Classical Horizontal Inequity and Reranking: an Integrated Approach

by

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Abstract

The last 20 years have seen a significant evolution in the literature on horizontal inequity (HI) and have led to two major and "rival" methodological strands, namely, classical HI and reranking. We propose in this paper a class of ethically flexible tools that integrate these two strands. This is achieved using a measure of inequality that merges the well-known Gini coefficient and Atkinson index, and that allows a decomposition of the total redistributive effect of taxes and transfers into a vertical equity effect, a loss of redistribution due to classical HI, and a loss of redistribution due to reranking. An inequality-change approach and a money-metric cost-of-inequality approach are developed. The latter approach makes aggregate classical HI decomposable across socio-economic groups and percentiles. As in recent work, equals are identified through a nonparametric estimation of the joint density of gross and net incomes. An illustration using Canadian data from 1981 to 1994 shows a substantial, and increasing, robust erosion of redistribution attributable both to classical HI and to reranking, but does not reveal whether reranking is more or less important than classical HI, since this requires a judgement that is fundamentally normative.

Keywords Horizontal inequity, reranking, tax equity, inequality, Canadian tax and transfer system.

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1 Introduction

The assessment of tax systems draws on two fundamental principles: efficiency and equity. The former relates to the presence of distortions in the economic behaviour of agents, while the latter focuses on distributive justice. Persistent conflicts in the application of these principles often mean that an optimal tax system must strive to balance them.

In this paper we examine in more detail a specific aspect of the notion of equity: horizontal equity (HE) in taxation. Two main approaches to the measurement of HE are found in the literature, which has evolved substantially in the last twenty years. The classical formulation of the HE principle prescribes the equal treatment of individuals who share the same level of welfare before government intervention. HE may also be viewed as implying the absence of reranking: for a tax to be horizontally equitable, the ranking of individuals on the basis of pre-tax welfare should not be altered by a fiscal system.

This paper draws on this conceptual duality by proposing a new set of tools that allow one to study simultaneously both classical horizontal inequity (HI) and reranking. For this, we use a class of social evaluation functions that display aversion to riskiness in net incomes[1] as well as aversion to rank inequality and relative deprivation. The associated class of inequality indices combines the popular Gini coefficient and Atkinson index of inequality, and is related to functions found in rank-dependent expected utility theory. This dual functional structure of our social evaluation functions allows us to measure jointly classical HI and reranking on fundamentally separate normative bases, a new feature in the literature. As in several recent papers[2], we also integrate the measurement of vertical equity (VE) and HE, and decompose the redistributive effect of taxes and transfers into a VE effect, a classical HI effect, and a reranking effect. Unlike previous work, however, we allow for different parameters of aversion to classical HI and to reranking[3]. Separate normative weights on reranking, classical HI and VE are justified by the different normative underpinnings of these concepts, as argued in the next section, and can also lead to optimal tax outcomes that are different from when these criteria are identically weighted. Ethical flexibility in the weighting of these criteria can also enable sensitivity checks on comparisons of classical HI and reranking across time and/or distributions.

A change-in-inequality approach and a cost-of-inequality approach to the decomposition of total redistribution are developed. The change-in-inequality approach has been hithertho the most popular in the literature, probably because of its close link with the now conventional use of inequality indices in distributive analysis. The cost-of-inequality approach has, however, the advantage of being money-metric: it gives the amount of money

[1] Gross incomes are market incomes (pre-tax and pre-transfer incomes), and net incomes are disposable incomes (total incomes after taxes and transfers).

[2] See for instance Aronson, Johnson and Lambert (1994), Aronson and Lambert (1994) and Aronson, Lambert and Trippeer (1997).

[3] This is in the spirit of King (1983) and Chakravarty (1985) for VE and reranking, and Auerbach and Hasset (1999) for VE and classical HI.

that society would be willing to give up to eliminate inequality, to preserve VE or to eliminate classical HI and reranking, and therefore provides indicators that are comparable to other money-metric indicators of government performance (such as equivalent or compensating variations for the assessment of efficiency criteria). As we will see, classical HI indicators based on the cost-of-inequality approach are also additively decomposable across percentiles and subgroups for wide ranges of ethical parameter values, and with local weights that are "uncontaminated" by vertical equity considerations[4].

The empirical estimation of these indices must also be tackled. For the change-in-inequality approach, natural estimators are easily derived and computed, except for the estimation of the VE and classical HI indices, where a regression of net income against gross income is needed to estimate the impact of removing HI in the fiscal system. For the cost-of-inequality indices, straightforward estimators are again available for the assessment of the total redistributive and reranking effects, but indicators of the variability of net incomes around predicted net incomes are needed to estimate the local and global welfare costs of the presence of classical HI. We suggest doing these estimations non-parametrically, by using kernel estimation of the conditional distribution of net incomes at various percentiles of gross incomes (in the fashion of Duclos and Lambert (2000)). This makes estimators of classical HI statistically consistent and it avoids the need for somewhat arbitrary normative assumptions on the treatment of "near-equals".

An illustration using Canadian data from 1981 to 1994 shows a substantial, and increasing, erosion of redistribution attributable to both HI and reranking. We also notice a greater problem with HI among low income households. It is not possible to say, however, whether reranking is a greater problem than classical HI, since this also involves a normative and not solely an empirical judgement. These results can be explained reasonably well by macroeconomic shocks (particularly the two recessions that struck Canada during that period), by socio-demographic changes, and by reforms in the tax and transfer system that increased its bite and its complexity.

The rest of the paper runs as follows. Section 2 reviews in some detail the evolution of the concept and measurement of horizontal inequity. This review is important since it justifies the methodological approach followed subsequently in the paper. Section 3 introduces the notation and develops the two sets of measures of total redistribution, VE, classical HI and reranking. It also discusses the paper's methodological links with previous work. The estimation procedure is presented in section 4. Our methodology is illustrated, using Canadian data, in section 5. The last section briefly concludes.

2 Classical horizontal equity and reranking

Why should concerns for horizontal equity influence the design of an optimal tax system? Several answers have been provided, using either of two approaches. The traditional or "classical" approach defines HE as the equal treatment of equals (see Musgrave (1959)).

[4]Lambert and Ramos (1997) implemented this idea first, though in an environment with ethical "rigidity".

While this principle is generally well accepted, different rationales are advanced to support it. First, a tax which discriminates between comparable individuals is liable to create resentment and a sense of insecurity, possibly also leading to social unrest. This is supported by the socio-psychological literature which shows that exclusion and discrimination have an impact both on individual well-being and on social cohesion and welfare. For instance, status/role structure theory indicates that one's relative socio-economic position (and its variability) "may give rise to definable and measurable social and psychological reactions, such as different types of alienation" (Durant and Christian (1990), p.210).

Second, the principles of progressivity and income redistribution, which are key elements of most tax and transfer systems, are generally undermined by HI (as we shall see in our own treatment below). This has indeed been one of the main themes in the development of the reranking approach in the last decades (see for instance Atkinson (1979) and Jenkins (1988)). Hence, a desire for HE may simply derive from a general aversion to inequality, without any further appeal to other normative criteria. Feldstein (1976) also notes that when utility functions are identical across individuals, a utilitarian social welfare function is maximized when equal incomes are taxed equally. This result then makes the principle of HE become a corollary of the principle of VE. A separate justification of HE would, however, generally be required when preferences are heterogeneous (they usually are), and because in some circumstances a random tax can otherwise be found to be optimal (Stiglitz (1982)). HI may moreover suggest the presence of imperfections in the operation of the tax and transfer system, such as an imperfect delivery of social welfare benefits, attributable to poor targeting or to incomplete take-up (see Duclos (1995b)). It can also signal tax evasion, which can *inter alia* cost the government significant losses of tax revenue (see Bishop *et al.* (1994)).

Third, HE can be argued to be an ethically more robust principle than VE (VE relates to the reduction of welfare gaps between unequal individuals). The HE principle is often seen as a consequence of the fundamental moral principle of the equal worth of human beings, and as a corollary of the equal sacrifice theories of taxation. Depending on the retained specification of distributive fairness, the requirements of vertical justice can vary considerably, while the principle of horizontal equity remains essentially invariant (Musgrave (1990)). Plotnick (1982) also supports this view by arguing that HI in the redistributive process would cause a loss of social welfare relative to an horizontally equitable tax, regardless of any VE value judgments on the final distribution. This has led several authors (including Stiglitz (1982), Balcer and Sadka (1986) and Hettich (1983)) to advocate that HE be treated as a separate principle from VE, and thus to form one of the objectives between which an optimal trade-off must be sought in the setting of tax policy[5].

[5] As in all trade-offs, it is clear that violations of HE are often inevitable (although still regrettable), such as when some forms of behaviour are encouraged for efficiency or VE reasons. This seems to be the case, for instance, for the HI created by the tax breaks granted to encourage charitable giving, for mortgage interest tax relief to encourage owner-occupied housing, for the tax deductibility of political contributions to encourage political participation, for indirect tax rate differentiation to promote VE, etc... Other forms of HI may, however,

The value of studying classical HI has nonetheless been questioned by a few authors, among whom figures Kaplow (1989, 1995), who rejects the premise that the initial distribution is necessarily just (see also Atkinson (1979) and Lerman and Yitzhaki (1995)) and adds that utilitarianism and the Pareto principle may justify the unequal treatment of equals (as seen above)[6]. A number of authors have also expressed dissatisfaction with the classical approach to HE because of the implementation difficulties it was seen to present. Indeed, since no two individuals are ever exactly alike in a finite sample, it was argued (see *inter alia* Feldstein (1976) and Plotnick (1982,1985)) that analysis of equals had to proceed on the basis of groupings of unequals which were ultimately arbitrary and which represented "an artificial way to salvage empirical applicability" (Plotnick (1985), p. 241). The proposed alternative was then to link HI and reranking and to note that the absence of reranking *implies* the classical requirement of HE: "the tax system should preserve the utility order, implying that if two individuals would have the same utility level in the absence of taxation, they should also have the same utility level if there is a tax" (Feldstein (1976), p.94)[7].

Various other ethical justifications have also been suggested for the requirement of no-reranking. For normative consistency, King (1983) argues for adding "and treating unequals accordingly" to the classical definition of HE, by which it then becomes clear that classical HE also *implies* the absence of reranking. Indeed, if two unequals are reranked by some redistribution, then it must be that at a particular point in that process of redistribution, these two unequals became equals and were then made unequal (and reranked), thus violating classical HE. Hence, from the above, "a necessary and sufficient condition for the existence of horizontal inequity is a change in ranking between the ex ante and the ex post distributions" (King (1983), p. 102). King (1983) then proposes an additively separable social welfare function that is characterized by a parameter of aversion to horizontal and vertical inequality. This function decreases with the distance of each net income from its order-preserving level of net income. Chakravarty (1985) also argues that reranking causes an individual's utility to differ from what it would have been otherwise, and moreover suggests that this difference reduces the final level of individual utility, creating a loss of social welfare as measured again by a utilitarian welfare function.

The theory of relative deprivation (which is well documented in the socio-psychological literature) also suggests that people specifically compare their relative individual fortune with that of others in similar or close circumstances[8]. The first to formalize the theory of

be less intentional and be a sign of truly sub-optimal tax policy.

[6]The same is true for the reranking of individuals, which is discussed below. See also King(1983), who sees this implication as a flaw of strict utilitarianism since it ignores the fairness of the redistributive process.

[7]The requirement of no reranking further implies that marginal tax rates should not exceed 100%, which can be taken as a basic economic requirement for incentive preservation and efficiency. On this, see *inter alia* Lambert and Yitzhaki (1995).

[8]Relative deprivation has also been linked to rank-dependent measures of inequality. See Sen (1973), Yitzhaki (1979) and Hey and Lambert (1980) for the link between relative deprivation and the popular Gini coefficient; see also Duclos (2000) for the links to other rank-dependent measures, and for the material on which this paragraph draws.

relative deprivation, Davis (1959), expressly allowed for this by suggesting how comparisons with similar *vs* dissimilar others lead to different kinds of emotional reactions; he used the expression “relative deprivation" for “in-group" comparisons (*i.e.*, for HI), and “relative subordination" for “out-group" comparisons (*i.e.*, for VE) (Davis (1959), p.283). Moreover, in the words of Runciman (1966), another important contributor to that theory, “people often choose reference groups closer to their actual circumstances than those which might be forced on them if their opportunities were better than they are" (p.29).

In a discussion of the post-war British welfare state, Runciman also notes that “the reference groups of the recipients of welfare were virtually bound to remain within the broadly delimited area of potential fellow-beneficiaries. It was anomalies within this area which were the focus of successive grievances, not the relative prosperity of people not obviously comparable" (p.71). Finally, in his theory of social comparison processes, Festinger (1954) also argues that “given a range of possible persons for comparison, someone close to one’s own ability or opinion will be chosen for comparison" (p.121). In an income redistribution context, it is thus plausible to assume that comparative reference groups are established on the basis of similar gross incomes and proximate pre-tax ranks, and that individuals subsequently make comparisons of post-tax outcomes across these groups. Individuals would then assess their relative redistributive ill-fortune in reference groups of comparables by monitoring *inter alia* whether they are overtaken by or overtake these comparables in income status, thus providing a plausible “micro-foundation” for the use of no-reranking as a normative criterion.

This suggests that comparisons with close individuals (but not necessarily exact equals) would be at least as important in terms of social and psychological reactions than comparisons with dissimilar individuals, and thus that analysis of HI and reranking in that context should be at least as important as considerations of VE. It also says that, although classical HI and reranking are both necessary and sufficient signs of HI, they are (and will be perceived as) different manifestations of violations of the HE principle. Hence, it seems reasonable that VE and the no-reranking requirement be assessed separately[9] from the classical requirement of equal treatment of equals when assessing the impact of taxes and transfers on social welfare[10]. We now turn formally to this task.

3 Income Inequality and Tax Equity

Let $F_{X,N}(\cdot,\cdot)$ be the joint cumulative distribution function (cdf) of gross (X) and net (N) income, with support contained in the positive real orthant. For simplicity, we assume that this joint cdf is continuous, although we will indicate in section 4 below how we can

[9]This is also pointed out by Kaplow (1989) who observes that the goal of limiting reranking may conflict with that of limiting the unequal treatment of equals.

[10]In the words of Feldstein (1976): “The problem for tax design is therefore to balance the desire for horizontal equity against the utilitarian principle of welfare maximisation. Balancing these two goals requires an explicit measure of the departure from horizontal equity.” (p.83).

estimate the various indices that follow using a finite number of sample observations. Let $p = F_X(\cdot)$ be the marginal cdf for gross incomes, and let $X(p)$ be the quantile function for gross incomes, formally defined as $X(p) = \inf\{s > 0 | F_X(s) \geq p\}$ for $p \in [0,1]$. Mean gross income is then $\mu_X = \int_0^1 X(p)dp$. The $N(q)$ q-quantile function for net incomes and average net income μ_N can be defined analogously. Denote by $F_{N|X=x}(\cdot)$ the cdf of N conditional on $X = x$. The q-quantile function for net incomes conditional on a p-quantile value for gross incomes is then defined as $N(q|p) = \inf\{s > 0 | F_{N|X=X(p)}(s) \geq q\}$ for $q \in [0,1]$. $N(q|p)$ thus gives the net income of the individual whose net income rank (or percentile) is q, among all those whose rank is p in the distribution of gross incomes.

Let the utility of income be given by the isoelastic function $U_\epsilon(y)$, with $\epsilon \geq 0$:

$$U_\epsilon(y) = \begin{cases} \frac{y^{1-\epsilon}}{1-\epsilon}, & \text{when} \quad \epsilon \neq 1, \\ \ln(y), & \text{when} \quad \epsilon = 1. \end{cases} \tag{1}$$

Since $U_\epsilon(y)$ is concave (strictly so for $\epsilon > 0$), individuals will be averse to uncertainty in their net income level, with ϵ being their parameter of relative risk aversion. This forms the basis of our measure of classical HI. Our chosen social welfare function then aggregates these utilities across the population by using rank-dependent ethical weights, $w(p,v)$, where v is a parameter of aversion to rank inequality (to be discussed further below). For the distribution of gross incomes, this social welfare function can be expressed as:

$$W_X(\epsilon, v) = \int_0^1 U_\epsilon(X(p)) w(p,v)\, dp. \tag{2}$$

$W_N(\epsilon, v)$ is similarly defined by replacing $X(p)$ by $N(p)$ in (2).

The weights in (2) are defined such that:

$$w(p_i, v) \leq w(p_j, v) \quad \text{if} \quad p_i \geq p_j, \tag{3}$$

(for W to obey the Pigou-Dalton principle of transfers and to be concave in incomes) and such that:

$$\int_0^1 w(p,v)\, dp = 1. \tag{4}$$

A continuous specification for $w(p,v)$ which obeys conditions (3) and (4) is (see Kakwani (1980), Donaldson and Weymark (1983) and Yitzhaki (1983)):

$$w(p,v) = (1-p)^{(v-1)}, \quad v \geq 1, \tag{5}$$

which we also adopt throughout the rest of this paper[11].

[11] The combined specification of (2) and (5) was originally proposed by Berrebi and Silber (1981) in a discrete setting.

As Araar and Duclos (1998) show, the social evaluation functions defined by (2) and (5) can be interpreted as average utility corrected for relative utility deprivation in the population. Individual relative deprivation is the expected shortfall from the well-being of others in society (see Runciman (1966) for an influential definition). Araar and Duclos (1998) also show that for an integer $v \geq 2$, relative deprivation for (2) in the population is the expected deprivation of those who would find themselves the most deprived in a group of $v - 1$ randomly selected individuals[12]. An increase in v gives greater weight to the relative deprivation of the poorer in the aggregation of individual relative deprivation.

As Sen (1973, p.39) argues, $U_\epsilon(y_i)$ can be an individual utility function, or it can be the "component of social welfare corresponding to person i, being itself a strictly concave function of individual utilities". Sen also adds that "it is fairly restrictive to think of social welfare as a sum of individual welfare components" (p.39), and that one might feel that "the social value of the welfare of individuals should depend crucially on the levels of welfare (or incomes) of others" (p.41). This is what is done in (2), which also avoids King's (1983) purely additive approach in handling reranking. Moreover, as Ben Porath and Gilboa (1994, p.445) note, "the most salient drawback of linear measures [e.g., the traditional S-Gini's] is that the effect on the social welfare of a transfer of income from one individual to another depends only on the ranking of the incomes but not on their absolute levels". Equation (2) escapes this drawback, since $U(y)$ does not have to be affine in incomes[13].

The expected net income of those at rank p in the distribution of gross income is given by:

$$\bar{N}(p) = \int_0^1 N(q|p)dq. \tag{6}$$

Hence, if the tax system were horizontally equitable and if individuals at rank p in the distribution of gross income were granted $\bar{N}(p)$ in net incomes, social welfare would equal:

$$W_N^E(\epsilon, v) = \int_0^1 U_\epsilon(\bar{N}(p))w\,(p, v)\,dp. \tag{7}$$

The expected net income utility of those at rank p in the distribution of gross income is, however, equal to:

$$\bar{U}_\epsilon(p) = \int_0^1 U_\epsilon\,(N(q|p))\,dq. \tag{8}$$

Hence, if horizontal inequities involved no change in ranking, social welfare would equal:

$$W_N^P(\epsilon, v) = \int_0^1 \bar{U}_\epsilon(p)w\,(p, v)\,dp. \tag{9}$$

[12] This interpretation of v was originally given in another context by Muliere and Scarsini (1989).

[13] Equation (2) is also linked to rank-dependent expected utility theory, as noted in Chew and Epstein (1989) and Ben Porath and Gilboa (1994).

Let $\xi_X(\epsilon, v)$ be the equally distributed equivalent income for a distribution of gross incomes X: if $\xi_X(\epsilon, v)$ were enjoyed by all, it would generate the same social welfare as that generated by the current distribution of gross incomes. By definition, $\xi_X(\epsilon, v)$ is thus given by:

$$W_X(\epsilon, v) = \int_0^1 U_\epsilon(\xi_X(\epsilon, v))\, w(p, v)\, dp = U_\epsilon(\xi_X(\epsilon, v)). \tag{10}$$

For expositional simplicity, we will often not mention explicitly in what follows the dependence of ξ_X and other functions on ϵ and v. Definitions similar to (10) apply for ξ_N, ξ_N^E and ξ_N^P, using W_N, W_N^E, and W_N^P. Denoting the inverse of the utility function $U_\epsilon(\cdot)$ by

$$U_\epsilon^{-1}(y) = \begin{cases} (1-\epsilon)y^{\frac{1}{1-\epsilon}}, & \text{when} \quad \epsilon \neq 1, \\ \exp(y), & \text{when} \quad \epsilon = 1, \end{cases} \tag{11}$$

it then follows that $\xi_X = U_\epsilon^{-1}(W_X(\epsilon, v))$. Finally, it is conventional since Atkinson(1970) to measure inequality as the difference between ξ_X and μ_X as a proportion of μ_X:

$$I_X = 1 - \frac{\xi_X}{\mu_X}. \tag{12}$$

I_X then measures the cost of inequality as a proportion of total income[14]: it represents the percentage of total income that could be spent in removing inequality with no resulting loss in social welfare.

3.1 Change in Inequality Approach

The redistributive change in inequality that results from the effect of taxes and transfers can be expressed as:

$$\Delta I = I_X - I_N. \tag{13}$$

In a progressive tax system, this expression may be expected to take a positive value. ΔI may, however, underestimate the underlying progressivity and vertical equity of the tax system. Indeed, if the tax system created neither horizontal inequity nor reranking, it would be more efficient in inequality reduction ΔI while generating the same tax revenues. To measure the extent of this loss of redistribution, recall that I_N^E measures the inequality of net incomes under a horizontally equitable tax, where each individual at rank p in the distribution of gross incomes receives $\bar{N}(p)$, and that I_N^P measures the inequality of net incomes when the initial ranking is maintained. Using (13), we then write the following decomposition of $I_X - I_N$:

$$I_X - I_N = \underbrace{I_X - I_N^E}_{V} - \underbrace{\left(I_N^P - I_N^E\right)}_{H \geq 0} - \underbrace{\left(I_N - I_N^P\right)}_{R \geq 0}. \tag{14}$$

[14]Again, similar definitions extend to I_N, I_N^E, and I_N^P.

The decomposition has three parts. V represents the decrease in inequality yielded by a tax which treats equals equally, and therefore causes no reranking. Thus, V measures the underlying vertical equity or progressivity of the tax. H measures the increase in overall inequality attributable to the unequal treatment of pre-tax equals—this is the classical HI effect. The excess of I_N^P over I_N^E is due to the appearance of gaps within groups of equals during the calculation of I_N^P, groups which remain equal in the calculation of I_N^E. Alternatively, it is due to the fact that $W_N^E(\epsilon, v) \geq W_N^P(\epsilon, v)$ since, through risk aversion and uncertainty of net income, $U_\epsilon\left(\bar{N}(p)\right) \geq \bar{U}_\epsilon(p)$.

Finally, R measures the extent of reranking. The difference between I_N and I_N^P is technically due to the incorrect weighting $w\left(p, v\right)$ of net income in the computation of I_N^P. Indeed, because of reranking, too much normative weight is assigned to high incomes and too little weight to low incomes when the equally distributed equivalent income $\left(\xi_N^P\right)$ is calculated. Interpreted in the light of Duclos (2000), moving from pre-tax-ordered to post-tax-ordered net incomes increases population relative deprivation, since in ξ_N some relatively poor individuals find out their true (lower) relative standing in the distribution of net living standards. Consequently, ξ_N^P is an overestimate of ξ_N, and I_N^P underestimates I_N.

Another interpretation of R is to think of individuals as assessing whether they are overtaken by or whether they overtake others in the redistributive process (see again Duclos (2000)). We may think of individuals resenting being outranked by others, but enjoying outranking others, and then assessing their net feeling of resentment by the amount by which the utility of the richer (than themselves) actually exceeds what the utility of the richer class would have been if no 'new rich' had displaced 'old rich' in the distribution of net incomes. We can then show that $W_N^P(\epsilon, v) - W_N(\epsilon, v)$ is the expected net utility resentment of the poorest person in samples of $v - 1$ randomly selected individuals, and thus that R is an indicator of such net resentment in the population.

3.2 Cost of Inequality Approach

We now perform a decomposition similar to that previously presented, though with a slight difference in the implied distribution of income[15]. In the above change-in-inequality approach, average income is kept constant while comparing distributions, but social welfare varies (since inequality varies) across the distributions. In the cost of inequality approach, the social welfare associated with all specified distributions is kept the same, while the mean income required to attain this level of welfare varies. Each element of the decomposition thus also corresponds to a difference in means at equal social welfare, which is set to $W_N(\epsilon, v)$.

As in Atkinson (1970), the cost of inequality in the distribution of net income can be expressed as:

$$C_N = \mu_N - \xi_N = \mu_N I_N. \tag{15}$$

[15]See Kakwani and Lambert (1998) and Duclos and Lambert (2000) for a similar type of decomposition, which could be done across household types or provinces, for instance

C represents the level of per capita income that society could use for the elimination of inequality with no loss of social welfare. This amount could also be recuperated by the government as supplementary tax revenue, again with no loss of social welfare if inequality were eliminated. Notice that, unlike I, C is homogeneous of degree one in income.

Let C_F represent the cost of inequality subsequent to a proportional (or flat, and thus inequality neutral) tax on gross incomes that generates the same level of social welfare as the distribution of net incomes. The average income under this welfare-neutral flat tax equals μ_F. The effect of total redistribution on the cost of inequality then becomes:

$$\Delta C = C_F - C_N. \tag{16}$$

Since $\xi_N = \mu_N - C_N = \mu_F - C_F$, we also have:

$$\Delta C = \mu_F - \mu_N. \tag{17}$$

Also, since $\xi_N = \mu_N(1 - I_N) = \mu_F(1 - I_X)$, the mean of incomes subsequent to this flat tax can be calculated as

$$\mu_F = \frac{(1 - I_N)}{(1 - I_X)}\mu_N. \tag{18}$$

Thus:

$$\Delta C = \frac{(I_X - I_N)}{(1 - I_X)}\mu_N. \tag{19}$$

which is positive if $I_X > I_N$. Since a proportional tax does not alleviate inequality, the government cannot collect as much revenue as it can under a redistributive tax system. Consequently, more money must be left in the hands of taxpayers to compensate for the absence of redistribution, and $\mu_F > \mu_N$. This expression has a close connection with the tax progressivity index of Blackorby and Donaldson (1984), which is simply $\Delta C/\mu_N$. It is also related to an index of tax performance developed in Duclos (1995a)[16]. To see this, let τ be the excess proportional tax revenue that the government could collect on net incomes N to make social welfare equal to that yielded by an inequality neutral tax regime with average income μ_N. τ is thus implicitly defined as:

$$(1 - \tau)(1 - I_N)\mu_N \equiv (1 - I_X)\,\mu_N \tag{20}$$

and it can then be shown using (20) that:

$$\Delta C = \frac{\tau}{(1 - \tau)}\mu_N. \tag{21}$$

Thus, the larger the redistributive decrease in the cost of inequality, the more efficient is the tax system. Since HI tends to decrease ΔC, it will also be costly in terms of tax performance. Duclos (1995a,1997) also shows that, for an additive social evaluation

[16]This was also noted in Duclos and Lambert (2000) in the context of the additive social welfare functions.

function U_ϵ, the more progressive the tax system, the greater the value of τ, and that, for a progressive tax system, τ is increasing in the degree of inequality aversion ϵ [17]. This is also valid in the context of the more general social evaluation functions $W(\epsilon, v)$, since they can be interpreted as weighted averages of the U_ϵ. Thus, the more progressive the net tax system, $N(p) - X(p)$, the greater the value of ΔC. If the tax system is progressive, the greater the value of ϵ, the greater the redistributive fall in the cost of inequality.

The decomposition of the total variation in the cost of inequality can then be written as:

$$\Delta C = C_F - C_N = \underbrace{C_F - C_N^H}_{V^*} - \underbrace{\left(C_N^R - C_N^H\right)}_{H^* \geq 0} - \underbrace{\left(C_N - C_N^R\right)}_{R^* \geq 0}. \tag{22}$$

Thus, the redistributive index again decomposes into three effects. Let us examine the reranking effect first. C_N^R measures the cost of inequality in the pre-ordered net incomes that generate the same social welfare as the net income distribution. Under this distribution, average income equals:

$$\mu_N^R = \mu_N \frac{(1 - I_N)}{(1 - I_N^P)} \leq \mu_N. \tag{23}$$

When imputing initial weights $\omega(p, v)$ for each individual in the calculation of after-tax social welfare, the latter is, again, overestimated, giving too much weight to high incomes and too little weight to low incomes. To maintain social welfare at a level corresponding to that of the net income distribution, the government can tax each individual at a proportional rate of $1 - (1 - I_N)/(1 - I_N^P) \geq 0$. Since $\xi_N^P = \xi_N^R$, the reranking effect is thus expressed as:

$$R^* = C_N - C_N^R = \mu_N - \mu_N^R, \tag{24}$$

where R^* represents the additional income accruing to the government (with no change in social welfare) if it could redress the reranking of individuals, so that each remains in his or her original rank. In other words, R^* measures the loss of tax revenue attributable to disturbances created by reranking.

Once the reranking is accounted for, V^* and H^* are computed without regard to changes in rank, *i.e.*, these indices use the ranks of gross incomes for all distributions under consideration.

C_N^H is the cost of inequality under a horizontally equitable tax, and such that social welfare remains equal at each percentile to $\bar{U}_\epsilon(p)$. Incomes at percentiles p are then replaced by the certainty-equivalent level of income $\xi_N(p) = U_\epsilon^{-1}\left(\bar{U}_\epsilon(p)\right)$ as generated by the distribution, $N(q|p)$, within each pre-tax percentile group (see (8)). Since, within each group, this creates less inequality than $N(q|p)$, the government can increase the tax level at each percentile. Notice that for the computation of C_N^H, we attribute an equally distributed equivalent income $\xi_N(p)$ to each individual at pre-tax percentile p; whereas,

[17] A tax system is progressive if the elasticity of $N(p)$ with respect to $X(p)$ is everywhere below 1. The lower the elasticity, the more progressive the tax system.

for I_N^E in the inequality change approach, we attribute $\bar{N}(p)$ to each of these individuals. We then have:

$$\mu_N^H = \int_0^1 \xi_N(p)dp. \tag{25}$$

Since $\xi_N^R = \xi_N^H$, the measure of classical horizontal inequity H^* corresponds to:

$$H^* = C_N^R - C_N^H = \mu_N^R - \mu_N^H, \tag{26}$$

where μ_N^H can also be computed as:

$$\mu_N^H = \mu_N^R \frac{\left(1 - I_N^R\right)}{\left(1 - I_N^H\right)} \leq \mu_N^P. \tag{27}$$

H^* measures the per capita revenues forgone because of classical HI – creating inequality and, by extension, resulting in tax revenue losses in order to maintain social welfare.

Finally, we measure vertical equity as follows:

$$V^* = C_F - C_N^H = \mu_F - \mu_N^H. \tag{28}$$

This measures the difference in the cost of inequality of two horizontally equitable tax systems, the first being a flat tax system, and the second granting everyone his certainty equivalent level of net income, with both yielding the level of social welfare $W_N(\epsilon, v)$. V^* is positive if the tax system is progressive in an *ex ante* sense; that is, if the distribution across percentiles of the certainty-equivalent net incomes is more equal (and thus less inequality costly) than the *ex ante* distribution of gross incomes. V^* represents the additional per capita revenue that (relative to a flat tax) would be available if the tax system were horizonally equitable, and thus created neither classical HI nor reranking.

As for (17), ΔC can then be alternatively expressed as:

$$\Delta C = \mu_F - \mu_N = \underbrace{\mu_F - \mu_N^H}_{V^*} - \underbrace{\left(\mu_N^R - \mu_N^H\right)}_{H^* \geq 0} - \underbrace{\left(\mu_N - \mu_N^R\right)}_{R^* \geq 0}. \tag{29}$$

3.3 Decomposition of classical HI

We may wish to know at which percentile or for which population group HI is more pronounced, and by how much it contributes to total HI. In the case of reranking, this exercise is of limited use, since this concept is ill-suited to separability of the measure across exclusive groups. To obtain a decomposition of the classical HI term, we define the local cost of classical violations of HE, at p, as:

$$H^*(p) = \frac{\mu_N^R}{\mu_N} \bar{N}\left(p\right) - \xi_N\left(p\right), \tag{30}$$

where $H^*(p)$ is the risk-premium of net income uncertainty at percentile p, once the cost of reranking (μ_N^R/μ_N) has been accounted for. $H^*(p)$ is thus the money-metric cost of local classical HI. It represents the supplementary per capita amount local taxpayers would be willing to contribute, with no loss of expected utility, to be subject to a horizontally equitable local tax schedule. We can aggregate $H^*(p)$ using population weights, which by (25) and (30) yields H^*, the index of total classical HI:

$$\int_0^1 H^*(p)dp = \frac{\mu_N^R}{\mu_N}\mu_N - \mu_N^H = H^*. \tag{31}$$

As in Duclos and Lambert (2000), when the local measure $H^*(p)$ is aggregated into the global index H^*, the weights used ensure that the significance attributed to local inequity depends only on population weights, and not upon the standard of living at which HI is experienced[18].

3.4 Discussion

The approach developed above is linked to and generalises a number of other measures of HI. When $\epsilon = 0$, R yields for $v = 2$ the reranking indices of Atkinson (1979) and Plotnick (1981), and for $v \geq 1$ the class of reranking indices of Duclos (1993) and of Aronson *et al.* (1994) (when their "near-equal" bandwidths approach 0). The vertical equity indices V are those of Reynolds-Smolensky (1977) and Kakwani (1977, 1984) for $v = 2$ and of Pfähler (1987) for $v \geq 1$. When $v = 1$, H^* yields the class of classical HI indices introduced in Duclos and Lambert (2000) and the analogous class of discrimination indices found in Kakwani and Lambert (1998).

Setting $\epsilon = 0$ implies a null classical HI effect: $H = I_N^P - I_N^E = 0$. Thus, ϵ can be treated as an index of aversion to classical HI, although in our formulation it also affects a little the assessment of reranking and vertical equity. Increases in ϵ will increase V and V^* if, respectively, the expected tax system and the welfare-equivalent horizontally equitable tax system are progressive (as discussed above). Increases in ϵ will definitely increase H^*, and each of the $H^*(p)$, since $\xi_N(p)$ is a progressive transformation of the distribution $N(q|p)$. As for the effect of changes in ϵ on R^*, this is ambiguous, since continuing to give each individual his rank in the gross income distribution is not necessarily a uniformly progressive transformation of the income distribution.

Setting $v = 1$ results in the weight $w(p, v)$ being equal to 1 for all individuals; consequently, W now becomes a mean of utilities, the calculation of which no longer requires rankings. Thus, the reranking effect becomes nil when $v = 1$: $R = R^* = 0$. The larger the parameter v, the more weight we give to the reranking resentment of the poorest, which may or may not increase R and R^*.

[18] Again, as in Duclos and Lambert (2000, theorem 3), an additional decomposition of (31) can be performed across socio-demographic groups and percentiles, yielding a sum of classical HI within and across socio-demographic groups.

The values of v and ϵ most representative of social preferences can obtained by the "leaky bucket" experiment, if we interpret them in the light of vertical equity preferences. The underlying idea is to measure society's tolerance to costs incurred while transferring income from a rich to a poor individual—be they administrative costs or forgone efficiency (see King (1983)). These experiences suggest that values for ϵ situated between 0.25 and 1.0, and for v between 1 and 4, seem reasonable (see for instance Duclos (2000)), and it is therefore on these ranges of ethical parameter values that the illustration below will focus.

The classical HI indices H and H^* are such that they do not depend on rank dependent measures. This is different from what is done in Aronson *et al.* (1994), for instance, where it is the impact of HI on ranks that captures classical HI among pre-tax equals[19]. Making the assessment of classical HI "rank-independent" is seemingly more consistent with the traditional economic approach to modelling the cost of uncertainty from a "behind-the-veil" position. As for the reranking indices R and R^*, they clearly depend on a rank-dependent formulation for the social welfare function. This would seem more natural than a dependence on somewhat artificial adjustments to an additive formulation for social welfare functions, as is found for instance in King (1983) and Chakravarty (1985)).

The ethical parameters ϵ and v thus distinguish the assessment of classical HI and reranking. We could also specify a separate ethical parameter for the assessment of vertical equity, by having a social welfare function for aggregation across percentiles that differs from the sum of the individual utility functions used for groups of equals (as is done for instance in Auerbach and Hassett (1999)). This could involve replacing $\bar{U}_\epsilon(p)$ by $U_\rho\left(\xi(p)\right)$ in equation (9), where ρ would be the parameter of vertical inequity aversion. This parameter would serve to aggregate the certainty equivalent incomes across percentiles, these incomes themselves being found through the use of the classical HI aversion parameter, ϵ. The particular details of this are, however, outside the scope of this paper and are left for future research.

4 Estimation

Measuring classical HI requires the identification of pre-tax equals in the distribution of gross income. However, as we rarely find two identical individuals in a finite sample, empirical implementation of this concept was seen to pose severe difficulties, which in part led to the development of the reranking approach, as mentioned in section 2. Until recently, most authors attempted to solve this problem by fixing income bands and using them to group individuals as "equals". In addition to being fundamentally arbitrary, this approach also has the failing of yielding results that can be quite sensitive to the size of

[19]This is due to the choice of a rank-dependent social welfare function for the measurement of both classical HI and reranking.

the bands [20].

We therefore turn to the issues involved in estimating equations (14), (22) and (30). Using sample observations, estimating I_X, I_N and I_N^P is straightforward: we simply replace the population distribution by the empirical or sample distribution of incomes, effectively replacing the integral signs in the expressions of the kind of (2) by a summation across sample observations, divided by the number of sample observations. The asymptotic sampling distribution of such estimators can be obtained from the results of Davidson and Duclos (1997), which deal with quantile-dependent estimators, by replacing incomes by utility functions of income. As for I_N^E, we replace each observation in the sample by the predicted level of net income, $\hat{\bar{N}}(p)$, at the sample (gross income) percentile p of that observation. $\hat{\bar{N}}(p)$ can be estimated using non-parametric regressions, as we do in the illustration that follows, based for instance on Silverman (1986) and Härdle (1990).

The estimation of C_N^H and of $H^*(p)$ for classical HI is slightly more difficult, as it requires an estimate of $F_{N|X=X(p)}$ at various pre-tax quantiles $X(p)$ in order to estimate $\xi_N(p)$. This, however, can be done using techniques of non-parametric density estimation, which essentially only require that the conditional population distribution $F_{N|X=X(p)}(n)$ be a sufficiently smooth function of n and $X(p)$. In our illustration below (as in Duclos and Lambert (2000)), we simplify the computation involved by assuming a normal distribution for $F_{N|X=X(p)}$, and by using kernel estimation to assess its variance around the (previously) estimated expected net income $\hat{\bar{N}}(p)$. We then simulate a number of (conditional) net incomes around $\hat{\bar{N}}(p)$, which allows us to compute an estimate of $\xi_N(p)$, $H^*(p)$ and H^* at the various empirical quantiles of gross incomes.

It is important to stress that this exercise does not amount to a normative treatment of "near equals" or "unequals" as equals in estimating classical HI. Here, the statistical and normative procedures are separated. The first step is a purely statistical exercise: we estimate the conditional density of net incomes through kernel estimation. In this statistical exercise, no assumptions on the normative treatment of near equals are needed; all that is required is that the joint distribution evolves smoothly across income levels, and thus that the distribution of net incomes conditional on percentiles of gross incomes is also smooth[21]. The second step then computes normative indices of inequality for the (conditional) net income distributions of exact pre-tax equals. All simulated net incomes then come from the same conditional level of gross income. Thus, this second normative step does not involve treating pre-tax unequals as pre-tax equals.

[20] See Aronson, Johnson and Lambert (1994) for an illustration of this; as the size of the income bands shrinks, classical HI gradually disappears, leaving only an index of reranking.

[21] The estimators of the conditional net income distributions will then be statistically consistent.

5 Illustration

5.1 Data

We illustrate briefly the use of our methodology using Canadian data drawn from Statistics Canada's *Survey of Consumer Finances* of 1981, 1985, 1990 and 1994. These surveys, covering between 36,000 and 46,000 Canadian households, include over one hundred variables, including gross income, net income, the sum of taxes paid and assorted transfers received. The size and composition of families, including the number of adults and children, are also recorded. Each family present in the survey is assigned a statistical weight corresponding to the number of Canadian households it represents, which we multiply by the number of household members to obtain "grossing-up" weights. To transform household income data into adult-equivalent units, we use the equivalence scale of the OECD, which assigns a weight of 1 to the first adult, 0.7 to additional adults, and 0.5 to each child under 17 years of age. Finally, in order to facilitate the interpretation of the results, we normalize gross and net incomes by their means, which has however no substantive effect on inequality and cost of inequality comparisons, since all indices are homogeneous of either degree 0 or 1.

5.2 Change-in-inequality results

We first consider the change-in-inequality approach. Table 1 first shows estimates of the inequality of gross and net incomes, calculated for the four years of observations, and for $v = 1.5$ and $\epsilon = 0.4$, values which appear reasonable in light of the experiences mentioned in Section 3.4. Bootstrap standard deviations for the estimators are shown in parentheses, and their theoretical asymptotic standard deviations appear within brackets[22]. Table 1 indicates a fairly consistent increase in the inequality of gross incomes between 1981 and 1994. Inequality in net incomes also rose between 1981 and 1994, but most of this increase occurred between 1990 and 1994, as the variation in inequality was either negative or statistically insignificant between 1981 and 1990. The explanation for this becomes clear upon examination of ΔI, the decrease in inequality attributable to the tax system. Indeed, the absolute value of this measure increases with time, compensating in large part for the increase in I_X. Notice that the proportional redistributive effect $\Delta I/I_X$ follows essentially the same trend, increasing strongly over time. Many of the estimates and differences across years are statistically significant, given the large sample sizes, and the asymptotic and bootstrap standard errors are very close to each other.

Figure 1 presents 1994 gross $X(p)$ and expected net incomes $\bar{N}(p)$ as well as sample net incomes, represented by points and ordered by their corresponding gross income value. The fact that these points are dispersed around the curve of expected net incomes indicates the presence of classical HI and reranking. Indeed, net incomes do not always

[22]The asymptotic standard errors were computed using the software DAD, which can be freely downloaded from www.crefa.ecn.ulaval.ca/dad.

Table 1: Inequality and redistribution, 1981 to 1994 ($\epsilon = 0.4, v = 1.5$)

Bootstrap standard deviations in parentheses
Asymptotic standard errors in brackets

Year	I_X	I_N	ΔI	$\Delta I/I_X$
1981	0.3323	0.2211	0.1112	0.3346
	(0.0022)	(0.0015)	(0.0013)	(0.0027)
	[0.0022]	[0.0013]	[0.0012]	[0.0023]
1985	0.3680	0.2334	0.1346	0.3658
	(0.0029)	(0.0021)	(0.0016)	(0.0030)
	[0.0029]	[0.0022]	[0.0014]	[0.0029]
1990	0.3719	0.2267	0.1452	0.3904
	(0.0030)	(0.0020)	(0.0017)	(0.0029)
	[0.0027]	[0.0020]	[0.0017]	[0.0029]
1994	0.4415	0.2677	0.1738	0.3937
	(0.0028)	(0.0021)	(0.0016)	(0.0027)
	[0.0028]	[0.0021]	[0.0015]	[0.0027]

increase with the rank of gross income—indicating that changes of rank have occurred. Moreover, even though the sample does not include any two gross incomes that are perfectly identical, we observe that some points are practically situated on a vertical line, a symptom of classical HI.

The information shown in Figure 1, combined with the estimation techniques described in Section 4, allows the computation of V, H, and R for different values of ϵ and v. The indices, computed for $\epsilon = 0.4$ and $v = 1.5$, are presented in Table 2. As predicted by the theoretical model, the vertical effect V dominates ΔI, with non-negligible losses in redistribution attributable to classical HI and reranking. The point estimates of V, H, and R all increased during each of the periods under observation (except for H between 1985 and 1990) and these increases are almost everywhere statistically significant. In all three cases the greatest change was between 1990 and 1994. These results agree with the Canadian results of Duclos and Lambert (2000) on classical HI as well as with Duclos and Tabi (1999) on reranking in Canada. In contrast, Aronson, Lambert, and Trippeer (1997), using U.S. and British data, observe the opposite effects between 1981 and 1990. Lambert and Ramos (1997) find a decline for V and H, along with an increase in R between 1985 and 1990, for Spain.

We are also interested in relative values, i.e., in values normalised by the total redistributive change in ΔI. $V/\Delta I$ represents the proportion of the observed redistribution obtained in the absence of horizontal inequity and reranking, *i.e.*, the tax system's "poten-

Table 2: Vertical equity, horizontal inequity and reranking, 1981 to 1994

$\epsilon = 0.4, v = 1.5$
Bootstrap standard deviations in parentheses

Year	ΔI	V	H	R	$V/\Delta I$	$H/\Delta I$	$R/\Delta I$
1981	0.1112	0.1280	0.0084	0.0083	1.1507	0.0756	0.0751
	(0.0013)	(0.0008)	(0.0002)	(0.0000)	(0.0023)	(0.0017)	(0.0008)
1985	0.1346	0.1557	0.0105	0.0106	1.1566	0.0779	0.0787
	(0.0016)	(0.0012)	(0.0002)	(0.0001)	(0.0025)	(0.0016)	(0.0009)
1990	0.1452	0.1673	0.0103	0.0119	1.1525	0.0707	0.0818
	(0.0017)	(0.0015)	(0.0002)	(0.0001)	(0.0027)	(0.0015)	(0.0010)
1994	0.1738	0.2097	0.0216	0.0143	1.2069	0.1246	0.0823
	(0.0016)	(0.0010)	(0.0003)	(0.0002)	(0.0028)	(0.0019)	(0.0009)

tial" for redistribution. This measure has not varied significantly between 1981 and 1990, but has very significantly increased during the period 1990-1994. $H/\Delta I$ represents the adverse redistribution generated by classical HI as a share of the observed total redistribution. This measure did not vary significantly from 1981 to 1990, but grew significantly between 1990 and 1994, so that the net effect was a statistically significant increase in horizontal inequity between 1981 and 1994. As to reranking as a proportion of net redistribution $(R/\Delta I)$, it witnessed no statistically significant change throughout the period[23]. The combined effect of classical HI and reranking is not negligible – amounting to between 15% and 21% of the net redistributive effect.

Our results show divergences in the evolution of classical HI and reranking over time. This phenomenon may seem surprising at first in light of the strong conceptual link between the two concepts hitherto highlighted in the previous literature. For a given level of HI, however, the incidence of reranking depends on the proximity of the groups of equals. Thus, even if H grew rapidly between two periods, a substantial increase in the inequality in gross income would attenuate the impact of HI on R–this actually occurred between 1990 and 1994. The opposite effect would occur subsequent to a fall in I_X.

5.3 Sensitivity of results

As discussed above, measures of classical HI and reranking are sensitive to the ethical parameters ϵ and v. Figures 2 and 3 confirm, however, that the measure of classical HI is considerably more sensitive to the choice of ϵ than of v, and conversely for the index

[23]Once again our results are in opposite direction to those obtained by Aronson, Lambert and Trippeer (1997) for the United States and Great Britain, and those of Lambert and Ramos (1997) for Spain.

of reranking. When ϵ increases from 0 to 0.8, classical HI increases from 0 to close to 0.1, which says that classical HI alone can decrease net redistribution by up to 0.1 (which is high, considering that the inequality indices considered here are bounded by 0 and 1). Changes in v have little effects on H, but an increase in v from 1.0 to 3.0 raises R from 0 to 0.05, with a corresponding increase in redistributive costs.

But what about the relative magnitude of classical HI and reranking? Which of these two manifestations of horizontal inequity is more detrimental to redistribution? Our results cannot furnish a clear answer to this question, which requires a normative judgement. This is illustrated in Table 3, where the H/R ratio varies considerably with the choice of ϵ and v, ranging from 0 when $\epsilon = 0$ to infinity when $v = 1$. The Table also confirms that H/R increases with ϵ/v, which is consistent with the definition of the two parameters. Table 4 shows, however, that the H/R ratio fell between 1981 and 1990, but grew significantly between 1990 and 1994 and ended higher than its 1981 value and this, for all three pairs of values of ϵ and v shown. This is in fact observed regardless of the choices of ϵ and v (except of course for a choice of $\epsilon = 0$, when $H = 0$, or for a choice of $v = 1$, for which $R = 0$). Thus, as a proportion of reranking, classical HI seems considerably higher in 1994 than in 1981.

Table 3: H/R, 1994

	v						
ϵ	1	1.5	2	2.5	3	3.5	4
0.0	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	-	0.6619	0.3918	0.2974	0.2490	0.2168	0.1931
0.4	-	1.5105	0.8942	0.6759	0.5592	0.4864	0.4346
0.6	-	2.7162	1.5845	1.1927	0.9830	0.8520	0.7587
0.8	-	4.8187	2.7934	2.0866	1.7145	1.4831	1.3211
1.0	-	7.3706	4.2250	3.1476	2.5833	2.2391	1.9975

The measurement of HI must also tackle the issue of household heterogeneity in needs, which affects the pre-tax ranking of individuals as well as the impact of taxes and transfers on their economic well-being. To check the sensitivity of inequality, classical HI and reranking indices to the choice of equivalence scales, I, H, R and V were computed using three alternative equivalence scales. The OECD scale was already described above. The Cutler and Katz scale divides household incomes by the factor $(n_a + \theta n_c)^{\phi}$, where n_a is the number of adults and n_c is the number of children. Parameters θ and ϕ can generally take any value between 0 and 1, but for the purposes of this illustration we set $\theta = \phi = 0.5$. This scale therefore assigns a decreasing weight to each additional household member. The Statistics Canada equivalence scale is implicitly based on estimates of low-income thresholds, which depend on family size as well as population density in the

Table 4: H/R, 1981 to 1994

year	H/R		
	$\epsilon = 0.4, v = 1.5$	$\epsilon = 0.8, v = 2$	$\epsilon = 1, v = 4$
1981	1.0120	1.5789	1.1892
1985	0.9906	1.5025	1.0996
1990	0.8655	1.2545	0.9029
1994	1.5105	2.7934	1.9975

region of residence[24]. Table 5 shows results based on these three equivalence scales. I_X, I_N, ΔI and V appear to be relatively insensitive to the choice of equivalence scales. This is not the case for H and R, however, which as a proportion of ΔI vary from 8.8% to 12.5% for H and from 5.9% to 9.6% for R. The overall redistributive costs of horizontal inequity can thus vary from 14.7% to 21.7% of net redistribution, depending on the choice of equivalence scales.

Table 5: Inequality and inequity 1994 ($\epsilon = 0.4, v = 1.5$)

	Scale		
	OECD	Cutler and Katz $\theta = \phi = 0.5$	Statistics Canada
I_X	0.4415	0.4501	0.4082
I_N	0.2677	0.2657	0.2245
ΔI	0.1738	0.1844	0.1837
V	0.2097	0.2114	0.2236
H	0.0216	0.0163	0.0222
R	0.0143	0.0108	0.0176
$V/\Delta I$	1.2069	1.1470	1.2169
$H/\Delta I$	0.1246	0.0883	0.1211
$R/\Delta I$	0.0823	0.0587	0.0958

5.4 Cost of inequality results

Estimates of the cost-of-inequality indices are shown in Table 6, again for $v = 1.5$ and $\epsilon = 0.4$. Observations on the absolute value of the measures are basically the same as pre-

[24]See the Statistics Canada web page http://www.statcan.ca for further information.

viously, except that we can now usefully interpret these values as money-metric indicators of the benefits of VE and of the costs of classical HI and reranking. The estimates of V^*, H^*, and R^* increase in each sub-period, except for H^* between 1985 and 1990, although the increases are often not statistically significant between 1981 and 1990. The increases in VE, classical HI and reranking between 1990 and 1994 are, however, large and statistically significant. Over 15 years, the cost of classical HI rises from 0.6% to 1.3% of per capita income, and the cost of reranking increases from 1.1% to 1.9% of average income. These increases are, in fact, well correlated with the increases in net redistribution, whose money-metric benefit rises from 16.7% to 30.9% of per capita income. As a proportion of ΔC, H^* and R^* are indeed often statistically indistinguishable across the four years.

Table 6: Vertical equity, classical horizontal inequity, and reranking based on cost of inequality, 1981 to 1994; $\epsilon = 0.4, v = 1.5$; Bootstrap standard deviations in parentheses

Year	ΔC	V^*	H^*	R^*	$V^*/\Delta C$	$H^*/\Delta C$	$R^*/\Delta C$
1981	0.1670	0.1836	0.0060	0.0106	1.0994	0.0359	0.0636
	(0.0017)	(0.0010)	(0.0001)	(0.0001)	(0.0016)	(0.0007)	(0.0010)
1985	0.2139	0.2351	0.0075	0.0138	1.0993	0.0350	0.0644
	(0.0029)	(0.0029)	(0.0001)	(0.0002)	(0.0015)	(0.0006)	(0.0010)
1990	0.2306	0.2531	0.0073	0.0152	1.0978	0.0318	0.0659
	(0.0031)	(0.0031)	(0.0002)	(0.0002)	(0.0018)	(0.0007)	(0.0013)
1994	0.3090	0.3415	0.0128	0.0187	1.1052	0.0446	0.0606
	(0.0029)	(0.0029)	(0.0002)	(0.0002)	(0.0015)	(0.0007)	(0.0009)

Examination of Table 7 and Figure 4 (using $\epsilon = 0.4$ and $v = 1.5$) reveals that the local measure of classical HI, $H^*(p)$, generally declines with the rank of gross income. In particular, the values of $H^*(p)$ are particularly high for the 20% of the population with the lowest incomes. Thus, discrimination in the tax system between equals appears most pronounced among the poorest, particularly in 1994. This observation agrees with the conclusions in Duclos and Lambert (2000). This phenomenon is in fact observed for all positive values of ϵ.

5.5 Discussion of results

Reforms in the Canadian tax and transfer system, macroeconomic shocks, and socio-demographic changes have all strongly affected the distribution and redistribution of income since the early 1980's, and can help explain our main results or important increases

Table 7: Classical Horizontal inequity by percentile, 1981 to 1994

($\epsilon = 0.4, v = 1.5$)

Rank	1981	1985	1990	1994
0.05	0.0228	0.0227	0.0231	0.0418
0.15	0.0106	0.0145	0.0151	0.0400
0.25	0.0056	0.0084	0.0094	0.0208
0.35	0.0040	0.0058	0.0070	0.0120
0.45	0.0034	0.0044	0.0054	0.0082
0.55	0.0033	0.0037	0.0045	0.0060
0.65	0.0029	0.0035	0.0039	0.0051
0.75	0.0030	0.0031	0.0034	0.0043
0.85	0.0025	0.0032	0.0034	0.0037
0.95	0.0039	0.0033	0.0038	0.0030

in redistribution and in the HI costs to that redistribution[25]. The 1981 and 1987 tax reforms introduced a broadening of the tax base through restrictions in certain tax preferences (*e.g.*, higher capital gain inclusion rate, restrictions on tax deferral mechanisms, and the repeal of certain tax shelter provisions). Although such base-broadening measures tend to favour horizontal equity, some selective and targetted measures implemented during the 1981-1994 period have had the opposite effect; examples include the replacement of personal exemptions by personal tax credits, the move from a family-based to an individual-based tax system, the means-testing of child tax credits, and the introduction of a lifetime exemption for capital gains. Besides, despite sometimes lower marginal tax rates, income was increasingly subjected between 1981 and 1994 to higher average tax rates, partly prompted by increasing public deficits. Thus, the observed progression in classical HI and reranking can be imputed to a changing tax system and also to stronger fiscal intervention.

On the transfer side, HI stems in large part from the unemployment insurance, old-age benefit and social assistance programmes, whose parameters and size have evolved significantly over the last twenty years. These programmes often discriminate on the basis of sex, age, household type, region of residence, marital status, monthly income variability, etc., in a way which can cause reranking and classical HI, although they will also typically generate an increase in net redistribution. The higher classical HI observed among low income households can be explained by the fact that most transfers are aimed at this income group. Socio-demographic and economic changes have also affected income redistribution, mainly through transfers. As a consequence of population aging, more Canadians

[25] For more detailed evidence on this, see Duclos and Lambert (2000).

depend on old-age benefits. The growing number of divorces has resulted in the rise of single-parent families, many of which depend on social welfare. Regional and industrial shocks have made the unemployment system look more like a social assistance than a social insurance system for many regions and industries. These changes have typically led to greater government intervention, resulting in more significant, but also more imperfect, redistribution.

6 Conclusion

This paper presents a decomposition of the redistributive change in income inequality as a sum of vertical equity, classical horizontal inequity (HI) and reranking components. We separate the measurement of the last two components since we argue that although classical HI and reranking are both necessary and sufficient signs of violations of the principle of horizontal equity, they are different manifestations of those violations. The decomposition uses a non-additive social welfare function which combines the features of the well-known Atkinson and Gini social welfare indices. This allows classical HI and reranking to be assessed jointly, though on fundamentally separate functional bases. The index of reranking is based on a social welfare aversion to rank inequality and relative deprivation; the index of classical horizontal inequity is based on an aversion to net income riskiness, and its measure does not depend on a rank-dependent formulation of the social welfare function. This dual formulation also allows for the specification of different ethical parameter values for the measurement of classical HI and reranking. Two measurement approaches are developed: one in terms of changes in indices of inequality, and the other in terms of changes in the costs of inequality. Both approaches help us evaluate the cost of classical HI and reranking, either in terms of forgone redistributive effects or in terms of foregone tax revenues for the government. An illustration on Canadian data indicates that these losses can be significant, and that they have generally tended to grow between 1981 and 1994. Finally, although comparing classical HI and reranking at a given point in time requires a normative judgement, the two phenomena have clearly gone through a divergent evolution in Canada between 1981 and 1994, with classical HI as a proportion of reranking being considerably higher in 1994 than in 1981.

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Figure 1: Gross, net and expected net income, 1994

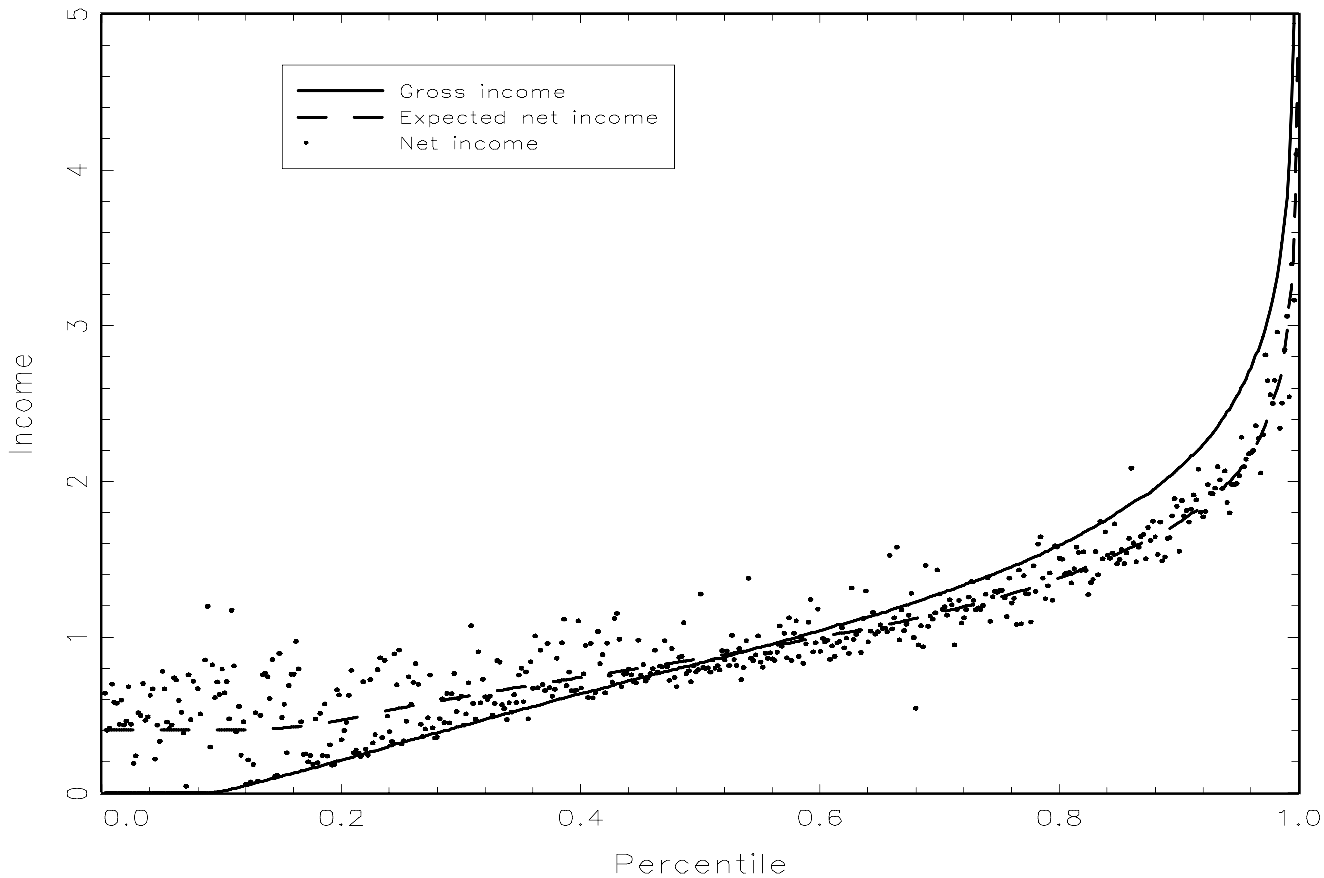

Figure 2: Index of classical horizontal inequity H by choice of parameter, 1994

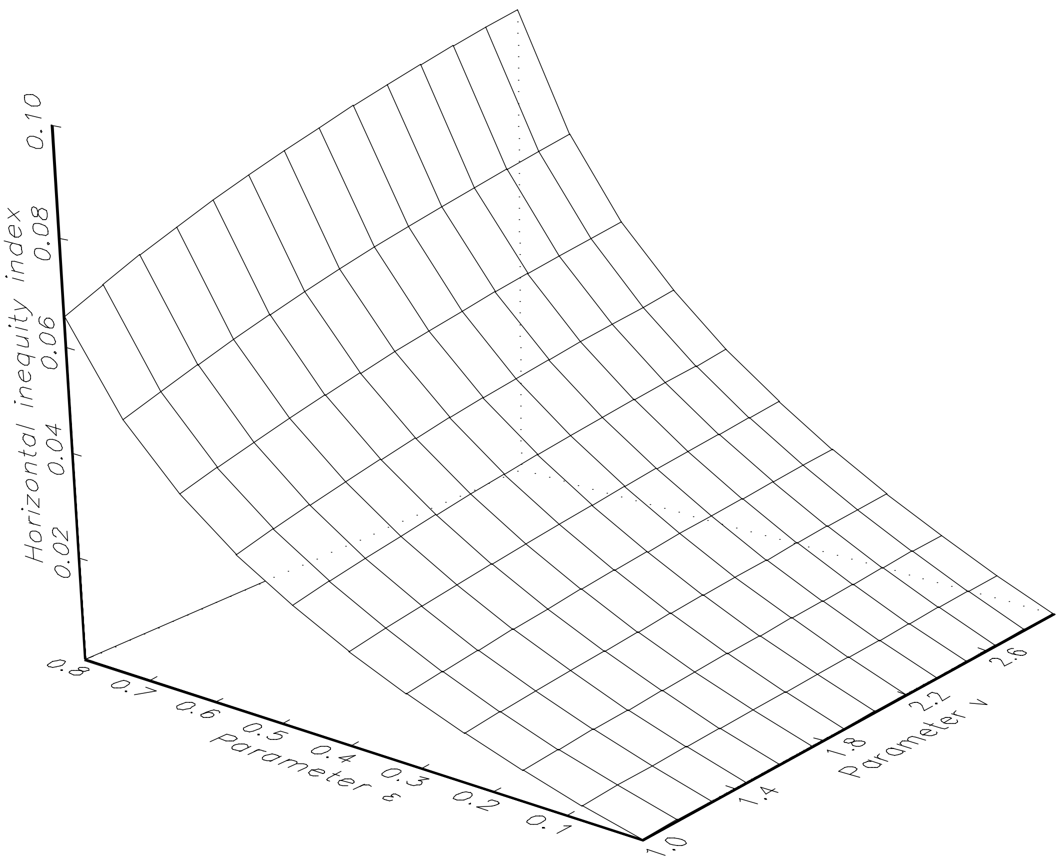

Figure 3: Index of reranking R by choice of parameter, 1994

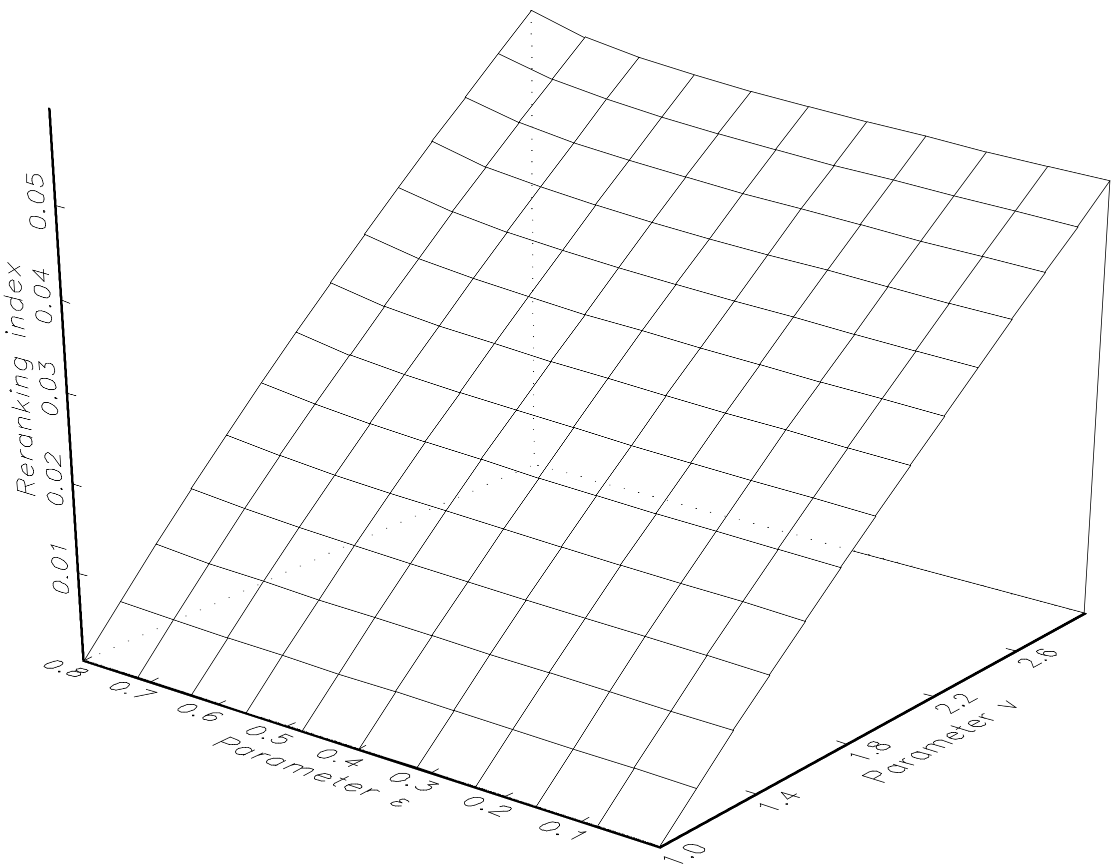

Figure 4: Classical horizontal inequity by percentile, 1981 to 1994 ($\epsilon = 0.4, v = 1.5$)