ON JUMPS AND ARCH EFFECTS IN NATURAL RESOURCE PRICES

AN APPLICATION TO STUMPAGE PRICES FROM PACIFIC NORTHWEST NATIONAL FORESTS*

by

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Abstract

Models used for natural resources prices usually preclude the possibility of large changes (jumps) resulting from discrete, unexpected events. To test for the presence of jumps and ARCH effects, we propose to use bounds and bootstrap test techniques, thus solving the unidentified nuisance parameter problem. We apply this approach to stumpage price time series from the Pacific Northwest and find evidence of jumps and ARCH effects. Using real options, we then develop a stopping model to assess the impact of neglecting jumps on the decision to harvest old-growth timber. Our numerical results show the importance of modeling jumps explicitly.

Keyword: jump processes; ARCH; bootstrap; stumpage prices; real options.

1. INTRODUCTION

An important empirical issue in economics is to adequately model the process generating market prices. A common basic assumption is that prices change smoothly on the premise that markets tend to anticipate the arrival of information and process it continuously. One of the most popular models in finance, capital theory, and natural resources, is the geometric Brownian motion (GBM) because it is compatible with the efficient market hypothesis and it often leads to analytically tractable solutions. In finance for example, the assumption that stocks follow a GBM is essential to derive the Black-Scholes (1973) formula. The GBM hypothesis is also central in a number of papers in the investment literature (e.g., MacDonald and Siegel, 1986; Pindyck, 1988; or Dixit, 1992) or the natural resources literature (Brennan and Schwartz, 1985; or Paddock, Siegel, and Smith, 1988). In forestry, Clarke and Reed (1989, 1990) and Reed (1993) assume that amenity and conversion value follow a GBM when they reconsider the tree-cutting problem under uncertainty. Morck, Schwartz, and Stangeland (1989) derive the value of a forestry lease when both timber price and inventory level follow a GBM. Conrad and Ludwig (1993) calculate the optimal stock of old-growth forest when the ratio of amenity to conversion value evolves according to a geometric Brownian motion.

While the usefulness of the geometric Brownian motion as a theoretical tool is well established, its adequacy for deriving practical decision rules has been questioned on both theoretical and empirical grounds. First, as explained by Merton (1982), assuming that a variable changes continuously, as for the GBM, precludes large changes, or jumps, which may result from the sudden arrival of "lumpy" information (possibly linked to an unexpected or catastrophic event). In forestry, these “jumps” can be caused, for example, by political decisions (a ban on imported timber), court rulings (e.g., a decision to restrict logging to protect endangered species), or natural causes (a forest fire, a disease, or a devastating storm). In addition, much empirical work has shown that the distributions of the increments of the logarithm of many economic and financial time series have tails heavier than the normal distribution (the so-called “fat tails”). This feature has been invoked to explain the discrepancies observed between actual pricing of financial options and theoretical predictions. More generally, ignoring "fat tails" could lead to ineffective hedging strategies or to mispricing assets.

A number of proposals have been advanced to model “fat tails”: (1) a stationary process with tails fatter than the normal, such as a Paretian stable distribution; (2) a distribution with time-varying parameters; and (3) a mixture of distributions such as a normal and a jump process. The first alternative is controversial, however, because Paretian distributions have infinite second moments (e.g., see Friedman and Vandersteel, 1982). The second alternative includes models with time-varying deterministic parameters, such as ARCH (Engle 1982), or with stochastic parameters (e.g., see Stein and Stein 1991). In the third class of models, a stochastic variable is represented by the combination of two processes: a diffusion, such as a GBM, and a jump process (usually a Bernoulli or a Poisson process) coupled with a distribution describing the size of the jumps (often a lognormal). The diffusion accounts for “ordinary” random fluctuations and the jump process models the arrival of "lumpy" information.

In this paper, we analyze four quarterly stumpage price time series from public forests in the Pacific Northwest region, but the approach we propose is widely applicable. It should be especially useful when dealing with small samples, which is often the case in natural resources. We find evidence of "fat tails" for all four time series, and we investigate both ARCH and mixture distributions as probable causes of their existence through jump-GBM and jump-ARCH-GBM specifications.

In the context of no-jump LR tests (with or without ARCH), deriving valid p-values is an econometric challenge that has often been overlooked in empirical work. First, testing the null hypothesis that the rate of arrival of jumps is zero is difficult because of possible distributional discontinuities on the parameter space boundary (see Brorsen and Yang, 1984). Inference is also complicated by the non-identification of nuisance parameters present under the null hypothesis. These problems are compounded in empirical work where we deal with finite samples, which are often fairly small in natural resources economics, and we try to estimate parameters that model relatively rare events (jumps) so the precision of our estimates can be greatly affected.

In addition, standard ARCH tests (such as Engle's 1982 test) are not necessarily appropriate in the presence of jumps (for a review of ARCH tests, see Bernard, Dufour, Khalaf and Genest, 1999). In this case, the parameters of the jump process (which intervene under the null and the alternative hypothesis) are identifiable if a *strict positivity* restriction is imposed on the rate of jump arrivals. Such a restriction implies, however, that the relevant nuisance parameters space include a *locally almost unidentified* (LAU) region (Dufour 1997) at the zero boundary, which may seriously distort the tests size. In practice, these issues imply that for all hypothesis tests at hand here, one must seriously guard against *spurious* rejections.

To circumvent these problems, we propose a methodology based on the Monte Carlo (MC) test technique (Dufour 1995), which is closely related to the parametric bootstrap. Simulation-based methods are typically employed in hope of obtaining improved approximations to finite-sample null distributions. Here, we show that the MC jumps and/or ARCH tests that we use are finite sample exact. To deal with nuisance parameters, we derive (when necessary) exact *bounds* cut-off points, to make sure rejections (which provide evidence in favor of ARCH and/or jumps) are conclusive. Although this implies conservative decision rules, no other procedure seems available which warrants statistically conclusive (non-spurious) evidence in finite samples. This approach is particularly relevant here because data samples available in natural resource economics are often fairly small (at least compared to finance): here, stumpage price data are usually available only on a quarterly basis (and sometimes on a monthly basis). We obtain empirical evidence against the no-jump and/or the no-ARCH hypotheses. In fact, two of our time series could be modeled by a jump-GBM process, and the other two by a GBM-ARCH process. The presence of jumps and ARCH effects in financial time series has by now been extensively documented (e.g., Ball and Torous 1985, Jorion 1988, Vlaar and Palm 1993, or Bates 1996), but these features seem to have been largely ignored in the natural resource economics literature. This is surprising because we can infer from finance that jumps and ARCH effects could impact the decision to harvest natural resources, forecasts of their price, and the valuation of natural resources investments.

To investigate the empirical impact of "jumps," we then reconsider the classical tree-cutting problem when stumpage prices for old growth forest follow a GBM with jumps. We formulate this infinite-horizon stopping problem in continuous time using a real options framework (Dixit and Pindyck). This leads to an integro-differential equation, which we solve with Galerkin's method (see Delves and Mohamed, 1985). This method, outlined in an appendix, can also easily be used when the price of a resource follows a mean reverting process, which may be more appropriate for other natural resource problems.

Another theoretical argument for rejecting the GBM for natural resource prices is mean reversion: when the price of a natural resource is low, high cost producers exit the market, and when it is high, they enter again (e.g., Schwartz, 1997). Pindyck and Rubinfeld (1991) found, however, that they could not reject the random walk hypothesis at the 5% level for yearly lumber prices over the period extending from 1870 to 1986. Here, we conduct the well known Perron (1989) unit root test for the time series considered and we find that we cannot reject the unit root model. It should be noted that the performance of unit root tests (including Perron's test) in the presence of jumps has not been formally assessed. In addition, it is well known that ARCH and/or breaks-in-variance can seriously affect these tests and lead to under- or over-rejections (e.g., see Hamori and Tokihisa, 1997, or Kim and Schmidt 1993). Since the processes we consider involve heteroscedasticity with related characteristics, this question deserves further consideration yet it is beyond the scope of this paper. We thus keep the GBM as our basic diffusion model but our approach can easily be extended to the case of a mean-reverting process.

This paper is organized as follows. In the next section, we present the econometric framework and the test statistics employed in the empirical analysis. Section III describes the data and reports the empirical results. In Section IV, we develop a simple stopping problem to assess the impact of neglecting jumps on the decision to harvest timber. The last section presents our conclusions. Details of the statistical and numerical procedures employed are presented in the Appendices.

2. METHODOLOGY

This section introduces the models considered and the test statistics used. Let S_t denote stumpage price per unit area at time t, and define $s_t \equiv Ln(S_t)$. If S_t follows a geometric Brownian motion with trend α and variance parameter σ (Dixit and Pindyck, 1994):

(1) $$dS_t = \alpha S_t dt + \sigma S_t dz$$

then $s_t - s_{t-1}$ is normally distributed with variance σ^2 and mean $\mu = \alpha - \frac{\sigma^2}{2}$. To allow for discontinuities in S_t, consider the mixed jump diffusion process:

(2) $$dS_t = \alpha S_t dt + \sigma S_t dz + S_t dq$$

where $\alpha\ dt$ is the expected change in S_t during dt when there is no jump; $\sigma^2 S_t^2$ is the instantaneous variance of S_t conditional on no jump; *dz* is an increment of a standard Wiener process (Dixit and Pindyck); and *dq* is a discrete increment in S_t due to a jump process with arrival rate λ. We assume that *dq* and *dz* are independent. If a jump occurs at T, define the random variable Y_T by $Y_T = \frac{S_{T+}}{S_{T-}}$, where $S_{T+} = \lim_{t \to T,\ t>T} S_t$ and $S_{T-} = \lim_{t \to T,\ t<T} S_t$; Y_T -1 is thus the percentage change in S_T due to a jump, and *dq* equals Y_T -1 if a jump occurs and 0 otherwise. Following Ball and Torous (1983, 1985), we assume that the arrival of jumps follows a Bernoulli

distribution and that jump sizes are i.i.d. lognormal with $\ln(Y_t)$~$N(\theta, \delta^2)$. In discrete time, this model can be written:

(3) $$s_t - s_{t-1} = \mu + \sigma\varepsilon_t + \ln(Y_t)n_t$$

where n_t is the Bernoulli variate which equals one when a jump occurs (with probability λ) in the interval $[t-1, t]$ and zero otherwise; $ln(Y_t)$ is the logarithm of the jump size; and ε_t is a standard normal deviate.

An alternative to the GBM with jumps is a GBM-ARCH(1) process with or without jumps. Using the notation above, the jump-ARCH model can be written:

(4) $$s_t - s_{t-1} = m + \sqrt{h_t}e_t + \ln(Y_t)n_t$$

where the conditional variance, h_t, is a deterministic function of the last squared innovation conditional on information at t-1:

(5) $$h_t = \alpha_0 + \alpha_1(s_{t-1} - s_{t-2} - \mu)^2 .$$

We estimate the parameters of these models by numerical maximization of the corresponding log-likelihood functions, noted $L(\vec{\Theta}, \vec{x})$, where $\vec{x}$ is the vector of observations on $s_t - s_{t-1}$ and $\vec{\Theta}$ is the vector of parameters to estimate. Expressions of the likelihood functions are given in Appendix A. For further reference, $\hat{\mu}_{GBM}$, $\hat{\sigma}^2_{GBM}$ and $\hat{\mu}_{ARCH}$, $\hat{\alpha}_0$, $\hat{\alpha}_1$ denote the no-jump maximum likelihood estimates (MLE) of the GBM and ARCH models respectively.

To provide evidence on ARCH/JUMP effects, we conduct a series of LR-based tests, following Jorion (1989) and Akgiray and Booth (1988) (among others) on "nesting" competing models. For ease of exposition, we introduce the notation:

$$(6) \qquad H_{ij} : \begin{cases} \alpha_1 & = 0, \quad if \quad i = 0 \\ & > 0, \quad if \quad i = 1 \\ \lambda & = 0, \quad if \quad j = 0 \\ & > 0, \quad if \quad j = 1 \end{cases} .$$

Conformably, we denote by $LR\begin{pmatrix} H_{ij} \\ H_{kl} \end{pmatrix}$ the statistic for testing H_{ij} (the null hypothesis) against H_{kl}, which is obtained from:

$$(7) \qquad LR\begin{pmatrix} H_{ij} \\ H_{kl} \end{pmatrix} = 2[\hat{L}_{H_{kl}} - \hat{L}_{Hij}],$$

$\hat{L}_{H_{ij}}$ and $\hat{L}_{H_{kl}}$ are respectively the maximum of the likelihood function under the null and the alternative hypothesis.

We perform the following tests: $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$, $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$and $LR\begin{pmatrix} H_{00} \\ H_{01} \end{pmatrix}$, *i.e.* the no-jump test in ARCH contexts, the no-ARCH test in the JUMP-ARCH framework and the no-jump test in the GBM model. Furthermore, we apply, in the GBM model, the Engle no-ARCH test (see Appendix B), which we denote by $LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix}$. The LM statistic is $\overset{asy}{\sim} \chi^2(1)$. However, it is well known that this test tends to under-reject if the latter critical point is used (e.g., see Bernard, Dufour, Khalaf and Genest 1999). It is also well known that the no-jump tests - specifically $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$and $LR\begin{pmatrix} H_{00} \\ H_{01} \end{pmatrix}$- may suffer from fundamentally more serious problems. As observed in Brorsen and Yang (1994), when the no-jump hypothesis is imposed, λ lies on the boundary of the parameter space and the nuisance parameter δ is not identified. Therefore, the standard χ^2

approximation to the null distribution of LR does not obtain, even asymptotically. Indeed, the statistic's limiting null distribution is non-standard and quite complicated (see Davies 1977, 1987, or Hansen 1996). Although this problem is well recognized in the econometric literature, χ^2-based critical points are often (inappropriately) used in empirical applications of the latter test. In the $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$ no-ARCH test case, the nuisance parameter λ and δ are "estimable" under the null and the alternative hypothesis, if the identifying restriction $\lambda > 0$ is imposed. Although this justifies the use of standard asymptotic cut-off points, it may also conceal important distributional problems. Indeed, the relevant nuisance parameter space includes a LAU region as λ approaches the zero boundary. As demonstrated in Dufour (1997), the distributions of test statistics are strongly affected by the presence of LAU nuisance parameters even if identifying restrictions are imposed, which implies that irrespective of sample size, severe size distortions may occur with standard critical points.[1] As a result, the question, for all tests considered, is how best to approximate the statistics' finite sample distribution under the null hypotheses.

In this paper, we use the MC test technique (Dufour 1995) to obtain improved p-values from a finite sample perspective and circumvent the unidentified nuisance parameter problem. When the test's null distribution can be simulated, the MC technique, which is highly related to the parametric bootstrap, provides a randomized version of the test that controls its size. The MC test technique may be implemented as follows (in the case of a right tailed test).

1. Using the observed sample, calculate the relevant test criterion, denoted by $STAT_0$.

2. Using draws from the null data generating process (DGP), generate N simulated samples. To obtain draws from the null DGP, it is usual practice to consider consistent constrained estimates of the intervening parameters (i.e. derived imposing the null in question). Here we use constrained MLEs: for $LR\begin{pmatrix} H_{00} \\ H_{01} \end{pmatrix}$ and $LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix}$, we draw from the $N(\hat{\mu}_{GBM}, \hat{\sigma}^2)$ distribution, and for $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$, we draw from an $ARCH(\hat{\mu}_{ARCH}, \hat{\alpha}_0, \hat{\alpha}_1)$ process.

3. For each simulated sample, compute the associated test statistic, which yields N replications of the statistic (imposing the null) $STAT_1$, … , $STAT_N$.

4. Obtain the rank $\hat{R}_N(STAT_0)$ of the observed statistic $STAT_0$ in the series $STAT_0$, $STAT_1$, … , $STAT_N$.

5. Reject the null hypothesis at level α if $\hat{R}_N(STAT_0) \geq (N+1)(1-\alpha)+1$.

   A MC p-value may then be obtained from: $\hat{p}_N(STAT_0) = 1 - \dfrac{\hat{R}_N(STAT_0) - 1}{N+1}$.

The validity of this procedure and how it solves the unidentified nuisance parameter problem is demonstrated in Khalaf, Saphores, and Bilodeau (2000). The key argument is that the MC test technique gives finite sample exact p-values if the simulated statistic is *pivotal* i.e. if its null distribution does not depend on unknown parameters. It can be easily seen that the MC p-value calculated as described above, drawing from no-jump processes, does not depend on δ and θ . This follows immediately from the implications of unidentification. With regards to $LR\begin{pmatrix} H_{00} \\ H_{01} \end{pmatrix}$, the invariance to location and scale (μ and σ) is straightforward. Because the statistic is pivotal, the MC test procedure yields exact p-values. The same arguments hold for the no-ARCH LM test. In the case of $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$, however, the ARCH parameters (α_0 and α_1) still intervene as (identified) nuisance parameters. The results of Dufour (1995) imply that the MC test we use here is only appropriate asymptotically (as $T \to \infty$), but it is important to remember that there is no tractable asymptotic theory for this test.[2] Below, we describe an exact bounds procedure which serves to assess whether the latter test rejections are conclusive (i.e. are not spurious).

For the $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$ test, the MC p-value drawing from a Jump-GBM null process is asymptotically valid, as argued above. Yet in the presence of the LAU nuisance parameter λ, rejections may not be reliable from a finite sample perspective. Note, however, that a non-significant MC test outcome may be considered reliable in this case. Indeed, Dufour (1995) argues that a non-rejection, in the context of a MC test conditional on a specific choice for the intervening parameters (which we denote a *local* MC (LMC) test), may be interpreted in an exact sense, so that non-rejections are compelling.[3] To obtain an overall reliable test, we apply the bounds MC procedure outlined in Khalaf, Saphores and Bilodeau (2000): we construct a MC p-value based on a conservative bounds criterion to obtain conclusive rejections. To understand the rationale underlying bounds tests, we first note the statistic at hand $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$ is nuisance parameter dependent yet *boundedly pivotal* i.e. we can come up with a pivotal statistic that is always $\geq LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$.[4] It is easy to see that $LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix}$ serves this purpose. On the one hand, it is pivotal because of location-scale invariance (the null sets α_1 and λ to zero, and δ does not intervene when $\lambda = 0$). On the other hand, since $H_{00} \subseteq H_{01}$, $LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix} \geq LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$. Now if we use the cut-off points associated with the bounding statistic, which (by construction) are always greater than or equal to the (unknown) cut-off point of $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$, we can be sure that rejections are conclusive.[5] It must be remembered that the bounding cut-off point must be obtained by simulation in this case since, as emphasized above, the null distribution of $LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix}$ is non-standard. Formally, the MC procedure outlined above should be modified as follows in the case of the MC bounds-test:

- Step (2): draws are generated from the "bounding" null DGP. i.e. imposing H_{00}.
- Step (3): from each simulated sample, the "bounding" statistic $LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix}$ is computed.

Note that step (1) is unaltered so that $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$ is still computed from the observed data. Note also that the same bounding statistic may be used in the context of a bounds procedure based on $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$. We have expressed above some concern about the possibility of non-conclusive rejections if the asymptotic LMC p-value is used in this case. Although the procedure is valid since the intervening nuisance parameters are identified, its reliability in finite samples is yet to be established. As emphasized above, non-rejections are not a problem here, so we focus on interpreting rejections using a bounds MC p-value: we apply the above bounds procedure using the observed $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$ in step 1; if the bounds p-value is significant, then the (exact) jump-test is significant in the presence of ARCH effects. With such a conservative approach, we can be sure that the GBM model is not spuriously rejected, even with our small sample sizes.

We also conduct diagnostic tests that are commonly used in the context of tests of random-walks: (i) the Jarque-Bera (skewness and kurtosis) tests, (ii) the Lo and MacKinlay variance ratio tests and (iii) Perron's (1989, 1993) unit root test. We use Perron's test instead of more "standard" tests like the Dickey-Fuller unit root test to account for a 1984 definition change that possibly creates a structural break in our data (see Section III.1 below). Overall, these tests consider deviations from GBM behavior of transforms of price data. The Jarque-Bera test checks for excess skewness and/or kurtosis. An important property of the random walk hypothesis is that the variance of random walk increments is linear in the sampling intervals. The variance ratio tests assess this property. For more details, see Appendix B or Campbell, Lo, and MacKinlay (1997, Chapter 2).

3. EMPIRICAL RESULTS

3.1 Stumpage Data

Quarterly stumpage prices for “Douglas Fir,” “Ponderosa and Jeffrey Pine,” “Western Hemlock,” and “True Fir” were obtained from the USDA Pacific Northwest Research Station, in Portland, Oregon. Over the last 23 years, the first two series have represented, on average, 42% and 16% respectively by volume of the timber harvested in National Forests in the Pacific Northwest while the shares of the last two species have varied substantially. Hence, the share of Western Hemlock has fallen to a few percents while the share of True Fir has increased from approximately 4% to between 13 and 15%. In the meantime, the total volume harvested has dropped from between 3 to 5 billion board feet annually to a few hundred million board feet, possibly as a result of the movement for the preservation of old growth forest.

Two types of prices are available from the National Forest service: “sold-stumpage price” and “cut-timber price”. The quarterly “sold-stumpage price” is a three months average of high-bid prices for the right to harvest timber from National Forests in Oregon and Washington. It represents current expectations of future prices of timber intended for future harvest. Since 1984, the “sold” price has included estimated purchaser road credit for road construction (Haynes and Warren, 1989). An alternative measure of stumpage prices is the so-called “cut price”, which is the high-bid price adjusted for rates actually paid for timber, when the logs are scaled after harvest. The “cut-price” thus represents the current value of harvested timber in the marketplace.

Many analysts feel that the “sold-price” is a better measure of the worth of timber. However, this “sold-price” is available only as an average for all species, whereas quarterly time series for “cut-price” are available from the Forest Service for various tree species, starting in 1973. For this study, we thus use quarterly “cut” stumpage prices from 1973 to the first quarter of 1997 deflated with wholesale price deflators from the Bureau of Economic Analysis. Figures 1 to 4 show graphs of the deflated stumpage prices (“cut-prices”) for the series analyzed.

In addition, Sohngen and Haynes (1994) report seasonal variations in stumpage prices: fall and winter stumpage prices tend to be higher, while summer stumpage prices tend to be lower than the yearly average. They explain this phenomenon by the increased difficulty of logging during the winter months and the cost of storing logs that could be harvested during the summer months. To account for these seasonal effects and the 1984 change of definition for “sold prices”, we follow the suggestion of follow Davidson and MacKinnon (1993, chapter 19). We thus perform a preliminary maximum likelihood estimation for all *maintained* models using a definition change dummy (one after the first quarter of 1984 and zero otherwise) and three seasonal dummies (recall that the series in question are in log-differences):

$$\text{Season1} = (1\ 0\ 0\ \text{-}1 \ldots 0\ \text{-}1)^t,$$
$$\text{Season2} = (0\ 1\ 0\ \text{-}1 \ldots 0\ \text{-}1)^t,$$
$$\text{Season3} = (0\ 0\ 1\ \text{-}1 \ldots 1\ \text{-}1)^t.$$

The residuals from the preliminary MLE yield a seasonally-adjusted series to which we fit our models. This is numerically equivalent to adding the above dummies to the models analyzed.

3.2 Results

Table 1 summarizes the diagnostic statistics for the stumpage price time series we analyze. First, we observe that in all cases (except for Western Hemlock) Perron’s test fail to reject the presence of a unit root at the 5% level.[6] Moreover, the skewness and kurtosis tests detect significant fat tails effects. The variance ratio rejects the random walk null for the Western Hemlock and the True Fir series.[7] As is well known, caution must be exercised in interpreting decisions from a battery of diagnostic tests. Yet on the whole, we can conclude that the GBM hypothesis seems to be soundly rejected and proceed by testing the ARCH-Jump hypotheses.

The parameters estimated by fitting a GBM, a jump-GBM, an ARCH, and a jump-ARCH to the four series considered are shown in Table 2. They were obtained by numerical maximization of the corresponding log-likelihood functions (see Appendix A), using GAUSS. Since it is well known (Ball and Torous, 1983 and 1985) that these functions may have more than one maximum, several starting points were considered each time. Table 2 also includes the statistics for the four tests described above, and exact MC p-values (based on bounds when necessary as explained above).

Looking first at Engle's LM test (with statistic $LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix}$), we find ARCH(1) effects over the GBM significant at the 2% level for all four series. The data do, however, also support jumps over a simple GBM (statistic $LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix}$), except for Ponderosa and Jeffrey Pine: we find significant jumps at the 1% level for Douglas Fir and Western Hemlock and at 6% for True Fir. As mentioned above, both ARCH effects and jumps could produce the "fat-tail" observed in the data. Both tests in question here were conducted exactly, so we are sure that rejections are statistically sound.

To sort out the contributions of ARCH and jumps, we test for ARCH effects in the presence of jumps (statistic $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$) and for the presence of jumps over ARCH(1) effects (statistic $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$). We find that we cannot exclude the presence of ARCH(1) over a Jump-GBM model, at the 1% level, for Ponderosa and Jeffrey Pine and at 2% for Western Hemlock. Moreover, we cannot exclude the presence of jumps in an ARCH(1) model for Douglas Fir (at 1%) and for Ponderosa and Jeffrey Pine (at 8%). Recall that our testing procedure in connection with both tests here is bounds-based. We have derived an exact nuisance-parameter-free bounds cut-off point (similar, in spirit, to the Durbin-Watson bound), so if a test is significant when referred to the bound, then it is certainly significant. Our rejections are thus statistically conclusive.

Given that bounds procedures may be thought as conservative, with negative implications on tests power, we also have re-examine the observed non-rejections in the case of $LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$ and $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$. As argued above, a non-significant parametric bootstrap test is reliable in this context. All (relevant) bootstrap p-values are consistent with the results reported in table 2.[8]

Several further points are worth noting here. First, observe that the LM and the LR tests for ARCH yield different results in the case of Douglas Fir: the former is significant whereas the latter fails to reject the no-ARCH null in the presence of jumps. Of course, the LM test does not take jumps into consideration. Yet there seems to be evidence in favor of jumps (imposing and ignoring ARCH). Alternatively, if ARCH effects - which seem present in the data - are not accounted for, jumps are not detected in the case of Ponderosa and Jeffrey Pine and are falsely detected in the case of Western Hemlock.

The True Firs case illustrates the fact that the pure-jump and pure- ARCH models may not be discernable in a small data set. Non-nested tests (needless to say that a reliable procedure given the inference problems documented above is not available for such problems) may be performed if a decision is required, yet the purpose of our study is not to discern between these two competing models. Instead, our goal is to test the GBM null against both jump and/or ARCH alternatives.

The frequency of jumps varies widely between Douglas Fir and True Firs: for the former, λ is approximately 0.2 (1 jump every 5 quarter), and for the later λ is around 0.5 (1 jump every 2 quarters). We also observe that allowing for jumps substantially decreases σ^2 estimated for a pure GBM as the volatility is redistributed between the diffusion and the jump components.

The *precision* of the parameter estimates is, of course, also effected by the small size of our data sets. We would like, however, to again warn readers against asymptotic standard errors and related t-tests when assessing the *precision* of estimates, even though we report them. Indeed, the theoretical literature cited above shows that it is impossible to control the size of these tests in the context of the models considered. This is why we use LR tests and strive to control the size (or at least the level) of all tests conducted.

4. IMPLICATIONS FOR STOPPING PROBLEMS

From results reported in the finance literature (e.g., see Ball and Torous, 1983, 1985; Jorion, 1988; Stein and Stein, 1991; Vlaar and Palm, 1993; Brorsen and Yang, 1994; or Bates, 1996), we anticipate that jumps and ARCH effects in the price of a natural resource can have important consequences on its management or on the valuation of portfolios relying on this resource. In the case of stumpage prices, these findings could, for example, affect the value and the investment strategy of a paper making firm, and impact the decision to cut or to preserve a public forest.

Investigating the consequences of ARCH effects and/or jumps in the price of an asset is, however, a difficult task (look again at the finance papers referenced above). Moreover, we cannot simply use results obtained in finance because the time horizon in natural resources problems tends to be much longer, often years instead of just a few months in finance. In this section, we thus focus on the impacts of neglecting jumps, when they are statistically significant, in the context of an infinite horizon, continuous time stochastic dynamic model based on the GBM. We leave the investigation of the impact of ARCH effects in this framework for further work because it would require analyzing two factor models (e.g. see Drost and Werker, 1996).

For illustrative purposes, we thus revisit the classic problem of the optimal timing of cutting a stand of old growth forest (e.g., see Clarke and Reed, 1989; Reed, 1993; Conrad and Ludwig, 1994). This stand generates a constant amenity A per unit area per time period, net of maintenance costs. Cutting this stand at time t would provide net revenues S_t from timber sales, where S_t varies stochastically with time. For simplicity, we assume that timber volume is constant, and that the value of bare land, L_t, can be neglected. We denote by r the discount rate.

We consider two models: in the first one, S_t follows a GBM; in the second model, it follows a GBM with jumps. For each model, we want to find S^*, which separates the values of S_t where the stand should be preserved (the continuation region, for "low" values of S_t), from the values of S_t where the trees should be cut (the stopping region, for "high" values of S_t). We thus have two infinite horizon, autonomous, optimal stopping problems, which we solve using a real options approach (for an introduction, see Dixit and Pindyck, 1994). While the case where S_t follows a GBM is easily solved analytically, the addition of jumps leads to a Bellman equation that is difficult to solve, even numerically. To tackle this problem, we use Galerkin's method, which, to our knowledge, has not been considered before in the economics literature.

First, let us suppose that S_t follows a GBM as given in Equation (1) (e.g., see Reed 1993). The Bellman equation in the continuation region is the second order differential equation:

$$(8) \qquad rV(S_t) = A + \alpha\, S_t V'(S_t) + \frac{\sigma^2}{2} S_t^2 V''(S_t)$$

The left side of this equation is the normal return expected by a social planner to hold the asset represented by the stand of old-growth forest. On the right side, A is the flow of forest amenities, assumed constant, and the other terms are the equivalent of expected capital gains. We write $V(S_t)$ as the sum of a particular solution, $V_P(S_t)$, which corresponds to the case where the trees are never cut:

$$(9) \qquad V_P(S_t) = \frac{A}{r}$$

and an option term, $\varphi(S_t)$, which is a solution to the homogeneous equation associated to (8). Since $\varphi(S_t)$ represents the value of the option to cut, it should be well defined at S_t=0 and increasing in S_t. Hence:

(10) $$\varphi(S_t) = V_0 S_t^{\rho}$$

V_0 is a constant to be determined jointly with S^*, and ρ is the positive root of:

(11) $$\frac{\sigma^2}{2} z^2 + (\alpha - \frac{\sigma^2}{2}) z - r = 0$$

A simple calculation shows that $\rho>1$ when $r>\alpha$, which is required if the stand is to be cut.

In the stopping region, $V(S_t)$ equals the net value of timber, since land value is neglected:

(12) $$V(S_t) = S_t$$

To find S^*, we apply the continuity and smooth-pasting conditions, which require V and its first derivative to be smooth across the stopping frontier (Dixit and Pindyck, 1994). We get:

(13) $$S^* = \frac{\rho}{\rho - 1} \frac{A}{r}$$

Let us now assume, instead, that net stumpage price follows a geometric Brownian motion with jumps, as given by Equation (2). For analytical convenience, we suppose that the arrival of jumps follows a Poisson process, which is a slight generalization of the Bernoulli process considered in Sections II and III (the Bernoulli process can be seen as a truncated Poisson process). Using Itô's lemma for mixed processes, it can be shown (Merton, 1976) that the Bellman equation in the continuation region is the linear integro-differential equation:

(14) $$rV(S_t) = A + \alpha\ S_t \frac{dV(S_t)}{dS_t} + \frac{\sigma^2}{2} S_t^2 \frac{d^2V(S_t)}{dS_t^2} + \lambda\ \varepsilon_{Y_t} \{V(S_t Y_t) - V(S_t)\}$$

The interpretation of Equation (14) is similar to that of Equation (8), and the random variable Y_t-1 gives the percentage change in S_t due to a jump at t. We again write $V(S_t)$ as the sum of a particular solution, $V_P(S_t)$ given by (9), and an option term, $\varphi(S_t)$, which represents the value of the option to cut. It verifies:

(15) $$r\varphi(S_t) = \alpha\ S_t \frac{d\varphi(S_t)}{dS} + \frac{\sigma^2}{2} S_t^2 \frac{d^2\varphi(S_t)}{dS_t^2} + \lambda\ \varepsilon_{Y_t} \{\varphi(S_t Y_t) - \varphi(S_t)\}$$

From (15), we see that if S_t ever becomes 0, it is zero forever so $\varphi(0)=0$. Moreover, in the stopping region, $V(S_t)$ equals S_t, so:

(16) $$\varphi(S_t) = S_t - \frac{A}{r}$$

Finally, from the continuity and smooth-pasting conditions, we have that:

(17) $$\varphi(S^*) = S^* - \frac{A}{r}, \text{ and } \frac{d\varphi}{dS}(S^*) = 1$$

We solve Equations (14) to (17) numerically using Galerkin's method for integro-differential equations (Delves and Mohamed, 1985). See Appendix C for details.

We then apply the two models described above to Douglas Fir and to True Fir, since we found significant jumps but no ARCH(1) effects over jumps in these two time series. Since our purpose is purely illustrative, we assume that amenity is unity in coherent units.

For both the GBM and the jump-GBM models, a necessary condition to have a finite stopping value S^* is that r be larger than the instantaneous expected rate of growth of stumpage price. For the GBM, the instantaneous expected rate of growth of stumpage price is the infinitesimal drift, given by α in Equation (1). For the jump-GBM model, we also need to take into account the contribution of the jumps, so the instantaneous expected rate of growth of stumpage price is (Merton, 1976):

$$(18) \qquad \varepsilon(\frac{dS_t}{S_t}) = \alpha + \lambda(e^{\theta + \delta^2/2} - 1)$$

Using parameter values reported in Table 2, we find that the annual infinitesimal trends are 24.2% and 40.0% respectively for Douglas Fir and True Firs for a pure GBM, while for the jump-GBM, the infinitesimal trends are 25.5% and 37.5% respectively. These high values may partly be explained by all the events that affected logging in the Pacific Northwest over the last decade or so. They force us to adopt high discount rates to make cutting worthwhile. A more realistic model should, for example, take into account the risk of destruction of old growth forest by fire or diseases, which would lower somewhat infinitesimal trends.

In Table 3, we report a few stopping values for each model as well as the relative change in stopping value ($S^*_{jump\text{-}GBM}$ - S^*_{GBM}) / $S^*_{jump\text{-}GBM}$) for different values of the social discount rate. From Clarke and Reed (1989), we know that the decision to cut a stand of old growth forest depends on the ratio of stumpage price to amenity value when land and existence values are small, which we assume. The relative change in S^* thus eliminates the impact of A. From the last column of Table 3, we see that neglecting jumps can either lead to cutting too early, in the case of Douglas Fir, or too late, in the case of Ponderosa and Jeffrey Pines. Moreover, the relative difference in stopping values between the GBM and the jump-GBM models decreases when the discount rate increases. To explain this difference between Douglas Fir and True Firs, recall that jumps are multiplicative and have a lognormal distribution so their expected value is $e^{\theta + 0.5\ \delta^2}$. From this expression, we find that on average jumps tend to increase S_t for Douglas Fir ($e^{\theta + 0.5\ \delta^2} = 1.17 > 1$) whereas they tend to decrease S for True Firs ($e^{\theta + 0.5\ \delta^2} = 0.90 < 1$). Waiting more thus increases expected net harvest value for Douglas Fir but decreases it for True Firs. Similar results are reported in the finance literature (e.g., see Jorion 1988).

5. CONCLUSIONS

In this paper, we reconsider the representation of natural resource prices by continuous processes by allowing for the presence of jumps, which can be due to the arrival of discrete events that cause large price changes. We also allow for ARCH effects, which have also been found to generate price increments with more extreme values than the normal distribution (fat tails). These models have been extensively used in finance but they do not seem to have been considered in natural resources economics.

Our contribution is two-fold: first, we present a methodology based on the bootstrap to obtain finite sample exact p-values when testing for jumps or ARCH effects using LR tests to avoid relying on asymptotic results. This is particularly useful in natural resources because sample sizes are often small (at least compared to finance). Second, we present a fairly easy way to solve autonomous, infinite horizon stopping problems with jump-diffusions based on Galerkin's method for integro-differential equations.

To illustrate this methodology, we analyze four quarterly time series of stumpage prices from Pacific Northwest National Forests. Controlling for seasonal variations, we find evidence of the presence of jumps in two of the series considered, while the other two exhibit ARCH effects. We then show that ignoring jumps, when they are indeed present, may lead to significantly suboptimal decisions to harvest old-growth timber.

REFERENCES

1. C. Ball and W. Torous, “A simplified Jump Process for Common Stock Returns”, *Journal of Financial and Quantitative Analysis* 18, 53-61 (1983).

2. C. Ball and W. Torous, “On Jumps in Common Stock Prices and Their Impact on Call Option Pricing”, *The Journal of Finance* 40, 155-173, (1985).

3. D. S. Bates, “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options”, *The Review of Financial Studies* 9, 69-107, (1996).

4. J.-T. Bernard, J.-M. Dufour, L. Khalaf and I. Genest, “MC homoscedastictity tests”, *Working Paper*, CRDE - *Université de Montréal*, GREEN - *Université Laval*, (1999).

5. F. Black and M. Scholes, “The Pricing of Options and Corporate Liabilities”, *The Journal of Political Economy* 81, 637-659, (1973).

6. M. J. Brennan and E. S. Schwartz, “Evaluating Natural Resource Investments”, *Journal of Business* 58, 135-157, (1985).

7. B. W. Brorsen and S. Yang, “Nonlinear Dynamics and the Distribution of Daily Stock Index Returns”, *The Journal of Financial Research* 17, 187-203, (1994).

8. J. Y. Campbell, A. W. Lo and A. C. MacKinlay, *The Econometrics of Financial Markets*, Princeton University Press, Princeton, New Jersey, (1992).

9. H. R. Clarke and W. J. Reed, “Land Development and Wilderness Conservation Policies under Uncertainty: A Synthesis”, *Natural Resource Modeling* 4, 11-37, (1990).

10. H. R. Clarke and W. J. Reed, “The Tree-cutting Problem in a Stochastic Environment”, *Journal of Economics Dynamics and Control* 13, 569-595, (1989).

11. J. M. Conrad and D. Ludwig, “Forest Land Policy: the Optimal Stock of Old-Growth Forest”, *Natural Resource Modeling* 8, 27-45, (1994).

12. R. B. D'Agostino and M. I. A. Stephens, *Goodness of fit techniques*, New York: Marcel Dekker, (1986).

13. R. Davidson and J.G. MacKinnon, *Estimation and Inference in Econometrics*, New York: Oxford University Press, (1993).

14. L. M. Delves and J. L. Mohamed, *Computational Methods for Integral Equations*, Cambridge, U.K.: Cambridge University Press, (1985).

15. A. Dixit, "Investment and Hysteresis", *The Journal of Economic Perspectives* 6, 107-132, (1992).

16. A. K. Dixit and R. S. Pindyck, *Investment under Uncertainty*, Princeton, New Jersey: Princeton University Press, (1994).

17. J.-M. Dufour, "Monte Carlo Tests in the Presence of Nuisance Parameters with Econometric Applications", *Working Paper*, CRDE - *Département de Sciences Économiques, Université de Montréal*, (1995).

18. J.-M. Dufour, "Some Impossibility Theorems in Econometrics with Applications to Structural and Dynamic Models", *Econometrica*, 65, 1365-1387, (1997).

19. J.-M. Dufour, A. Farhat, G. Gardiol and L. Khalaf, "Simulation-based Finite Sample Normality Tests in Regression Models", *The Econometrics Journal* 1, 154-173, (1998).

20. J.-M. Dufour and L. Khalaf, "Simulation-based Finite and Large Sample Inference Methods in Multivariate Regressions and Seemingly Unrelated Regressions", *Cahier* 309, CRDE - *Université de Montréal*, (1997).

21. R. Engle, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica* 50, 987-1007, (1982).

22. D. Friedman and S. Vandersteel, "Short-Run Fluctuations in Foreign Exchange Rates: Evidence from the Data 1973-79", *Journal of International Economics* 13, 171-186, (1982).

23. S. Hamori and A. Tokihisa, "Testing For a Unit Root in the Presence of a variance Shift", *Economic Letters* 57, 245-253, (1997).

24. R. W. Haynes and D. D. Warren, "Volume and Average Prices of Stumpage Harvested from the National Forests of the Pacific Northwest Region, 1973-1987", *U.S.D.A. Forest Service, Pacific Northwest Research Station*, (1989).

25. P. Jorion, "On Jump Processes in the Foreign Exchange and Stock Markets", *The Review of Financial Studies* 1, 427-445, (1988).

26. L. Khalaf, J.-D. Saphores and J.-F. Bilodeau, “Simulation Based Exact Tests for Structural Discontinuities with Unidentified Nuisance Parameters: An Application to Commodities Spot Prices”, *Working Paper*, GREEN - *Université Laval*, (2000).

27. K. Kim and P. Schmidt, “Unit root tests with conditional heteroscedasticity”, *Journal of Econometrics* 59, 287-300, (1993).

28. A. W. Lo and A. C. MacKinlay, “Stock Market Prices do not Follow Random Walks: Evidence from a Simple Specification Test”, *The Review of Financial Studies* 1, 41-66, (1988).

29. R. MacDonald and D. Siegel, “The Value of Waiting to Invest”, *The Quarterly Journal of Economics* 101, 707-728, (1986).

30. R. C. Merton, *Continuous-Time Finance*, Paperback ed. Cambridge, Massachusetts, and Oxford, UK: Blackwell Publishers Ltd, (1992).

31. R. C. Merton, “On the Mathematics and Economics Assumptions of Continuous-Time Models”, *Financial Economics: Essays in Honor of Paul Cootner. Ed. W.F. and C.M. Cootner Sharpe*, Englewood Cliffs, NJ: Prentice-Hall, (1982).

32. R. C. Merton, “Option Pricing when Underlying Stock Returns are Discontinuous”, *Journal of Financial Economics* 3, 125-144, (1976).

33. R. E. Morck, E. Schwartz and D. Stangeland, “The Valuation of Forestry Resources under Stochastic Prices and Inventories”, *Journal of Financial and Quantitative Analysis* 24, 473-487, (1989).

34. J. L. Paddock, D. R. Siegel and J. L. Smith, “Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases”, *The Quarterly Journal of Economics* 103, 479-508, (1988).

35. E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Cambridge, U.K.: Biometrika Trustees at the University Press, (1970).

36. P. Perron, “The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis”, *Econometrica* 57, 1361-1401, (1989).

37. P. Perron, “The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis: ERRATUM”, *Econometrica* 61, 248-249, (1993).

38. R. S. Pindyck, “Irreversible Investment, Capacity Choice, and the Value of the Firm”, *The American Economic Review* 79, 969-985, (1988).

39. R. S. Pindyck and D. L. Rubinfeld, *Econometric Models and Economic Forecasts*, New York, NY: McGraw-Hill (1991).

40. W. J. Reed, "The Decision to Conserve or Harvest Old-Growth Forest", *Ecological Economics* 8, 45-69, (1993).

41. J.-D. Saphores, "Option Value and the Timing of Environmental Policy - An Application to Pesticides, Pollutants, and Forestry," *Ph.D. dissertation, Cornell University*, Ithaca, NY, (1996).

42. E. S. Schwartz, "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging," *The Journal of Finance* 52, No. 3, 923-973, (1997).

43. G. D. Smith, *Numerical Solution of Partial Differential Equations: Finite Difference Methods*, Third ed: Oxford University Press (1985).

44. B. L. Sohngen and R. W. Haynes, "The Great Price Spike of '93: An Analysis of Lumber and Stumpage Prices in the Pacific Northwest", *Research paper PNW-RP-476, USDA*, (1994).

45. E. M. Stein and J. C. Stein, "Stock Price Distributions with Stochastic Volatility: An Analytic Approach", *The Review of Financial Studies* 4, 727-752, (1991).

46. P. J. G Vlaar and F. C. Palm, "The message in Weekly Exchange Rates in the European Monetary System: Mean Reversion, Conditional Heteroscedasticity, and Jumps", *The Journal of Business and Economic Statistics* 11, 351-360, (1993).

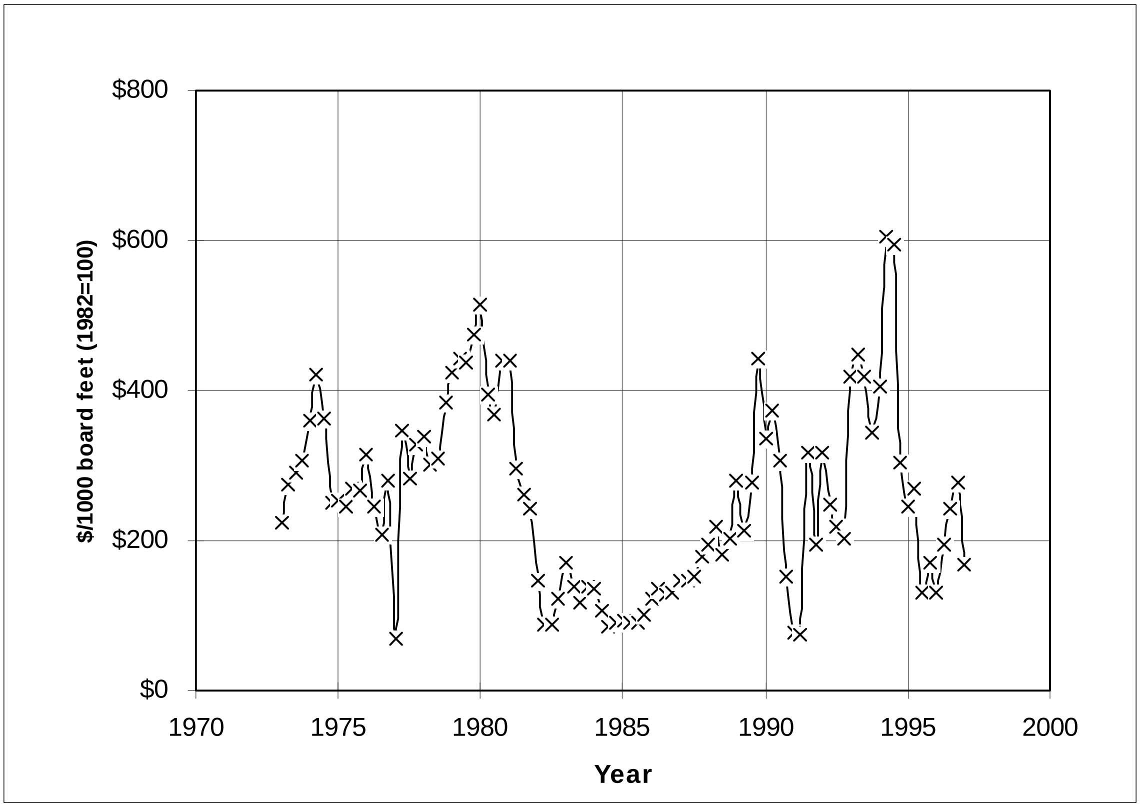

FIG. 1. Quarterly Real Stumpage Prices for Douglas Fir.

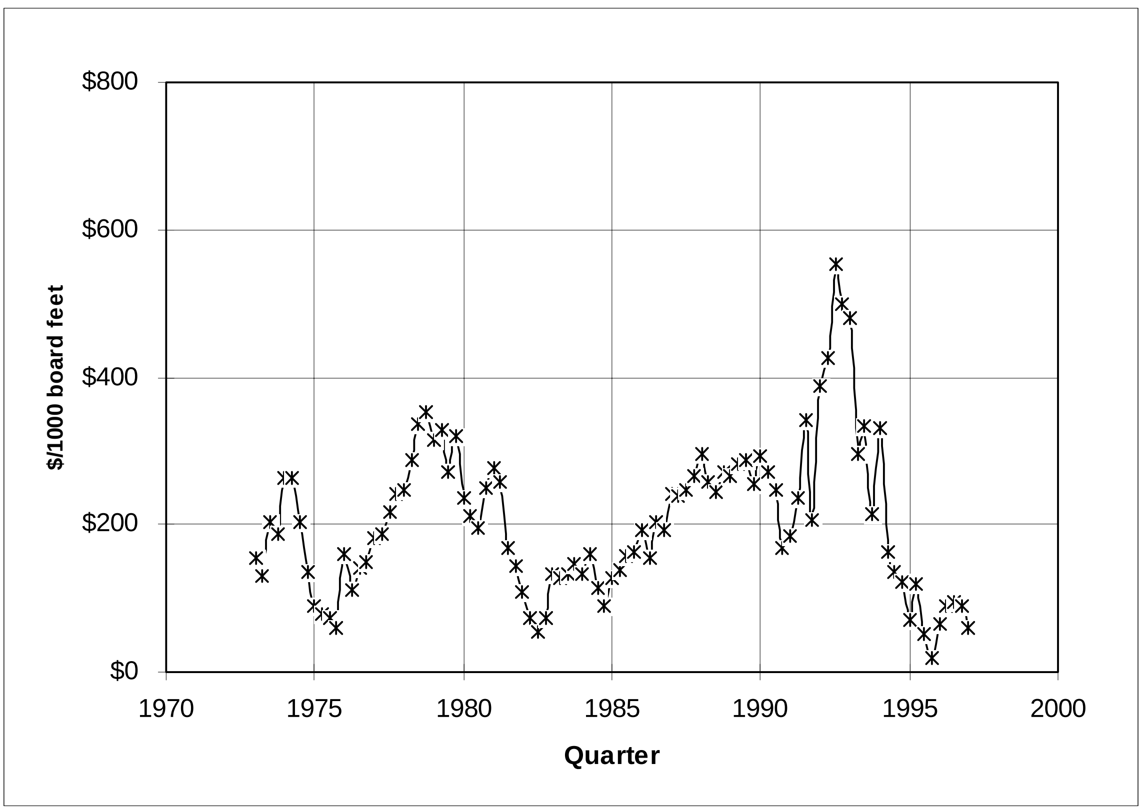

FIG. 2. Quarterly Real Stumpage Prices for Ponderosa and Jeffrey Pines.

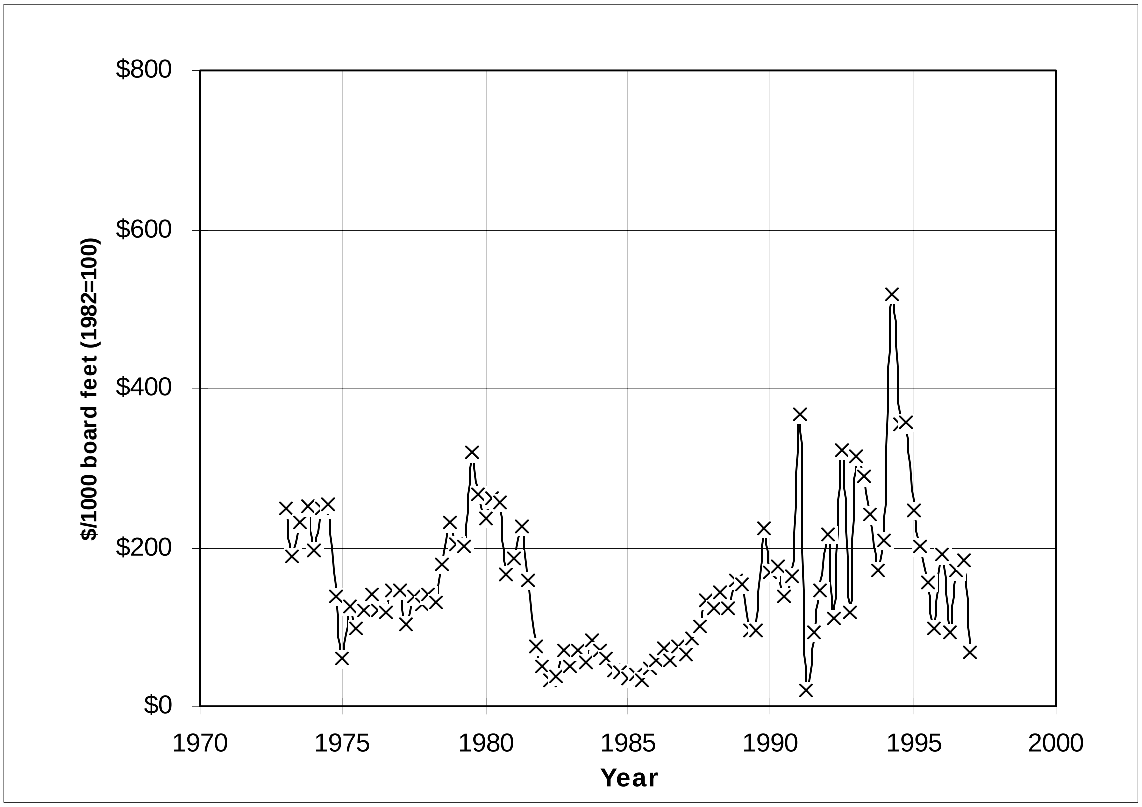

FIG. 3: Quarterly Real Stumpage Prices for Western Hemlock.

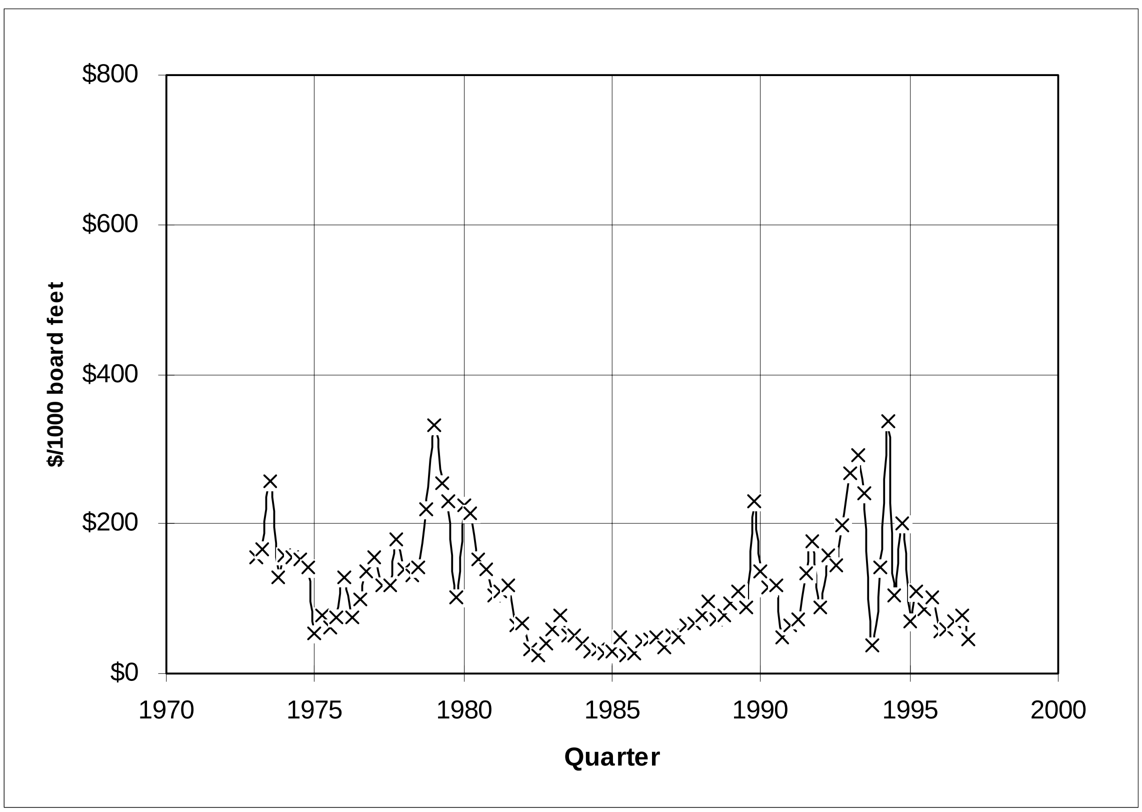

FIG. 4. Quarterly Real Stumpage Prices for True Firs.

TABLE 1
Summary Statistics[a]

Statistic	5% Critical Points	Douglas Fir	Ponderosa & Jeffrey Pine	Western Hemlock	True Firs
Skewness	±0.479	0.728	-0.006	-1.276	-0.684
Kurtosis	3.52	8.907	4.698	10.686	4.710
Jarques-Bera	5.99	148.052	11.527	262.357	19.195
Perron (1989)	-3.93[b]	-3.55	-2.44	-4.15	-3.79
Autocorrelations					
Lag 1		-0.255	-0.119	-0.420	-0.341
Lag 2		0.018	-0.007	0.049	-0.056
Lag 3		-0.105	0.058	-0.062	0.016
Lag 4		-0.040	-0.190	0.111	-0.184
Variance Ratios					
$Z^*(2)$	±1.96	-1.302	-0.793	-2.021	-2.126
$Z^*(4)$	±1.96	-1.232	-0.618	-1.814	-1.972

[a] Test statistics are presented in Appendix B. The sample size is 97.

[b] The corresponding Dickey-Fuller cut-off point is –3.41. In connection, see Perron (1998, page 1378).

TABLE 2

Maximum Likelihood parameters and related tests

	Continuous Process				Jump Process			Test Statistic (MC p-value)	
	μ	σ	α_0	α_1	λ	θ	δ		
Douglas Fir									
GBM	-0.012 (0.038)	0.368 (0.027)						$LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix}$	8.30 (0.02)
Jump/GBM	0.003 (0.029)	0.191 (0.032)			0.219 (0.106)	-0.068 (0.170)	0.670 (0.164)	$LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$	11.97 (0.16)
ARCH	-0.047 (0.033)		0.097 (0.017)	0.274 (0.154)				$LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix}$	34.56 (0.01)
Jump/ARCH	-0.014 (0.023)		0.023 (0.007)	0.258 (0.114)	0.199 (0.085)	-0.161 (0.163)	0.590 (0.141)	$LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$	31.31 (0.01)
Ponderosa And Jeffrey Pine									
GBM	-0.002 (0.032)	0.319 (0.023)						$LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix}$	24.10 (0.01)
Jump/GBM	-0.060 (0.042)	0.160 (0.061)			0.514 (0.241)	-0.113 (0.088)	0.376 (0.063)	$LR\begin{pmatrix} H_{01} \\ H_{11} \end{pmatrix}$	25.36 (0.01)
ARCH	0.051 (0.024)		0.044 (0.010)	0.614 (0.227)				$LR\begin{pmatrix} H_{00} \\ H_{11} \end{pmatrix}$	9.60 (0.14)
Jump/ARCH	0.116 (0.013)		0.002 (0.002)	0.724 (0.250)	0.630 (0.153)	-0.151 (0.048)	0.208 (0.037)	$LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$	12.71 (0.08)

(Continued next page.)

Western Hemlock

GBM	-0.026	0.516						$LM\begin{pmatrix}H_{00}\\H_{10}\end{pmatrix}$	8.34
	(0.053)	(0.037)							(0.02)
Jump/GBM	-0.005	0.407			0.042	-0.485	1.472	$LR\begin{pmatrix}H_{01}\\H_{11}\end{pmatrix}$	20.65
	(0.047)	(0.052)			(0.062)	(1.303)	(0.807)		(0.02)
ARCH	0.013		0.096	0.714				$LR\begin{pmatrix}H_{00}\\H_{11}\end{pmatrix}$	22.13
	(0.036)		(0.019)	(0.214)					(0.01)
Jump/ARCH	0.031		0.028	0.718	0.380	-0.018	0.415	$LR\begin{pmatrix}H_{10}\\H_{11}\end{pmatrix}$	4.92
	(0.035)		(0.017)	(0.230)	(0.223)	(0.115)	(0.113)		(0.67)

True Firs

GBM	-0.025	0.475						$LM\begin{pmatrix}H_{00}\\H_{10}\end{pmatrix}$	8.97
	(0.049)	(0.034)							(0.02)
Jump/GBM	0.106	0.228			0.503	-0.259	0.559	$LR\begin{pmatrix}H_{01}\\H_{11}\end{pmatrix}$	6.10
	(0.052)	(0.054)			(0.156)	(0.136)	(0.082)		(0.57)
ARCH	-0.040		0.185	0.171				$LR\begin{pmatrix}H_{00}\\H_{11}\end{pmatrix}$	12.06
	(0.044)		(0.032)	(0.108)					(0.06)
Jump/ARCH	0.105		0.060	0.123	0.368	-0.419	0.484	$LR\begin{pmatrix}H_{10}\\H_{11}\end{pmatrix}$	11.34
	(0.061)		(0.027)	(0.073)	(0.176)	(0.211)	(0.107)		(0.14)

Note: $LM\begin{pmatrix}H_{00}\\H_{10}\end{pmatrix}$ is the LM test for ARCH(1) effects over a GBM (see Appendix B). $LR\begin{pmatrix}H_{01}\\H_{11}\end{pmatrix}$ refers to the no-ARCH test in the Jump-GBM model; the statistics $LR\begin{pmatrix}H_{00}\\H_{11}\end{pmatrix}$ and $LR\begin{pmatrix}H_{10}\\H_{11}\end{pmatrix}$ test for jumps in the GBM and ARCH models respectively. All MC p-values reported are finite sample exact. For $LR\begin{pmatrix}H_{01}\\H_{11}\end{pmatrix}$ and $LR\begin{pmatrix}H_{10}\\H_{11}\end{pmatrix}$, we report the bounds Monte Carlo p-value. Refer to Section II for definitions and formulae. The MC tests are applied with N=99 replications.

TABLE 3

Comparison of Stopping Values[c]

Discount rate Annual	Quarterly	S* GBM	S* GBM w/jumps	Relative Change
Douglas Fir				
25.6%	5.9%	746	4617	83.8%
26.0%	5.9%	577	1526	62.2%
28.0%	6.4%	268	345	22.2%
30.0%	6.8%	173	192	9.9%
32.0%	7.2%	127	133	3.9%
True Firs				
39.0%	8.6%	- -	639	- -
42.0%	9.2%	592	219	-170.6%
44.0%	9.5%	292	151	-92.8%
46.0%	9.9%	193	115	-67.1%
50.0%	10.7%	114	78	-46.6%

[c] In this table, S* represents the net revenue from harvesting old growth forest on a unit area of land, when S_t follows a GBM or a Jump-GBM process. Results are obtained with parameter values shown in Table 2 above. We also assume that non-timber value per unit area is unity.

APPENDIX A: LIKELIHOOD FUNCTIONS

We use the notations defined in Section II to define the likelihood functions associated with the various models considered. We consider a sample $y_1,...,y_T$, with $\mathrm{y_t} = \ln(S_t) - \ln(S_{t-1})$.

- GBM model: $L_{GBM} = -\frac{T}{2}\ln(2\pi) + \sum_{t-1}^{T} \ln\left[\frac{1}{\sqrt{\sigma^2}}\exp\left(\frac{-(y_t-\mu)^2}{2\sigma^2}\right)\right]$

- Jump-GBM model:

$$L_{Jump/GBM} = \sum_{t=1}^{T} \ln\left[\frac{(1-\lambda)}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(y_t-\mu)^2}{2\sigma^2}\right) + \frac{\lambda}{\sqrt{2\pi(\sigma^2+\delta^2)}}\exp\left(\frac{-(y_t-\mu-\theta)^2}{2(\sigma^2+\delta^2)}\right)\right]$$

- ARCH(1) model: $L_{ARCH} = -\frac{T}{2}\ln(2\pi) + \sum_{t-1}^{T} \ln\left[\frac{1}{\sqrt{h_t}}\exp\left(\frac{-(y_t-\mu)^2}{2h_t}\right)\right]$

- Jump-ARCH(1) model:

$$L_{Jump/ARCH} = \sum_{t=1}^{T} \ln\left[\frac{(1-\lambda)}{\sqrt{2\pi h_t}}\exp\left(\frac{-(y_t-\mu)^2}{2h_t}\right) + \frac{\lambda}{\sqrt{2\pi(h_t+\delta^2)}}\exp\left(\frac{-(y_t-\mu-\theta)^2}{2(h_t+\delta^2)}\right)\right]$$

For both L_{ARCH} and $L_{Jump/ARCH}$, $h_t = \alpha_0 + \alpha_1 (y_{t-1} - \mu)^2$.

APPENDIX B: SUMMARY STATISTICS

We consider again a sample $y_1,...,y_T$, with $\mathrm{y_t} = \ln(S_t) - \ln(S_{t-1})$. The symbol $\overset{asy}{\sim}$ means "*under the null hypothesis and relevant regularity conditions, the statistic's asymptotic distribution is*"

- Normality test statistics

Jarque-Bera $= T\left[\frac{1}{6}Skewness^2 + \frac{1}{24}(Kurtosis-3)^2\right] \overset{asy}{\sim} \chi^2(2)$

Skewness = $\frac{1}{T}\sum_{t=1}^{T}\left(\frac{y_t}{\hat{\sigma}}\right)^3$, $Kurtosis = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{y_t}{\hat{\sigma}}\right)^4$, $\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left(y_t^2\right)$

Critical points for the Skewness and Kurtosis tests are taken from D'Agostino and Stephens (1986, Table 9.3, p. 379 and Table 9.5, p. 385).

- Unit root tests

It is well know that standard unit root tests (e.g. Dickey-Fuller) are not reliable in the presence of structural breaks. We thus apply the unit root test proposed by Perron (1989, Model B) when an exogenous break point occurring at a known date T_B is present. We first de-trended each series using a preliminary regression (in log-levels) on a time trend and the structural change dummy variable $DT_t^* = t - T_B$. If x_t^* denotes the residuals from this regression, we then regress the difference $\left(x_t^* - x_{t-1}^*\right)$ on a constant, the seasonal dummies, and x_{t-1}^*. The standard t statistic associated with x_{t-1}^* yields a valid unit root test criterion, provided cut-off points form Perron (1993, Table I) are used. Perron shows that these cut-off points are typically farther in the tails than the corresponding Dickey-Fuller tests critical points.

- Variance ratio tests

We use the heteroscedasticity-robust *Z*(q)* variance ratio tests from Lo and MacKinlay (1988). The index q>1 is an integer such that the number of observations is $nq+1$. Let $s_0, ..., s_{nq}$ denote the logarithm of the observations as defined above. Consider the “variance” ratio:

$$VR(q) = \frac{\bar{\sigma}_c^2(q)}{\bar{\sigma}_a^2},$$

where $\bar{\sigma}_c^2(q)$ is q times an unbiased estimate of the variance of q-differences of the time series, and $\bar{\sigma}_\mathrm{a}^2$ is the estimated variance of the first difference of the time series, defined by:

$$\sigma_c^2(q) = \frac{1}{m}\sum_{t=q}^{nq}(s_t - s_{t\text{-}q} - q\hat{\mu})^2, \;\; \sigma_a^2 = \frac{1}{nq\text{-}1}\sum_{t=1}^{nq}(s_t - s_{t\text{-}q} - \hat{\mu})^2,$$

with $m = q(nq - q + 1)(1 - \frac{1}{n})$ and $\hat{\mu} = \frac{1}{nq}(s_{nq} - s_0)$. Now let :

$$\hat{\theta}(q) = \sum_{j=1}^{q\text{-}1}\left[\frac{2(q\text{-}j)}{q}\right]^2 \hat{\delta}(j), \;\; \hat{\delta}(j) = \frac{\sum_{k=j+1}^{nq}(s_k - s_{k\text{-}1} - \hat{\mu})^2(s_{k\text{-}j} - s_{k\text{-}j\text{-}1} - \hat{\mu})^2}{\left[\sum_{k=1}^{nq}(s_k - s_{k\text{-}1} - \hat{\mu})^2\right]^2}.$$

Under the null hypothesis of uncorrelated increments (and specific regularity conditions allowing for heteroskedasticity), the test statistic $Z^*(q) = \frac{VR(q) - 1}{\sqrt{\hat{\theta}(q)}} \overset{asy}{\sim} N(0,1)$.

- <u>Test for ARCH(1)</u>

The (Lagrange Multiplier) test statistic, which we denote $LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix}$, is TR^2 from the regression of y_t^2 on a constant and y_{t-1}^2. $LM\begin{pmatrix} H_{00} \\ H_{10} \end{pmatrix} \overset{\text{asy}}{\sim} \chi^2(1)$.

APPENDIX C: GALERKIN'S METHOD

Galerkin's method (Delves and Mohamed, 1985) allows to numerically solve for the unknown function *f* in the integro differential equation:

(C1) $$P(x)f''(x)+Q(x)f'(x)+R(x)f(x)+\int_{-1}^{1}k(x,u)f(u)du=g(x),\ \ -1\le x\le 1$$

with boundary conditions:

(C2) $$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} f(-1) \\ f(1) \end{pmatrix}+\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}\begin{pmatrix} f'(-1) \\ f'(1) \end{pmatrix}=\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

P(x), Q(x), R(x), k(x,u), and *g(x)* are "well behaved" functions.

To implement Galerkin's method, we start by replacing *P(x), Q(x), R(x), g(x), f(x), f'(x),* and *f''(x)* by their truncated Chebychev decomposition:

(C3) $$\begin{aligned}&P(x)=\sum_{0}^{N}{}' p_jT_j(x),\ Q(x)=\sum_{0}^{N}{}' q_jT_j(x),\ R(x)=\sum_{0}^{N}{}' r_jT_j(x),\ g(x)=\sum_{0}^{N}{}' g_jT_j(x)\\ &\text{and } f(x)=\sum_{0}^{N}{}' a_jT_j(x),\ f'(x)=\sum_{0}^{N}{}' a_j^{'}T_j(x),\ f''(x)=\sum_{0}^{N}{}' a_j^{''}T_j(x)\end{aligned}$$

$T_j(x)=\cos(\ j\ \arccos(x)\ )$ is the j^{th} Chebychev polynomial. A "prime" superscript following the summation sign indicates that the first term is halved. A "double prime" indicates that both the first and the last terms are halved. As an example, to find the Chebychev decomposition of *g(x)*, we calculate, for $1\le i\le N$: $g_i=\frac{2}{N}\sum_{k=0}^{N}{}'' g(\cos\frac{kp}{N})\cos\frac{ikp}{N}$. A simple derivation shows that $a_j=\frac{1}{2}(a_{j-1}^{'}-a_{j+1}^{'})$, and $a_j^{'}=\frac{1}{2}(a_{j-1}^{''}-a_{j+1}^{''}),\ j\ge 1$. In vector form:

(C4) $$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}=A\begin{pmatrix} a_0^{'} \\ a_1^{'} \\ \vdots \end{pmatrix}.$$

with $A_{i,i} = \dfrac{1}{2i}$, $A_{i,i+2} = \dfrac{-1}{2i}$ and 0 otherwise.

The boundary conditions (Equation (C2)) provide two relationships linking a_0 to the a'_j and a'_0 to the a''_j:

$$\text{(C5)}\quad \begin{cases} a_0 = \dfrac{2e_1}{c_{11}+c_{12}} - \left[\begin{pmatrix} d_{11} & d_{12} \end{pmatrix} T + \begin{pmatrix} c_{11} & c_{12} \end{pmatrix} T_1 A \right] \vec{a}\,' \\ a_0^{'} = 2(f_{21}e_1 + f_{22}e_2) + \left[\vec{h} - 2\begin{pmatrix} f_{11} & f_{12} \end{pmatrix} \left\{ (CT_2 + DT_1)A + D\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \vec{h} \right\} \right] \vec{a}\,'' \end{cases}$$

In the above, $T = \begin{pmatrix} .5T_0(\text{-}1) & T_1(\text{-}1) & T_2(\text{-}1) \cdots \\ .5T_0(1) & T_1(1) & T_2(1) \cdots \end{pmatrix}$, T_i is the matrix T without its first i columns, A_{11} is A without its first line and column, $\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} = \left\{ \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} 0.5 & -1 \\ 0.5 & 1 \end{pmatrix} + \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \right\}^{-1}$,

and $\vec{h} = (\ 0\ \ 0.25\ \ 0\ \ \text{-}0.25\ 0\ \ldots\ 0\ )$. $\vec{a}$, $\vec{a}\,'$, and $\vec{a}\,''$ are the vectors of Chebychev coefficients for f, f', and f'' respectively. We find:

$$\text{(C6)}\quad \vec{a} = \mathbf{A}'\vec{a}' + \vec{\mu},\ \ \vec{a}' = \mathbf{A}''\vec{a}'' + \vec{\eta}$$

with $\vec{\mu} = \begin{pmatrix} \dfrac{2e_1}{c_{11}+c_{12}} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $\vec{\eta} = \begin{pmatrix} 2(f_{21}e_1 + f_{22}e_2) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

The second stage of Galerkin's method consists in multiplying the resulting equation by $\dfrac{T_i(x)}{\sqrt{1-x^2}}$ and integrating between *-1* and *1*, for $0 \le i \le N$. We obtain the algebraic equation:

$$\text{(C7)}\quad P\vec{a}'' + Q\vec{a}' + (R + \lambda\ B)\vec{a} = \vec{g}$$

For $i \geq 0$, $j>0$, the coefficients of P, Q, R and B are: $P_{i0} = \frac{p_i}{2}$, $Q_{i0} = \frac{q_i}{2}$, $R_{i0} = \frac{r_i}{2}$,

$B_{i0} = \frac{\pi}{p^2} \sum_{s=1}^{p-1} \sin(\frac{s\pi}{p}) \sum_{r=0}^{p}{}'' k(\cos(\frac{r\pi}{p}), \cos(\frac{s\pi}{p})) \cos(\frac{r\, i\pi}{p})$, $P_{ij} = \frac{p_{i+j} + p_{|i-j|}}{2}$, $R_{ij} = \frac{r_{i+j} + r_{|i-j|}}{2}$,

$Q_{ij} = \frac{q_{i+j} + q_{|i-j|}}{2}$, and $B_{ij} = \frac{2\pi}{p^2} \sum_{s=1}^{p-1} \cos(\frac{sj\pi}{p}) \sin(\frac{s\pi}{p}) \sum_{r=0}^{p}{}'' k(\cos(\frac{r\pi}{p}), \cos(\frac{s\pi}{p})) \cos(\frac{r\, i\pi}{p})$.

Substituting (C6) into (C7) gives the linear system in $\vec{a}\,''$:

(C8) $[P + (Q + (R + \lambda\, B)A')A''\,]\,\vec{a}'' = \vec{g} - (Q + (R + \lambda\, B)A')\vec{\eta} - (R + \lambda\, B)\vec{\mu}$

Once $\vec{a}\,''$ is known, we calculate $\vec{a}$ from: $\vec{a} = A'(A''\vec{a}'' + \vec{\eta}) + \vec{\mu}$.

In this paper, we want to solve numerically for φ and S^* in (we omit time subscripts here):

(C9)
$$\begin{cases} r\varphi(S) = \alpha\, S\varphi'(S) + \frac{\sigma^2}{2} S^2 \varphi''(S) + \lambda\, \varepsilon_Y \{\varphi(SY) - \varphi(S)\},\ 0 \leq S \leq S^* \\ \varphi(0) = 0 \\ \varphi(S) = S - \frac{A}{r},\ S \geq S^* \\ \varphi'(S^*) = 1 \end{cases}$$

To simplify the numerical scheme, we perform two changes of variables. The first one is:

(C10) $s \equiv Ln(S)$, $\psi(Ln(S)) \equiv \varphi\ (S)$, $Ln(Y) \equiv Z$

We replace $-\infty$ by a very small value of s, denoted by $s_{\inf}$, and assume that $Z \sim N(\theta,\delta^2)$.

The second change of variables is:

(C11) $w \equiv \frac{2}{s^* - s_{\inf}} s - \frac{s^* + s_{\inf}}{s^* - s_{\inf}}$, $f(w) \equiv \psi(s)$, $g(w) \equiv G(s)$

It brings the argument of the unknown function between -1 and 1. Our problem becomes:

$$
\text{(C12)} \quad \begin{cases} P(w)f''(w) + Q(w)f'(w) + R(w)f(w) + \int_{-1}^{1} k(w,u)f(u)du = g(w), \quad -1 \le w \le 1 \\ f(-1) = 0 \\ f'(1) = \dfrac{s^* - s_{\inf}}{2} e^{s^*} \\ f(1) = e^{s^*} - \dfrac{A}{r} \end{cases}
$$

with:

$$
\text{(C13)} \quad P(w) = \frac{\sigma^2}{2}\left(\frac{2}{s^* - s_{\inf}}\right)^2, \; Q(w) = \left(\alpha - \frac{\sigma^2}{2}\right)\frac{2}{s^* - s_{\inf}}, \; R(w) = -(\lambda + r)
$$

$$
\text{(C14)} \quad k(w,u) = \frac{s^* - s_{\inf}}{2\delta}\phi\left(\frac{s^* - s_{\inf}}{2\delta}(u - w) - \frac{\theta}{\delta}\right)
$$

$$
\text{(C15)} \quad \begin{aligned} g(w) = {} & \lambda\, e^{0.5\{(s^* - s_{\inf})w + s^* + s_{\inf} + 2\theta + \delta^2\}}\left[\Phi\left(\frac{s^* - s_{\inf}}{2\delta}(1 - w) - \frac{\theta}{\delta} - \delta\right) - 1\right] + \\ & \lambda\frac{A}{r}\left[1 - \Phi\left(\frac{s^* - s_{\inf}}{2\delta}(1 - w) - \frac{\theta}{\delta}\right)\right] \end{aligned}
$$

$\phi(z)$ and $\Phi(z)$ are respectively the density and the cumulative distribution of the standard normal.

Compared to (C1), we have one extra boundary conditions; it is needed here because s^* is unknown. To solve, we pick a value of s^*, find f from the first three equations of (C12), and check if the fourth is satisfied. If it is, we are done; otherwise, we repeat with a different s^*.

To implement this procedure, we wrote a program in GAUSS and checked it against the examples in Babolian and Delves (1981). With s_{inf} = -4.0, we found that 60 to 80 terms in the Chebychev expansions of *g(w)* and *k(w,u)* give satisfactory numerical convergence.

[1] This issue was first recognized in instrumental regressions when the LAU region implies poor instruments; see for example Staiger and Stock (1997), Dufour (1997), and the references cited there.

[2] N is explicitly taken into consideration in MC tests so that no asymptotic argument on the number of MC replications is needed to establish the validity of the method. This is the principal difference between MC tests and the (highly related) parametric bootstrap, which is formally valid for $N \to \infty$.

[3] To see this, observe that to obtain a MC exact test in the sense of level control, Dufour (1995) shows that that MC p-value must be maximized with respect to the relevant nuisance parameters. This yields the decision rule: reject the null (at level α) if the largest MC p-value does not exceed α. It is clear that, if the MC p-value exceeds α for a specific nuisance parameter value, then the maximal MC p-value most certainly exceeds α. Hence, if the LMC is not significant, then the exact test based on the largest p-value is most certainly not significant.

[4] The "boundedly pivotal" test property is formally defined in Dufour (1997) for general test procedures.

[5] This is the basic reasoning behind the Durbin-Watson auto-correlation bounds test. In connection, note that one solution that has received renewed attention in instrumental regressions (where LAU parameters raise problems related to the ones we are dealing with here) is also bounds-based (e.g., see Wand and Zivot (1998).

[6] Note, however, that in this case, Perron's test is not significant at 2.5%. This result must be qualified given that the performance of unit root tests in the presence of jumps has not been formally assessed.

[7] The Ljung-Box auto-correlation test is also significant for these two series. This effect however may be due to the presence of heteroscedasticity that distorts the test's size in smaller samples (see Jorion, 1988, page 432).

[8] The bootstrap p-values for $LR\begin{pmatrix} H_{10} \\ H_{11} \end{pmatrix}$ are respectively .08 and .06 for True Firs and Ponderosa and Jeffrey Pine, whereas the bounds p-values are .14 and .08. Again, results of the bootstrap test should be interpreted with caution here, since the same theoretical problems that cause the failure of standard asymptotic may cause to reject spuriously. To avoid this problem, we interpret the bootstrap in conjunction with an exact bounds procedure.