A NORMATIVE APPROACH TO MEASURING CLASSICAL HORIZONTAL INEQUITY

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December 1996

* The idea for this paper came from a remark made by David Donaldson when Lambert presented a paper on horizontal inequity at the University of British Columbia in June 1995. Work began when Lambert was a Visiting Scholar at the International Monetary Fund, and this was continued when both authors attended the Canadian Public Economics Study Group in Quebec in June 1996. We owe our thanks to these organizations, and to many individuals, including particularly Michael Hoy, Vito Tanzi and Matthew Turner for their helpful observations. Duclos also acknowledges FCAR and SSHRC grants as well as the research assistance of Philippe Grégoire and Manon Rouleau. All inadequacies in the paper are ours alone.

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Résumé

Nous nous intéressons dans ce document au problème tenace de la mesure de l'iniquité horizontale. Nous proposons tout d'abord une mesure locale d'iniquité horizontale, que nous agrégeons ensuite en un indice global. À la différence d'autres approches, qui s'intéressent au gain en bien-être social qu'il y aurait à éliminer l'iniquité horizontale en gardant constant le revenu de l'État, notre indice global mesure le gain fiscal per capita qui échoirait au gouvernement si l'iniquité horizontale était éliminée sans changement dans le niveau de bien-être social. Lorsque ce gain fiscal est exprimé en proportion des revenus moyens nets, notre mesure constitue un élément (négatif) dans l'indice de progressivité de Blackorby et Donaldson (1984); elle quantifie alors la perte de performance verticale causée par un traitement fiscal discriminatoire vis-à-vis d'individus qui étaient égaux avant l'intervention de l'État. Notre indice étant exprimé en unités monétaires, son interprétation est facile et intuitive. Nous proposons finalement des procédures d'estimation non-paramétrique pour résoudre le problème important de l'identification des individus égaux dans une distribution de bien-être. À notre connaissance, il s'agit d'ailleurs de la première solution statistiquement cohérente au problème classique de la mesure de l'iniquité horizontale. La méthode est appliquée à la distribution canadienne des revenus bruts et nets en 1981 et 1990.

Abstract

This paper makes a new attack on the old problem of measuring horizontal inequity (HI). A local measure of HI is proposed, and aggregated into a global index. Whilst other approaches have captured the welfare gain which would come from eliminating HI revenue-neutrally, our global index provides a measure of the revenue gain *per capita* which would come from eliminating HI welfare-neutrally. When expressed as a fraction of mean post-tax income, the measure can be viewed as a negative component in the Blackorby and Donaldson (1984) index of tax progressivity, quantifying the loss of vertical performance arising from differences in the tax treatment of equals. Being money-metric, the measure can also be easily and intuitively interpreted. We propose non-parametric estimation procedures to obviate the important 'identification of equals' problem. To our knowledge, this provides the first consistent statistical solution to measuring classical horizontal inequity. The method is applied to the Canadian distributions of gross and net incomes in 1981 and 1990.

1. Introduction

Horizontal equity (HE) demands that like individuals be treated alike, whilst vertical equity is a command to differentiate appropriately among unlike individuals. These principles can be extended to households and families. Violations of HE can arise, for instance, from the many tax breaks granted for charitable giving, mortgage interest, capital gains, dividends, political contributions or retirement savings, or due to tax evasion, incomplete benefit take-up, arbitrariness in the allocation of state benefits, or differentiation in indirect tax rates[1]. In general, indeed, it can be argued that any form of government intervention which impacts on relative prices in a world of heterogeneous individual consumption and investment preferences will lead to violations of horizontal equity.

In this paper, we outline a new procedure for measuring the extent of horizontal inequity (HI) and for decomposing the distributional impact of the income tax[2] into vertical and horizontal components. The starting point for our procedure is a local HI measure capturing the dispersion of post-tax welfare among pre-tax equals. Specifically, we adopt the 'cost-of-inequality' approach which has been described by Kay and King (1984).[3] When the local measure is aggregated into a global index, using a weighting scheme which ensures that the importance attributed to a local inequity does not depend upon the welfare level at which it is experienced, a global index results which, we show, measures the revenue gain *per capita* that would come from eliminating HI with no loss of social welfare in any equals group.

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[1] For such instances, see, e.g., Gravelle (1992), Bishop et al. (1994) and Duclos (1995b).

[2] Our methodology allows for negative income taxes, that is, for the presence of transfers. Our illustration below indeed features a number of state benefits as well as a positive income tax.

[3] "How much commission would we pay Robin Hood to transfer £1 from the rich to the poor? The answer will depend on our view of inequality .. we can imagine a continuing series of such transfers which eventually bring us to a wholly egalitarian outcome and measure the amount of income which we would be willing to give up in order to bring about this result. This total amount is the "cost of inequality": the reduction in aggregate income which we would accept in order to achieve complete equality in its distribution .. the size of these costs depends on how much we are offended by inequality" *(p. 221).*

As we explain, other recent approaches based on the local-to-global aggregation procedure, those of Aronson *et al.* (1994) and Lambert and Ramos (1996), have an opposite but symmetrical property: they capture the overall welfare gain that would come from eliminating HI revenue-neutrally within every equals group.

The new HI index enjoys a close connection with the Blackorby and Donaldson (1984) index of progressivity in the tax system. This index, we show, decomposes into two components. One, being positive, measures the distributional characteristic of the hypothetical (or reference) tax system in which the local inequities have been eliminated as described above; the other, negative, is our HI index measured as a fraction of mean post-tax income. By means of this decomposition, the analyst can describe the progressivity of the tax system in terms of vertical and horizontal contributions, and for different assumed values of the inequality aversion parameter.

We propose a non-parametric estimation procedure to assess the distribution of horizontal inequities. This allows us to solve statistically the important normative 'identification of equals' problem, according to which few exact equals can be found in small samples (and none from a continuous population distribution of welfare). The procedure is applied to the Canadian distribution of gross and net incomes in 1981 and in 1990.

The structure of the paper is as follows. In Section 2, we lay out briefly the necessary prerequisites for the analytical framework we shall adopt. In Section 3, the measurement system is specified and the theoretical results already indicated are made explicit and proven. In Section 4, we discuss implementation difficulties and also statistical and modelling issues. Section 5 contains the application to the Canadian income distribution for 1981 and 1990, and Section 6 concludes.

2. The analytical framework

Let x be income, not necessarily denominated in standard monetary terms; for example, it could be equivalized income, and could also encompass various forms of imputable income and non-market sources of utility and disutility. Let $\mathbf{x}$ be the income distribution vector and W($\mathbf{x}$) a homothetic social evaluation function, for which the equally distributed equivalent (henceforth EDE) income is ξ: $W(\xi\mathbf{1}) = W(\mathbf{x})$. For the empirical application to follow, $W(\mathbf{x})$ will be average utility, where $U_e(x)$ is the utility-of-income function which displays constant relative inequality aversion with parameter e:

$$U_e(x) = x^{1-e}/(1-e) \quad \text{if} \quad 0 < e \neq 1 \,, \quad U_1(x) = \ell n\ x \tag{1}$$

In this case, $U_e(\xi)$ measures average utility across $\mathbf{x}$ and ξ is a generalized mean of $\mathbf{x}$. Whatever the evaluation function, the cost of inequality in $\mathbf{x}$ is, in *per capita* terms:

$$C = \mu - \xi \tag{2}$$

where μ is the mean of $\mathbf{x}$. The Atkinson (1970) index of inequality for $\mathbf{x}$ is:

$$I(e) = C/\mu = 1 - \xi/\mu \tag{3}$$

Now suppose that $\mathbf{x^b}$ and $\mathbf{x^a}$ are distributions of income before and after application of an income tax, with Atkinson indices I^b and I^a respectively. The Blackorby and Donaldson (1984) index of progressivity may be defined in terms of the inequality reduction occurring in the transition from pre- to post-tax income distribution, as:

$$\Pi = [I^b - I^a] \,/\, [1 - I^b] \tag{4}$$

If we defined an index of *equality* as 1-I, Π would measure the percentage change in equality generated by the move from the pre-tax to the post-tax income distribution. Let the relationships between the pre- and post-tax means and EDE incomes be these:

$$(1-g)\mu^b = \mu^a \,, \quad (1-\gamma)\xi^b = \xi^a \tag{5}$$

Here, g is the rate of a proportional (henceforth *flat*) tax raising the same revenue as the actual tax system, whilst, by the linear homogeneity of ξ, γ is the rate of a flat tax for which post-tax welfare is the same as after the actual tax.[4] Substituting in (4) from (3) and (5), we have:

$$\Pi = [g - \gamma] / [1 - g] \tag{6}$$

Let C^a and C^f be the cost-of-inequality measures for the distributions of income $\mathbf{x^a}$ after the actual tax and $\mathbf{x^f} = (1-\gamma)\mathbf{x^b}$ after the equal-welfare flat tax. From (2) we have:

$$C^a = \mu^a - \xi^a \quad , \quad C^f = (1-\gamma).\mu^b - \xi^a \tag{7}$$

It follows using (5) that:

$$\Pi = [C^f - C^a] / \mu^a \tag{8}$$

That is, Π measures the percentage of after-tax income that a social decision-maker (SDM) with inequality aversion e would pay to convert a flat tax system with the same after-tax welfare into the actual one.[5] Or, said differently, Π measures the additional percentage of post-tax income which the actual tax system yields to the tax authority as compared to the yield of a proportional tax system, with identical social welfare from both tax systems.

3. The measurement system

The equals or like individuals in the HE command are, according to Feldstein (1976), those with the same utility. The injunction to treat like individuals alike has

[4] Duclos (1995a) defines a performance index τ for the tax system, which captures the distinction between g and γ. Specifically, τ is the proportional surcharge on post-tax incomes $\mathbf{x^a}$ which would reduce post-tax welfare to that after revenue-neutral flat tax: $(1-\tau)\xi^a = (1-g)\xi^b$. It follows from (5) that $(1-\tau) = (1-g) / (1-\gamma)$.

[5] The social decision maker would clearly pay a positive amount if the actual tax system is progressive (inequality-reducing): the more progressive, the more he would pay. It is also the case that the more inequality-averse he is, the more he would pay. Duclos (1995a,1996) demonstrates this, in respect of his performance index τ, which is related to Π by $\Pi = \tau/(1-\tau)$: compare (6) with the formula in the previous footnote.

also been extended to households and families.[6] The first step for our analysis is to turn the business of identifying the equals into a unidimensional problem. We shall require income units' pre-tax incomes, or living standards, x^b, to be measured on a scale which identifies the equals: *equals will be those having the same pre-tax income x^b.*

For all that is to follow, we assume that an appropriate scale has been devised[7]. The income unit may be the individual, the family, the household or the equivalent adult. For convenience we refer to an income unit as a person henceforth.

The starting point for empirical analysis is a pair of vectors $\mathbf{x} = \mathbf{x^b}$ and $\mathbf{x^a}$ representing the pre- and post-tax distributions. If a scatterplot is drawn, these typically evidence an unsystematic relationship. See Figure 1, which depicts a sample of Canadian individuals for the year 1990.[8] HI occurs when points on this scattergraph are vertically aligned: this depicts well the situation in which pre-tax equals have different post-tax incomes.

Let Ω_x denote the group of persons having exactly x before tax: this is the 'equals group' located at point x. Locally, we seek to capture as HI the magnitude of unequal tax treatment among these people. We may thus think of local HI as *inequality introduced by the tax system* where there was none before - that is, within each Ω_x. We use the cost-of-inequality approach, requiring the input of a specified degree of inequality aversion e on the part of an SDM as described in Section 2.

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[6] See Manser (1979, p. 224), Habib (1979, p. 286) and Steuerle (1983, p. 81).

[7] Money income would serve for x if the population under investigation consisted of people with identical tastes, needs and abilities. Manser (1979) discusses the modelling of household objectives including different leisure times of their members, and Rosen (1976) demonstrates an empirical procedure which, given rich enough microdata, will "generate two vectors, one of family utilities before tax and one of family utilities after tax", and he goes on to say that "the real problem in measuring horizontal equity is to summarize the differences between these vectors in a meaningful way" *(p. 314)*. Steuerle (1983) advocates equivalization as the means to provide "a working definition of equity" across family sizes. Jenkins (1988) argues against equivalizing, seeing the business of identifying the equals as an essentially multidimensional issue, and adopts instead a partial approach in which he refrains from making identifications across distinct socioeconomic subpopulations, capturing HI within each in terms of rank changes induced by the tax.

[8] The description of the data used to plot Figure 1 can be found in Section 5 ahead.

Thus let μ^b and ξ^b be the mean and EDE income levels overall before tax, and let μ^a and ξ^a be those after tax, as in Section 2. Further, let μ^a_x and ξ^a_x be the mean and EDE income after tax within equals group Ω_x. The SDM would give up an amount:

$$H_x = \mu^a_x - \xi^a_x \tag{9}$$

of post-tax income *per capita* within Ω_x to have that group's HE violations removed with no loss of welfare. This is our measure of local HI at x.

The next step is to aggregate the H_x , $x \in \mathbf{R}$, into a global index, call it H, using a weighting scheme. There are plenty of possibilities; we choose to use population shares as weights. Thus:

$$H = \Sigma_x\, p_x .\, H_x \tag{10}$$

where p_x is the proportion of the overall population who are located at point x on the pre-tax income scale: $p_x = N_x / N$, where $N_x = |\Omega_x|$ and $N = \Sigma_x\ N_x$ .This construction ensures that the importance attributed to a local HE violation does not depend upon the income level at which it is experienced. That is, H is not polluted with vertical considerations, heeding Musgrave's (1990) warning to avoid "inappropriate comparisons between unequals" *(pp. 117-8)* in constructing a global HI index.[9] There are also other benefits from choosing this particular weighting scheme, which will emerge shortly.

Defining:

$$T_{wn}(x) = x - \xi^a_x \quad \forall x \tag{11}$$

as the tax people would pay to remove HI with no loss of social welfare in Ω_x, we may call $T_{wn}(\bullet)$ the **welfare-neutral HE replacement tax**. Our first result shows that overall HI, as defined in (10), may be interpreted as the additional tax revenue *per capita* that would flow from replacing the actual tax system by this welfare-neutral replacement schedule:

[9] Global indices of HI proposed by Habib (1979), Berliant and Strauss (1985) and Aronson *et al.* (1994) use explicitly income-dependent weights for local inequities. For more on the last of these, see on.

Theorem 1

H measures the *per capita* gain in revenue that would come from substituting the tax system by $T_{wn}(\bullet)$.

Proof : This follows directly from the choice of weighting scheme. The income saving which would come from eliminating post-tax inequality with social indifference in Ω_x is H_x, and therefore the income saving overall, or additional tax revenue generated, is $\Sigma_x N_x . H_x / N = \Sigma_x p_x . H_x = H$. *Q.E.D.*

Of course there would be winners and losers from this hypothetical process of HI elimination. It is not a policy recommendation of this paper that we should identify and substitute $T_{wn}(\bullet)$ for the actual tax system; $T_{wn}(\bullet)$ serves as the yardstick against which the social cost of the HE violations in actual taxes can be assessed. An attractive feature of our index H is that it sets a *dollar value* upon HI, conditioned by the assumed inequality aversion e of the SDM: as e is increased, the SDM becomes willing to pay more to eliminate unequal tax treatment of equals, and measured HI therefore increases.

The locus of points (x,μ^a_x) as x varies traces out the pre-tax/post-tax income relationship which would obtain if tax payments were averaged at each pe-tax income level x. The schedule $T_{rn}(\bullet)$, defined by:

$$T_{rn}(x) = x - \mu^a_x \tag{12}$$

would collect the same revenue as the actual tax system from each equals group: we call it the **revenue-neutral HE replacement tax**. Such averaging out produces a welfare gain for the membership of Ω_x as well as an inequality reduction (Atkinson, 1970). Hence, comparing the two HE replacement tax schedules $T_{rn}(\bullet)$ and $T_{wn}(\bullet)$, the one delivers a better welfare performance than the actual system and the same revenue, whilst the other delivers the same welfare and more revenue.[10] Both schedules are illustrated for Canada in 1990 in Section 5 ahead. We then have that:

$$H_x = T_{wn}(x) - T_{rn}(x)$$

[10] The global HI indices of Aronson *et al.* (1994) and Lambert and Ramos (1996) capture the welfare gain from replacing the tax system by $T_{rn}(\bullet)$, to accord with Musgrave's (1990) view that: "applied to any one group of equals, HE performance is measured .... over what it would have been with equal division of liability within the group" *(p.117)*.

Our global HI index H can be interpreted as a loss of performance, where we measure the performance of an income tax relative to equal-welfare flat taxes.[11] Recall from Section 2 that γ is the rate of the equal-welfare flat tax, and that C^a and C^f, as defined in (7), measure the income savings that would arise if all inequality were to be eliminated with social indifference in the income distributions $\mathbf{x^a}$ and $\mathbf{x^f} = (1-\gamma)\mathbf{x^b}$ after application of the actual tax and the flat one respectively. Define as C^* the cost of inequality which remains after application of the tax $T_{wn}(\bullet)$. Then the two values:

$$P = C^f - C^a\,, \qquad P_{wn} = C^f - C^* \tag{13}$$

measure the performance of the actual tax system and of $T_{wn}(\bullet)$ respectively in reducing inequality: they are the amounts *per capita* which an SDM with inequality aversion e would pay to convert the flat (inequality-neutral) tax into the actual one and into $T_{wn}(\bullet)$ respectively. The result which secures the interpretation of H as a performance loss is this:

Theorem 2

$$P = P_{wn} - H$$

Proof : If all inequality of post-tax income were to be eliminated with social indifference, by moving from $\mathbf{x^a}$ to the distribution in which everybody got ξ^a, the income saving *per capita* would be C^a; whilst if we moved to perfect equality from the distribution after application of $T_{wn}(\bullet)$, in which people in Ω_x get ξ^a_x $\forall x$, the income saving would be C^*. Equating the overall income saving with the sum of those arising *(i)* from application of $T_{wn}(\bullet)$ and *(ii)* from subsequent equalization,[12] we have:

$$C^a = H + C^* \tag{14}$$

[11] In Aronson *et al.* (1994) and Lambert and Ramos (1996), the performance of the tax system is measured relative to equal-*revenue* flat taxes - that is, by redistributive effect.

[12] (14) is readily validated in formal terms as follows: use the formulae for C^a and H in (7) and (10); note that $C^* = \mu^* - \xi^a$, where $\mu^* = \Sigma_x\, p_x.\, \xi^a{}_x$ is the mean after application of $T_{wn}(\bullet)$; and use $\mu^a = \Sigma_x\, p_x.\, \mu^a_x$. This is a particular case of the general between-and-within-groups decomposition due to Blackorby *et al.* (1981).

The after-tax cost of inequality is therefore the cost of inequality after an horizontally equitable tax plus the cost of HI. The result of Theorem 2 follows by subtracting each side of (14) from C^f. *Q.E.D.*

It is both natural and convenient to measure the cost of HI as a fraction of the total income in the post-tax distribution.[13] By this, we turn H into an Atkinson-type index, say:

$$H_1 = H/\mu^a \tag{15}$$

which is, of course, unit-free and between 0 and 1.[14] In fact, H_1 is a weighted sum of the Atkinson indices of post-tax inequality across equals groups:

$$H_1 = \Sigma_x q_x.I^a_x \tag{16}$$

where q_x is the post-tax income share of the people in Ω_x.[15] The weighting scheme here is *non-pure*: to measure local HI as inequality I^a_x rather than cost of inequality H_x means, for example, that a 1% deviation of post-tax incomes around their mean in a 'richer' equals group counts more towards global HI than the comparable dispersion in a poorer one.[16]

The decomposition of Theorem 2 can be modified, to make H_1 into a measure of performance loss. For this, performance is measured not by P and P_{wn} but by the corresponding Blackorby and Donaldson (1984) indices of tax progressivity:

Theorem 3

Let Π and Π_{wn} be the Blackorby-Donaldson progressivity indices for the actual tax system and for $T_{wn}(\bullet)$ respectively. Then:

$$\Pi = \theta.\Pi_{wn} - H_1$$

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[13] David Donaldson suggested this approach, which is motivated by the work of Blewitt (1982).

[14] In the absence of HI, $H_1 = 0$. If a very large number of people all had the same pre-tax income, and one of them got all of the post-tax income, then $H_1 \to 1$.

[15] This follows because $H_x = \mu^a_x.\ I^a_x$ and $q_x = p_x\,\mu^a_x / \mu^a$, *i.e.* $q_x\, I^a_x = p_x\, H_x / \mu^a$. Now use (10).

[16] The Gini-based global HI index of Aronson *et al*. (1994) has a similar property. It takes the form $H = \Sigma\, p_x\, q_x\, G^a_x$, where G^a_x is the Gini coefficient of post-tax income in the equals group Ω_x. Lambert and Ramos's (1996) index, based on the mean logarithmic deviation, has pure weights.

where θ is the ratio of mean income after $T_{wn}(\bullet)$ to mean post-tax income μ^a ($\theta \leq 1$).

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Proof : Divide by μ^a in the result of Theorem 2, noting that $\Pi = P / \mu^a$ from (8) and $\Pi_{wn} = P_{wn} / \mu^*$ similarly. The result follows, with $\theta = \mu^* / \mu^a = \Sigma_x p_x. \xi^a{}_x / \Sigma_x p_x. \mu^a{}_x$ which is less than 1 because the EDE income $\xi^a{}_x$ is less than the mean $\mu^a{}_x$, $\forall$x. *Q.E.D.*

Because everything is expressed at constant welfare, there is a size effect in eliminating HI; this is why the value θ appears in the decomposition.

4. Implementation: statistical and modelling issues

If, as will generally be the case, one's sample micro-data is drawn from an (approximately) continuous joint population distribution of individual incomes x^a and x^b, the sample probability of observing exact equals is virtually zero. This is the 'identification of equals' problem which led in the late 1970s to the emergence of the 'reranking' approach to measuring HI.[17] However, using recent statistical advances, we can estimate non-parametrically and consistently the continuous population distribution of x^a and x^b using the empirical joint distribution of the two variables, and integrate over that estimated distribution to yield consistent estimates of the HI indices (H and H_1) and performance indices (P, P_{wn}, Π and Π_{wn}).

To be more precise, a consistent estimator $\hat{f}_{xa|x}$ of the conditional density function for x^a (given pre-tax income x) can be used to generate natural consistent estimators of $\xi^a{}_x$, $\mu^a{}_x$, $T_{wn}(x)$, $T_{rn}(x)$, and H_x by integration over the conditional distribution of x^a using $\hat{f}_{xa|x}$. From the definition of the EDE income for equals group Ω_x, we will have, for instance, that a natural estimator $\hat{\xi}^a{}_x$ for $\xi^a{}_x$ is given by

$$U_e(\hat{\xi}^a_x) = \int_{\Omega_x} U_e(x^a)\, \hat{f}_{x^a|x}\, dx^a \tag{17}$$

[17] On the reranking approach, see, for instance, Feldstein (1976), Atkinson (1979), Plotnick (1982) and Duclos (1993). For a criticism of that approach, see Kaplow (1989) and Musgrave (1990). For a comparison of the reranking and classical approaches using simulation, see Lambert and Ramos (1997).

Similarly, a natural estimator $\hat{\mu}^a_x$ for μ^a_x is given by

$$\hat{\mu}^a_x = \int_{\Omega_x} (x^a)\, \hat{f}_{x^a|x}\, dx^a \tag{18}$$

By (9), this yields

$$\hat{H}_x = \hat{\mu}^a_x - \hat{\xi}^a_x \tag{19}$$

If $\hat{f}_{xa|x}$ is continuous over x, these estimators will also be continous across x. Integrating $\hat{H}_x$ over x will give an estimator of the overall cost of HI:

$$\hat{H} = \int_0^\infty \hat{H}_x\, \hat{f}(x)\, d(x) \tag{20}$$

where $\hat{f}(x)$ is the estimator of the implied marginal density function for pre-tax income. Alternatively, if income x is estimated to be at percentile p, so that $p=\int_0^x \hat{f}(z)dz$, then we may write $\hat{H}_x=\hat{H}(p)$ and

$$\hat{H} = \int_0^1 \hat{H}(p)\, dp \tag{21}$$

In the application to follow, we use the non-parametric kernel estimation procedure, with Gaussian kernel and bandwith chosen to minimise the mean integrated square error in measuring the shape of a wide range of possible population densities.[18] The estimated distribution tends asymptotically to the true one if the latter is continuous.[19]

This approach enables us to 'reconstruct' indices of local HI, of the cost of inequality and of EDE income as functions of pre-tax income x. This is a statistically preferable procedure to the use of discrete banding into 'close equals groups', which

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[18] See Silverman (1986), p.48. Another approach would be to choose the bandwidth parameter to minimise the square error in measuring H. This is left to future investigation.

[19] See Silverman (1986), pp.71-72.

has been the practice in some previous literature on the measurement of classical HI.[20] The kernel method replaces the arbitrariness of the close equals groupings by kernel bandwidths possessing known statistical properties, and provides smooth estimates with automatic convergence to the true population values under weak regularity conditions.

5. Canada, 1981 and 1990

The Canadian Survey of Consumer Finances provides sample micro-data annually on pre-tax and pre-benefit family incomes, provincial and federal personal income taxes and cash transfers received from the provincial and federal governments.[21] The 1981 and 1990 data sets comprise respectively 38,000 and 45,000 observations. To adjust these data for heterogeneity in the size and the composition of families, we equivalised using the OECD equivalence scale. We also removed families reporting negative gross or net incomes.

The kernel estimate of the joint density function for gross and net equivalent incomes is shown in Figure 2.[22] If there were no HI in the tax and transfer system, this joint density would be positive only above a single line, showing a deterministic relationship between gross and net income. The flatter the conditional density of net income (given a level of gross income), the greater the HI at that gross income level. Alternatively, the more unequal are net incomes conditional on a particular level of gross income, the more HI there is.[23]

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[20] Berliant and Strauss (1985) and Lambert and Ramos (1996) do this explicitly. Such an approach could also be devised for the indices and decompositions expounded in this paper: see Lambert (1995).

[21] Income includes wages and salaries, self-employment income, private pensions and investment income. Transfers include Federal and Québec family and youth allowances, Child Tax Credits, Old Age Security Pensions and Guaranteed Income Supplement, Canada/Québec Pension Plan Benefits, Unemployment Insurance Benefits, Social Assistance Benefits and provincial income supplements, various tax credits and grants to individuals, veterans' pensions, pensions to widows, and workers' compensation.

[22] For expositional convenience, we have normalized gross and net incomes by their means for Figure 1 and all subsequent Figures and Tables.

[23] This suggests that a test of 'conditional HE dominance' of one joint distribution over another might be constructed, in a manner analogous to the well-known tests of Lorenz dominance for the measurement of inequality. We have

In Figure 3, some features of the joint distribution of gross and net income are shown for Canada in 1990. For each centile point in the distribution of gross income, the unbroken line shows the value of gross income x (normalized by the mean), the dotted line shows the predicted value μ^a_x of net income (also normalized), and the dots show the sample values of net income (these dots form a subsample of the dots in Figure 1). The estimated mean logarithmic deviation (MLD) of net income at each centile point is also shown (scale on the left vertical axis).[24] The difference between the lines of gross income and predicted net income shows the vertical impact of the tax and benefit system: relative to a system of tax and benefit allocation that would be proportional to gross income, the poor benefit and the rich lose. The MLD decreases over most of its range, which indicates that the conditional inequality of net income is greater the lower is gross income. This is not unexpected, since benefits affect largely those at the bottom end of the distribution. The dispersion of the dots indicates a significant degree of income reranking in the tax and benefit allocation.

To indicate the cost of HI at different gross income levels, we display in Figure 4 the levels of revenue-neutral taxation, $T_{rn}(x)$, and the differences, $H_x = T_{wn}(x)-T_{rn}(x)$, between welfare-neutral and revenue-neutral taxation (scale on the right vertical axis) for 1990 Canada. The size of $T_{rn}(x)$ in Figure 4 corresponds to the distance between predicted net incomes μ^a_x and gross incomes in Figure 3. $T_{rn}(x)$ is negative for values of x up to about 80% of per capita mean gross income, and varies in size from -45% to 100% of it. $T_{wn}(x)-T_{rn}(x)$ is positive everywhere, ranging between 5% and 0.5% of per capita gross income for e=0.75 and between 1.2% and 0.2% for e=0.25. This excess of welfare-neutral over revenue-neutral tax is larger at lower values of gross income and is higher for a greater degree of inequality aversion (recall footnote 4). At average gross income, for instance, we could replace the actual tax system by an horizontally equitable one with the same welfare but which would yield an additional revenue to the government of the order of 0.8% of per capita income (for e=0.75; 0.3% for e=0.25).

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earmarked this as a topic for future research.

[24] The MLD for a distribution with density function f(x) and mean μ is $\int(\ln\mu-\ln x)f(x)dx$.

Figure 5 displays the cost of HI, H(p), at different centiles in the Canadian income distribution in 1981 and 1990, for e=0.25 and e=0.75. The figure also shows the Atkinson index of inequality for 1990 at e=0.75 (scale on right vertical axis). When multiplied by μ^a_x, this index yields the cost of HI. Hence, the greater the value of μ^a_x, the greater the HI cost associated with a given degree of (conditional) net income inequality; this accounts for the growing gap between the inequality (continuous line) and cost of HI (dotted line) as centiles and μ^a_x both increase (1990, e=0.75). HI for 1990 is generally greater than for 1981, except in the bottom and top percentiles, for both values of e.

The areas under the curves H(p) for 1981 and 1990 give the overall money-metric costs of HI (recall (21)). These values are shown in Table 1 as percentages of per capita income (i.e. as the index H_1 of equation (15)). Table 1 also shows the cost C^f of inequality after the welfare-equivalent flat tax (at rate γ), the excess tax revenue which the actual tax system generates in comparison to this flat tax as a fraction of the mean (i.e. $(g-\gamma)/(1-g)$, which from (6) measures the performance of the tax system), the cost C^a of after-tax inequality, and the cost C^* of inequality of income net of the horizontally equitable tax $T_{wn}(x)$ (all measured relative to mean post-tax income). As fractions of mean post-tax income, C^f in 1990 is significantly greater than in 1981, and C^a is conversely less in 1990 than in 1981. Hence, the performance of the tax and transfer system is significantly better in 1990 than in 1981, by (4.66-3.45=) 1.21% of per capita income for e=0.25 and by 5.74% of per capita income for e=0.75. Had the actual tax and transfer system been horizontally equitable, the cost of after-tax inequality would have been C^* (= C^a-H). Hence, in the absence of HI in 1981 for e=0.25, the state would have been able to collect 0.4% more of per capita income in taxes with no adverse effect on social welfare. This goes up to 1.61% of per capita income for e=0.75 and for 1990.

In Table 2, the net performance Π of the tax and transfer system is decomposed in terms of the Blackorby and Donaldson progressivity index Π_{wn} for the horizontally equitable tax $T_{wn}(x)$ and the aggregate index of horizontal inequity H_1, as in Theorem 3. The first column shows that compared to flat tax, the actual system generates between 3.45% and 28.49% more of per capita income in revenue. The second column indicates that the average income after application of $T_{wn}(x)$ would be between 99.6% and 98.39% of the actual post-tax mean. The third and fourth columns display the Blackorby-Donaldson tax progressivity index and performance contribution of $T_{wn}(x)$. The difference is accounted for by HI. For e=0.25, HI decreases the overall performance of the tax system by about 10% (and by around 0.4% of per capita income); for e=0.75, performance is reduced by about 5% (and by 1.3% to 1.6% of per capita income).

Table 3 summarises these results and compares them with those obtained following the approach advocated by Lambert and Ramos (1996), which uses the MLD. Both approaches agree that overall redistribution and vertical equity are greater in 1990 than in 1981. The cost-of-inequality approach indicates that HI is greater in 1990 than in 1981, but the MLD approach suggests the converse, although by a small (and probably statistically insignificant) margin. By both the MLD and the e=0.25 decompositions, HI represents a loss of about 10% of the vertical equity exerted by the tax and transfer system.

An alternative and final decomposition is shown in Table 4 using a change-in-inequality approach. This approach is often encountered in applied work when assessing by how much inequality is affected by the tax and transfer system. The overall redistributive change in inequality, I^b-I^a, equals the change in inequality brought about by the vertical impact of the tax and transfer system, I^b-$I^*\mu^*/\mu^a$, minus the level of HI (inequality) introduced by the system. The tax and transfer system is again found to be significantly more redistributive in 1990 than in 1981, for both e=0.25 and e=0.75, and net redistribution is about 10% lower than it would have been in the absence of HI.

6. Overview and conclusion

There is a need among tax policymakers, as well as tax designers and administrators, for meaningful summary indicators of HI to guide the process of reform. This paper offers a systematic and normatively sound approach, according to which HI is measured by the amount an inequality-averse SDM would pay to have it removed, both in dollars and as a percentage of total income. Being money-metric, the cost of HI can be compared to the money-metric increase in efficiency which a government rule or intervention may generate, or to the money-metric fall in the cost of inequality brought about by the vertical redistribution of a tax and benefit system. We can then determine ethically if the dollar increase in efficiency (or fall in inequality) possibly exerted by some government policy is worth the dollar cost of HI which this policy may cause. The degree of inequality aversion must be specified by the analyst, and this offers the opportunity to test robustness of conclusions using sensitivity analysis - of itself a new development in the HI measurement literature[25]. The methodology also shows that HI can be seen as 'loss of performance': a new decomposition of Blackorby and Donaldson's (1984) index of tax progressivity demonstrates this.

We have discussed the implementation difficulties arising from the identification problem, and have proposed statistically attractive procedures to estimate HI, both locally and globally, and to compare the HI characteristics of alternative tax and transfer systems[26]. Our illustrative application showed that HI in the Canadian tax and transfer system increased from 1981 to 1990, being responsible in 1990 for about a ten percent loss of vertical equity.

The methodology has wide applicability, both as a means to investigate the performance and the equity of the tax and transfer system in itself and in comparisons between countries or levels of government, and over time. We may also use it to decompose total HI into socio-economic groups and individual taxes and transfers. Is it socially less costly, in foregone tax dollars, for government to collect its revenue using a range of taxes, or by engaging a single tax instrument like the expenditure tax? How socially costly is the HI (and inequality) introduced by private sector rules and markets (e.g., by gender or racial discrimination)? Our measures can inform topical questions such as this. Indeed, the technology we have described is capable of extracting from any scattergram of normatively significant variables, such as that in Figure 1, a horizontal-vertical characterization of the underlying data generating process. It may be of interest in biometrics, for example, in characterizing biodiversity (Polasky and Solow, 1995), as well as in economics.

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[25] Note that this degree of inequality aversion can very well differ for vertical and horizontal inequality. For instance, a "minimal state" SDM could be insensitive to the exercise of vertical equity and to levels of vertical inequality but ethically very sensitive to violations of horizontal inequity by the state.

[26] Future work must nevertheless be undertaken to check the statistical reliability of our results (possibly using numerical simulation techniques), as well as their sensitivity to the presence of measurement errors and to the choice of equivalence scales.

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Table 1

Cost of Vertical and Horizontal Inequality

(as % of post-tax per capita income)

		C^f/μ^a	$(g-\gamma)/(1-g)$	C^a/μ^a	C^*/μ^a	$H_1=\int H(p)dp/\mu^a$
e=0.25	1981	7.09	3.45	3.64	3.24	0.40
	1990	8.13	4.66	3.47	3.00	0.47
e=0.75	1981	33.82	22.75	11.07	9.78	1.29
	1990	39.13	28.49	10.64	9.03	1.61

Table 2

Tax Performance and Blackorby-Donaldson Indices of Tax Progressivity

(as % of post-tax per capita income)

		P/μ^a (= Π)	$\theta=\mu^*/\mu^a$	Π_{wn}	P_{wn}/μ^a (= $\theta\Pi_{wn}$)	H_1
e=0.25	1981	3.45	99.60	3.87	3.85	0.40
	1990	4.66	99.53	5.13	5.15	0.47
e=0.75	1981	22.75	98.71	24.35	24.04	1.29
	1990	28.49	98.39	30.59	30.10	1.61

Table 3

Redistribution, Vertical Equity and Horizontal Inequity

Using the Cost of Inequality and the Mean Logarithmic Deviation Decompositions

	Cost of Inequality Decomposition				Mean Logarithmic Deviation Decomposition		
	Inequality Aversion	Overall Redistribution	Vertical Equity	Horizontal Inequity	Overall Redistribution	Vertical Equity	Horizontal Inequity
1981	e=0.25	0.0345	0.0385	0.0040	0.3530	0.3920	0.0391
	e=0.75	0.2275	0.2404	0.0129			
1990	e=0.25	0.0466	0.0513	0.0047	0.4298	0.4678	0.0380
	e=0.75	0.2849	0.3010	0.0161			

Table 4

Redistribution, Vertical Equity and Horizontal Inequity

Using the Change in Inequality Decomposition

		I^b-I^a	I^b-$I^*\mu^*/\mu^a$	H_1
e=0.25	1981	0.0321	0.0361	0.0040
	1990	0.0430	0.0477	0.0047
e=0.75	1981	0.1648	0.1777	0.0129
	1990	0.1982	0.2142	0.0161

Figure 1: Scatter Plot of Gross and Net Income, Canada 1990

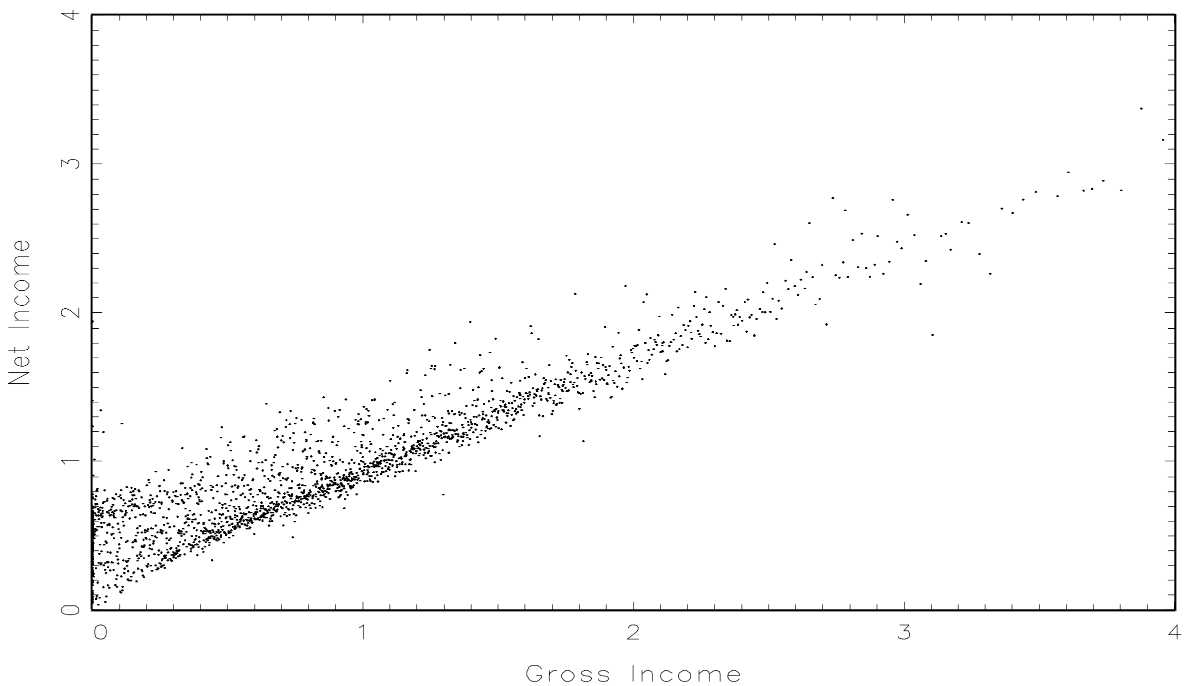

Figure 2: Joint density of Gross and Net Incomes, Canada 1990

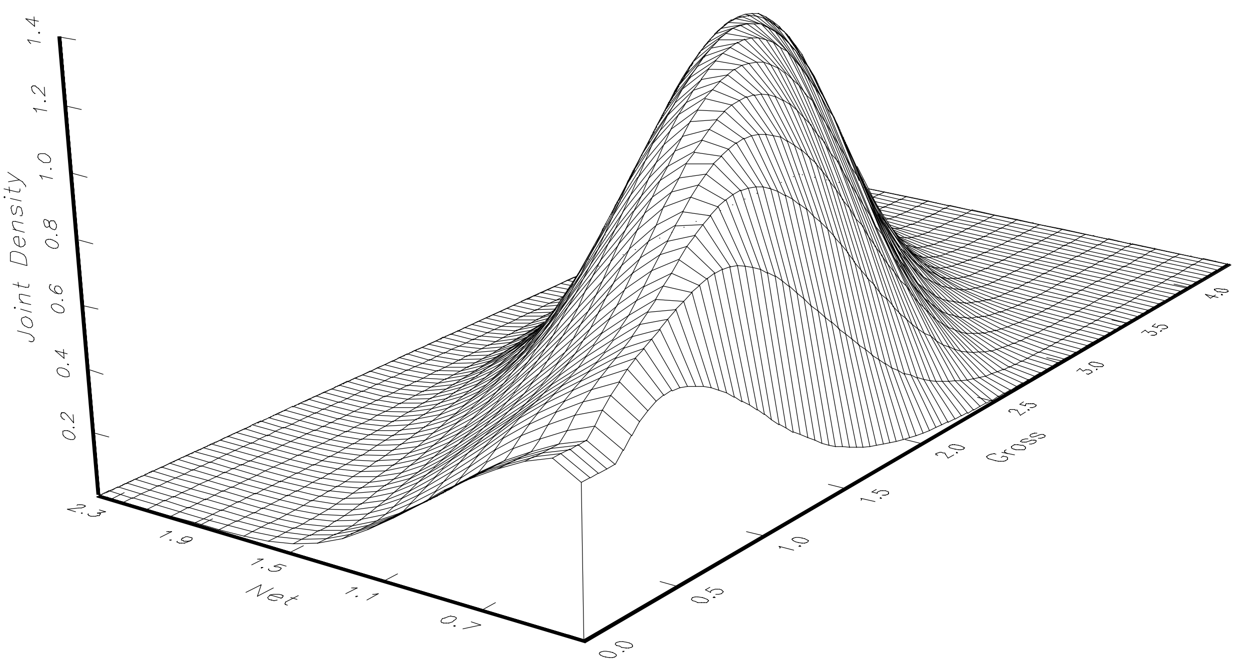

Figure 3

Distribution of gross and net incomes

Canada, 1990

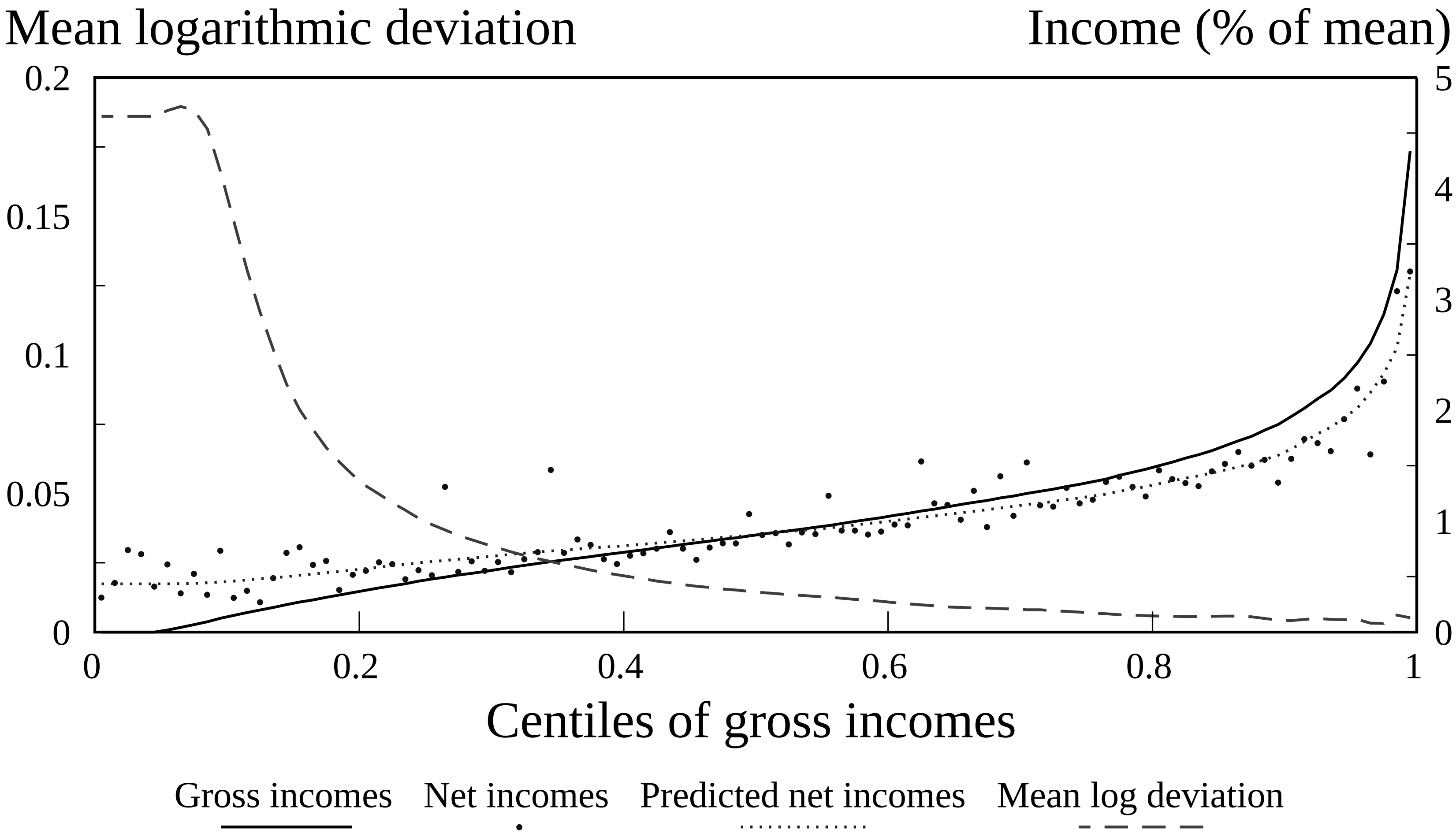

Figure 4

Revenue-neutral and welfare-neutral taxation

Canada, 1990

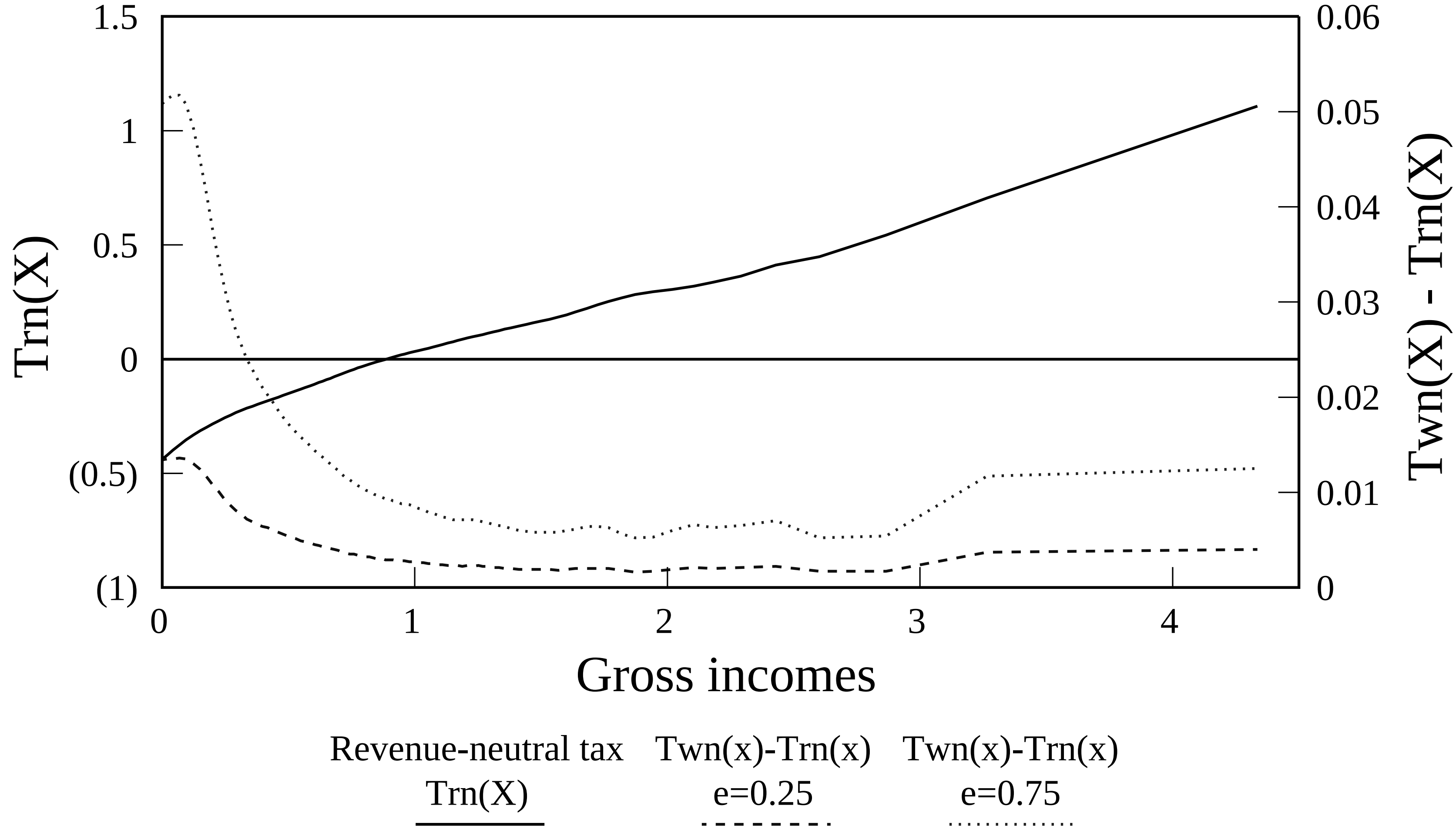

Figure 5

Cost of horizontal inequity