Sequential Screening with Renegotiation

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This version, June 3, 2000

Abstract

In this paper, I consider the problem of designing an optimal screening contract for a principal facing an agent whose type comes as a sequence that unfolds through time. Formally, the agent has a private ex ante type that stands for the expected value of his private ex post type. Under full commitment, the principal will first try to separate agents with respect to their ex ante type and will subsequently try to separate them with respect to their realized ex post type. While at the last period a traditional instrument is used to separate optimally the agent types, it is shown that such separation is achieved by resorting to a more or less efficient ex ante contracting scheme in the first period. Under no commitment, pooling always occurs with respect to ex ante types while in the case of commitment with renegotiation, the introduction of an ex ante adverse selection problem is shown to have a non trivial effect on the possibility of interim renegotiation in the second period.

Keywords: adverse selection, screening, mechanism design, renegotiation.

JEL Classification Numbers: D42, D82.

*This paper is adapted from the second chapter of my dissertation and was previously simply titled *Sequential Screening*. I have given it a new title after being alerted of the existence of Courty and Hao' paper which was issued a few months before mine. This research was undertaken while I was at CIRANO. I wish to thank Michel Poitevin for his relevant direction. All errors are mine. `pgon@ecn.ulaval.ca`.

1 Introduction

In this paper, I address the normative question of designing optimal screening complete contracts when agents have *sequential types*, that is, types that unfold dynamically as a sequence. The analysis is made under the assumption that, at any point in time, the agent behavior is resumed by a single dimensional characteristic.

The economic analysis of screening contracts with sequential types is a useful generalization of the standard model where all private information is resumed by a single static characteristic. Assuming sequential types allows for situations where the private information has a more complex nature. The celebrated trade-off between efficiency in production space and informational rent extraction, in the single static type set up, is transposed, for sequential types, in a trade-off between efficiency in *contract* space and informational rent extraction.

With sequential types, screening is achieved using the direct mechanism proposed by Myerson [10], for which he demonstrates that a form of revelation principle holds. This mechanism requires that players announce privately their type sequence as it unfolds and that they follow a proper action after each announcement. In the initial stages of the mechanism, these actions will take the form of a choice of a binding contract to be followed afterward. In my set-up, it is only in the final stage of the mechanism that a particular choice of a physical action (consumption, production, etc.) takes place. Transfers are usually conditioned on the whole history of type announcements.

I limit myself to two-stage sequences: this is enough to analyze issues like ex post vs. ex ante pricing or the effect of lack of commitment on screening contracts. A more serious restriction of this paper is that most of the results are derived for situations where the players only have two possible types at each stage of the game.

Sequential types screening contracts have been explored before by Baron and Besanko [1] in a principal-agent framework. The hypotheses I make in my model are closely related to that of their Theorem 3. The main difference between both papers lies in the scope of the analysis. In particular, Baron and Besanko recognize but bypass the issue of global incentive compatibility (that is, the fact that no player should be tempted to misreport his type ex ante) while I handle it in a systematic fashion. All the results presented in Theorem 3 of their paper are based on the heroic assumption that global incentive compatibility constraints do not bind at the optimum and they give an example of a type distribution for which this is the case. In most of the examples I study here, some of these incentive constraints are binding at the optimum.

An assumption that explains these differences is that consumption and production take place in each period in Baron and Besanko's model while all such activities are relegated at the end of the second period in my model. As a result, in their model, there are much stronger incentives for the principal to induce type revelation in the first period since that has, loosely speaking, a "first-order" effect on first-period production. This, in turn, relaxes the "second-order" incidence of global incentive compatibility constraints.

The economic interpretation of sequential screening contracts is very different in both papers. Baron and Besanko emphasize the trade-off, when sequential types are correlated, between inducing information revelation ex post or ex ante. I emphasize that at each period, there is a trade-off between efficiency in the contract space and rent extraction. While their model can be seen as a useful multi-period generalization of the one-period model, it is unsatisfactory with respect to the analysis of the combination of incentives one can expect when information is more complex by nature.

There are but a few papers where some form of sequential screening is developed. An early contribution is that of Gales and Holmes [5] who model the optimal pricing and refund policy for airline tickets. In their model, the ex post incentive compatibility constraints turn out to be non binding. Problems of sequential screening have also been identified in the pricing of calling plans in the telecommunication industry [3, 9]. A more recent contribution is that of Courtny and Hao [4] who also apply their model to the problem of airline tickets. Like that of Gale and Holmes, their analysis is made in a continuous type setting but involves consumers with a unitary demand and separation of consumers is achieved by the way of varying the probability of delivery.

In my model, I consider the relationship between a principal and an agent that has private costs. The analysis is restricted to discrete type supports but I do have full menus of pairs of actions and transfers. The main contribution of this paper is in the resolution strategy. The previous papers transform the dynamic problem into a static multi-product price discrimination problem. All incentive constraints are thus handle at once. I choose the dynamic programming approach, that is to solve first the ex post problem for some level of utility expected by the agent and then the ex ante problem with respect to these utility levels. Incentives constraints are thus handle sequentially. As it turns out, this strategy makes easier the analysis of commitment issues like renegotiation in this context. Sequential screening contracts are very likely to be constrained by renegotiation issues: as time goes by, the distortions induced in the contracting process to achieve early separation loose their initial value to both parties and renegotiation is unavoidable. This is particularly the case with random delivery rules where the surplus between the seller and the buyer still exists ex post in the event of a non delivery. This paper present new results with respect to the effect of renegotiation on sequential screening contracts. In particular, I show that the effect of renegotiation on the qualitative nature of these contracts critically depends on the moment when the possibility of renegotiation is considered.

The rest of this paper is divided as follows. The model is presented in the next section. In section 5, the analysis is completed under the assumption that the agent has two possible ex post types. Section 6 describes the optimal contract under various commitment assumptions. The conclusion follows.

2 The Model

I consider a principal-agent relationship in a two-period economy with incomplete information. Ex ante, in period 1, the agent has a private type $m \in \mathcal{M} \equiv \{\underline{m}, \ldots, \overline{m}\}$ where $\mathcal{M}$ has M elements. Ex post, in period 2, the agent is in a random state $\theta \in \Theta$ where Θ is an ordered set of N elements, independent of m. At times, I will refer to the agent's individual state as his *ex post type*. The type m of the agent characterizes his distribution probability $f(m)$ over Θ, represented as a $N \times 1$ vector $f(m) = [f_\theta(m)]_\Theta$, and his ex ante reservation utility (best external opportunity) $\overline{u}(m)$.[1] The state θ affects the ex post cost function $c(x, \theta)$ of the agent to produce some good in quantity $x \geq 0$. c is assumed strictly convex and the lower the θ, the lower the cost:

$$-c_\theta(x, \theta) \leq 0 \tag{1}$$

where the equality stands only when $x = 0$. Furthermore, the l.h.s. of (1), that is the marginal saving of having a better type, is assumed a decreasing strictly concave function of x:

$$-c_{x\theta} < 0 \quad \text{and} \quad -c_{xx\theta} < 0. \tag{2}$$

[1] In section 6, I address the issue of defining ex post reservation utility. It is subsumed here by the certainty equivalent of the expected ex post opportunities.

This is the familiar marginal rate of substitution ordering (single crossing) condition (see Matthews and Moore [8]).

The principal does not observe neither the ex ante type m, nor the ex post state θ, nor the actual costs borne by the agent. Nevertheless, he has a strictly positive Bayesian prior $p(m)$ about the probability that the agent is of ex ante type m. Both all the $p(m)$ and the $f(m)$ are common knowledge to both players. I assume that the principal has all the bargaining power over the duration of the relationship. Various commitment capabilities, that of full commitment, no commitment and commitment with renegotiation are explored in the analysis.

The agent can communicate freely with the principal at all times. Since new information about θ is revealed to the agent at the end of period 1, I model this economy as a two-stage game with communication [10]. More specifically, the course of the game is as follows. At the beginning of period one, the principal offers a contract (to be defined later) to the agent. The agent then accepts or refuses the contract. If he refuses, then the game ends and both players get their reservation utility (normalized to zero for the principal). If he accepts, the game moves to period 2 once the uncertainty about θ has been resolved. There, depending on the commitment assumption, the contract may be renegotiated. After that, production and exchange take place according to the provisions of the final contract. The economic problem is to compute the optimal contract that will be offered to the agent by the principal under these various commitment assumptions.

Both the agent and the principal are assumed to be risk-neutral with respect to income. If we normalize the price of the good to 1, the principal's payoff is $x - t$, where t is a transfer from the principal to the agent, while the agent gets $u = t - c(x, \theta)$. I decompose the transfer t into actual cost and utility u for the agent so that the principal's ex post payoff becomes $x - c(x, \theta) - u$.

A set of behavioral strategies for the agent is a binary rule, to accept or to refuse the initial contract and a choice of messages to be sent to the principal about his ex ante type and the state θ that will be realized. These communication possibilities are resumed by invoking the revelation principle for multistage games under which the agent truthfully announces his type, at each stage, and is then asked to follow the prescriptions of a (possibly randomized) history-dependent scheme. I also assume that the agent accepts any contract that is individually rational. A strategy for the principal is an offer of a complete contract at the beginning of period one. Ultimately, a contract should specify a production level and a monetary transfer.

Under these hypotheses, the agent accepts any contract that leaves him with at least his reservation utility and truthfully reveals his type m and the state θ he is in, if the contract satisfies the usual incentive compatibility constraints. The principal's payoff out of a contract evaluated at the initial contracting stage is thus $\mathrm{E}(x - c(x,\theta) - u)$ while the agent expects $\mathrm{E}(u|m)$.

3 The equilibrium

Our solution concept is to look for a Perfect Bayesian Equilibrium (PBE) where the principal offers the contract that maximizes his payoff, under the relevant participation and incentive constraints. In the appendix, I show that we can restrict our attention to non randomized schemes. Because utility is transferable, a contract in that class is simply a pair of functions that map the type space $\mathcal{M} \times \Theta$ into the production and utility spaces. These functions will be represented by pairs of $N \times 1$ vectors, one for each m, that is,

$$(x, u) \equiv \{(x(m), u(m))\}_{m \in \mathcal{M}}$$

where $x(m)$ and $u(m)$ are vectors of dimension N. For now on, (x, u) will refer to a *contract*, that is, a collection of such pairs of vectors $x(m)$ and $u(m)$. One such pair of vectors (ξ, v) will be referred to as an *ex post* contract; hence, a contract is a collection or a menu of ex post contracts.

Let Γ be the set of contracts that satisfy global incentive compatibility (to be defined later). Under full commitment, the optimal contract solves[2]

$$\max_{(x,u) \in \Gamma} \mathrm{E}(x - c(x,\theta) - u) \tag{3}$$
$$\text{subject to} \quad \mathrm{E}(u(m)|m) \geq \overline{u}(m) \quad \forall m \in \mathcal{M}.$$

[2]I use the following convention: $\mathrm{E}(z)$ where z is a vector means $\mathrm{E}(\tilde{z})$ where $\tilde{z}$ is a random variable whose support are the elements of z. In what follows, unconditional expectations are taken with respect to p and f while conditional expectation (on m) are taken with respect to f. More precisely:

$$\mathrm{E}(u) \equiv \sum_{m \in M} p(m) \sum_{\theta \in \Theta} f_\theta(m) u_\theta(m) \quad \text{and} \quad \mathrm{E}(u|m) \equiv \sum_{\theta \in \Theta} f_\theta(m) u_\theta(m).$$

I will take the less direct approach of dynamic programming first suggested by Townsend [11]. More specifically, I rewrite (3) as

$$\max_u \mathrm{E}(s(u,\tilde{m})-u), \tag{4}$$

$$\begin{aligned} \text{subject to} \quad & u(m)\in U \quad \forall m\in\mathcal{M}, \\ & \mathrm{E}(u(m)-u(m')|m)\geq 0 \quad \forall (m,m')\in\mathcal{M}^2, \qquad (5) \\ & \mathrm{E}(u(m)|m)\geq \overline{u}(m) \quad \forall m\in\mathcal{M}. \qquad (6) \end{aligned}$$

Here, the notation $\tilde{m}$ emphasizes that the agent's ex ante type is random from the principal perspective; U is the set of vectors of payoffs v *feasible* under ex post incentive compatibility (see the next section) and

$$s(v,m)=\max_{x\in X(v)} \mathrm{E}(\xi-c(\xi,\tilde{\theta})|m). \tag{7}$$

There, $X(v)$ is the set of vectors ξ that are *feasible* under ex post incentive compatibility once the vector of payoffs v has been promised to the agent (see the next section). Hence, this problem accepts a recursive formulation where we first associate a vector ξ of production plans to every possible v and we then choose the optimal collection u of such vectors v. In the next section, I characterize U, $X(v)$ and $s(v,m)$, so that I can pursue the analysis in the reduced contract space $R^{M\times N}$ where the optimal menu u lies.

4 The Reduced Contract Space

Let $r_{ij}(z)=c(z,\theta_i)-c(z,\theta_j)$ be the *rent function.* Because of assumption (2), each rent function is strictly monotonous and $r_{ij}=-r_{ji}$. A vector of payoffs $v\in R^N$ is defined to be *feasible* ex post if there exists a vector $\xi\in R^N$ such that the ex post incentive compatibility constraints are satisfied. These constraints can be written as

$$v_i\geq v_j+r_{ji}(\xi_j) \quad \forall (i,j)\in I^2. \tag{8}$$

Here the indices i and j in $I=\{1\dots N\}$ refer to the elements of the $N\times 1$ vectors v and ξ. From then on, I assume that θ_i increases with i. $X(v)$ is then the set of ξ that satisfy (8). A vector of payoffs v is feasible if $X(v)$ is not empty.

The constraints (8) have the usual interpretation: they state that an agent of ex post type θ_i prefers to truthfully announce his type, in which case he produces ξ_i and ends up with transfer $t_i = \upsilon_i + c(\xi_i, \theta_i)$, than to announce θ_j for which transfer t_j was devised, given that he would then have to support the cost differential $r_{ji}(\xi_j) = c(\xi_j, \theta_j) - c(\xi_j, \theta_i)$ to produce ξ_j.

Since each of the constraints (8) that define $X(\upsilon)$ is monotonous in each ξ_j, all the constraints are quasi-concave in ξ; this makes $X(\upsilon)$ a convex set. Since the maximand of program (7) is strictly concave, its associated argmax is a well defined function of $u(m)$ into the space of production vectors. Since that relation will hold at the optimum of (4), there is no loss of generality in considering only contracts for which ξ solves (7). Since these contracts are completely defined by u, I can talk of u as a *contract* and of $u(m)$ as an ex post contract intended at type m.

I note U the set of feasible contracts. It is never empty: consider, for instance, υ such that $\upsilon_i = \overline{u} - c(\overline{x}, \theta_i)$, where $\overline{x}$ and $\overline{u}$ are positive numbers that satisfy $\overline{u} - c(\overline{x}, \theta_N) > 0$; it is straightforward then that υ satisfies (8) for $\overline{x}$.

Multiplying (8) by $(\theta_j - \theta_i)$ and using (1), I get the following obvious property of feasible contracts

$$(\upsilon_i - \upsilon_j)(\theta_i - \theta_j) \leq 0 \quad \forall (i,j) \in I^2, i \neq j; \tag{9}$$

with equality only when $\xi_j = x_0$. Hence, all non trivial feasible contracts must guarantee that utility decreases with the ex post type.

The set of self-selecting constraints (8) can be rearranged by pairs:

$$r_{ji}(\xi_i) \geq \upsilon_i - \upsilon_j \geq r_{ji}(\xi_j) \quad \forall i \in I, \forall j \in I_i, \tag{10}$$

where $I_i = I \setminus \{1 \dots i\}$. Using (1) and (9), it is obvious from (10) that ξ_i decreases with θ_i (with i).

Rearranging (10), a necessary condition for υ to be feasible is thus

$$\upsilon_i - \upsilon_j \geq \upsilon_j - \upsilon_k \quad \forall i \in I \setminus \bar{i}, \forall j \in I_i, \forall k \in I_j. \tag{11}$$

I say that a vector of payoffs υ is *internally consistent* if it satisfies these constraints. I then show the following.

Proposition 1. *A vector of payoffs υ is feasible if and only if it decreases with θ and it is internally consistent. The set U of feasible vector of payoffs (contracts) is a convex cone.*

Proof. All missing proofs are in the appendix. □

Let ξ^* be the vector of efficient production values that, for each state i, equates marginal cost $c_x(x_i^*, \theta_i)$ to marginal (unitary) benefit for the principal and let $U^* \subseteq U$ bet the set of feasible contracts that allow efficient production ξ^*. When money is no object, we can always implement the efficient allocation ξ^* and induce truth telling from the agent as it is established in the following proposition.

Proposition 2. *U^* is closed, convex and of non empty interior.*

Let $\pi(\upsilon, m)$ be the value to the principal of a feasible vector of payoffs υ offered to some ex ante type m. This value is the expected difference between the maximal ex post expected surplus $s(\upsilon, m)$ he might gain from υ and the expected utility compensation $\mathrm{E}(\upsilon|m)$; hence,

$$\pi(\upsilon, m) = s(\upsilon, m) - \mathrm{E}(\upsilon|m). \tag{12}$$

Working ex ante with function π is useful since it gives us the principal's ex post valuation of any ex ante promise of a vector of payoffs υ. Arbitrage between informational rent extraction and efficiency can then be made ex ante in the space of contracts. Put differently, given any level of utility $E(\upsilon|m)$ the principal may wish to confer to some agent m, the principal will have the choice to offer a set of more or less efficient ex post contracts $\{u(m)\}_{m\in\mathcal{M}}$ as the next proposition shows.

Proposition 3. *For any expected utility level $\mathrm{E}(\upsilon|m)$, there exists a $\upsilon^* \in U^*$ such that $\mathrm{E}(\upsilon^*|m) = \mathrm{E}(\upsilon|m)$. Likewise, for any expected utility level $\overline{\pi} = \pi(\upsilon, m)$, we can find a contract $\upsilon^* \in U^*$ that yields $\overline{\pi}$ to the principal.*

Given the decomposition (12) of the principal's expected utility from any contract υ, I get the following insight on the form of the principal's indifference surfaces over U.

Proposition 4. *Let $\overline{\pi} = \pi(\upsilon_0, m)$ for some feasible contract $\upsilon_0 \in U$ and let $\Gamma_{\overline{\pi}} = \{\upsilon \in U | \pi(\upsilon, m) = \overline{\pi}\}$ be the set of feasible contracts among which the principal is (ex post) indifferent. By proposition 3, we know that $U^* \cap \Gamma_{\overline{\pi}}$ is not empty. Let υ^* be a point in that intersection. Then, for all $\upsilon \in \Gamma_{\overline{\pi}}$, $\mathrm{E}(\upsilon|m) \leq \mathrm{E}(\upsilon^*|m)$ and the equality stands only if $\upsilon \in U^*$.*

Corrolary 4.1. *If $\upsilon_0 \in \operatorname{argmax}_\upsilon \pi(\upsilon, m)$, subject to $\mathrm{E}(\upsilon|m) \geq \overline{u}(m)$, then $\upsilon_0 \in U^*$.*

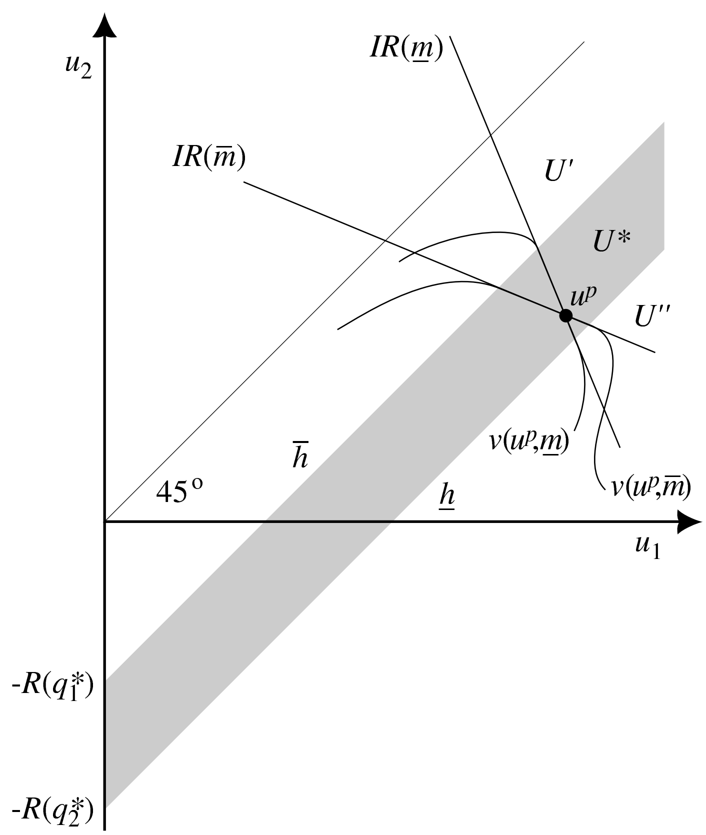

Figure 1: Efficient contract.

Proposition 4 simply states that the indifference surface $\Gamma_{\overline{\pi}}$ associated to some utility level $\overline{\pi}$ is everywhere below the hyperplane $\{v|\,\mathrm{E}(v - v^*|m) = 0, \pi(u^*, m) = \overline{\pi}\}$ is common with that hyperplane everywhere on $\Gamma_{\overline{\pi}} \cap U^*$. Corollary 4.1 states that if a contract maximizes the principal's payoff under only an ex ante participation constraint of the agent, then it is efficient. Such an indifference surface (curve) is represented in figure 1, when $N = 2$. It bends inward, outside U^*, because these contracts are distorted and cannot achieve the optimal level of social surplus. For any given level of utility $\overline{\pi}$ for the principal, that distortion must be compensated by a reduction in the expected payment to the agent, hence in his expected consumption level $\mathrm{E}(v|m)$. That distortion is minimized to zero for efficient contracts in U^*.

5 Ex Ante Contracting Under Full Commitment

I now considerably restrict the scope of the analysis by assuming that m and θ can take only two values, that is $\mathcal{M} = \{\underline{m}, \overline{m}\}$ and $\Theta = \{\theta_1, \theta_2\}$. That simplification is made to avoid having to establish a set of sufficient conditions under which function π is quasi-concave on a relevant portion of its domain.[3] Let $f(m) = \mathrm{Prob}(\theta_1|m)$ such that $f(\underline{m}) > f(\overline{m})$. Hence, $\underline{m}$ is the "good" ex ante type since he is most likely to have a "good" ex post type θ_1. The proportion of ex ante good type $\underline{m}$ is simply noted p and the rent function r_{21} is simply noted r.

When $N = 2$, internal consistency is satisfied if $v_1 \geq v_2$ (where the index $i = 1$ refers to the ex post type θ_1); hence U amounts to the cone under the 45° line in the non-negative orthant of R^2. Consider now U^*; that set is given by all $v \in U$ that satisfy (8) at ξ^*, that is,

$$r(\xi_1^*) \geq v_1 - v_2 \geq r(\xi_2^*). \tag{13}$$

Hence, U^* is the set delimited by the hyperplanes $\overline{h}$ and $\underline{h}$

$$\overline{h} : v_2 = v_1 - r(\xi_1^*)$$
$$\underline{h} : v_2 = v_1 - r(\xi_2^*),$$

[3]Remember that the analysis of adverse selection models is usually done under some assumption about the probability distribution of the types (for instance, the monotone likelihood ratio assumption in the continuous case); such kind of assumption is needed here for higher dimensional setting. This will be the object of future research. See footnote 4.

is the ex post rent function. U is thus composed of three disjoint sets: U', between the 45° line and $\overline{h}$, U^* and U'', outside U^* on the $\underline{h}$'s side (see figure 1). Contracts in U' are those where the ex post type θ_2 is asked to produce inefficiently; those in U'' require that the good ex post type θ_1 produces inefficiently. This leads us to the following proposition.

Proposition 5. *When $N = 2$, π is quasi-concave over $U' \cup U^*$ and strictly quasi-concave over U' for all distributions f.*

Corrolary 5.1. *Suppose $\upsilon \notin U''$ and let $\upsilon^* \in U^*$ such that* $\mathrm{E}(\upsilon|m) = \upsilon^*$. *Let υ_α be any convex combination of υ and υ^*,*

$$\upsilon_\alpha = \alpha\upsilon + (1-\alpha)\upsilon^* \quad \alpha \in (0,1).$$

Then $\pi(\upsilon_\alpha, m) \geq \pi(\upsilon, m)$ with equality if and only if $\upsilon \in U^*$.[4]

Corollary 5.1 implies that the valuation of contracts is well behaved in U': if we keep the expected transfer constant, then contracts become more and more valuable to the principal as they get closer to U^*. The principal has strictly concave indifference curves over U'. This makes him locally behave ex ante like a risk averse player although such behavior was ruled out on the basis of preferences alone. This is because the principal prefers to pay the agent any given expected transfer with a non-degenerated lottery (based on θ) than with a sure amount of money; for lotteries, contingent on the agent's performance, allow provisions for ex post efficient incentives. Put differently,

[4]Proposition 5 and corollary 5.1 are used in proposition 7 to show that, given any expected level of utility $E(\upsilon|m)$ that contracts must yield to the agent, the principal will always prefer contracts that are closer to U^*. Quasi-concavity of π is obviously sufficient for that purpose but it is by no mean necessary and it is difficult to establish when $N > 2$. All that is needed to get the required result on the principal's preferences, is to show that the function s of equation (7) satisfies a weak form of uniform monotonicity. Uniform monotonicity (see Kranich [7]) of a function $s : U \to R$ over some domain $U \subseteq R^N$ states that, for any $(\upsilon', \upsilon) \in U^2$,

$$s(\upsilon') \geq s(\upsilon) \quad \Longrightarrow \quad s(\upsilon') \geq \alpha s(\upsilon') + (1-\alpha)s(\upsilon), \quad \forall \alpha \in [0,1]. \tag{14}$$

Assume that the function s reaches a maximum on U and let $U^* \subseteq U$ be the points were that happens. Then relax the criteria of uniform monotonicity to functions such that, for any $\upsilon \in U$, there exists a $\upsilon^* \in U^*$ such that (14) is satisfied. Showing that the function s of equation (7) belongs to that class is then a sufficient step to generalize proposition 7 to higher dimensional cases.

the principal is willing to pay a positive premium to get rid of a lottery and thus behaves like a risk averse agent.

Once a value $\pi(u(m)|m)$ has been assigned to any contract offered (and chosen) by an agent of ex ante type m, computing the best self-selecting ex ante contract amounts to solve program (4):

$$\max_u \mathrm{E}(\pi(u(\tilde{m}), \tilde{m})) \tag{15}$$

subject to $u(m) \in U \quad \forall m \in \mathcal{M}$ and

$$IC: \quad \mathrm{E}(u(m) - u(m')|m) \geq 0 \quad \forall (m, m') \in \mathcal{M}^2, \tag{16}$$

$$IR: \quad \mathrm{E}(u(m)|m) \geq \overline{u}(m) \quad \forall m \in \mathcal{M}. \tag{17}$$

where the expectation in the maximand is based on the principal's prior p about the ex ante type population and $\overline{u}(m)$ is the reservation utility level of agent m.

Ex ante informational asymmetries do not necessarily command the use of inefficient contracts. When the ex ante efficient agent has a high reservation utility, the efficient allocation may be implemented despite the agent's capacity of mimicking the bad type. This is illustrated in figure 1. There, the ex ante participation constraint of the efficient agent $IR(\underline{m})$ is so high that it crosses that of the inefficient agent, $IR(\overline{m})$, within U^*. Hence, the principal can optimally offer a single ex ante pooling contract u^p at that point, and achieve efficiency. That contract leaves no expected informational rent to any agent. The link between pooling contracts and U^* is exposed in the following proposition.

Proposition 6. *If the optimal contract u is pooling ex ante types, that is, $u(\underline{m}) = u(\overline{m}) = \upsilon^p$, then it is efficient: $\upsilon^p \in U^*$.*

It should be noted that the pooling contract associated to υ^p is pooling types ex ante but not ex post. Since $\upsilon^p \in U^*$, then ex post types are asked to produce at their own efficient level, hence the contract is separating types ex post. In fact, what proposition 6 tells us is that if it is optimal to pool ex ante types, then it must be that we are separating efficiently ex post types.

As long as the reservation utility of the ex ante good type $\underline{m}$ is not too high with respect to that of type $\overline{m}$, he will be offered an efficient contract as the following proposition shows.

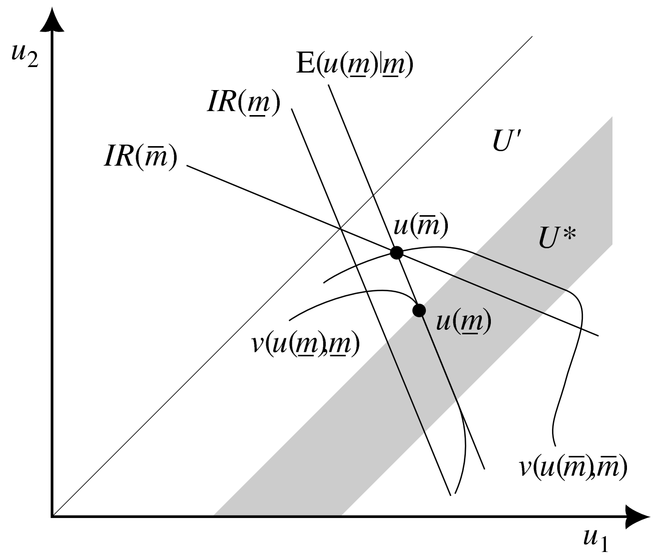

Figure 2: Sequential Screening Contracts.

Proposition 7. *When $M = N = 2$ and the individual rationality constraints of both agents do not cross in U'', then agent $\underline{m}$ is offered an efficient contract. Furthermore, $IR(\overline{m})$ is binding at an optimum.*

The following corollary extend the scope of proposition 6 when $M = N = 2$.

Corrolary 7.1. *Assume that the individual rationality constraints of both agents do not cross in U''. Then if the optimal contract $(u(\underline{m}), u(\overline{m}))$ is efficient, there exists an efficient pooling contract $v^p \in U^*$ that is optimal as well.*

When $\overline{u}(\underline{m})$ is sufficiently low, so that $IR(\underline{m})$ is not binding at an optimum, the optimal ex ante contract realizes the marginal trade off between rent extraction and efficiency so familiar in the one period setting except that it takes place here in the contract space. Such a contract is represented in figure 2. If $IR(\underline{m})$ is not binding, then $IC(\underline{m})$ must be binding; hence, given that the efficient agent is offered an efficient contract, the program becomes

$$\begin{aligned}
&\max_{u} p\pi(u(\underline{m}), \underline{m}) + (1-p)\pi(u(\overline{m}), \overline{m}) \\
&\text{s.t. } \mathrm{E}(u(\underline{m}) - u(\overline{m})|\underline{m}) = 0 \\
&\qquad \mathrm{E}(u(\overline{m})|\overline{m}) = 0.
\end{aligned}$$

After substituting for the constraints and taking the constant term $p(s^*(\underline{m}) - \overline{u}(\underline{m}))$ out of the maximand, the principal is maximizing

$$\max_{u(\overline{m})} -pb(u(\overline{m})) + (1-p)s(u(\overline{m}), \overline{m}). \tag{18}$$

where $b(u(\overline{m})) = \mathrm{E}(u(\overline{m})|\underline{m}) - \overline{u}(\underline{m})$ is the ex ante expected rent left to agent $\underline{m}$. Increasing the ex post efficiency $s(u(\overline{m}), \overline{m})$ of the bad ex ante type contract $u(\overline{m})$ increases the rent $\mathrm{E}(u(\overline{m})|\underline{m}) - \overline{u}(\underline{m})$ that must be left to the good type. If the reservation utility of the efficient agent is low, that process is costly and the inefficient agent is offered a very inefficient contract, that is a contract, close to the 45° line, that involves little risk, hence, little ex post type separation.[5] Program (18) trades off these rents in the first period because the the information revealed can then be used to devise an optimal self-selecting scheme in the second period (at least for type $\underline{m}$). Under such contract, an agent who is inefficient ex ante (type $\overline{m}$) but efficient ex post (type θ_1) will be offered to produce inefficiently. The traditional trade off between informational rent extraction and efficiency applies here to the extent that efficiency is measured with respect to contracts, not production.[6]

6 Contracting Under No Commitment

The analysis so far was performed under the assumption that the principal and the agent could commit themselves to a long-term contract. In some cases, the players cannot credibly commit themselves ex ante to abide by any contract. Even when such commitment is possible, ex ante contracting might

[5] I do not provide a full characterization of what is going on when the IR curves cross in U''. This case will generally result in the agent $\underline{m}$ being offered and efficient contract while the ex ante efficient type $\overline{m}$ will be offered an inefficient contract that involves, ex post, overproduction to a point where marginal cost is higher than marginal utility.

[6] The analysis is made under the implicit assumption that the principal never wishes to shut down the ex ante inefficient agent. That is ensured if the proportion of ex ante type $\overline{m}$ is not too low, that is,

$$p \leq \frac{\pi(u(\overline{m}), \overline{m})}{\pi(u(\overline{m}), \overline{m}) + b(u(\overline{m}))},$$

where $u(\overline{m})$ is the optimal separating contract for type $\overline{m}$. Otherwise, (in the case where the reservation utilities are the same for both ex ante types) the principal will maximize his payoff by offering the efficient agent $\underline{m}$ an efficient contract in U^* that sets him on his participation constraint.

leave the players ex post in a position where Pareto improving amendments of the original contract are available. These are the cases of non commitment or default and of commitment with renegotiation. Both cases will be treated by imposing additional constraints on the set of feasible ex ante contracts. Contracts robust to the possibility of default must satisfy ex post individual rationality (or participation constraints) while those robust to renegotiation must be *renegotiation-proof*. I examine the impact of these constraints in turn.[7]

The non commitment hypothesis does not trivialize ex ante contracting. On the agent's side, it means that we must deal with as much participation constraints as there are ex post states θ since the ultimate decision to abide by the contract will be taken ex post. The expected individual rationality constraints (17) are replaced by

$$IR: \quad u(m) \geq \overrightarrow{u}(m) \quad \forall m \in \mathcal{M}.$$

where $\overrightarrow{u}(m)$ is the $N \times 1$ vector of best payoffs an agent of ex ante type m can gain by leaving the relationship in period 2. Geometrically, the effect of such constraints is to replace each half-space $E(u(m)|m) \geq \overline{u}(m)$ by a perpendicular convex cone whose vertices are given by $\overrightarrow{u}(m)$. The literature on how to deal with these constraints is still in its infancy and is beyond the scope of this paper[8] so I will restrict myself to the specific familiar case where $\overrightarrow{u}(m)$ is a vector composed of a single constant $\overline{u}(m)$. In that case, the cone points on the 45° line. When $N = 2$, imposing ex post participation constraints amounts to rotate back to the horizontal the ex ante participation constraint of the agent. This will restrict the set of contracts available to the principal. Obviously, it must be assumed that the principal bears the same commitment incapacity or that the agent has serious liquidity constraints for that restriction to be effective. Otherwise, the principal can simply post a bond, whose payments are contingent on the agent's ex post behavior, that the agent will buy ex ante.

[7]The possibility of default or renegotiation introduces new nodes in the course of the game. Care must be taken in handling the players beliefs at these nodes while constructing a PBE. But PBE are not very restrictive with respect to the admissible ex post beliefs off equilibrium paths: to pin down an equilibrium, I assume that if the agent initially refuses the contract or if he proposes an unexpected renegotiation at the interim stage, then the principal is therefore convinced that the agent is of the most efficient type ex ante and in the most favorable state θ.

[8]See Julien [6].

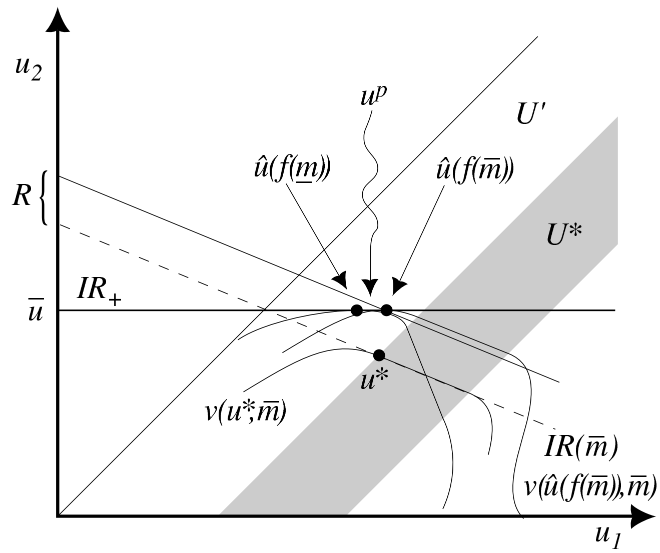

Figure 3: The Classical Principal-Agent Problem.

In figure 3, for instance, I represent the classical principal-agent problem when there is a single ex ante type m and $N = 2$ where $\phi = f(m)$. When a full commitment contract is available, then full efficiency can be achieved with a contract υ^*, anywhere along the intersection between IR and U^* where the utility of the principal is maximized at $\pi(\upsilon^*, m)$. But if the contract has to be individually rational ex post or, equivalently, if we assume that the initial contract is not binding, then, in effect, we are restricting the contract space to contracts above the horizontal line IR_+. In this case, the principal's utility is maximized at $\hat{\upsilon}(\phi)$ yielding[9] $\pi(\hat{\upsilon}(\phi), m)) < \pi(\upsilon^*, m)$. This contract gives an expected informational rent of R to the agent and is not efficient since it lies outside U^*. Contract $\hat{\upsilon}(\phi)$ is easily identified by solving the first-order condition $\pi_1(\hat{\upsilon}(\phi), m) = 0$, that is,

$$(1-\phi)(1-c_x(\hat{\xi}(\hat{\upsilon}(\phi)), \theta_2))\frac{\partial\hat{\xi}}{\partial\upsilon}(\hat{\upsilon}(\phi)) - \phi = 0,$$

where $\hat{\xi}$ is the inverse of the rent function (see the proof of proposition 5). Its derivative will equal the inverse of the derivative of the rent function so

[9]Actually noted $\hat{u}(f(m))$ on the figure.

that

$$(1-\phi)(1-c_x(\hat{\xi}(\hat{v}(\phi)),\theta_2)-\phi(c_x(\hat{\xi}(\hat{v}(\phi)),\theta_2)-c_x(\hat{\xi}(\hat{v}(\phi)),\theta_1))=0.$$

Hence, contract $\hat{v}(\phi)$ realizes the familiar trade off between marginal expected production inefficiency $(1-\phi)(1-c_x(\hat{\xi}(\hat{v}(\phi)),\theta_2)$ and the marginal expected informational rent to be left to ex post efficient type θ_1, $\phi(c_x(\hat{\xi}(\hat{v}(\phi)),\theta_2)-c_x(\hat{\xi}(\hat{v}(\phi)),\theta_1))$. As ϕ increases – say, from $f(\overline{m})$ to $f(\underline{m})$) – the contract is pushed away from U^* along IR_+:

$$\frac{\partial \hat{u}_1}{\partial \phi}(\phi)=\frac{1}{(1-\phi)\pi_{11}}<0. \tag{19}$$

with $\hat{u}_2(\phi)=\overline{u}$. If there are many ex ante types and commitment is a problem, the principal will not try to separate ex ante types as the following proposition shows.

Proposition 8. *Suppose that $M=2$, $\overline{u}(\underline{m})=\overline{u}(\overline{m})$, $N=2$ and that both players can't credibly commit themselves in the long run. Then the optimal contract $(u(\underline{m}),u(\overline{m}))$ is a pooling contract (v^p,v^p) where v^p is inefficient and lies somewhere between $\hat{v}(f(\underline{m}))$ and $\hat{v}(f(\overline{m}))$.*

Full commitment contracts achieve higher levels of efficiency by using ex post penalties as an instrument to induce ex ante type separation, while keeping expected transfers low. These penalties are no longer available under no commitment so that separating ex ante types is no longer an interesting option.

The difference between contracts $\hat{v}(f(\underline{m}))$ and $\hat{v}(f(\overline{m}))$ lies in the amount of distortion that is put on the production plan of the ex post type θ_2 (the distance from the 45° line). As usual, that distortion is imposed in order to separate the ex post types while minimizing the expected rent to be left to the ex post efficient type θ_1. The distortion is greater for the ex ante type $\underline{m}$ because he is more likely to be ex post of type θ_1. If both ex ante type are present, at least one ex ante type m will obviously be offered a contract v^p on IR_+. Strict ex ante separation could then be achieved by offering a contract $u(m')$ to the other ex ante type that lies above IR_+ and his indifference curve going through v^p. All these contracts yield less utility to the principal than v^p so that (v^p,v^p) is to be expected as an ex ante pooling contract. Hence, ex ante separation does not occur not because it is infeasible but because it does not maximize the principal's payoff.

The contract υ^p does separate ex post types θ. It is efficient with respect to weighting the need for ex post efficiency in production and ex post rent minimization under ex post revelation constraints. The weight used to locate υ^p is the unconditional probability of facing an ex post efficient type

$$f_p = pf(\underline{m}) + (1-p)f(\overline{m})$$

which lies between $f(\overline{m})$ and $f(\underline{m})$. In effect, the absence of commitment yields a similar outcome to the one we would get by postponing the contracting date once the uncertainty about θ had resolved. Then, both ex ante types $\underline{m}$ and $\overline{m}$ would be behaviorally equivalent, regardless of their ex post type, and the principal would offer a single menu based on his unconditional prior f_p.

7 Renegotiation-proof Contracts

In this section, I explore the issue of renegotiation in my framework. I assume that any offer of renegotiation is to be made by the principal and that it only needs to provide the agent as much utility as he can expect under the status quo (the ex ante contract) to be accepted. Since the principal has no private information, offers of renegotiation won't carry any signalling feature. Renegotiation may be considered at different points in time in my model:

interim-m: just after the contract has been signed ex ante but prior to the agent announcing his type m;

ex post-m: just after the agent has announced his type m but before the ex post type θ is realized;

interim-θ: just after θ is realized but before the agent has announced his ex post type;

ex post-θ: just after the agent has announced his ex post type but prior to production taking place.

The first and the last forms of renegotiation are of lesser interest here since they don't affect the nature of the final contract once all the proper incentives, induced by the common knowledge possibility of renegotiation, have been incorporated in the analysis[10].

For a contract to be robust to the possibility of renegotiation, it should always prescribe allocations that are on the Pareto frontier at any node where renegotiation is assumed possible. Consider first the possibility of ex post-m renegotiation. The contract pairs I derive in section 5, when the IR curves cross in U', are not renegotiation-proof when they prescribe an inefficient contract for type $\overline{m}$, that is $u(\overline{m}) \notin U^*$. A contract that is robust to ex post-m renegotiation will thus have to be in U^*. By corollary 7.1, for any such contract, there exists a pooling contract v^p in U^* that does just as well. Hence, any optimal ex post-m renegotiation-proof contract $(u(\underline{m}), u(\overline{m}))$ will solve

$$\mathrm{E}(\pi(u(\tilde{m}), \tilde{m})) = \max_{\substack{v^p \in U^* \\ \mathrm{E}(v^p|m) \geq \overline{u}(m), \\ \forall m \in \mathcal{M}}} \mathrm{E}(\pi(v^p, \tilde{m})).$$

It is straightforward to see that any such contract, conditionally on belonging to U^* and on satisfying the participation constraints, will in fact minimize the expected transfers to the agent,

$$\max_{\substack{v^p \in U^* \\ \mathrm{E}(v^p|m) \geq \overline{u}(m), \\ \forall m \in \mathcal{M}}} \mathrm{E}(\pi(v^p, \tilde{m})) = \mathrm{E}(\xi^* - c(\xi^*, \tilde{\theta})) \; - \min_{\substack{v^p \in U^* \\ \mathrm{E}(v^p|m) \geq \overline{u}(m), \\ \forall m \in \mathcal{M}}} \mathrm{E}(v^p).$$

The effect of renegotiation constraints is to make separation more costly or, put differently, rent extraction more difficult to the principal. Hence, unless his participation constraint is so high that pooling in U^* would have had occurred anyway (when the IR constraints of both agents crosses in U^*), the ex ante efficient agent will gather more surplus when ex post-m renegotiation is possible.

If a contract is robust to ex post-m renegotiation, it is naturally robust to interim-θ renegotiation. To the extent that the possibility of renegotiation is costly ex ante to the principal (it constrains the set of feasible contracts), he might wish to devote resources to improve his commitment capabilities and relax the ex post-m renegotiation-proof constraints. Besides, it is plausible that such commitment decreases over time. This raises the possibility that the principal can commit himself not to renegotiate the contract in the short

[10]In particular, the possibility of ex post-θ renegotiation is eliminated if we assume that the announcement of the ex post type is implicitly made by the very irreversible act of producing. See Beaudry and Poitevin [2].

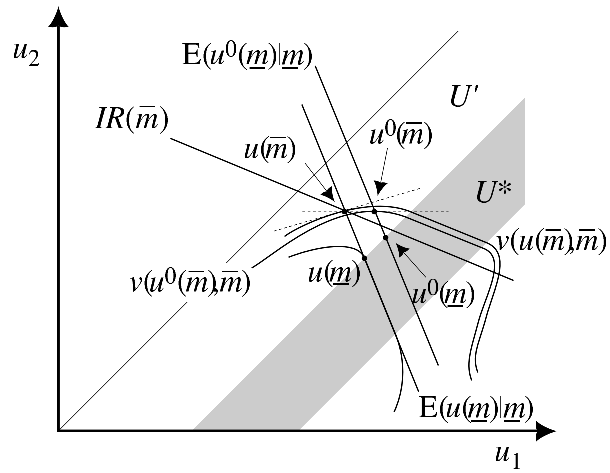

Figure 4: Interim-θ Renegotiation.

term (ex post-m type of renegotiation) but not in the long term (interim-θ type of renegotiation).

I define a interim-θ renegotiation-proof contract as a pair of contracts $(u(\underline{m}), u(\overline{m}))$ such that $\pi(u(m), m) \geq \pi(u(m)+t)$, for all non-negative $N \times 1$ vectors t. The idea is that it should not be possible to improve the principal utility, once the types m have been separated, simply by giving more utility to the agent with respect to what he can expect under the original contract he chose. Equivalently, one can states that $u(m)$ is interim-θ renegotiation-proof for ex ante type m if

$$0 \in \operatorname*{argmax}_{t \geq 0} \pi(u(m) + t, m).$$

Hence, assuming differentiability of π at $u(m)$, $\pi_i(u(m)) \leq 0$, for all $i \in I$, is a necessary condition for $u(m)$ to be renegotiation-proof. When $N = 2$, this implies that the indifference curve of the principal at $u(m)$ should not be upward sloping. Obviously, all contracts U^* are interim-θ renegotiation-proof so that only contracts designed for type $\overline{m}$ may be subject to that kind of renegotiation. Hence, to find the best interim-θ renegotiation-proof contract, I add the constraint

$$\frac{\pi_1}{\pi_2}(u(\overline{m}), \overline{m}) \geq 0$$

to program (15). Since[11] $\pi_2 = -(1+\pi_1)$, for the optimal separating contract, the interim-θ renegotiation-proof constraint will bind. That is, we will need to set $\pi_1(\overline{m}, \overline{m}) = 0$ like we did for the no commitment case.

The effect of ex-post-θ renegotiation is illustrated in figure 4. There, I assume that the IR curve of the efficient agent is sufficiently low so that it is not binding. The optimal self-selecting pair of contracts is to offer an inefficient contract $u(\overline{m})$ to type $\overline{m}$ so that the marginal production distortion cost of that contract equals the marginal informational rent that has to be left to the efficient type, for any contract u^* in the intersection of the indifference curve that goes through $u(\overline{m})$ and U^*. But $u(\overline{m})$ is not interim-θ renegotiation-proof because the indifference curve of level $\pi(u(\overline{m}), \overline{m})$ that goes through it is upward-sloping at $u(\overline{m})$. Once the agent has committed himself to $u(\overline{m})$, the principal would then try to renegotiate at $u^0(\overline{m})$. That renegotiation would be anticipated by $\underline{m}$ and he will pretend to be of type $\overline{m}$ because $\mathrm{E}(u^0(\overline{m}) - u(\underline{m})|\underline{m}) > 0$. Contract $(u^0(\underline{m}), u^0(\overline{m}))$, on the other hand, is renegotiation-proof and it specifies, for the ex ante inefficient type $\overline{m}$, a similar contract to the non commitment case.

The interim-θ renegotiation constraint is binding for agent $\overline{m}$ because the ex ante separation of types $\underline{m}$ and $\overline{m}$ distorts the interim efficiency of the ex post contract offered to him. This does not occur with standard models of renegotiation since the global incentive compatibility constraints, with respect to the revelation of the ex ante type, are not present.

The possibility of ex post-m renegotiation implies that contracts should lie in U^* while that of interim-θ renegotiation only implies that contracts should lie on decreasing portions of the principal's indifference curves. Since all contracts in U^* satisfy that last property anyway, the effect of ex post-m renegotiation on the principal expected profit is more stringent than that of interim-θ renegotiation.

The possibility of interim-θ renegotiation is less harmful to the principal than that of ex post-m renegotiation because it does not preclude separation with respect either to the ex ante type or the ex post type. Ex post-m renegotiation precludes such separation for ex ante types. Unlike the case of ex post-m renegotiation, interim-θ renegotiation still allows the principal to devise distortions in production plans to better extract rent from the ex post efficient agent. In both types of renegotiation, the ex ante type m is announced to the principal, on the equilibrium path, prior the opportunity

[11] See the proof of proposition 5 in the appendix.

to renegotiate arises. That announce has a dramatic effect when the opportunity to choose an ex ante efficient contract in U^* is still available but it only leads to a revision of the ex post principal's prior with respect to the ex post type in the case of interim-θ renegotiation.[12]

8 Conclusion

In this paper, I have sketched a theory of sequential screening contracts in a principal-agent framework where the agent learns his two-dimensional type sequentially. Contrary to Baron and Besanko [1], I have handled the global incentive compatibility constraints at the initial contracting stage. These constraints turn out to be binding at the optimum. I have shown how the trade-off between efficiency and rent extraction is transposed in the contract space with sequential types. The form of the optimal contract depends crucially on the nature of the commitment assumption. Under full commitment, the optimal contract will often imply ex ante as well as ex post separation of types. Ex ante pooling will occur only if ex post efficiency in production can be ensured for all ex ante types. By contrast, under no commitment, the optimal contract will pool types ex ante and will be inefficient ex post. Two forms of commitment with renegotiation were analyzed. If the contract can be renegotiated once the ex ante type has been announced but prior to the agent learning his ex post type, then the renegotiation-proof contract will be an efficient contract that pools type ex ante. If the contract can only be renegotiated once the agent has learned his ex post type, ex ante type separation might still be optimal but the renegotiation constraints will be binding and that will reduce the set of feasible contracts. The ex ante inefficient agent will be offered a contract similar to the one of the non commitment case while the efficient agent will be offered an efficient contract.

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A Appendix

A.1 Non optimality of randomized schemes

In section 3, I state that we can restrict our attention to non randomized contract schemes. According to the revelation principle for multistage games, the general contractual process should be subject to the outcome of two lotteries, one for each stage, whose distributions depend on the history of the announcements made by the agent and the outcomes of the lotteries. Let L_1 and L_2 be the supports of these lotteries and α_1 and α_2 their outcome. A contract is then a set of four functions; two that map history and announcements into the spaces ΔL_1 and ΔL_2 of distributions onto L_1 and L_2, while the last two map into R. Formally, δ is represented by

$$\begin{aligned}
\delta_1 &: M \to \Delta L_1 \\
\delta_2 &: M \times L_1 \times \Theta \to \Delta L_2 \\
\xi_\delta &: M \times L_1 \times \Theta \times L_2 \to R \\
t_\delta &: M \times L_1 \times \Theta \times L_2 \to R
\end{aligned}$$

where $\delta_1(m)$ and $\delta_2(m, \alpha_1, \theta)$ are the lotteries.

Yet, I claim that the optimal contract won't be randomized. To see this, suppose δ is an optimal randomized self-selecting contract and let F be the joint distribution of m and θ ($dF(\theta, m) = p_m f_m(\theta)$). δ must then satisfy the following equations:

$$\begin{aligned}
\delta \in \underset{\delta \in \Gamma}{\operatorname{argmax}} \int_{\Theta \times M} \int_L \int_L &(\xi_\delta(m, \alpha_1, \theta, \alpha_2) - c(\xi_\delta(m, \alpha_1, \theta, \alpha_2), \theta) \\
&- u_\delta(m, \alpha_1, \theta, \alpha_2)) d\delta_2(\alpha_2; m, \alpha_1, \theta) d\delta_1(\alpha_1; m) dF(\theta, m) \quad (20)
\end{aligned}$$

subject to

$$IC: \quad m \in \underset{m' \in \mathcal{M}}{\operatorname{argmax}} U(\delta, m, m') \quad \forall m \in \mathcal{M}, \quad (21)$$

where

$$U(\delta, m, m') = \int_\Theta \int_L \int_L u_\delta(m', \alpha_1, \theta, \alpha_2) d\delta_2(\alpha_2; m', \alpha_1, \theta) d\delta_1(\alpha_1; m') f_m(\theta),$$

and

$$IR: \quad \max_{m' \in \mathcal{M}} U(\delta, m, m') \geq \overline{u}(m) \quad \forall m \in \mathcal{M}, \quad (22)$$

$$IC_2: \quad \theta \in \underset{\theta' \in \Theta}{\text{argmax}} \int_L (u_\delta(m, \alpha_1, \theta', \alpha_2) + c(\xi_\delta(m, \alpha_1, \theta', \alpha_2), \theta') - c(\xi_\delta(m, \alpha_1, \theta', \alpha_2), \theta)) d\delta_2(\alpha_2; m, \alpha_1, \theta') \geq 0 \quad \forall m \in \mathcal{M}, \quad \forall \alpha_1 \in L, \quad \forall \theta \in \Theta. \quad (23)$$

Equation (20) states that δ is optimal, (21) and (23) are the ex ante and ex post incentive compatibility constraints and (22) is the participation constraint of the agent. Consider the non randomized contract $\overline{\delta}$

$$\xi_{\overline{\delta}}(m, \theta) = \int_L \int_L \xi_\delta(m, \alpha_1, \theta, \alpha_2) d\delta_2(\alpha_2; m, \alpha_1, \theta) d\delta_1(\alpha_1; m)$$
$$u_{\overline{\delta}}(m, \theta) = \int_L \int_L u_\delta(m, \alpha_1, \theta, \alpha_2) d\delta_2(\alpha_2; m, \alpha_1, \theta) d\delta_1(\alpha_1; m)$$

that specifies the expected instruments of δ conditional on the agent's type. Contract $\overline{\delta}$ is strictly preferred, by the principal, to δ because of the strictly convex costs. By simple substitution, it is then straightforward to see that (21) and (22) are unaffected by switching to $\overline{\delta}$. Finally, if θ maximizes (23) for all possible values of α_1, it does so for any convex combination over L so that, once I integrate the maximand times $d\delta_1(\alpha_1; m)$ over L, θ still maximizes it. Simple substitution then shows that $\overline{\delta}$ satisfies the same requirements as δ for (23). Since any randomized contract is strictly dominated by a non randomized scheme, there is no loss of generality to restrict our attention to the latter.

Proof of proposition 1.
I have already shown the necessary part. To see that these two conditions are sufficient, I only need to find a ξ that complies with incentive compatibility. Consider ξ such that its elements ξ_i's solve

$$r_{Ni}(\xi_i) = v_i - v_N \quad \forall i \in I.$$

Such a ξ unambiguously exists and is unique because of assumption (1). The incentive compatibility constraints (10) become

$$v_i - v_N \geq v_i - v_j \geq v_j - v_N \quad \forall i \in I \setminus \bar{i}, \forall j \in I_i.$$

The l.h.s. is trivially true because utility decreases with i while the r.h.s. is true because u satisfies internal consistency.

Hence U is completely defined by (9) and by the internal consistency constraints (11). These are linear weak inequalities so that U is closed and convex. Clearly, if υ and υ' satisfy these conditions, so is their sum $\upsilon + \upsilon'$ and $\alpha\upsilon$, the product of υ by a non negative scalar α. This makes U a convex cone. □

Proof of proposition 2.
Simply build the set of contracts associated to ξ^* such that

$$\upsilon_i^* = b_i + \overline{u} + x_i^* - c(\xi_i^*, \theta_i), \tag{24}$$

where $\overline{u}$ is an arbitrary constant and the b_i's belong to an open ball $B(\epsilon)$ of radius ϵ around $\tilde{\upsilon} \in U$, that is $\sum_I b_i^2 < \epsilon^2$. Take $\epsilon = 0$ so that $u^* = \tilde{\upsilon}$ and suppose now that $\tilde{\upsilon} \notin U^*$. Then, at least for some i, (8) must be violated for some j. This would yields

$$\xi_i^* - c(\xi_i^*, \theta_i) < \xi_j^* - c(\xi_j^*, \theta_i), \tag{25}$$

which is impossible since x^* maximizes social surplus so that ξ_i^* should creates at least as much surplus than ξ_j^* for type θ_i. Hence, $\tilde{\upsilon}$ belongs to U^*. Suppose now that there is no ball $B(\epsilon)$ around $\tilde{\upsilon}$ so that all points in $B(\epsilon)$ belong to U^*. Then one of the incentive compatibility constraints has to be binding at $\tilde{\upsilon}$. But this implies that (25) must hold with equality for some (i, j) which is also impossible. Hence, such a ball $B(\epsilon)$ exists and that makes U^* of non-empty interior. Closeness and convexity comes from the fact that U^* is described by the weak inequalities (8) evaluated at x^*.

Finally, if $\upsilon^* \in U^*$, then it accepts decomposition (24) where

$$b_i + \overline{u} = \upsilon_i^* + c(\xi_i^*, \theta_i) - x_i^*.$$

Hence, equation (24) does completely characterizes all contracts in U^*.

Note that if a vector of payoffs υ is in U^*, then shifting υ by a constant does not affect that property; that is, if $\upsilon \in U^*$, then $\upsilon + \overline{u}e \in U^*$ where $\overline{u}$ is an arbitrary constant and e is a vector of ones. □

Proof of proposition 3.
Taking the conditional expectation on (24) yields

$$\mathrm{E}(\upsilon^*|m) = \mathrm{E}(b|m) + \overline{u} + s^*(m)$$

where $s^*(m)$ is defined like in equation (28) of proposition 6. It suffices then to choose b and $\overline{u}$ so that

$$\mathrm{E}(b|m) + \overline{u} = \mathrm{E}(u(m)|m) - s^*(m).$$

Likewise, any utility level of the principal $\pi(u(m), m)$ can be reached through the use of an efficient contract by setting

$$\mathrm{E}(b|m) + \overline{u} = -\pi(u(m), m).$$

□

Proof of proposition 4.
Along the indifference curve $\Gamma_{\overline{\pi}}$, we have

$$\pi(u(m), m) = \pi(v^*, m)$$
$$s(u(m), m) - \mathrm{E}(u(m)|m) = s(v^*, m) - \mathrm{E}(v^*|m).$$

Hence,

$$\mathrm{E}(v^*|m) - \mathrm{E}(u(m)|m) = s(v^*, m) - s(u(m), m) \geq 0.$$

Suppose the equality stands and $u(m) \notin U^*$; then, by proposition 3, there is a $u^{**} \in U^*$ such that $\mathrm{E}(u(m)|m) = \mathrm{E}(v^{**}|m)$. Since v^{**} is efficient, it must be strictly preferred by the principal to $u(m)$, hence $\pi(v^{**}, m) > \pi(u(m), m) = \pi(v^*, m)$ but that implies $s(u^{**}, m) > s(v^*, m)$ which is impossible.

To prove the corollary, let $\overline{\pi} = \pi(v_0, m)$ and suppose that $v_0 \notin U^*$. Take $v^* \in \Gamma_{\overline{\pi}}$. By proposition 3, we must have $\mathrm{E}(v_0|m) < \mathrm{E}(v^*|m)$ but that is impossible since $\pi(v_0, m) = \pi(v^*, m)$ and $s(v^*, m)$ is maximal. □

Proof of proposition 5.
For any given $v \in U$, let $\hat{\xi}_1(v)$ and $\hat{\xi}_2(v)$ be the solution to program (7). These values must satisfies the self-selecting constraints (10) that define $X(v)$, that is,

$$r(\hat{\xi}_1(v)) \geq v_1 - v_2 \geq r(\hat{\xi}_2(v)). \tag{26}$$

By proposition 4, we know that the principal has linear indifference curves over U^* so that π is trivially quasi-concave on that portion of its domain. I want to prove that

$$\pi(v) = f(\xi_1(v) - c(\xi_1(v), \theta_1) - v_1) + (1 - f)(\xi_2(u) - c(\xi_2(u), \theta_2) - v_2),$$

is strictly quasi-concave over U'.

By assumption (2), the rent function r is monotonously strictly increasing. Hence, it has a well defined inverse $\hat{\xi}$ which I apply on constraints (26) to get:

$$\xi_1(v) \geq \hat{\xi}(v_1 - v_2) \geq \xi_2(v).$$

Depending on the value of $\hat{\xi} = \hat{\xi}(v_1 - v_2)$, the solution of (7) can be of three types:

$$(\xi_1(v), \xi_2(v)) = \begin{cases} (\hat{\xi}, \xi_2^*) & \text{if } \hat{\xi} > \xi_1^* \quad (v \in U'') \\ (\xi_1^*, \xi_2^*) & \text{if } \xi_1^* \geq \hat{\xi} > \xi_2^* \quad (v \in U^*) \\ (\xi_1^*, \hat{\xi}) & \text{if } \xi_2^* \geq \hat{\xi} \quad (v \in U') \end{cases} \tag{27}$$

where the (ξ_1^*, ξ_2^*) stands for the unconstrained solution that maximizes social surplus.

Since $\hat{\xi}$ is a function of the difference $v_1 - v_2$ alone, I can write

$$\frac{\partial \hat{\xi}}{\partial v_1} = -\frac{\partial \hat{\xi}}{\partial v_2} = \hat{\xi}' > 0.$$

One can check that the second derivative,

$$\hat{\xi}'' = -(\hat{\xi}')^3 (c_{xx}(\hat{\xi}, \theta_2) - c_{xx}(\hat{\xi}, \theta_1)) < 0,$$

is negative using assumption (2) so that $\hat{\xi}$ is a strictly concave function. Strict quasi-concavity of π can be checked by verifying if its bordered determinant is positive,

$$|B| = \begin{vmatrix} 0 & \pi_1 & \pi_2 \\ \pi_1 & \pi_{11} & \pi_{12} \\ \pi_2 & \pi_{21} & \pi_{22} \end{vmatrix} > 0.$$

When $u \in U'$, we have

$$\begin{aligned} \pi_1 &= (1 - f)(1 - c_x(\hat{\xi}, \theta_2))\hat{\xi}' - f \\ &= -\pi_2 - 1 \\ \pi_{11} &= -(1 - f)c_{xx}(\hat{\xi}, \theta_2)(\hat{\xi}')^2 - (\pi_1 + f)\hat{\xi}'' \\ &= -\pi_{12} = -\pi_{21} = \pi_{22}. \end{aligned}$$

and $|B| = -\pi_{11}$. That value is positive since $\pi_1 + f = (1-f)(1-c_x(\hat{\xi}, \theta_2))\hat{\xi}' > 0$ for $\hat{\xi} < \xi_2^*$, when $u \in U'$.

The proof of corollary 5.1 is trivial if $v \in U^*$. Suppose that $v \in U'$ and, without loss of generality, let $v^* \in \overline{h}$ so that $v_\alpha(m) \in U'$. Since v and v^* imply the same expected payment, it must be that $\pi(v, m) < \pi(v^*, m)$ and since π is strictly quasi-concave on U', this implies that $\pi(v_\alpha, m) > \pi(v, m)$. □

Proof of proposition 6. Suppose that the optimal contract is a pooling contract and that $\upsilon^p \notin U^*$. Consider offering with υ^p an efficient contract intended at type $\underline{m}$. Built that contract by adding payment $t(\underline{m})$ to υ^p such that

$$t_i(\underline{m}) = \mathrm{E}(\upsilon^p|\underline{m}) - \upsilon_i^p + \xi_i^* - c(\xi_i^*, \theta_i) - s^*(\underline{m}),$$

where

$$s^*(m) = \mathrm{E}(\xi^* - c(\xi^*, \theta)|m). \tag{28}$$

Such additional transfers worth zero for agent $\underline{m}$ while they have an expected value of

$$s^*(\overline{m}) - s^*(\underline{m}) + \mathrm{E}(\upsilon^p|\underline{m}) - \mathrm{E}(\upsilon^p|\overline{m}) \tag{29}$$

for agent $\overline{m}$. If (29) was non-positive, then the contract $\upsilon^p + t(\underline{m})$ could be offered to agent $\underline{m}$ without affecting any incentive compatibility constraint. The contract $(\upsilon^p + t(\underline{m}), \upsilon^p)$ would obviously be preferred by the principal and that would lead to a contradiction since (υ^p, υ^p) was assumed optimal. Hence, it must be that (29) is strictly as positive. Yet, we could construct a similar contract intended at type $\overline{m}$ so that (29) should be strictly negative. Since both conditions cannot be met by υ^p at the same time, it must be that $\upsilon^p \in U^*$. □

Proof of proposition 7. The proof is trivial when the IR lines cross in U^*. Besides, if the optimal contract was pooling ex ante type, then agent $\underline{m}$ would be offered an efficient contract by proposition 6. Hence, I want to show that the proposition holds even if the optimal contract separates ex ante types. Consider first that, for each agent, either his incentive compatibility or individual rationality constraint is binding, or both, at the optimum. Otherwise, reducing his expected payment by the minimum strictly positive slack value of these two constraints would have no effect on the agent performance, would not create any bad incentive and, yet, would improve the principal's payoff so that the original contract would not be optimal after all. Now, unless the contract is pooling ex ante types, the incentive compatibility constraints for types $\underline{m}$ and $\overline{m}$ cannot be binding at the same time since the 2×1 vector $u(\underline{m}) - u(\overline{m})$ can only be orthogonal to a single vector $[f(m), 1 - f(m)]$. Furthermore, the incentive compatibility constraint of agent $\overline{m}$ cannot be binding at an optimum. To see this, suppose that $IC(\overline{m})$ is binding so that $IC(\underline{m})$ is free. This implies that $u(\overline{m})$ and $u(\underline{m})$ belong to the same hyperplane that goes through some point $\upsilon^* \in U^*$ that yields an identical payoff to agent $\overline{m}$. Since $IC(\underline{m})$ is free, $u(\underline{m})$ is lying between $u(\overline{m})$

and υ^*. Corollary 5.1 then tells us that $\pi(u(\underline{m}), \overline{m}) > \pi(u(\overline{m}), \overline{m})$. Hence, the principal would be better off if $\overline{m}$ would choose contract $u(\underline{m})$. Since the original contract is dominated by a pooling contract that offers $u(\underline{m})$, it was not optimal in the first place. Since $IC(\overline{m})$ is free, it must be that $IR(\overline{m})$ is binding. Now, suppose that $u(\underline{m})$ is not efficient. Since $IC(\overline{m})$ is free at the optimum, proposition 3 allows us to assign to $\underline{m}$ a contract $\upsilon^* \in U^*$ that brings him the same expected transfer as $u(\underline{m})$ and strictly improves the principal's payoff. Hence, $u(\underline{m})$ must be efficient. □

Proof of corollary 7.1.
By proposition 7, we know that $IR(\overline{m})$ is binding at the optimum and that $IC(\underline{m})$ and $IR(\underline{m})$ cannot be free at the same time. Obviously, $u(\underline{m})$ must lie on or below the hyperplane delimited by $IR(\overline{m})$ otherwise, choosing $u(\overline{m})$ on $IR(\overline{m})$ would be a dominated strategy for $\overline{m}$. Suppose that $IC(\underline{m})$ is binding, then the pooling contract such that $\upsilon^p = u(\overline{m})$ is optimal as well. If it is $IR(\underline{m})$ that binds, consider the pooling contract υ^p where both $IR(\underline{m})$ and $IR(\overline{m})$ bind. That contract must be in U^* otherwise $IC(\underline{m})$ could not be satisfied in the first place. □

Proof of proposition 8.
Since both agents $\underline{m}$ and $\overline{m}$ have the same ex post reservation utility, both contracts $u(\underline{m})$ and $u(\overline{m})$ must lie on or above the same horizontal hyperplane IR_+. Obviously, at least one of these two contracts, say $u(m)$, must lie on IR_+ otherwise the transfers of both contracts could be reduced by a same amount, at the principal benefit.

Suppose now that, for some $m \in \mathcal{M}$, $u(m)$ lies in U'' at the optimum; then the contract designed for $m' \neq m$ would be sought by maximizing $\pi(u(m'), m')$ subject to the incentive compatibility constraints and IR_+: $u(m')$ would be set in U^* but the resulting separating contract would be dominated by a pooling contract υ^* in U^* (see figure 5). Now, any incentive compatible separating contract where $u(m)$ is not in U'' is weakly dominated by a pooling contract that offers $u(m)$ so that we can restrict our search for an optimum to pooling contracts that lies on IR_+. In that class, the optimal pooling contract (υ^p, υ^p) solves,

$$\pi(\upsilon^p, \tilde{m}) = \max_{\upsilon^p} \mathrm{E}(\pi(\upsilon^p, m)) \quad \text{s.t.} \quad (\upsilon^p, \upsilon^p) \in IR_+.$$

where the distribution associated to $\tilde{m}$ is $f_p = pf(\underline{m}) + (1-p)f(\overline{m})$. Since υ_1^p is monotonic with respect to f from (19), it must be that υ^p lies between contracts $\hat{\upsilon}(f(\underline{m}))$ and $\hat{\upsilon}(f(\overline{m}))$ on IR_+. □

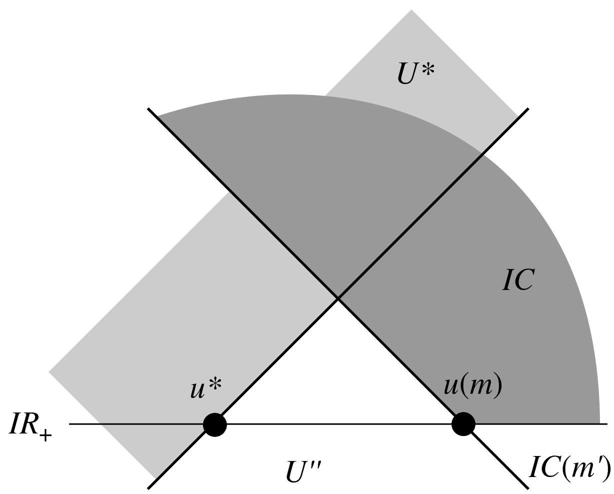

Figure 5: Proof of proposition 8.