A Test of Collective Rationality for Multi-Person Households[*]

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Abstract

This paper provides a test of efficiency of consumption decisions in households with many decision-makers. It also presents a method of determining the number of these decision-makers. Information on some distribution factors is needed to implement this approach.

KEYWORDS: Intra-household allocation, collective model, polygamy, extended family, Pareto optimality.

1 Introduction

Much effort has recently been put into the search for ways of empirically testing so-called "collective rationality", according to which intra-household decisions are Pareto-efficient (e.g., Chiappori 1992, Browning et al. 1994, Udry 1996 and Fortin and Lacroix 1997). Even in a very general setting allowing for private commodities and consumption externalities, Browning and Chiappori (1998, hereafter BC) have shown that Pareto-efficiency may impose testable restrictions on consumption behavior. When there are two potential decision-makers in the household, they show that the (Pseudo-)Slutsky matrix is the sum of a symmetric negative semi-definite matrix and a matrix that has, at most, rank one. They also show how this condition can be generalized to the case of a household with more than two decision-makers. This extension is important since it is likely that in many households adult children who live with their parents influence the family decision

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process. Moreover, polygamous or extended families are quite common in many developing countries. As a by-product of their analysis, BC also provide a simple test which allows the number of decision-makers in a multi-person household to be determined.

These tests face two limitations, however. First, they cannot be performed when only cross-sectional data (with no observed variability in regional prices) are available. Second, from BC results, it is easy to show that these tests cannot be implemented when the number of observed commodities is less than two times the number of intra-household decision-makers. In this case, the symmetry plus rank restrictions are always satisfied. This means, for instance, that they have no implication on the standard labor supply model with one Hicksian consumption good, two leisure commodities and two decision makers.

Fortunately, a complementary approach, based on so-called distribution factors (see Browning et al. 1994), provides tests that are less subject to these limitations. These tests can be implemented with cross-sectional data and, as shown below, only require having a number of observed commodities larger than the number of intra-household decision-makers. A distribution factor is a variable that influences the decision process within the household, but which doesn't influence preferences or the household budget set. In the recent literature, the share of exogenous income under the control of one household member (e.g., Browning et al. 1994) and the state of the marriage market, as proxied for instance by the sex ratio (Chiappori et al. 1998), have been used as distribution factors.[1]

In the case of a household where decision-makers are limited to two, Bourguignon et al. (1995) have shown that the restrictions imposed by distribution factors stem from the fact that they influence consumption choices only through their effect on the relative weight of one individual in the household utility function. However, for each additional individual involved in the decision process there is an associated relative weight. Therefore, the one-dimensional effect of distribution factors is lost, so that their result does not extend trivially to the case of multi-person households. This paper generalizes the distribution factors test to households where there are potentially more than two persons who participate in the decision process. It also provides a simple method of determining the number of decision-makers when the intra-household consumption decisions process is efficient.

[1] An important class of distribution factors are the «extra-environmental parameters» (EEPs) discussed by McElroy (1990).

2 The Theoretical Framework

The convention used throughout this note is to denote vectors and matrices using letters in boldface. Also, the expression $D_z f(z)$ denotes the partial derivatives matrix of any vector-valued differentiable function $f(z)$ with respect to z, whose mnth entry is $\partial f_m(z)/\partial z_n$.

Let's consider a household with $I + 1$ members participating in the decision process (with $I \geq 1$). Each draws his/her well-being from the consumption of N market commodities, which can take a private form, a public form or both (e.g., part of home electricity consumption can be used to heat each household adult's home office and part can be used to heat the common rooms). Define $x \equiv [x_1; x_2; ...; x_N]'$ as the N-vector representing household consumption. All prices are normalized to one. The household budget constraint is therefore given by: $\iota' x = m$, where ι is a unity vector of dimension N while m holds for the level of household income.[2] Each member i, for $i = 1; ...; I + 1$, has preferences given by a strongly concave and twice continuously differentiable utility function $U_i(x)$.

Axiom 1 The outcomes of the decision process are (weakly) Pareto-efficient.

Axiom 2 The decision process depends on a set of K variables $y \equiv [y_1; y_2; ...; y_K]'$ which are independent of individual preferences and which do not affect the overall household budget constraint.

More precisely, the household behaves as though it were maximizing the following program:

$$\begin{aligned} \underset{x \in R^N_+}{\text{Max}} \quad & \mu_I(m; y)' U_I(x) + U_{I+1}(x) \qquad \text{(P)} \\ \text{subject to} \quad & \iota' x = m; \end{aligned}$$

where $\mu_I(m; y)$: $R^{K+1} \to R^I_{++}$ and $U_I(x)$: $R^N \to R^I$. Thus the household utility function to be maximized in this program is a weighted sum of the decision-makers' utility functions, with the vector $\mu_I(m; y)$ holding for the relative utility weights of the I first decision-makers with respect to the $I + 1$ th's participant.[3] One important characteristic of this collective approach is that the I relative utility weights are not constant in general, but are functions of the overall household income and of the distribution factors y.

The demand system under collective rationality, as obtained from solving the program (P) for x; can be written as: $x = \xi(m; \mu_I(m; y))$. This system shows that the distribution factors influence household consumption choices only through the I relative utility weights entering the household

[2] This assumes that the household does not produce any of these N goods, or if not, that the markets for these goods are perfect.

[3] These weights could also be interpreted as the Lagrangean multipliers associated with the inegality constraints in the program: $\text{Max}_{\{x \in R^N_+\}} U_{I+1}(x)$ subject to $U_I(x) \geq v_I(m; y)$ and $\iota' x = m$, where $v_I(m; y)$: $R^{K+1} \to R^I$.

utility function. This is a consequence of the fact that the distribution factors do not affect the Paretian frontier (which depends only on preferences and the household budget constraint) but only the point chosen by the household on this frontier. The basic issue therefore, is to find a way to test whether the household demand system can be written as $\hat{x}(m; \mu_I(m; y))$. The problem is that this function is unobservable since the relative utility weights are unobservable. Rather, what is actually observed is the function $\xi(m; y)$ which must satisfy:

$$\xi(m; y) = \hat{x}(m; \mu_I(m; y)) \tag{1}$$

and adding-up: $\pi'\xi(m; y) = m$: In order to keep the presentation as simple as possible, we shall drop m from all functions for the remaining of the paper. Thus (1) becomes:

$$\xi(y) = \hat{x}(\mu_I(y)): \tag{2}$$

Now, based on a particular type of conditional demand system generalizing the approach suggested by Bourguignon et al., it is possible to derive a (local) test of collective rationality. We shall consider partitions $x = [x_1'; x_2']'$ of the demand system and $y = [y_1'; y_2']'$ of the distribution factors, with $x_1$and $y_1$having the same dimension k. Given such a partition, (2) can be written as:

$$x_1 = \xi_1(y_1; y_2) = \hat{x}_1(\mu_I(y_1; y_2)); \tag{3}$$

$$x_2 = \xi_2(y_1; y_2) = \hat{x}_2(\mu_I(y_1; y_2)): \tag{4}$$

Lemma 1 Let $N \geq I + 1$ and $K \geq I + 1$ and consider a $y^* \in R^K$ at which $\xi(y)$ is differentiable. Next, consider partitions of x and y such that $D_{y_1}\xi_1(y^*)$ is non-singular and let $x_1^* = \xi_1(y_1^*; y_2^*)$: Then, there exists a neighborhood $V(y_2^*; x_1^*) \subset R^{K-k}$ and a differentiable function $\varphi_1 : V(y_2^*; x_1^*) \to R^k$ such that:

$$x_1^* = \xi_1(\varphi_1(x_1^*; y_2); y_2) = \hat{x}_1(\mu_I(\varphi_1(x_1^*; y_2); y_2)) \ \forall \ y_2 \in V(y_2^*; x_1^*): \tag{5}$$

Proof. Use the implicit function theorem. ■

Under the conditions of Lemma 1, one can define the function $\bar{x}_2 : R^{K-k} \to R^{N-k}$ by:

$$\bar{x}_2(x_1^*; y_2) = \hat{x}_2(\mu_I(\varphi_1(x_1^*; y_2); y_2)): \tag{6}$$

The following theorem now generalizes a result by Bourguignon et al.:

Theorem 1 Let the conditions of Lemma 1 hold and suppose that, in addition, $\mu_I(y)$ and $\hat{x}(y)$ are also differentiable at respectively y^* and $\mu_I(y^*)$. Then, for $k = I$ the demand system for x satisfies:

$$D_{y_2}\bar{x}_2(x_1^*; y_2^*) = 0; \tag{7}$$

where 0 is a null matrix of dimension $(N - I) \times (K - I)$:

Proof. Taking the derivatives of (5) and (6) with respect to y_2 at y_2^* and using $y^* = [\tilde{\psi}_1(x_1^*; y_2^*); y_2^*]$, one obtains:

$$\begin{aligned} 0 &= D_{\mu_I}\hat{x}_1(\mu_I(y^*))\,[D_{y_1}\mu_I(y^*)D_{y_2}\tilde{\psi}_1(x_1^*; y_2^*) + D_{y_2}\mu_I(y^*)]\,; \\ D_{y_2}\bar{x}_2(x_1^*; y_2^*) &= D_{\mu_I}\hat{x}_2(\mu_I(y^*))\,[D_{y_1}\mu_I(y^*)D_{y_2}\tilde{\psi}_1(x_1^*; y_2^*) + D_{y_2}\mu_I(y^*)]: \end{aligned}$$

Now consider the system of I equations in I variables $D_{\mu_I}\hat{x}_1(\mu_I(y^*))\,z = 0$: For $D_{y_1}\tilde{\psi}_1(y^*)$ to be non-singular, it is necessary that $D_{\mu_I}\hat{x}_1(\mu_I(y^*))$ be also non-singular. Thus, the only solution of this system is $z = 0$; from which we obtain $[D_{y_1}\mu_I(y^*)D_{y_2}\tilde{\psi}_1(x_1^*; y_2^*) + D_{y_2}\mu_I(y^*)] = 0$ and thus $D_{y_2}\bar{x}_2(x_1^*; y_2^*) = 0$. ■

The intuition behind this result is that demands for the $N - I$ commodities, as given by $\bar{x}_2(x_1^*; y_2^*)$, are conditioned on as many commodities as there are relative utility weights in the household utility function (that is, I). Therefore, adjustments in y_1 will compensate for any change in y_2 so as to keep x_1 constant in a way that will leave the I relative utility weights unchanged. However, if μ_I stays constant when y_2 changes, then x_2 must also stay constant, and therefore $D_{y_2}\bar{x}_2(x_1^*; y_2^*) = 0$. Note that when $N = I + 1$, one has $\bar{x}_2(x_1^*; y_2^*) = m - \iota' x_1^*$ from the adding-up constraint. Therefore, (7) is always satisfied in this case. This implies that $N > I + 1$ is required to provide a test of the collective setting.

Corollary 1 Assume that the preferences of the decision-makers all differ. Then, under the conditions of Lemma 1, the number of decision-makers in the household is given by the smallest number of goods under which demand functions must be conditioned in order to satisfy restrictions (7), plus one.

Assuming that conditions of Lemma 1 apply to all observed y and with due account of preference variables and household income, (7) provides a (local) test of collective rationality. Furthermore, under collective rationality, Corollary 1 gives a method of determining the number of actual decision-makers within the household.

3 Econometric Considerations

In practice, econometricians only observe K^o ($\leqslant K$) distribution factors and N^o ($\leqslant N$) commodities. It is possible to verify whether some restrictions imposed by collective rationality are satisfied, only if $K^o \geqslant I + 1$ and $N^o \geqslant I + 1$ for $N^o < N$ or $N^o > I + 1$ for $N^o = N$. Before testing collective rationality, one must check that each of the K^o variables significantly affects each of the N^o unconditional demands, which is required for these variables to be distribution factors. With regards to the test itself, the choice of the elements of x_1 on which the demand sub-system is conditioned should not influence the result.[4] Furthermore, note that the estimation of this conditional sub-system raises an identification issue even when y and m are exogenous, since the x_1 variables are endogenous. However, since the number of exogenous variables excluded (that is, the number of elements in vector y_1) is equal to the number of right-hand side endogenous variables included ($= I$), the order criterion for exact identification in a linear model is satisfied. Finally, one problem with the test proposed is that it requires one more observable distribution factor for each additional decision-maker, which may appear quite demanding. However, this may not be a limitation as long as each of the I decision makers' share of total household income can be considered as a distribution factor. Econometric work to implement this test using household data from Burkina Faso (where polygamous families are quite frequent) is currently ongoing.

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[4] The reason for this is that, regardless of whether Pareto-efficiency holds or not, if equation (7) holds for a given order of elements of x and y, it will also hold for any other order for which the regularity conditions are satisfied.