

Specific investment, absence of commitment and observability

Patrick González*

I consider the problem of the design of an optimal self-selecting contract scheme for a principal who is buying a good from an agent which has the opportunity of making a cost-reducing unobservable investment prior to the contracting stage. Because of a hold-up problem, the agent will randomizes on his investment level. This forces the principal to spend informational “rents” to achieve screening. In equilibrium, these “rents” match the investment costs and the resulting contract yields a price schedule such that the marginal revenue of the agent equals his long run marginal cost curve. Since the agent’s “type” is an endogenously determined characteristic, I argue that informational “rents” should be interpreted as quasi-rents that stand as a payment factor for investment.

*Université Laval, GREEN and CIRANO, Ste-Foy (QC), Canada, G1K 7P4, pgon@ecn.ulaval.ca.

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1. Introduction

Consider a firm that must sink an investment in order to produce a good. The profitability of such a venture depends on whether the firm expects or not to recover its sunk costs from future sales. Once the investment is sunk, the firm is exposed to the risk that its clients reduce their demand or the price they are willing to pay. Even if the good is very valuable, it may still be advantageous for the clients to never pay the firm more than its variable costs. When the good and the investment are specific to a client, and have a lesser intrinsic value outside the relationship, the firm has little option but to accept such a proposal. There is then less incentive for the firm to invest in the first place and the social benefits of investment may be lost.

This is an illustration of Williamson's (1983) classic "hold-up" problem. There are two sets of circumstances when the hold up problem can generally be solved; vertical integration and commitment with binding contracts. If the client and the firm vertically integrate, the issue of who shall absorb the investment costs becomes economically irrelevant. But vertical integration is often an unrealistic option. It creates problems of its own by substituting internal management of resources, which can be subject to costly moral hazard effects, for market transactions. Hence, the general analysis of investment usually implicitly assumes that binding contracts are possible. With such contracts, the firm's clients or possibly some institutions like banks, can commit themselves to buy today the firm's future production at a price that internalizes investment costs.

Efficient investment can then be achieved under various assumptions about the information structure (Rogerson, 1992). Yet, in many cases, such contracts are unmanageable. For instance, with respect to international business transactions, it may prove difficult or even impossible for a local firm to efficiently sue a foreign firm for a breach of contract (Thomas and Worrall, 1994; Choi and Esfahani, 1998). Firms doing business with the government may reasonably doubt that the return they expect from some specific long-term project will effectively be paid fully in the future, under all circumstances, because of the government's sensitivity to public opinion and its ability to change the law (Vickers, 1993). In other cases, the client may not even be identified at the investment stage (consider the development of a new product). Even when binding contracts are effective, enforcing them usually involves the judiciary system and that can have a very costly and unpredictable outcome. For instance, if the "specific" good involved is a common good of a "specific" quality that can be observed by the firm and its client but not by the courts. The firm would still be able to obtain a reduced price from the market but would lose the specific value added in quality.

One way of looking at the hold-up problem is to point out the wedge between the sharing rules that are used to distribute investment costs ex ante and investment returns ex post. Another approach to mitigate the problem is then to identify specific bargaining subgames whose outcomes reduce that wedge. Tirole (1986) pioneered this route by showing that investment incentives could be increased under asymmetric information because the ex post sharing rule that results from a bargaining subgame is generally sensible to the information structure.

In this paper, I highlight the fact that the privacy of the investment decision provides the party who makes that investment with a sufficient strategic advantage to protect the return on investment from a hold-up. There is a principal who wants her agent to invest in a costly technology in order to reduce the variable cost of producing some good. The principal is limited to short-term contracts and cannot sign a binding contract prior to the investment stage. Under perfect information, the agent would invest too little because of the hold up problem: he would justifiably fear that the principal would refuse to pay for the investment cost at the contracting stage. But if the agent invests privately and if that piece of information is valuable to the principal, then the principal will be willing to yield informational rents to the agent, through a screening mechanism. These rents, in turn, will indirectly finance investment. My model can be interpreted as a classical principal-agent model (Guesnerie and Laffont 1984) to which I add an initial investment stage where the agent has the opportunity of choosing his “type”, at a price (the cost of investment). In equilibrium, the agent randomizes on his investment support thus inducing a common-knowledge “type” distribution that is the basis of the subsequent play.¹ Being “tough” with the agent is an option for the principal only when he has good knowledge of the firm’s cost structure. Without this knowledge, she runs the risk of making an unacceptable offer to the agent that can jeopardize the ex post realization of the gains from trade. An unobserved mixed strategy allows the agent to “hide” his investment behind a veil of endogenously created noise. An uninformed principal then has a weaker bargaining position which might reduce the ex ante incidence of the hold-up problem. In Tirole’s model, the principal’s reply to this randomization is to increase the probability of disagreement ex post. My approach extends Tirole’s analysis to the case where the principal can use the production level as an instrument to screen the agent at the contracting stage. In equilibrium, the parties always reach an agreement. When the possibility of renegotiating the contract is added, screening becomes impossible and the probability of disagreement increases.

The equilibrium contract I obtain has a deceptively simple structure: it amounts to paying the agent a nonlinear price such that his marginal revenue curve is equal to his long run marginal cost curve (*LRMC*). Contrary to most models of asymmetric information cast in a Bayesian framework, the distribution of “types” is endogenous in my model (the outcome of an equilibrium mixed strategy) so that observable variables like production and contracts (“prices”) are functions only of taste and technological parameters like in classical economics.

Laffont and Tirole (1993) have proposed an explanation of the hold-up problem under asymmetric information that does not involve a mixed strategy for investment. In their model, investment affects the distribution of variable costs but not their support. Since variable costs depend on an exogenous random variable, the principal will try to screen the agent ex post. But they make the implicit assumption that it is not possible to contract after investment has taken place but prior costs are realized. One can show (González, 1997) in that context that it is always optimal for the principal to screen agents with respect to their ex post variable costs but to pool them with respect to their investment level. Since the agent’s payoff function is strictly concave in investment and all investment levels are treated equally (pooled) by the principal, the agent will then play a pure strategy by choosing the unique maximizer on his investment set.

In a recent paper, Gul (1997) analyzes a model of bargaining between a seller and a buyer in an environment very similar to the one presented here. In Gul’s paper, the buyer has the opportunity ex ante to make an investment that increases the gains from trade ex post. By allowing the investment to be made privately by the buyer and by considering a sequential bargaining subgame of offers and counter-offers, Gul comes to a surprising conclusion. Not only does the unobservability create a need for the seller to screen the buyer using time as an instrument, but the outcome of the whole game will come arbitrarily close to efficiency as the delay between offers goes to zero even if the seller gathers all the surplus. In my model, the principal will also screen the agent because of the equilibrium induced randomized strategy and will gather all the surplus realized in the relationship, but I do fall short of efficiency. Not only does the agent invest suboptimally but he generally chooses an inefficient level of production given his investment.

In Bayesian models, where the distribution of types is exogenous, unobservability of the agent characteristics usually diminishes social welfare as parties engage in wasteful rent-seeking behavior. When the distribution is endogenous, unobservability actually prevents the distribution of types to collapse on the least efficient type – an even worse outcome – so that unobservability actually helps to maintain social welfare.

The rest of the paper is divided as follows. The model is presented in the next section and solved in section 3. Three analytical examples are proposed in section 4 to illustrate the links between the agent's cost function and his equilibrium mixed strategy. In section 5, I address the welfare implications of the unobservability of investment. In section 6, I add the possibility of renegotiation. The last section concludes with a discussion about the empirical predictive power of principal-agent models based on incomplete information games *vs* those, like the one in this paper, that rely on games of complete but imperfect information.

2. The model

Consider a two-period relationship between two risk-neutral players. There is a firm (hereafter, the agent) which produces a good that can be sold at a unit price of $p \in (0, 1)$ on the market and a potential client (the principal) with linear preferences. I assume that the firm can produce two varieties of the good, one being tailored to the specific needs of the principal.

Let q_M and q_P be the quantities produced by the firm for the market and the principal where q_P is composed only of the variety preferred by the principal. Both varieties would be perceived as identical on the market and would sell at price $p < 1$ but the principal values each unit of q_M at p and each unit of q_P at one. I assume that there is little if no chance that the principal could procure himself at price p through the market the good of the specific variety he values the most.²

I assume that there are limited economies of scope in using the firm's installed capacity for a joint production of both varieties by letting the total variable cost of producing q_M and q_P be a function of their sum $q = q_M + q_P$ alone.

To reduce production costs, the agent has the opportunity of making an investment $e \geq 0$. The cost of producing the joint output q is then $c(q, e)$ where c is a strictly convex function with³ $c_q > 0$. Investment e is irreversible; hence, once undertaken, total costs $c(q, e)$ can be decomposed in fixed costs $c(0, e)$ and variable costs $c(q, e) - c(0, e)$. I also assume that $c_e(q, e) - c_e(0, e) < 0$ and $c_{ee}(q, e) - c_{ee}(0, e) > 0$ for all q so that investment reduces variable costs at a decreasing rate.

I will also need a natural sorting condition $c_{qe} < 0$ which says that investment decreases marginal cost at any given level of production (that is, investment increases capacity) and two technical conditions: $c_{qqe} < 0$ and $c_{qqq} \geq 0$. The first one insures that the second order conditions for the principal's program are always satisfied while the second one allows us to disregard stochastic contracting schemes.

The game. The principal observes neither investment nor costs, but is aware of the production set of the agent; that is, he knows c . I also assume that he cannot sign binding contracts prior to the investment stage and that it is common knowledge that he holds all of the bargaining power during the entire game.

Because both varieties of the good have a market value, there always exists an incentive for the agent to invest. Nevertheless, efficiency requires that the agent deals with the principal who has a higher valuation than the market for a specific variety that is a perfect substitute in output to the common variety. If the agent expects a fair deal with the principal to be difficult, he might focus his attention only on the market and invest suboptimally. The problem is then to provide the agent with incentives to invest optimally. More precisely, I want to show to what extent the unobservability of investment does provide such incentives.

The course of events is as follows: in the first period, the agent privately invests e . Once e has been sunk, the agent is said to be of “type” e and both players enter the second stage⁴ where they must agree, through some bargaining subgame, on an allocation that includes a production plan $q \in Q$ and a monetary transfer $t \in T$ from the principal to the agent (the sets Q and T are non negative values of q and the real line for T).

The precise bargaining subgame that is played at that stage is of crucial importance in determining the equilibrium of the entire game and, in particular, the investment strategy that will be played by the agent. For instance, if the agent expects that he will be able to make a take-it-or-leave-it offer to the principal at the second stage, it is easy to see that he will undertake the socially optimal amount of investment and that he will offer to produce the socially optimal level of the good for a transfer that will cover both fixed and variable costs and all the gains from trade.

In that case, the player that invests reaps all the ex post gains from trade and will thus optimally equalize the marginal cost of investment to its marginal return on these gains when making ex ante his investment decision. A hold-up problem occurs when the bargaining procedure does not share the ex post gains from trade commensurately to the ex ante investment costs that have to be born by the players.

The bargaining subgame beginning in the second stage is an integral part of the game and cannot conceptually be modelled as an endogenous choice.⁵ Hence, the ex post bargaining procedure is given exogenously here by way of the class of contracts that the principal can offer on a take-it-or-leave-it basis. Nevertheless, the agent always has the external opportunity to use his capacity to sell on the competitive market at price p . As p is increased, the specific gains from trade that can be realized with the principal are decreased and the option value for the agent of going to the market is increased. Hence, p can be viewed as a measure of the incidence of the hold-up problem in this economy.⁶

Strategies and payoffs. A pure strategy for the principal is an offer of a contract δ (to be defined later) at the bargaining stage. Since investment costs are sunk at this stage, only variable costs matter to the agent in this subgame. If the agent refuses the contract proposed by the principal, the game ends: the principal pays nothing and receives nothing.⁷ The agent can use all of his capacity to produce and sell on the market at price p . That outcome yields a feasible payoff of

$$\hat{w}(e, p) \equiv \max_{q \in Q} (pq - c(q, e)) \quad (1)$$

to the agent. For all given $p \geq 0$, this is a well defined strictly concave function of e . Furthermore, since there is a unique solution to the r.h.s. of (1), we can define the argmax function $\hat{q}(e, p)$. Throughout this paper, I will use the caret notation to indicate efficient level values; hence $\hat{q}(e, b)$ is the efficient level of production that equates marginal benefit b (either p or 1) to marginal cost c_q given investment level e . To insure (and simplify) the construction of an equilibrium, I also assume that,

$$\forall b, \exists \epsilon(b) \text{ such that } \hat{w}(e, b) < 0, \forall e > \epsilon(b). \quad (2)$$

Hence, investing a “large” amount can never be an option. To lighten the notation, I will write $\hat{w}(e) = \hat{w}(e, 1)$ and $\hat{q}(e) = \hat{q}(e, 1)$ when no confusion arises. Likewise, I note

$$\hat{e}(q) \equiv \operatorname{argmin}_{e \geq 0} c(q, e) \quad (3)$$

the level of investment that minimizes total cost of producing q . Note that $c(q, \hat{e}(q))$ is the traditional long run cost curve of the firm. Finally, when e and q are simultaneously chosen, I note

$$w^*(b) \equiv \max_{e \geq 0, q \in Q} (bq - c(q, e)). \quad (4)$$

and let $\{e^*(b), q^*(b)\}$ be the solution to the r.h.s. of (4). Existence and uniqueness of these values is insured by the global convexity of the program. I assume that for all $p > 0$, (4) with $b = p$ has an interior solution. It follows that we have an interior solution in $b = 1$; that $\hat{e}(q) > 0$ for all $q > 0$ and it is then straightforward to verify that $e^*(b) = \hat{e}(q^*(b))$ and $q^*(b) = \hat{q}(e^*(b), b)$.

Without loss of generality, we can assume that the principal will never propose to pay more than an average price of 1 per unit of the good produced by the agent. It follows that the agent cannot expect to make a profit greater than $\hat{w}(e, 1)$ for some e . Since the agent can forego the expected relationship with the principal and guarantee himself a market payoff of $\hat{w}(e, p) > 0$, he will never invest more than $\epsilon(1)$. Hence, it is common knowledge that the agent will invest some $e \in [0, \epsilon(1)] \equiv E$.

Suppose now that the agent has invested privately and consider the subgame that begins in the second stage when the principal, holding all the bargaining power, negotiates a contract with the unidentified agent. Let \tilde{F} be the Bayesian prior of the principal about the level of investment made by the agent over E . By the Revelation principle, the equilibrium outcome of any Bayesian rational communication subgame played by the principal and the agent can be reached by the take-it-or-leave-it offer of a direct mechanism that satisfies individual rationality (*IR*) and incentive compatibility (*IC*).

Formally, one should define a contract δ as a mapping from the “type” (message) space E into the general class of stochastic contracts, that is the space $\Delta(Q \times T)$ of distribution functions over $Q \times T$. A mixed strategy can then be defined as a randomization into this space. I restrict the scope of the analysis by constructing an equilibrium where the principal plays a pure strategy in the contract space.⁸ These contracts can be represented as pairs $\delta(e)$ of functions $(q(e), t(e))_E$ and form a menu in which the agent may select a particular allocation $\delta(m) = (q(m), t(m))$ after having sent a message m about his type e . If allocation $\delta(e)$ is selected, then the principal gets $q(e) - t(e)$.

Following the offer made by the principal, the agent either accepts or refuses. In both cases, he may proceed to the market where he can sell at will at unit price p . Once the contract negotiation has been settled and once the agent has completed the booking of orders on the market, production and exchange take place. Since refusing the contract and going to the market can be replicated with a contract that specifies zero production and no transfer, any agent that has invested e would behave rationally by accepting any contract that satisfies the following *IR* constraint for type e :

$$r(e) \equiv t(e) - \hat{w}(e, p) + \max_{q \in Q} (pq - c(q(e) + q, e)) \geq 0. \quad (IR)$$

Here, $r(e)$ is the “rent” gathered by an agent of type e from a self-selecting contract with respect to his best alternative $\hat{w}(e, p)$.⁹ This formulation encompasses the case where the agent accepts the principal’s contract but still wishes to sell the output of his remaining capacity on the competitive market.

The agent accepts any self-selecting contract that satisfies (IR) given his type e . For these contracts, the IC constraints are summarized by

$$r(e) \geq t(m) - \hat{w}(e, p) + \max_{q \in Q} (pq - c(q(m) + q, e)), \quad \forall e \in E \text{ and } \forall m \in E. \quad (IC)$$

Whenever an agent is left with an excess capacity ($q(e) < \hat{q}(e, p)$), he will choose to be active on the market. By doing so, the agent will equalize his marginal rate of substitution MRS between q and money to p ; it follows that the MRS is constant for these types and the sorting condition (that the MRS does not decrease with e) is trivially satisfied. When $q(e) \geq \hat{q}(e, p)$, the solution in the r.h.s. of (IC) is zero and the agent produces only for the principal. In this case, the sorting condition is given by

$$-q'(e)c_{qe}(q(e), e) \geq 0.$$

This condition is always satisfied provided that $q(e)$ is non-decreasing. Because the agent's payoff function is quasi-linear, the sorting condition is also sufficient to insure global IC . It follows that the local approach for IC can be applied, which yields

$$r'(e) = \begin{cases} 0 & \text{if } q(e) < \hat{q}(e, p), \\ c_e(\hat{q}(e, p), e) - c_e(q(e), e) \geq 0 & \text{if } q(e) \geq \hat{q}(e, p). \end{cases} \quad (5)$$

In the second regime, the agent is left with no excess capacity for the market; accordingly, his rent pattern reflects the (variable) cost savings that can be achieved with a better type (adjusted for the fact that his reservation payoff would be modified).

To interpret the first case (excess capacity), suppose that the principal is locally paying a fixed price \tilde{p} for each of the $q(e)$ units produced for him by the agent, that is $t(e) = \tilde{p}q(e)$; for $dq = q'(e)de = 1$, marginal transfer is thus \tilde{p} ; differentiating (IR) on both sides and using (5) we get $\tilde{p} = c_q(\hat{q}(e, p), p) = p$; that is, the principal must pay the market price for each unit he buys. Obviously, as the next lemma shows, that regime is never played as the principal will always prefer that an agent of type e produce at least $\hat{q}(e, p)$.

Lemma 1. In an optimal self-selecting contract, we can assume that the agent produces only for the principal, that is $q(e) \geq \hat{q}(e, p), \forall e \in E$.

Proof. Consider first the following function s and its derivative with respect to θ :

$$s(\theta, e) \equiv p\theta + \max_{q \in Q} (pq - c(\theta + q, e));$$

$$s_{\theta}(\theta, e) = \begin{cases} 0 & \text{if } \theta < \hat{q}(e, p), \\ p - c_q(\theta, e) \leq 0 & \text{otherwise;} \end{cases}$$

$$\leq 0; \quad \text{so that } s \text{ is non-increasing in } \theta.$$

Then assume we have an optimal self-selecting contract that satisfies *IR* and *IC* and for which $q(e) < \hat{q}(e, p)$ for some type e . Consider raising $q(e)$ to $\hat{q}(e, p)$ and $t(e)$ by $p(\hat{q}(e, p) - q(e))$ for type e . That modification can only increase the payoff to the principal while keeping the agent's payoff constant. Clearly, to disprove the lemma, for some optimal self-selecting contract, this type of modification should be impossible. In other words, it must disrupt the *IC* constraint (*IC*) for at least some type e' . Yet, for any e' , the r.h.s. of that constraint is decreased by

$$s(q(e), e') - s(\hat{q}(e, p), e') > 0,$$

so that the constraint is actually relaxed, a contradiction. *Q.E.D.*

Because of lemma 1, we know that equation (5) simplifies to

$$r'(e) = c_e(\hat{q}(e, p), e) - c_e(q(e), e)$$

for an optimal self-selecting contract.

3. The equilibrium

Even if we assume that the principal plays a pure strategy in a Nash equilibrium, it is easy to see that the agent will be randomizing over his investment set. To show this, suppose the agent plays a pure strategy $e \in E$ in equilibrium. Anticipating this strategy to be \tilde{e} , the principal will invite the agent to produce $\hat{q}(\tilde{e}, 1) > \hat{q}(\tilde{e}, p)$ but will be willing to pay only the market price p for the first $\hat{q}(\tilde{e}, p)$ units (the revenue the agent could get on the market) plus a premium no more than the excess in variable costs needed to accomodate the higher marginal value of the principal:

$$t(\tilde{e}) = p\hat{q}(\tilde{e}, p) + c(\hat{q}(\tilde{e}, 1), \tilde{e}) - c(\hat{q}(\tilde{e}, p), \tilde{e}).$$

Anticipating this contract, the agent can choose to minimize the cost of producing $\hat{q}(\tilde{e}, 1)$ by setting his investment level to $\hat{e}(\hat{q}(\tilde{e}, 1))$ or he can plan to deviate on the market and to set e according to $e^*(p)$.

To obtain an equilibrium in pure strategies where the contract is accepted, we need $\tilde{e} = e^*(1)$ so that

$$\hat{e}(\hat{q}(\tilde{e}, 1)) = \hat{e}(\hat{q}(e^*(1), 1)) = \hat{e}(q^*(1)) = e^*(1),$$

and the expectation of the principal matches the actual strategy played by the agent. But then, whenever $p < 1$, a deviation would give the agent $\hat{w}(e^*(p), p) - \hat{w}(e^*(1), p) > 0$, and the equilibrium would not hold.

Since, in equilibrium, the agent randomizes on his investment level, the principal will find it profitable to try to screen the agent with respect to his variable cost structure using the production level as an instrument. The easiest way to construct a Nash equilibrium is to check whether a given pair of strategies are best replies one to another. Following that route, I construct the following equilibrium.

Proposition 1. There exists a Nash equilibrium where the agent randomizes with F over $E^* = [e^*(p), e^*(1)]$ and the principal offers a non stochastic contract $\delta = (q(e), t(e))_{e \in E^*}$ where $q(e)$ solves

$$c_e(q(e), e) = 0; \tag{6}$$

transfers cover total costs and provide an equivalent to the market payoff $t(e) = c(q(e), e) + w^*(p)$; and F is a continuous distribution function on E^* given by

$$F(e) = 1 - \exp\left(-\int_{e^*(p)}^e (1/h(\bar{e}))d\bar{e}\right) \quad (7)$$

where

$$h(e) = -\frac{1 - c_q(q(e), e)}{c_{qe}(q(e), e)}. \quad (8)$$

Proof. Let $E^* \subseteq E$ be the actual bounded support of the distribution played by the agent and let \underline{e} be its lower bound (obviously, $\underline{e} > 0$ since the agent can always produce for the market). Define a mixed behavioral strategy for investment as a (right-hand continuous) distribution function F where $F(e) = 0$, for all $e < \underline{e}$. As in Gul (1997), define $\underline{e} = \sup\{e | F(e) = 0\}$ to be an extremum of E^* and let e be an *increasing point* of F if either F is discontinuous in e or, for all $\epsilon > 0$, $F(e + \epsilon) > F(e)$. We have $e \in E^*$ iff e is an increasing point of F and all $e \in E^*$ must yield the same expected return. By construction, \underline{e} is a point of increase of F .

Let δ be the contract offered in equilibrium by the principal. The ex post return from δ to an agent that has invested e is given by (IR) and evolves with e according to (5).

In any right-hand neighborhood $[\underline{e}, \underline{e} + \epsilon]$ of \underline{e} we can find a point of increase e (perhaps \underline{e} itself) that is played in equilibrium. By lemma 1, we can assume that the agent will only produce for the principal; it follows that his ex ante payoff (relative to his best ex ante alternative) when investing e is

$$t(e) - c(q(e), e) - w^*(p) = r(e) - (w^*(p) - \hat{w}(e, p)) \geq 0. \quad (9)$$

This payoff is the difference between the ex post rent $r(e)$ from the contract, which compensates better types for the marginal variable cost savings they could appropriate for themselves by lying about their type, and the ex ante opportunity cost of investment $w^*(p) - \hat{w}(e, p)$. This payoff must be non negative otherwise the agent would simply choose to produce for the market. For any decreasing sequence of (e) that converges

toward \underline{e} , the corresponding sequence $(r(e))$ must converge to $r(\underline{e}) = 0$ otherwise, since rents are non decreasing with e , the principal could increase her payoff by augmenting δ with a fixed strictly positive payment no less than $r(\underline{e})$ which would not disrupt any *IC* or *IR* constraint. It follows that the non negative sequence of differences $(w^*(p) - \hat{w}(e, p))$ must converge to zero, which implies $\underline{e} = e^*(p)$.

I will establish later that $\bar{e} = e^*(1)$. For the time being, assume that F is a continuously differentiable distribution function on $E^* = [e^*(p), \bar{e}]$ with density f . I compute the optimal contract δ given that the agent randomizes with F on E^* (Guesnerie and Laffont, 1984).

Ex post *IC* requires that $q(e)$ be a non decreasing function and we know by lemma 1 that $r'(e) = c_e(\hat{q}(e, p), e) - c_e(q(e), e)$. The optimal contract then solves

$$\max_{\delta} \int_{e^*(p)}^{\bar{e}} (q(e) - r(e) - c(q(e), e))f(e)de$$

subject to the monotonous condition $q'(e) \geq 0$ and

$$\begin{aligned} r'(e) &= c_e(\hat{q}(e, p), e) - c_e(q(e), e), \\ r(e^*(p)) &= 0. \end{aligned}$$

Leaving aside the monotonous condition for the moment, the Hamiltonian function of this program is

$$H(e) = (q(e) - r(e) - c(q(e), e))f(e) + \mu(e)(c_e(\hat{q}(e, p), e) - c_e(q(e), e)).$$

The first-order conditions for $H(e)$ are:

$$\frac{\partial H(e)}{\partial q(e)} = f(e)(1 - c_q(q(e), e)) - \mu(e)c_{qe}(q(e), e) = 0 \quad (10)$$

$$\mu'(e) = -\frac{\partial H(e)}{\partial r(e)} = f(e) \quad (11)$$

where $\mu(e) \leq 0$ is the shadow cost of increasing the rent of agent e .

The boundary condition at $e = \bar{e}$ is unconstrained, hence $\mu(\bar{e}) = 0$. Integrating (11) I get

$$-\mu(e) = \mu(\bar{e}) - \mu(e) = \int_e^{\bar{e}} \mu'(\bar{e})d\bar{e} = \int_e^{\bar{e}} f(\bar{e})d\bar{e} = 1 - F(e). \quad (12)$$

The optimal quantity $q(e)$ for type e is thus implicitly given by

$$f(e)(1 - c_q(q(e), e)) + (1 - F(e))c_{qe}(q(e), e) = 0 \quad (13)$$

which implicitly identifies $q(e)$.

Once $q(e)$ is determined, the necessary rents can be computed:

$$r(e) = \int_{e^*(p)}^e (c_e(\hat{q}(\tilde{e}, p), \tilde{e}) - c_e(q(\tilde{e}), \tilde{e})) d\tilde{e}. \quad (14)$$

Finally, as $e \rightarrow \bar{e}$, the term $1 - F(e)$ vanishes so that (13) becomes

$$f(\bar{e})(1 - c_q(q(\bar{e}), \bar{e})) = 0$$

which implies

$$q(\bar{e}) = \hat{q}(\bar{e}, 1); \quad (15)$$

that is, the classical efficiency-at-the-top result.

I now impose equilibrium conditions to determine the value of \bar{e} and to characterize completely the equilibrium contract. For F to be played in equilibrium over E^* , I need (9) to hold with equality over E^* (this implies the transfer equation directly). Hence, differentiating (9), I must have

$$-c_e(q(e), e) = 0 \quad \text{over } E^*. \quad (16)$$

To reconcile this equilibrium condition with (15) I need $\bar{e} = e^*(1)$ so that $E^* = [e^*(p), e^*(1)]$. Equation (16) gives $q(e)$ as a function of e . Differentiating it yields

$$q'(e) = -\frac{c_{ee}(q(e), e)}{c_{qe}(q(e), e)} > 0,$$

which is positive under our convexity assumptions. Hence, the monotonous condition is satisfied in equilibrium. Substituting the implicit equilibrium solution $q(e)$ of (16) in (13) yields an ordinary linear differential

equation of the form

$$h(e)y' + y = 1, \quad (17)$$

where $y = F(e)$, $y' = f(e)$, the function $h(e)$ (the inverse of the hazard rate) is given by (8) and with initial condition $F(e^*(p)) = 0$. Note that h vanishes only at $e^*(1)$. This implies that (17) has a unique solution, over any subset $[e^*(p), e^*(1) - \epsilon]$, $\epsilon > 0$, given by (7) which is continuous and differentiable. Taking then the limit of $F(e^*(1) - \epsilon)$ as $\epsilon \rightarrow 0$ yields 1, so F is a cumulative distribution function on E^* . *Q.E.D.*

Given the equilibrium strategy F for the agent, the contract offered by the principal is the standard screening contract à la Guesnerie and Laffont (1984) which yields the best possible weighting for the principal between efficiency in production and (ex post) “rent” extraction. Since all pure strategies in the support of the mixed strategy played by the agent must yield the same payoff, these rents must match the investment cost of having a more or less ex post efficient type. I shall talk of *quasi rents*, in the Marshallian sense, since these rents are nothing more than a minimum fair return on past investment in capital.

The equilibrium contract has a simple and very intuitive structure which is illustrated in figure 1 with the traditional envelope representation (Viner, 1952) of short-run average costs (AC) curves.¹⁰ It amounts to paying the agent a marginal price of p for each of the first $q^*(p)$ units he agrees to produce, and equating marginal revenue (MR) to the agent’s long run marginal cost ($LRMC$) for all additional units. To see this, consider equation (6) which defines the optimal production plans, that is, the function $q(e)$. This equation is the first-order condition of program (3) that is related to points on the $LRMC$ and $LRAC$ curves. The average revenue curve of the agent, that is, the unit price he will receive for his production, is then increasing from p at $q^*(p)$ and is below his MR curve. With such a scheme, the agent will produce until his ex post (short-run) marginal cost curve $MC(e)$ crosses his MR curve. This happens at $q(e)$ where his short-run average cost curve $AC(e)$ is tangent to his $LRAC$ curve; hence, the agent’s chosen production plan is cost efficient. At that point, the agent’s payoff (average revenue minus average cost times his production level; that is, the sum of the two darker areas) is equal to his market payoff (the sum of the two lighter areas). Hence, from an ex ante point of view, the agent gets the same payoff from each investment level in $[e^*(p), e^*(1)]$, and this induces him to randomize on that support.

Since the equilibrium contract guarantees the market payoff to the agent, no hold-up will occur and the agent can invest with confidence. The complex part of the contract lies in the purchase policy $q(e)$ that prescribes lowering the principal's demand from a poorly capitalized firm ($e < e^*(1)$). It is given by (6) and manages to equate ex ante marginal opportunity cost of investment and ex post marginal savings on variable costs. Under full commitment, the optimal contract would equate marginal utility (1) of the principal to the long-run marginal cost at $q^*(1)$. That point is both ex ante and ex post efficient. In the absence of commitment and under full observability of the investment decision, the agent would simply refuse to invest more than $e^*(p)$. Efficient bargaining would yield an ex post efficient production level at $\hat{q}(e^*(p), 1)$. If the investment decision is kept private, then the equilibrium contract allows the agent to choose any production level in $[q^*(p), q^*(1)]$, say $q(e)$, and to receive a unit price of $AR(e)$. It is easy to see that this contract is ex post self-selecting since the most economical way to produce $q(e)$ and to realize sales of $AR(e)q(e)$ is to be of type e so that costs are minimized as the (short-run) average cost curve $AC(e)$ reaches the $LRAC$ curve in $q(e)$. Keeping investment private and producing at any $q(e)$ is both ex ante inefficient with respect to $q^*(1)$ and ex post inefficient with respect to $\hat{q}(e, 1)$ but it is cost efficient.

Simple static comparative analysis reveals that if the good has no value outside the relationship ($p = 0$) or if the market is competitive so that the agent's reservation payoff is achieved at the minimum of his $LRAC$ curve, the contract (the AR curve) will follow the $LRAC$ curve so that the transfer will match only total costs. As p is increased, $w^*(p)$ will rise so that all transfers will be shifted by an identical amount. Nevertheless $q(e)$ is unaffected as long as it is no less than $q^*(p)$. It follows that, for a given variation in p , the unit price $t(e)/q(e)$ always changes more for low levels of production, that is for low e . In a sense, high levels of production orders are relatively more anchored to the principal's valuation of 1 than to the market price.

4. Analytical examples

The shape of the distribution F depends on the derivative of h since

$$\frac{f'(e)}{f(e)} = -\frac{1 + h'(e)}{h(e)}.$$

If $h' \leq -1$ for all e , then the distribution will be skewed to the right toward $e^*(1)$ and the agent will invest almost optimally most of the time; since the ex post inefficiency $1 - c_q(q, e)$ is reduced as e is

increased, we will get almost optimal production most of the time. On the other hand, both ex ante and ex post inefficiencies will be exacerbated if $h' \geq -1$ as the distribution will be skewed toward $e^*(p)$. Other possible cases are that of $h'(e) = -1$ which implies a uniform distribution over $[e^*(p), e^*(1)]$ and $h'(e)$ first lesser (greater) than -1 , then equal to -1 at some interior \tilde{e} , then greater (lesser) than -1 which would yield a bell-shaped (U-shaped) distribution. Since $q(e)$ increases with e , the distribution of quantities will share the same characteristics.

I present below three analytical examples that illustrate the dependence of the distribution F upon the cost function c . In all these examples, the reservation market price is set to $p = 1/2$. For each example, I have graphed the Viner representation of the firm's cost structure for three investment levels, that of $e^*(p)$, $e^*(1)$ and the median investment level e such that $F(e) = 1/2$. The short-run marginal and average cost functions (MC and AC) are drawn for these three levels. The contract provision $(q(e), t(e)/q(e))$ is represented by the lowest thick line between $q^*(p)$ and $q^*(1)$.

Example 1. Let the cost function be

$$c(q, e) = \exp(E - Q)(\exp(q - e) - 1) + \exp(e - E) - \exp(-E).$$

With this specification, the parameters E and Q are the ex ante efficient levels of investment and production $\hat{e} = E$ and $\hat{q}(E, 1) = q^*(1) = Q$. If the agent expects to sell only on the market, he will set $e^*(p) = E + \ln(p)$ and $q^*(p) = Q + 2 \ln(p)$ for a profit of $p^2/2$.

The firm's short-run marginal cost function is $c_q(q, e) = \exp(q - Q + E - e)$ so that $c_{qe}(q, e) = -\exp(q - Q + E - e)$. Under observability, the firm would invest $e^*(p) = E + \ln(p)$ and would produce $q^*(p) = Q + 2 \ln(p)$ to get the market payoff of $w^*(p) = pq^*(p) - c(q^*(p), e^*(p))$. The optimal equilibrium contract under unobservability manages to keep this payoff constant over $[E + \ln(p), E]$ by having $q(e) = Q - 2(E - e)$ and by paying a unit price of

$$\begin{aligned} \frac{t(e)}{q(e)} &= \frac{w^*(e)}{q(e)} + \frac{c(q(e), e)}{q(e)}, \\ &= \frac{p(2 \ln(p) + Q - 2) + 2 \exp(e - E)}{Q - 2(E - e)}, \end{aligned}$$

which converges toward the $LRAC$ curve as $q(e)$ is increased. The inverse of the hazard rate resumes to

$h(e) = \exp(E - e) - 1$ and is decreasing with $h'(e) \leq -1$ so that the distribution will be skewed toward E . Using (7), the equilibrium distribution is given by

$$F(e) = \frac{\exp(e - E) - p}{1 - p}. \quad (18)$$

This case is illustrated in figure 2 for parameters $E = 1$ and $Q = 3$. At $p = 1/2$, the price is set higher than the long-run breakeven point¹¹. If investment were observable, the agent would produce with certainty $q^*(1/2) = 3 - 2 \ln(2) \simeq 1.6137$ with an investment of $e^*(1/2) = 1 - \ln(2) \simeq .3069$. From (18), we can compute the percentile function

$$\epsilon(z) = E + \ln((1 - z)p + z)$$

where $\epsilon(z)$ is the value of e that solves $F(e) = z$. The tick marks on the abscissa represent the tenth percentiles of production. Hence, the median investment level is at $e = \ln(3/4) + 1$ which yields $q(e) = 3 + 2 \ln(3/4) \simeq 2.4246$. The skewness of the distribution toward E is apparent as more ticks are concentrated near $q^*(1)$.

Example 2. In this second example, the cost function is

$$c(q, e) = q^2 - 2eq + 2e^2.$$

With this function, investment decreases costs whenever $q \geq 2e$. The market solution would be for the firm to produce $q^*(p) = p$ with an initial investment of $e^*(p) = p/2$ and resulting profit of $p^2/2$. The first best solution is $q^*(1) = 1$ and $e^*(1) = 1/2$. The equilibrium contract sets $q(e) = 2e$ and a transfer scheme of $t(e)/q(e) = p^2/4e + e$. The agent then randomizes according to $h(e) = 1/2 - e$. Since h is linear in e , we have $h'(e) = -1$ and the distribution will be uniform with $f(e) = 2/(1 - p)$ over $[p/2, 1]$. The cost structure is depicted in figure 3 where the uniform nature of the randomization is apparent by the equidistant ticks on the abscissa (q is also distributed uniformly since it is a linear function of e).

Example 3. In this last example, the distribution will be skewed toward $e^*(p)$. Let

$$c(q, e) = \frac{q^2}{2e} + \frac{e^2}{4}.$$

The inverse of the hazard rate function is $h(e) = \sqrt{e} - e$ and has derivative $h'(e) = -1 + 1/(2\sqrt{e}) > -1$.

It follows that the distribution

$$F(e) = 1 - \left(\frac{1 - \sqrt{e}}{1 - p} \right)^2$$

will be skewed to the left. See figure 4.

5. Welfare consequences

Little mention has been made up to now of the consequences of unobservability on expected welfare. We know that expected investment will rise but so will the incidence of ex post inefficiencies in production. Under perfect information, the agent inefficiently never invests more than $e^*(p) < e^*(1)$ but always produces efficiently given $e^*(p)$. Under imperfect information, the agent randomizes over E^* and will invest efficiently some of the time. However he will produce inefficiently $q(e) < \hat{q}(e, 1)$ most of the time.

Since the agent receives a payoff of zero (net of his market payoff) whether the game is played under asymmetric or symmetric information and since the principal can insure himself to pay no more than the market price for $q(e^*(p))$ units of variety 1 of the good (the amount he would get under full observability), it is clear that unobservability can only increase social welfare. This is in sharp contrast with Bayesian games of incomplete information where unobservability of types diminishes social welfare as players engage in rent seeking behavior.

This important difference comes from the fact that traditional Bayesian models assume that the distribution of types is exogenous so that it is not affected by the observability issue. It follows that going from unobservable to observable types increases welfare as all inefficiencies associated with bargaining under asymmetric information are resolved. When types are endogenous, observability would cause the type distribution to collapse to $e^*(p)$ at a great cost in social welfare. Unobservability then allows more types to be played, so much that the presence of more efficient types overcomes the fact that most types will now produce inefficiently.

One may wonder if this overall gain is attributable to the fact the agent now almost always invests a “little more” (as $e \geq e^*(p)$) or to the fact that almost efficient investment with almost efficient production is now possible with some probability as e approaches $e^*(1)$? Let $w(e) = q(e) - c(q(e), e)$. With observable investment, social welfare amounts to $\hat{w}(e^*(p))$. With unobservable investment, the agent randomizes on E^* with F . I want to compare $E_F(w)$ with $\hat{w}(e)$. Observe that $w(e^*(1)) = \hat{w}(e^*(1))$. I then derive the following

$$\begin{aligned}
E_F(w) - \hat{w}(e^*(p), 1) &= \int_{E^*} w(e)f(e)de - \hat{w}(e^*(p)), \\
&= \int_{E^*} ((w(e)F(e))' - w'(e)F(e))de - \hat{w}(e^*(p)), \\
&= w(e^*(1)) - \int_{E^*} w'(e)F(e)de - \hat{w}(e^*(p)), \\
&= \int_{E^*} (\hat{w}'(e) - w'(e)F(e))de = \int_{E^*} \phi(e)de, \tag{19}
\end{aligned}$$

where

$$\begin{aligned}
\phi(e) &= \hat{w}'(e) - w'(e)F(e), \\
&= (1 - F(e))\hat{w}'(e) - F(e)(w'(e) - \hat{w}'(e)).
\end{aligned}$$

One can check that $\phi(e^*(p)) = \hat{w}'(e^*(p)) > 0$ and $\phi(e^*(1)) = 0$ but $\phi'(e^*(1)) > 0$ so that ϕ reaches zero in $e^*(1)$ from below¹² and it has at least another zero in the interior of E^* . Since ϕ is positive for low values of e but negative for high values, the overall positive contribution to welfare of unobservability can be traced to the fact that the agent now always invests a little more than $e^*(p)$.¹³ Consider the contribution to welfare of a marginal increase in e under unobservability. As e is increased, total feasible surplus is increased by $\hat{w}'(e)$ each time the agent invests at least e , that is, $1 - F(e)$ of the time. On the other hand, an increase in the support of e implies more distortions on the production plan of lower types (in proportion $F(e)$), that is, a marginal increase in the amount of welfare destruction $\hat{w}(e) - w(e)$ for each of these types in order to prevent higher types to mimic these types. When e is low, a little more investment has an important positive marginal effect (since e is low) and the negative effect is low since production plans need to be distorted only a little. When e is high, the opposite applies and the marginal contribution is negative. $\phi(e^*(1)) = 0$ since additional distortions become less necessary at the margin as $q(e) \rightarrow \hat{q}(e)$ when $e \rightarrow e^*(1)$.

6. Renegotiation

In this section, I extend the game of section 3 by allowing the agent to propose a take-it-or-leave-it offer of renegotiation after he has accepted the initial contract δ proposed by the principal and committed himself to some allocation $\delta(e) = (q(e), t(e))$ in δ . To simplify the analysis, I will assume that the market price is 0 so that both the ex ante and the ex post reservation payoffs of the agent are always zero with an initial investment of $e^*(0) = 0$. As before, a refusal at the initial contracting stage terminates the relationship but if the principal refuses the renegotiation offer, then allocation $\delta(e)$ is to be implemented. I will refer to this game as renegotiation conditional on acceptance (RCA).

If the contract is accepted and $\delta(e)$ is selected, then the agent can renegotiate to any allocation that is rationally acceptable to the principal given that $\delta(e)$ can be costlessly enforced. Assume that the agent randomizes its investment decision on some subset E^* . Given that the initial contract is accepted and that the agent has committed himself to some allocation $\delta(m)$, it is easy to see that an agent that has invested e will propose the new allocation $(\hat{q}(e), t(m) + \hat{q}(e) - q(m))$ which gives the same payoff to the principal and is thus accepted. In a self-selecting renegotiation-proof contract, the initial contract is not renegotiated and the agent tells the truth about his investment level so that $q(e) = \hat{q}(e)$. Anticipating the outcome of the renegotiation, the initial contract will be self-selecting if and only if

$$t(e) - c(\hat{q}(e), e) \geq t(m) + \hat{q}(e) - \hat{q}(m) - c(\hat{q}(e), e) \quad \forall e, m \in E^*.$$

Since that relation must be true for all types played in equilibrium, it follows that

$$t(e) - \hat{q}(e) = t(m) - \hat{q}(m) = -\pi \quad \forall e, m \in E^*,$$

where π is the principal's payoff in that game when a contract is accepted and carried out. I expect a contract to be of the form $\delta_\pi = (\hat{q}(e), \hat{q}(e) - \pi)_{e \in E^*}$. Such a contract is accepted by any agent e for which

$$\hat{q}(e) - \pi - c(\hat{q}(e), e) = \hat{w}(e) - \pi \geq 0;$$

that is, whenever the payoff π the principal asks for herself is less than the realizable ex post surplus $\hat{w}(e) + c(0, e)$ as fixed cost $c(0, e)$ are to be paid anyway. I then obtain the following proposition.

Proposition 2. In the RCA game, there is no equilibrium in pure strategies. There exists a Nash equilibrium where the agent randomizes on $E^* = \{0, e^*(1)\}$ with $F = \text{Prob}(e = e^*(1)) = w(0)/(w^*(1) + c(0, e^*(1)))$ and the principal randomizes on $\Delta = \{\delta_{w(0)}, \delta_{w^*(1)}\}$ with $1 - G = \text{Prob}(\delta_\pi = \delta_{w(0)}) = c(0, e^*(1))/(w^*(1) - w(0) + c(0, e^*(1)))$. In equilibrium, both players enjoy the same payoffs that they would have had investment been observable.

Proof. The ex post realizable surplus function $\hat{w}(e) + c(0, e)$ is strictly increasing with derivative $-c_e(\hat{q}(e), e) + c_e(0, e) > 0$. It follows that it is minimal at $w(0)$. Hence, there is no point for the principal to offer any contract δ_π for which $\pi < w(0)$ since these contracts are strictly dominated by $\delta_{w(0)}$. Because of assumption (2), the agent never invests more than ϵ . We can then restrict the strategy space of the principal to contracts δ_π for which $\pi \in [w(0), w(\epsilon) + c(0, \epsilon)]$. Given any such contract δ_π , the agent's ex ante payoff is given by the maximum between the minimum loss in fixed cost $-\min_E c(0, e)$ and the maximum gains from trade $\max_E (\hat{w}(e) - \pi)$. Depending on π , that payoff is maximized either in $e = 0$ or in $e = e^*(1)$; it follows that the agent will randomize on the discrete support $\{0, e^*(1)\}$ with $F = \text{Prob}(e = e^*(1))$. If the agent plays $e = 0$, then the principal should play $\pi = w(0)$. If the agent plays $e = e^*(1)$, then the principal should play $\pi = w^*(1) + c(0, e^*(1))$. It follows that the principal will play in the discrete support $\{w(0), w^*(1) + c(0, e^*(1))\}$. No pure strategy equilibrium will exist as $e = 0$ is a strictly best response (s.b.r) for the agent to $\pi = w^*(1) + c(0, e^*(1))$ which is a s.b.r. to $e = e^*(1)$ which is a s.b.r. to $\pi = w(0)$ which is a s.b.r. to $e = 0$. Let $G = \text{Prob}(\pi = w^*(1) + c(0, e^*(1)))$. Given these numbers, the players must be indifferent over their best response strategies. This yields the same expected payoffs for the agent and the principal as under full observability:

$$\begin{aligned} -Gc(0, e^*(1)) + (1 - G)(w^*(1) - w(0)) &= G \cdot (0) + (1 - G) \cdot 0 = 0, \\ F(w^*(1) + c(0, e^*(1))) + (1 - F) \cdot (0) &= Fw(0) + (1 - F)w(0) = w(0), \end{aligned}$$

and these equations solve for F and G . That is, the principal gets all the surplus $w(0)$ that could be created under full observability and the agent gets none of it (with a market price of zero as an option). *Q.E.D.*

Although the nature of the mixed strategies is different, the fact that unobservability has no effect on welfare in proposition 2 is related to proposition 1 in Gul (1997). The possibility of renegotiation destroys the power of incentive contracts. Without these contracts, simple bargaining subgames are not sufficient to provide enough incentives for expected welfare-enhancing investment to take place.

7. Conclusion

My results apply generally to Bayesian principal-agent models with adverse selection. In these models, transfers are decomposed into “costs” that remunerate factors of production and “informational rents” that are left to the agent as an incentive to make him reveal his type. Here, I challenge this interpretation by acknowledging that an agent’s “type” is most likely to be the result of an ex ante maximizing choice. Hence, we should not talk of “informational rents” but of quasi-rents. This interpretation is important both on normative and positive grounds. From a normative point of view, the mechanism design strategy followed here is robust to the endogenous formation of types and requires only a good knowledge of cost functions to be implemented as a practical compensating scheme.

From a positive point of view, Bayesian models have very little predictive power since the contracts they predict are supposed to be functions of an elusive (at least for the econometrician) “type” distribution. One can express doubts concerning the coherence of the assumption that types are unobservable with the assumption of a common knowledge distribution of types. The resolution of this paradox is to be found in Harsanyi’s (1967) classical exposition of incomplete information model where the “type” distribution emerges as the result of some thought equilibrium process that resolves the discrepancies between the players’ various beliefs. In a sense, the approach followed in this paper substitutes that process with what can rationally be expected given factor prices on the market and the available technology. Since the contracts described in this paper depend only on market data, the theory should have more predictive power.

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Notes

¹See Fudenberg and Tirole (1990) for an application of this technique in a moral hazard problem.

²In what follows, the principal is given all the bargaining power so that her participation constraint is never binding. What matters is that the agent cannot expect to find a customer, other than the principal, ready to pay a price higher than p for his production of the specific variety.

³Suscripts denote partial derivatives for functions with more than one argument. For functions with a single argument, the prime notation will be used.

⁴The *interim* stage in the terminology of Holmström and Myerson (1983); that is, when the agent knows his type.

⁵One could think of a game where parties choose together initially to play a particular bargaining subgame at the interim stage, but that would amount to assuming that commitment is possible.

⁶The severity of the hold-up problem increases as p decreases. Note that if full enforcement contracts were available at the investment stage, efficiency would require that all trade be conducted with the principal. Hence, interestingly, the first best benchmark is unaffected by p .

⁷Since the principal has linear preferences, there are no wealth effects and the principal purchases on the market will not depend on whether he contracted successfully or not with the agent.

⁸Under my convexity assumptions, one can show that stochastic contracts are dominated by non stochastic ones. See Laffont and Tirole (1993).

⁹ $\hat{w}(e, p)$ includes fixed costs. These are irrelevant ex post, but are cancelled out with the same implicit fixed costs in $c(q(e) + q, e)$.

¹⁰This figure matches the second analytical example discussed below.

¹¹Consider the cost-minimizing investment level for production level q along the *LRAC* curve. That level is given by $E - (Q - q)/2$ whenever the value is positive and, as q is decreased, by zero otherwise ($e \geq 0$ is then binding). Furthermore, given our parametrization, one can show that $B(e) = \min_{q \in Q} c(q, e)/q$

increases with e . It follows that the breakeven point is given at $B(0) = \lim_{q \rightarrow 0} c(q, 0)/q \simeq .1353$.

¹²We have

$$\phi'(e) = \hat{w}''(e) - w''(e)F(e) - w'(e)f(e).$$

In $e = e^*(1)$, $F(e^*(1)) = 1$, $w'(e^*(1)) = 0$ and $\hat{q}(e^*(1)) = q(e^*(1)) = q^*(1)$, it follows that

$$\begin{aligned} \phi'(e^*(1)) &= \hat{w}''(e^*(1)) - w''(e^*(1)) \\ &= \frac{(c_{qe})^2}{c_{qq}} + c_{qq} \frac{(c_{ee})^2}{(c_{qe})^2} > 0. \end{aligned}$$

¹³This heuristic argument, based on the function ϕ , and what follows in the rest of the paragraph, depends a lot on unspecified accounting conventions. If $\int \phi(e)de = k$, then so does $\int \tilde{\phi}(e)de$ where $\tilde{\phi}(e) = 2kf(e) - \phi(e)$.

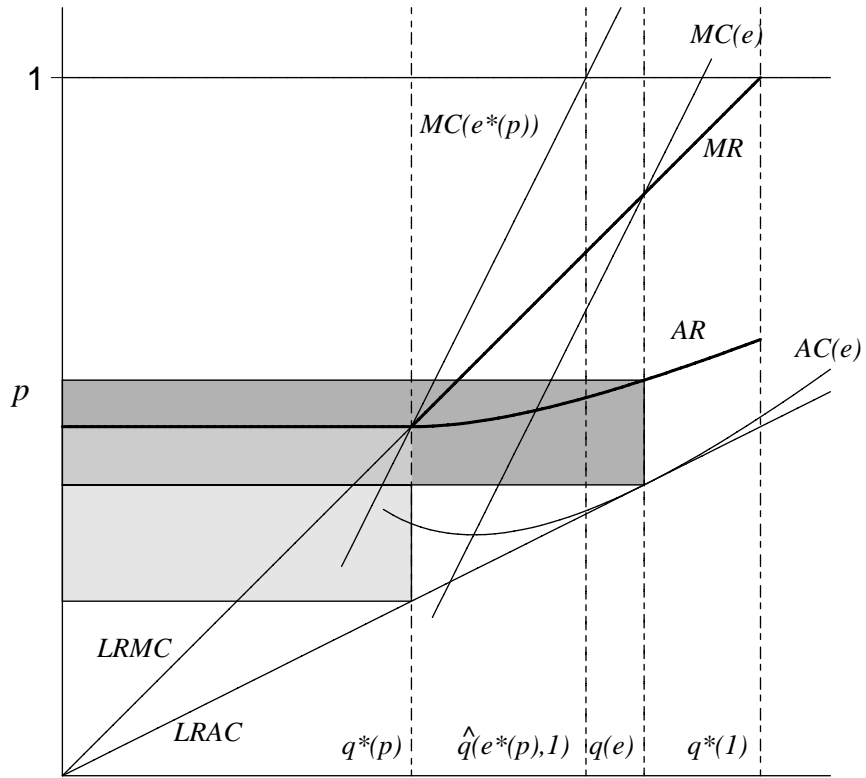


Figure 1: The equilibrium contract.

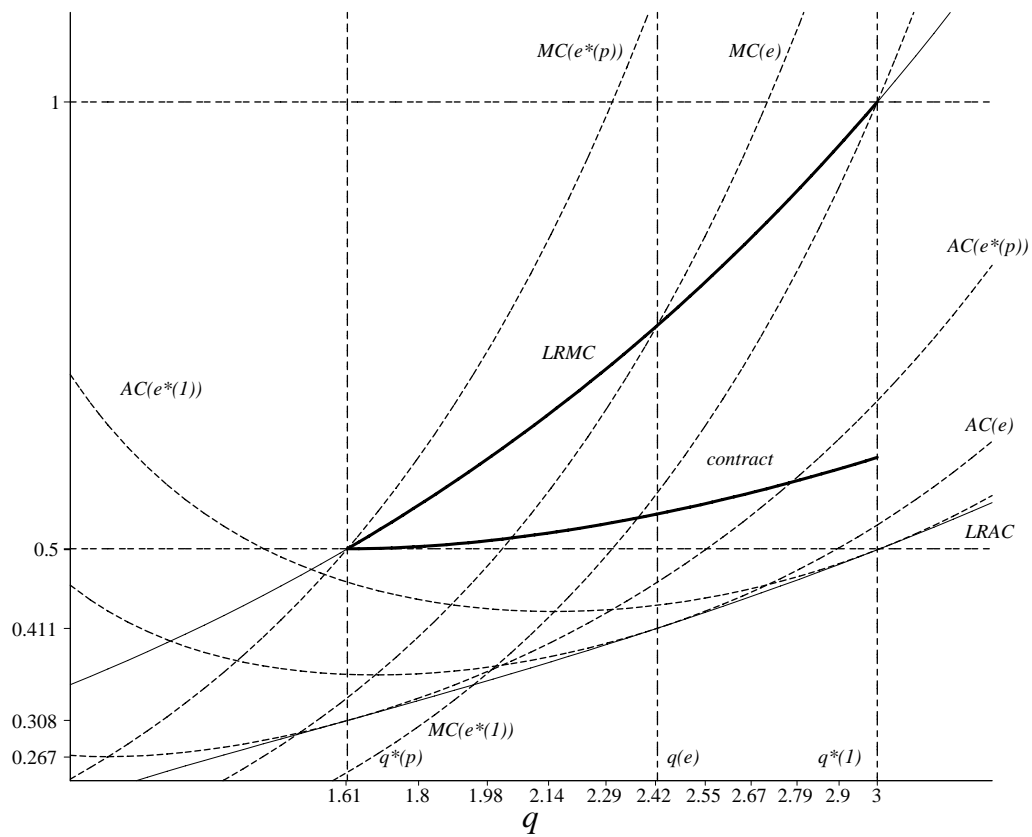


Figure 2: The equilibrium contract of example 1.

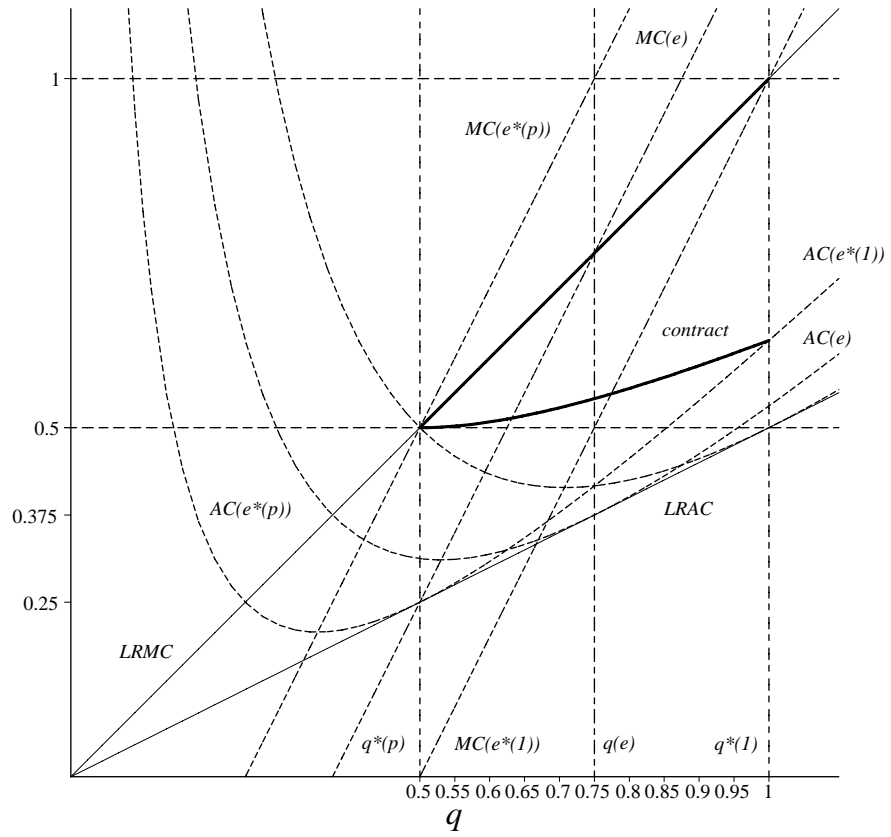


Figure 3: The equilibrium contract of example 2.

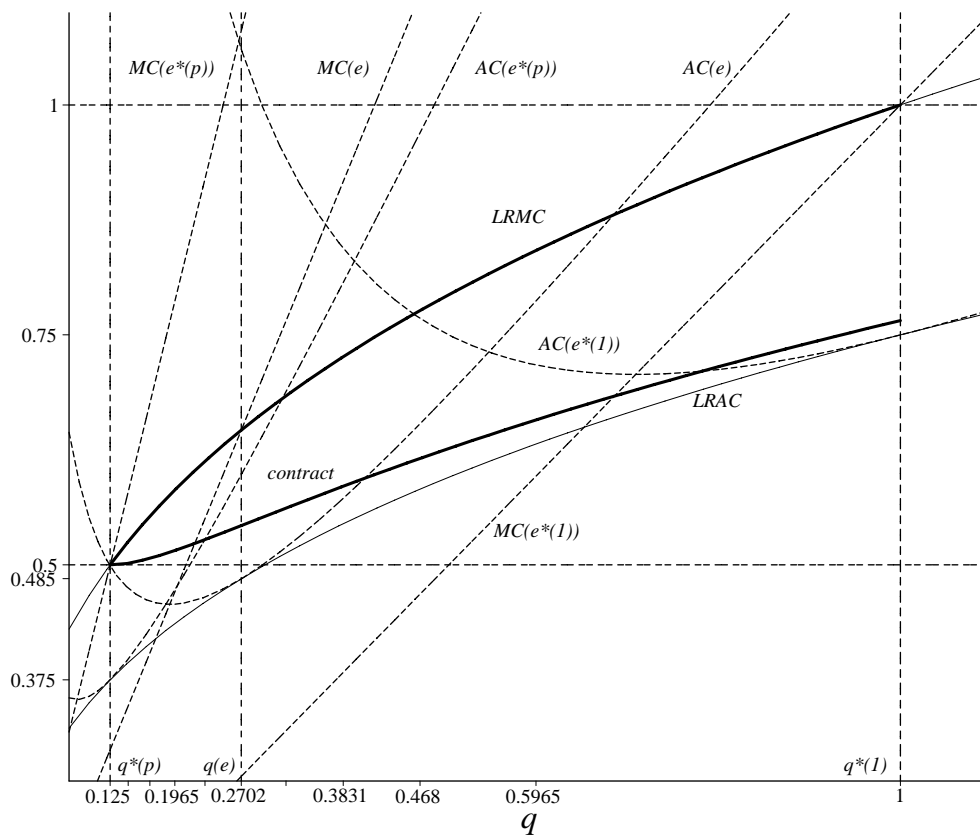


Figure 4: The equilibrium contract of example 3.