# Environmental Impact Assessment and Investment under Uncertainty
# An Application to Power Grid Interconnection.

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**Abstract**

We consider a firm that must undergo a costly and time-consuming regulatory process before making an irreversible, lagged investment whose value varies randomly. We analyze two cases: regulatory approval is valid forever or it expires after some time. We apply our model to Hydro-Québec's project of building a 1250 megawatts interconnection with Ontario. We find that the firm may start the regulatory process earlier if regulatory approval is valid long enough or if uncertainty is high enough; it postpones it otherwise. When to start the regulatory process and when to invest depend on the duration of the regulatory green light.

## 1. Introduction

From the point of view of the firm, regulatory requirements prior to starting a project are a costly investment lag. Since the work of Bar-Ilan and Strange (1996), the theoretical and practical importance of investment lags has been increasingly acknowledged, as projects often need several years before they become operational. This lag is of course needed for project design, planning, and construction, but also to obtain the necessary regulatory approval, often for environmental reasons. Environmental impact assessment can be particularly long (compared to the building stage) for energy project such as natural gas and oil pipelines, power plants and high voltage transmission lines. Distinguishing between the different causes of a project lag is important because a firm typically has little control over the cost, the duration, and even the probability of success of a regulatory review. Moreover, once secured, the regulatory green light may be valid only for a limited time. Thus far, these considerations appear to have received little attention in the economics literature on regulation.

The purpose of this paper is thus to analyze how the parameters of a regulatory review (duration, cost, and probability of success) affect the decision to invest in a simple model of investment under uncertainty. We consider a firm that contemplates making a lagged, irreversible investment, whose value varies stochastically. The firm must incur an upfront cash outlay to start the regulatory process, whose outcome is uncertain. Once regulatory approval has been granted, we consider two possible cases: 1) it is valid forever; or 2) it expires. The firm thus faces a sequential investment problem, where an initial investment is required to get the option of investing. To ground our numerical results, we apply our model to Hydro-Québec's recent project proposal to add a 1250 MW (megawatts) interconnection to the Ontario power grid.[1]

Our paper makes two contributions. First, we show that the duration of the validity of regulatory approval significantly impacts the decisions to start the regulatory process and to proceed with the project. For small durations of the regulatory approval or for low volatility, the firm delays starting the regulatory process, but this is reversed for long approval durations or for high volatility. In spite of the presence of an investment lag, the firm delays investing (once approval has been secured) when uncertainty increases, because we suppose that the flow of investment benefits admits a lower reflecting boundary. This assumption, which is common in the investment literature, is appropriate if there is no risk that the investment opportunity disappears. It seems reasonable here for our application to a power grid interconnection. These results underscore the need for transparent regulatory policies.

Second, our paper is one of the few applications of real options to regulation and to energy projects; one exception is Chaton (2001). There is also a growing literature on electricity

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[1] This interconnection will allow Hydro-Québec to perform arbitrage operations on the Ontario electricity market through the use of its large hydro power sites.

spot prices and futures: see for example Bystrom (2003), Bessembinder and Lemmon (2002) and Lucia and Schwartz (2002)). For transmission lines, the only study dealing with investment under uncertainty is by Martzoukos and Teplitz-Sembitzky (1992). Our results thus help understand the impact of regulations on the decision to invest under uncertainty, which is useful both to firms and to regulatory agencies.

This paper is organized as follows. In the next section, we present our model. In Section 3, we apply our model to Hydro-Québec's proposed capacity addition to the interconnection of the Ontario/Quebec power grid; this project is presently under review by Quebec's environmental regulatory agency. Section 4 offers concluding comments and discusses some possible extensions.

## 2. The Model

We consider a firm interested in developing a project such as a high voltage power line that interconnects two power grids. We suppose that before it can make the investment, the firm's project is subject to regulatory proceedings (such as an environmental impact assessment) that take time $T_R$ and cost $C_R$, which is sunk. Both $T_R$ and $C_R$ are assumed known for simplicity. The firm may start the regulatory review at any time.

The firm cannot proceed with its project if the regulatory outcome is negative. If it is successful (with probability $q \in (0,1)$), however, the firm gains the possibility of investing a known, sunk amount $C_B$ to start its project, which begins yielding a flow of benefits after a period $T_B$ (when construction is complete, for example).[2] For tractability, we suppose that the flow of net project benefits, denoted by $X$, follows the geometric Brownian motion (GBM):

$$dX = \mu X dt + \sigma X dz, \qquad (1)$$

where: $\mu > 0$ but $\mu < \rho$ ($\rho$ is the relevant interest rate) in order for the present value of expected project benefits to be finite; $\sigma > 0$; and $dz$ is the increment of a standard Wiener process (Karlin and Taylor 1981).

As emphasized by Dixit and Pindyck (1994), because of uncertainty and irreversibility, a standard cost-benefit approach is likely to yield incorrect decisions for the timing of the regulatory process and of the investment. Here there are two sources of uncertainty: the risk of being turned down by the regulatory body, and the randomness of project benefits. Likewise, irreversibility stems from the sunk costs associated with the regulatory process and from the implicit assumption that, once the project is built, the firm cannot recoup its investment, $C_B$, if it turns out that it made a mistake. This assumption seems especially relevant for investments in power lines because they typically have little residual value (basically the value of scrap metal).

In this context, it is fruitful to formulate this problem using concepts from the theory of real options. Thus, we see the possibility of initiating the regulatory process as a compound perpetual American option (we assume that the possibility to start the regulatory process does not expire); if it is exercised, it gives (with probability $q$) the firm the option (also American) of building its project.[3] The firm thus needs to solve a compound stopping problem. First, let us

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[2] For simplicity, $C_B$ also includes the present value of maintenance and operation costs, which are assumed known.

[3] An American option can be exercised at any time until it expires. By contrast, a European option can be exercised only at maturity (i.e., at the end of its life). For more on options, see for example Wilmott, Howison, and Dewynne (1995).

analyze the decision to start the regulatory process.

### 2.1 The decision to start the regulatory process

It is intuitively clear that the firm should start this process when $X$ is high enough (recall that $X$ is the flow of net project benefits). The firm then exchanges the option to start the regulatory process for $V_R(X)$, the expected net present value of the project itself, corrected for the risk of not getting regulatory approval and for the cost of the regulatory process. Thus, for $x$ "large enough,"

$$\Phi_R(x) = V_R(x), \tag{2}$$

where

$$V_R(x) = -C_R + qe^{-\rho T_R} \int_0^{+\infty} \Phi_B(y, T_R) f(y, T_R; x) dy. \tag{3}$$

In the above,

- $-C_R$ is the present value of the cost to the firm of the regulatory proceedings;
- $q \in (0,1)$ is the probability that the regulatory outcome is positive;
- $\rho$ is the firm's risk adjusted interest rate;
- $T_R$ is the time required to complete the regulatory process;
- $\Phi_B(x,t)$ is the value of the option to build the project at time $t$ and for $X$=$x$. $\Phi_B(.)$ depends explicitly on time only if the regulatory approval expires. We adjust the time clock here so that the regulatory review starts at time 0; and
- $f(y, T_R; x)$ is the value at $y$ of the probability density of $X$ at $T_R$ time units after $X$=$x$.

Since we assume that $X$ follows a GBM, $f(y, T_R; x)$ is the lognormal density

$$f\left(y,T_R;x\right)=\frac{1}{y\sigma\sqrt{2\pi T_R}}e^{\frac{-1}{2\sigma^2 T_R}\left(\ln\left(\frac{y}{x}\right)-mT_R\right)^2}, \tag{4}$$

where for convenience, we introduce

$$m\equiv\left(\mu-\frac{\sigma^2}{2}\right). \tag{5}$$

However, the firm is better off waiting when $X$ is "too low". Let $x_R^*$ denote the frontier between the high values of $X$ for which the regulatory process should be started and the low values of $X$ for which waiting is optimal. For $x\in(0,x_R^*)$, the return on $\Phi_R\left(x\right)$ per unit of time should equal its expected capital gains so, after using Ito's lemma, we find that $\Phi_R\left(x\right)$ verifies the well-known second-order linear ordinary differential equation (a Bellman equation):[4]

$$\rho F\left(x\right)=\mu xF^{'}\left(x\right)+\frac{\sigma^2 x^2}{2}F^{"}\left(x\right). \tag{6}$$

It is easy to check that the general solution to (6) is a combination of two power functions: one is negative and the other is positive. Since the option term should be finite when $x$ tends towards 0, we eliminate the negative power function and find

$$\Phi_R\left(x\right)=A_0 x^{\theta}, \tag{7}$$

where $A_0$ is an unknown constant, and

$$\theta=\frac{-m+\sqrt{m^2+2\rho\sigma^2}}{\sigma^2}>1. \tag{8}$$

$\theta>1$ because we assume that $\rho>\mu$.

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[4] This approach is correct here because we assume that X=0 is a lower reflecting boundary; see Saphores 2002.

Since we solve simultaneously for $x_R^*$ and $A_0$ in (7), we need another condition in addition to (2); it is the well-known "smooth-pasting" condition (Dixit and Pindyck 1994):

$$\frac{d\Phi_R}{dx}\Big|_{x=x_R^*} = qe^{-\rho T_R}\frac{d}{dx}\left(\int_0^{+\infty}\Phi_B\left(y,T_R\right)f\left(y,T_R;x\right)dy\right)\Big|_{x=x_R^*}. \tag{9}$$

Equations (2), (3), (7) and (9) enable us to find $\Phi_R$ once $\Phi_B$ is known.

## 2.2 The decision to start construction

Let us now consider the decision to build assuming that the regulatory green light has been obtained at time $T_R$. We consider two possibilities: 1) the regulatory green light is valid forever; and 2) it is valid only for a limited time $T_A$. This constraint reflects the need to revisit regulatory approval following changes in economic, political, or environmental conditions. When the authorization to build expires, the firm looses the opportunity to invest.

*Case 1: the regulatory green light is valid forever.*

This assumption simplifies the problem. First, the option to build does not depend explicitly on time (it is a perpetual American option), so we denote it by $\tilde{\Phi}_B(x)$ instead of $\Phi_B(x,t)$. Based on arbitrage considerations, $\tilde{\Phi}_B(x)$ also verifies Equation (6) for $x \in (0,\tilde{x}_B^*)$. Since $\tilde{\Phi}_B(x)$ should be finite when $x$ is close to 0, we keep only the solution of (6) which is a positive power function, so that (see (8) for an expression of $\theta$)

$$\tilde{\Phi}_B(x) = B_0 x^{\theta}. \tag{10}$$

Second, the frontier between values of $X$ for which the firm should build and those for which it should wait is again independent of time since we have an autonomous problem (time intervenes only through discounting). We denote this frontier by $\tilde{x}_B^*$. As before, the firm should wait if $X$ is not high enough, and it should invest immediately otherwise. Thus, for $x \geq \tilde{x}_B^*$, the option to build equals $\tilde{V}_B(x)$, the sum of building costs plus the present value of the flow of expected net revenues:

$$\tilde{\Phi}_B(x) = \tilde{V}_B(x), \tag{11}$$

where

$$\tilde{V}_B(x) = -C_B + e^{-\rho T_B} \int_0^{+\infty} PV(y) f(y, T_B; x)\, dy. \tag{12}$$

In the above:

- $T_B$ is the time required to complete the project;
- $PV(y)$ is the present value of the flow of project benefits when $X=y$; and
- $f(y, T_B; x)$ is the value at $y$ of the probability density function of $X$, $T_B$ time units after $X=x$.

A simple integration shows that the expected value of $X$ at time $t$ given that $X(0)=y$ is $\int_0^{+\infty} \xi f(\xi, t; y)\, dy = y e^{\mu t}$, so $PV(y) = \int_0^{+\infty} y e^{-(\rho-\mu)t} dt = \dfrac{y}{\rho - \mu}$. Another integration then shows

that the integral in (12) is $\int_0^{+\infty} PV(y) f(y, T_B; x)\, dy = \dfrac{x e^{\mu T_B}}{\rho - \mu}$. Hence, for $x \geq \tilde{x}_B^*$,

$$\tilde{\Phi}_B(x) = \tilde{V}_B(x) = -C_B + \frac{x e^{-(\rho-\mu)T_B}}{\rho - \mu}. \tag{13}$$

From (13), the “smooth-pasting” condition now becomes

$$\frac{\partial \tilde{\Phi}_B(x)}{\partial x}\Big|_{x=\tilde{x}_B^*} = \frac{e^{-(\rho-\mu)T_B}}{\rho - \mu}. \tag{14}$$

When we insert (10) into (13) and (14) to solve for $\tilde{x}_B^*$ and for $B_0$, we find

$$\tilde{x}_B^* = \frac{\theta}{\theta-1}(\rho-\mu)e^{(\rho-\mu)T_B}C_B, \tag{15}$$

and

$$B_0 = \frac{C_B}{\theta-1}\tilde{x}_B^{*\,-\theta}. \tag{16}$$

A comparative statics analysis shows that, as expected, $\tilde{x}_B^*$ increases with $T_B$, $C_B$ and $\sigma$. Conversely, it decreases with $\mu$: a faster growth rate allows the firm to start the project earlier and thus to discount less heavily future benefits (recall that $\mu<\rho$). With these results, we can derive an equation that defines $\tilde{x}_R^*$ implicitly ((A.7) in Appendix A). We solve it numerically.

*Case 2: the regulatory green light is valid only for a limited duration $T_A>0$.*

In this case, for $t \in [T_R, T_R + T_A]$, the frontier that separates the values of $X$ for which building is optimal from those for which waiting is preferable depends explicitly on time; we denote it by $x_B^*(t)$. Again, it is optimum for the firm to start building $X$ is high enough, and to wait otherwise. Thus, for $x \geq x_B^*(t)$ at $t \in [T_R, T_R + T_A]$ , the value of the option to build equals the sum of the present value of costs plus the present value of the expected flow of project revenues. So, $\forall t \in [T_R, T_R + T_A],\ \forall x \geq x_B^*(t)$,

$$\Phi_B(x,t) = -C_B + \frac{xe^{-(\rho-\mu)T_B}}{\rho-\mu}, \tag{17}$$

and the "smooth-pasting" condition becomes,

$$\frac{\partial \Phi_B(x,t)}{\partial x}\Big|_{x=x_B^*(t)} = \frac{e^{-(\rho-\mu)T_B}}{\rho-\mu}. \tag{18}$$

As for $\Phi_R(x)$, the return per unit of time for $\Phi_B(x,t)$ should equal its expected capital gains. Time now intervenes explicitly in $\Phi_B(x,t)$ so, when we apply Ito's lemma, we obtain the second order, linear partial differential equation, valid for $t \in [T_R, T_R + T_A]$ and for $x \le x_B^*(t)$:

$$\rho\Phi_B(x,t) = \frac{\partial \Phi_B(x,t)}{\partial t} + \mu(x)\frac{\partial \Phi_B(x,t)}{\partial x} + \frac{\sigma^2(x)}{2}\frac{\partial^2 \Phi_B(X,t)}{\partial x^2}. \tag{19}$$

Moreover, we know from Saphores (2002) that a lower barrier, even when it is unattainable (i.e., it cannot be reached in finite time), can be important for the decision to invest. We thus assume that 0 is the limit of a reflecting boundary for *X*, so that

$$\forall t \in [T_R, T_R + T_A],\ \lim_{L \to 0} \frac{\partial \Phi_B(x,t)}{\partial x}\Big|_{x=L} = 0. \tag{20}$$

Finally, at time $T_R$+$T_A$, when the option to build expires, $\Phi_B$ is exercised on the basis of a simple cost-benefit analysis since the firm no longer has the flexibility to delay the project. Thus,

$$\Phi_B(X(T_R + T_A), T_R + T_A) = \max\left[0, -C_B + \frac{X(T_R + T_A)e^{-(\rho-\mu)T_B}}{\rho - \mu}\right]. \tag{21}$$

Equations (17) to (21) fully define $\Phi_B(X,t)$. Since there is no explicit solution for $\Phi_B(X,t)$, we solve numerically using finite difference methods (see Appendix B).

### 2.3 The deterministic case

To better assess the impact of uncertainty on the decision to invest, let us now examine the deterministic case. Since $\sigma$=0 in Equation (1), a simple integration shows that the flow of project benefits increases at a constant rate:

$$X(t) = X(0)e^{\mu t}. \tag{22}$$

In this situation, if the project is worthwhile, the firm starts building as soon as regulatory approval has been secured because waiting would just reduce discounted net revenues. A limit in the duration of the regulatory green light is thus irrelevant here. Let $T_1$ be the time at which the firm triggers the regulatory proceedings. Using (22), the firm's objective function is[5]

$$\max_{0\leq T_1} e^{-\rho T_1}\left\{-C_R + qe^{-\rho T_R}\left[-C_B + e^{-\rho T_B}\frac{X(0)e^{\mu(T_1+T_R+T_B)}}{\rho-\mu}\right]\right\}. \tag{23}$$

We suppose again that $\mu<\rho$ because otherwise waiting forever would be optimal. Solving the corresponding necessary first order condition for an interior solution ($T_1>0$), we find

$$T_1 = \frac{1}{\mu}\ln\left(\frac{\rho(C_R + qC_B e^{-\rho T_R})}{qX(0)e^{-(\rho-\mu)(T_R+T_B)}}\right). \tag{24}$$

$\rho(C_R + qC_B e^{-\rho T_R})$ represents annualized project costs and $qX(0)e^{-(\rho-\mu)(T_R+T_B)}$ is the value of the flow of project benefits at the end of the construction phase if $X$=$X$(0) at $T_1$. Both costs are expressed in \$ at the start of the regulatory process. When we combine (24) and (22) to get the value of $X$ at which the regulatory process should start, we see that $T_1$ is simply the time necessary for the flow of project benefits to equal annualized project costs:

$$x_{R0}^{*} = \frac{\rho(C_R + qC_B e^{-\rho T_R})}{q\,\mathrm{e}^{-(\rho-\mu)(T_R+T_B)}}. \tag{25}$$

Finally, inserting (24) into (23), we find that $Q^*$, the optimal net profit of the firm, is

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[5] For the stochastic problem, the decision to start the regulatory process does not depend explicitly on time but on the level of $X$; we don't know what value $X$ will have at a given time in the future, so we start the clock when the firm starts the regulatory process. By contrast, in the deterministic case, the firm knows perfectly how $X$ changes over time and it can immediately make all necessary decisions.

$$Q^* = \frac{\mu}{\rho-\mu}\frac{qX(0)e^{-(\rho-\mu)(T_R+T_B)}}{\rho}\left[\frac{X(0)}{x_{R0}^*}\right]^{\frac{\rho}{\mu}}. \qquad (26)$$

A comparative statics analysis shows that:

- $T_1$ is an increasing function of $C_R$, $C_B$, $T_B$, and $\rho$. Indeed, with higher regulatory or construction costs, the project is initially less attractive so the firm waits until project benefits increase over time. A longer time to build has the same impact because it reduces the present value of project benefits. Similarly, a higher discount rate decreases future benefits more than project costs since these are incurred upfront.
- $T_1$ is a decreasing function of $q$ and $\mu$. This also makes sense intuitively: as the probability of regulatory success increases, so does the value of the project and the firm acts earlier. Likewise, if $X$ increases faster, the firm acts earlier as the discount rate is larger than the growth rate ($\rho>\mu$).
- Interestingly, however, $T_1$ first decreases and then increases as $T_R$, the time to complete the regulatory review, increases. The reason is simple: for "small" values of $T_R$ (i.e., for $T_R < \frac{1}{\rho}\ln\left(\frac{\mu}{\rho-\mu}\frac{qC_B}{C_R}\right)$), a small increase in the length of the regulatory process leads to a slight discount of future project benefits but this effect is more than offset by the increase in project benefits ($\mu>0$); acting sooner is therefore optimal. The reverse is true for "large" values of $T_R$ (i.e., for $T_R > \frac{1}{\rho}\ln\left(\frac{\mu}{\rho-\mu}\frac{qC_B}{C_R}\right)$).
- Since $x_{R0}^* = X(0)e^{\mu T_1}$, $x_{R0}^*$ and $T_1$ vary similarly with $C_R$, $C_B$, $T_R$, $T_B$, $q$, and $\rho$. The same holds for $\mu$ since $\frac{\partial T_1}{\partial \mu} = -\frac{1}{\mu}\left[T_1 + T_R + T_B\right] < 0$ and $\frac{\partial x_{R0}^*}{\partial \mu} = -\left[T_R + T_B\right]X(0)e^{\mu T_1} < 0$.

- Finally, for obvious reasons, $Q^*$ is a decreasing function of $C_R$, $C_B$, $T_R$, $T_B$, and $\rho$, and an increasing function of $q$ and $\mu$.

## 3. Application to the decision to build a power line

### 3.1 Data

We apply our model to one of Hydro-Québec's (HQ) recent project proposals (1998) to build a 1250 MW interconnection in the Outaouais region. This interconnection would unable HQ, a Quebec-owned utility, to link Quebec's transmission line network to Ontario's and to the other networks connected to Ontario's power grid (they include western New York and Michigan).

Two reasons are given to justify the proposed investment. First, this interconnection would insure a more secure electricity supply for Quebec. HQ's domestic production and transmission capacity is concentrated, which makes it vulnerable to catastrophic events. Access to Ontario could provide additional supply to Quebec in case of a domestic breakdown.

Second, the interconnection would provide more exchange opportunities in the deregulated wholesale electricity market of Ontario. HQ's main activity would be to import electricity when outside power prices are low. Water supplies in hydroelectric complexes would be used to export electricity when outside prices are high.[6] For this illustration, we focus on the commercial aspects of the project because the assessment of the value of an additional supply source in the case of an emergency adds further complex issues that are not addressed here.

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[6] Most of HQ's production capacity is hydroelectric. Because this technology is flexible, HQ can easily adjust its production, which is not the case for producers in Ontario and in parts of United States; they rely on thermal (fossil and nuclear fuels) power plants.

According to Report 143 of the "Bureau d'Audiences Publiques sur l'Environnement" (BAPE), HQ estimates that it would take two years ($T_B$=2) and approximately 185 million dollars ($C_B$=185) to build the interconnection.[7] Before HQ can invest, however, it must go through an environmental regulatory process to obtain approval. According to the Quebec Ministry of Natural Resources (QMNR) which supervises HQ's operations, this process can take as long as two years ($T_R$=2) and cost approximately 2 million dollars ($C_R$=2). The QMNR does not want to speculate on the probability of success for this kind of proposal (note that this project is currently under review), so we assume here that HQ has a 50% chance of getting approval ($q$=0.5).

Four more parameters are needed: the discount rate $\rho$; the infinitesimal growth rate $\mu$ and the variance parameter $\sigma$ for $X$; and the expiry time of the regulatory approval $T_A$. For $\rho$, the QMNR estimates that for this type of investment, HQ requires a 10% annual rate of return. In BAPE's Report 143, HQ does not detail its revenues from arbitrage activities (it is private information), so it is not possible to construct a time series for $X$ in order to estimate $\mu$ and $\sigma$. We thus choose an arbitrary but plausible value for $\mu$ of 2% per year, and we vary $\sigma$ between 0.15 and 0.8 per $\sqrt{\text{year}}$.

Finally, the QMNR gives no official expiry time for the regulatory approval. However, if HQ waits too long before investing, the QMNR can force HQ to undergo another regulatory review. To round up the list of parameters for our base case, we suppose that $T_A$ equals 7 years.

Since there is some substantial uncertainty concerning the value of some of our model parameters, we conduct an extensive sensitivity analysis. We consider the following parameter values: $q \in \{0.25, 0.5, 0.75\}$; $T_A \in \{3, 5, 7, 10, +\infty\}$, in years; $C_R \in \{1, 2, 4\}$, in \$ million;

[7] All \$ amounts in this paper are in Canadian dollars.

$C_B \in \{150,\ 185,\ 220\}$, in $ million; and $\mu \in \{0.01,\ 0.02,\ 0.044\}$, per year, in addition to systematically varying $\sigma$ between 0.15 and 0.8 per $\sqrt{\text{year}}$.

### 3.2 Results

Our main results are summarized on Figures 1 to 3. We are particularly interested in how uncertainty combines with the regulatory parameters ($q$, $C_R$, and $T_A$) to influence the optimal regulatory threshold $x_R^*$.

Figure 1 illustrates how $x_R^*$ varies with the volatility in net arbitrage revenues, $\sigma$, for three different values of the probability of regulatory approval, $q$. As for the deterministic case, we observe that, as $q$ increases, $x_R^*$ decreases. It is interesting to note, however, that $x_R^*$ is not a monotonic function of $\sigma$: initially it increases and then it decreases as $\sigma$ increases; this is more apparent for higher values of $q$. To understand this result, take the logarithm of $X$; from Ito's lemma, the logarithm of $X$ follows a Brownian motion with trend $\mu - \frac{\sigma^2}{2}$ and variance $\sigma^2$. A large volatility is thus akin to considering a project whose value is trending down (its trend would be $\mu - \frac{\sigma^2}{2} < 0$), so it is optimal for the firm to act earlier.

The same phenomenon is apparent when we analyze $x_R^*$ for different values of the cost of the regulatory process, $C_R$, or of the duration of the regulatory process, $T_R$ (these results are not shown). As for the deterministic case, when $C_R$ increases, so does $x_R^*$: the firm needs to wait for a larger value of the benefits flow before the project is worthwhile. For the range of parameters explored here, $x_R^*$ decreases when $T_R$ increases: as for the deterministic case, for "small" values of $T_R$, a small increase in the length of the regulatory process discounts slightly future project benefits but this effect is more than offset by the increase in project benefits so acting sooner is optimal. In addition, the difference between the $x_R^*$ s for different values of $C_R$ or of $T_R$ increases with $\sigma$, just as it does for different values of $q$: uncertainty thus exacerbates errors if $q$, $C_R$ or $T_R$ are known imperfectly. Moreover, $x_R^*$ is consistently above the deterministic threshold $x_{R0}^*$, for smaller values of $\sigma$. If uncertainty is ignored in that case, the regulatory process is thus started prematurely.

Figure 2 shows how the duration of the validity of regulatory approval, $T_A$, affects $x_R^*$. We observe that for $\sigma$ given, as $T_A$ decreases $x_R^*$ increases: as the firm has a smaller window of opportunity to invest, it waits for a more attractive potential payoff. In addition, while the action threshold to start the regulatory process first increases with $\sigma$ (before decreasing when uncertainty is large enough), $x_R^*$ is monotonically decreasing when $T_A = +\infty$ for the range of variances considered: assuming that $T_A = +\infty$ when it is finite, even large (e.g., $T_A$ =10 years), may thus lead to sub-optimum decisions.

Figure 3 illustrates the variations of the optimal threshold to start investment $x_B^*$ with uncertainty for different values of $T_A$. Of course, $x_B^*$ depends neither on $T_R$, $C_R$, nor on $q$. Whereas $x_R^*$ is a concave function of $\sigma$ that first increases and then decreases when uncertainty is large enough, $x_B^*$ is convex increasing. Indeed, a higher volatility increases both the incentive to wait and the opportunity cost of not investing because the risk of small values for $X$ is bounded from below: our assumption that $X$ admits a lower reflecting barrier guarantees that a higher level of uncertainty will lead to potentially higher payoffs.[8] This explains that our results differ from the findings of Bar-Ilan and Strange (1996) because they consider a lower absorbing barrier where the project is abandoned. Our results also show that their findings depend more on the nature of the lower absorbing barrier than on the existence of investment lags (on the importance of barriers when investing, see Saphores 2002). In addition, we note that $x_B^*$ increases with $T_A$: when the regulatory green light lasts longer, the firm can invest at higher levels of the flow of benefits.

Finally, our sensitivity analysis (results not included here) shows that both $x_R^*$ and $x_B^*$ decrease when $\mu$ increases, although they both vary relatively little when $\mu$ changes from 0.01 to 0.03 per year. In addition, both $x_R^*$ and $x_B^*$ decrease with $C_B$ and increase with $T_B$. The intuition for these results is the same as for the deterministic case.

## 4. Conclusions

In this paper, we extend a simplified version of Bar-Ilan and Strange's model (1996) to analyze the optimal decisions of a firm that needs to undergo a costly and time-consuming regulatory review in order to make a lagged, irreversible investment whose value varies randomly. Our numerical application to Hydro-Québec's proposal to build a 1250 megawatts interconnection to the Ontario power grid shows that the decision to start the regulatory review and the decision to start the project after regulatory approval has been secured vary quite differently with uncertainty. Indeed, the first one is a non-monotonic, concave function of uncertainty, while the

[8] McDonald and Siegel (1986), for example, also obtain this result but they do not make clear that it depends on a hidden assumption regarding the nature of the zero boundary.

second is an increasing convex function of uncertainty; both are sensitive to the duration of the regulatory green light. These results underscore the need for transparent regulatory proceedings. They are a first step towards understanding the impact of regulatory requirements such as environmental impact assessments on the decision to invest.

Future research could consider a number of extensions, including: stochasticity in the duration and/or the cost of the regulatory process (i.e., $T_R$ and/or $C_R$ are random); alternative stochastic processes for $X$; uncertainty in building costs (i.e., $C_B$ stochastic); and a general equilibrium analysis of the impact of environmental impact assessments. In addition, it would be instructive to model construction in details along the lines suggested in Majd and Pindyck (1987) or Milne and Whalley (2000). Finally, as more information becomes available and as deregulated electricity markets stabilize, it would be very useful to estimate actual arbitrage revenues from interconnecting power grids.

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**Appendix A: Implicit Equation for $\tilde{x}_B^*$**

$\tilde{x}_B^*$ is the value of *X* above which construction should start when regulatory approval is valid forever ($T_A = +\infty$). We first derive an expression for $V_R(x)$ that incorporates (15) and (16). As emphasized above, $\tilde{\Phi}_B(x) = B_0 x^\theta$ is valid only for $x \in (0, \tilde{x}_B^*)$. For $x \geq \tilde{x}_B^*$, construction should start immediately and the option to build is $\tilde{\Phi}_B(x) = -C_B + \frac{xe^{-(\rho-\mu)T_B}}{\rho - \mu}$ (see (13)). Thus, if the firm were to start the regulatory review when *X*=*x*, the net present value of the whole project would be

$$V_R(x) = -C_R + qe^{-\rho T_R}\left[\int_0^{\tilde{x}_B^*} B_0 y^\theta f(y, T_R; x)\,dy + \int_{\tilde{x}_B^*}^{\infty}\left(-C_B + \frac{ye^{-(\rho-\mu)T_B}}{\rho-\mu}\right) f(y, T_R; x)\,dy\right]. \tag{A.1}$$

For $\gamma \geq 0$, let us first calculate

$$I_\gamma(x) \equiv \int_0^{\tilde{x}_B^*} y^\gamma f(y, T_R; x)\,dy, \tag{A.2}$$

where $f(y, T_R; x)$ is the lognormal distribution given by (4). After the change of variables $z = \frac{\ln(y) - \ln(x) - (m + \gamma\sigma^2)T_R}{\sigma\sqrt{T_R}}$ with $m \equiv \left(\mu - \frac{\sigma^2}{2}\right)$ in (A.2), we obtain

$$I_\gamma(x) \equiv x^\gamma e^{\left(\gamma m + \frac{\gamma^2\sigma^2}{2}\right)T_R}\,\Phi\left(\frac{\ln(\tilde{x}_B^*) - \ln(x) - (m + \gamma\sigma^2)T_R}{\sigma\sqrt{T_R}}\right). \tag{A.3}$$

$\Phi(.)$ is the standard normal cumulative distribution function. Thus,

$$\int_{\tilde{x}_B^*}^{+\infty} f\left(y, T_R; x\right) dy = 1 - \int_0^{\tilde{x}_B^*} f\left(y, T_R; x\right) dy = 1 - I_0(x), \tag{A.4}$$

and using the expression of the expected value of a lognormally distributed random variable,

$$\int_{\tilde{x}_B^*}^{+\infty} yf\left(y, T_R; x\right) dy = xe^{\left(m+0.5\sigma^2\right)T_R} - \int_0^{\tilde{x}_B^*} yf\left(y, T_R; x\right) dy = xe^{\mu T_R} - I_1(x). \tag{A.5}$$

Combining (A.1), (A.4), and (A.5) leads to:

$$V_R\left(x\right) = -C_R + qe^{-\rho T_R}\left\{B_0 I_\theta(x) - C_B\left[1 - I_0(x)\right] + \frac{e^{-(\rho-\mu)T_B}}{\rho - \mu}\left[xe^{\mu T_R} - I_1(x)\right]\right\}. \tag{A.6}$$

If we now combine (A.6) with (2), (9), and (7), we get the equation verified by $\xi = \frac{x_R^*}{\tilde{x}_B^*}$:

$$\begin{aligned} &g_\theta(\xi)\xi^\theta + \theta\left[(\theta-1)[e^{\mu T_R} - G_1(\xi)] - g_1(\xi)\right]\xi \\ &+(\theta-1)\left\{\theta(G_0(y)-1) + g_0(y) - \theta\frac{C_R}{C_B}\frac{e^{\rho T_R}}{q}\right\} = 0, \end{aligned} \tag{A.7}$$

where for $\gamma \in \{0,1,\theta\}$,

$$\begin{aligned} G_\gamma(\xi) &\equiv e^{\left(\gamma m + \frac{\gamma^2\sigma^2}{2}\right)T_R}\Phi\left(\frac{-\ln(\xi) - (m+\gamma\sigma^2)T_R}{\sigma\sqrt{T_R}}\right), \\ g_\gamma(\xi) &\equiv \frac{e^{\left(\gamma m + \frac{\gamma^2\sigma^2}{2}\right)T_R}}{\sigma\sqrt{T_R}}\phi\left(\frac{-\ln(\xi) - (m+\gamma\sigma^2)T_R}{\sigma\sqrt{T_R}}\right). \end{aligned} \tag{A.8}$$

$\phi(.)$ is the standard normal density function.

**Appendix B: Numerical Solution of $\Phi_B(x,t)$**

To obtain $x_R^*$, we first need to approximate $\Phi_B(x,t)$. Following Wilmott, Howison and Dewynne (1995) and Saphores (2000), we transform Equations (17) to (21) by making the change of variables:

$$X = C_B e^z,\ t = T_A - \frac{2\tau}{\sigma^2},\ \Phi_B(X,t) = C_B e^{\alpha z + \beta\tau} u(z,\tau), \tag{B.1}$$

where $\alpha$ and $\beta$ solve the system:

$$\begin{cases} \beta = \alpha^2 + (k-1)\alpha - w, \\ 0 = 2\alpha + (k-1), \end{cases} \tag{B.2}$$

with $k = \dfrac{2\mu}{\sigma^2}$ and $w = \dfrac{2\rho}{\sigma^2}$.

We obtain the non-dimensional heat diffusion problem:

$$\begin{cases} \dfrac{\partial u}{\partial \tau} = \dfrac{\partial^2 u}{\partial z^2},\ \text{for } z < z_B^*(\tau), \\ u(z,\tau) = h(z,\tau),\ \text{for } z \geq z_B^*(\tau), \end{cases} \tag{B.3}$$

where

$$h(z,\tau) = e^{\frac{1}{4}\left[(k-1)^2 + 4w\right]\tau} \max\left[0,\ -e^{\frac{1}{2}(k-1)z} + e^{\frac{1}{2}(k+1)z - (\rho-\mu)T_B}\right]. \tag{B.4}$$

The initial condition is

$$u(z,0) = h(z,0) = \max\left[0, -e^{\frac{1}{2}(k-1)z} + e^{\frac{1}{2}(k+1)z - (\rho-\mu)T_B}\right]. \tag{B.5}$$

Because of the lower reflecting barrier at *X*=0, we also need:

$$\lim_{z \to -\infty} \left[ e^{(\alpha-1)z+\beta\tau} \left( \alpha u(z,\tau) + \frac{\partial u(z,\tau)}{\partial z} \right) \right] = 0. \tag{B.6}$$

In addition, to prevent arbitrage opportunities, the following constraint is required:

$$u(z,\tau) \geq e^{\frac{1}{4}\left[(k-1)^2+4w\right]\tau} \max\left[ 0, -e^{\frac{1}{2}(k-1)z} + e^{\frac{1}{2}(k+1)z-(\rho-\mu)T_B} \right]. \tag{B.7}$$

Finally, $u$ and $\partial u/\partial z$ must be continuous at $z = z_B^*(\tau)$.

To avoid tracking the free boundary, we use the linear complementary formulation

$$\begin{cases} \left( \frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial z^2} \right) \geq 0, \quad \left( u(z,\tau) - h(z,\tau) \right) \geq 0, \\ \left( \frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial z^2} \right) \cdot \left( u(z,\tau) - h(z,\tau) \right) = 0, \end{cases} \tag{B.8}$$

Both expressions in the first part of (B.8) are equalities at the free boundary.

We solve the above problem using the Crank-Nicholson finite difference scheme and the projected successive over-relaxation (PSOR) algorithm (see Wilmott, Howison and Dewynne 1995) with $\delta z = 0.01$; a grid for $z$ with $N^- = 1400$ and $N^+ = 800$; and time steps equivalent to 3 days. Details are available from the authors. To obtain the option value function, we reverse the change of variables.

Once we have an approximation of $\Phi_B(x,0)$, we combine the continuity and smooth-pasting conditions to approximate $x_R^*$. Taking their ratio, we find that $x_R^*$ satisfies:

$$\frac{dV_R\left(x_R^*\right)}{dx} \frac{x_R^*}{V_R\left(x_R^*\right)} = \theta. \tag{B.9}$$

We use the bisection method to find the zero of (B.9).

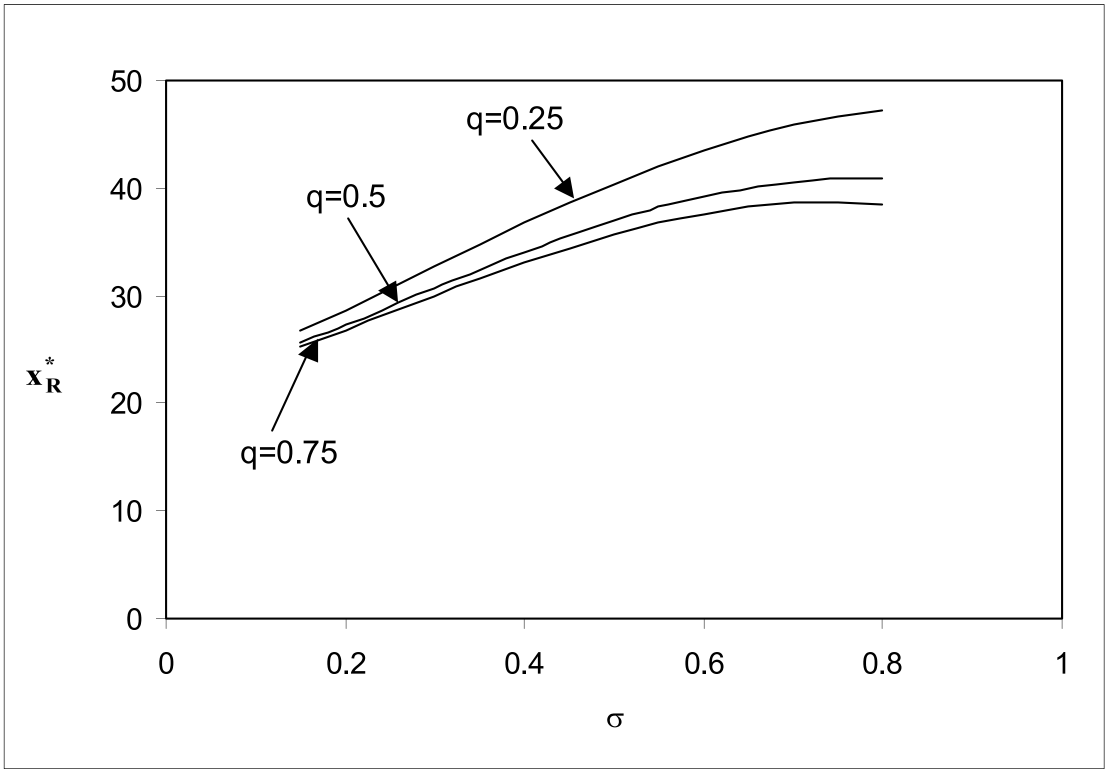


Figure 1: $x_R^*$ versus $\sigma$ for different probabilities of regulatory success.

Notes: Results above are generated with $T_B$=2 years, $C_B$=\$185 million, $T_R$=2 years, $C_R$=\$ 2 million, $\mu$=0.02 per year, $\rho$=0.1 per year, and $T_A$= 7 years. $q$ is the probability of regulatory approval.

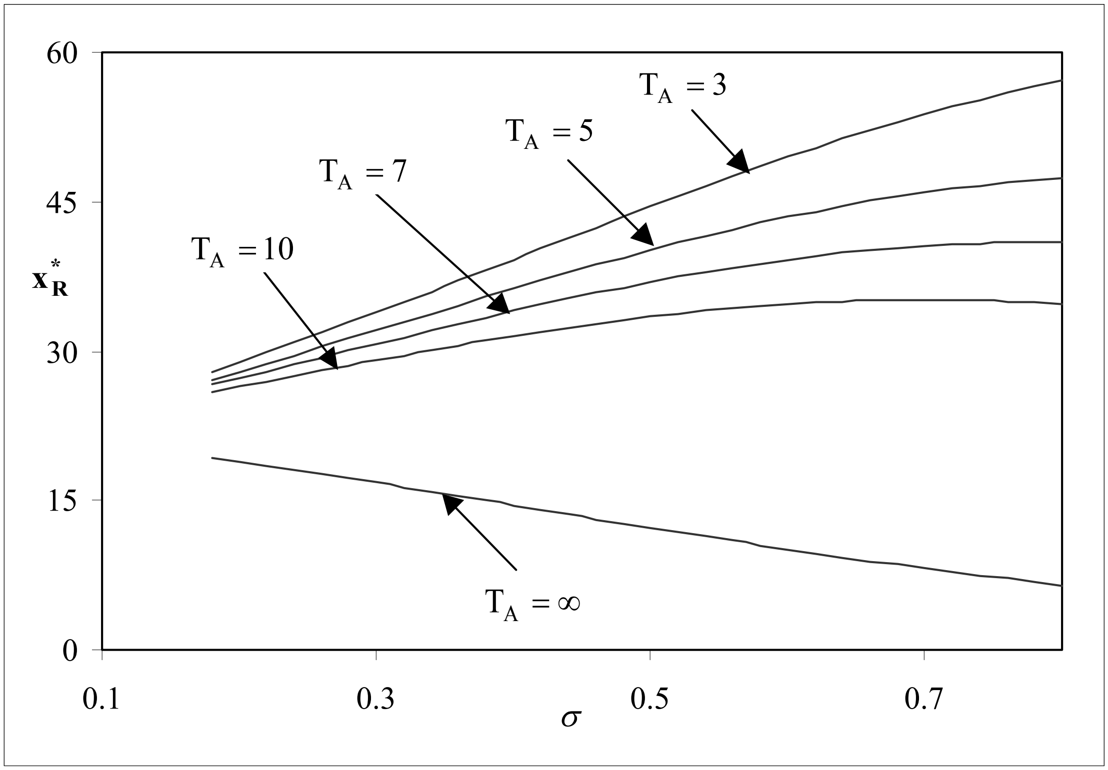


Figure 2: $x_R^*$ versus $\sigma$ for different durations of the regulatory authorization.

Notes: Results above are generated with $T_B$=2 years, $C_B$=\$185 million, $T_R$=2 years, $C_R$=\$2 million, $q$=0.5, $\mu$=0.02 per year, and $\rho$=0.1 per year. $T_A$ is the duration of the regulatory green light to proceed with the project.

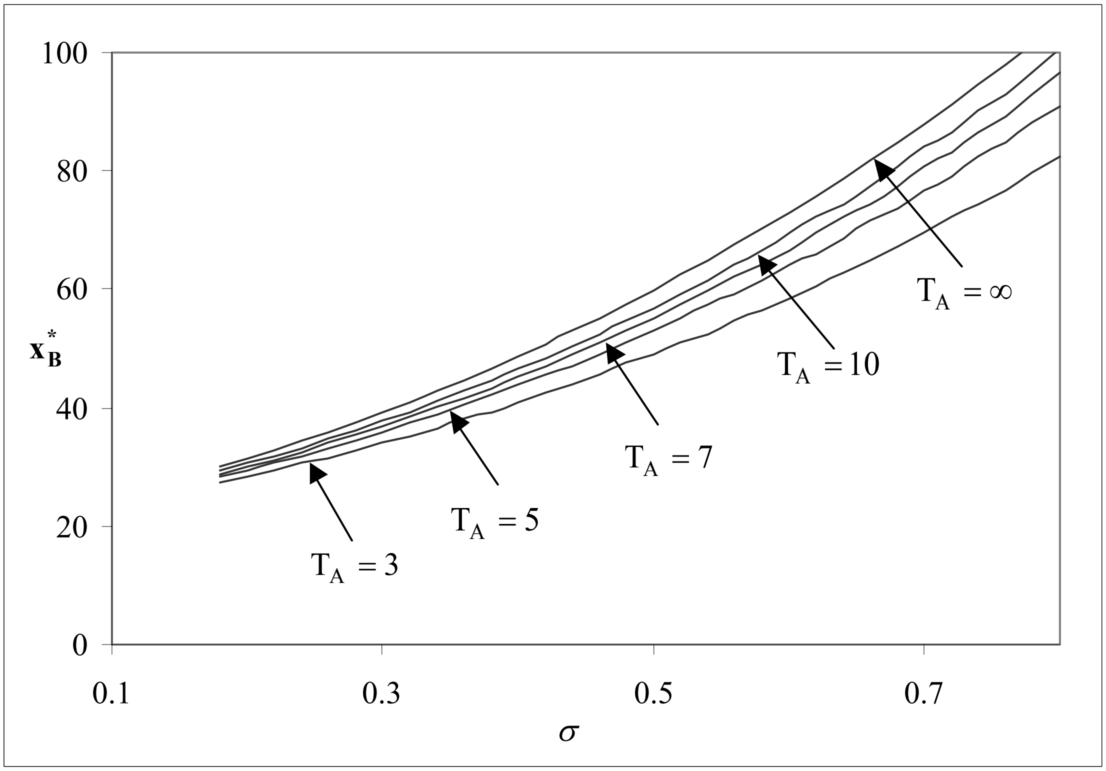


Figure 3: $x_B^*$ versus $\sigma$ for different durations of the regulatory authorization.

Notes: Results above are generated with $T_B$=2 years, $C_B$=\$185 million, $\mu$=0.02 per year, and $\rho$=0.1 per year. $T_A$ is the duration of the regulatory green light to proceed with the project.