Business Cycles in a Small Open Economy with Agency Costs

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Abstract

Open economy extensions of otherwise typical DGE models have met with some difficulties. It is hard for example to replicate the correlation between output and the trade balance, as well as the variance of the latter variable. The correlation between the trade balance and the terms of trade is also problematic. Capital adjustment costs have been suggested to resolve some of these problems. In this paper, we propose a dynamic general equilibrium model which incorporates asymmetry in information and agency costs as an alternative. The model considers the possibility, associated with Irving Fisher’s (1933) “debt-deflation” story of the great depression, that entrepreneurs may be limited in their investment activities by their amount of net worth. This limitation implies that the level of internal financing available for projects will influence aggregate economic activity. The main conclusion is that the proposed model is able to replicate the Canadian stylized facts fairly well. Moreover, compared to a typical DGE model, its predictions regarding the autocorrelation functions of output growth and investment are closer to those observed in the data.
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I Introduction

Financial variables, such as entrepreneurs’ net worth, play no important role in standard Dynamic General Equilibrium (DGE) models because investment can be financed indifferently through the use of internal funds or external borrowing. In this idealized framework, the Modigliani-Miller theorem prevails and the balance between internal and external funding is irrelevant for investment. This is because they entail the same cost. Hence, business cycle dynamics are unaffected by “financial” variables. Calibrated versions of standard DGE models have been fairly successful in replicating first and second moments of important economic time series, but have met some difficulties in generating realistic autocorrelation functions for variables such as output growth and investment.

In contrast, Bernanke and Gertler (1989) proposes a model where entrepreneurs have an informational advantage over lenders. Only the former group can costlessly observe the output of their projects. The implied agency costs, imposed on the newly created capital, increase with the amount of external financing required. In this framework, a negative shock to entrepreneurs’ net worth leads to lower investment, creating a link between real and financial variables. This is very much in the spirit of Fisher’s (1933) debt-deflation story of the great depression. Moreover, following a positive aggregate productivity shock, the model predicts a hump-shaped behavior for investment and output which is consistent with the empirical findings in Cogley and Nason (1995) regarding the autocorrelation functions of these variables.

Fuerst (1995) and Carlstrom and Fuerst (1997) have introduced in a DGE environment the type of informational asymmetry and agency costs present in Bernanke and Gertler (1989). Their

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1Bernanke (1983) and Mishkin (1978) also linked the severity of the great depression to financial variables such as low entrepreneurs’ net worth.
2CF hereafter.
3For a complete review of models related to the economics of information, see Stiglitz (2000).
objective was to provide a quantitative measure of the importance of this type of financial constraint on business cycle dynamics. The simulation exercises performed in CF are based on a closed economy model calibrated on United States data. The authors are able to reproduce the hump-shaped behavior of output, hours of work and investment following a temporary but persistent productivity shock. This is significant since standard DGE models are unable to reproduce hump-shaped dynamics. These promising results invite further investigation of the role played by agency costs in the propagation of economy wide shocks.

In this paper, we build a dynamic general equilibrium model which incorporates asymmetry in information and agency costs similar to those proposed in Bernanke and Gertler (1989), Greenwald and Stiglitz (1993) and CF. We extend their analysis by considering a small open economy model in order to allow for the presence of exogenous terms of trade shocks in addition to the usual productivity shocks. Open economy extensions of otherwise typical DGE models have met with some difficulties. It is hard for example to replicate the correlation between output and the trade balance, as well as the variance of the latter variable. The correlation between the trade balance and the terms of trade is also problematic. Mendoza (1991) suggests capital adjustment costs to resolve some of these problems. When calibrated to a small open economy, namely Canada, the proposed model makes realistic predictions with respect to output, investment and the trade balance. In this paper, we consider agency costs as an alternative to capital adjustment costs. A detailed comparison of all predicted moments with the data is performed. Moreover, we consider the issue of the hump-shaped behavior of output, labor hours and investment, not addressed in Mendoza (1991).

Our main conclusions can be summarized in the following way. First, the proposed model is

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There are other ways of introducing credit rationing. Recently, Wasmer and Weil (2000) considers a model where credit market imperfections are introduced in a symmetrical way to labor market frictions by using search and a matching function between lenders and entrepreneurs similar to the one affecting employers and workers. Aiyagari and Williamson (1999) also analyzes credit rationing in a random matching model.
able to replicate the Canadian stylized facts fairly well. The predicted correlation between the trade balance and the terms of trade is positive and close to one half as in the data. The model also replicates very closely the high variance of the trade balance. It is unable however to reproduce the small negative correlation between the trade balance and GDP observed in Canadian data. The model predicts a small positive correlation. Second, terms of trade shocks are the main source of disturbances influencing the dynamics of the model via, among other things, its influence on net worth. In particular we observe that approximately 80% of the fluctuations in output can be accounted by this variable. Third, compared to a standard DGE model, the predictions of the proposed model regarding the autocorrelation functions of output growth and investment are closer to those observed in the data.

The rest of the paper proceeds as follows. Section II presents an overview of the complete model. In section III, the structure of the contract between lenders and entrepreneurs is discussed. The problems facing consumers, entrepreneurs and firms are also presented and resolved. The proposed model is then calibrated to Canadian data in Section IV. Results from simulation exercises are reported and discussed in Section V. Finally, conclusions are drawn in the last section.

II The Environment

We consider a small open economy composed of three types of agents, consumers/lenders, firms and entrepreneurs. The consumers/lenders maximize lifetime utility. Over their lifetime, consumers accumulate/decumulate wealth in the form of domestic capital goods and international lending/borrowing. They earn income by supplying labor and by renting capital to domestic firms. They also invest in a domestic mutual fund that finances the economy’s entrepreneurs. The nature and role of this mutual fund are explained in greater detail below. Firms maximize profits and produce tradable and nontradable goods with a constant return to scale technology subjected to
exogenous technology shocks. The third type of agent, labeled entrepreneurs, operates the technology required to produce the economy’s capital stock. More specifically, it is assumed that new capital goods cannot be imported from abroad and must be produced locally using a simple linear technology which combines domestic and imported goods as inputs.

Entrepreneurs use their net worth and borrow from domestic financial intermediaries to finance their purchase of domestic and imported inputs. No direct external borrowing is allowed. To keep the model manageable, it is assumed that entrepreneurs financial transactions are carried out through a capital mutual fund and are limited to within period transactions. The sequence of events during a typical period is as follows. At the beginning of the period, the technology shock and terms of trade are observed by everyone. Firms hire labor and rent capital inputs from consumers and entrepreneurs to produce domestic consumption goods. Consumers decide on their consumption level, labor effort, capital accumulation, international lending/borrowing and on the loan made to entrepreneurs through the mutual fund. Entrepreneurs use all their net worth and the resources borrowed from the mutual fund to buy the combination of perishable domestic and imported goods required to produce the domestic capital good. Parameter values are selected to make net worth small enough to ensure borrowing.

A distinctive feature of the model is that entrepreneurs are the only ones to costlessly observe their output which is subject to a random outcome. Others cannot privately observe an entrepreneur’s output without incurring an auditing cost. After observing his project outcome, an entrepreneur decides whether to repay the mutual fund or to default on his loan. In case of default, the financial intermediary audits the loan and recovers the project outcome less monitoring.

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5 This feature of the model can be motivated by the assumption that monitoring costs for foreign mutual funds are too high. In general equilibrium, new capital can however be financed abroad, indirectly, through consumers/lenders borrowing in the world capital market and lending to the local mutual fund.

6 There is a moral hazard problem since in the absence of monitoring the entrepreneur would have an incentive to report low outcomes.
costs. All of these events occur within the period and the mutual fund has no meaningful role to play between periods.

Interactions with the rest of the world are the following. To produce new capital goods, entrepreneurs must import foreign goods. The economy is small in the sense that the relative price of foreign goods – the terms of trade – is exogenous. To pay for imports, the tradable good produced locally can be exported. Preferences are such that the consumer/lender consumes both local goods (tradable and non-tradable) and the imported goods, while the entrepreneur specializes in consumption of the imported good. Moreover, individual consumers can borrow from (or lend to) the rest of the world at the world market interest rate. The capital mutual fund has no direct link with the outside world. It can best be seen as a local cooperative that facilitates financial transactions between the residents of the small economy. No outside borrowing or lending is made by this institution.

III Interactions between Firms, Entrepreneurs and Lenders

In this section, agents optimization problems are discussed in greater details. The financial contract between the mutual fund and the entrepreneurs are also presented. The economy is inhabited by infinitely lived agents. In order to preclude entrepreneurs from ever accumulating enough net worth to render borrowing unnecessary, it is assumed that they discount the future more heavily than consumers. Entrepreneurs’ subjective discount factor will be modeled as a positive fraction \( \gamma \) of lenders’ subjective discount factor \( \beta \). Let us now turn to the complete description of the optimization problems beginning with the firm’s problem.

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7By assumption, random monitoring is ruled out. As demonstrated in Gale and Hellwig (1985) and Williamson (1987), a debt contract with default in some states of the world is the optimal contract between the two parties in this type of setup.

8CF made a similar hypothesis.
III.1 The Firms

We assume that firms produce both types of goods and allocate factors of production between sectors so as to maximize net receipts, $\Pi_t$, expressed in terms of the domestically produced tradable good (the *numéraire*):

$$\Pi_t = F(\vartheta^K_t \cdot K_t, \vartheta^L_t \cdot L_t, A_t) + p_n t \cdot G((1 - \vartheta^K_t) \cdot K_t, (1 - \vartheta^L_t) \cdot L_t, A_t) - r_t K_t - w_t L_t, \quad (1)$$

where, $p_n t$, $r_t$ and $w_t$ are respectively the price of nontradable goods, the rental rate of capital and the wage rate, all measured in terms of the *numéraire*. $F(\cdot)$ and $G(\cdot)$ are the production functions for tradable and nontradable goods respectively, $K_t$ and $L_t$ measure capital and labor inputs, while $\vartheta^K_t$ and $\vartheta^L_t$ give the shares of inputs used in the tradable sector. Finally, $A_t$ is a vector of the other factors affecting production in both sectors. The production functions are Cobb-Douglas and exhibit constant returns to scale in capital and labor.

$$F(K_t, L_t, A_t) = A_t K_t^{\varphi} L_t^{1 - \varphi} \quad (2)$$

$$G(K_t, L_t, A_t) = A_t K_t^{\upsilon} L_t^{1 - \upsilon} \quad (3)$$

where, $\varphi$ and $\upsilon$ represent the shares of capital in the tradable and nontradable sectors respectively.

Under the assumption that firms behave competitively in goods and factors markets, optimal choices of $L_t$, $K_t$, $\vartheta^L_t$ and $\vartheta^K_t$ must satisfy the following necessary conditions.\(^9\)

$$w_t = (1 - \varphi) \cdot A_t \cdot \left( \frac{\vartheta^K_t \cdot K_t}{\vartheta^L_t \cdot L_t} \right)^{\varphi} \quad (4)$$

$$r_t = \varphi \cdot A_t \cdot \left( \frac{\vartheta^L_t \cdot L_t}{\vartheta^K_t \cdot K_t} \right)^{1 - \varphi} \quad (5)$$

$$(1 - \varphi) \cdot A_t \cdot \left( \frac{\vartheta^K_t \cdot K_t}{\vartheta^L_t \cdot L_t} \right)^{\varphi} = (1 - \upsilon) \cdot p_n t \cdot A_t \cdot \left( \frac{(1 - \vartheta^K_t) \cdot K_t}{(1 - \vartheta^L_t) \cdot L_t} \right)^{\upsilon} \quad (6)$$

\(^9\text{Where we postulate that firms are always at an interior solution.}\)
Equations (4) and (5) are the familiar static conditions for factor demands that equates the value of marginal products to factor prices in each period. The following two equations state that the optimal allocation of labor and capital between sectors must equalize the marginal products of each factor.

\[ \varphi \cdot A_t \cdot \left( \frac{\partial L_t}{\partial K_t} \right)^{1-\varphi} = v \cdot p_{nt} \cdot A_t \cdot \left( \frac{(1 - \varphi_L) \cdot L_t}{(1 - \varphi_K) \cdot K_t} \right)^{1-v} \]  

(7)

III.2 The Entrepreneurs

This section presents the entrepreneur’s decision problem in greater details. We proceed in two steps. First, Section III.2(i) develops the intra-period loan contract between a typical entrepreneur and the financial intermediary, taking the perspective of an entrepreneur having \( n_t \) units of net worth. Then, Section III.2(ii) looks at the question of the optimal accumulation of net worth over time.

III.2(i) The Contract

This section adapts CF’s contractual arrangement between entrepreneurs and the mutual fund to the case where the production of new capital goods partly uses imported goods. As will be seen below, this modification introduces the terms of trade as an additional variable influencing the model’s investment supply function. This creates a new channel, working through the supply side of the model, by which terms of trade shocks can induce economic fluctuations. The main features of the contractual arrangement are as follows. It is assumed that entrepreneurs produce the new capital goods with a simple linear technology that uses a composite good, \( i_t \), made of domestic \( (i_t^d) \) and imported \( (i_t^f) \) goods, as input. More specifically, the composite good \( i_t \) is a Leontief function, \( \min[\kappa \cdot i_t^d, \kappa \cdot i_t^f] \), of \( i_t^d \) and \( i_t^f \), where \( \kappa^d \) and \( \kappa^f \) are the parameters determining the optimal mix \( i^d \) and \( i^f \) in the composite investment good. \( i_t \) units of the composite good invested
by the entrepreneur produces \( \omega_t \cdot i_t \) units of new domestic capital, where \( \omega_t \) is a random component affecting this production.

The assumption that the composite investment good is a Leontief function of domestic and foreign goods is made to preserve the linearity required for consistent aggregation among entrepreneurs. Linearity of new capital formation is preserved with this assumption because costs minimization will induce all entrepreneurs, regardless of net worth, to use domestic and foreign inputs in the same proportion, i.e. \( i_t^f = \frac{\kappa^d}{\kappa^f} \cdot i_t^d \). Uncertainty in the capital production technology exists at the entrepreneur level but not the aggregate level; \( \omega_t \) is i.i.d. across entrepreneurs and time. It cannot take a negative value and has a mean of one. The distribution and density functions of \( \omega_t \) will be denoted \( \Phi(\omega_t) \) and \( \phi(\omega_t) \) respectively. For the calibration exercise performed in Section IV, \( \omega \) will be assumed to obey a lognormal distribution. By assumption, the realized value of \( \omega_t \) is private information to the entrepreneur. Others can privately observe the project outcome at a cost equal to the destruction of \( \nu \cdot i_t \) units of the capital good. Parameters are set to insure that an entrepreneur’s net worth, \( n_t \), measured in units of the numéraire, always falls short of the project’s cost, \( (1 + \kappa^d \kappa^f \cdot p_t) \cdot i_t^d \), where \( p_t \) is the terms of trade.\(^{10}\) As a result, the typical entrepreneur will be looking to finance part of his project externally. There exists a domestic financial intermediary that specializes in making risky loans to entrepreneurs. An entrepreneur who borrows an amount equal to \( (1 + \kappa^d \kappa^f \cdot p_t) \cdot i_t^d - n_t \) of the numéraire agrees to repay the financial intermediary \( (1 + r_k^k) \cdot [(1 + \kappa^d \kappa^f \cdot p_t) \cdot i_t^d - n_t] \) units of new capital at the end of the period, where \( r_k^k \) is the loan’s interest rate. Loans are risky because entrepreneurs default when project outcomes \( \omega_t \cdot \kappa^d \cdot i_t^d \) do not cover loan repayments \( (1 + r_k^k) \cdot [(1 + \kappa^d \kappa^f \cdot p_t) \cdot i_t^d - n_t] \).\(^{11}\) Default induces the financial intermediary

\(^{10}\)By convention, the terms of trade, \( p_t \), is the number of units of domestically produced tradable goods (the numéraire) required to purchase one unit of foreign good. As a result, it costs \( i_t^d + p_t \cdot i_t^d \) units of the numéraire to invest \( i_t \) units of the composite good in the linear technology. Given the cost minimizing mix of domestic and foreign goods, project costs can alternatively be expressed as \( (1 + \kappa^d \kappa^f \cdot p_t) \cdot i_t^d \).

\(^{11}\)The cost minimizing mix of domestic and foreign inputs implies that \( \kappa^d \cdot i_t^d = \kappa^f \cdot i_t^f \). Here, we exploit this property to replace \( i_t \) by \( \kappa^d \cdot i_t^d \).
to audit the projects and to recoup the project outcomes $\omega_t \cdot \kappa^d \cdot i_t^d$ less the audit costs $\nu \cdot \kappa^d \cdot i_t^d$.

One can define a critical value for $\omega_t$ below which an entrepreneur will default.

$$\bar{\omega}_t = (1 + r_t^F) \cdot [(1 + \frac{\kappa^d}{\kappa_F} \cdot p_t) \cdot i_t^d - n_t] \cdot \frac{1}{\kappa^d \cdot i_t^d}$$  \hspace{1cm} (8)

Define $f(\bar{\omega}_t)$ and $g(\bar{\omega}_t)$ as the expected income shares accruing to entrepreneurs and lenders. Then, expected income of entrepreneurs and lenders can be defined as:\(^{12}\)

$$q_t \cdot \kappa^d \cdot i_t^d \cdot f(\bar{\omega}_t) = q_t \cdot \kappa^d \cdot i_t^d \cdot \left\{ \int_{\bar{\omega}_t}^{\infty} \omega_t \Phi(d\omega_t) - [1 - \Phi(\bar{\omega}_t)] \cdot \bar{\omega}_t \right\}$$  \hspace{1cm} (9)

and,

$$q_t \cdot \kappa^d \cdot i_t^d \cdot g(\bar{\omega}_t) = q_t \cdot \kappa^d \cdot i_t^d \cdot \left\{ \int_{0}^{\bar{\omega}_t} \omega_t \Phi(d\omega_t) - \Phi(\bar{\omega}_t) \cdot \nu + [1 - \Phi(\bar{\omega}_t)] \cdot \bar{\omega}_t \right\}$$  \hspace{1cm} (10)

Where $q_t$ is the market price of new capital goods. Observe that expected income shares do not sum to unity because of expected monitoring costs $\Phi(\bar{\omega}_t) \cdot \nu$.

Under the additional assumptions that all the economic rent goes to entrepreneurs and that entrepreneurs expected income from carrying out their project is at least as high as their invested net worth, the optimal contract implies maximization of the entrepreneurs’ capital income subject to the condition that lenders’ income be no less than what they would get by simply retaining the funds. The optimal contract therefore involves the following two conditions:

$$q_t \cdot \kappa^d \cdot \left\{ 1 - \Phi(\bar{\omega}_t) \cdot \nu + \phi(\bar{\omega}_t) \cdot \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \cdot \nu \right\} = 1 + \frac{\kappa^d}{\kappa_F} \cdot p_t$$  \hspace{1cm} (11)

and,

$$i_t^d = \left\{ \frac{1}{1 + \frac{\kappa^d}{\kappa_F} \cdot p_t - q_t \cdot \kappa^d \cdot g(\bar{\omega}_t)} \right\} n_t$$  \hspace{1cm} (12)

\(^{12}\)Additional details concerning the $f(\bar{\omega}_t)$ and $g(\bar{\omega}_t)$ functions can be found in CF.
Together, conditions (11) and (12) imply that investment supply is an increasing function of the price of capital, \( q_t \), of net worth, \( n_t \), and a decreasing function of the terms of trade, \( p_t \).\(^{13}\) CF had already highlighted the relationship between \( q_t \), \( n_t \) and \( i_t \). Our analysis reveals that the terms of trade, \( p_t \), is an additional variable that impinges on investment supply in an open economy context. As a result, there will be an additional channel, going through the supply side of the model, by which terms of trade shocks will induce economic fluctuations in our framework. The relationship between investment and net worth is where the Modigliani-Miller theorem breaks down in this framework. In general equilibrium, the price of capital and entrepreneurs’ net worth are two endogenous variables and the rest of the model will seek to determine how they are affected by exogenous factors such as terms of trade and technology shocks.

**III.2(ii) Entrepreneurs’ Capital Accumulation Decisions**

Entrepreneurs are assumed to be risk neutral and to maximize expected discounted lifetime consumption. For simplicity, it is assumed that they consume imported goods \( (e_f) \) only.\(^{14}\) The objective at the end of time \( t \) of a typical entrepreneur owning \( k_t^e \) units of capital is

\[
V(k_t^e) = \max \ e_{ft} + \gamma \cdot \beta \cdot E_t[V(k_{t+1}^e)]
\]  

(13)

where,

\[
p_t \cdot e_{ft} = r_t^e \cdot n_t - q_t \cdot k_{t+1}^e
\]  

(14)

\[
n_t = (r_t + (1 - \delta) \cdot q_t) \cdot k_t^e
\]  

(15)

\[
r_t^e = \frac{q_t \cdot f(\bar{\omega}_t) \cdot \kappa^d \cdot i_t^d}{n_t} = \frac{\kappa^d \cdot f(\bar{\omega}_t) \cdot q_t}{1 + \frac{\kappa^d \cdot f(\bar{\omega}_t) \cdot q_t}{\kappa^d \cdot p_t - \kappa^d \cdot g(\bar{\omega}_t) \cdot q_t}}
\]  

(16)

\(^{13}\)Recall that \( p_t \) is defined as the price of imports divided by the price of exports, and that investment goods are imported.

\(^{14}\)Alternatively, it could be assumed that entrepreneurs’ preferences are of the Leontief type.
Recall that the entrepreneur’s subjective discount factor is a fraction $\gamma$ of the consumer’s discount factor. This assumption is made to ensure that entrepreneurs never accumulate enough net worth to dispense from external investment finance altogether. Equation (14) is the entrepreneur’s budget constraint. It says that a successful entrepreneur (i.e. non bankrupted) having invested $n_t$ units of net worth in his capital producing technology receives $r^e_t \cdot n_t$ as investment income at the end of the period, where $r^e_t$ is the rate of return of internal fund. This income is then used to purchase $e f_t$ units of foreign consumption goods and $k^e_{t+1}$ units of capital bought at prices $p_t$ and $q_t$ respectively.

Equation (15) states that the entrepreneur’s net worth comes from two sources: the rental income, $r^e_t \cdot k^e_t$, earned from renting $k^e_t$ units of capital to firms producing goods, and the undepreciated value of his beginning of period capital stock $(1 - \delta) \cdot q_t \cdot k^e_t$.\(^{15}\) Equation (16) defines the expected return on internal funds. Intuitively, an entrepreneur investing $n_t$ units of net worth, in a project expected to yield $q_t \cdot f(\bar{\omega}_t) \cdot \kappa^d \cdot i^d_t$, earns a return of $\frac{q_t \cdot f(\bar{\omega}_t) \cdot \kappa^d \cdot i^d_t}{p_t}$ on his investment.

The optimal choice of $k^e_{t+1}$ gives rise to the following Euler condition:

$$
\gamma \cdot \beta \cdot E_t \{ [(r^e_t + (1 - \delta) \cdot q_{t+1}) \cdot r^e_{t+1}] / p_{t+1} \} - (q_t / p_t) = 0 \tag{17}
$$

which represents the usual tradeoff between current and future expected marginal utility of consumption, expressed here directly in units of good since the entrepreneur is risk neutral. We now turn to the choices made by the other group of agents.

### III.3 The Consumers/Lenders

The consumers/lenders maximize expected discounted lifetime utility. Instantaneous utility is assumed to depend on consumption of domestic and imported goods as well as on leisure time. Agents

\(^{15}\)In practice, entrepreneurs should also accumulate net worth through labor income to ensure positive net worth in all states of the world. Here, we follow Carlstrom and Fuerst (1998) and we abstract from entrepreneur’s labor supply in order to simplify the presentation.
earn income from their work effort, from renting their capital goods to firms and from their investment in the world bond market. At each period, they can accumulate (liquidate) assets by acquiring (selling) domestic capital or by investing (borrowing) in foreign bonds. Consequently, the representative consumer/lender faces the following problem at time t:

$$V(b_t, k_t) = \max u(c_{d,t}, c_{f,t}, c_{n,t}, 1 - l_t) + \beta E_t[V(b_{t+1}, k_{t+1})]$$

subject to

$$r_t \cdot k_t + w_t \cdot l_t + q_t \cdot (1 - \delta) \cdot k_t + b_{t+1} \cdot R_{t+1} - b_t - c_{d,t} - p_{n,t} \cdot c_{n,t} - p_t \cdot c_{f,t} - q_t \cdot k_{t+1} = 0$$

where $u(\cdot)$ is the instantaneous utility function, $c_{d,t}$ is consumption of the domestically produced tradable good, $c_{f,t}$ is consumption of the foreign good, $c_{n,t}$ is consumption of the domestic nontradable good and $l_t$ is work effort. Time is normalized to one, so leisure is $(1 - l_t)$. Note that $k_{t+1}$ and $b_{t+1}$ refer to capital and bond holding decisions made in period $t$ for period $t+1$. Moreover, note the convention that a positive value for $b_t$ represents an external debt (expressed in terms of the numéraire). Capital goods are bought at the market price $q_t$ and international borrowing is made at the discount rate $R_{t+1}$.16

Optimal choices of $c_{d,t}$, $c_{f,t}$, $c_{n,t}$, $l_t$, $b_{t+1}$ and $k_{t+1}$ give rise to the following first-order conditions:

$$u_{c_{d,t}}(\cdot) - \lambda_t = 0$$

$$u_{c_{f,t}}(\cdot) - p_t \cdot \lambda_t = 0$$

$$u_{c_{n,t}}(\cdot) - p_{n,t} \cdot \lambda_t = 0$$

$$u_h(\cdot) + w_t \cdot \lambda_t = 0$$

16In other words, the real rate of interest on international loans made between periods $t$ and $t+1$ equals $\frac{1}{R_{t+1}} - 1$. 

\[ R_{t+1} \cdot \lambda_t - \beta \cdot E_t[\lambda_{t+1}] = 0 \]  

(24)

\[ \beta E_t[((1 - \delta) \cdot q_{t+1} + r_{t+1}) \cdot \lambda_{t+1}] - q_t \cdot \lambda_t = 0 \]  

(25)

The first four static conditions state the rate at which the consumer is willing to substitute within period the consumption of domestic tradable and nontradable goods, the foreign good and leisure. The next two conditions pertain to the optimal intertemporal allocation of international bond and domestic capital. Finally, for strictly positive values of the Lagrange multiplier, \( \lambda_t > 0 \), the budget constraint (19) is also binding. In the calibration exercise performed below the following functional form for the instantaneous utility function is used:

\[
 u(c_{dt}, c_{ft}, c_{nt}, 1 - l_t) = \left[ (cd_t^\theta \cdot cf_t^{1-\theta} - \mu + cn_t^{-\mu}) \right]^{1-\varepsilon} + \psi \cdot \log(1 - l_t) 
\]

(26)

where \( \theta \) reflects the share of domestic goods in consumption of tradables, \( \mu \) determines the consumer’s willingness to substitute tradables and nontradables in consumption, while \( (1/\varepsilon) \) is the elasticity of intertemporal substitution in consumption. Finally, \( \psi \) determines the share of leisure in the global basket of consumption. For simplicity, leisure and goods consumption are assumed separable in utility. We now turn to the task of closing the model.

### III.4 The General Equilibrium

The general equilibrium involves the simultaneous resolution of equations (4)-(7) of the firm’s problem, equations (11) and (12) of the optimal debt contract problem, equations (15), (16) and (17) of the entrepreneur’s problem, and equations (19) to (25) of the consumer/lender’s problem, together with the goods and factors market clearing conditions.  

Aggregate population is normalized to unity, with a continuum of agents divided between \( \eta \) entrepreneurs and \( (1 - \eta) \) consumers. Therefore, the market clearing conditions of the labor market
clearing the rental market of capital requires that the demand for capital services be equal to the supply, namely:

\[ K_t = (1 - \eta) \cdot k_t + \eta \cdot k^e_t \]  \hspace{1cm} (28)

in a small open economy, clearing the goods market requires two conditions. first, domestic demand and supply of nontradable must always be equalized. second, the economy’s trade balance, \( TB_t \), must reflect the difference between exports and imports.

that is,

\[ G \left( (1 - \vartheta^K_t) \cdot K_t, (1 - \vartheta^L_t) \cdot L_t, A_t \right) = (1 - \eta) \cdot cn_t \]  \hspace{1cm} (29)

\[ TB_t = F \left( \vartheta^K_t \cdot K_t, \vartheta^L_t \cdot L_t, A_t \right) - (1 - \eta) \cdot (cd_t + pt \cdot cf_t) - \eta \left( id_t + pt \cdot (ef_t + if_t) \right) \]  \hspace{1cm} (30)

finally, one must also take into account the law of motion of the aggregate capital stock:

\[ K_{t+1} = (1 - \delta) \cdot K_t + \eta \cdot \left[ 1 - \Phi(\bar{\omega}_t) \cdot \nu \right] \cdot i_t \]  \hspace{1cm} (31)

this equation reflects the fact that a fraction of new capital production, given by \( \Phi(\bar{\omega}_t) \cdot \nu \cdot i \), is lost in monitoring costs.\(^{17}\)

to close the model, one must specify the stochastic processes governing the terms of trade, \( p_t \), and the productivity shock, \( A_t \). for simplicity, we make the usual assumption that the logarithm of both shocks follow stationary independent AR(1) processes.

\[ \ln p_t = \rho^p \cdot \ln p_{t-1} + \epsilon_t \]  \hspace{1cm} (32)

\(^{17}\)recall that the production of new capital contributing to capital accumulation is limited to the sum of \( f(\bar{\omega}_t) \cdot \nu \cdot i \) and \( g(\bar{\omega}_t) \cdot \nu \cdot i \).
and,
\[ \ln A_t = \rho^A \cdot \ln A_{t-1} + \zeta_t \] (33)

Where innovations, \( \epsilon_t \) and \( \zeta_t \), are independent, centered on zero and have constant variances. Implicit in (32) and (33) is the assumption that steady state values of \( A_t \) and \( p_t \) are normalized to unity.

It is well known that external debt is indeterminate in small open economy versions of the representative agent model when \( \beta \) and the world interest rate are exogenous. In a deterministic setting, agents would borrow or lend indefinitely depending on whether \( \beta < R \) or \( \beta > R \), with resulting infinite debt accumulation or decumulation. While, the small country international indebtedness would stay constant at its exogenously given initial value if \( \beta = R \). To side-step this feature of the model and obtain a determinate level for the country’s external debt, we make the *ad hoc* but reasonable assumption that the implicit interest rate at which domestic consumers can borrow from the rest of the world depends on the country’s aggregate external debt \( B_t \) in the following way.\(^{18}\)

\[ R_{t+1} = R^* \cdot e^{-\xi B_t - \chi [B_{t+1} - B_t]} \] (34)

This equation states that the interest rate at which individual consumers can borrow internationally depends negatively on the world benchmark discount factor \( R^* \) and positively on the level and the change in the country’s aggregate outstanding debt \( B \).\(^{19}\) With this assumption, the world benchmark factor is only available to consumers in countries with no outstanding debt \( (B_t = 0) \) and zero current aggregate borrowing \( (B_{t+1} - B_t = 0) \).\(^{20}\)

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\(^{18}\)There exists in the literature two alternative solutions to make external debt determinate. Firstly, one can follow Obstfeld (1981) and make \( \beta \) respond to agent’s wealth in a way that precludes infinite debt accumulation or decumulation. Secondly, one can adopt Blanchard (1985) perpetual youth model. As in our setup, both alternatives are not without problems. Obstfeld’s solution, although intuitive, leaves open the question of the exact functional form to use. On the other hand, aggregation issues limit severely the form of utility in Blanchard’s model.

\(^{19}\)Recall that \( R_{t+1} \) is one divided by one plus the real rate of interest.

\(^{20}\)Senhadji (1997) makes a fairly similar assumption in his study of the sources of debt accumulation in small open economies.
IV  Calibration

IV.1  Business Cycles Facts in a Small Open Economy

The model is calibrated to reproduce the stylized facts from a typical small open economy, Canada. All of the relevant data has been obtained from the CANSIM database provided by Statistics Canada, except for the entrepreneur internal rate of return which comes from the Canadian Financial Markets Research Centre database. Seasonnally adjusted quarterly data is used and the sample period is 1961:1-2001:4, making 164 observations.

Table 1 reports various statistics of interest pertaining to the canadian economy. All variables are evaluated at domestic prices and have been subjected to the following transformations. They are expressed in logarithm, with the exception of the trade balance, and the Hodrick-Prescott filter was applied to remove the trend.\(^{21}\) To facilitate comparisons with the existing literature, we use two alternative definitions for the trade balance. The first measure\((tb^1)\), due to Stockman and Tesar (1995), is the difference between hpfiltered exports and imports. Alternatively, Mendoza (1991) reports statistics related to the ratio of the trade balance to GDP. We also present statistics calculated with this second definition that we refer to as \(tb^2\).

The first column of Table 1 reports the standard deviation of real \(GDP\), private consumption, investment, exports, imports, the trade balance and hours of work. The next column presents the standard deviation in proportion to the standard deviation of \(GDP\). Column three summarizes the correlation between each variable and output. In the fourth column, correlations with the terms of trade are presented. Lastly, the fifth column shows the first autocorrelation coefficient for the same series.

Columns one and two of Table 1 reveal that private consumption is nearly as variable as pro-

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\(^{21}\)The smoothing parameter was chosen to be 1600.
duction in Canada when a broad measure of consumption which includes the purchase of durable goods is used. This high variability of consumption should test severely intertemporal models based on the principle of consumption smoothing. It can next be observed that the standard deviation of investment is higher by a factor of 4.5 compared to real GDP, which is fairly standard. The next four lines of the table pertain to the external sector and they highlight some interesting additional features of the data. In particular, both imports and exports are more variable than output with imports having the highest variance, again a prediction that would normally not result from a typical model with consumption smoothing. Lastly, the variance of the trade balance is either higher or roughly equal to the variability of output depending on the definition used. It is higher for Stockman and Tesar’s definition, and the same for Mendoza’s ratio.

One often finds that the trade balance is counter-cyclical in industrialized country. See for instance Backus, Kehoe and Kydland (1992). As shown in the third column of Table 1, this feature is also present in Canadian data since our two measures agree on the counter-cyclical behaviour of the Canadian trade balance. It should be noted that, for the same consumption smoothing reason, standard DGE models have met great difficulties replicating a counter-cyclical trade balance. Finally, the last line of Table 1 indicates that the measured correlation between the cyclical components of hours and production is at 0.62. This correlation is a bit lower that the value of 0.69 found by Backus, Kehoe and Kydland (1995) for Canada, but their sample period was ten years shorter than ours.

Also of interest in an open economy context, is the instantaneous correlation between the terms of trade and the trade balance. For the period considered, it is positive for both definitions of the trade balance. Finally, the first-order autocorrelation coefficients have mean values ranging between 0.50 and 0.92, which is similar to frequently reported values. The other statistics generally

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22 Removing durables from our definition of consumption reduces the variability of consumption somewhat.
conform to what is known about other countries’ business cycles.

Table 1: Canadian Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_i$</th>
<th>$\sigma_i/\sigma_y$</th>
<th>$\rho_{i,y}$</th>
<th>$\rho_{i,p}$</th>
<th>$\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.015</td>
<td>1.00</td>
<td>1.00</td>
<td>-0.147</td>
<td>0.838</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.012</td>
<td>0.835</td>
<td>0.819</td>
<td>-0.233</td>
<td>0.764</td>
</tr>
<tr>
<td>Investment</td>
<td>0.066</td>
<td>4.51</td>
<td>0.791</td>
<td>-0.351</td>
<td>0.717</td>
</tr>
<tr>
<td>Exports</td>
<td>0.038</td>
<td>2.60</td>
<td>0.614</td>
<td>0.152</td>
<td>0.718</td>
</tr>
<tr>
<td>Imports</td>
<td>0.048</td>
<td>3.27</td>
<td>0.725</td>
<td>-0.339</td>
<td>0.805</td>
</tr>
<tr>
<td>Trade balance (tb$^1$)</td>
<td>0.041</td>
<td>2.83</td>
<td>-0.273</td>
<td>0.531</td>
<td>0.753</td>
</tr>
<tr>
<td>Trade balance (tb$^2$)</td>
<td>0.016</td>
<td>1.08</td>
<td>-0.161</td>
<td>0.329</td>
<td>0.919</td>
</tr>
<tr>
<td>Hours of work</td>
<td>0.008</td>
<td>0.467</td>
<td>0.618</td>
<td>0.023</td>
<td>0.494</td>
</tr>
</tbody>
</table>

Note. $\sigma_i$ = standard deviation of variable $i$, $\rho_{i,y}$ = correlation of $i$ with GDP, $\rho_{i,p}$ = correlation of $i$ with the terms of trade defined as the price of imports divided by the price of exports, and $\rho_i$ = coefficient of autocorrelation at lag one. $tb^1$ is the difference between logarithmically filtered exports and imports, and $tb^2$ is the ratio of the trade balance to GDP. Seasonally adjusted quarterly data is used and the sample period is 1961:1-2001:4 (except for hours of work with sample 1976:1-2001:4). The ratios in column 2 may differ from those obtained by dividing the standard deviations in column 1 due to rounding.

IV.2 Setting Parameter Values

The parameter settings have been based, as much as possible, on the existing literature. In the case where this was impossible, they have been estimated from the data, or calibrated to replicate specific sample moments.

The first group of parameters to be discussed are those drawn from the existing literature. The world benchmark discount factor $R^*$ has been fixed to 0.99 which implies a world annual real interest rate of 4%. This corresponds to the value generally used in the DGE literature. The depreciation rate of capital, $\delta$, has been set at 2.6% per quarter. Once again this is a value close to what is generally found in the literature for this parameter. The income share of capital in the tradable ($\varphi$) and nontradable ($\upsilon$) sectors have been set respectively to 0.43 and 0.28 which are the values estimated by Macklem (1993) on Canadian data. The elasticity of intertemporal substitution in
consumption $\varepsilon$ is set to 1.0, a value also used in CF. Following Stockman and Tesar (1995), $\theta$ and $\mu$ which determine the share of domestic goods in the basket of tradable goods and the willingness to substitute tradable and nontradable goods in consumption are fixed at 0.5 and 1.273.\textsuperscript{23} There is no real consensus on the cost of bankruptcy in the literature. We follow CF and set $\nu$ at 0.25, a value that is roughly in the middle of the range of existing estimates.

The second group of parameters has been selected to make the model’s steady-state equilibrium compatible with observed stylized facts. One generally finds that households allocate 33% of time endowment to work effort. This requires that $\psi$ be set to 1.717 in the artificial economy. The value of $\beta$ and $\xi$ were selected to reproduce two stylized facts about the Canadian economy. First, Macklem (1993) reports that Canada’s net foreign indebtedness is around 35% of GDP. Second, over the sample period, the Canadian annual real rate of interest has been, on average, 111 basis points higher than the US real rate. Setting $\beta$ at 0.987 and $\xi$ at 0.004 makes the model replicate exactly these moments.\textsuperscript{24}

We set $\gamma$ and $\sigma$ to match the quarterly default rate and the return on internal funds. However, no direct measure of the default rate exists for the Canadian economy and we have used CF’s estimate of 0.974%. Given the similarities of the Canadian and US economies, this value should be close to the true Canadian default rate. Our target for the steady-state return on internal funds is 5.3%, a value based on the Canadian equity premium estimated with data from the Canadian Financial Markets Research Centre database. Matching these two moments requires $\gamma$ and $\sigma$ to be set respectively at 0.949 and 0.229.\textsuperscript{25}

As mentioned previously, the exogenous state variables are assumed to follow independent AR(1) processes. The parameters of the stochastic process governing the terms of trade was estimated

\textsuperscript{23}The implied elasticity of substitution between tradable and nontradable goods, $\frac{1}{1+\mu}$, is therefore 0.44.

\textsuperscript{24}Conditional on the values of the other parameters.

\textsuperscript{25}These match closely CF’s values for these parameters.
by ordinary least square. The Canadian terms of trade was measured as the ratio of import to export price deflators. The estimated persistence parameter ($\rho^p$) is 0.87 with an associated standard deviation of 0.013 for the terms of trade innovation.\textsuperscript{26} The productivity shock is calibrated so as to replicate the variance and persistence of GDP, given the parameters of the model and the process governing $p_t$. Consequently, the persistence parameter $\rho^A$ is set at 0.25 and the standard deviation of the innovation at 0.005.\textsuperscript{27}

We are left with $\kappa^d$, $\kappa^f$, $\chi$ and $\eta$ as the last parameters to fix. The latter, $\eta$, is simply a normalization parameter and was set at 0.5. We set $\kappa^d$ and $\kappa^f$ at 2.0.\textsuperscript{28} This implies that 38 percent of imports goes for capital formation in steady state equilibrium.\textsuperscript{29} Finally, $\chi$, determines the sensitivity of the individual international borrowing rate to current aggregate borrowing ($B_{t+1} - B_t$). Given the absence of strong empirical evidence on this coefficient, a value was picked arbitrarily. Our benchmark simulations are base on a value of 0.10 for $\chi$. This value implies that if the country wanted to borrow internationally an additional amount (from steady state) equal to 10% of its steady state debt level, its borrowing rate would increase by sixty basis points. We perform a sensitivity analysis to assess the robustness of our results to different values for $\chi$. Table 2 summarizes the parameter settings used in the simulations reported below.

\textsuperscript{26}Backus et al. (1995) reports similar estimates for a smaller sample period.

\textsuperscript{27}With a highly persistent and large terms of trade shock, the productivity shock required to reproduce the output serial correlation and variance is relatively small and has a low persistence parameter. Mendoza (1991) reports a value of 0.36 for $\rho^A$.

\textsuperscript{28}The values of $\kappa^d$ and $\kappa^f$ was constrained by numerical issues. In particular, we were not successful in finding the steady-state equilibrium when $\kappa^d$ or $\kappa^f$ was set below 2.0.

\textsuperscript{29}This is somewhat lower than the number reported in the World Development Report (1994). For instance, Table 14 of that report revealed that in 1992, fifty percent of Canadian merchandise imports were made of machinery and equipment. The World Bank statistic refers to the share in merchandise imports however. On can presume that the statistic would have been lower had all types of imports been considered.
Table 2: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.026</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$R^*$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.43</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.273</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.10</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.28</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.229</td>
</tr>
<tr>
<td>$\kappa^f$</td>
<td>2.0</td>
</tr>
<tr>
<td>$\rho^p$</td>
<td>0.87</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.717</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.949</td>
</tr>
<tr>
<td>$\kappa^d$</td>
<td>2.0</td>
</tr>
<tr>
<td>$\rho^A$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.987</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.004</td>
</tr>
</tbody>
</table>

These parameter values are those used in the benchmark model from which the simulation results presented in the paper are obtained.

V The Model’s Predictions

V.1 Replicating the Stylized Facts

As a first task, it is important to verify whether the artificial economy replicates well the business cycles facts exposed above. Table 3 reports the business cycle statistics derived from the artificial economy with agency costs. The model’s numerical solution is obtained with the King, Plosser and Rebelo (1987) algorithm. All statistics refer to population moments derived from the model’s numerical solution.\(^{30}\) Columns one to five of this table report the standard deviation, the standard deviation in proportion to GDP, the correlation coefficients with GDP and the terms of trade, and the first autocorrelation coefficient of the variables pertaining to the artificial economy. The model has predictions about more variables than those observed in the data, some of these additional statistics are also presented in this table.

Closed economy models generally predict that the variance of consumption is smaller than the variance of GDP. Here, the access to international markets implies that consumers/lenders have an even greater opportunity to smooth out consumption than in a closed economy setting. However, in the presence of asymmetric information and agency costs, entrepreneurs behave very differently. For example, a negative terms of trade shock (a fall in the price of imports) induces

\(^{30}\text{Additional details on the method used to compute population moments can be found on pages 41 and 42 of King et al. (1987).}\)
them to produce more capital goods. Since their production activity is limited by their level of net worth, they temporarily consume less in order to carry out their investment plans, and then consume more again. This makes their consumption level very volatile as shown in the bottom part of Table 3. But since their consumption level is very small, it contributes only marginally to the variance of aggregate consumption. The model predicts a ratio $\sigma_c/\sigma_y$ of roughly seventy-five percent for aggregate consumption. This number is only a bit smaller than what is observed in the data.

Another moment that is reasonably well matched by the data is the variance of investment. This is an interesting feature of the model since this is the mechanism through which agency costs operate. The data suggests a ratio $\sigma_I/\sigma_y$ of 4.5, while the agency cost model predicts 5.2. This number is still too high, but that aspect of the model does a lot better than a simple DGE model would. For example, Mendoza (1991) reports a predicted ratio of 7.5. As suggested by the latter, introducing capital adjustment costs can also bring this statistic in line with the data.

Table 3: Business Cycle Statistics in an Artificial Economy with Agency Costs

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_i$</th>
<th>$\sigma_i/\sigma_y$</th>
<th>$\rho_{i,y}$</th>
<th>$\rho_{i,p}$</th>
<th>$\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.015</td>
<td>1.00</td>
<td>1.00</td>
<td>-0.568</td>
<td>0.838</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.011</td>
<td>0.748</td>
<td>0.616</td>
<td>0.140</td>
<td>0.785</td>
</tr>
<tr>
<td>Investment</td>
<td>0.078</td>
<td>5.18</td>
<td>0.408</td>
<td>-0.880</td>
<td>0.927</td>
</tr>
<tr>
<td>Exports</td>
<td>0.029</td>
<td>1.95</td>
<td>0.680</td>
<td>0.143</td>
<td>0.704</td>
</tr>
<tr>
<td>Imports</td>
<td>0.029</td>
<td>1.92</td>
<td>0.491</td>
<td>-0.631</td>
<td>0.728</td>
</tr>
<tr>
<td>Trade balance ($tb^1$)</td>
<td>0.045</td>
<td>2.98</td>
<td>0.139</td>
<td>0.496</td>
<td>0.750</td>
</tr>
<tr>
<td>Trade balance ($tb^2$)</td>
<td>0.015</td>
<td>0.968</td>
<td>0.130</td>
<td>0.502</td>
<td>0.751</td>
</tr>
<tr>
<td>Hours of work</td>
<td>0.008</td>
<td>0.501</td>
<td>0.773</td>
<td>-0.811</td>
<td>0.773</td>
</tr>
<tr>
<td>Entrepreneurs</td>
<td>0.494</td>
<td>33.0</td>
<td>0.260</td>
<td>0.032</td>
<td>0.258</td>
</tr>
<tr>
<td>Lenders Consumption</td>
<td>0.010</td>
<td>0.638</td>
<td>0.603</td>
<td>0.151</td>
<td>0.959</td>
</tr>
<tr>
<td>Entrepreneurs Net Worth</td>
<td>0.070</td>
<td>4.66</td>
<td>0.334</td>
<td>-0.764</td>
<td>0.872</td>
</tr>
<tr>
<td>Bankruptcy Probability</td>
<td>0.140</td>
<td>9.36</td>
<td>0.435</td>
<td>-0.776</td>
<td>0.418</td>
</tr>
</tbody>
</table>

Note. $\sigma_i = \text{standard deviation of variable } i$, $\rho_{i,y} = \text{correlation of } i \text{ with } GDP$, $\rho_{i,p} = \text{correlation of } i \text{ with the terms of trade}$ and $\rho_i = \text{coefficient of autocorrelation at lag one}$. The ratios in column 2 may differ from those obtained by dividing the standard deviations in column 1 due to rounding.
The model underestimates the standard deviations of exports and imports taken separately, but is very good at replicating the variability of the trade balance. Canadian data suggest that imports are slightly more volatile than exports, a feature that the model is not able to reproduce. Consumption smoothing is operative in this setup leading to a more severely underestimated variance of imports and a predicted slightly pro-cyclical behavior of the trade balance that is not found in the data.\textsuperscript{31} It can be observed however that the correlation between the trade balance and the terms of trade is predicted to be positive at roughly one half, as is the case for the Canadian economy. Overall it can be said that with respect to replicating the moments presented in Table 1, the agency cost model performs fairly well.

Two other interesting predictions of the agency cost model are presented in Table 3. First, entrepreneurs’ net worth is predicted to be nearly five times more volatile than output. Following a serially correlated negative terms of trade shock for example, entrepreneurs will reduce current consumption in order to increase their net worth and produce more capital goods. This behavior makes net worth and entrepreneurs’ consumption very volatile. Second, the predicted probability of bankruptcy, \( \Phi(\bar{\omega}_t) \), has a high variance and is pro-cyclical. This result is inherent to the contract specified. One should note that this prediction does not arise from the open economy extension performed in this analysis, since it would be present in a closed economy context as well.

In order to evaluate the impact of changing some of the parameters for which no direct evidence was available, the results from a sensitivity analysis are reported. The main statistics for these alternative specifications are reported in Table 4. The first line shows what happens when the elasticity of intertemporal substitution, \((1/\varepsilon)\), is halved. The next two lines vary the parameter affecting the interest rate when the amount borrowed internationally changes. The parameter \( \chi \) is

\textsuperscript{31}Mendoza (1991) indicates that small adjustment costs can lead to a predicted negative correlation between the trade balance and output. Adding liquidity constraints on the consumers side as in Carmichael, Keita and Samson (1999) also seems to produce more realistic variances and correlations.
Table 4: Sensitivity Analysis

<table>
<thead>
<tr>
<th>ε</th>
<th>σ_c/σ_y</th>
<th>σ_I/σ_y</th>
<th>σ_x/σ_y</th>
<th>σ_im/σ_y</th>
<th>σ_h/σ_y</th>
<th>ρ_{tb^1,y}</th>
<th>ρ_{tb^1,p}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.691</td>
<td>5.7</td>
<td>2.24</td>
<td>2.06</td>
<td>0.473</td>
<td>0.179</td>
<td>0.489</td>
</tr>
<tr>
<td>0.25</td>
<td>0.842</td>
<td>4.19</td>
<td>1.44</td>
<td>1.34</td>
<td>0.576</td>
<td>0.154</td>
<td>0.477</td>
</tr>
<tr>
<td>0.05</td>
<td>0.737</td>
<td>6.55</td>
<td>2.59</td>
<td>2.71</td>
<td>0.507</td>
<td>0.139</td>
<td>0.453</td>
</tr>
</tbody>
</table>

Where, c = consumption, I = investment, x = exports, im = imports, h = hours of work, and tb = trade balance. Using \( tb^2 \) instead of \( tb^1 \) does not change the results significantly.

first increased to 0.25 and then decreased to 0.05. No significant changes are apparent from the benchmark case discussed previously with the possible exception that reducing \( \chi \) makes imports slightly more variable than exports as observed in the data.

V.2 Simulations

This section presents the results from two simulation exercises. We consider the impact of temporary but persistent disturbances that move the economy away from the steady state for a certain period of time. We focus on the effects of productivity and terms of trade disturbances. The autocorrelation coefficient being positive in both cases, we consider below the impact of shocks that disappear only gradually, but more so in the case of the external shock. The impact of this terms of trade shock is first considered. Recall that since the economy is small, it takes the behavior of this variable as given.

V.2(i) Terms of Trade Shock

The first experiment considers the impact of a positive 1% terms of trade shock, which represents a rise in the relative price of imports. This shock persists for some time due to the associated positive autocorrelation coefficient, 0.87, in the \( p_t \) equation. The impulse response of the economy is depicted in the various panels of Figure 1. The lines drawn in each panel reproduce the immediate
percentage changes in the variables from the initial steady state and the paths describing the return of each variable to this steady state. The shock occurs in period four.

The increase in the terms of trade makes the composite investment good more expensive. This reduces entrepreneurs’ capacity to produce new capital, at all levels of net worth, and it generates a shift to the left of in the investment supply function. As a result, the price of new capital is pushed up at the period of the shock. This causes a decline in investment demand and results in a smaller amount of capital good being produced. The fall in investment is accompanied, in general equilibrium, by an increase in consumers/lenders consumption, even though the foreign component of consumption is negatively affected by the shock, thanks to the higher relative price of imports. Higher consumption induces households to reduce labor supply, which in turn makes aggregate output fall. In order to achieve their planned consumption in the face of a lower income, households borrow temporarily from the rest of the world.

Entrepreneurs see their consumption of the foreign good increase in the first period. There are two forces behind this phenomenon. First, the foreign good being more expensive they would normally like to consume less of it, however, since they cannot substitute for the domestic good or leisure this impact is not significant. Second, since the price of new capital is higher, they prefer to accumulate less capital for the future and to consume more in the present. The combined rises in households and entrepreneurs consumption is responsible for the period four movement of the trade balance towards a deficit.

Entrepreneurs’ net worth is affected positively at impact because of the higher price of capital. The fall in investment leads however to a smaller capital stock in period five. As a result, entrepreneurs’ net worth start declining the period following the shock, which leads to still lower future investment supply and higher future price of new capital. Rising capital price stimulates consumption spending, particularly households’, and discourages capital accumulation. The lower
Figure 1: Terms of trade shock

\[ q_t = \left(1 + \frac{\kappa_d}{\kappa_f} \cdot p_t \right) \cdot s_t \]

- \( q_t \)
- \( (1 + \frac{\kappa_d}{\kappa_f} \cdot p_t) \cdot s_t \)
- \( cd_t + pm_t \cdot cn_t + pt \cdot cf_t \)
- \( cf_t \)
- \( h_t \)
- \( PIB_t \)
- \( K_t \)
- \( tb_t \)
- \( ce_t \)
- \( nt \)
capital stock held by entrepreneurs will imply another fall in their level of net worth the following period and another fall in investment. Given the temporary nature of the shock, these variables eventually start returning to their steady state.

Figure 1 highlights some interesting features of this open economy model with agency costs. Firstly, a terms of trade shock leads to hump shape responses for investment and GDP. The dynamic response of GDP can easily be traced to the paths of the capital stock and of hours of work. Secondly, a depreciation of the terms of trade produces a J-curve type responses for the trade balance that is somewhat similar to those observed in the data. Here, the initial deficit of the trade balance comes from the consumption burst caused by the high price of capital. Thirdly, the impulse responses depicted in Figure 1 reveal that agency costs affect the dynamics of the model, particularly during the early periods following the shock. Carlstrom and Fuerst (1997) and (1998) studied extensively the impulse responses of a closed economy following a persistent productivity shock. Many of the distinctive features highlighted by these authors are preserved in an open economy setting with a terms of trade shock accounting for most of the fluctuations.

V.2(ii) Productivity Shock

The second experiment considers the impact of a positive 1 % productivity shock. This shock persists only for a short time due to the associated small autocorrelation coefficient, 0.25, in the $A_t$ equation. The impulse response of the economy is depicted in the various panels of Figure 2. As previously, the lines drawn in each panel reproduce the immediate percentage changes in the variables from the initial steady state and the paths describing the return of each variable to this steady state. Once again, the shock occurs in period four.

The effect of a positive productivity shock in a DGE model are well known to depend on two opposing forces. On one hand, higher productivity raises the level of output and induces a
pro-saving effect as households attempt to smooth out the extra consumption over many future periods. However, the assumed persistence of the shock generates, on the other hand, an opposing pro-borrowing effect because households simultaneously want to install new capital to capture the benefit of higher productivity, as soon as possible. Obstfeld (1986) and Finn (1990) show that the strength of the pro-borrowing effect increases with the degree of persistence of the shock because the expected marginal product of capital is higher over a longer horizon, making capital accumulation more attractive. In a closed economy setup, these opposing effects are brought in line by the movement of the real interest rate. As a result, saving and investment are essentially the same decision in closed economy model, investment being simply the share of output that is left unconsumed. In a small open economy model, the real interest rate is exogenous and the relative strength of these opposing effects determine instead the cyclical nature of the trade balance. The pro-saving effect makes the trade balance pro-cyclical while the pro-borrowing effect induces a counter-cyclical movement. Here, the calibrated productivity shock is not persistent enough to make the second effect dominant and the trade balance is, accordingly, predicted to be pro-cyclical.

A positive productivity shock raises the marginal product of labor and makes leisure more costly. Consumers/lenders respond by increasing their work effort. They also increase their demand for all types of goods. These consumers also want to save part of their increased income in order to smooth out consumption. They have two means of saving, capital good accumulation (investment), and international lending (or reduction in external debt). For the benchmark scenario, the immediate effects go in the direction of more investment demand and less borrowing. At the period of the shock, investment demand increases more than investment supply because entrepreneurs ability to produce new capital is limited by their net worth. Therefore, the price of capital must rise to bring demand in line with the limited supply.

Given the low persistence of the shock, the higher marginal product of capital does not last
Figure 2: Productivity shock

\begin{align*}
q_t &= (1 + \kappa d p_t) \cdot \beta^d_t \\
\text{cd}_t + \text{pm}_t \cdot \text{cn}_t + p_t \cdot \text{cf}_t &\quad \text{ct}_t \\
\text{ht}_t &\quad \text{PIB}_t \\
\text{K}_t &\quad \text{tb}_t \\
\text{ce}_t &\quad \text{n}_t
\end{align*}
long enough to induce entrepreneurs to accumulate capital. Contrary to consumers/lenders, entre-
preneurs find optimal to reduce their end-of-period capital stock and to increase their consump-
tion by more than 5% at impact. CF find instead that entrepreneurs’ consumption falls at impact.
This difference results from the divergence in the assumed persistence of productivity shocks. The
longer the productivity shock lasts, the longer is the horizon over which the demand for new capital
is affected positively. With investment depending on agency costs, entrepreneurs’ ability to produce
the required capital is severely limited by their net worth. As a result, entrepreneurs have a strong
incentive to build net worth at a faster pace to rip the reward brought by the improved productivity
of capital. At impact, this can be done by reducing their consumption. Here, productivity shocks
do not last long enough to induce this behavior.

The interesting aspects related to the serial correlation of important variables noted in the case
of the terms of trade shock - and in CF for a productivity shock - are not present here because of
the very small persistence of the disturbance. The variables adjust at impact and then the return
to the steady state starts the following period. No hump-shaped behavior is observed in this case.
It should be recalled however that most of the fluctuations in output are accounted for by terms of
trade movements when the model is calibrated to Canadian data.

VI Conclusion

This paper has put the emphasis on informational considerations. We have built a dynamic general
equilibrium model which incorporates asymmetry in information and agency costs. We extend the
analysis in Carlstrom and Fuerst (1997) by considering a small open economy model in order to
allow for the presence of exogenous terms of trade shocks in addition to the usual productivity
shocks. Open economy extensions of otherwise typical DGE models have met with some difficulties
when trying to replicate moments related to the trade balance in particular. Capital adjustment
costs have been introduced in the literature to solve this problem. In this paper, we have considered agency costs as an alternative to capital adjustment costs.

Our main conclusions can be summarized in the following way. First, the proposed model is able to replicate the stylized facts of the economy considered, Canada, quite well. The predicted correlation between the trade balance and the terms of trade is positive and close to one half as in the data. The model also replicates very closely the high variance of the trade balance. It is unable however to reproduce the small negative correlation between the trade balance and GDP observed in the Canadian data. Second, terms of trade shocks are the main source of disturbances influencing the dynamics of the model via, among other things, its influence on net worth. Third, compared to a standard DGE model, the predictions of the proposed model regarding the autocorrelation functions of output growth and investment are closer to those observed in the data.

In summary, capital adjustment costs are better at replicating the counter-cyclical behavior of the trade balance, but they do not produce the hump-shaped behavior of output and investment agency costs generate following a terms of trade shock. Combining these two predictions must still be done and is an avenue for future research.
References


