Nash Implementable Liability Rules for Judgement-Proof Injurers

Patrick González*
Université Laval
CIRANO and GREEN

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Abstract

I provide a complete characterization of Nash implementable allocations of spending in prevention by judgement-proof injurers. This characterization is used to identify the optimal rule that allows for the maximum total spending in prevention. The optimal rule amounts to apply the negligence rule to the "deep-pocket" (or the "victim"), that is the injurer who responds the most to monetary incentives under the strict liability rule, and the strict liability rule to everybody else.

Keywords: negligence rule, limited liability, multiple injurers, Nash implementation.

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1 Introduction

In this paper, I provide a complete characterization of Nash implementable allocations of spending in prevention by judgement-proof injurers (hereafter *players*). This characterization is used to identify the rule that allows for the maximum total spending in prevention. Models of liability rules for two players go back to Brown (1973). Liability rules for more than two heterogeneous players can be found in Shavell (1987) and Emons and Sobel (1991). In a series of papers, Kornhauser and Revesz (1989, 1990, 1994) have pointed out that the characterization of liability rules for many judgement-proof injurers, that is injurers with a limited liability, is problematic. Limited liability is a real concern in liability cases involving life or environmental matters where the magnitude of damages can quickly skyrocket well beyond the actual capacity of paying of injurers. A liability rule designed to provide incentives to players to undertake due care must take into account these constraints.

Kornhauser and Revesz' analysis is restricted to two-player situations. They consider the equilibria induced by *ad hoc* liability rules that have been proposed in law and economics or that are actually used in real life legal disputes. By contrast, the number of players here is arbitrary and I cast the problem as one of mechanism design so that the whole set of feasible rules can be analyzed in a single step.

My analysis is related to that of Bergstrom, Blum and Varian (1986, 1992) who study the voluntary private provision of a public good by players with different wealth endowments. These authors show how the voluntary provision to a public good is affected by a redistribution of wealth when the redistribution modifies the subset of net contributors. In the present context, liability plays the role assigned to wealth in the public good problem.

My main result states that to achieved the maximum total spending in prevention, it is strictly efficient to make all players strictly liable except one that I call the "deep-pocket" or the "victim". This player is identified as the one who is the most responsive to monetary incentives under the strict liability rule. He is to be subjected to the negligence rule: he shall evade liability only if he has undertaken a due amount of spending in effort. Actually, the money gathered from the strictly liable players is used to provide additional monetary incentives to the deep-pocket. Under that regime, all spending in effort are undertaken by the deep-pocket only.

This is a striking result. The typical analysis of liability rules deals with the problem of disciplining a single player and leads to the conclusion that the negligence rule is strictly better than the strict liability rule when the player has a low solvency. The analysis here shows that when a *group* of potential players is involved, the negligence rule should be applied only on a single subsidized player; all the others should be subjected to the strict liability rule.

Who shall be that subsidized player is problematic: on one hand, since monetary incentives are scarce, we would like him to be highly responsive under the negligence rule. Players who are the more responsive under the strict liability rule are also those who are the most responsive under the negligence rule. Yet, these players are also the ones who have the highest solvency, hence the ones who would provide more monetary incentives under the strict liability regime. I show that the first effect will dominate favoring the deep-pocket.

The rest of this paper is structured as follows. In the next section, I present the formal model. In section 3, I show that implementation in dominant strategy is not possible with judgement-proof players. This justify the focus on Nash implementation. In section 4, I recast the classical result of the dominance of the negligence rule over the strict liability rule to discipline a judgement-proof player. In section 5, I generalize this result to the multiplayer setting by characterizing the set of allocations of spending that can be achieved under Nash implementation. With this characterization at hand, I identify the rule that provides the maximum incentives to spend in prevention. This is done in the last section which also provides two interpretations of this rule depending on whether the subsidized party is perceived as a "victim" or as a "deep-pocket".

2 The Model

There are N players indexed with i. These players spend in prevention to reduce the probability of an accident. If an accident occurs, the courts applies a liability rule that specifies the different compensating damages to be paid or received by the players.

All players are assumed to have quasi-linear preferences. Player i's utility in the no accident state is V_i. The indexes are chosen so that

$$V_1 \leq V_2 \leq \ldots \leq V_N.$$

I assume that the last inequality is strict so that $V_N > V_i$ for $i < N$. Player i's utility in the accident state is a measure of the value of his seizable assets. It is denoted U_i. I assume that $U_i \geq 0$ and strictly so for at least one player j. The cost of an accident is thus $C_i = V_i - U_i$. I assume that an accident is bad for all players ($C_i \geq 0$) and strictly so for at least one player. Because of player j above, it follows that $V_N \geq V_j \geq U_j > 0$; hence $V_N > 0$.[1]

After an accident, player i must pay compensatory damages L_i. This raises his cost of an accident to $C_i + L_i$. This payment could actually lead to a reduction of that cost if $L_i < 0$; that is, if i is a "victim" that gets compensated.

Player i's strategy amounts to choose a level of spending $X_i \geq 0$ in prevention to decrease the likelihood of the accident state. Let $X = [X_1, \ldots, X_N]$ denotes the strategy profile. I use the standard game-theoretic notation for alternate profiles

$$X_{-i} = [X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N],$$
$$[X_{-i}, \chi] = [X_1, \ldots, X_{i-1}, \chi, X_{i+1}, \ldots, X_N].$$

Beside, $x = \sum X_i$ is the total sum of spending in accident prevention and $x_{-i} = x - X_i$ is the share of this sum supported by the players other than i. A similar notation is used for the variables U_i, V_i, C_i and L_i. For instance, it was assumed above that $V \geq U \geq \mathbf{0}$ (the null vector) and that $u > 0$ and $c > 0$.

The higher x, the lower the probability $P(x)$ of an accident. Hence the spending of all players are perfect substitutes in the prevention technology. To account for decreasing marginal returns in prevention, P is assumed to be differentiable, strictly decreasing, strictly convex and to satisfy the standard Inada conditions: $\lim_{x \to 0} P'(x) = -\infty$, $\lim_{x \to \infty} P'(x) = 0$ and $\lim_{x \to \infty} P(x) = 0$.

Define player i's (expected) private cost as

$$\phi_i(X, L_i) = P(x)(C_i + L_i) + X_i.$$

Private cost strictly increases with L_i. Prevention is a public good whose provision may be problematic: Notwithstanding its effect on the allocation of liabilities, a raise by player i of his contribution X_i reduces not only his (expected) private cost of an accident – his private benefit – but it reduces that of the other players as well – an external effect.

[1] The same implication results from $V_N > V_i \geq U_i \geq 0$ for $i < N$.

I assume that the accident state entails an additional external cost a to society. Expected social cost is defined as the expected sum of private costs plus a and minus the sum of transfers

$$P(x)(a-l) + \sum \phi_i(X_i, L_i) = P(x)(a+c) + x.$$

Notice that, given x, social cost is independent of X.

Consider the function $P(\chi)K + \chi$ where K is a parameter. This function is strictly quasiconvex: If $K > 0$, it is strictly convex; if $K \leq 0$, it is strictly increasing. In both cases, strict quasiconvexity follows. Private and social costs are special cases of this function with respectively $K = C_i + L_i$ and $K = a + c$. We shall repeatedly encounter the case where $K = V_i$ so it is worthwhile to define

$$\psi_i(\chi) = P(\chi)V_i + \chi,$$

which is also strictly quasiconvex by the same argument. Strict quasiconvexity ensures a unique minimum; hence define

$$\psi_i^* = \min_{\chi \geq 0} \psi_i(\chi) = \psi_i(\xi_i).$$

The value ξ_i is player i's *maximum level of spending.* Indeed, should player i expect to be fully liable ($L_i = U_i$) and to spend alone in prevention ($x_{-i} = 0$), his cost of an accident would be $C_i = V_i$ and he we would choose to spend ξ_i to minimize his private cost. A straightforward application of the envelope theorem establishes that ξ_i increases with V_i, hence with i: player N has the highest maximum level of spending. Furthermore, since $V_N > 0$ and $P'(0) \to -\infty$, that level is strictly positive: $\xi_N > 0$. I call player N the *deep-pocket* or the *victim* (both interpretations are discussed in the conclusion).

Applying again the envelope theorem to the minimization of social cost above, we see that, as the external cost a changes, any level $x \geq 0$ may be rationalized as socially efficient.

A *liability rule* is a function R that maps the courts' available information into a vector of liabilities L to be imposed to the players in the accident state. In this paper, I assume that the courts have ex post perfect information but I shall only make explicit the liability rule dependence on X by writing $L = R(X)$. A liability rule is *separable* (with respect to X) if L_i depends on X through X_i alone. It is *admissible* if, for all $X \geq \mathbf{0}$, it satisfies the limited liability constraints

$$R(X) \leq U,$$

and budget balance

$$\sum R_i(X) \geq 0,$$

so that the courts are not a net contributor[2].

All the relevant information (a, P, U, etc) is common knowledge among the players when they choose their strategy profile X. In particular, they commonly know which rule R will be applied in the accident state. A liability rule is then a *mechanism* that structures the prevention game through the payoff functions $-\phi_i(\cdot, R_i(\cdot))$.

In this paper, I characterize the admissible rule that provides the best incentives to minimize expected social cost. The choice of an optimal mechanism depends on the solution concept assumed to give a good description of how the game will be played. Among the solution concepts encountered in the literature, those of *(weakly) dominant strategy equilibrium* and of *Nash equilibrium* are the most common place.

An allocation X may be implemented in dominant strategies (DS) if there exists a rule such that X_i is a (weakly) dominant strategy for each player i. An allocation X is Nash implementable (NI) if there exists a rule such that X_i is a best reply to X_{-i} for each player i. An admissible NI allocation is a NI allocation that can be implemented with an admissible rule.

3 Implementation in dominant strategy

If the limited liability constraints are discarded, any allocation X may be implemented in DS as follows. Let f_i be any function such that

$$f_i(\chi) \geq \phi_i([\mathbf{0}_{-i}, \chi], 0),$$

and that reaches its minimum at X_i. Define the rule

$$R_i(X) = \frac{f_i(X_i) - X_i}{P(x)} - C_i,$$

so that player i's private cost becomes

$$\phi_i(\chi, R_i([X_{-i}, \chi])) = f_i(\chi).$$

[2]When this inequality is strict, the money collected is used to restore the resource and/or is redistributed among the general public.

By construction, whatever the value x_{-i}, spending X_i in prevention minimizes his private cost. Notice that budget balance is ensured since

$$R_i([X_{-i}, \chi]) \geq \frac{P(\chi) - P(x_{-i} + \chi)}{P(x_{-i} + \chi)} C_i \geq 0.$$

However, this rule does not satisfy the liability constraints. If player i believes that the other players will invest a lot so that the probability of an accident becomes small, he knows that in any event his liability will be bounded at U_i. Since he does not expect the accident state to occur anyway, he will invest zero. But players are similar and investing a lot and not investing can't concurrently be dominant strategies. This result is generalized in the next proposition (all proofs are in the appendix).

Proposition 1. *The set of allocations that can be implemented in dominant strategies by an admissible rule is empty.*

The scope of proposition 1 goes beyond stating that it is difficult to handle the crowding out problem (giving player i incentives to invest reduces the same incentives for the other players). It emphasizes that there is also a coordination problem since no equilibrium in dominant strategies would exist even if no liability was imposed ($R \equiv \mathbf{0}$).

Proposition 1 relies a lot on the Inada conditions imposed on the prevention technology. Arguably, in less stringent environments, one could find allocations implementable in DS with an admissible rule. But since existence of such allocations is not guaranteed, DS is not an attractive concept to study liability rules. On the other hand, existence is not an issue with Nash implementation. Besides, the set of NI allocations obviously includes that of allocations implementable in DS. Identifying the set of NI allocations is thus an important step to devise a sensible liability rule (see footnote 3).

4 The strict liability and negligence rules

Suppose that R implements X as a Nash equilibrium:

$$X_i \in \operatorname*{argmin}_{\chi \geq 0} \phi_i([X_{-i}, \chi], R_i([X_{-i}, \chi])), \quad \forall i. \tag{1}$$

Then we can always define the separable rule

$$R'_i(\chi) = R_i([X_{-i}, \chi]),$$

that implements X as well. Furthermore, if R is admissible, so is R'. Hence, without loss of generality, we can focus on separable rules[3].

I begin the analysis of admissible separable rules by fixing X_{-i} and the liabilities $L_j = R_j(X_j)$ for $j \neq i$. This leaves X_i and the function R_i to be specified. To ensure that best replies are well defined, I shall restrict R_i to be lower semi-continuous. Besides, admissibility imposes that

$$-l_{-i} \leq R_i(X_i) \leq U_i. \tag{2}$$

A classical result in law and economics is the weak dominance of the negligence rule over the strict liability rule under limited liability (see Shavell, 1986). These two rules differ in the definition of the event in which player i is liable. Under the negligence rule, player i's liability is conditional on the event that he has spent less than some standard of care X_i (see below). Under the strict liability rule, player i is fully liable for the damage in any event. When the value of the damage is greater than the value of his assets, player i's liability binds at U_i. Because I am interested in cases where there is under-provision of spending in prevention, that is in cases where player i's liability is likely to bind, I associate the strict liability rule with the constant rule $R_i(X_i) \equiv U_i$ that specifies the same maximum payment regardless of X_i. Player i's lost in the accident state is then raised to $C_i + L_i = V_i$ and his expected cost becomes

$$\phi_i([X_{-i}, X_i], U_i) = \psi_i(x_{-i} + X_i) - x_{-i}. \tag{3}$$

That cost is minimized in

$$X_i^* = \max\{0, \xi_i - x_{-i}\},$$

to

$$\begin{aligned} \phi_i^* &= \phi_i([X_{-i}, X_i^*], U_i), \\ &= \begin{cases} \psi_i(x_{-i}) - x_{-i} & \text{if } x_{-i} > \xi_i, \\ \psi_i^* - x_{-i} & \text{if } x_{-i} \leq \xi_i. \end{cases} \end{aligned}$$

[3] To go back to the issue of robustness provided by DS, if a separable rule R' induces X as a Nash equilibrium, then investing X_i is optimal for i given x_{-i} but not if i expects $x'_{-i} \neq x_{-i}$. Proposition 3 states that, generally, there does not exist an admissible rule that works for *all* possible deviation x'_{-i} but the rule could be made robust against many. That is, we can enrich R' by defining a non-separable rule R'' for which investing X_i is optimal against many possible deviations x'_{-i}. Nevertheless, $R''([X_{-i}, X_i]) = R'(X_i)$ is still a necessary condition for R'' to be implementable.

That is, either the other players spend more than player i's maximum level of spending ($x_{-i} > \xi_i$) so that i spends zero and bears the expected cost $P(x_{-i})V_i = \psi(x_{-i}) - x_{-i}$, or player i is willing to contribute $X_i^* = \xi_i - x_{-i} \geq 0$ to raise x to ξ_i and bears the expected cost $P(\xi_i)V_i + X_i^* = \psi_i^* - x_{-i}$. If we define

$$\xi_i^* = \max\{x_{-i}, \xi_i\},$$

then we may write

$$X_i^* = \xi_i^* - x_{-i} \quad \text{and} \quad \phi_i^* = \psi_i(\xi_i^*) - x_{-i}.$$

Consider now the constant rule where player i always receives $-l_{-i}$ in the accident state so that his expected cost is $\phi_i(X, -l_{-i})$. Let χ_i^* minimize this cost and define the lower contour set

$$\mathbf{X}_i = \{\chi \geq 0 : \phi_i([X_{-i}, \chi], -l_i) \leq \phi_i^*\}.$$

By (2), player i's cost is reduced under this ruled so that $\mathbf{X}_i$ is not empty. Since ϕ_i is quasiconvex and continuous in X_i, the set $\mathbf{X}_i$ is a closed interval

$$\mathbf{X}_i = [X_i^{\min}, X_i^{\max}]$$

where both ends are implicitly defined as the solutions of

$$\phi_i([X_{-i}, \chi], -l_i) = \phi_i^*. \tag{4}$$

I define the negligence rule as

$$R_i(\chi) = \begin{cases} -l_{-i} & \text{if } (\chi - X_i)(X_i - \chi_i^*) \geq 0, \\ U_i & \text{else.} \end{cases} \tag{5}$$

Hence, when a high level of care is expected ($X_i > \chi_i^*$), player i receives l_{-i} if he has spent at least X_i and pays U_i otherwise[4].

Proposition 2 establishes the weak optimality of the negligence rule since it can implement any level o spending that could be implemented with any other rule.

[4]Again, this rule is somewhat different than the classical negligence rule because it always specifies the maximum amount the player can pay in the event of default, regardless of the actual damage, and the maximum reward otherwise. Again, this simplification is made because I am interested in cases where damages are large so that the liability constraints bind.

Proposition 2.

1. *X_i can be implemented with a rule bounded by* (2) *if and only if* $X_i \in \mathbf{X}_i$.

2. *Any $X_i \in \mathbf{X}_i$ may be implemented with the negligence rule.*

By construction, player i's expected cost is minimized in X_i with the negligence rule. Hence, his liability is given by $L_i = R_i(X_i) = -l_{-i}$. (Notice though that when $-l_{-i} = U_i$, the negligence rule and the strict liability rule are confounded.) Proposition 2 is illustrated in Figure 1. There, the two constant rules where player i always pays U_i or $-l_i$ induce two U-shaped expected cost functions $\phi_i([X_{-i}, \chi], U_i)$ and $\phi_i([X_{-i}, \chi], -l_i)$. Any other admissible rule $\tilde{R}$ induces an expected cost function between those two. Hence, the minimum cost with such rule is necessarily reached within a set $\mathbf{X}_i$ delimited by the lower contour set of $\phi_i([X_{-i}, \chi], -l_i)$ at ϕ_i^*. By construction, the negligence rule induces an expected cost function that is discontinuous at X_i (the thick line). When $X_i \geq X_i^*$, it follows $\phi_i([X_{-i}, \chi], U_i)$ for $\chi < X_i$ and $\phi_i([X_{-i}, \chi], -l_i)$ thereafter. It is minimized in X_i, a value that may be set anywhere in $\mathbf{X}_i$.

5 Nash Implementation

Up to now, X_{-i} and L_{-i} were assumed fixed. I now consider the case where all players choose their level of spending simultaneously given a liability rule R. Proposition 2 states that for any player i, and given X_{-i} and L_{-i}, there is no loss in generality in imposing the negligence rule to that player. Hence, the only unknowns to be specified are the allocation X of standards and the liabilities L. Hence, in what follows, I resume the description of a liability rule by the vector L where it is understood that, given X and L, R is given by (5).

With many players, there is a dilution of incentives. Suppose that all players are liable under the strict (constant) liability rule; $L = U$. In that case, each player minimizes (3)

$$\min_{\chi \geq 0} \psi_i(x_{-i} + \chi) - x_{-i}$$

Let M be the subset of m players who spend in prevention in a Nash equilibrium. Since player N's maximum level of spending ξ_N is strictly positive, we know that $m \geq 1$ because if nobody else would spend, player N would. Then

$$\begin{aligned} x = \sum_{i \in M} X_i &= \sum_{i \in M} \left(\xi_i - x_{-i}\right), \\ &= \sum_{i \in M} \left(\xi_i - x + X_i\right), \\ &= \sum_{i \in M} \xi_i - (m-1)x, \\ &= \frac{1}{m} \sum_{i \in M} \xi_i. \end{aligned} \tag{6}$$

The only way (6) may hold is if $M \equiv \{N\}$, $m = 1$ and $x = \xi_N$; that is, if player N is the only one who spends in prevention. In what follows, I note $X^i(x)$ such allocation where player i alone spends the total amount x in prevention:

$$X^i(x) = [0, \ldots, 0, x, 0, \ldots, 0],$$

with x in the i^{th} position. Hence, under the strict liability rule, the spending in prevention by player N crowds out the incentives for the other players to spend as well. This rule generates excess liability since $l = u > 0$. This suggests a better rule where that money could be used to provide additional incentives to player N. I will show that such a rule is indeed optimal.

To get this result, we need to define the function $F : \mathbb{R}_+^N \to \mathbb{R}_+$,

$$F(X) = \sum \psi_i(\xi_i^*).$$

Notice that F is a continuous function. Because all players have quasi-linear preferences, we get the following characterization of admissible NI allocations.

Proposition 3. *An allocation X is NI with an admissible rule if and only if*

$$P(x)c + Nx \leq F(X). \tag{7}$$

Besides, if (7) *holds, then*

$$L_i^+ = U_i - \frac{\psi_i(x) - \psi_i(\xi_i^*)}{P(x)}$$

is admissible and implements X as a Nash equilibrium.

Proposition 3 gives a characterization of admissible NI allocations that does not depend on the rule actually used to implement them. As we shall see, not all levels of spending x can be achieved with an admissible NI allocation and two allocations that provide the same level of total spending x, hence the same level of expected social cost, may not be both admissible NI allocations because the distribution of spending among players matters for implementation. Inequality (7) makes a clear distinction between the total level of spending on the l.h.s. and the distribution of spending on the r.h.s. In particular, it is clear that given x, (7) is easier to satisfy when F is maximized.

The liabilities L^+ are set so that (1) holds (X_i is a best reply for each player). The rule induced by L^+ is admissible by construction:

- If $\xi_i \geq x_{-i}$, then $\psi_i(\xi_i^*) = \psi_i^*$ and $L_i^+ \leq U_i$ since ψ_i has ψ_i^* for minimum.
- If $\xi_i < x_{-i}$, then $\psi_i(\xi_i^*) = \psi_i(x_{-i})$. Both x and x_{-i} are two values to the right of the minimum ξ_i of ψ_i. Since that function is quasiconvex, the difference $\psi_i(x) - \psi_i(x_{-i})$ is positive. It follows that $L_i^+ \leq U_i$.

Multiplying L_i^+ by $P(x)$ and summing over i then yields

$$P(x)l^+ = F(X) - (P(x)c + Nx),$$

so that budget balance holds if (7) holds.

6 Providing the maximum incentives

Recall that, depending on the external cost a, any x may be rationalized as a socially efficient amount of spending in prevention when designing a liability scheme. In this section, I use Proposition 3 to determine the maximum level of spending x^* that can be achieved with a NI admissible allocation and the optimal liability rule that implements this allocation.

As a corollary of Proposition 3, notice that for $X^N(\xi_N)$, inequality (7) becomes

$$\begin{aligned} P(\xi_N)c + N\xi_N &\leq P(\xi_N)v + N\xi_N, \\ -P(\xi_N)u &\leq 0, \end{aligned} \tag{8}$$

which is true so that $X^N(\xi_N)$ is NI with an admissible rule (the strict liability rule for every player) as it has already been suggested at the beginning of section 5. It follows that $x^* \geq \xi_N$. Since both sides of (7) are continuous functions of X, it is clear that x^* is reached when (7) holds with equality and F is maximized given x. As the next lemma shows, F is maximized in $X^N(x)$ when x is sufficiently large.

Lemma 1. *For $x > \xi_N$, $X^N(x)$ uniquely maximizes F subject to $\sum X_i = x$.*

If (8) did hold with equality, we would have found x^* but it does not since $u > 0$. Hence $x^* > \xi_N$ and we may use Lemma 1 in (7) to reach x^*:

$$\begin{aligned} P(x^*)c + Nx^* &= F(X^N(x^*)), \\ P(x^*)(C_N + c_{-N}) + Nx^* &= \psi_N(\xi_N) + \sum_{i<N} \psi_i(x^*), \\ P(x^*)(C_N + v_{-N} - u_{-N}) + Nx^* &= \psi_N^* + P(x^*)v_{-N} + (N-1)x^*, \\ P(x^*)(C_N - u_{-N}) + x^* &= \psi_N^*, \\ \phi_N(X^N(x^*), -u_{-N}) &= \phi_N^*. \end{aligned} \tag{9}$$

Comparing (9) with (4), we see that x^* is implemented with the liabilities

$$L^* = (U_1, U_2, \ldots, U_{N-1}, -u_N);$$

that is, by providing player N with the maximum level of (admissible) incentives $l_{-N} = u_{-N}$ under the negligence rule and by setting a maximal standard at the top of $\mathbf{X}_N$ with these incentives. This important result is formalized in the next proposition.

Proposition 4. *Let x^* solve* (9). *Then, within the class of liabilities rules defined by* (5), *$X^N(x^*)$ and L^* uniquely implement x^*.*

Since $-l_i = U_i$ for all $i < N$, the optimal multiplayer liability rule puts every player under a strict liability regime except player N who stays under the negligence rule and who actually receives the money collected from the other players when an accident happens.

It is easy to understand proposition (4) if we relate it to the classical problem of the private provision of a public good first analyzed by Warr (1983) and Bergstrom, Blum, and Varian (1986). These authors show that the amount of public good provided is independent of the distribution of income unless the set of contributors is affected by the distribution. The optimal multiplayer rule achieves this by concentrating all the ex post wealth (the compensatory damages from the liable agents) into the hands of a single player. That player is then disciplined through the negligence rule. Because there is a single player who spends in prevention, there is no dilution of incentives and a maximum of spending is undertaken.

Under this rule, the deep pocket is expected to undertake all spending. Again, the comparison with the public good problem helps to understand the result: if all wealth is to be given to a single player to spend on a public good and if any player would spend less than the socially optimal amount, then it makes sense to give the wealth to the player who values the most the public good. If we concentrate all incentives upon a single player i which we submit to the negligence rule, then the cost of an accident for this player becomes

$$C_i + L_i = V_i - U_i + (-u_{-i}) = V_i - u.$$

Hence, the most responsive player under that rule is the "deep pocket" for whom the cost of an accident under strict liability (V_i) is the greatest.

I have suggested that there are two interpretations of player N as the "deep pocket" or as the "victim". By definition,

$$V_i \equiv C_i + U_i.$$

Hence, the cost of an accident and the ex post liability of a player are jointly identified in this model. The "deep-pocket" interpretation is natural when there is little variation in the C_is relatively to the U_is. Then, all players would be similarly careless in absence of a liability regime but player N is highly motivated to produce the required amount of care once his assets U_i are at stake. He is then chosen because he is the most responsive to monetary incentives under the negligence rule.

When there is a lot of variation in the C_is relatively to the U_is, interpreting player N as a "victim" is more natural. Then, all players have roughly the same ability to pay ex post but player N has an higher ex ante incentive to spend in prevention because of his higher cost of an accident.

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A Appendix

The proofs of the propositions and Lemma 1 follow.

Proof of proposition 1. First, define

$$z(K) = \min_{\chi \geq 0} P(\chi)K + \chi.$$

Because $P'(0) \to -\infty$, $z(K) = 0$ implies that $K \leq 0$.

Suppose that X can be implemented in DS with R. Then either $x = 0$ (so that $X = \mathbf{0}$) or $x > 0$.

Suppose that $x = 0$ and consider a rule R that implements $X = \mathbf{0}$. For $X_i = 0$ to be dominant when $x_{-i} = 0$, it must be that

$$z(C_i + R_i(\mathbf{0})) = 0,$$

which implies

$$C_i + R_i(\mathbf{0}) \leq 0, \tag{10}$$

as above. Summing (10) over i yields

$$l = \sum R_i(\mathbf{0}) \leq -c < 0;$$

a violation of budget balance.

Suppose that $x > 0$. Then there exists a player i for which setting $X_i > 0$ is a weakly dominant strategy at least as good as investing nothing:

$$P(x)(C_i + R_i([X_{-i}, X_i])) + X_i \leq P(x_{-i})(C_i + R_i([X_{-i}, 0])), \quad \forall X_{-i}.$$

Since R_i is bounded above by U_i, as $x_{-i} \to \infty$, the probability of an accident vanishes on both sides and this inequality yields $X_i \leq 0$; a contradiction. □

Proof of proposition 2.

1. To prove sufficiency, consider any lower semi-continuous rule R_i bounded by (2). Notice that ϕ_i is increasing in its second argument. Hence, for any best reply X_i to any lower semi-continuous rule R_i, one has

   $$\phi_i(X, -l_i) \leq \phi_i(X, R_i(X_i)) \leq \phi_i([X_{-i}, \chi], R_i(\chi)) \leq \phi_i([X_{-i}, \chi], U_i),$$

   for any $\chi \geq 0$. In particular, for $\chi = X_i^*$,

   $$\phi_i(X, -l_i) \leq \phi_i([X_{-i}, X_i^*], U_i) = \phi_i^*,$$

   so that $X_i \in \mathbf{X}_i$. To prove necessity, consider point 2 below and the fact that the negligence rule is a lower semi-continuous rule.

2. Assume that we want to implement $X_i \in \mathbf{X}_i$. We verify that the negligence rule incites player i to invest X_i.

   If $X_i = \chi_i^*$, then $R_i(\chi) \equiv -l_i$ and cost are minimized by setting $\chi = \chi_i^* = X_i$. If $X_i > \chi_i^*$, then

$$R_i(\chi) = \begin{cases} -l_i & \text{if } \chi \geq X_i, \\ U_i & \text{else.} \end{cases}$$

   Playing $\chi < X_i$ yields at least ϕ_i^* while playing $\chi \geq X_i$ minimizes cost to

$$\min_{\chi \geq X_i} \phi_i([X_{-i}, \chi], -l_i) = \phi_i(X, -l_i) \leq \phi_i^*.$$

   The minimum is in X_i because the unconstrained solution is $\chi_i^* < X_i$ and $\phi_i([X_{-i}, \chi], -l_i)$ is quasiconvex. If $X_i < \chi_i^*$, a similar argument applies and cost are also minimized in X_i.

□

Proof of Proposition 3. Suppose that X is NI and that L implements X. Then, for all i, $X_i \in \mathbf{X}_i$ so that

$$\begin{aligned} \phi_i(X_i, L_i) &\leq \phi_i^*, \\ P(x)(C_i + L_i) + X_i &\leq \psi_i(\xi_i^*) - x_{-i}, \\ P(x)(C_i + L_i) + x &\leq \psi_i(\xi_i^*). \end{aligned} \tag{11}$$

Clearly, if (11) holds for all i then L implements X as a Nash equilibrium. Summing (11) over i yields

$$P(x)(c + l) + Nx \leq F(X).$$

From (11), it is clear that if $L \leq U$ implements X, so does any $L' \leq L$. Hence if $l > 0$, we can always find $L' \leq U$ that implements X as well and such that $l' = 0$. Hence (7) holds.

For the sufficiency part: given X such that (7) holds, L^+ solve (11) with equality so that X is NI. The discussion in the text establishes that $L^+ \leq U$. Besides, since (7) holds, $l^+ \geq 0$ and L^+ is admissible. □

Proof of Lemma 1. Since F is continuous, it reaches its minimum on the compact set defined by $X \geq \mathbf{0}$ and $\sum X_i = x$. Define

$$\theta_{ij}(\chi) = F([X_{-i-j}, \chi, X_i + X_j - \chi]).$$

Then, for any pair (i, j), the allocation χ of spending $X_i + X_j$ between i and j should be optimal. It follows that

$$X_i \in \underset{0 \leq \chi \leq X_i + X_j}{\operatorname{argmax}} \; \theta_{ij}(\chi) \tag{12}$$

is a necessary condition for X to maximize F. More in details:

$$\theta_{ij}(\chi) = \sum_{k \neq i, k \neq j} \psi_k(\xi_k^*) + \psi_i(\max\{x - \chi, \xi_i\}) + \psi_j(\max\{x - X_i - X_j + \chi, \xi_j\})$$

The function θ is convex over $\chi \geq 0$. It is the sum of a constant and two functions. Recall that ψ_i is convex. Then the first function is

$$\psi_i(\max\{x - \chi, \xi_i\}) = \begin{cases} \psi_i(x - \chi) & \text{if } 0 \leq \chi < x - \xi_i, \\ \psi_i^* & \text{if } x - \xi_i \leq \chi. \end{cases}$$

It is convex since $\psi_i(x - \chi)$ decreases toward the minimum ψ_i^* as χ is increased. The second function is

$$\begin{aligned} &\psi_j(\max\{x - X_i - X_j + \chi, \xi_j\}) \\ &\qquad = \begin{cases} \psi_j^* & \text{if } 0 \leq \chi < \xi_j - x + X_i + X_j. \\ \psi_j(x - X_i - X_j + \chi) & \text{if } X_i + X_j - (x - \xi_j) \leq \chi. \end{cases} \end{aligned}$$

It is convex since $\psi_j(x - X_i - X_j + \chi)$ increases from the minimum ψ_j^* as χ is increased. The sum of convex functions is convex so that θ_{ij} is convex.

It follows that, in our search for an X that maximizes F, we may assume that, for any pair (i, j), either X_i or X_j equals zero. For this to be true for every possible pair it must be that $X = X^i(x)$. Then, when $x > \xi_N > \xi_i$,

$$\begin{aligned} F(X^i(x)) &= \psi_i(\xi_i) + \sum_{j \neq i} \psi_j(x), \\ &= \sum_j \psi_j(x) - [\psi_i(x) - \psi_i(\xi_i)], \end{aligned}$$

That last expression is maximized when the bracketed term is minimized. Suppose that $i < N$; then

$$\psi_i(x) - \psi_i(\xi_i) = \int_{\xi_i}^{x} (P'(\chi) V_i + 1) d\chi.$$

To the right of ξ_i, $P'(\chi)V_i + 1$ is a positive function; hence,

$$\begin{aligned} &> \int_{\xi_N}^{x} (P'(\chi)V_i + 1) d\chi, \\ &> \int_{\xi_N}^{x} (P'(\chi)V_N + 1) d\chi, \\ &= \psi_N(x) - \psi_N(\xi_N). \end{aligned}$$

The inequalities are strict because $x > \xi_N > \xi_i$ and $V_N > V_i$. Hence, if we restrict X to $X^i(x)$, F is uniquely maximized in $X^N(x)$.

Finally, consider the possibility that X maximizes F and that $X_i > 0$ for a group M of more than one player. With $x > \xi_N$ and for (12) to hold, it must be that

$$M \equiv \{i : X_i > 0\} = \{i : x - X_i < \xi_i\}.$$

To understand this step, go back to the definition of θ_{ij} for i and j in M. If $x - X_i \geq \xi_i$, then $\psi_i(\max\{x - \chi, \xi_i\})$ has a strictly convex (decreasing) portion on the left when $0 \leq \chi < x - \xi_i$. If $x - X_j \geq \xi_j$, then $\psi_j(\max\{x - X_i - X_j + \chi, \xi_j\})$ has a strictly convex (increasing) portion on the right when $X_i + X_j - (x - \xi_j) \leq \chi \leq X_i + X_j$. If any of these two portions is present, then θ_{ij} is maximized either in $\chi = 0$ so that $X_i = 0$, or in $\chi = X_1 + X_2$ so that $X_2 = 0$. In both cases we get a contradiction.

Then

$$F(X) = \sum_{i \in M} \psi_i^* + \sum_{k \notin M} \psi_k(x). \tag{13}$$

Now, for i and j in M, consider lowering X_j to zero and raising X_i to $X_i + X_j$. We would still have $i \in M$ but $j \notin M$. Then F would be raised by $\psi_j(x) - \psi_j^* > 0$. Hence (13) is not a maximum. □

Proof of proposition 4. We have already shown that $X^N(x^*)$ and L^* implements x^*. To get to x^*, we had to maximize F. Since $x^* > \xi_N$, Lemma 1 implies that $X^N(x^*)$ uniquely implements x^*. Proposition 2 establishes that L^* uniquely implements $X^N(x^*)$ with a negligence rule. □

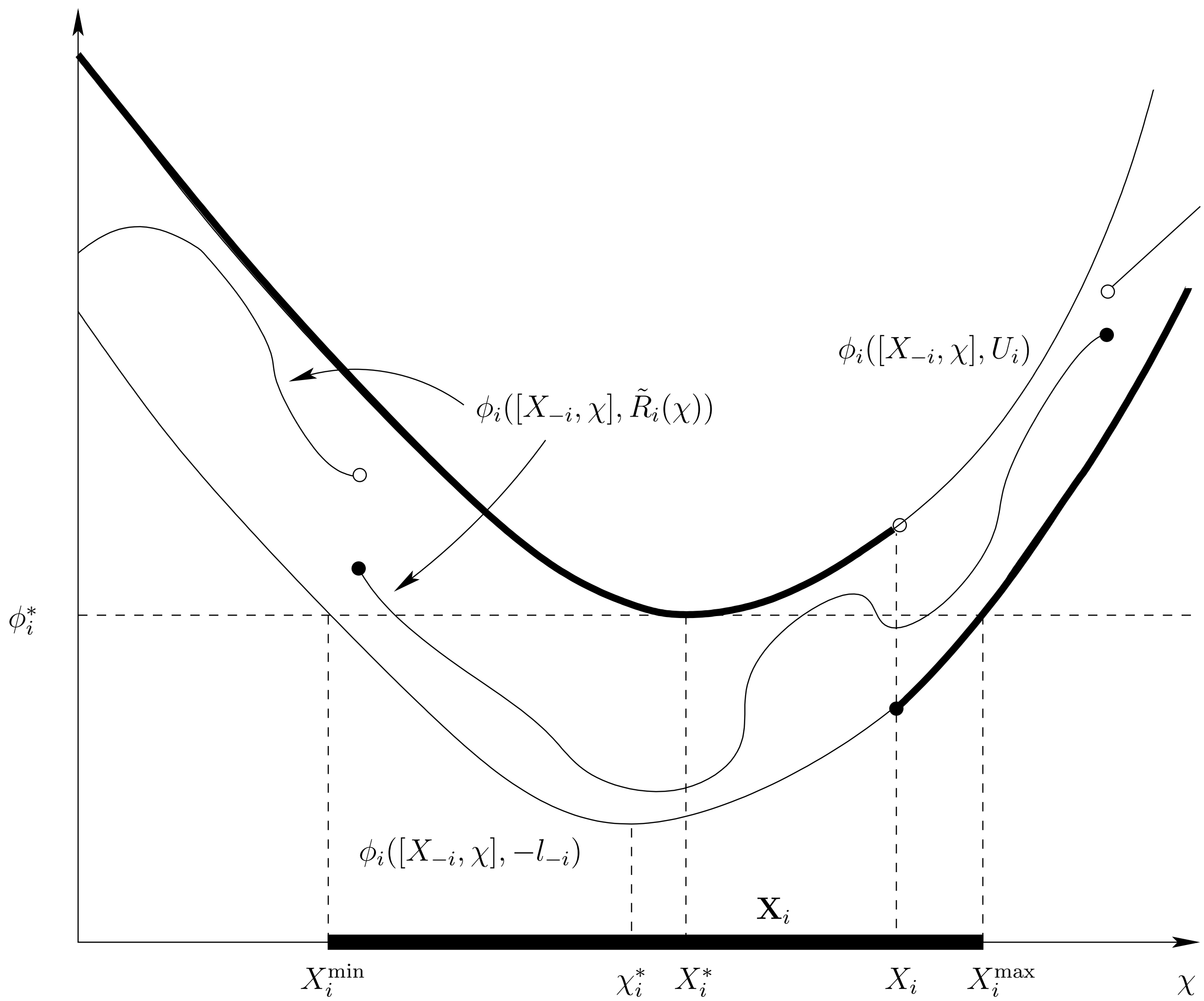

Figure 1: The optimality of the negligence rule.