

Optimal Assignment of Liabilities

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Abstract

I characterize a generalization of the negligence rule to assign compensating damages in an accident involving multiple tortfeasors. These tortfeasors have the opportunity to undertake spending in prevention and the rule is designed to provide them with the best incentives to do so. I study the case where liability is constraint in the sense that the optimal amount of effort (not constrained by liability) cannot be implemented. The optimal multi-player rule is to apply the negligence rule to the most liable player (the “deep-pocket” or the “victim”, defined as the player who is the most responsive to monetary incentives under the strict liability rule) and the strict liability rule to everybody else.

Keywords: negligence rule, limited liability, multiple tortfeasors.

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1 Introduction

In this paper, I characterize the optimal multi-players liability rule when players have limited liability. Models of liability rules for two players go back to Brown (1973). Liability rules for more than two heterogeneous players can be found in Shavell (1987) and Emons and Sobel (1991). In a series of papers, Kornhauser and Revesz (1990, 1994) have pointed out that the characterization of liability rules under limited liability is quite different than when liability is not constrained. The limited solvency of tortfeasors is a real concern in liability cases involving life or environmental matters where the magnitude of damages can quickly skyrocket well beyond the actual capacity of paying of injurers. A liability rule designed to provide incentives to players to undertake due care must thus take into account these constraints.

Kornhauser and Revesz' analysis is restricted to two players situations. They consider the equilibria induced by *ad hoc* liability rules that have been proposed in law and economics or that are actually used in real life legal disputes. By contrast, the number of players here is arbitrary and I consider an *optimal* rule that provide the maximum possible incentives for the players to undertake effort given their liability constraints. Since this optimal rule is not considered by Kornhauser and Revesz, the results presented here are not directly comparable to theirs.

My analysis is related to that of Bergstrom, Blum and Varian (1986, 1992) who analyze the voluntary private provision to a public good by players with different wealth endowments. These authors show how the voluntary provision to a public good is affected by a redistribution of wealth when the redistribution modifies the subset of net contributors. In the present context, liability plays the role assigned to wealth in the public good problem. An optimal multi-player liability rule will affect the subset of players upon which liability will be put in order to incite an efficient amount of spending in care.

My main result states that when solvency is scarce (in the sense that the first-best allocation cannot be implemented) it is strictly efficient to make all players strictly liable except the “deep-pocket” or the “victim”. The deep-pocket is defined as the player who is the most responsive to monetary incentives under the strict liability rule. He is to be subjected to the negligence rule: he shall evade liability only if he has undertaken a required amount of spending in effort. Actually, the money gathered from the strictly liable players is used to provide additional monetary incentives to the deep-pocket. Under that regime, all spending in effort are undertaken by the

deep-pocket only. Hence, the second-best is achieved by concentrating all monetary incentives to affect the precautionary behavior of a single player.

This is a striking result. The typical analysis of liability rules deals with the problem of disciplining a single tortfeasor and leads to the conclusion that the negligence rule is strictly better than the strict liability rule when the tortfeasor has a low solvency. The analysis here shows that when a *group* of potential tortfeasors is involved, the negligence rule should be applied only on a single subsidized tortfeasor (who is expected to have undertaken all spending in effort); all the others should be subjected to the strict liability rule.

Who shall be that subsidized player is problematic: on one hand, since monetary incentives are scarce, we would like him to be highly responsive under the negligence rule. Players who are the more responsive under the strict liability rule are also those that are the most responsive under the negligence rule. Yet, these players are also the ones who have the highest solvency, hence the ones who would provide more monetary incentives under the strict liability regime. I show that the first effect will dominate favoring the deep-pocket.

The rest of this paper is structured as follows. In the next section, I present the formal model which is solved under dominant strategy with no liability constraint. In section 3, I recast the classical result of the dominance of the negligence rule over the strict liability rule in the single player case with a liability constraint. In section 4, I show that this result cannot be extended to the multi-players case with dominant strategy implementation. Finally, the optimal multi-players rule with Nash equilibrium implementation is characterized. A brief conclusion follows.

2 The Model

Consider a two periods game with N players indexed with i . In the first period, these players may spend resource in prevention to reduce the probability of an accident in the second period. In the second period, the state of nature is revealed to be either the accident state or the no accident state. In the accident state, I assume that the courts observe at no cost the identities of these players, their motives, the spending in prevention that they undertook, the underlying risk and the amount of actual damages. On the basis of this information, the courts then apply a liability rule that specifies

the different compensating damages to be paid or received by the players. The rule applied by the courts is known beforehand by the players and affect their ex ante choice of spendings in prevention. There is no involvement of the courts in the no accident state.

All players are assumed to have quasi-linear preferences. Player i 's utility in the no accident state is V_i while his utility in the accident state is U_i . The opportunity cost of an accident $V_i - U_i$ is noted C_i . No player wishes an accident to happen so that $C_i \geq 0$ for all i . Without loss of generality, assume that the players are indexed from 1 to N in increasing order with V_i .

In the first period, each player chooses a level of spending $X_i \geq 0$ in prevention to decrease the likelihood of the accident state¹. After an accident, player i is required to pay (or to receive) a compensating damage L_i that raises the opportunity cost of an accident to $C_i + L_i$.

The utility U_i is to be interpreted as a measure of the value of player i 's seizable assets so that $U_i \geq 0$. Hence, player i 's liability L_i is bounded above by U_i .

$$L_i \leq U_i. \quad (LC_i)$$

I refer to these constraints as the liability constraints.

Let $X = [X_1, \dots, X_N]$; X_{-i} is the vector X with the i^{th} component suppressed; $x = \sum X_i$ and $x_{-i} = x - X_i$. A similar notation will be use for the variables U_i , V_i , C_i and L_i to which are associated the vectors U , V , C and L . For instance, the liability constraint holds for all i if $L \leq U$.

A one-sided budget balance constraint will be imposed throughout so that the courts are not a net contributor:

$$l \geq 0.$$

When $l > 0$, the money collected is either used to restore the resource and/or to be distributed among the general public.²

The sum of spending x determines the probability $P(x)$ that an accident will occur. To account for decreasing marginal returns in accident prevention the function P is assumed to be strictly decreasing and strictly convex and to satisfy

¹The fact that spending in effort are bounded below imposes an implicit limit on the effect of moral hazard: a player who decides to be negligent say, by increasing production to increase his profits, is equivalent here to a player who would reduce his X_i . That can be done up to a limit $X_i = 0$. I thank Karine Gobert for pointing this to me.

²I will actually show that the optimal liability rule is such that $l = 0$.

$$\text{A1: } \lim_{x \rightarrow 0} P'(x) = -\infty,$$

$$\text{A2: } \lim_{x \rightarrow \infty} P'(x) = 0,$$

$$\text{A3: } \lim_{x \rightarrow \infty} P(x) = 0.$$

If player i has a liability L_i , his private expected opportunity cost is

$$P(x)(C_i + L_i) + X_i. \quad (PC_i)$$

I assume that the accident state entails an additional external cost a to society. Expected social cost is defined as value of the sum of expected private costs plus the expected external cost a and minus the expected sum of transfers

$$P(x)(a - l) + \sum PC_i = P(x)(a + c) + x. \quad (\text{SC})$$

Private cost (PC_i) is a strictly quasiconvex function of X_i . If $C_i + L_i \leq 0$, then it is monotonously increasing; otherwise, it is strictly convex; in both cases, quasiconvexity follows. The same result applies to (SC) which is a strictly convex function of x . It follows that the following function is well defined

$$\xi(z) = \underset{x \geq 0}{\operatorname{argmin}} P(x)z + x.$$

Because of assumption A1, notice that $\xi(z) = 0$ implies that $z \leq 0$. The value $\xi(V_i)$ will be simply noted ξ_i .

Conditionally on the occurrence of an accident, the courts are assumed to have (ex post) perfect information. Thus a *liability rule* is a function that maps a , the function P , and the vectors U , V and X into a vector of liabilities L . Ex ante, the social planner who designs the rule is not in relation with the players and is not considered to be a player in the game induced by the liability rule. This explains why the liability rule is silent in the no accident state. Think of a more general setting where there are infinitely many potential players so that the ex ante probability that any given set of players will end up in court is nearly zero while the probability that *some* set of players will end up in court is significant. Ex ante, some players may know that there is a significant probability that they will end up in court (namely $P(x)$) while the social planner is unaware of it. Any form of communication between these players and the social planner prior

the state of the world is realized is implicitly assumed away³. A liability rule will be written as a vector R of N functions of X alone (its dependency on the parameters is implicit) so that $L_i = R_i(X)$. The notation $R_i(X)$ will at times be substituted by $R_i(X_i, X_{-i})$.

The parameters a , V and U , the accident technology P and the liability rule R are common knowledge among the players when they choose (simultaneously and independently) their spending levels X . The purpose of this paper is to characterize the liability rule that provides the best incentives to minimize the social cost (SC). In that context, a liability rule R is a *mechanism* that structure a game then played by the potential tortfeasors. As usual, the choice of an *optimal* mechanism depends on the solution concept considered to give a good description of how the game is to be played. Among the solution concepts encountered in the literature, those of (*weakly*) *dominant strategy equilibrium* and of *Nash equilibrium* are the most common place.

A liability rule implements X in dominant strategies if, for each player i , playing X_i minimizes player i 's private cost *for all* X_{-i} . A liability rule implements X as a Nash equilibrium if, for each player i , playing X_i minimizes his private cost *given* X_{-i} . The set of allocations X that are implementable as a Nash equilibrium thus includes the allocations that are implementable in dominant strategies.

If the liability constraints are discarded, any allocation X^* may be implemented in dominant strategies as follows. Let f_i be a function such that

$$f_i(X_i) \geq P(X_i)C_i + X_i$$

and that reaches its minimum at X_i^* . Define the rule

$$R_i(X) \equiv \frac{f_i(X_i) - X_i}{P(x)} - C_i.$$

Notice that the one-sided budget balance condition is ensured since

$$R_i(X) \geq \left[\frac{P(X_i) - P(x)}{P(x)} \right] C_i \geq 0.$$

With such rule, whatever player i 's expectations about x_{-i} might be, the minimization of his expected private cost yields

$$\min_{X_i \geq 0} P(x)(C_i + R_i(X)) + X_i = \min_{X_i \geq 0} f_i(X_i) = f_i(X_i^*).$$

³See Brown (1973) for an early discussion of that point and Emons and Sobel (1991) for a discussion about the links with the implementation literature.

However, that rule does not satisfy the liability constraints. For instance, if player i believes that the other players will invest $x_{-i} \rightarrow \infty$ (so that $P(x) \rightarrow 0$) and if he chooses to invest less than X_i^* so that $R_i(X) > 0$, he knows that his liability will bound at U_i . Since he does not expect the accident state to occur anyway, he will then minimize his private cost by investing zero. That result is easily generalized in the next proposition.

Proposition 1. *Under budget balance and limited liability, the set of allocations that can be implemented in dominant strategies is empty.*

Proof. Suppose that X^* can be implemented in dominant strategies with R . We have two cases: either $x^* = 0$ (so that $X^* = \mathbf{0}$ is the nul vector) or $x^* > 0$.

Suppose that $x^* = 0$. Given a liability rule R , any player i who expects $x_{-i}^* = 0$ will spend an amount X_i such that $X_i = \xi(C_i + R_i(X_i, \mathbf{0}))$. If $X_i = X_i^* = 0$, then $C_i + R_i(\mathbf{0}) \leq 0$. Summing yields $\sum R_i(\mathbf{0}) = l \leq -c < 0$ which is a violation of budget balance.

Suppose that $x^* > 0$. Then there exists a player i for which setting $X_i^* > 0$ is a weakly dominant strategy at least as good as investing nothing:

$$P(x_{-i} + X_i^*)(C_i + R_i(X_i^*, X_{-i})) + X_i^* \leq P(x_{-i})(C_i + R_i(0, X_{-i})), \quad \forall X_{-i}.$$

Since R_i is bounded above by U_i , as $x_{-i} \rightarrow \infty$ and the probability of an accident vanishes on both sides, this inequality yields $X_i^* \leq 0$; a contradiction. \square

The scope of proposition 1 goes beyond stating that it is difficult to handle the moral hazard problem. It emphasizes that there is also a coordination problem since no equilibrium in dominant strategies would exist even if there was no liability rule. This is to bear in mind since I will show that the optimal NI rule solves that fundamental coordination problem in a rather crude way.

3 The strict liability and negligence rules

A classical proposition in law and economics is to establish the weak dominance of the negligence rule over the strict liability rule when a player has a limited liability. Let x_{-i} and l_{-i} be fixed. The strict liability rule and the negligence rule differ only in the definition of the event where player i shall pay a compensation to the injured party. According to the strict liability

rule, it is the event of an accident while under the negligence rule, it must also be that player i spent less than a required standard⁴ X_i . Under both rules, limited liability caps the amount of money player i may be required to pay. Likewise, the liability constraints for all the other players $-i$ limit the compensating damage player i may receive: using (LC_i) and budget balance, we get

$$\begin{aligned} L_i &= l - l_{-i}, \\ &\geq -l_{-i}. \end{aligned}$$

Hence

$$-l_{-i} \leq L_i \leq U_i. \quad (1)$$

Consider the class of liability rules that are lower semi-continuous and bounded by (1). Given that X_{-i} is fixed here, a rule r in this class will later translate to the multi-players setting through $r(X_i) \equiv R_i(X_i, X_{-i})$.

Under the *strict liability rule*, a tortfeasor is responsible for any damage he may have cause even if he undertook an appropriate level of accident prevention. When the liability constraint binds, the value of the damage is greater than the value of the player's assets. In that event, one may associate the strict liability rule to the constant rule $r(X_i) = U_i$. Private cost with that rule becomes

$$c^1(X_i) = P(x_{-i} + X_i)V_i + X_i.$$

Player i will choose $X_i \geq 0$ in order to minimize $c^1(X_i)$: he wants ξ_i to be spent and will thus spend $X_i^1 = \max\{0, \xi_i - x_{-i}\}$ by himself.

Since $V_i > 0$ and $P'(0) = -\infty$, we have $\xi_i > 0$ for all $i \neq 0$. Since P is strictly convex, P' is strictly increasing so that ξ_i increases with V_i , hence with i . Player N wants the highest total level of spending under the strict liability rule. I call that player the *deep-pocket* or the *victim* (both interpretations are discussed later).

Consider now the constant rule where the player pays L_i in the accident state. Under that rule, player i expected cost becomes

$$c^0(X_i) = P(x_{-i} + X_i)(V_i - U_i + L_i) + X_i.$$

⁴Different players may be required to satisfy different standards (Emons and Sobel 1991). According to Grady (1990), courts usually consider the peculiar setting of a case to establish the standard of care. Besides, from the moment that players have different solvency constraints, it seems unlikely that the same level of care will be expected from each player. It follows from this assumption that two heterogeneous agents may have to comply with a different standard of care to avoid liability.

Let c^0 be minimized in X_i^0 . Define the lower contour set

$$\mathcal{X}_i = \{X_i \geq 0 : c^0(X_i) \leq c^1(X_i^1)\}.$$

Proposition 2 below establishes the optimality of the negligence rule since it can implement any level of effort that could be implemented with an other rule (all proofs are in the appendix).

Proposition 2.

1. X_i may be implemented with a lower semi-continuous rule bounded by (1) if and only if $X_i \in \mathcal{X}_i$.
2. Any $X_i \in \mathcal{X}_i$ may be implemented with the negligence rule

$$r(z) = \begin{cases} L_i & \text{if } (z - X_i)(X_i - X_i^0) \geq 0, \\ U_i & \text{else.} \end{cases}$$

By construction, the agent expected cost is minimized in X_i with this rule. Hence, the liability for agent i under the negligence rule is given by L_i . Notice that, when $L_i = U_i$, then this “negligence” rule is equivalent to the strict liability rule in a limited liability context where the agent always pays the damages up to his limited liability U_i .

4 The multi-players case

In proposition 2, both x_{-i} and l_{-i} were assumed fixed. I now consider the case where all players choose their level of spending simultaneously given a liability rule R .

As in section 2, the problem of the choice of a multi-players liability rule is one of mechanism design. A liability rule R will implement X in dominant strategy if, for all i , we have $X_i \in \mathcal{X}_i$, for all $x_{-i} \geq 0$. A liability rule R will implement X as a Nash equilibrium if, for all i , we have $X_i \in \mathcal{X}_i$. Because of proposition 1, we know that the concept of dominant strategies implementation is of little help here.

With the Nash equilibrium implementation concept, the liability rule R needs only to be defined at the equilibrium point X : $L = R(X)$. Hence, let a pair (X, L) be associated to a multi-players liability rule which specifies for

each player i the liabilities L_i and U_i paid by the player if he has provided or not his required standard of spending X_i as in proposition 2. If $X_i \in \mathcal{X}_i$ for all i , the pair is said to be Nash implementable (NI).

The multi-players set-up generates a dilution of incentives. Consider the case where everybody is liable under the strict liability rule; that is $L = U$. In that case, everybody will set $X_i = X_i^1$ in order to solve $\min_{X_i \geq 0} c^1(X_i)$. Let I be the group of M players who spend in prevention in equilibrium. We know that $M \geq 1$ because if nobody else was providing effort, agent N would since $\xi_N > 0$. Then

$$\begin{aligned} x &= \sum_{i \in I} X_i = \sum_{i \in I} (\xi_i - x_{-i}) = \sum_{i \in I} (\xi_i - x_{-i} - X_i + X_i), \\ &= \sum_{i \in I} (\xi_i - x + X_i) = \sum_{i \in I} \xi_i - (M - 1)x, \end{aligned}$$

so that

$$x = \frac{\sum_{i \in I} \xi_i}{M}. \quad (2)$$

The only way (2) may hold is if $I \equiv \{N\}$, $M = 1$ and $x = \xi_N$. Hence, under the strict liability rule, the spending in prevention by player N crowds out the incentives for the other agents to spend as well. That rule generates a lot of excess liability since $l_{-0} = u_{-0} = u > 0$. This suggests that a better multi-players rule where that excess liability is used to provide additional monetary incentives to player N . I will show that such a rule is indeed optimal.

Because all players have quasi-linear preferences, we get the following characterization of a NI pair.

Lemma 1. *A pair (X, L) is NI if and only if*

$$\min_{z \geq -X_i} P(x + z)V_i + z \geq P(x)(C_i + L_i),$$

for all players.

Let (X, L) be a NI pair that satisfies budget balance ($l = 0$). With such a pair, the players make a total of x in spendings. Let the set of such implementable sums be

$$\mathcal{X} = \{x : \exists (X, L), x = \Sigma X_i, \forall i X_i \in \mathcal{X}_i, l = 0\}.$$

Lemma 2. *\mathcal{X} is non empty, closed and bounded above by $\hat{x} > 0$.*

The social problem is to choose a budget balanced NI liability rule that minimizes social cost (SC) over \mathcal{X} . Given a , let x_a^* be the unconstrained minimum of (SC) over \mathbb{R}_+ . Remember that (SC) is strictly quasiconvex in x . Hence, if a is high enough, then $\hat{x} < x_a^*$ and \hat{x} will be chosen. This corresponds to the interesting case where liability is *scarce* so that a only a *second-best* solution will be feasible. Assuming that this is the case, the problem I study is then to characterize how \hat{x} may be implemented.

Proposition 3. *When $\hat{x} < x_a^*$, \hat{x} is uniquely implemented with (X, L) where $X_{-N} = 0$, $X_N = \hat{x}$, $L_{-N} = U_{-N}$ and $L_N = -u_{-N}$.*

Hence, the optimal multi-players liability rule puts every agent under a strict liability regime except player N who stays under the negligence rule and who actually receives money collected from the other players when an accident happens. With this solution, \hat{x} is given by the top of \mathcal{X}_N , as defined in proposition 2, when $L_N = -u_{-N}$.

It is easy to understand proposition (3) if we relate that problem with the classical problem of the private provision of a public good first analyzed by Warr (1983) and Bergstrom, Blum, and Varian (1986). These authors shows that the amount of public good provided is independent of the distribution of income unless the set of contributors is affected by the distribution. The optimal multi-player rule achieves that by concentrating all the ex post wealth (the collected fines from the liable agents) into the hand of a single player. That player is then disciplined through the negligence rule. Because there is a single player who spends resources into prevention, there are no dilution of incentives and a maximum of spending is undertaken.

Under this rule, the deep pocket is expected to undertake all spendings. Again, the comparison with the public good problem helps to understand that result: if all wealth is to be given to a single player to spend on a public good and if any player would spend less than the socially optimal amount, then it makes sense to give the wealth to the player who values the most the public good. If we concentrate all monetary incentives upon a single player i which we submit to the negligence rule, then the opportunity cost of an accident for this player becomes

$$C_i + L_i = V_i - U_i + (-u_{-i}) = V_i - u.$$

Hence, the most responsive player under that rule is the one for which the opportunity cost of an accident under strict liability (V_i) is the greatest, the deep pocket.

To understand the two different interpretations of player N as the “deep pocket” or the “victim”, consider the following. By definition,

$$V_i \equiv C_i + U_i,$$

Hence, the opportunity cost of an accident and the ex post liability of a player are jointly identified in this model. The “deep-pocket” interpretation is natural when there is little variation in the $C_i \simeq \bar{C}$ (relatively to the U_i). Then, all players would be similarly careless in absence of a liability regime but player N is highly motivated to produce the required amount of care once his assets U_i are at stake. He is then chosen because he is the most responsive monetary incentives under the negligence rule.

When there is a lot of variation in C_i and $U_i \simeq \bar{U}$, interpreting of player N as a “victim” is more natural. Then, all players have roughly the same ability to pay ex post but player N has an higher ex ante incentive to spend in prevention because of his higher opportunity cost.

5 Manipulation of Liabilities

Liability rules work to the extent that the players cannot evade punishment by placing their assets U_i out of the reach of the courts. Yet, liabilities may be manipulated in another way by artificially raising or lowering the opportunity cost of an accident. Assuming that lowering U_i is impossible, this is done by raising or lowering V_i .

A minimal requirement that a liability rule should satisfy is to be immune to such manipulation. In a single-player context, both the strict liability rule and the negligence rule share that property⁵. As it turns out, so does the

⁵For the strict liability rule the proposition is trivial. For the negligence rule, one must show that the expected cost rises with V_i . Remember that with limited liability, \hat{x} is set by solving

$$P(\hat{x})(V_i - U_i) + \hat{x} = P(\xi_i)V_i + \xi_i.$$

We gather that

$$\frac{\partial \hat{x}}{\partial V_i} = \frac{P(\xi_i) - P(\hat{x})}{P'(\hat{x})(V_i - U_i) + 1} > 0.$$

The denominator is positive because increasing spending in prevention beyond the required standard of care would increase at the margin the player private cost. The numerator is positive because this implies that the standard of care involves more spendings than the player would otherwise undertake in absence of a liability system.

optimal multi-player rule. To see this, consider the incentive of a player i other than the deep-pocket to increase his stake V_i to $V_N + \epsilon$ in equilibrium so that he becomes the deep-pocket.

For this to be a profitable move, he should get a lower expected cost (on the r.h.s.) than what he obtains under the strict liability rule (on the l.h.s.); hence

$$P(\hat{x})V_i > P(\hat{x}_\epsilon)(V_N + \epsilon - u) + \hat{x}_\epsilon.$$

Since the negligence rule is robust to such manipulation

$$P(\hat{x}_\epsilon)(V_N + \epsilon - u) + \hat{x}_\epsilon \geq P(\hat{x})(V_N - u) + \hat{x}.$$

By definition of \hat{x} , where

$$P(\hat{x})(V_N - u) + \hat{x} = P(\xi_N)V_N + \xi_N \geq 0.$$

It follows that

$$P(\hat{x})V_i > P(\xi_N)V_N + \xi_N \geq 0.$$

This is certainly false for a small V_i . Could it be true for $V_N - \epsilon$? Then we would get

$$0 > (P(\xi_N) - P(\hat{x}))V_N + P(\hat{x})\epsilon + \xi_N \geq 0,$$

a contradiction. Hence, the multi-players optimal liability rule is robust to an upward manipulation of liabilities. Intuitively, although player N is certainly better off if an accident happens, nobodies envy him ex ante since he has to spend \hat{x} to maintain his status as a deep-pocket and the level \hat{x} is computed with $V_N - u = \max_i V_i - u$ so that it is barely acceptable to player N .

6 Conclusion

I have given a characterization of the optimal multi-player liability rule that provide the players a maximum amount of incentives to produce a join effort in the case when all players have a limited liability.

At the margin, an increase in V_i thus raises the expected cost

$$P'(\hat{x})(V_i - U_i) \frac{\partial \hat{x}}{\partial V_i} + P(\hat{x}) > 0.$$

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A Appendix

The proofs of the propositions and lemmas follow.

Proof of proposition 2. Since $X_i^1 \in \mathcal{X}_i$ and is c^0 is quasiconvex, \mathcal{X}_i is a non empty, closed and convex set. Now assume that an arbitrary lower semi-continuous rule \hat{r} , bounded by (1), is applied. If X_i may be implemented with this rule, then X_i must be an optimal response to it⁶. Hence, for all $z \geq 0$,

$$P(x_{-i} + z) \left[C_i + \hat{r}(z) \right] + z \geq P(x_{-i} + X_i) \left[C_i + \hat{r}(X_i) \right] + X_i. \quad (3)$$

Since $\hat{r}(z) \leq U_i$, for all z we have

$$\begin{aligned} c^1(z) &= P(x_{-i} + z)V_i + z, \\ &\geq P(x_{-i} + z) \left[V_i - U_i + \hat{r}(z) \right] + z. \end{aligned} \quad (4)$$

Hence, combining (4) and (3),

$$\begin{aligned} c^1(X_i^1) &\geq P(x_{-i} + X_i^1) \left[C_i + \hat{r}(X_i^1) \right] + X_i^1, \\ &\geq P(x_{-i} + X_i) \left[C_i + \hat{r}(X_i) \right] + X_i, \end{aligned}$$

and since $L_i \leq \hat{r}(X_i)$,

$$\begin{aligned} c^1(X_i^1) &\geq P(x_{-i} + X_i)(C_i + L_i) + X_i, \\ &\geq c^0(X_i), \end{aligned}$$

so that $X_i \in \mathcal{X}_i$.

Now assume that we want to implement $X_i \in \mathcal{X}_i$. We must verify that r as specified in the proposition will do the trick.

- If $X_i = X_i^0$, then then the player always pays L_i and minimizes cost by setting $X_i = X_i^0$.

⁶An optimal response exists because the rule is lower semi-continuous.

- If $X_i > X_i^0$ and $z > X_i$, then cost are minimized down to $c^0(X_i)$ by lowering z up to X_i .
- If $X_i > X_i^0$, and $z < X_i$, then cost are no less than $c^1(X_i^1)$ which is no less than $c^0(X_i)$. Hence, cost would be no higher by having $z = X_i$ like in the previous argument.
- If $X_i < X_i^0$, a similar argument applies.

□

Proof of Lemma 1. Suppose that (X, L) is NI. Then, for all i , $X_i \in \mathcal{X}_i$ or

$$\min_{y \geq 0} P(x_{-i} + y)V_i + y \geq P(x_{-i} + X_i)(C_i + L_i) + X_i,$$

subtracting X_i on both sides and making a change of operand $z = y - X_i$,

$$\min_{z \geq -X_i} P(x + z)V_i + z \geq P(x)(V_i - U_i + L_i). \quad (5)$$

Now suppose that (5) holds for all i . Proceed backward to show that X is NI. □

Proof of Lemma 2. Let Y be the set of NI pairs (X, L) . To prove the lemma, it is sufficient to show that Y is also non empty, closed and that there exist $(X, L) \in Y$ such that $X \neq 0$. Because the sum $x = \Sigma X_i$ is a continuous mapping, \mathcal{X} also share these properties.

Formally

$$Y = \{(X, L) : \forall i \quad X_i \in \mathcal{X}_i, L \leq U, l \geq 0\}.$$

For any given L , the choice of X by the players is a Nash equilibrium of a game of private contributions (the X_i) to a public good (the $P(x)$ component of the players' payoff functions). Bergstrom, Blum, and Varian (1986) have shown that these games have a unique Nash equilibrium. Hence, letting $L = 0$, I conclude that there exists at least one pair $(X, 0) \in Y$.

Let Y^C be the complement of Y in $\mathbb{R}^{2(N+1)}$. Each component L_i is bounded by (1). Each \mathcal{X}_i is bounded below by zero. Now if $(X, L) \in Y$, then for all $z \geq -X_i$

$$P(x + z)V_i + z \geq P(x)(C_i + L_i).$$

Hence if X_i is not bounded above and if $X_i \rightarrow \infty$, then

$$z \geq 0, \quad \forall z \in \mathbb{R},$$

which is certainly false for an arbitrarily low value of z . It follows that each \mathcal{X}_i is bounded above, that Y is bounded and that \mathcal{Y}^C is non empty.

Consider $y = (X, L)$, $y \in Y^C$. By definition, there exists some $i(y)$ and $z(y) \geq -X_i$ such that

$$P(x + z(y))V_{i(y)} + z(y) < P(x)(V_{i(y)} - U_{i(y)} + L_{i(y)}).$$

Since P is continuous, there exists $\delta(y) > 0$ such that if $\|y' - y\| < \delta(y)$, where $y' \equiv (X', L')$, then

$$P(x' + z(y))V_{i(y)} + z(y) < P(x')(V_{i(y)} - U_{i(y)} + L'_{i(y)}),$$

so that $y' \in Y^C$. It follows that y is an interior point of Y . Since y was chosen arbitrarily, I conclude that Y^C is open so that Y is closed.

To show that $\hat{x} > 0$ consider the feasible rule $L = U$ that makes all agents are strictly liable and that implements $x = \xi_N > 0$. \square

Proof of proposition 3. The proof is lengthy and proceeds in a series of lemmas. First, let (X, L') implements \hat{x} and, given X , define

$$L_i^+ = U_i - \frac{(P(\hat{x}) - P(\xi_i)) V_i + \hat{x} - \xi_i}{P(\hat{x})},$$

$$L_i^- = U_i - \frac{(P(\hat{x}) - P(\hat{x} - X_i)) V_i + X_i}{P(\hat{x})},$$

and

$$\mathcal{L}_X = \operatorname{argmax}_L l \quad \text{s.t. } L \leq U \text{ and } (X, L) \text{ is NI.}$$

Then for all $L \in \mathcal{L}_X$, (X, L) implements \hat{x} as well. Without loss of generality, I consider an implementation (X, L) of \hat{x} such that $L \in \mathcal{L}_X$.

Lemma 3.1 establishes that, although l is linear, it has a unique maximizer over the set of vectors L that satisfy both the limited liability constraints and NI.

Lemma 3.1. $\mathcal{L}_X = \{L\}$ where

$$L_i = \begin{cases} L_i^- & \text{if } \xi_i < \hat{x} - X_i, \\ L_i^+ & \text{if } \xi_i \geq \hat{x} - X_i. \end{cases}$$

Proof. By definition of ξ_i we have:

$$P(\hat{x})V_i + \hat{x} \geq P(\xi_i)V_i + \xi_i, \quad (6)$$

and

$$P(\hat{x} - X_i)V_i + \hat{x} - X_i \geq P(\xi_i)V_i + \xi_i. \quad (7)$$

Rewrite (6) as

$$(P(\hat{x}) - P(\xi_i))V_i + \hat{x} - \xi_i \geq 0. \quad (6')$$

Multiply (7) by -1 and add $P(\hat{x})V_i + \hat{x}$ on both sides to get

$$(P(\hat{x}) - P(\xi_i))V_i + \hat{x} - \xi_i \geq (P(\hat{x}) - P(\hat{x} - X_i))V_i + X_i. \quad (7')$$

From (6'), we gather that $L_i^+ \leq U_i$.

Nash implementability constrains L_i to satisfy

$$\min_{z \geq -X_i} P(\hat{x} + z)V_i + z \geq P(\hat{x})(V_i - U_i + L_i). \quad (8)$$

The unconstrained solution to the program on the l.h.s. is $z^* = \xi_i - \hat{x}$. There are two cases:

1. If $\xi_i - \hat{x} \geq -X_i$, then the solution is unconstrained and (8) yields

$$P(\xi_i)V_i + \xi_i - \hat{x} \geq P(\hat{x})(V_i - U_i + L_i). \quad (9)$$

2. If $\xi_i - \hat{x} < -X_i$, then the solution is constrained in $-X_i$ and (8) yields

$$P(\hat{x} - X_i)V_i - X_i \geq P(\hat{x})(V_i - U_i + L_i). \quad (10)$$

Now if L belongs to \mathcal{L}_X , then all the L_i must be set as large as possible given X . In the first case, (9) holds and its r.h.s. strictly increases with L_i . Raise L_i so that (9) holds as an equality. Then $L_i = L_i^+$. Likewise, in the second case, raise L_i so that (10) holds as an equality. Then $L_i = L_i^-$. Notice

that in the latter case, the candidate solution $z = 0$, although feasible, was discarded when solving the l.h.s. of (8). This implies that

$$P(\hat{x})V_i \geq P(\hat{x} - X_i)V_i - X_i,$$

or

$$(P(\hat{x}) - P(\hat{x} - X_i))V_i + X_i \geq 0,$$

which implies $L_i^- \leq U_i$. Hence, $L_i = L_i^-$ also satisfies the limited liability constraint in that case. \square

The first part of the next lemma is the core of the proof. Basically, it states that there is at most one player who will be given incentives to spend in prevention.

Lemma 3.2. *There is at most one player i such that $\xi_i < \hat{x} - X_i$ and $X_i > 0$. There is at least one player j such that $\xi_j \geq \hat{x} - X_j$ and $X_j > 0$.*

Proof. First part: Suppose that for $k \in \{i, j\}$, we have $\xi_k < \hat{x} - X_k$, which implies $L_k = L_k^-$, and $X_k > 0$. Let $X_i + X_j = \bar{X}$ so that $X_j \equiv \bar{X} - X_i$. Then X_i belongs to an interval $[0, \bar{X}]$ of strictly positive measure. If L belongs to \mathcal{L}_X , then $L_i^- + L_j^-$ should be maximized. Equivalently, using the definition of L_k^- with \bar{X} fixed, hence \hat{x} fixed, X_i should maximize

$$P(\hat{x} - X_i)V_i + P(\hat{x} - \bar{X} + X_i)V_j, \tag{11}$$

over $[0, \bar{X}]$. Yet, (11) is strictly convex in X_i so that it does not have an interior maximum on $[0, \bar{X}]$. It follows that either X_i or X_j is zero and we get a contradiction.

Second part: Suppose there are no player j such that $\xi_j \geq \hat{x} - X_j$ and $X_j > 0$. Since $\hat{x} > 0$, then all spending is provided by some player i such that $X_i = \hat{x}$ and $\xi_i < \hat{x} - X_i = 0$. This can't be because $\xi_i \geq 0$ for all i . \square

At this point, it is easy to establish that, given the liability structure, there is no player who does not feel compelled to spend in prevention who would actually do so. This is done in the next lemma.

Lemma 3.3. *There is no player such that $\xi_i < \hat{x} - X_i$ and $X_i > 0$.*

Proof. Suppose there is such a player. Then he has liability a L_i^- which strictly decreases with X_i and is maximized to U_i in $X_i = 0$. From Lemma 3.2, we know that there is a player j such that $\xi_j \geq \hat{x} - X_j$ and $X_j > 0$ who pays L_j^+ . Reduce X_i to zero and increase X_j by X_i so that \hat{x} stays constant. Since L_j^+ is independent of X_j , the liability of player j has not changed while that of player i has strictly increased. Hence total liability l has increased and $L \notin \mathcal{L}_X$; a contradiction. \square

At this point, we have established that a solution (X, L) is such that there is a group (possibly empty) of players who, in equilibrium, do not want and do not spend in prevention, that is for which $\xi_i < \hat{x}$; while the other group (certainly not empty) have $\xi_j \geq \hat{x} - X_j$ with $X_j > 0$, willingly contribute. In the last lemma, it is established that this group is resumed by player N alone.

Lemma 3.4. *Let (X, L) implements \hat{x} . Then all spending is provided by player N .*

Proof. Suppose there are more than one player who spend in prevention. These players can be ranked with their index i (which strictly increase with ξ_i). Reduce the spending of the lowest ranked player, say player i , to zero and increase that of the highest ranked player, say player j , by the same amount to keep \hat{x} constant. Liabilities are left unchanged except perhaps that of the player i if $\xi_i - \hat{x} \geq -X_i$ but $\xi_i - \hat{x} < 0$. In that case, player i 's liability has strictly increased (since $\xi_i \neq \hat{x}$) from L_i^+ to U_i . Repeat the operation for the second-lowest ranked player and so on. Eventually, all spending will be provided by the highest ranked player.

We have a configuration for a solution where a single player produces all the effort and all the others players pay the maximum liability that entails providing zero spending as a best-response. Since \hat{x} is a constrained optimum, all the money gather this way should be used to provide the maximum amount of incentive ($-u_{-j}$) under the negligence rule to the player who spends in prevention. Such a solution may be implemented as a sequence:

1. Collect all the players their U_i to get a maximum u in incentives to distribute.
2. Provide some incentives to some player i who then implements $X_i = \hat{x}$.
3. Provide the remaining incentives to all other players j such that $\xi_j \geq \hat{x}$ to entail a zero best-response in spendings.

It is clear that we should select player N in step 2 because he provides the highest \hat{x} at stage 2 and the higher the \hat{x} , the lesser the number of players that have to be subsidized in step 3 (that is, none with player N). With this solution⁷

$$L = [U_1, U_2, U_3, \dots, U_{N-1}, -u_{-N}]. \quad \square$$

□

⁷Being under the negligence rule, player N is indifferent between providing \hat{x} as required to receive u_{-N} in case of an accident or shirking with $\xi_N < \hat{x}$ (otherwise \hat{x} would not be a constrained optimum) and being fined up to U_N . It follows that

$$\begin{aligned} P(\hat{x})(V_N - U_N - u_{-N}) + \hat{x} &= P(\xi_N)V_N + \xi_N, \\ (P(\hat{x}) - P(\xi_N))V_N + \hat{x} - \xi_N &= P(\hat{x})u. \end{aligned}$$

Hence, $L_N = L_N^+ = U_N - u = -u_{-N}$ as stated.