribution and redistribution of income in Canada in 1981 and in 1990. We find that generalised Gini indices and average feelings of relative deprivation have increased significantly between 1981 and 1990 for the distribution of gross incomes, but that they fell for the distribution of net incomes for the standard Gini and all other Ginis with a greater focus on the relative deprivation feelings of the poorer. Taxes and benefits reduce these average feelings of deprivation by about 23% in 1981 and 29% in 1990. Unemployment benefits and social assistance amount, together, to about 25% of total redistribution, and Old Age and pension benefits for at least 35%, whatever the value of the ethical parameter. The estimated contribution of income taxes to total redistribution decreases from more than 25% to about 10% as we become more sensitive to the relative welfare of the poor. All taxes and benefits are (except family allowances) significantly more redistributive in 1990 than they were in 1981, whatever the value of our ethical parameter; public pensions, social assistance benefits, unemployment benefits, and old age benefits witnessed the greatest increases in that decade. Ill-performance feelings and indices of reranking and horizontal inequity increased by about 50% between 1981 and 1990; they averaged between 1% and 2% of per capita income during these two years.

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Table 1

Impact of parameters v and q on inequality $G^*(v,q)$ and on the ethical weights $k^*(p)$

	v->0	0<v<1	v=1	v>1	v->∞
q=0	$G^*=1$	$k^*(p)$ larger for low p	G^*=Gini coefficient	$k^*(p)$ larger for large p	$G^*=0$
q	$G^*=1$ if $q<0.5$ $G^*=0$ if $q>0.5$	$k^*(p)$ larger for p close to q	G^*=Gini coefficient	$k^*(p)$ larger for p far from q	$G^*=0$ if $q<0.5$ $G^*=1$ if $q>0.5$
q=1	$G^*=0$	$k^*(p)$ larger for large p	G^*=Gini coefficient	$k^*(p)$ larger for low p	$G^*=1$

Table 2

Limit to the socially tolerable bucket leaks α

for different values of the ethical parameter v

p_i = rank of transfer receiving individual; p_j = rank of tax paying individual

v	p_i=0.2 and p_j=0.8	p_i=0.25 and p_j=0.75	p_i=0.33 and p_j=0.67
0.0	0	0	0
0.1	0.129	0.104	0.067
0.5	0.500	0.423	0.293
1.0	0.750	0.667	0.500
2.0	0.938	0.889	0.750
3.0	0.984	0.963	0.875
4.0	0.996	0.988	0.938
5.0	0.999	0.996	0.969

Table 3

Kakwani and Reynolds-Smolensky indices for the 1990 and 1981 tax and benefit systems (v=1)
(asymptotic standard errors in parentheses).

Kakwani indices for 1990 and 1981 tax and benefit systems [$\pi(1)$].								
	TAX	FAAL	CHILD	OLD	PEN	UNEMP	SOC-ASS	OTHER
1981	0.114 (0.001)	-0.507 (0.003)	-0.743 (0.004)	-0.900 (0.007)	-0.687 (0.011)	-0.576 (0.008)	-1.149 (0.007)	-0.687 (0.015)
1990	0.102 (0.001)	-0.505 (0.003)	-0.906 (0.003)	-0.890 (0.006)	-0.682 (0.007)	-0.564 (0.006)	-1.182 (0.005)	-0.774 (0.010)
Average tax rates for tax and benefit groups (as % of gross income).								
	TAX	FAAL	CHILD	OLD	PEN	UNEMP	SOC-ASS	OTHER
1981	16.7	-1.0	-0.5	-3.1	-1.1	-1.9	-1.0	-0.7
1990	22.1	-0.8	-0.5	-3.5	-2.1	-2.7	-1.4	-1.3
Reynolds-Smolensky indices for 1990 and 1981 tax and benefit systems [$\rho_m^*(1)$].								
	TAX	FAAL	CHILD	OLD	PEN	UNEMP	SOC-ASS	OTHER
1981	0.021 (0.000)	0.006 (0.000)	0.004 (0.000)	0.030 (0.001)	0.009 (0.000)	0.012 (0.000)	0.013 (0.000)	0.006 (0.000)
1990	0.025 (0.000)	0.005 (0.000)	0.006 (0.000)	0.035 (0.001)	0.020 (0.000)	0.017 (0.000)	0.018 (0.000)	0.011 (0.000)

Table 4

Generalised Reynolds-Smolensky indices [$\rho^*_m(v)$] for the 1990 tax and benefit system (asymptotic standard errors in parentheses)

v	TAX	FAAL	CHILD	OLD	PEN	UNEMP	SOCASS	OTHER
0.2	0.011 (0.000)	0.002 (0.000)	0.002 (0.000)	0.009 (0.000)	0.006 (0.000)	0.006 (0.000)	0.005 (0.000)	0.003 (0.000)
0.4	0.017 (0.000)	0.003 (0.000)	0.003 (0.000)	0.017 (0.000)	0.010 (0.000)	0.010 (0.000)	0.008 (0.000)	0.005 (0.000)
0.6	0.021 (0.000)	0.003 (0.000)	0.004 (0.000)	0.023 (0.000)	0.014 (0.000)	0.013 (0.000)	0.012 (0.000)	0.008 (0.000)
0.8	0.023 (0.000)	0.004 (0.000)	0.005 (0.000)	0.029 (0.001)	0.017 (0.000)	0.015 (0.000)	0.015 (0.000)	0.009 (0.000)
1.0	0.025 (0.000)	0.0005 (0.000)	0.006 (0.000)	0.035 (0.001)	0.020 (0.000)	0.017 (0.000)	0.018 (0.000)	0.011 (0.000)
1.2	0.026 (0.000)	0.005 (0.000)	0.006 (0.000)	0.039 (0.001)	0.023 (0.000)	0.019 (0.000)	0.021 (0.000)	0.012 (0.000)
1.4	0.027 (0.000)	0.005 (0.000)	0.007 (0.000)	0.044 (0.001)	0.025 (0.000)	0.020 (0.000)	0.023 (0.001)	0.014 (0.000)
1.6	0.027 (0.000)	0.0006 (0.000)	0.007 (0.000)	0.048 (0.001)	0.027 (0.000)	0.022 (0.000)	0.026 (0.001)	0.015 (0.000)
1.8	0.027 (0.000)	0.006 (0.000)	0.008 (0.000)	0.052 (0.001)	0.029 (0.001)	0.023 (0.000)	0.028 (0.001)	0.016 (0.000)
2.0	0.027 (0.000)	0.006 (0.000)	0.008 (0.000)	0.055 (0.001)	0.030 (0.001)	0.023 (0.000)	0.030 (0.001)	0.017 (0.000)
2.2	0.027 (0.000)	0.006 (0.000)	0.008 (0.000)	0.059 (0.001)	0.032 (0.001)	0.024 (0.001)	0.033 (0.001)	0.018 (0.000)
2.4	0.027 (0.000)	0.006 (0.000)	0.009 (0.000)	0.062 (0.001)	0.033 (0.001)	0.025 (0.001)	0.035 (0.001)	0.018 (0.001)
2.6	0.027 (0.000)	0.007 (0.000)	0.009 (0.000)	0.065 (0.001)	0.034 (0.001)	0.025 (0.001)	0.037 (0.001)	0.019 (0.001)
2.8	0.027 (0.000)	0.007 (0.000)	0.009 (0.000)	0.068 (0.001)	0.036 (0.001)	0.026 (0.001)	0.039 (0.001)	0.020 (0.001)
3.0	0.027 (0.000)	0.007 (0.000)	0.009 (0.000)	0.071 (0.001)	0.037 (0.001)	0.026 (0.001)	0.041 (0.001)	0.021 (0.001)

Table 5

Rankings of the generalised Reynolds-Smolensky indices $[\rho^*_m(v)]$ for the different tax and benefit groups for 1990

	TAX	FAAL	CHILD	OLD	PEN	UNEMP	SOCASS	OTHER
TAX		>	>	If v=0.2: > If v>0.4: <	If v<1.6: > If v>1.6: <	If v<2.8: >	If v<1.6: > If v>1.8: <	>
FAAL			If v>0.4: <	<	<	<	<	<
CHILD				<	<	<	<	<
OLD					>	>	>	>
PEN						If v>0.4: >	If v<1.6: > If v>2.2: <	>
UNEMP							If v<0.8: > If v>0.8: <	>
SOCASS								>

N.B.: If the sign > is used: {lign} > {column},
if the sign < is used: {lign} < {column}.

Table 6

Rankings of the generalised Reynolds-Smolensky indices $[\rho^{*}_{m}(v)]$ for the different tax and benefit groups for 1981

	TAX	FAAL	CHILD	OLD	PEN	UNEMP	SOCASS	OTHER
TAX		>	>	If v>0.2: <	>	>	If v<2.2: > If v>2.4: <	>
FAAL			>	<	If v>0.2: <	<	<	If v=1.6 or v>1.8: <
CHILD				<	<	<	<	<
OLD					>	>	>	>
PEN						<	<	If v>0.2: >
UNEMP							If v=0.2: > If v>0.6: <	>
SOCASS								>

N.B.: If the sign > is used: {lign} > {column},
if the sign < is used: {lign} < {column}.

Table 7

S-Gini coefficients and redistributive impact for the 1981 and 1990 tax and benefit systems (asymptotic standard errors in parentheses)

	1981				1990			
v	Gini for gross income	Gini for net income	Redist. impact	Horiz. inequity	Gini for gross income	Gini for net income	Redist. impact	Horiz. inequity
0.2	0.121 (0.001)	0.095 (0.001)	0.027 (0.000)	0.003 (0.000)	0.133 (0.002)	0.094 (0.002)	0.039 (0.000)	0.004 (0.000)
0.4	0.211 (0.001)	0.164 (0.001)	0.047 (0.000)	0.005 (0.000)	0.228 (0.003)	0.161 (0.002)	0.066 (0.001)	0.007 (0.000)
0.6	0.280 (0.001)	0.217 (0.001)	0.063 (0.001)	0.007 (0.001)	0.301 (0.004)	0.213 (0.003)	0.087 (0.001)	0.011 (0.001)
0.8	0.337 (0.001)	0.260 (0.001)	0.077 (0.001)	0.009 (0.001)	0.360 (0.004)	0.255 (0.003)	0.105 (0.001)	0.014 (0.001)
1.0	0.384 (0.002)	0.296 (0.001)	0.088 (0.001)	0.011 (0.001)	0.409 (0.004)	0.289 (0.003)	0.119 (0.001)	0.017 (0.001)
1.2	0.424 (0.002)	0.326 (0.001)	0.098 (0.001)	0.013 (0.001)	0.450 (0.004)	0.319 (0.003)	0.132 (0.001)	0.019 (0.001)
1.4	0.459 (0.002)	0.352 (0.001)	0.107 (0.001)	0.015 (0.001)	0.486 (0.004)	0.344 (0.003)	0.142 (0.001)	0.022 (0.001)
1.6	0.489 (0.002)	0.374 (0.001)	0.115 (0.001)	0.016 (0.001)	0.518 (0.003)	0.366 (0.003)	0.152 (0.001)	0.025 (0.001)
1.8	0.516 (0.002)	0.394 (0.001)	0.122 (0.001)	0.018 (0.001)	0.546 (0.003)	0.386 (0.003)	0.160 (0.001)	0.027 (0.001)
2.0	0.540 (0.002)	0.412 (0.002)	0.128 (0.001)	0.020 (0.001)	0.571 (0.003)	0.403 (0.003)	0.168 (0.001)	0.030 (0.001)
2.2	0.562 (0.002)	0.428 (0.002)	0.134 (0.001)	0.021 (0.001)	0.594 (0.003)	0.419 (0.003)	0.175 (0.001)	0.032 (0.001)
2.4	0.582 (0.002)	0.443 (0.002)	0.139 (0.001)	0.023 (0.001)	0.614 (0.003)	0.434 (0.002)	0.181 (0.001)	0.035 (0.001)
2.6	0.600 (0.002)	0.456 (0.002)	0.144 (0.001)	0.025 (0.002)	0.633 (0.003)	0.447 (0.002)	0.186 (0.001)	0.037 (0.002)
2.8	0.617 (0.002)	0.468 (0.002)	0.149 (0.001)	0.026 (0.002)	0.650 (0.003)	0.459 (0.002)	0.192 (0.001)	0.040 (0.002)
3.0	0.632 (0.002)	0.480 (0.002)	0.153 (0.001)	0.028 (0.002)	0.666 (0.003)	0.470 (0.002)	0.196 (0.001)	0.042 (0.002)

Figure 1
Generalised Reynolds-Smolensky indices
for the 1990 tax and benefit system

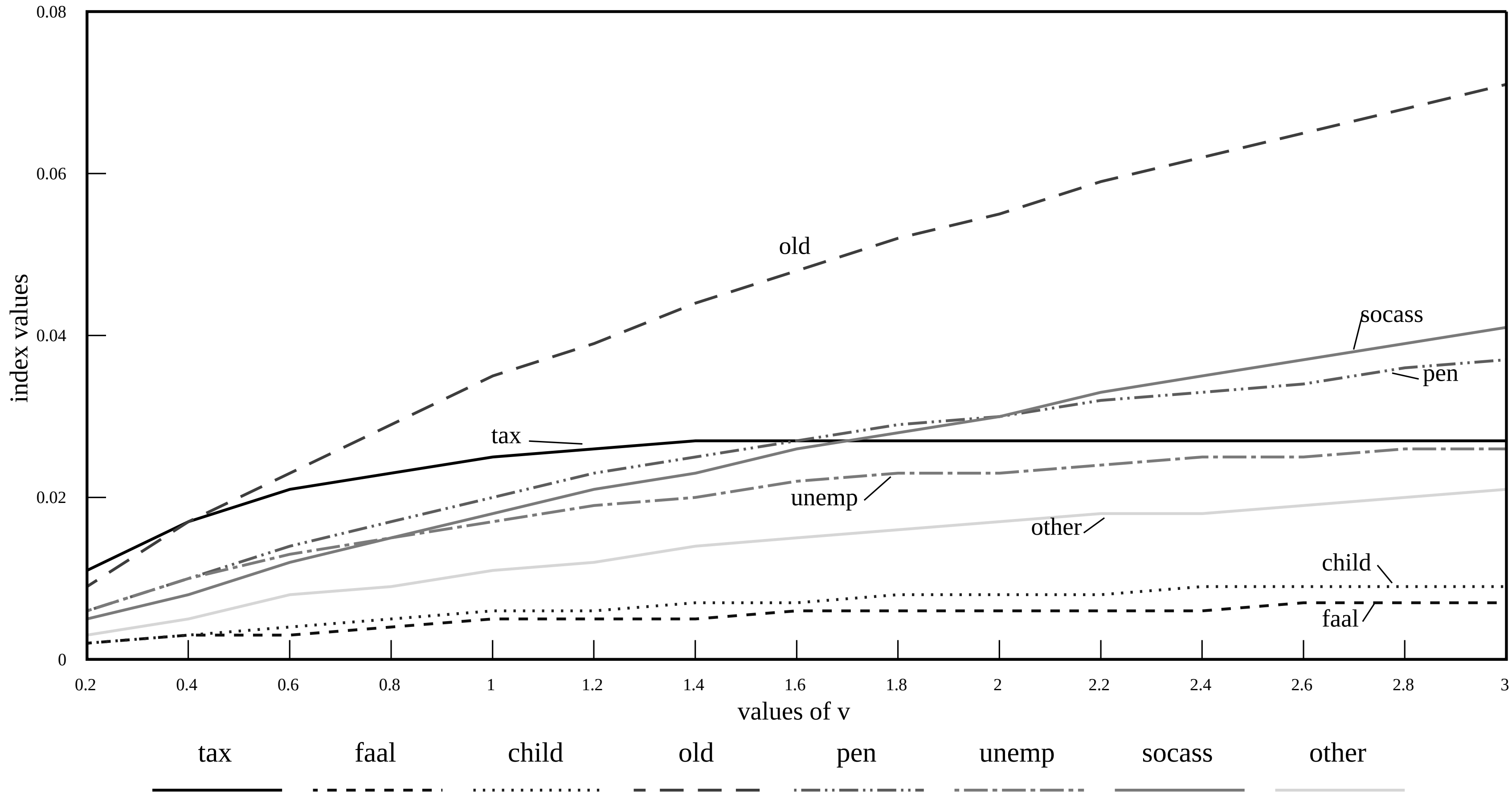

Figure 2
Importance of tax and benefit groups in total Reynolds-Smolensky indices for 1990

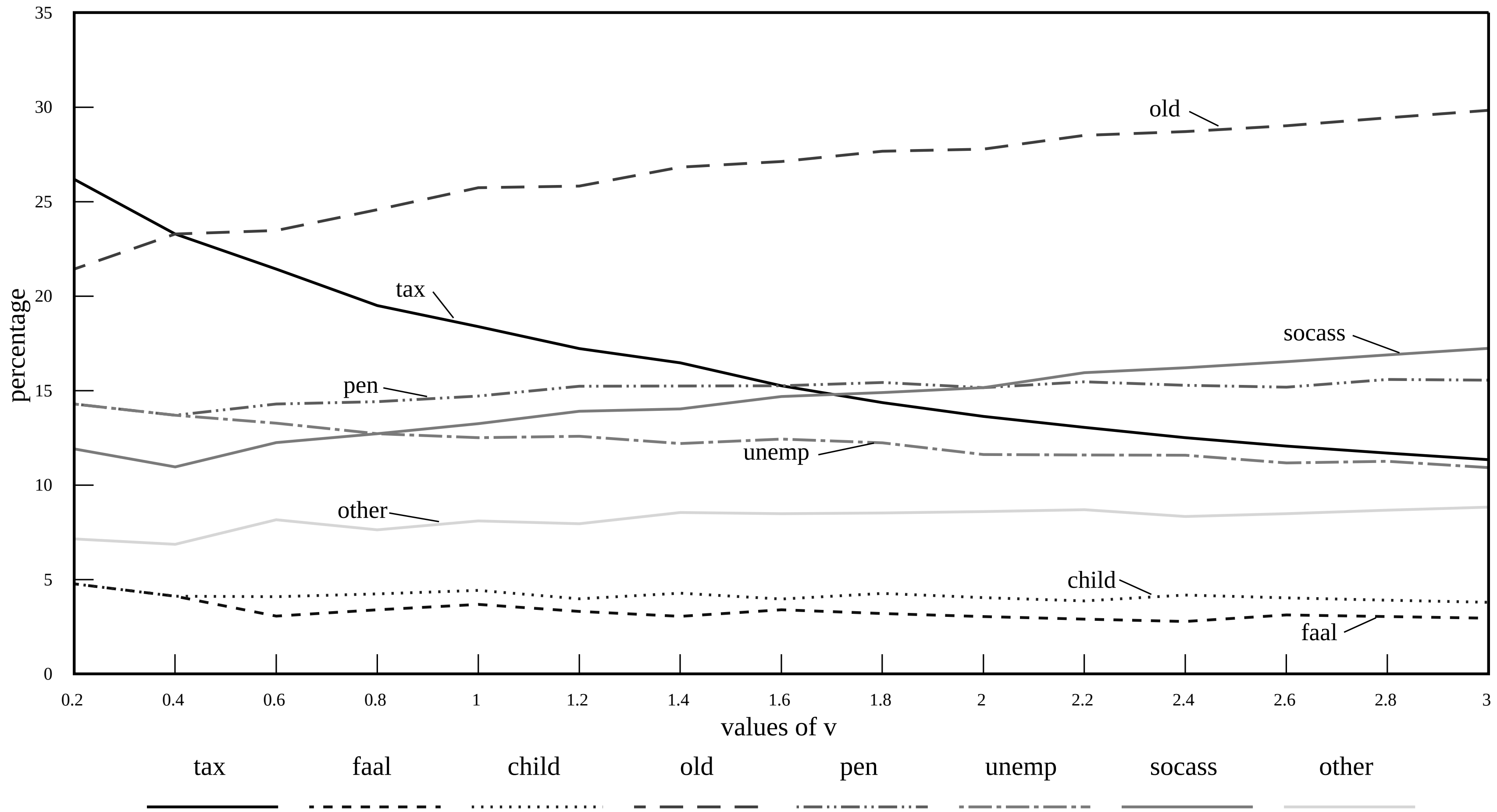

Figure 3
Differences in Reynolds-Smolensky indices of the tax and benefit systems of 1990 and 1981

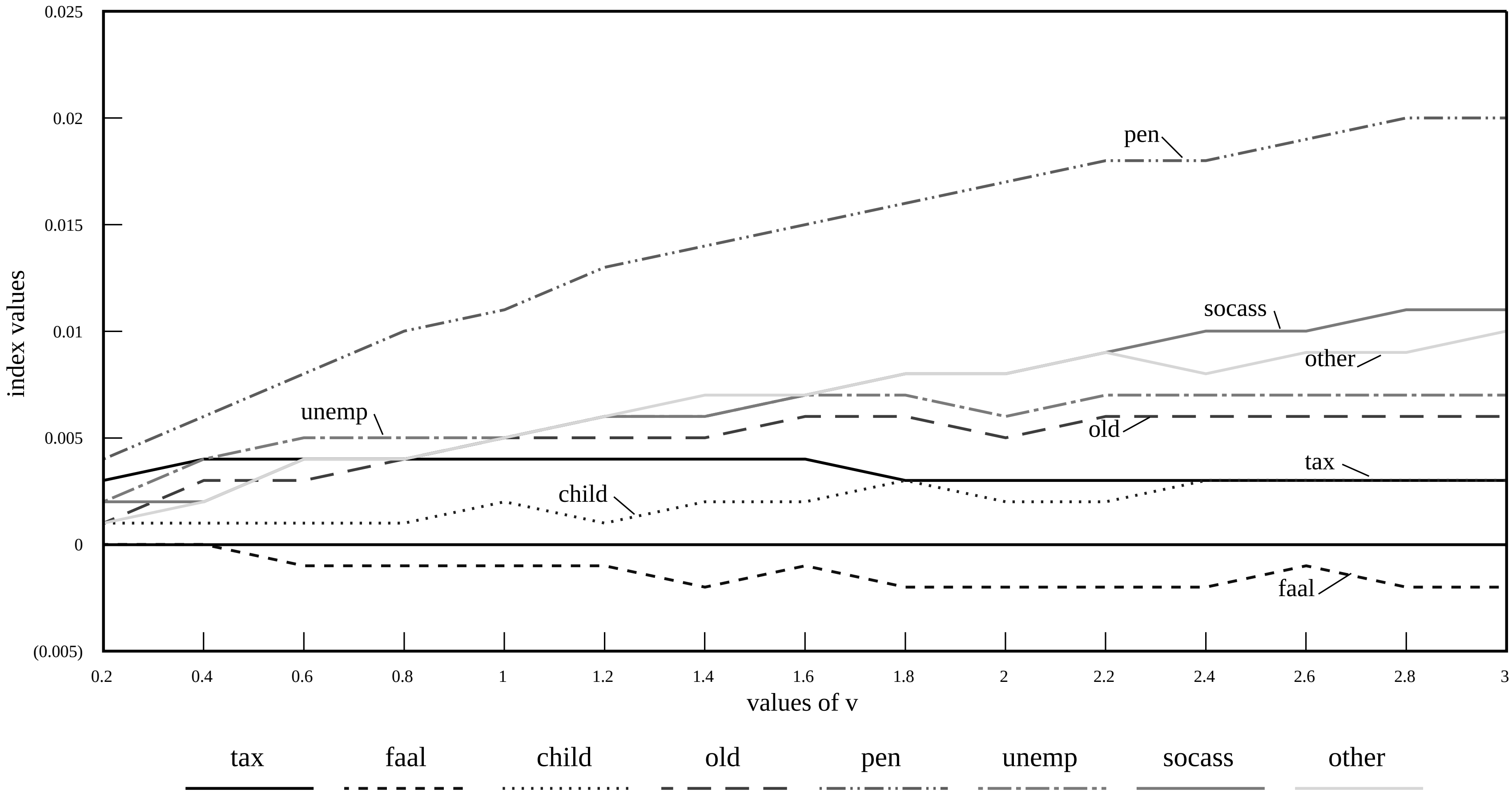

Figure 4
S-Gini coefficients and redistributive impact for the 1981 and 1990 tax and benefit systems